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UNIT 07

Kinematics

INTRODUCTION

- **Kinematics** is the branch of mechanics that describes the motion of objects without considering the causes of motion.
- In this unit, we will explore how objects move along a straight line by studying key physical quantities such as displacement, velocity, and acceleration. Through sketching and interpreting displacement-time and velocity-time graphs, we will develop a visual understanding of motion.
- The unit also introduces the use of rates of change to analyze how displacement, velocity, and acceleration vary over time. Using differentiation and integration with respect to time, we will solve simple problems related to motion. For situations involving constant acceleration, we will apply appropriate kinematic formulae to determine unknown quantities.
- Finally, the concepts learned will be applied to real-life situations, such as the motion of vehicles, free fall, vertical projectile motion, relative motion, and the creation of realistic movement in animation.

Content Outline

- Rectilinear Motion
  - Displacement, Average Velocity and Acceleration
  - Displacement-Time Graph
  - Velocity-Time Graph
  - Motion with Constant Acceleration
  - Motion of a Free Particle Moving Along a Vertical Line

RECTILINEAR MOTION

The motion of a particle along a straight line is called rectilinear motion.

Scalars:

Scalar quantities are those that are completely described by magnitude only (size or numerical value). Scalars do not have direction.

Examples:

- **Distance:** Total length of the path travelled, regardless of direction.
- **Speed:** Rate at which distance is covered per unit time, irrespective of direction.

Vectors:

Vector quantities are described by both magnitude and direction.

Examples:

- **Displacement:** Displacement is change in position of a particle represented by a vector from initial to final position.
- **Velocity:** Rate of change of displacement with respect to time in a specified direction (i.e., speed in a given direction). It is vector quantity and its magnitude gives speed.
- **Acceleration:** Rate of change of velocity with respect to time, including direction.

Displacement, Average Velocity and Acceleration

When a particle undergoes rectilinear motion, its displacement, velocity, and acceleration can have only one of two possible directions, along the line of motion.

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In such cases, we do not use arrows or other vector symbols to represent direction; instead, we use positive and negative signs to indicate it:

- One direction along the line is taken as positive.
- The opposite direction is taken as negative.

This simplifies the representation of displacement, velocity, and acceleration in rectilinear motion.

Average Velocity: The average velocity (v) of an object is defined as the total displacement divided by the total time taken:

$$v_{avg} = \text{Average velocity} = \frac{\text{Total displacement}}{\text{Total Time}}$$

Average Speed: The average speed of an object is defined as the total distance travelled divided by the total time taken:

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total Time}}$$

Recall!

A body goes and come back starting position,

- Displacement = 0
- Distance ≠ 0

Example 1: A man walks 100 meters east in 2 minutes, then turns around and walks 60 meters west in 1 minute. Find his speed and velocity.

Solution:

Total distance = 100 + 60 = 160 meters

Total time = 2 + 1 = 3 minutes

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{160}{3} = 53.3 \text{ m/min}$$

Taking east as positive and west as negative:

Total displacement = 100m (East) − 60m (West)

= 40 (East)

$$v_{avg} = \frac{\text{Total Displacement}}{\text{Total Time}}$$

$$= \frac{40}{3} = 13.33 \text{ m/min (eastward)}$$

Instantaneous Velocity:

The instantaneous velocity is the rate of change of displacement with respect to time at a given instant. If $x(t)$ is the displacement of a particle at any time t, then instantaneous velocity $v(t)$ at any time t is given by:

$$v(t) = \frac{dx}{dt}$$

Instantaneous Speed:

Instantaneous speed is the magnitude of instantaneous velocity:

$$\text{Instantaneous speed} = |v(t)| = \left|\frac{dx}{dt}\right|$$

Example 2: Let $x(t) = t^3 - 6t^2$ be the position of a particle moving along a straight line at any time t, where x is in meters and t is in seconds. Find the velocity and the speed of the particle at any time t and at $t = 1$ second.

Solution:

Given: $x(t) = t^3 - 6t^2$

Differentiate w.r.t. 't'

$$\frac{d}{dt}(x) = \frac{d}{dt}(t^3 - 6t^2)$$

$$\frac{dx}{dt} = 3t^2 - 6(2t)$$

Velocity at any time t is:

$$v(t) = 3t^2 - 12t$$

Speed at any time t is:

$$\text{Speed} = |v(t)| = |3t^2 - 12t|$$

At $t = 1$:

$$v(1) = 3(1)^2 - 12(1) = 3 - 12 = -9 \text{ m/s}$$

$$\text{Speed} = |v(1)| = |-9| = 9 \text{ m/s}$$

Acceleration:

Rate of change of velocity with respect to time is called acceleration. If $v(t)$ is velocity of a particle at any time t, then its acceleration $a(t)$ is given by:

$$a(t) = \frac{dv}{dt}$$

Since $v = \frac{dx}{dt}$, the above expression can also be written as:

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

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If velocity v of particle is given as function of displacement x:
Using the Chain Rule:

$$a = \frac{d}{dt}(v(x)) = \frac{dv}{dx}\frac{dx}{dt}$$

But since $v = \frac{dx}{dt}$, we get:

$$a = v\frac{dv}{dx}$$

Units of Acceleration:

If displacement is measured in meters and time is in seconds, then unit of acceleration is meters per second square i.e. m/s^2.

Displacement-Time Graph

Displacement-time graphs are important tools in kinematics, because they provide a visual representation of an object's motion. Here's why we discuss them:

- They give a clear picture of how an object's position changes over time.
- They help us easily determine whether an object is moving, stationary or accelerating.

In a displacement-time graph, the displacement (x) is plotted on the y-axis, and time (t) is plotted on the x-axis.

Key Features of a Displacement-Time Graph

Slope of the graph $= \left(\frac{dx}{dt}\right)$, represents velocity of the particle.

- If velocity is uniform i.e.,

$$v = \frac{dx}{dt} = \text{Slope of the graph} = \text{constant},$$

the displacement-time graph is a straight line.

- If $v = \frac{dx}{dt} = 0$, the particle is at rest and the displacement-time graph is a horizontal line, which reflects that displacement remains constant with time. In Figure 7.1, the particle is at rest from point B to point C.
- If $v = \frac{dx}{dt} = \text{constant} > 0$, the particle moves forward, and the displacement-time graph is a rising straight line; that is, displacement increases as time increases. In Figure 7.1, the particle is moving with positive uniform velocity from point A to point B.

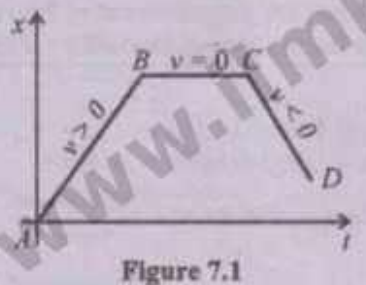

Figure 7.1

Recall!
- Positive slope ⟶ forward motion.
- Negative slope ⟶ backward motion.

- If $v = \frac{dx}{dt} = \text{constant} < 0$, the particle moves in the opposite direction, and the displacement time graph is a falling straight line; that is, displacement decrease as time increases. In Figure 7.1, the particle is moving with uniform negative velocity from point C to point D.

Example 3: A cyclist rides in a straight line for 20 minutes. He then stops and waits for 30 minutes, after which he returns in a straight line to the starting point in 10 minutes. Figure 7.2 shows the displacement-time graph for his journey.

(a) Calculate the average velocity for each stage of the journey in km/h.

(b) Calculate the average velocity for the whole journey.

(c) Find the average speed for the whole journey.

Solution:

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(a) Journey from O to A:

Time = 20 mins, displacement = 4 km

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$= \frac{4}{20} = \frac{1}{5}\text{ km/min} = 12\text{ km/h}$$

Journey from A to B:

Time = 30 mins, displacement = 0 km

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$= \frac{0}{30} = 0\text{ km/min}$$

$$= 0 \times 60 = 0\text{ km/}h \Rightarrow \text{object is stationary.}$$

Journey from B to C:

Time = 10 mins, displacement = –4 km

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$= \frac{-4}{10} = \frac{-2}{5}\text{ km/min}$$

$$= -24\text{ km/h}$$

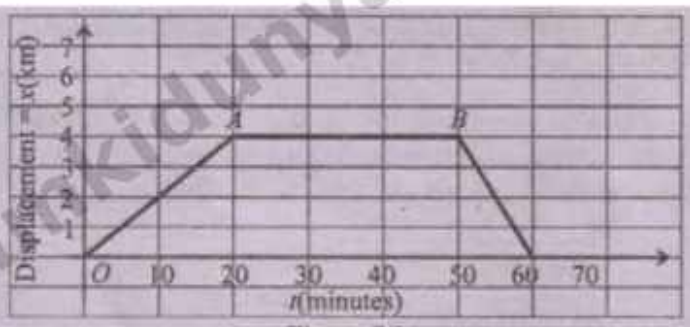

Figure 7.2

(b) Displacement for whole journey = 4 + 0 + (–4) = 0 km

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$= \frac{0}{10} = 0\text{ km/h}$$

(c) Total time = 60 mins = 1 h ; Total distance = 4 + 4 = 8 km

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{8}{1}\text{ km/h} = 8\text{ km/h}$$

Example 4: Javed drives from his home to Murree. He drives for 2 hours at an average velocity of km/h. He then stops for lunch before continuing to Murree. Figure 7.3 shows a displacement-time graph for his journey.

(a) Work out the displacement of Murree from Javed's home.

(b) Work out his average velocity for his whole journey.

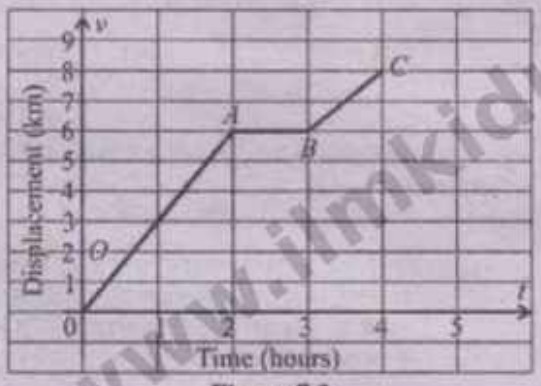

Figure 7.3

Solution:

(a) Journey from O to A:

Time = 2 h: average velocity = 50 km/h

$$\text{Since, average velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$50 = \frac{\text{Displacement}}{2}$$

$$\text{Displacement} = 50 \times 2 = 100\text{ km}$$

Scale on vertical axis from graph:

6 units = 100 km

$$\text{So, 1 units} = \frac{100}{6} = \frac{50}{3}\text{ km}$$

Journey from B to C:

Vertical rise = 2 units

$$\text{Displacement} = 2 \times \frac{50}{3} = \frac{100}{3}\text{ km}$$

Total displacement of Murree from Javed's home

$$= 100 + \frac{100}{3} = \frac{400}{3} \approx 133.3\text{ km}$$

(b) Average velocity:

Time = 2 h (first drive) + 1 h (lunch stop) + 1 h (second drive) = 4 h

Total displacement = 133.3 km

$$\text{Average velocity} = \frac{133.3}{4}$$

$$\approx 33.33\text{ km/h}$$

Velocity-Time Graph

Velocity-time-graph helps us visually understand how the velocity of an object changes with time. It is used to:

- Analyze uniform or non-uniform motion
- Determine acceleration
- Calculate displacement
- Study motion in different time intervals clearly and effectively

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In a velocity-time graph, the velocity (v) is plotted on the y-axis, and time (t) is plotted on the x-axis.

Key Features of Velocity-Time Graph

1. Slope of the graph $\left(\frac{dv}{dt}\right)$, represents acceleration of the particle.

- If a particle is moving with constant acceleration, its velocity-time graph is a straight line.
  - If $a = \frac{dv}{dt} = 0$, the velocity-time graph is a horizontal straight line. In Figure 7.4, the particle is moving with uniform velocity from point B to point C.
  - If $a = \frac{dv}{dt}$ = constant > 0, the velocity of the particle is increasing with increase in time, and the velocity time graph is a rising straight line. In Figure 7.4, the particle is moving with positive uniform acceleration from point A to point B

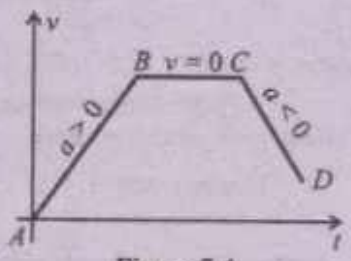

Figure 7.4

Recall!
- Displacement = Area under velocity-time graph
  Acceleration = Slope of velocity time graph

  - If $a = \frac{dv}{dt}$ = constant < 0, the velocity of the particle decreases with increase in time, and the velocity-time graph is a falling straight line. In Figure 7.4, the particle is moving with uniform negative acceleration (retardation) from point C to point D.

- If a particle is moving with variable acceleration, then its velocity-time graph is not a straight line. (See Figure 7.5)

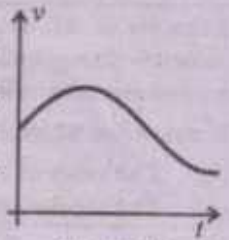

Figure 7.5

2. If A is the area under the velocity-time graph and above the time-axis from $t = t_1$ to $t = t_2$, then from integral calculus:

$$A = \int_{t_1}^{t_2} v(t)dt$$

Recall!
If $y = f(x) \geq 0$ for $a \leq x \leq b$, then the area under the curve $y = f(x)$, above the x-axis, and between the vertical lines $x = a, x = b$ is equal to $\int_a^b y dx$.

Since $v = \frac{dx}{dt}$,

We have:

$$A = \int_{t_1}^{t_2} \frac{dx}{dt} dt = \int_{t_1}^{t_2} dx = x(t_2) - x(t_1)$$

Thus, A = displacement of the particle during the time interval from t_1 to t_2.

Note

If velocity-time graph lies both above and below the time axis, and A_1 is area above time axis and graph, while A_2 is the area below time axis and graph, then:

Displacement $= A_1 - A_2$

Distance $= A_1 + A_2$

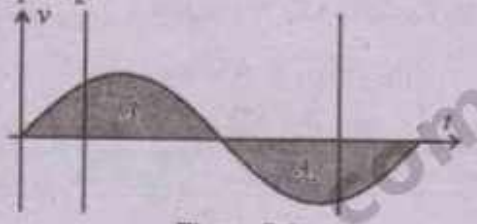

Figure 7.6

Example 5: Figure 7.7 is the velocity-time graph for a cyclist traveling in a straight line. (The time is in minutes and velocity is in meters per minute).

(a) Calculate the total displacement of the cyclist from his starting position at the end of the 12 minutes.

(b) Determine the uniform deceleration during the final 4 minutes of motion.

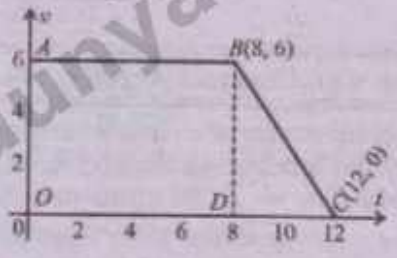

Figure 7.7

Solution:

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(a) Displacement = Area of trapezium $OABC$

$$= \frac{1}{2}(a+b)h \qquad \ldots(1)$$

where, $a = |\overline{OC}| = 12, b = |\overline{AB}| = 8$

h = Perpendicular distance between $\overline{OC}$ and $\overline{AB} = 6$

Putting values in (1), we have

$$\text{Displacement} = \frac{1}{2}(12+8)\times 6$$

$$= 20 \times 3 = 60 \text{ meters}$$

(b) Let

a = Acceleration during the final 4 minutes of motion

= Slope of $\overline{BC}$

$$= \frac{0-6}{12-8} = \frac{-6}{4} = \frac{-3}{2}$$

$$a = -1.5 \text{ m/min}^2$$

$$\text{Deceleration} = -(-1.5)$$

$$= 1.5 \text{ m/min}^2$$

Area of Trapezium

A trapezium (trapezoid) is a four-sided plane figure that has exactly one pair of parallel sides. If a and b are the lengths of parallel sides and h is the perpendicular distance between them, then the area is given by:

$$\text{Area} = \frac{1}{2}(a+b)\times h$$

Example 6: Ali drives a car 400 km in a straight line. Figure 7.8 is a sketch of a velocity- time graph representing his motion. Ali starts with a velocity of u km/h and accelerates to a velocity of 70 km/h in 3 hours. He then maintains his velocity for 3 hours, after which he slows down uniformly to rest over the next $\frac{1}{2}$ hour. Find:

(a) The value of u

(b) The acceleration of Ali during the first 3 hours of his drive.

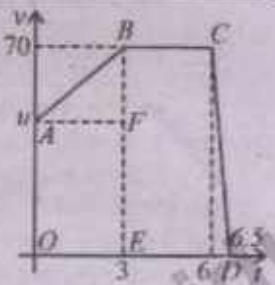

Figure 7.8

Solution:

(a) Given: Displacement = 400 km

$\Rightarrow$ Area of trapezium $OABE$ + Area of trapezium $BCDE = 400$

$$\frac{1}{2}(a+b)h_1 + \frac{1}{2}(c+d)h_2 = 400 \qquad \ldots(1)$$

where, $a = |\overline{OA}| = u, b = |\overline{BE}| = 70, h_1 = |\overline{OE}| = 3$

and

$c = |\overline{BC}| = 6-3 = 3, d = |\overline{DE}| = 6.5-3 = 3.5, h_2 = |\overline{BE}| = 70$

Putting values in eq. (1), we get

$$\frac{1}{2}(u+70)(3)+\frac{1}{2}(3+3.5)(70) = 400$$

$$\frac{1}{2}(3u+210+455) = 400$$

$$3u + 665 = 400 \times 2$$

$$3u = 800 - 665$$

$$u = \frac{135}{3}$$

$$\boxed{u = 45 \text{ km/h}}$$

(b) Let a be the acceleration during the first 3 hours of his drive.

a = Slope of $\overline{AB}$

$$a = \frac{\text{Rise}}{\text{Run}} = \frac{|\overline{BF}|}{|\overline{AF}|}$$

$$a = \frac{70-u}{|\overline{OE}|} \qquad \because |\overline{AF}| = |\overline{OE}|$$

$$a = \frac{70-45}{3}$$

$$a = \frac{25}{3}$$

$$\boxed{a = 8.33 \text{ km/h}^2}$$

Motion with Constant Acceleration

To study the motion of a particle moving with constant acceleration, we consider four quantities: displacement, time, velocity, and acceleration. Usually, some of these quantities are known and the remaining are to be determined. To relate these quantities, we use the equations of motion.

Suppose a particle moves along a straight line with constant acceleration a. At time $t = 0$, the particle is at point O (i.e, $x = 0$) and has initial velocity u.

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Since acceleration is the instantaneous rate of change in velocity with respect to time, we have:

$$\frac{dv}{dt} = a \quad \Rightarrow \quad dv = a\,dt$$

23 Integrating both sides:

$$\int dv = \int a\,dt$$

$v = at + c$ (where c is the constant of integration) ...(1)

Using the initial condition $v = u$ at $t = 0$, substitute into equation (1):

$$u = 0 + c \Rightarrow c = u$$

Substitute back into equation (1):

$$\boxed{v = u + at}$$ (1st equation of motion) ...(2)

Since velocity is the instantaneous rate of change of displacement with respect to time:

$$v = \frac{dx}{dt} \quad \Rightarrow \quad \frac{dx}{dt} = u + at \quad \Rightarrow \quad dx = (u + at)dt$$

Integrating both sides:

$$\int dx = \int (u + at)dt$$

$$x = ut + a\frac{t^2}{2} + c_1 \qquad ...(3)$$

Using the initial condition $x = 0$ at $t = 0$, substitute into equation (3):

$$0 = 0 + 0 + c_1 \Rightarrow c_1 = 0$$

Substitute back into equation (3):

$$\boxed{x = ut + \frac{1}{2}at^2}$$ (2nd equation of motion) ...(4)

Now from equation (2):

$$t = \frac{v - u}{a}$$

Substitute this into equation (4) to eliminate t:

$$x = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

On simplification, we get:

$$\boxed{2ax = v^2 - u^2}$$ (3rd equation of motion)

The three equations of motion for a particle moving with constant acceleration are:

- $v = u + at$
- $x = ut + \frac{1}{2}at^2$
- $2ax = v^2 - u^2$

These equations are frequently used to describe and analyze the motion of a particle under constant acceleration.

Example 7: A car starts from rest and accelerates uniformly at m/s². Find the velocity of the car after 5 seconds.

Solution:

Given that:

$u = 0$ ∵ Car starts from rest.

$a = 3$ m/s², $t = 5$

Using the 1st equation of motion:

$v = u + at$

$= 0 + 3(5)$

$\boxed{v = 15 \text{ m/s}}$

Example 8: A car accelerates from 10 m/s to 30 m/s in 4 seconds. Find the distance covered during this time.

Solution:

Given that:

$u = 10$ m/s, $v = 30$ m/s, $t = 4$s

Using the 1st equation of motion:

$v = u + at$

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$$30 = 10 + a(4)$$

$$30 - 10 = 4a \Rightarrow 4a = 20 \Rightarrow a = \frac{20}{4}$$

$$\boxed{a = 5\ m/s^2}$$

Using the 3rd equation of motion:

$$2ax = v^2 - u^2$$

$$2(5)x = 30^2 - 10^2$$

$$10x = 900 - 100$$

$$x = \frac{800}{10}$$

$$\boxed{x = 8\ \text{meters}}$$ (Distance covered)

Example 9: Find the position and the velocity of a particle moving along a straight line at any time t seconds, given that it starts from rest at $t = 0$ and is subject to an acceleration $a(t) = t^2 + \cos t$.

Solution:

Given that: $a(t) = t^2 + \cos t$

$$\frac{dv}{dt} = t^2 + \cos t \qquad \because a(t) = \frac{dv}{dt}$$

$$dv = (t^2 + \cos t)dt$$

Integrating both sides:

$$\int 1dv = \int (t^2 + \cos t)dt$$

$$v(t) = \frac{t^3}{3} + \sin t + c \qquad \ldots(1)$$

Using the initial condition: $v = 0$ at $t = 0$

$$0 = \frac{0^3}{3} + \sin(0) + c \Rightarrow c = 0$$

Put $c = 0$ in eq. (1), we get

$$v(t) = \frac{1}{3}t^3 + \sin t + 0$$

$$\boxed{v(t) = \frac{1}{3}t^3 + \sin t}$$

$$\frac{dx}{dt} = \frac{1}{3}t^3 + \sin t \qquad \because v(t) = \frac{dx}{dt}$$

$$dx = \left(\frac{1}{3}t^3 + \sin t\right)dt$$

Integrating both sides

$$\int 1dx = \int \left(\frac{1}{3}t^3 + \sin t\right)dt$$

$$x(t) = \frac{1}{3}\cdot\frac{t^4}{4} - \cos t + c_1$$

$$x(t) = \frac{1}{12}t^4 - \cos t + c_1 \qquad \ldots(2)$$

Assuming $x = 0$, $t = 0$

$$0 = \frac{1}{12}(0)^4 - \cos 0 + c_1 \Rightarrow 0 - 1 + c_1 = 0 \Rightarrow c_1 = 1$$

Put $c_1 = 1$ in eq. (2), we get

$$\boxed{x(t) = \frac{1}{12}t^4 - \cos t + 1}$$

Example 10: The acceleration (in m/s^2) of a particle is given by $a(t) = \cos t + \sin t$. Given that the velocity is 2 m/s at $t = 0$, find the expression for velocity $v(t)$.

Solution:

Given that: $a(t) = \cos t + \sin t$

$$\frac{dv}{dt} = \cos t + \sin t \qquad \because a(t) = \frac{dv}{dt}$$

$$dv = (\cos t + \sin t)dt$$

Integrating both sides:

$$\int 1dv = \int (\cos t + \sin t)dt$$

$$v(t) = \sin t - \cos t + c \qquad \ldots(1)$$

Using initial condition: $v = 2$, at $t = 0$

$$2 = \sin 0 - \cos 0 + c$$

$$2 = 0 - 1 + c \Rightarrow c = 3$$

Put $c = 3$ in eq. (1), we get

$$\boxed{v(t) = \sin t - \cos t + 3}$$

Motion of a Free Particle Moving Along a Vertical Line

When a particle moves under the influence of gravity alone and air resistance is neglected, the motion is called free fall or free vertical motion. This motion takes place along a vertical straight line, either upward or downward. The acceleration due to gravity is denoted by g and is approximately:

$$g \approx 9.8\ m/s^2$$

If a particle is projected upward, its velocity decreases uniformly due to gravity and becomes zero at the highest point. If the particle moves downward, it accelerates toward the ground at the rate g. The equations of motion for constant acceleration can also be used in vertical motion.

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Example 11: (Free Fall from Rest)

A stone is dropped from the top of a tower 45 m high. Find the time taken to reach the ground and the final velocity on impact.

Solution:

Given that: $u = 0, x = 45$ m, $a = g = 9.8$ m/s^2

Using the 2nd equation of motion:

$$x = ut + \frac{1}{2}gt^2$$

$$45 = (0)t + \frac{1}{2}(9.8)t^2$$

$$45 = 4.9t^2 \Rightarrow t^2 = \frac{45}{4.9}$$

$$\sqrt{t^2} = \sqrt{9.1837}$$

$$t = 3.03\ s$$

Using the 1st equation of motion:

$$v = u + gt$$

$$v = 0 + (9.8)(3.03)$$

$$\boxed{v = 29.7 \text{ m/s}}$$

Example 12 (Upward Throw):

A ball is thrown vertically upward with a speed of m/s. Find, the maximum height attained and the time to reach the maximum height.

Solution:

Given that: $u = 20$ m/s ; $v = 0$ m/s (at max height)

$a = g = -9.8$ m/s^2

Using the 1st equation of motion:

$$v = u + gt$$

$$0 = 20 + (-9.8)t \Rightarrow 9.8t = 20 \Rightarrow t = \frac{20}{9.8}$$

$$t \approx 2.04s$$

Using the 3rd equation of motion:

$$2ax = v^2 - u^2$$

$$2(-9.8)x = 0^2 - (20)^2$$

$$x = \frac{-400}{-19.6}$$

$$\boxed{x \approx 20.41 \text{ m}}$$

EXERCISE 7

Q1. Kamran covers 100 km in the first 2 hours, stops for 1 hour, and then drives 60 km in the next 2 hours. The displacement-time graph is shown in Figure 7.9.

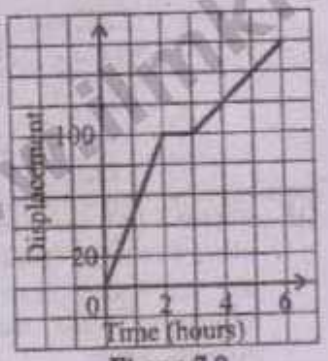

Figure 7.9

(a) What is Junaid's average velocity?

Solution:

Total time = 2 h + 1 h + 3 h + 6 h

Total displacement = 100 km + 0 km + 60 km = 160 km

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$= \frac{160}{6} = 26.67 \text{ km/h}$$

(b) What is his velocity during the last phase of the journey?

Solution:

Let v be the velocity during the last phase of journey

Displacement = 60 km

Time = 3 h

$$v = \frac{\text{Displacement}}{\text{Time}}$$

$$= \frac{60}{3}$$

$$\boxed{v = 20 \text{ km/h}}$$

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Q2. The displacement-time graph of a moving car is shown in Fig. 7.10. The car changes its direction during the journey, and its motion occurs in three distinct phases.

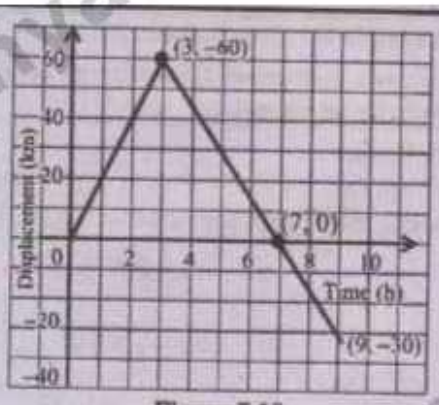

Figure 7.10

(a) What is the car's velocity in each phase?

Solution:

Phase-1 (0 to 3 hours)

Let $O(0, 0)$, $A(3, 60)$

$$\text{Velocity} = \text{Slope of } \overline{OA}$$

$$= \frac{60-0}{3-0}$$

$$= \frac{60}{3}$$

$$= 20 \text{ km/h}$$

Phase-2 (3 to 7 hours)

Let $A(3, 60)$, $B(7, 0)$

$$\text{Velocity} = \text{Slope of } \overline{AB}$$

$$= \frac{0-60}{7-3} = -\frac{60}{4} = -15 \text{ km/h}$$

Phase-3 (7 to 9 hours)

Let $B(7, 0)$, $C(9, -30)$

$$\text{Velocity} = \text{Slope of } \overline{BC}$$

$$= \frac{-30-0}{9-7}$$

$$= \frac{30}{2}$$

$$= -15 \text{ km/h}$$

(b) When did the car pass its starting point again?

Solution:

From the graph, it is clear that the car passes its starting point after 7 hours.

(c) What is the total distance travelled?

Solution:

Total distance = 60 km + 60 km + 30 km = 150 km

(d) What is the average velocity?

Solution:

Total displacement = 60 km – 90 km = –30 km

Total time = 9 hours

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$= \frac{-30}{9} = -3.33 \text{ km/h}$$

Q3. Figure 7.11 is the velocity-time graph of Aslam's motion as he runs along a straight track.

For the first 5 seconds, he accelerates uniformly from rest to a velocity of 8 m/s. He then maintains this velocity for a further 8 seconds. Find:

(a) The rate at which Aslam accelerates.

(b) The displacement from the starting point of Aslam after 13 seconds.

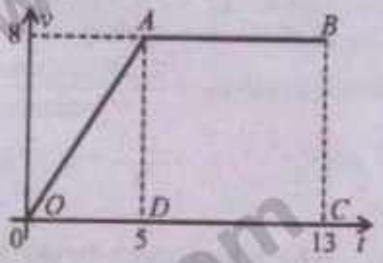

Figure 7.11

Solution:

(a) Let a be the rate of acceleration

$$a = \text{Slope of } \overline{OA}$$

$$= \frac{\text{Rise}}{\text{Run}} = \frac{8}{5}$$

$$\boxed{a = 1.6 \text{ m/s}^2}$$

(b) Displacement = Area of trapezium $OABC$

$$= \frac{1}{2}(a+b)h \quad \ldots(1)$$

where, $a = |\overline{AB}| = 13-5 = 8$, $b = |\overline{OC}| = 13$, $h = |\overline{AD}| = 8$

Putting values in eq. (1), we have

$$\text{Displacement} = \frac{1}{2}(8+13)\times 8$$

$$= 21 \times 4$$

$$= 84 \text{ meters}$$

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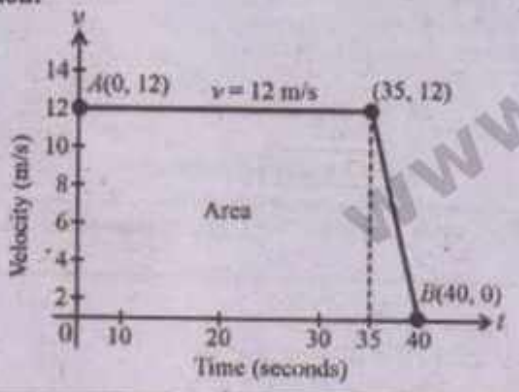
Q4. A car is travelling along a straight road. At $t = 0$ seconds, the car passes a point A with a velocity of 12 m/s, and this velocity is maintained up to $t = 35$ seconds. The driver then applies the brakes, and the car decelerates uniformly, coming to rest at the point B at $t = 40$ seconds.

(a) Sketch a velocity-time graph to illustrate the motion of the car.

Solution:



(b) Find the displacement from A to B.

Solution:

Displacement = Area of trapezium $OAPB$

$$= \frac{1}{2}(a+b)h \qquad \ldots(1)$$

where, $a = |\overline{AP}| = 35, b = |\overline{OB}| = 40, h = |\overline{PQ}| = 12$

Putting values in eq. (1), we have

$$\text{Displacement} = \frac{1}{2}(35+40)\times 12$$

$$= 75 \times 6$$

$$= 450 \text{ meters}$$

Q5. Figure 7.12 shows the velocity-time graph of an object moving in a straight line over a 12 -second time interval.

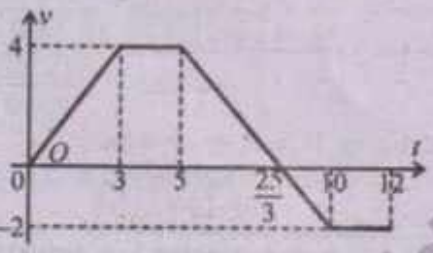


Figure 7.12

(a) At what time did the object change its direction of motion?

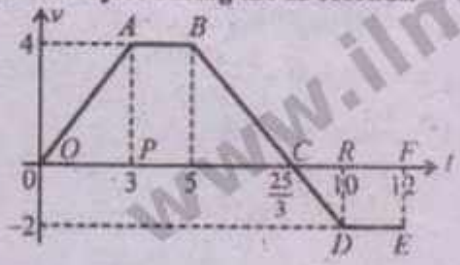
Solution:

The object changes its direction when its velocity changes sign i.e., when the graph crosses the time-axis.

$$\Rightarrow \quad t = \frac{25}{3}s \approx 8.33s$$

(b) Calculate the total distance travelled and the displacement of the object during the 12 seconds.

Solution:



A_1 = Area above time-axis

= Area of trapezium $OABC$

$$= \frac{1}{2}(a+b)h_1$$

where, $a = |\overline{AB}| = 5-3 = 2, b = |\overline{OC}| = \frac{25}{3}, h_1 = |\overline{AP}| = 4$

$$\Rightarrow \quad A_1 = \frac{1}{2}\left(2+\frac{25}{3}\right)\times 4 = \frac{31}{3}\times 2 = \frac{62}{3}\text{m}$$

A_2 = Area below time-axis

A_2 = Area of trapezium $CDEF$

$$A_2 = \frac{1}{2}(c+d)h_2$$

where, $c = |\overline{DE}| = 12-10 = 2, d = |\overline{CF}| = 12-\frac{25}{3} = \frac{11}{3}$,

$h_2 = |\overline{DR}| = 2$

$$\Rightarrow \quad A_2 = \frac{1}{2}\left(2+\frac{11}{3}\right)\times 2 = 2+\frac{11}{3} = \frac{17}{3}m$$

Displacement = $A_1 - A_2$

$$= \frac{62}{3} - \frac{17}{3}$$

$$= \frac{62-17}{3} = \frac{45}{3}$$

$$= 15\text{m}$$

Distance = $A_1 + A_2$

$$= \frac{62}{3} + \frac{17}{3}$$

$$= \frac{62+17}{3} = \frac{79}{3}$$

$$= 26.33\text{m}$$

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Q6. A vehicle moving with a speed of 36 km/h is stopped by applying brakes and comes to rest in 10 seconds. Find the retardation and distance covered during this time.

Solution:

Given that: $u = 36\ \text{km/h} = \frac{36\times1000}{3600}\ \text{m/s} = 10\ \text{m/s}$

$v = 0\ \text{m/s}, t = 10s$

Using the 1st equation of motion:

$v = u + at$

$0 = 10 + a(10) \Rightarrow 10a = -10$

$a = -1\ \text{m/s}^2$

Retardation $= -a = -(-1\ \text{m/s}^2)$

$\boxed{\text{Retardation} = 1\ \text{m/s}^2}$

Using the 3rd equation of motion:

$2ax = v^2 - u^2$

$2(-1)x = 0^2 - 10^2$

$-2x = -100 \Rightarrow x = \frac{-100}{-2}$

$\boxed{x = 50\ \text{meters}}$ (Distance)

Q7. A train moving at 54 km/h is accelerated uniformly to 90 km/h in 20 s . Find the acceleration and the distance covered in this time.

Solution:

Given that: $t = 20s$

$u = 54\ \text{km/h} = \frac{54\times1000}{3600}\ \text{m/s} = 15\ \text{m/s}$

$v = 90\ \text{km/h} = \frac{90\times1000}{3600}\ \text{m/s} = 25\ \text{m/s}$

Using the 1st equation of motion:

$v = u + at$

$25 = 15 + a(20)$

$25 - 15 = 20a \Rightarrow a = \frac{10}{20}$

$\boxed{a = 0.5\ \text{m/s}^2}$

Using the 3rd equation of motion:

$2ax = v^2 - u^2$

$2(0.5)x = 25^2 - 15^2$

$x = 625 - 225$

$\boxed{x = 400\ \text{meters}}$ (Distance)

Q8. A particle moving in a straight line starts from rest and is accelerated uniformly to attain a velocity of 30 km/h in 5 seconds. Find the distance travelled by the particle in the last three seconds.

Solution:

Given that: $u = 0$ (Starts from rest), $t = 5s$

$v = 30\text{km/h} = \frac{30\times1000}{3600}\ m/s = \frac{25}{3}\ m/s$

Using the 1st equation of motion:

$v = u + at$

$\frac{25}{3} = 0 + a(5)$

$a = \frac{25}{3\times5}$

$\boxed{a = \frac{5}{3}\ m/s^2}$

Using 2nd equation of motion:

Total distance covered $= x_5$

$x_5 = ut + \frac{1}{2}at^2$

$x_5 = (0)(5) + \frac{1}{2}\left(\frac{5}{3}\right)(5)^2$

$x_5 = \frac{125}{6} = 20.83\text{m}$

Distance in first 2 seconds $= x_2$

$x_2 = ut + \frac{1}{2}at^2$

$x_2 = (0)(2) + \frac{1}{2}\left(\frac{5}{3}\right)(2)^2$

$x_2 = \frac{20}{6} = 3.33\text{m}$

Distance in last 3 seconds $= x_5 - x_2$

$x = 20.83 - 3.33$

$\boxed{x = 17.5\ \text{meters}}$

Q9. A particle is projected vertically upwards with a velocity 100 m/s, and another is let fall from a height of 200 m at the same time. Find the height of the point where they meet each other.

Solution:

For Particle 1 (Projected upward)

Let height at any time $t = h_1$

Initial velocity: $u_1 = 100\ \text{m/s}$

Acceleration: $a_1 = -g$

$h_1 = u_1t + \frac{1}{2}a_1t^2$

$h_1 = 100t - \frac{1}{2}gt^2$ ...(1)

For Particle 2 (Dropped from a height)

Let height at any time $t = h_2$

Initial velocity: $u_2 = 0\ \text{m/s}$

Acceleration: $a_2 = g$

$h_2 = u_2t + \frac{1}{2}a_2t^2$

$h_2 = 0t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$ ...(2)

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According to the statement, we have

$$h_1 + h_2 = 200$$

$$100t - \frac{1}{2}gt^2 + \frac{1}{2}gt^2 = 200$$

$$100t = 200 \quad t = \frac{200}{100} \Rightarrow t = 2s$$

Put $t = 2$ in eq. (1), we get

$$h_1 = 100(2) - \frac{1}{2}(9.8)^2(2)^2$$

$$h_1 = 200 - 19.6$$

$\boxed{h_1 = 180.4\text{m}}$ (Required Height)

Q10. A ball is thrown vertically upward with a velocity of 20 m/s. Find, the maximum height reached, and the time taken to return to the thrower's hand.

Solution:

Given that: $u = 20$ m/s, $v = 0$ (at max. height)

$$a = -g = -9.8 \text{ m/s}$$

Using the 3rd equation of motion:

$$2ax = v^2 - u^2$$

$$-2gx = 0^2 - 20^2$$

$$-2(9.8)x = -400 \Rightarrow x = \frac{400}{19.6}$$

$\boxed{x = 20.4\ m}$ (Max. Height)

Let 't' be the time to reach max. height

Using the 1st equation of motion:

$$v = u + at$$

$$0 = 20 - gt \Rightarrow gt = 20$$

$$9.8t = 20 \Rightarrow t = \frac{20}{9.8} \approx 2.04s$$

Return time = $t_{\text{upward}} + t_{\text{downward}}$

$= 2.04 + 2.04$

$= 4.085$ seconds

Q11. A rock is dropped from a cliff. One second later, another rock is dropped from the same height.

(a) What is the distance between the two rocks 2 seconds after the first rock is dropped?

Solution:

Using the 3rd equation of motion:

$$x = ut + \frac{1}{2}gt^2$$

$$x = \frac{1}{2}gt^2 \quad \ldots(1) \because u = 0$$

For Rock 1: $t = 2s$, $g = 9.8$ m/s^2

$$x_1 = \frac{1}{2}(9.8)(2)^2$$

$$= (9.8)(2) = 19.6m$$

For Rock 2: $t = 1s$, $g = 9.8$ m/s^2

$$x_2 = \frac{1}{2}(9.8)(1)^2$$

$$= 4.9\text{m}$$

Distance between rocks $= x_1 - x_2$

$= 19.6 - 4.9$

$= 14.7$ m

(b) At what time will the distance between them be 14.7 m ?

Solution:

Distance between the rocks is:

$$\Delta x = x_1 - x_2$$

$$14.7 = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2 \quad \text{Using (1)}$$

$$14.7 = \frac{1}{2}g(t^2 - t^2 - 1 + 2t)$$

$$14.7 \times 2 = g(2t - 1)$$

$$29.4 = 9.8(2t - 1)$$

$$2t - 1 = \frac{29.4}{9.8}$$

$$2t = 3 + 1 \Rightarrow 2t = 4$$

$\boxed{t = 2s}$

Q12. The acceleration of a particle is $a(t) = \frac{1}{1+t}$ for $t \geq 0$. If the velocity is 0 m/s when $t = 0$, find the velocity at $t = 5$ seconds.

Solution:

Given that: $a(t) = \frac{1}{1+t}, t \geq 0$

$$\frac{dv}{dt} = \frac{1}{1+t} \quad \because a(t) = \frac{dv}{dt}$$

$$dv = \frac{1}{1+t}dt$$

Integrating both sides:

$$\int 1dv = \int \frac{1}{1+t}dt$$

$$v = \ln(1+t) + c \quad \ldots(1)$$

Using initial condition: $v = 0$ at $t = 0$

$$0 = \ln(1+0+c \Rightarrow 0 = \ln 1 + c$$

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$0 = 0 + c \Rightarrow c = 0$

Put $c = 0$ in eq. (1), we get

$$v(t) = t\ln(1+t)$$

At $t = 5s$

$$v(5) = t\ln(1+5)$$
$$= t\ln 6 \approx 1.79 \text{ m/s}$$

Q13. The motion of a robotic arm moving along a straight track is described by the velocity function

$$v(t) = 6 + t - t^2$$

where $v(t)$ is in meters per second and t is the time in seconds, valid for $0 \le t \le 5$. Find the distance travelled by the arm.

Solution:

Given that: $v(t) = 6 + t - t^2,\ 0 \le t \le 5$

For t-intercept, put $v = 0$

$$6 + t - t^2 = 0 \Rightarrow 6 + 3t - 2t - t^2 = 0$$
$$3(2+t) - t(2+t) = 0 \Rightarrow (3-t)(2+t) = 0$$
$$\Rightarrow \quad 3 - t = 0 \quad ; \quad 2 + t = 0$$
$$t = 3 \quad ; \quad t = -2 \notin [0, 5]$$

As A_1 is the area above the time axis on [0, 3] and A_2 is the area below the time axis on [3, 5]

Distance $= A_1 + A_2$

$$= \int_0^3 v(t)dt + \left|\int_3^5 v(t)dt\right|$$

$$= \int_0^3 (6+t-t^2)dt + \left|\int_3^5 (6+t-t^2)dt\right|$$

$$= \left[6t + \frac{t^2}{2} - \frac{t^3}{3}\right]_0^3 + \left|\left[6t + \frac{t^2}{2} - \frac{t^3}{3}\right]_3^5\right|$$

$$= \left[\left(6(3) + \frac{3^2}{2} - \frac{3^3}{3}\right) - 0\right] + \left|\left(6(5) + \frac{5^2}{2} - \frac{5^3}{3}\right) - \left(6(3) + \frac{3^2}{2} - \frac{3^3}{3}\right)\right|$$

$$= \left(18 + \frac{9}{2} - 9\right) + \left|30 + \frac{25}{2} - \frac{125}{3} - 18 - \frac{9}{2} + 9\right|$$

$$= \left(9 + \frac{9}{2}\right) + \left|21 + \frac{25}{2} - \frac{9}{2} - \frac{125}{3}\right|$$

$$= \frac{27}{2} + \left|\frac{126 + 75 - 27 - 250}{6}\right| = \frac{27}{2} + \left|-\frac{76}{7}\right|$$

$$= \frac{27}{2} + \frac{38}{3} = \frac{81 + 76}{6} = 26.17m$$

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Multiple Choice Questions (MCQs)

1. **Kinematics is the branch of mechanics that describes the motion of objects without considering the:**
   A) causes of motion
   B) mass of the object
   C) shape of the object
   D) energy of the object

2. **The motion of a particle along a straight line is called**
   A) circular motion
   B) rectilinear motion
   C) projectile motion
   D) rotational motion

3. **Speed is defined as the:**
   A) rate of change of displacement
   B) total distance travelled divided by total time
   C) rate of change of velocity
   D) change in position per unit acceleration

4. **Instantaneous velocity v(t) in terms of displacement x(t) is:**
   A) $\frac{dx}{dt}$
   B) $\frac{dt}{dx}$
   C) $\frac{d^2x}{dt^2}$
   D) $\frac{x}{t^2}$

5. **If the slope of a displacement-time graph is zero, the particle is:**
   A) moving with positive acceleration
   B) moving backward
   C) at rest
   D) moving with maximum speed

6. **A negative slope on a displacement-time graph represents:**
   A) forward motion
   B) backward motion
   C) zero displacement only
   D) positive acceleration only

7. **For a displacement-time graph, the slope represents:**
   A) acceleration
   B) velocity
   C) distance
   D) time

8. **For a velocity-time graph, the slope represents**
   A) displacement
   B) speed
   C) acceleration
   D) distance

9. **If a velocity-time graph is a horizontal straight line, the acceleration is**
   A) constant positive
   B) constant negative
   C) zero
   D) infinite

10. **If $\frac{dv}{dt}$ is constant and positive, the velocity-time graph is a:**
    A) falling curve
    B) horizontal line
    C) rising straight line
    D) vertical line

11. **If $\frac{dv}{dt}$ is constant and negative, the velocity-time graph is a**
    A) rising straight line
    B) falling straight line
    C) horizontal curve
    D) vertical line

12. **If a particle starts from rest, its initial velocity u is:**
    A) −1 m/s
    B) 0 m/s
    C) 1 m/s
    D) 9.8 m/s

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13. A car starts from rest and accelerates uniformly at 3 m/s² for 5 s. Its velocity is:

A) 8 m/s B) 15 m/s

C) 18 m/s D) 30 m/s

14. A car accelerates from 10 m/s to 30 m/s in 4 s. Its acceleration is:

A) 2 m/s² B) 4 m/s²

C) 5 m/s² D) 10 m/s²

15. If $a(t) = t^2 + \cos t$ and $v(0) = 0$, then $v(t)$ is:

A) $\frac{t^3}{3} + \sin t$ B) $\frac{t^3}{3} + \sin t - 1$

C) $t^2 + \sin t$ D) $t^3 + \cos t$

16. When a particle is projected upward, its velocity:

A) increases uniformly B) decreases uniformly due to gravity

C) remains constant D) becomes infinite

17. For vertical motion with upward positive direction, the constant acceleration is:

A) $+g$ B) $-g$

C) 0 D) $\frac{1}{g}$

18. A stone is dropped from rest from a height of 45 m. Neglecting air resistance, the time to reach the ground is approximately

A) 2.14 s B) 3.03 s

C) 4.50 s D) 9.80 s

19. If a journey has total displacement 8 km and total time 1 h, its average velocity is

A) 4 km/h B) 8 km/h

C) 16 km/h D) 0 km/h

20. A trapezium with parallel sides a and b and perpendicular distance h has area:

A) $(a+b)h$ B) $\frac{1}{2}(a+b)h$

C) $\frac{1}{2}ab+h$ D) $\frac{ab}{h}$

ANSWER KEY

1.	A	2.	B	3.	B	4.	A	5.	C	6.	B	7.	B	8.	C	9.	C	10.	C
11.	B	12.	B	13.	B	14.	C	15.	A	16.	B	17.	B	18.	B	19.	B	20.	B

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