

Chapter # 11

Special Theory of Relativity

STUDENT'S LEARNING OUTCOMES (SLO's)

After studying this unit, the students will be able to:

- ✓ Distinguish between inertial and non-inertial frames of reference.
- ✓ Describe the significance of Einstein's assumption of the constancy of the speed of light.
- ✓ Describe that if c is constant then space and time become relative.
- ✓ State the postulates of Special theory of relativity
- ✓ Explain qualitatively and quantitatively the consequences of special relativity
- ✓ Specifically in the case of:
 - a. The relativity of simultaneity.
 - b. The equivalence between mass and energy.
 - c. Length contraction.
 - d. Time dilation.
 - e. Mass increase
- ✓ State that spacetime is a mathematical model in relativity that treats time as a fourth dimension of the traditional three dimensions of space
(It can be thought of as a metaphorical sheet of paper that can bend, and when it bends it can cause effects such as stretching and compression seen when gravitational waves pass through objects.)

11.1 RELATIVE MOTION



1. What is relative motion? How does it differ from absolute motion? Explain.

Ans. Relative Motion

Definition

Relative motion is the motion of an object as observed from a particular frame of reference, which may itself be in motion.

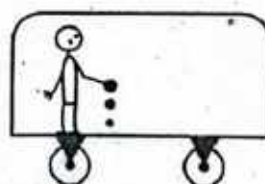
Explanation:

Consider throwing a ball to your right. For someone facing you, this direction appears to his left. This illustrates that direction is a relative concept. Similarly, the state of rest or motion of an object depends on the observer. For example, the walls of a moving train seem stationary to passengers inside the train but appear to be moving to someone standing on the ground. Thus, we cannot definitively say whether an object is absolutely at rest or in motion; all motions are relative to the observer or to the reference frame being used.

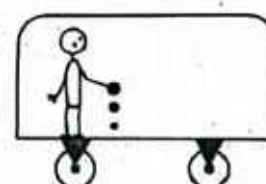
Example: This becomes evident with the following example:

An observer in a closed train compartment uses the compartment as his frame of reference. To determine the train's motion, the observer drops a ball and measures the horizontal distance travelled by the ball, keeping the vertical distance the same in each case. It is assumed that the vertical distance is covered in " t " seconds in all scenarios.

Case (a) Suppose the train is stationary. In this case, the horizontal velocity of the ball will be zero, and the horizontal distance travelled will also be zero. In this scenario, the observations made by the observer inside the train and by someone outside the train will be identical. The ball will have fallen to a point on the floor directly below the point from where it was dropped; Fig. 1(a).



Observer in the train



Observer outside the train

Fig. 1(a)

Case (b) The train is moving with a uniform velocity v_0 . When the ball is released, it has an initial horizontal velocity v_0 and behaves like a projectile. It travels a horizontal distance $v_0 t$ in time t which it takes for the ball to reach the floor. Since both the train and the observer inside it are moving with the same velocity v_0 , they both travel the same horizontal distance $v_0 t$ in the same time t . Therefore, the observer inside the train sees that the ball falls to a

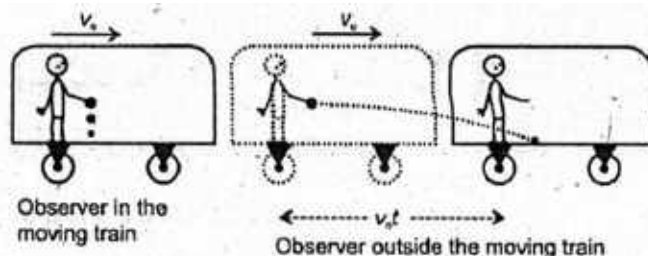


Fig. 1(b)

point on the floor directly below where it was dropped. In contrast, an observer outside the train will see the ball following a projectile path; Fig. 1(b). Thus, observers in different frames of reference, moving with uniform velocity relative to each other, will perceive motion differently.

Difference between Relative Motion and Absolute Motion

Relative motion is described in relation to another object or frame of reference. In relative motion there is no universal or fixed frame, all motion is compared to something else. Whereas absolute motion is described with respect to a fixed, unmoving reference point or space. The concept of absolute motion assumes that there is an absolute space or rest frame.

11.2 FRAMES OF REFERENCE



2. What is meant by the frame of reference? Distinguish between inertial frame of reference and non-inertial frame of reference by giving examples.

Ans. Frames of Reference

Definition

A frame of reference is a set of coordinates or a viewpoint used to measure and observe physical phenomena like position, Velocity, and acceleration.

Example: The position of a table in a room can be described relative to the walls of the room, making the room the frame of reference. Similarly, the laboratory is the reference frame for measurements taken there. If the same experiment is performed in a moving train, the train becomes the frame of reference. The position of a spaceship can be described relative to the positions of distant stars. A coordinate system based on these stars is then the frame of reference.

Inertial Frame of Reference

Definition

An inertial frame of reference is a coordinate system in which Newton's first law or the law of inertia is valid.

This means a body at rest remains at rest unless acted upon by an unbalanced force that produces acceleration. Other laws of nature also apply in such a system. For instance, a body placed on the Earth remains at rest unless an unbalanced force acts upon it, indicating that the Earth can be considered an inertial frame of reference. A body in a car moving with uniform velocity relative to the Earth also remains at rest, so the car is also an inertial frame of reference. Thus, any frame of reference moving with uniform velocity relative to an inertial frame is also an inertial frame.

Non-Inertial Frame of Reference

A frame of reference that is accelerating is known as non-inertial frame of reference.

It is not at rest and not moving at constant velocity.

In non-inertial frame of reference, Newton's law of motion do not hold unless you introduced fictitious forces to explain the motion.

For Your Information

Relativity is the study of the way in which observers from moving frame of reference affect your perception of the world.

Examples of Non-Inertial frame of Reference

- 1. A turning car:** Passengers feel pushed outward. This force (Centripetal force) is fictitious and appears because the car is accelerating in a circle.
- 2. An elevator starting to move:** You feel heavier or lighter depending on the direction of acceleration.
- 3. A rotating merry-go-round:** Objects seem to move outward or curve, again due to fictitious forces like Coriolis force and centripetal force.

1. **Relative motion is defined as the motion of one object:**
(A) w.r.t. the ground (B) w.r.t. time
(C) w.r.t. one another ✓ (D) w.r.t. itself
2. **If two bodies move with equal speed in the same direction, the relative speed is:**
(A) Equal to their individual speed
(B) Zero ✓
(C) The sum of their speeds
(D) Infinity
3. **In relative motion, if object A is stationary and object B is moving, the velocity of B relative to A is:**
(A) Equal to velocity of B ✓ (B) Zero
(C) Equal to velocity of A (D) Negative of velocity of A
4. **Relative velocity between two bodies moving in opposite directions is:**
(A) Sum of their velocities ✓
(B) Product of their velocities
(C) Difference of their velocities
(D) Zero
5. **When the relative motion of two objects is zero, it means:**
(A) They are moving at the same speed
(B) They are at rest
(C) They are moving towards each other
(D) They are moving with equal velocities in the same direction ✓
6. **The coordinate system in which law of inertia is valid is called:**
(A) Special frame of reference
(B) Inertial frame of reference ✓
(C) Non-inertial frame of reference
(D) Standard frame of reference
7. **An inertial frame of reference is one in which:**
(A) Newton's laws of motion are not valid
(B) Newton's first law is valid ✓
(C) Objects always accelerate
(D) The speed of light is variable
8. **A frame of reference that is accelerating w.r.t. an inertial frame is called:**
(A) Inertial (B) Stationary
(C) Non-inertial ✓ (D) Rotational
9. **Which of the following is a non-inertial frame of reference?**
(A) A train moving at constant speed on a straight track
(B) A stationary table
(C) A car accelerating forward ✓
(D) A spaceship drifting in deep space with no forces acting
10. **A freely falling elevator is an example of:**
(A) Inertial frame (B) Non-inertial frame ✓
(C) Rotational frame (D) Stationary frame
11. **Which of the following is true for an inertial frame?**
(A) It must be at rest (B) It must rotate
(C) It must be accelerating
(D) It must move with constant velocity ✓
12. **A rotating merry-go-round is an example of:**
(A) Inertial frame (B) Static frame
(C) Non-inertial frame ✓ (D) Geostationary frame
13. **In a non-inertial frame, what kind of force must be introduced to apply Newton's laws correctly?**
(A) Gravitational force (B) Frictional force
(C) Fictitious force ✓ (D) Magnetic force
14. **The Earth is considered an inertial frame for most purposes because:**
(A) It is at absolute rest (B) It does not rotate
(C) Newton's laws do not work on Earth
(D) Its acceleration is negligible compared to objects on its surface ✓
15. **A ball dropped inside a moving train (with constant velocity) will fall:**
(A) Straight down relative to the train ✓
(B) Backward down relative to the train
(C) Forward down relative to the train
(D) Sideways down relative to the train

11.3 SPECIAL THEORY OF RELATIVITY



3. What is special theory of relativity? Write down its postulates.

Ans. Special Theory of Relativity

The special theory of relativity proposed by Albert Einstein in 1905, is a foundational of theory in Physics that describes how space and time behave at high speeds, especially close to the speed of light.

Postulates of Special Theory of Relativity

The special theory of relativity is based on two postulates:

1. The laws of Physics are the same in all inertial frames of reference (i.e., observers moving at constant velocity).
2. The speed of light in a vacuum is constant for all observers, regardless of their motion or the light source's motion.

Do you know?



Explanation of Postulates

The first postulate generalizes the fact that all physical laws are the same in frames of reference moving with uniform velocity relative to one another. If the laws of Physics differed for observers in relative motion, those observers could determine which was stationary and which was moving. However, such a distinction does not exist, implying that there is no way to detect absolute uniform motion.

The second postulate states the experimental fact that the speed of light in free space is a universal constant, denoted as ' c ' ($c = 3 \times 10^8 \text{ m s}^{-1}$). Since c is constant, space and time become relative. For example, if you are sitting in a train moving at the speed of light and you hold up a mirror in front of you at arm's length, you will still see your reflection in the mirror. This is because, according to the principle of relativity, no experiment can detect the constant motion of the train relative to the person inside it.

The speed of light c emitted by the flashlight is measured same by two observers, one moving in the car with speed v and other standing on the road.



4. What is meant by relativity of simultaneity? Explain.

Ans. Relativity of Simultaneity

Relativity of simultaneity is one of the key concepts in Einstein's special theory of relativity. It means that: "Whether two events happen at the same time depends on the observer's frame of reference."

Explanation

If two events in different locations are observed by one observer to be simultaneous, they will generally not be observed as simultaneous by another observer in a different frame of reference moving relative to the first observer. In other words, whether two events are seen as simultaneous depends on the observer's frame of reference.

If two events in different locations are observed by one observer to be simultaneous, they will generally not be observed as simultaneous by another observer in a different frame of reference moving relative to the first observer. In other words, whether two events are seen as simultaneous depends on the observer's frame of reference. Doors opening simultaneously. However, an observer outside the train will see the back door open before the front door. This is because the back door is moving towards the light waves, while the front door is moving away from the light waves.

For Your Information

If you are in a frame of reference moving at constant velocity from which you cannot see any other frame of reference, there is no way to know if you are moving or at rest.



5. What are the consequences / results of special theory of relativity? Explain.

Ans. Consequences of special Theory of Relativity

Some interesting results of special theory of relativity can be summarized as follows:

1. Time Dilation

According to the special theory of relativity, time is not an absolute quantity; it depends on the motion of the frame of reference.

Suppose an observer is stationary in an inertial frame and measures the time interval between two events in this frame. Let this time interval be t_0 . **This is known as proper time.** If the observer is moving with respect to the frame of events with relativistic velocity v , or if the frame of events is moving with respect to the observer with a uniform relativistic velocity v , the time measured by the observer will not be t_0 , but rather t , given by

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As the quantity $\sqrt{1 - \frac{v^2}{c^2}}$ is always less than one, so t is greater than t_0 , i.e., time has dilated or stretched due to the relative motion of the observer and the frame of reference of the events. This astonishing result applies to all timing processes (physical, chemical, and biological). Even the aging process of the human body is slowed by motion at very high speeds or relativistic speeds.

For example, if a traveler on a plane moving at $0.8c$ picks up and opens a book, the event takes one second as measured by the traveler. However, to a person standing outside the plane, the same event takes 1.7 seconds.

2. Length Contraction

The distance from Earth to a star measured by an observer in a moving spaceship would appear smaller than the distance measured by an observer on the Earth. In other words, if you are in motion relative to two points that are a fixed distance apart, the distance between the two points appears shorter than if you were at rest relative to them. **This effect is known as length contraction.**

Length contraction occurs only along the direction of motion; no such contraction is observed perpendicular to the direction of motion. The length of an object or the distance between two points measured by an observer who is at rest relative to them is called the proper length l_0 . If an object and an observer are in relative motion with speed v , then the contracted length ' l ' is given by

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

Let a train that is measured to be 100 metres long when at rest travel at 80% of the speed of light ($0.8c$). A person inside the train will measure its length as 100 metres. However, a person standing by the side of the track will observe the train to be only 60 metres long. This effect of relativity, which is the shortening of length in the direction of motion, is due to length contraction.

3. Mass Variation

According to the special theory of relativity, the mass of an object is a variable quantity that depends on the object's speed. An object whose mass is measured at rest is called its rest mass m_0 , will have an increased mass m when observed to be moving at speed v . They are related by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The increase in mass indicates the increase in inertia that an object has at high speeds. As v approaches c , it requires a greater force to change the object's speed.

As $v \rightarrow c, \frac{v}{c} \rightarrow 1$ therefore, $\sqrt{1 - \frac{v^2}{c^2}} \rightarrow 0$

Thus $m \rightarrow \infty$

An infinite mass would require an infinite force to accelerate it. Since infinite forces are not available, an object cannot be accelerated to the speed of light c in free space.

11.4 THE EQUILIBRIUM BETWEEN MASS AND ENERGY



6. Describe the Einstein's mass-energy equation; why cannot we observe its effects in everyday life?

Ans. Einstein's Mass-energy Equation

The Einstein's mass-energy equation is:

$$E = mc^2$$

This equation states that energy (E) is equal to mass (m) multiplied by the speed of light squared (c^2). It reveals that mass and energy are interchangeable i.e., mass can be converted into energy, and vice versa.

Why we do not Observe its Effects in Everyday Life?

1. c^2 is a huge number

Since the speed of light squared (c^2) is a huge number, even a small amount of mass contains a massive amount of energy. However, converting mass into energy requires extreme conditions (like those in nuclear reactions), which we do not usually encounter.

2. Mass is conserved in normal processes

In typical chemical reactions (e.g., burning fuel), mass is not converted into energy to any significant degree. The energy released comes from changes in chemical bonds, not mass itself.

3. Effects are only noticeable in extreme environments

The mass-energy equivalence becomes significant in nuclear fission / fusion, particle collisions in accelerators; or astrophysical events (like in stars and black holes); far beyond daily human experiences.

In short, although the equation is fundamentally and underpins much of modern Physics, the conditions needed to observe meaningful mass-energy conversion are not present in our ordinary lives.



7. What are the significant consequences of Einstein's mass-energy equation?

Ans. Significant Consequences of Mass-Energy Equation

Einstein's mass-energy equivalence equation; $E = mc^2$, has profound and wide-ranging consequences in Physics and beyond. Some of the most significant consequences of Einstein's mass-energy equation are:

1. Mass can be converted Into Energy

- This equation shows that mass is a form of energy.
- Even a small amount of mass can be converted into a tremendous amount of energy due to the large value of c^2 (the speed of light).

Application: This principle is the basis for nuclear energy, where mass is converted to energy in nuclear reactions (e.g., fission and fusion reactions).

2. Nuclear Power and Weapons

- Nuclear power plants utilize nuclear fission (splitting of heavy nuclei like uranium or plutonium), releasing energy as per $E = mc^2$.
- Nuclear bombs (both fission and fusion types) release massive energy by converting small amounts of massive energy.

3. Understanding Stellar Processes

- Stars, including the sunshine due to nuclear fusion reactions in their cores where hydrogen nuclei fuse into helium, releasing energy (light and heat).
- $E = mc^2$ explains how mass is lost in the fusion process and emitted as energy.

4. Particle Physics and Anti-matter

- Particle accelerators demonstrate that energy can be converted into mass (e.g., creation of particle anti-particle pairs).
- Conversely, when matter and anti-matter annihilate, their entire mass is converted into energy, its perfect example is $E = mc^2$.

5. Mass Defect in Atoms

- In nuclei, the total mass of bound protons and neutrons is less than the sum of their individual masses.
- This missing mass (mass defect corresponds) to the binding energy holding the nucleus together, as give by $E = mc^2$.

6. Cosmological and Astrophysical Implications

- In black holes, neutron stars, and the early universe, mass-energy equivalence plays a central rule in understanding structure, formation, and evolution.
- In cosmology, the total energy (including mass) influences the shape and fate of the universe.

7. Advancement in Technology

Though indirectly, understanding mass-energy equivalence has driven technological progress in medical imaging (e.g., PET scans), space propulsion concepts, and energy generation.

11.5 SPACE-TIME IN RELATIVITY



8. State the Einstein's concept about the space-time. Describe the view of gravity according to this concept.

Ans. Einstein's Concept about Space-Time

Space is said to be a three-dimensional extent in which all objects and events occur. It provides a framework to define the position and motion of various objects under the influence of some force.

Time measures the sequence and duration of events. In the theory of relativity, time is not absolute; it is considered the fourth dimension. For example, oscillatory motion, such as that of a swinging pendulum, relies on time to determine the frequency of oscillations. Another example is time dilation, a phenomenon discussed earlier in this chapter, where time passes more slowly for an observer moving at extremely high speeds compared to one at rest.

The special theory of relativity explains that space and time are related to each other. It describes how space and time are influenced by gravity and speed, such as the bending of light around massive objects like stars.

Space-time is, in fact, a mathematical model that unifies space-time into a single continuum. It is a concept used to describe all points of space and time and their relation to each other. According to Einstein's theory, space-time is curved especially near massive bodies and for speeds approaching the speed of light. We can hypothetically visualize this as a fabric sheet. If a heavy ball is placed over this sheet, it curves; Fig. 2.

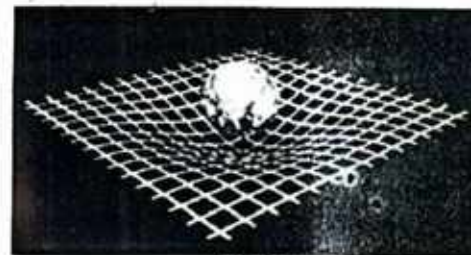


Fig. 2

Objects such as stars and planets cause space-time to curve around themselves, much like an elastic fabric deforms when holding a ball! The more massive the object, the deeper the curve.

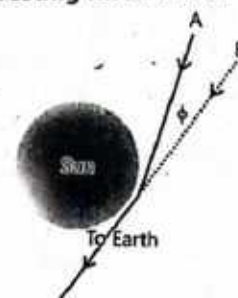
Consequently, we do not speak of a force of gravity acting on bodies; instead, we say that bodies and light rays move along geodesics (analogous to straight lines in plane geometry) in curved space-time. Thus, a body at rest or moving slowly near a massive object would follow a geodesic toward that object.

Einstein's Concept about Gravity

Einstein's theory provides a physical picture of how gravity works. Newton discovered the inverse square law of gravity but explicitly stated that he offered no explanation for why gravity should follow this law. Einstein's theory also describes gravity as following an inverse square law (except in strong gravitational fields), but it explains why this is so. This is why Einstein's theory is considered an advancement over Newton's, even though it encompasses Newton's theory and yields the same results as Newton's theory in all but very strong gravitational fields.

The bending of starlight caused by the Sun's gravity was measured during a solar eclipse in 1919. The results matched Einstein's theory rather than Newton's, leading to Einstein's theory being hailed as a scientific triumph. Another success of Einstein's theory was the detection of gravitational waves, produced by some celestial events causing disturbances (squeezes and stretches) in the curvature of space-time. These waves were detected in 2015 and announced in 2016.

Interesting Information



Bending of starlight by the Sun. Light from the star A is deflected as it passes close to the Sun on its way to Earth. We see the star in the apparent direction B, shifted by the angle ϕ . Einstein predicted that $\phi = 1.745$ seconds of angle which was found to be the same during the solar eclipse of 1919.

Hypothetical Example of Space-Time

Let a spaceship be travelling to a star with half of the speed of light. Let it takes eight years to reach to the star, from the point of view of the observer on the Earth. From the Earth's point of view, the clocks on the spaceship are moving slowly, so that less time passes on the spaceship compared to the Earth.

For the spaceship occupants, the length of the journey has contracted which they cover in less time. The occupants of the spaceship record 7 years to reach their destination, rather than 8 years.

Example 11.1: The period of a pendulum is measured to be 3.0 s in the inertial reference frame of the pendulum. What is its period measured by an observer moving at a speed of 0.95 c with respect to the pendulum?

Solution:

Given that;

$$t_0 = 3.0 \text{ s}, \quad v = 0.95 c.$$

To Find:

$$t = ?$$

Calculations:

Using
$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{3.0 \text{ s}}{\sqrt{1 - \frac{(0.95c)^2}{c^2}}} = \frac{3.0 \text{ s}}{\sqrt{1 - (0.95)^2}} = 9.6 \text{ s} \text{ Ans.}$$

Example 11.2: A bar 1.0 m in length and located along x-axis moves with a speed of 0.75 c with respect to a stationary observer. What is the length of the bar as measured by the stationary observer?

Solution:

Given that;

$$l_0 = 1.0 \text{ m}, \quad v = 0.75 c$$

To Find:

$$l = ?$$

Calculations:

Using
$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$l = 1.0 \text{ m} \times \sqrt{1 - \frac{(0.75c)^2}{c^2}} \\ = 1.0 \text{ m} \times \sqrt{1 - (0.75)^2} = 0.66 \text{ m} \text{ Ans.}$$

Example 11.3: Find the mass m of a moving object with speed 0.8 c.

Solution:

Given that;

$$v = 0.8 c$$

To Find:

$$m = ?$$

Calculations:

Using
$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or
$$m = \frac{m_0}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}} = \frac{m_0}{\sqrt{1 - (0.8)^2}} = 1.67 m_0$$

or
$$m = 1.67 m_0 \text{ Ans.}$$

For Your Information

The faster you are moving or close to a strong source of gravity, the slower the time goes for you.

Interesting Information

If you are on some spaceship moving extremely fast through space near a black hole like in movie, "Interstellar" then you could miss 7 years on the Earth in every hour.



- The equation; $E = mc^2$ expresses the idea that:**
 - Energy is proportional to m
 - Mass and energy are equivalent ✓
 - Energy is inversely proportional to m
 - Light has no mass
- A person travelling near the speed of light will experience:**
 - Increased mass
 - Slower passage of time
 - Decreased length in the direction of motion
 - All of these ✓
- The length contraction in special relativity occurs:**
 - In the direction perpendicular to motion
 - In all directions equally
 - Only in the direction of motion ✓
 - Not at all
- An object's mass increases with:**
 - Speed approaching the speed of light ✓
 - Decrease in energy
 - Increase in temperature
 - Time
- Which of the following is a postulate of special theory?**
 - Time is absolute
 - Force is equal to mass times acceleration
 - Speed of light is constant in all inertial frames ✓
 - All frames of reference are non-inertial
- According to special theory, time measured in a moving frame:**
 - Runs faster
 - Remains the same
 - Runs slower (time dilation) ✓
 - Stops completely

7. Length contraction occurs in the direction of:
 (A) Perpendicular to motion
 (B) Any direction
 (C) Along the direction of motion ✓
 (D) Opposite to gravitational pull
8. Which of the following is not consistent with special theory of relativity?
 (A) Light speed is the same in all inertial frames
 (B) Time is absolute for all observers ✓
 (C) Simultaneity is relative
 (D) Moving clocks tick slower
9. Which of the following remains constant in all inertial frames, according to special theory of relativity?
 (A) Time (B) Length
 (C) Mass (D) Speed of light ✓
10. If an object moves at the speed of light, its mass would:
 (A) Become zero (B) Remain constant
 (C) Become infinite ✓ (D) Double
11. Two events that are simultaneous in one inertial frame:
 (A) Are simultaneous in all frames
 (B) Must occur at the same place
 (C) May not be simultaneous in another frame ✓
 (D) Cannot happen at different locations
12. When speed of an object is half of the speed of light, then:
 (A) $t = t_0$ (B) $t = 2t_0$
 (C) $t = \frac{3t_0}{\sqrt{1}}$ (D) $t = \frac{2}{\sqrt{3}} t_0$ ✓
13. For what speed of an object, is its relativistic time double of the proper time?
 (A) $2c$ (B) $\sqrt{2}c$
 (C) $\frac{1}{4}c$ ✓ (D) $\frac{\sqrt{3}}{2}c$
14. In an expression for time dilation, the quantity $\sqrt{1 - \frac{v^2}{c^2}}$ is always:
 (A) Equal to zero (B) Greater than one
 (C) Equal to one (D) Less than one ✓
15. If velocity of a body becomes equal to "c", then its mass becomes:
 (A) 0 kg (B) $m = m_0$
- (C) $m \rightarrow \infty$ ✓ (D) $m = \frac{m_0}{2}$
16. The speed of light ($3 \times 10^8 \text{ m s}^{-1}$) in free space is:
 (A) Variable (B) Less than that in air
 (C) Unpredictable (D) Constant ✓
17. The special theory of relativity is applicable to the object moving with maximum velocity equal to:
 (A) Speed of light (B) More than speed of light
 (C) Less than speed of light ✓
 (D) Double the speed of light
18. At what speed would the mass of an electron become double of its rest mass?
 (A) $0.5c$ (B) $0.707c$
 (C) $0.866c$ ✓ (D) $0.99c$
19. In 1905, the special theory of relativity was proposed by:
 (A) Maxwell (B) de Broglie
 (C) Bohr (D) Einstein ✓
20. All motions are:
 (A) Absolute (B) Uniform
 (C) Relative ✓ (D) Variable
21. Two photons approach each other, their relative speed will be:
 (A) $2c$ (B) Zero
 (C) Less than c ✓ (D) c
22. The mass of an object will be double at speed:
 (A) $2.5 \times 10^8 \text{ ms}^{-1}$ ✓ (B) $1.6 \times 10^8 \text{ ms}^{-1}$
 (C) $3.6 \times 10^8 \text{ ms}^{-1}$ (D) $0.6 \times 10^8 \text{ ms}^{-1}$
23. 0.1 kg is equivalent to the energy of:
 (A) $5 \times 10^6 \text{ J}$ (B) $6 \times 10^{16} \text{ J}$
 (C) $9 \times 10^{16} \text{ J}$ (D) $9 \times 10^{15} \text{ J}$ ✓
24. The velocity at which the mass of a body becomes double is:
 (A) $\frac{3}{2}c$ (B) $\frac{\sqrt{2}}{3}c$
 (C) $\frac{\sqrt{3}}{2}c$ ✓ (D) c
25. The concept of space-time combines:
 (A) Space and gravity (B) Mass and velocity
 (C) Space and time into a four-dimensional continuum ✓
 (D) Time and space
26. In space relativity, simultaneity is:
 (A) Absolute (B) Relative ✓
 (C) Irrelevant (D) Always preserved

ADDITIONAL SHORT ANSWER QUESTIONS



Q.1 What is relative motion?

Ans. It is the motion of an object as observed from a particular frame of reference, which may itself be in motion.

Q.2 Can an object be at rest in one frame and in motion in another?

Ans. Yes, motion is relative, so an object can be at rest in one frame and moving in another.

Q.3 Why is relative motion important in physics?

Ans. Because there is non absolute frame of reference, all motion must be described relative to some observer.

Q.4 What is the relative velocity of an object w.r.t. itself?

Ans. Zero. An object is always at rest relative to itself.

Q.5 Can two objects moving with constant speeds have zero relative velocity?

Ans. Yes, if they move in the same direction with the same speed.

Q.6 What is a frame of reference?

Ans. It is a coordinate system from which motion is observed and measured.

Q.7 Is the Earth an inertial frame of reference?

Ans. Approximately yes, for most practical purposes though technically it is non-inertial due to its rotation and revolution.

Q.8 Give an example of non-inertial frame of reference.

Ans. A car that is accelerating or turning is a non-inertial frame of reference.

Q.9 Give an example of inertial frame of reference.

Ans. A train moving at constant speed in a straight line.

Q.10 What is time dilation?

Ans. It is the effect where time runs slower for an object moving at high speeds relative to a stationary observer.

Q.11 What is the length contraction?

Ans. It is the shortening of an object in the direction of motion as seen by a stationary observer.

Q.12 Can any object with mass travel at the speed of light?

Ans. No, objects with mass require infinite energy to reach the speed of light.

Q.13 Is it possible for an object to move with speed of light? Justify your answer. (GRW-19)

Ans. Relativistic Mass:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This shows that the increase in mass shows the increase in inertia at high speeds.

Condition: If the speed of the material object is equal to the speed of light i.e., $v = c$

Then

$$m = \frac{m_0}{\sqrt{1 - \frac{c^2}{c^2}}}$$

or

$$m = \frac{m_0}{\sqrt{1 - 1}}$$

or

$$m = \frac{m_0}{0}$$

or

$$m = \infty$$

Result: As infinite mass requires an infinite force to accelerate it. As the infinite force is not available. So, an object cannot be accelerated, to the speed of light in free space.

Q.14 What are the measurements on which two observers in relative motion will always agree upon?

Ans. Two observers in relative motion will always agree upon the following measurements.

- Speed of light in free space is same for both observers.
- The magnitude of relative velocities of the observers with respect to each other is same.

Q.15 Does the dilation means that time really passes more slowly in moving system or that its only seems to pass more slowly?

Ans. According to special theory of relativity, time is not an absolute quantity. It depends upon the motion of frame of reference. The relation for the time dilation is given by:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As knowledge of science is based upon observation and measurements only. So we can say that time is measured to dilate. In other words, it passes more slowly.

Q.16 If you are moving in a spaceship at a very high speed relative to the earth, would you notice a difference
 (a) in your pulse rate (b) in the pulse rate of people on earth?

Ans. According to $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ the time will dilate or clock of space ship will appear slow as compared to clock on

Earth. So,

(a) Your pulse rate remain same because there is no relative motion between you and your pulse in the space craft.

(b) The pulse rate of people on the earth decreases with respect to you due to relative motion as you are moving in the space craft.

Example: For observer on spaceship pulse rate is $\frac{N}{t_0}$ and for an observer on earth it is $\frac{N}{t}$

where $t > t_0$ $\left(\therefore \frac{N}{t} < \frac{N}{t_0} \right)$

Q.17 If the speed of light were infinite, what would be equations of special theory of relativity reduce to?

Ans. If the speed of light were infinite, there will be no relativistic effects (time dilation, length contraction and mass variation at all).

Explanation:

If speed of light $c = \infty$, then $\frac{v^2}{c^2} = \frac{v^2}{\infty} = 0$, so we have

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1 - 0}} = m_0$$

or $m = m_0$

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}} = l_0 \sqrt{1 - 0} = l_0 \sqrt{1} = l_0$$

or $l = l_0$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t_0}{\sqrt{1 - 0}} = \frac{t_0}{\sqrt{1}} = t_0$$

or $t = t_0$

Also, $E = mc^2 = \infty$ $\therefore c = \infty$

Q.18 Since mass is a form energy, can we conclude that a compressed spring has more mass than the same spring when it is not compressed?

Ans. Yes, the compressed spring has more than the same spring when it is not compressed.

Reason: Mass is a form of energy, according to the theory of relativity, yes there would be increase in the mass of compressed spring. However, this increase is too small to be measured.

According to special theory of relativity mass is a form of energy.

$$\therefore E = mc^2$$

or $\Delta E = \Delta mc^2$

or $\Delta m = \frac{\Delta E}{c^2}$

ΔE is very small therefore Δm is too small to be measured.

Q.19 What does mass-energy equivalence imply about mass?

Ans. Mass is a form of energy and even a small amount of mass can be converted into a large amount of energy.

Q.20 Give an example of mass being converted to energy.

Ans. In nuclear reactions, such as fission in nuclear reactors, mass is converted to energy.

Q.21 Can energy be converted to mass?

Ans. Yes, under the right conditions, such as in high energy particle collisions.

Q.22 Is the mass of a system always conserved?

Ans. No, total energy is conserved, but mass cannot be converted to energy and vice versa.

Q.23 How does gravity affect time in general relativity?

Ans. Time runs slower in stronger gravitational fields; this is called gravitational time dilation.

Q.24 What is the significance of space-time curvature in general relativity?

Ans. It explains gravity as the effect of mass curving space-time, rather than as a force.

Q.25 What is space-time?

Ans. It is a four dimensional continuum that combines the three dimensions of space with the dimension of time.

Q.26 What is meant by mass variation?

Ans. Mass variation means the increases in the mass of an object as its speed approaches the speed of light.

SOLVED EXERCISE

MULTIPLE CHOICE QUESTIONS

Tick (✓) the correct answer.

- 11.1 Relativistic mechanics yields results different from classical mechanics for objects moving with:
(a) low velocity (b) velocity equal to that of sound waves
(c) velocity greater than sound waves (d) velocity approaching that of light ✓
- 11.2 If an observer is moving in the same direction as a sound wave, the velocity of the wave seems to be:
(a) more (b) less ✓ (c) constant (d) sum of the two velocities
- 11.3 If the rest mass of a particle m_0 increases to m due to its high speed, then its kinetic energy is:
(a) $\frac{1}{2} mc^2$ (b) $\frac{1}{2} mv^2$ (c) $(m - m_0) c^2$ ✓ (d) $\frac{1}{2} (m - m_0) c^2$
- 11.4 The speed of beam light of a car while moving with high speed as compared to its rest position is:
(a) greater (b) less (c) same ✓ (d) zero
- 11.5 A photon is a particle of light. What is its mass when it moves with $0.9 c$?
(a) $9.1 \times 10^{-31} \text{ kg}$ (b) $1.67 \times 10^{-19} \text{ kg}$ (c) $1.67 \times 10^{-27} \text{ kg}$ (d) Zero ✓

SHORT ANSWER QUESTIONS

11.1 What is meant by inertial frame of reference and a non-inertial frame of reference?

Ans. An inertial frame of reference is a reference frame in which an object remains at rest or in uniform motion unless acted upon by an external force. In such a frame, Newton's laws of motion hold true. Essentially, it is a non-accelerating frame of reference.

A non-inertial frame of reference, on the other hand, is one that is accelerating. In these frames, Newton's laws of motion do not directly apply unless fictitious forces are introduced to account for the acceleration. For example, a rotating frame of reference or a car turning a corner would be considered non-inertial.

11.2 What are the two postulates of special theory of relativity?

Ans. The two postulates of Einstein's special theory of relativity are:

- The Principle of Relativity:** The laws of physics are the same for all observers in uniform motion relative to one another, meaning that there is no preferred inertial frame of reference. No inertial observer can distinguish their state of rest or uniform motion.
- The Constancy of the Speed of Light:** The speed of light in a vacuum is always constant (approximately $299,792,458 \text{ meters per second}$ or $3 \times 10^8 \text{ ms}^{-1}$) for all observers, regardless of their motion or the motion of the light source.

11.3 Describe why it is impossible for a material particle to move with speed of light.

Ans. It is impossible for a material particle to move at the speed of light due to the following reasons rooted in the theory of special relativity:

- Mass and Energy Relationship:** According to Einstein's famous equation $E = mc^2$, a particle with mass requires energy to accelerate. As a material particle approaches the speed of light, its relativistic mass increases, meaning that more and more energy is required to continue accelerating. As it nears the speed of light, the energy needed approaches infinity, which is not physically attainable with any finite amount of energy. Hence, it becomes impossible for any particle with mass to reach the speed of light.

2. **Relativistic Time Dilation:** As an object moves faster, time appears to slow down relative to an outside observer. At the speed of light, time would theoretically stop entirely for the object in motion, which creates a paradox because this cannot happen for objects with mass. This time dilation becomes extreme as the speed approaches c , but it never reaches a point where the object can actually achieve c .
3. **Space Contraction:** As a particle approaches the speed of light, lengths in the direction of motion contract according to the Lorentz transformation. At the speed of light, this contraction would cause the length to shrink to zero, which is physically impossible for any material object.
4. **Photon Behaviour:** Particles that do travel at the speed of light, such as photons, are massless. Only massless particles can travel at this speed, while material particles, which have mass, cannot. Thus, a material particle with mass cannot reach the speed of light.

11.4 Does theory of relativity contradicts Newton's laws of motion? Explain briefly.

Ans. No, the theory of relativity does not contradict Newton's laws of motion; rather, it extends and refines them. Newton's laws are accurate for everyday speeds and situations where gravitational fields are not extreme. However, Einstein's theory of relativity provides a more comprehensive framework, particularly for objects moving at speeds close to the speed of light or in strong gravitational fields.

In relativity:

1. Newton's laws work well at low speeds (compared to the speed of light) and in weak gravitational fields.
 2. Einstein's theory takes into account the effects of spacetime curvature and time dilation, which become significant at high speeds or in intense gravitational fields.
- Relativity reduces to Newton's laws in these "normal" conditions, meaning that Newton's laws are still valid as an approximation in everyday circumstances.

11.5 What is meant by proper time, and proper length?

Ans. Proper time and proper length are concepts in the theory of relativity that refer to measurements made in a specific reference frame.

1. Proper time refers to the time interval measured by a clock that is at rest relative to the observer and located at the same place as the event being observed. It is the time measured in the reference frame where the object or observer is at rest. In simpler terms, it's the time experienced by an observer who is moving along with the event. For example, if you are traveling in a spaceship, the time you experience on board as you move is the proper time for you. It is the shortest time interval between two events and is measured along the worldline of an observer.
2. Proper length is the length of an object measured in the rest frame of the object, meaning in the reference frame where the object is not moving. In this frame, the object's length is measured without any relativistic effects like length contraction. If an object is moving relative to an observer, the observer will measure the object's length to be contracted along the direction of motion. Proper length is the length that would be measured by an observer who is at rest with respect to the object.

11.6 What is meant by relativistic mass, length and time?

Ans. Relativistic mass, length, and time are concepts in the theory of special relativity, formulated by Albert Einstein, that describe how the measurements of these quantities change when objects move at speeds close to the speed of light.

1. Relativistic Mass:

This refers to the increase in mass that an object experiences as its velocity approaches the speed of light. According to special relativity, the mass of an object is not constant and increases with speed. The equation for relativistic mass is:

$$m_{rel} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As the object approaches the speed of light (c), its relativistic mass grows without bound, making it impossible to accelerate to the speed of light.

2. Relativistic Length (Length Contraction):

Length contraction is the phenomenon where the length of an object moving at high speeds appears shorter to an observer at rest. The contracted length is given by:

$$L = \frac{L_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Objects appear to contract only in the direction of motion as their speed increases.

3. Relativistic Time (Time Dilation):

Time dilation refers to the phenomenon where time passes more slowly for an object moving at high speeds relative to an observer at rest. The time elapsed for a moving clock is given by:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As the object's speed approaches the speed of light, time for the object (as measured by an external observer) slows down.

11.7 Why mass of a moving object increases?

Ans. The "increase" in mass that you are referring to happens due to the relativistic effect described in Einstein's theory of special relativity. When an object moves at high speeds (close to the speed of light), its relativistic mass increases. This is different from the object's rest mass, which remains constant.

The concept is based on the fact that as an object moves faster, it requires more and more energy to keep accelerating it. The energy required to accelerate the object contributes to its total energy, and in relativity, total energy and mass are related. As the object's speed increases, its energy increases, and thus, its relativistic mass also increases.

Mathematically, the relativistic mass m_{rel} is given by the equation:

$$m_{rel} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As the velocity V approaches the speed of light c , the denominator becomes smaller, causing the relativistic mass to increase.

It's important to note that this increase in mass only applies to the object's energy and how it interacts with forces. The object's rest mass (the mass when it's not moving) does not change, only its relativistic mass increases with speed.

11.8 All motion are relatives. Does space-time is absolute? Explain briefly.

Ans. No, space-time is not absolute. According to Einstein's theory of relativity, space and time are interconnected and relative, meaning they depend on the observer's frame of reference. This is known as spacetime curvature. Events that are simultaneous for one observer may not be for another moving at a different velocity. Space and time can stretch and bend due to gravity and relative motion, leading to phenomena like time dilation and length contraction. So, unlike the classical Newtonian view, where space and time were absolute, relativity shows they are dynamic and relative to the observer's motion and position in the universe.

11.9 Explain that speed of light is an ultimate limit for any object.

Ans. The speed of light is $3 \times 10^8 \text{ ms}^{-1}$ (in a vacuum), is considered an ultimate speed limit for any object or information in the universe, according to Einstein's theory of relativity. This is because as an object with mass approaches the speed of light, its relativistic mass increases, meaning it requires more and more energy to continue accelerating.

At the speed of light, the object's mass would become infinite, and the amount of energy required to accelerate it further would also become infinite. This makes it physically impossible for anything with mass to reach or exceed the speed of light. Photons, the particles of light, are massless, which is why they can travel at this ultimate speed limit.

Additionally, relativity tells us that time slows down for an object moving at speeds close to the speed of light, leading to effects like time dilation. This reinforces the idea that the speed of light acts as a barrier, beyond which the normal rules of time and space break down.

11.10 Give examples where the results of special theory of relativity have been verified.

Ans. The special theory of relativity, proposed by Albert Einstein in 1905, has been experimentally verified in numerous ways. Some of the most notable examples include:

- 1. Time Dilation (Muons in Cosmic Rays):** Muons, which are unstable subatomic particles, are produced when cosmic rays collide with the Earth's atmosphere. These particles have a short lifespan, but due to time dilation, they live longer when moving at high speeds close to the speed of light. This has been confirmed by measuring the number of muons detected at the Earth's surface, which matches the prediction of special relativity.
- 2. GPS System:** The Global Positioning System (GPS) relies on a network of satellites in orbit. The clocks on these satellites experience time differently than clocks on the Earth's surface due to their high speeds (special relativity) and the weaker gravitational field (general relativity). The system must account for both effects to provide accurate positioning data. The success of GPS navigation confirms the validity of relativity.
- 3. Relativistic Mass Increase (Particle Accelerators):** In particle accelerators, particles are accelerated to speeds close to the speed of light. As their velocity increases, their relativistic mass increases, and the energy required to continue accelerating them also increases. This effect has been measured directly in particle accelerators, such as at CERN.
- 4. The Behaviour of Electromagnetic Waves:** The speed of light in a vacuum is constant and independent of the motion of the observer, as predicted by special relativity. Experiments measuring the speed of light in various frames of reference have confirmed this prediction.
- 5. Relativistic Doppler Effect:** The relativistic Doppler effect describes the change in frequency (and thus energy) of light or other electromagnetic radiation due to the relative motion of the source and the observer. This effect has been observed in the light emitted from moving stars and galaxies, as well as in laboratory experiments. These and other experiments demonstrate the ongoing success of special relativity in explaining and predicting physical phenomena at high velocities and provide strong evidence for its accuracy.

CONSTRUCTED RESPONSE QUESTIONS

11.1 Speed of sound is affected by relative motion between the observer and the source. Does this apply to speed of light as well? Describe briefly.

Ans. No, the speed of light is not affected by the relative motion between the observer and the source. Unlike sound, which travels through a medium (like air), light does not require a medium and always at a constant speed of 3×10^8 m in a vacuum regardless of the motion of the source or observer. In contrast, the speed of sound depends on the motion of the source, the observer, and the medium it is travelling through.

11.2 Is it ever possible to see a star moving away from us at a uniform velocity equal to the velocity of light?

Ans. No, it is not possible to see a star moving away from us at a uniform velocity equal to the speed of light. According to the theory of relativity, as an object approaches the speed of light, its relativistic mass increases, and it would require an infinite amount of energy to accelerate it to the speed of light. Therefore, no object with mass, including stars, can ever reach or exceed the speed of light.

In the context of the expanding universe, distant galaxies can appear to move away from us at velocities greater than the speed of light due to the expansion of space itself, but this doesn't violate relativity because the galaxies are not moving through space at superluminal speeds. Rather, the space between us and those galaxies is expanding. This effect is described by the metric expansion of space in cosmology. However, this does not mean that the galaxy itself is moving faster than light within the local spacetime.

11.3 If the speed of light is just 50 m s^{-1} , how would every day events appear to?

Ans. If the speed of light were 50 meters per second (much slower than the actual speed of light, which is $3 \times 10^8 \text{ ms}^{-1}$), everyday events would appear drastically different. Here's how some situations might look:

- 1. Seeing Light:** You would notice light traveling across the room rather than instantaneously. For example, if someone turned on a light switch, the light would appear to "move" from the bulb to your eyes at a speed of 50 metres per second, which is about as fast as a running athlete.
- 2. Delays in Communication:** Any kind of communication, like phone calls, video chats, or even text messages, would have significant delays. A simple sentence could take seconds to travel, and a longer distance (e.g., across a city or country) would have noticeable lag.
- 3. Watching Motion:** For moving objects, such as cars or people, there would be a noticeable delay in the movement of their image. If someone waved their hand, you might see the hand start to move and then the image of the hand arriving at its new position after a noticeable delay.

4. **Astronomical Events:** Observing distant stars, planets, and even the sun would have a huge delay. A sunset or sunrise would appear much slower, and stargazing would involve seeing objects as they were minutes or even hours ago, depending on the distance.
5. **Sports and Entertainment:** Anything involving rapid movement, like watching sports or a fast-paced movie, would seem unnaturally slow. A sprint or a car race would appear as if the athletes or vehicles were moving in slow motion.

11.4 If the speed of light were infinite, what would the equations of special theory of relativity reduce to?

Ans. If the speed of light were infinite, the equations of special relativity would simplify significantly. In special relativity, the speed of light c plays a crucial role in shaping the relationships between space, time, and mass. The key equations would reduce as follows:

1. Lorentz transformation:

The Lorentz transformations describe how space and time coordinates change between observers in relative motion. When c becomes infinite, the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

simplifies to $\gamma = 1$, because $\frac{v^2}{c^2}$ becomes zero. This implies that time dilation and length contraction would no longer occur. Essentially, space and time would behave in a way similar to classical Newtonian mechanics, where both space and time are absolute.

2. Energy-mass relationship:

The famous equation $E = mc^2$ relates energy and mass. If c is infinite, the equation becomes meaningless since it would tend to infinity for any non-zero mass. However, in a practical sense, this suggests that the concept of relativistic energy would no longer be relevant, and we might revert to a classical expression for kinetic energy,

$$E = \frac{1}{2}mv^2, \text{ for speeds much less than infinity.}$$

3. Relativity of simultaneity:

In the context of infinite light speed, events that are simultaneous in one reference frame would also be simultaneous in all other reference frames. The relativity of simultaneity would disappear because signals would travel instantaneously across all space.

4. Causality:

The principle of causality (the idea that causes precede effects) would be preserved. However, since the speed of light would no longer be a limiting factor, any signal could travel instantaneously, potentially leading to non-causal connections across spacetime.

11.5 According to Einstein's equation; $E = mc^2$, is it possible to create a single electron from energy? Explain.

Ans. Einstein's famous equation, $E = mc^2$, relates energy (E) and mass (m), with c being the speed of light in a vacuum. This equation shows that energy and mass are interchangeable, meaning energy can be converted into mass and vice versa.

However, creating a single electron from energy is not as simple as directly applying $E = mc^2$. Here is why:

1. **Mass-Energy Equivalence:** The equation indicates that a certain amount of energy is required to create mass, but the process of creating an electron requires more than just energy. The energy must be in a specific form; typically, high-energy photons (like gamma rays) can be converted into particle-antiparticle pairs, including electrons and positrons, through a process called pair production.
2. **Energy Threshold:** According to quantum mechanics and special relativity, the energy needed to create an electron and a positron (since an electron alone cannot spontaneously appear without violating conservation laws) is at least the equivalent of the rest mass energy of the two particles. The rest mass energy of an electron is approximately 0.511 MeV (million electron volts). Therefore, the energy required for creating a single electron is around 0.511 MeV, but this does not account for other necessary conditions like conservation of momentum and the presence of sufficient energy in a suitable environment.
3. **Pair Production:** To create a single electron, the energy is typically provided in the form of high-energy photons (like those from gamma rays). When a photon with energy above 1.022 MeV (the combined rest mass

energy of an electron and positron) interacts with a nucleus or another particle, it can transform into an electron and a positron.

COMPREHENSIVE QUESTIONS

11.1 What is meant by the "frame of reference"? Distinguish between inertial frame of reference and non inertial frame of reference by giving examples.

Ans. See Q. 2

11.2 Describe the Einstein's mass-energy equation; why cannot we observe its effects in everyday life? What are its significant consequences? Give examples.

Ans. See Q. 6 and Q. 7

11.3 State the Einstein's concept about the space-time. Describe the view of gravity according to this concept.

Ans. See Q. 8

NUMERICAL PROBLEMS

11.1 An electron is accelerated to a speed of $0.995c$ which passes down an evacuated tube 500 m long. How long will the tube appear to the electron?

Solution:

Given data:

Speed of electron $= v = 0.995c$

Length of tube $= l_0 = 500\text{ m}$

To Find:

How long the tube appears $= l = ?$

Calculations: To find how long the tube appears to the electron, we need to apply the concept of length contraction.

Using length contraction formula;

$$l = \frac{l_0}{\gamma} \quad \dots\dots (i)$$

where

l = Contracted length (length observed by the electron)

l_0 = Proper length (length of the tube), 500 m

γ = Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Putting the values

$$\gamma = \frac{1}{\sqrt{1 - (0.995)^2}} = \frac{1}{\sqrt{1 - 0.990025}}$$

$$\gamma = \frac{1}{\sqrt{0.009975}} = \frac{1}{0.099875} = 10.01$$

Putting the values of γ in Eq. (i)

$$l = \frac{500\text{ m}}{10.01} \approx 49.95\text{ m}$$

$l = 50\text{ m}$ **Ans.**

Thus, the tube appears approximately 50 m long to the electron.

11.2 A neutron, being not a stable particle, disintegrates in 20 minutes on the average. How long will it seem to exist if shoots out from a nucleus with a speed of $0.8c$?

Solution:

Given data:

Proper lifetime of neutron $= t_0 = 20\text{ min} = 1200\text{ s}$

Speed of neutron $= v = 0.8c$

To Find:

How long the neutron seems to exist $= t = ?$

Calculations: Using time dilation formula:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Putting the values

$$t = \frac{1200\text{ s}}{1 - \frac{(0.8c)^2}{c^2}} = \frac{1200\text{ s}}{1 - (0.8)^2} = \frac{1200\text{ s}}{\sqrt{1 - 0.64}}$$

$$= \frac{1200\text{ s}}{0.36} = \frac{1200}{0.6} = 2000\text{ s} = 33.3\text{ min}$$

$t = 33.3\text{ min}$ **Ans.**

So, to an observer in the laboratory, the neutron seems to live for about 33.3 minutes due to time dilation.

11.3 A spaceship is measured 100 m long while it is at rest with respect to an observer, if this spaceship now flies by the observer with a speed of $0.99c$, what length will the observer find for the spaceship?

Solution:

Given Data:

Length of spaceship at rest $= L_0 = 100\text{ m}$

Relative speed of spaceship $= v = 0.99c$

Speed of light $= c = 3.0 \times 10^8\text{ m/s}$

To Find:

The contracted length of spaceship $= L = ?$

Calculation: Using the Formula:

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

Putting the values

$$\begin{aligned} L &= 100 \text{ m} \sqrt{1 - \frac{(0.99c)^2}{c^2}} \\ &= 100 \text{ m} (1 - 0.9801) \\ &= 100 \text{ m} (0.0199) \end{aligned}$$

$$\boxed{L = 14 \text{ m}}$$

The observer will measure the length of spaceship as 14 m.

11.4 The rest mass of an electron is $9.11 \times 10^{-31} \text{ kg}$. Calculate the corresponding rest-mass energy.

Solution:

Given data:

Rest mass of an electron $= m_0 = 9.1 \times 10^{-31} \text{ kg}$

Speed of light $= c = 3.0 \times 10^8 \text{ ms}^{-1}$

To Find:

Rest mass energy $= E_0 = ?$

Calculation: Using the Formula:

$$E_0 = m_0 c^2$$

Putting the values

$$E_0 = (9.1 \times 10^{-31} \text{ kg}) \times (3.0 \times 10^8 \text{ ms}^{-1})^2$$

$$E_0 = 8.2 \times 10^{-14} \text{ J}$$

Rest-mass energy E_0 in MeV:

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$E_0 (\text{MeV}) = \frac{8.2 \times 10^{-14} \text{ J}}{1.602 \times 10^{-19} \text{ J}}$$

$$\boxed{E_0 (\text{MeV}) = 0.51 \text{ MeV}}$$

11.5 An electron is accelerated to a speed $v = 0.85 c$. Calculate its total energy and kinetic energy in electron volt.

Solution:

Given data:

Speed of an electron $= v = 0.85 c$

Rest mass of an electron $= m_0 = 9.11 \times 10^{-31} \text{ kg}$

Speed of light $= c = 3.0 \times 10^8 \text{ ms}^{-1}$

To Find:

(i) Total energy $= ?$

(ii) Kinetic energy $= ?$

The total energy E of a particle is:

$$E = \gamma m_0 c^2$$

where γ is the Lorentz factor, $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

The kinetic energy is the difference between the total energy and rest energy.

$$\text{K.E.} = E - m_0 c^2$$

Step-by-Step calculation:

• **Lorentz factor:**

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.85c)^2}{c^2}}} = \frac{1}{\sqrt{1 - (0.85)^2}} \\ &= \frac{1}{\sqrt{1 - 0.7225}} = \frac{1}{\sqrt{0.2775}} \approx 1.906 \end{aligned}$$

• **Total energy E**

$$E = \gamma m_0 c^2$$

$$= 1.906 \times (9.11 \times 10^{-31} \text{ kg}) (3.0 \times 10^8 \text{ ms}^{-1})^2$$

$$= 1.906 \times 9.11 \times 10^{-31} \times 9.0 \times 10^{16} \text{ m}^2 \text{s}^{-2}$$

$$\approx 1.55 \times 10^{-13} \text{ J}$$

• **Rest energy:**

$$E_0 = m_0 c^2 = 9.11 \times 10^{-31} \text{ kg} \times (3.0 \times 10^8 \text{ ms}^{-1})^2$$

$$= 9.11 \times 10^{-31} \text{ kg} \times 9.0 \times 10^{16} \text{ m}^2 \text{s}^{-2}$$

$$E_0 \approx 8.199 \times 10^{-14} \text{ J}$$

• **Kinetic energy**

$$\text{K.E.} = E - E_0 = 1.55 \times 10^{-13} \text{ J} - 8.199 \times 10^{-14} \text{ J}$$

$$\text{K.E.} \approx 7.351 \times 10^{-14} \text{ J}$$

• **Conversion of energy in electron volt**

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$\text{Total energy} = E (\text{eV}) = \frac{1.55 \times 10^{-13} \text{ J}}{1.602 \times 10^{-19} \text{ J}}$$

$$E (\text{eV}) \approx 9.68 \times 10^5 \text{ eV}$$

$$E (\text{MeV}) \approx .968 \text{ MeV}$$

$$E (\text{MeV}) \approx 0.97 \text{ MeV}$$

$$\text{K.E. (eV)} \approx \frac{7.351 \times 10^{-14} \text{ J}}{1.602 \times 10^{-19} \text{ J}} \approx 4.59 \times 10^5 \text{ eV}$$

$$\boxed{\text{K.E. (MeV)} = 0.459 \text{ MeV}}$$

11.6 At what speed, would the mass of a proton in a particle accelerator be tripled?

Solution: To determine the speed at which the relativistic mass of a proton is tripled in a particle accelerator, we use the equation for relativistic mass:

$$m_{\text{rel}} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Hence, $m_{\text{rel}} = 3 m_0$, so

$$3 m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or

$$3 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Squaring on both sides

$$9 = \frac{1}{1 - \frac{v^2}{c^2}}$$

or $1 - \frac{v^2}{c^2} = \frac{1}{9}$

or $\frac{v^2}{c^2} = 1 - \frac{1}{9} = \frac{8}{9}$

• Taking square root

$$\frac{v}{c} = 0.943$$

Thus, the speed of proton would need to 94.3% of the speed of light ($v = 0.943c$) for its relativistic mass to triple.

11.7 The period of pendulum is measured to be 3 s in an inertial frame of reference. What will be the period measured by an observer in a spaceship with a constant speed of $0.95c$ with respect to the pendulum?

Solution:

Given data:

Period of the pendulum = $t_0 = 3$ s

Speed of spaceship = $v = 0.95c$

Speed of light = $c = 3 \times 10^8 \text{ ms}^{-1}$

To Find:

Period measured by the observer = $t = ?$

Calculations: The measured period of the pendulum will increase due to time dilation, as observer from the spaceship moving at a constant speed of $0.95c$ relative to the pendulum, time dilation is given by the Formula:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Putting the values

$$t = \frac{3s}{\sqrt{1 - \frac{(0.95c)^2}{c^2}}} = \frac{3s}{\sqrt{1 - (0.95)^2}} = 9.6s$$

11.8 Hypothetically, if a ball of mass 0.5 kg is projected with a velocity of $0.9c$, what will be its mass in flight?

Solution:

Given data:

Mass of ball = $m_0 = 0.5$ kg

Velocity of a ball = $v = 0.9c$

Speed of light = $c = 3 \times 10^8 \text{ ms}^{-1}$

To Find:

Relativistic mass of a ball = $m = ?$

Calculation: Using the formula

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Putting the values

$$m = \frac{0.5 \text{ kg}}{\sqrt{1 - \frac{(0.9c)^2}{c^2}}} = \frac{0.5 \text{ kg}}{\sqrt{1 - 0.81}} = \frac{0.5 \text{ kg}}{\sqrt{0.19}} = 1.147 \text{ kg}$$

or

$$m = 1.15 \text{ kg}$$