

OSCILLATIONS

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

Investigate the motion of an oscillator using experimental, analytical and graphical methods.

Show that the motion of mass attached to a spring is simple harmonic.

Understand that the motion of simple pendulum is simple harmonic and to calculate its time period.

Understand and use the terms amplitude, time period, frequency, angular frequency and phase difference.

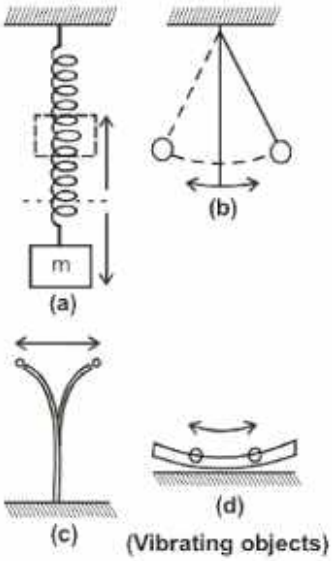
Know and use of solutions in the form of $x = x_o \cos \omega t$ or $y = y_o \sin \omega t$.

Describe the interchange between kinetic and potential energies during SHM.

Describe practical examples of free and forced oscillations.

Q.1 Define oscillatory motion.

Ans. OSCILLATORY MOTION



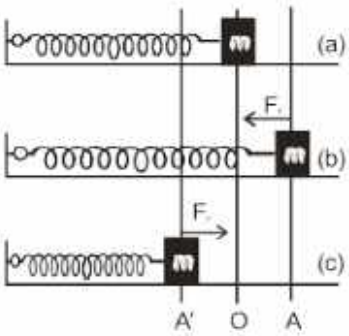
"Such a motion in which a body moves to and fro about a mean position, is called oscillatory or vibratory motion."

The oscillatory motion is called periodic motion when it repeats itself after regular intervals of time.

Examples

1. A mass, suspended from a spring, when pulled down and then released starts oscillating.
2. The bob of a simple pendulum, when displaced from its rest position and released vibrates.
3. A steel ruler clamped at one end to a bench oscillates when the free end is displaced sideways.
4. A steel ball rolling in a curved dish.

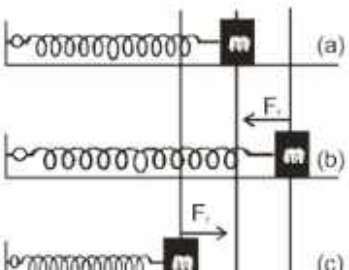
Thus to get oscillations, a body is pulled away from its rest or equilibrium position and then released. The body oscillates due to a restoring force. Under the action of this restoring force, the body accelerates and it overshoots the rest position due to inertia. The restoring force then pulls it back. The restoring force is always directed towards the rest position and so the acceleration is also directed towards the rest or mean position.



It is observed that the vibrating bodies produce waves. There are many phenomena in nature whose explanation requires the understanding of the concepts of vibrations and waves. Although many large structures, such as skyscrapers and bridges, appear to be rigid, they actually vibrate. The architects and the engineers who design and build them, take this fact into account.

Q.2 Explain simple harmonic motion. (OR) Derive an expression for the acceleration of a body vibrating under elastic restoring force.

Ans. SIMPLE HARMONIC MOTION



Consider a body of mass 'm' attached with a spring of spring constant (K). The other end of the spring attached with the fixed support and the spring is placed on the smooth horizontal surface. When the body is pulled towards right from the mean position through a displacement 'x', a deforming force F = Kx is needed. When the body is released, an elastic restoring force which is equal and opposite to the deforming force comes into play and restore the position of the body. But due to inertia, the body over shoot the mean position and

Thus under the action of restoring force and inertia, the body continue its vibratory motion for a long time between these two positions A and A'. The body speeds up while moving towards mean position and slows down while moving away from mean position. This means that the acceleration of the body is always directed towards the mean position. This acceleration can be calculated as follows:

$$F = -Kx \quad \text{(Elastic restoring force)}$$

$$F = ma \quad \text{(Newton's IInd law of motion)}$$

Comparing

$$ma = -Kx$$

$$a = -\frac{Kx}{m}$$

Where $\frac{K}{m}$ is constant

then, $a = \text{Constant}\ (-x)$

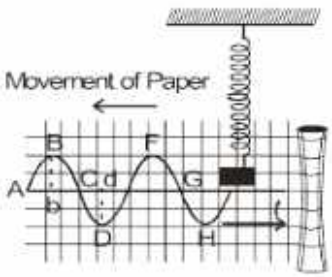
$$a \propto -x$$

Thus such a motion in which acceleration of the body is always directed towards the mean position and is directly proportional to the displacement called simple harmonic motion.

Q.3 Define instantaneous displacement and amplitude.

Ans. **INSTANTANEOUS DISPLACEMENT AND AMPLITUDE OF VIBRATION**

"When a body is vibrating, its displacement from the mean position changes with time. The value of its distance from the mean position at any time is known as its instantaneous displacement." It is zero at the instant when the body is at the mean position and is maximum at the extreme positions.



It is zero when the body is at mean position and is maximum at the extreme positions. It is denoted by x.

Amplitude

"The maximum value of displacement (where Hook's law is valid), on either side of mean position is called amplitude. It is denoted by x_0.

Vibrations

"A complete round trip of body in motion vibrating motion is called vibration."

Time Period

"It is the time required to complete one vibration. It is denoted by 'T'."

Frequency

"Number of vibrations completed in one second is called frequency. It is denoted by 'f'."

$$f = \frac{1}{T}$$

Angular Frequency

If 'T' is the time period of a body executing SHM, its angular frequency is

$$\omega = \frac{2\pi}{T}$$

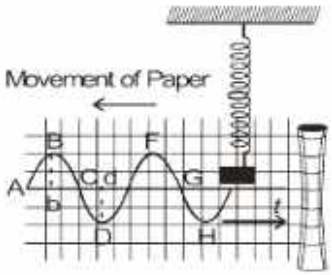
Since $\frac{1}{T} = f$

$\therefore \quad \omega = 2\pi f$

ω	=	Angular frequency
θ	=	ωt
s	=	Vt
$2\pi r$	=	$r\omega T$
2π	=	ω T
ω	=	$\frac{2\pi}{T}$

Wave Form of SHM

The arrangement as shown in figure can be used to record the vibration in displacement with time for a mass-spring system. The strip of paper is moving at a constant speed from right to left, thus providing a time scale on the strip. A pen is attached with the vibrating mass, which records its displacement against time as shown in figure.



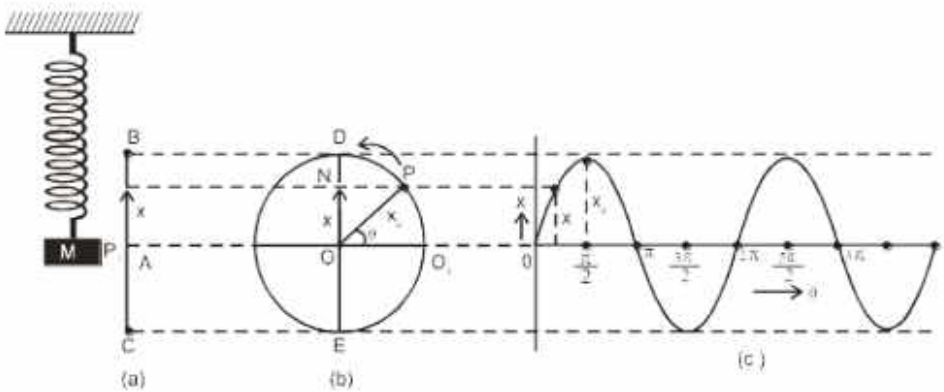
It can be seen that the curve showing the variation of displacement with time is a sine curve. It is usually known as wave-form of SHM.

Q.4 Discuss simple harmonic motion on the bases of uniform circular motion. Also calculate displacement and instantaneous velocity.

Ans. **SIMPLE HARMONIC MOTION AND UNIFORM CIRCULAR MOTION**

Consider a mass 'm', attached with the end of a vertically suspended spring, vibrate simple harmonically with period 'T', frequency 'f' and amplitude 'x_0'. The motion of mass is displayed by the 'P_1' on the line 'BC' with 'A' as mean position, and 'B', 'C' as extreme position, as shown in figure.

Assuming 'A' as the position of the pointer at t = 0, it will move so that it is at 'B', 'A', 'C' and back to 'A' at instants T/4, T/2, 3T/4 and T respectively. This will complete one cycle of vibration with amplitude of vibration being x_0 = AB = AC.

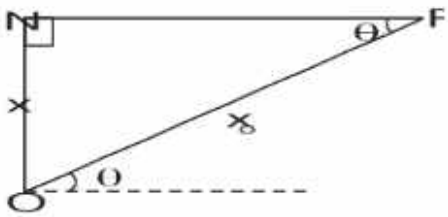


Consider another point 'P' moving on a circle of radius 'x_0' with a uniform angular frequency $\omega = 2\pi/T$, where 'T' is the time period of the vibration of the pointer. It may be noted that the radius of the circle is equal to the amplitude of the pointer's motion.

Consider the motion of the point 'N', the projection of 'P' on the diameter 'DE' drawn parallel to the line of vibration of the pointer as shown in figure. Note that the level of points 'D' and 'E' is the same as the points 'B' and 'C'. As 'P' describes uniform circular motion with a constant angular speed 'ω', 'N' oscillates to and fro on the diameter 'DE' with time period T. Assuming O_1, to be the position of P at t = 0, the position of the point N at the instants 0, T/4, T/2, 3T/4 and T will be at the points O, D, O, E and O respectively. A comparison of the motion of N with that of the pointer P_1 shows that it is a replica of the pointer's motion. Thus the expressions of the displacement, velocity and acceleration for the motion of N also hold good for the pointer P_1, executing SHM.

Displacement

If we count the time t = 0 from the instant when P is passing through O_1, the angle, which the radius 'OP' sweeps out in time 't', is $\theta = \omega t$. The displacement x of the point N at instant will be



$$x = ON = OP \sin \angle QOP$$

Since $OP = x_0$

$$\angle O_1OP = \theta = \omega t$$

$$x = x_0 \sin \omega t$$

This will be also the displacement of the pointer at the instant 't'.

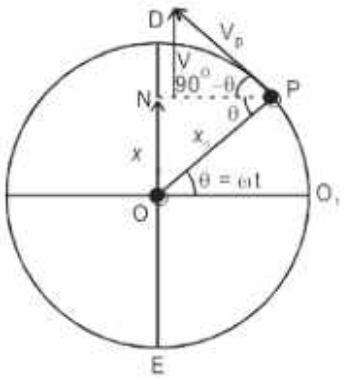
Instantaneous Velocity

The velocity of point 'P' at the instant 't' will be directed along the tangent to the circle at 'P' and its magnitude will be

$$V = r\omega$$

$$V_P = x_0 \omega \quad (\because r = x_0) \qquad \ldots\ldots\ldots (i)$$

As the motion of 'N' on the diameter 'DE' is due to the motion of 'P' on the circle, the velocity of 'N' is actually the component of 'V_P' in a direction parallel to the diameter 'DE'.



From figure this component is

$$\frac{V}{V_P} = \sin (90^\circ - \theta)$$

$$V = V_P \cos \theta \quad \because \sin (90^\circ - \theta) = \cos \theta$$

Putting the value of V_P,

$$\therefore \quad V = x_0 \omega \cos \theta \qquad \ldots\ldots\ldots (ii)$$

The direction of velocity of 'N' depends upon the value of phase angle θ. When 'θ' is between 0° to 90° the direction is from 'O' to 'D', for 'θ' between 90° to 270°, its direction is

$$\frac{\text{Base}}{\text{Hypt}} = \cos\theta$$

$$\frac{NP}{OP} = \cos\theta$$

$$\frac{NP}{x_0} = \cos\theta \qquad \ldots\ldots\ldots \text{(iii)}$$

By using Pythagorean theorem,

$$(H)^2 = (P)^2 + (B)^2$$

$$x_0^2 = x^2 + (NP)^2$$

$$(NP)^2 = x_0^2 - x^2$$

$$NP = \sqrt{x_0^2 - x^2}$$

Putting this value in equation (iii).

$$\therefore \qquad \cos\theta = \frac{\sqrt{x_0^2 - x^2}}{x_0}$$

Putting this value in equation (ii).

$$\therefore \qquad V = x_0\,\omega\,\frac{\sqrt{x_0^2 - x^2}}{x_0}$$

$$\boxed{V = \omega\sqrt{x_0^2 - x^2}}$$

This equation shows that at mean position, where $x = 0$, the velocity is maximum and at the extreme positions where $x = x_0$, the velocity is zero.

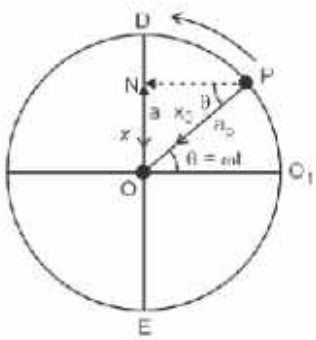
Q.5 Derive the relation for acceleration in terms of ω.

Ans. **ACCELERATION IN TERMS OF ω**

When the point P is moving on a circle, it has an acceleration

$$a_p = x_o\omega^2 \qquad \text{because} \qquad \boxed{a_p = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2}$$

$$a_p = \frac{V_p^2}{r} \quad \ldots \text{(i)} \quad (\because a_c = a_p)$$



At instant t, its direction will be along PO. The acceleration of the point N will be component of the acceleration a_p along the diameter DE on which N moves due to motion of P. The value of this component is

$$a_p \sin\theta = x_o\omega^2 \sin\theta$$

Thus the acceleration of N is

$$a = x_o\omega^2 \sin\theta$$

$$\frac{a}{a_p} = \sin\theta$$

$$a = a_p \sin\theta$$

Putting value of a_p

$$\therefore \quad a = x_0 \omega^2 \sin\theta \qquad \ldots\ldots\ldots \text{(ii)}$$

Considering ΔONP

$$\frac{ON}{OP} = \sin\theta$$

$$\therefore \quad \sin\theta = \frac{x}{x_0}$$

Putting this value in equation (ii)

$$\therefore \quad a = x_0 \omega^2 \left(\frac{x}{x_0}\right)$$

$$\vec{a} = \omega^2 \vec{x}$$

Since acceleration of N is directed towards mean position. So

$$\therefore \quad \vec{a} = -\omega^2 \vec{x}$$

or $$\vec{a} \propto -\vec{x}$$

This shows that the acceleration is proportional to displacement and is directed towards the mean position, which is the characteristics of SHM. Thus N is executing SHM with same amplitude, period and instantaneous displacement as the pointer P_1. This conforms that the motion of N is just a replica, of the pointers motion.

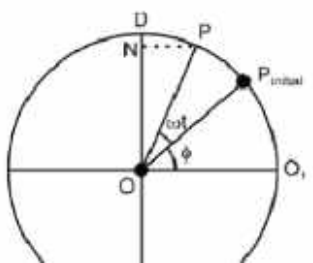
Q.6 What is meant by phase angle?

Ans. PHASE

"The angle $\theta = \omega t$, which specifies the displacement as well as the direction of motion of the point, executing SHM is known as phase."

It is the angle, which the rotating radius 'OP' makes with reference direction 'OO_1' at any instant 't'.

At 't' = 0, the position of rotating radius 'OP' is along 'OO_1' so that 'N' is at its mean position and the displacement is 'O'. In general at t = 0, 'OP' can make any angle 'ϕ' with the reference 'OO_1' as shown in Fig. 1.



In time 't' the radius will rotate by 'ωt'. So, now 'OP' would make an angle ($\omega t + \phi$) with 'OO_1' as the instant 't' and displacement

Now the phase angle is

$$0 = \omega t + \phi$$

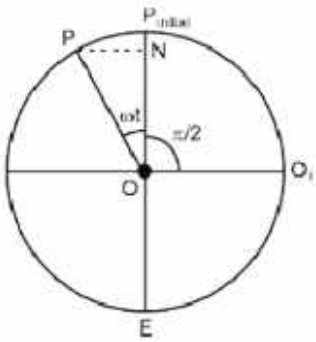
at $t = 0$

$$\therefore \quad \theta = \phi$$

So ϕ is the initial phase. If we take initial phase as 90° then,

$$x = x_0 \sin(\omega t + 90^\circ)$$

$$x = x_0 \cos \omega t$$



This also gives the "displacement" of SHM, but in this case the 'N' is starting its motion from extreme position instead of mean position as shown in figure. Now shadow 'N' is moving along horizontal diameter.

Q.7 Discuss the motion of mass attach with one end of spring placed on horizontal surface. A horizontal mass spring system also derive the expressions for time period, instantaneous displacement and velocity.

Ans. **A HORIZONTAL MASS-SPRING SYSTEM**

Consider the vibrating mass attached to a spring as shown whose acceleration at any instant is given by

$$a = -\frac{k}{m}x \qquad \ldots\ldots\ldots (1)$$

As k and m are constant. We see that the acceleration is directly proportional to the displacement x and its direction is towards the mean position. In case of circular motion, the acceleration is

$$a = -\omega^2 x \qquad \ldots\ldots\ldots (2)$$

Comparing (1) and (2)

$$-\omega^2 x = -\frac{k}{m}x$$

$$\omega^2 = \frac{k}{m}$$

Taking square root

$$\omega = \sqrt{\frac{k}{m}}$$

As $T = \dfrac{2\pi}{\omega}$

Time Period

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Instantaneous Displacement

The instantaneous displacement of mass is given by

$$x = x_0 \sin \omega t$$

Putting value of ω

$$x = x_0 \sin \sqrt{\frac{k}{m}}\, t$$

Instantaneous Velocity

The instantaneous velocity V of mass m is given by

$$V = \omega\sqrt{x_0^2 - x^2}$$

Putting value of ω

$$V = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x^2}$$

$$V = \sqrt{\frac{k}{m}}\sqrt{x_0^2\left(1 - \frac{x^2}{x_0^2}\right)}$$

$$V = x_0\sqrt{\frac{k}{m}}\ \sqrt{1 - \frac{x^2}{x_0^2}} \qquad \ldots\ldots\ldots (3)$$

This equation shows that velocity of mass gets maximum equal to V_0 when $x = 0$ i.e. at mean position.

$$V_0 = \sqrt{\frac{k}{m}}\sqrt{x_0^2(1-0)}$$

$$V_0 = \sqrt{\frac{k}{m}}\sqrt{x_0^2}$$

$$V_0 = \sqrt{\frac{k}{m}}\,(x_0)$$

$$V_0 = x_0\sqrt{\frac{k}{m}}$$

Putting $V_0 = x_0\sqrt{\frac{k}{m}}$ in equation (3)

Note: The formula derived for displacement and velocity are also valid for vertically suspended mass-spring system provided air friction is not considered.

Q.8 What is simple pendulum. Show that its motion is S.H.M. Also derive the expression for time period and frequency.

Ans. **SIMPLE PENDULUM**

It is an arrangement in which a heavy bob is suspended with light and inextensible string. "A simple pendulum consists of a small heavy mass 'm', suspended by a light inextensible string of length '*l*', fixed at its upper end," by a frictionless support as shown in the figure.

Explanation

When such a pendulum is displaced from its mean position through a small angle '0' to position 'B' and released. It starts oscillating to and fro.

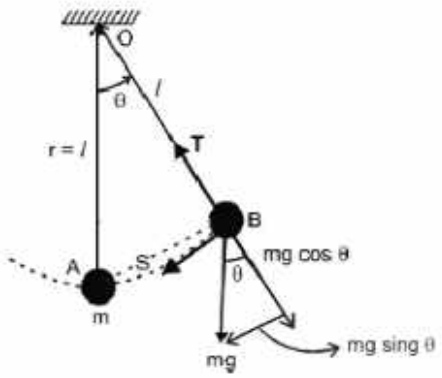
Two forces are acting on bob

(i) tension 'T' along the string.

(ii) Its weight 'mg' downwards. The weight 'mg' can be resolved into two components.

(i) mg sin θ along the tangent at B.

(ii) mg cos θ along the string to balance the tension in the string.



i.e. $T = mg \cos \theta$

Therefore 'mg sin θ' is responsible for the motion of the bob. The restoring force at 'B' will be,

$$F = -mg \sin \theta$$

As $F = ma$

$\therefore$ $ma = -mg \sin \theta$

$$a = -g \sin \theta \quad \ldots\ldots\ldots (1)$$

If θ is very small then $\sin \theta \approx \theta$ (in radian)

Equation (1) becomes

$\therefore$ $$a = -g\theta \quad \ldots\ldots\ldots (2)$$

As θ is very small.

$\therefore$ Arc x = chord x

As $s = r\theta$

Arc length = s

$\because$ $r = l$ and $s = x$

$\therefore$ $x = l\theta$

$$\theta = \frac{x}{l}$$

Putting this value in equation (2).

$\therefore$ $$a = -g\frac{x}{l}$$

$$a = -\frac{g}{l}x \qquad \ldots\ldots\ldots (3)$$

Since, $\frac{g}{l}$ is constant

So, $a = -$ Constant x

$\therefore$ $a \propto -x$

which shows that the acceleration of a simple pendulum is directly proportional towards the displacement and is always directed toward the mean position. So the motion of simple pendulum is a simple harmonic motion.

But for simple harmonic motion

$$a = -\omega^2 x \qquad \ldots\ldots\ldots (4)$$

From (3) and (4)

$$-\omega^2 x = -\frac{g}{l} x$$

$$\omega^2 = \frac{g}{l}$$

Taking square root

$$\therefore \quad \omega = \sqrt{\frac{g}{l}}$$

Time Period

$\therefore$ time period of simple pendulum is

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{g/l}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}}$$

"This equation shows that the time period depends only on the length of the pendulum and the acceleration due to gravity. It is independent of the mass of the bob."

It is also independent of amplitude.

Q. What is the length of the second's pendulum?

Ans. **As, T = 2 sec**

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}}$$

Squaring the both sides

$$T^2 = 4\pi^2\left(\sqrt{\frac{l}{g}}\right)$$

$$l = \frac{T^2 g}{4\pi^2}$$

$$l = \frac{(2)^2 \times 9.8}{4\,(3.14)^2}$$

$$l = \frac{4 \times 9.8}{4 \times 9.86}$$

$$l = \frac{9.8}{9.86}$$

$$= 0.9939 \text{ m}$$

$$l = 99.39 \text{ cm}$$

Second's Pendulum

"A second's pendulum is a pendulum which completes one vibration in two seconds."

Hence time period of such a pendulum is 2 second.

As, $T = 2$ sec

$$\therefore \quad f = \frac{1}{T}$$

$$f = \frac{1}{2}$$

$$f = 0.5 \text{ Hz}$$

Q.9 Explain the energy conservation in S.H.M.

Ans. **ENERGY CONSERVATION IN SHM**

When the mass 'm' is pulled slowly, the spring is stretched by an amount 'x_0' against the elastic restoring force 'F'. It is assumed that the stretching is done slowly, so that acceleration is zero.

According to Hook's law:

$$F = k x$$

$$F = \text{Average force}$$

$$= \frac{0 + k x_0}{2}$$

$$= \frac{1}{2} k x_0$$

Work done in moving the mass 'm' through 'x_0' is

$$W = F d$$

$$W = \frac{1}{2} k x_0 \times x_0$$

$$W = \frac{1}{2} k x_0^2$$

This work appears as elastic P.E. of spring.

Hence, Elastic P.E. $= \frac{1}{2} k x_0^2$

This gives maximum P.E. at the extreme position.

$$P.E_{max} = \frac{1}{2} k x_0^2$$

Energy at Extreme Position

At extreme position,

$$P.E. = \frac{1}{2} k x_0^2$$

$$K.E. = 0$$

$\therefore$ Total energy $= E = P.E. + K.E.$

$$E = \frac{1}{2} k x_0^2 + 0$$

So, $E = \frac{1}{2} k x_0^2$

Energy at any Instant 't'

At any instant 't', if the displacement is 'x', then P.E. at that instant is

$$P.E. = \frac{1}{2} k x^2$$

the velocity at that instant is;

K.E. at that instant is

$$\text{K.E.} = \frac{1}{2} m V^2$$

Putting the value of 'V'.

$$\text{K.E.} = \frac{1}{2} m x_0^2 \left(\frac{k}{m} \left(1 - \frac{x^2}{x_0^2} \right) \right)$$

$$\text{K.E.} = \frac{1}{2} k x_0^2 \left(1 - \frac{x^2}{x_0^2} \right)$$

$$\therefore \quad E = \text{P.E.} + \text{K.E.}$$

$$= \frac{1}{2} k x^2 + \frac{1}{2} k x_0^2 \left(1 - \frac{x^2}{x_0^2} \right)$$

$$= \frac{1}{2} k x^2 + \frac{1}{2} k x_0^2 - \frac{1}{2} k x_0^2 \left(\frac{x^2}{x_0^2} \right)$$

$$= \frac{1}{2} k x^2 + \frac{1}{2} k x_0^2 - \frac{1}{2} k x^2$$

$$E_{total} = \frac{1}{2} k x_0^2$$

Energy at Mean Position

K.E. is maximum at mean position.

i.e., $x = 0$

$$\text{K.E.}_{max} = \frac{1}{2} k x_0^2 \left(1 - \frac{(0)^2}{x_0^2} \right)$$

$$= \frac{1}{2} k x_0^2 \left(1 - \frac{0}{x_0^2} \right)$$

$$= \frac{1}{2} k x_0^2 (1 - 0)$$

$$\text{K.E.}_{max} = \frac{1}{2} k x_0^2$$

Also $\text{P.E.} = 0$

$$\therefore \quad E = \text{P.E.} + \text{K.E.}$$

$$E = 0 + \frac{1}{2} k x_0^2$$

$$= \frac{1}{2} k x_0^2$$

"Hence the total energy of the vibrating mass and spring system is constant." When the K.E of the mass is maximum, the P.E of the spring is zero. On the other hand, when the P.E of the spring is maximum, the K.E of the mean is zero.

Q.10 Define free and forced oscillations.

Ans. FREE AND FORCED OSCILLATIONS

Free Oscillations

"A body is said to be executing free vibrations, when it oscillates without the interference of an external force. the frequency of these free vibrations is known as its natural frequency."

Example

A simple pendulum when slightly displaced from its mean position vibrates freely with its natural frequency that depends only upon the length of the pendulum.

Forced Oscillations

"If a freely oscillating system is subjected to an external force, then forced vibrations will take place."

Example

When the mass of a vibrating pendulum is struck repeatedly, then forced vibrations are produced.

The vibrations of a factory floor caused by running of heavy machinery.

DRIVEN HARMONIC OSCILLATOR

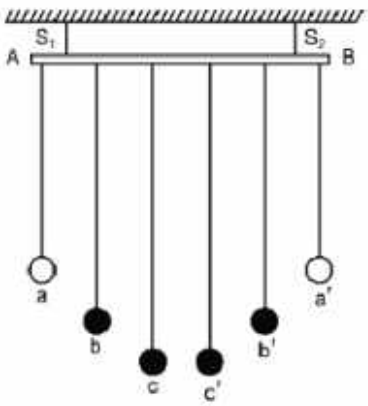
"A physical system under going forced vibrations is known as driven harmonic oscillator."

Q.11 Explain the phenomenon resonance with example.

Ans. RESONANCE

Definition

"The resonance occurs, when the frequency of the applied force is equal to one of the natural frequencies of vibration of the forced or driven harmonic oscillator."



To demonstrate the resonance effect, an apparatus is shown in the Fig. A horizontal rod 'AB' is supported by two strings S_1 and S_2. Three pairs of pendulums aa', bb' and cc' are suspended to this rod. The length of each pair is the same but is different for different pairs. If one of these pendulums, says c, is displaced in a direction perpendicular to the plane of the paper, then its resultant oscillatory motion causes in rod AB a very slight disturbing motion, whose period is the same as that of c'.

other pendulums remain small through out the subsequent motions of c and c′, because their natural periods are not the same as that of the disturbing force due to rod AB.

Example 1

A swing is a good example of mechanical resonance.

Example 2

The column of soldiers, while marching on a bridge of long span are advised to break their steps. Their rhythmic march might set up oscillations of dangerously large amplitude in the bridge structure.

Example 3

Tuning a radio is the best example of electrical resonance.

Example 4

The heating and cooking of food very efficiently and evenly by microwave oven.

Do You Know?

All structures are likely to resonate at one or more frequencies. This can cause problem. It is especially important to test all the components in helicopters and aeroplanes; resonance in an aeroplane's wing or a helicopter rotor could be very dangerous.

Interesting Information



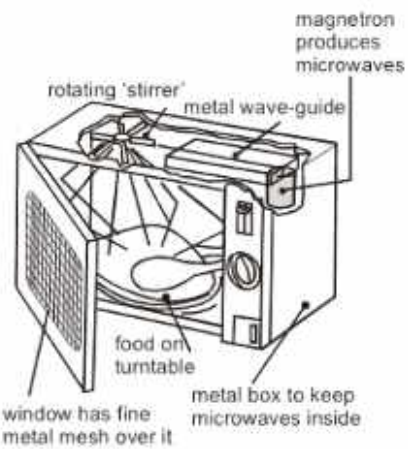
The collapse of Tacoma Narrow Bridge (USA) is suspected to be due to violent resonance oscillations.

Advantages and Disadvantages of Resonance

We come across many examples of resonance in every day life. A swing is a good example of mechanical resonance. It is like a pendulum with a single natural frequency depending on its length. If a series of regular pushes are given to the swing, its motion can be built up enormously. If pushes are given irregularly, the swing will hardly vibrate.

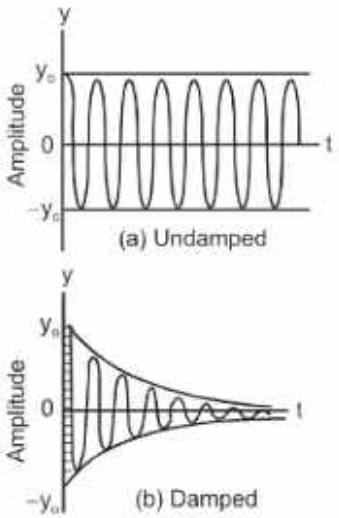
The column of soldiers, while marching on a bridge of long span are advised to break their steps. Their rhythmic march might set up oscillations of dangerously large amplitude in the bridge structure.

Tuning a radio is the best example of electrical resonance. When we turn the knob of a radio, to tune a station, we are changing the natural frequency of the electric circuit of the receiver, to make it equal to the transmission frequency of the radio station. When the two frequencies match, energy absorption is maximum and this is the only station we hear.



Another good example of resonance is the heating and cooking of food very efficiently and evenly by microwave oven figure. The waves produced in this type of oven have a wavelength of 12 cm at a frequency of 2450 MHz. At this frequency, the waves are absorbed due to resonance by water and fat molecules in the food, heating them up and so cooking the food.

This is a common observation that the amplitude of an oscillating simple pendulum decreases gradually with time till it becomes zero. Such oscillations, in which the amplitude decreases steadily with time, are called damped oscillations.



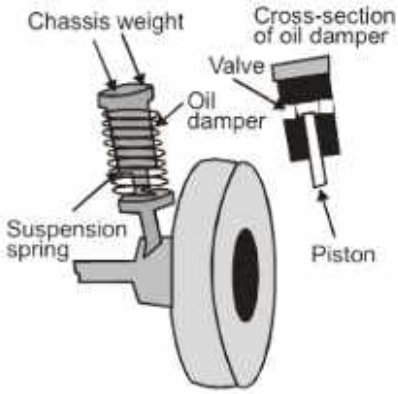
We know from our everyday experience that the motion of any macroscopic system is accompanied by frictional effects. While describing the motion of a simple pendulum, this effects was completely ignored. As the bob of the pendulum moves to and fro, then in addition to the weight of the bob and the tension in the string, bob experiences viscous drag due to its motion through the air. Thus simple harmonic motion is an idealization (Fig. a). In practice, the amplitude of this motion gradually becomes smaller and smaller because of friction and air resistance because the energy of the oscillator is used up in doing work against the resistive forces. Fig. b shows how the amplitude of a damped simple harmonic wave changes, with time as compared with an ideal un-damped harmonic wave. Thus we see that

Damping is the process whereby energy is dissipated from the oscillating system.

An application of damped oscillations is the shock absorber of a car which provides a damping force to prevent excessive oscillation.

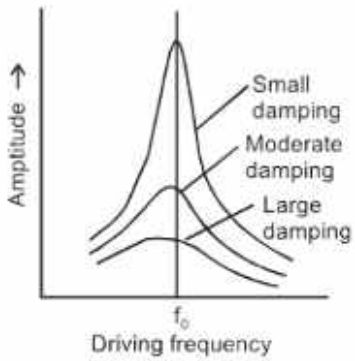
Q.13 What is sharpness of resonance?

Ans. **SHARPNESS OF RESONANCE**



At resonance, the amplitude of vibration becomes very large when damping is small. Thus, damping prevents the amplitude from becoming excessively large. The amplitude decreases rapidly at a frequency slightly different from the resonant frequency. Where as a heavily damped system has a fairly flat resonance curve as is shown in an amplitude frequency graph in figure.

The effect of damping can be observed by attaching a very light mass such as pith ball, and another of the same length carrying a heavy mass such as lead bob of equal size, to a rod as shown in figure (1). They are set into motion (vibrations) by a third pendulum of equal length attached to the same rod. It is observed that the amplitude of lead bob is much greater than that of pith ball. The


Amplitude →
Small damping
Moderate damping
Large damping
f_0
Driving frequency

SOLVED EXAMPLES

EXAMPLE 7.1

A block weighing 4.0 kg extends a spring by 0.16 m from its un–stretched position. The block is removed and a 0.5 kg body is hung from the same spring. If the spring is now stretched and then released, what is the period of vibration?

Data

Mass of block = m = 4 kg

Extension in spring = x = 0.16 m

Mass of new body = m' = 0.50 kg

To Find

Time period = T = ?

SOLUTION

Using: $F = k\,x$

$$\frac{F}{x} = K$$

Putting the values

$$K = \frac{4 \times 9.8}{0.16}$$

$$K = 245 \text{ N/m}$$

Using, $$T = 2\pi\sqrt{\frac{m'}{k}}$$

Putting the values

$$T = 2\pi\sqrt{\frac{0.5}{245}}$$

$$T = 0.285 \text{ sec}$$

Result

Time period = T = 0.285 sec

EXAMPLE 7.2

What should be the length of a simple pendulum whose period is 1.0 second at a place where $g = 9.8\ ms^{-2}$? What is the frequency of such a pendulum?

Data

Value of g = $9.8\ m/s^2$

To Find

Frequency = f = ?

SOLUTION

Using, $T = 2\pi\sqrt{\frac{l}{g}}$

Squaring both sides

$$T^2 = 4\pi^2\left(\frac{l}{g}\right)$$

$$\frac{T^2 g}{4\pi^2} = l$$

Putting the values,

$$l = \frac{(1)^2 (9.8)}{4 \times \pi^2}$$

$$l = 0.25\ m$$

and $f = \frac{1}{T}$

$$f = \frac{1}{l}$$

$$f = 1\ Hz$$

Result

Frequency of vibration = f = 1 Hz

EXAMPLE 7.3

A spring whose spring constant is 80.8 Nm^{-1} vertically supports a mass of 1.0 kg in the rest position. Find the distance by which the mass must be pulled down so that on being released, it may pass the mean position with a velocity of 1.0 ms^{-1}.

Data

Spring constant = K = $80.0\ Nm^{-1}$

Mass = m = 1.0 kg

Maximum velocity = V_{max} = $1.0\ ms^{-1}$

To Find

Distance = x_0 = ?

$$K.E._{max} = \frac{1}{2} k x_0^2$$

$$\frac{1}{2} m V_{max}^2 = \frac{1}{2} k x_0^2$$

$$m V_{max}^2 = k x_0^2$$

Putting the values;

$$(1)(1)^2 = (80)(x_0)^2$$

$$\frac{1}{80} = x_0^2$$

Taking square root

$$x_o = \frac{1}{\sqrt{80}}$$

$$x_o = 0.11 \text{ m}$$

Result

Distance $= x_o = 0.11$ m



OSCILLATIONS

Each question has four possible answers, encircled the correct answer:

1. To and fro motion of a body about its mean position is known as:

(a) Linear motion **(b)** Rotatory motion

(c) Angular motion **(d)** Vibratory motion

2. A motion which repeats itself in equal intervals of time is:

(a) Rotatory motion **(b)** Periodic motion

(c) Oscillatory motion **(d)** Translatory motion

3. In SHM, the acceleration of a body is directly proportional to:

(a) Applied force **(b)** Displacement

(c) Restoring force **(d)** Amplitude

4. The law which derived in SHM by:

(a) Hook's law **(b)** Ampere's law

(c) Dalton's law **(d)** Newton's law

5. The wave form of SHM is:

(a) Square wave **(b)** Sine wave

(c) Cosine wave **(d)** None of these

6. The maximum distance of the vibrating body from the mean position is called:

(a) Displacement **(b)** Time period

(c) Frequency **(d)** Amplitude

7. The number of vibrations completed in one second is called:

(a) Amplitude **(b)** Frequency

(c) Time period **(d)** Revolution

8. The relation between time period and frequency is:

(a) $f = 2\pi T$ **(b)** $f = \frac{1}{2\pi T}$

(c) $f = \frac{T}{2\pi}$ **(d)** $f = \frac{1}{T}$

9. The time taken to complete one revolution is called:

(a) Frequency **(b)** Time period

(c) Time **(d)** Displacement

10. A force 20 N is applied on an elastic spring. If the extension produced in the spring is 10 cm, the spring constant k is:

(a) 20 N/m **(b)** 40 N/m

(c) 10 N/m **(d)** 200 N/m

11. An example of SHM is:

(a) Motion of earth around the sun **(b)** Motion of simple pendulum

(c) Motion of cricket ball **(d)** None of these

12. A body attached to a spring is pulled to a distance of 20 cm. If the spring constant k is 48 N/m then find the force applied:

(a) 5.3 N **(b)** 9.6 N

(c) 96 N **(d)** 192 N

13. The main cause of an oscillatory motion of an elastic spring is:

(a) Mass of the spring **(b)** Weight of spring

(c) K.E of the spring **(d)** Restoring force of the spring

14. The restoring force is directly proportional to the displacement within elastic limit, this is the statement of:

(a) Hook's law **(b)** Newton's law

(c) Meld's law **(d)** Youngs law

15. The unit of frequency is:

(a) m/s **(b)** Hz

(c) N/m **(d)** None of these

16. A particle performing SHM has displacement equal to:

(a) $x_0 \sin \omega t$ **(b)** $x_0 \cos \omega t$

(c) $x_0 \cos^2 \omega t$ **(d)** $x_0 \sin^2 \omega t$

17. Instantaneous acceleration of a system executing SHM is directed:

(a) Towards the mean position

(b) Away from the mean position

(c) Perpendicular to the mean position upward

(d) Perpendicular to the mean position downward

18. The angular speed of mass attached with spring is:

(a) $\omega = \frac{1}{2\pi}\sqrt{\frac{m}{m}}$ **(b)** $\omega = \sqrt{\frac{k}{m}}$

(c) $\omega = \frac{2\pi}{T}$ **(d)** $\omega = 2\pi\sqrt{\frac{m}{k}}$

19. The time period of mass attached with the spring is:

(a) $T = 2\pi\sqrt{\frac{m}{k}}$ **(b)** $T = 2\pi\sqrt{\frac{k}{m}}$

(c) $T = \frac{2\pi}{\omega}$ **(d)** $T = \frac{1}{2\pi}\sqrt{\frac{l}{g}}$

20. The velocity of a body in SHM is maximum at the:

(a) Extreme position **(b)** Between mean and extreme position

(c) Mean position **(d)** Between extreme and mean

21. The velocity of the body is minimum at the:

(a) Mean position **(b)** Extreme position

(c) Between mean and extreme position **(d)** None of the above

22. The acceleration of projection of a point P on the diameter moving on a circle is:

(a) $-\omega^2 x$ **(b)** ωx^2

(c) $-\omega x^2$ **(d)** $\omega^2 x$

23. The time period of simple pendulum depends upon:

(a) Thickness of the thread **(b)** Mass of the pendulum

(c) Length of the pendulum **(d)** Amplitude

24. The time period of simple pendulum is:

(a) 1 second **(b)** 1.5 second

(c) 2 second **(d)** None of these

25. The phase angle $\theta = \omega t$ of a body performing SHM indicates:

(a) Only the magnitude of displacement **(b)** Only the direction of the displacement

(c) Both magnitude and direction **(d)** None of these

26. A body performing SHM has a displacement X given by the equation $X = 30 \sin 50\ t$, what is the frequency of oscillation:

(a) 0.020 Hz **(b)** 0.13 Hz

(c) 8.0 Hz **(d)** 50 Hz

27. In vibratory motion:

(a) P.E remains constant **(b)** K.E remains constant

(c) Total energy remains constant **(d)** None of these

28. When a particle is moving along a circular path, its projection along the diameter executes:

(a) S.H.M **(b)** Angular motion

(c) Linear motion **(d)** Rotatory motion

29. The instantaneous speed of the projection on the diameter for a particle moving in a circle is:

(a) $\omega^2\sqrt{x_o^2 - x^2}$ **(b)** $\omega^2\sqrt{x_o - x}$

(c) $\omega\sqrt{x_o^2 - x^2}$ **(d)** None of these

30. The maximum K.E of the mass attached with spring is given by:

(a) $(K.E)_{max} = \frac{1}{2}kx_0^2$ **(b)** $(K.E)_{max} = \frac{1}{2}Kx_0$

(c) $(K.E)_{max} = \frac{1}{2}Kx^2$ **(d)** $(K.E)_{max} = \frac{1}{2}Kx$

31. The maximum velocity V_0 of the mass attached to the end of an elastic spring is:

(a) $v_0 = x_0\sqrt{\frac{m}{k}}$ **(b)** $v_0 = x\sqrt{\frac{k}{m}}$

(c) $v_0 = x\,.\sqrt{\frac{m}{k}}$ **(d)** $v_0 = x_0\sqrt{\frac{k}{m}}$

32. The total energy of a body executing S.H.M is directly proportional to:

(a) The amplitude **(b)** Square of amplitude

(c) Square root of amplitude **(d)** None of these

33. The total energy of a mass attached with spring is:

(a) Remain constant **(b)** Increased

(c) Decreased **(d)** None of these

34. The force which is responsible for the motion of simple pendulum is:

(a) $-mg\sin\theta$ **(b)** $-mg\cos\theta$

(c) mg **(d)** $mg\tan\theta$

35. The time period of simple pendulum is given as:

(a) $T = 2\pi\sqrt{\frac{l}{g}}$ **(b)** $T = 2\pi\sqrt{\frac{g}{l}}$

(c) $T = \frac{1}{2\pi}\sqrt{\frac{l}{g}}$ **(d)** $T = \frac{1}{\pi}\sqrt{\frac{l}{g}}$

36. The time period of a simple pendulum is directly proportional to the:

(a) $\sqrt{\frac{1}{l}}$ **(b)** $\sqrt{l}$

(c) $\sqrt{g}$ **(d)** $\sqrt{\frac{1}{g}}$

37. If the mass of the bob of simple pendulum is doubled, its time period is:

(a) One half **(b)** Double

(c) Remains constant **(d)** One fourth

38. The time period of a simple pendulum is independent of its:

(a) Mass **(b)** Length

(c) Acceleration due to gravity **(d)** Restoring force

39. The frequency of the second pendulum is:

(a) 0.5 Hz **(b)** 15 Hz

(c) 2 Hz **(d)** 1 Hz

40. The frequency of second pendulum is given by:

(a) $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ **(b)** $f = 2\pi\sqrt{\frac{l}{g}}$

(c) $T = \frac{1}{2\pi}\sqrt{\frac{l}{g}}$ **(d)** None of these

41. ♀ The length of second's pendulum is:

(a) 0.99 m **(b)** 0.6 m

(c) 3 m **(d)** 2 m

42. When the bob of simple pendulum is at extreme position, it has:

(a) Potential energy **(b)** Kinetic energy

(c) Both P.E and K.E **(d)** None of these

43. When the bob of simple pendulum is at mean position then it has:

(a) Potential energy **(b)** Kinetic energy

(c) Both P.E and K.E **(d)** None of these

44. ♀ Total energy of a particle executing SHM at any displacement X is given by:

(a) $T.E = kx$ **(b)** $T.E = \frac{1}{2}kx_0^2$

(c) $T.E = \frac{1}{2}kx_0$ **(d)** $T.E = \frac{1}{2}kx^2$

45. At the centre of the earth, the simple pendulum will:

(a) Vibrate with double time period **(b)** Vibrate with half time period

(c) Not move **(d)** None of these

46. The value of g is calculated from:

(a) $g = \frac{\pi^2 l}{T^2}$ **(b)** $g = \frac{2\pi^2}{lT^2}$

(c) $g = \sqrt{\frac{4\pi^2 l}{T}}$ **(d)** $g = \frac{4\pi^2 l}{T^2}$

47. The length of simple pendulum is calculated from the expression:

(a) $l = \frac{Tg^2}{4\pi^2}$ **(b)** $l = \frac{T^2 g}{4\pi^2}$

(c) $l = \frac{T^2 g^2}{4\pi^2}$ **(d)** $l = \frac{4\pi^2}{T^2 g^2}$

48. The acceleration of the body having SHM depends upon its:

(a) Velocity **(b)** Mass

(c) Displacement from mean position **(d)** None of these

49. The time period of a simple pendulum depends upon:

(a) Mass of bob **(b)** Length of thread

(c) Height of bob **(d)** None of these

50. The time period of second's pendulum is:

(a) 4 seconds **(b)** 1 seconds

(c) 3 seconds **(d)** 2 seconds

51. The time period of the mass attached with spring executing S.H.M is:

(a) $T = 2\pi\sqrt{\frac{m}{k}}$ **(b)** $T = \pi\sqrt{\frac{k}{m}}$

(c) $T = 2\pi\sqrt{mk}$ **(d)** None of these

52. A heating and cooking of food evenly by Microwave oven is an example of:

(a) S.H.M **(b)** Damped oscillations

(c) Resonance **(d)** None of these

53. The angular velocity and angular frequency is related by the relation:

(a) $\omega = 2\pi f$ **(b)** $f = 2\pi\omega$

(c) $f = \frac{2\pi}{\omega}$ **(d)** $\omega = \frac{2\pi}{f}$

54. Work done during horizontal mass spring system by the average force is:

(a) $\frac{1}{2}kx$ **(b)** $\frac{1}{2}F_{avc}\,x$

(c) $\frac{F_{avc}}{x}$ **(d)** $\frac{1}{2}kx^2$

55. The direction of both acceleration and restoring force in SHM is:

(a) Same direction **(b)** Opposite direction

(c) Perpendicular to each other **(d)** None of these

56. If an oscillating body is subjected to an external force then it is said to be executing:

(a) Free oscillations **(b)** Forced oscillations

(c) Mixed oscillations **(d)** Damping

57. If a body vibrates with its natural frequency without the effect of an external force then it is said to be:

(a) Free oscillations **(b)** Forced oscillations

(c) Mixed oscillations **(d)** Oscillations

58. Loud music produced by sounding wooden boards of strings instruments is an example of:

(a) Free oscillations **(b)** Beats

(c) Forced oscillations **(d)** Damped oscillations

59. When damping is small, the amplitude of vibration in resonance will be:

(a) Unchanged **(b)** Large

(c) Small **(d)** None of these

60. How long must be the length of a simple pendulum in order to have a period of one second:

(a) 0.50 m **(b)** 0.25 m

(c) 1 m **(d)** 3 m

61. Shock absorber in automobiles is a practical form of:

(a) SHM **(b)** Damped oscillations

(c) Pascal's law **(d)** None of these

62. Damping effect applied on an aeroplane wing is:

(a) For more speed **(b)** To push upward

(c) To overcome resonance effect **(d)** To overcome gravity

63. The amplitude of the lead ball is much greater than that of the:

(a) Pitch ball **(b)** Iron ball

(c) Plastic ball **(d)** None of these

64. When potential energy of the mass is maximum, the kinetic energy of the spring is:

(a) Zero **(b)** Maximum

(c) Minimum **(d)** None of these

65. If the position of oscillating object is given by the equation $X = \sqrt{2}\cos\left(\frac{\pi}{8}t\right)$ then its displacement after 2 second is:

(a) 3 m **(b)** 2 m

(c) 1 m **(d)** 0 m

66. The projection of a particle moving along a circular path executes:

(a) Simple motion **(b)** Angular motion

(c) Translatory motion **(d)** S.H.M

67. In simple harmonic motion, the acceleration is always directed:

(a) Towards its mean position **(b)** Away from mean position

(c) Along the tangent **(d)** None of these

68. In oscillating motion:

(a) P.E remains constant **(b)** K.E remains constant

(c) Total energy remains constant **(d)** None of these

69. The body oscillates due to:

(a) Gravitational force **(b)** Frictional force

(c) Restoring force **(d)** Deforming force

70. The oscillatory motion which does not repeat after regular interval of time is called:

(a) Periodic motion **(b)** Circular motion

(c) Non-periodic motion **(d)** Orbital motion

71. Acceleration of spring mass system is:

(a) Uniform

(b) Variable due to both change in magnitude and direction

(c) Variation due to change in direction

(d) Variation due to change in magnitude

72. In an isolated spring mass system, total energy is:

(a) Variable **(b)** Constant

(c) Low **(d)** High

73. The formula $T = 2\pi\sqrt{\frac{l}{g}}$ of a simple pendulum holds only if:

(a) Amplitude of the motion should be small **(b)** Length of pendulum is small

(c) Length of pendulum is large **(d)** Mass of pendulum is small

74. Potential energy of spring mass system is stored in:

(a) Spring **(b)** Mass

(c) Length **(d)** None of these

75. A quantity which indicates the state and direction of motion of a vibrating body is known as:

(a) Amplitude **(b)** Displacement

(c) Phase angle **(d)** Time period

76. In microwave oven, heating is produced by phenomenon of:

(a) Harmonic vibration **(b)** Forced vibration

(c) Free vibration **(d)** Resonance

77. The frequency of waves produced in microwave oven is:

(a) 2450 MHz **(b)** 1435 MHz

(c) 1760 MHz **(d)** 2550 MHz

78. The wavelength of the waves produced in microwave oven is:

(a) 8 cm **(b)** 10 cm

(c) 14 cm **(d)** 12 cm

79. The sharpness of the resonance curve depends on:

(a) Loss of energy **(b)** Loss of potential energy

(c) Loss of kinetic energy **(d)** Frictional loss of energy

80. The amplitude of a vibrating body at resonance in vacuum is:

(a) Maximum **(b)** Infinite

(c) Minimum **(d)** None of these

81. If F = 0.4 N and x = 2 cm then k =

(a) 10 N/m **(b)** 20 N/m

(c) 3 N/n **(d)** 30 N/n

82. At what place motion of simple pendulum will be slowest at:

(a) Equator **(b)** Poles

(c) On the surface of earth **(d)** At the centre of the earth

83. If mass attached to spring increases, then its time period:

(a) Increases **(b)** Decreases

(c) No change **(d)** None of these

84. Frequency of a vibratory motion is:

(a) Less than time period **(b)** Equal to time period

(c) Reciprocal of time period **(d)** None of these

85. At which of the following places motion of a simple pendulum is fatest:

(a) Karachi **(b)** Rehim Yar Kan

(c) Lahore **(d)** Islamabad

86. A cheap pendulum clock will:

(a) Lose time in summer and gain time in winter

(b) Gain time in summer and lose time in winter

(c) Keep correct time

(d) None of these

87. A simple pendulum is vibrating in an evacuation chamber it will:

(a) Oscillate forever with the same amplitude frequency

(b) Come to rest eventually

(c) Oscillate with same amplitude with frequency decreasing with time

(d) None of these

88. Total energy of a particle performing S.H.M. is directly proportional to:

(a) Square root of the amplitude **(b)** Amplitude

(c) Reciprocal of amplitude **(d)** Square of amplitude

89. When the mass and speed of a body are doubled, the K.E becomes:

(a) 5 times **(b)** 4 times

(c) 8 times **(d)** 16 times

90. Solar cells are made up from the material called:

(a) Iron **(b)** Oxygen

(c) Carbon **(d)** Silicon

91. At present, the hydroelectric generating capacity in Pakistan amounts to about:

(a) 4000 mega watt **(b)** 3000 mega watt

(c) 5000 mega watt **(d)** None of these

92. The consumption of energy by a 60 watt bulb in 2s in:

(a) 120 J **(b)** 100 J

(c) 90 J **(d)** 0.02 J

93. In SHM, the restoring force is directly proportionate to:

(a) Velocity **(b)** Acceleration

(c) Displacement **(d)** Time period

94. A student made a simple pendulum of time period 1 sec. The string used is of length 1 m, in order to make a simple pendulum of time period 2 sec, he should use a string of length.

(a) 2 m **(b)** 3 m

(c) 4 m **(d)** 4 sec.

95. Time period of second pendulum at moon is:

(a) 1 sec. **(b)** 2 sec.

(c) 3 sec. **(d)** 4 sec.

96. If the period of oscillation of mass M suspended from a spring is 1 sec., then period of mass 4 M will be:

(a) $\frac{1}{4}$ sec. **(b)** $\frac{1}{2}$ sec.

(c) 2 sec. **(d)** 4 sec.

97. A girl is swinging on a swing in the sitting position. How will the period of swing be affected if she stands up?

(a) The period will now be shorter

(b) The period will now be longer

(c) The period will remain unchanged

(d) The period may become longer or shorter depending upon the height of girl

98. A simple harmonic oscillator has time period T. The time taken by it to travel from the extreme position to half the amplitude is:

(a) $\frac{T}{6}$ **(b)** $\frac{T}{4}$

(c) $\frac{T}{8}$ **(d)** $\frac{T}{2}$

99. The frequency of oscillation of a simple pendulum of length L mounted in a cabin that is falling freely under gravity is:

(a) Infinity **(b)** Zero

(c) $\sqrt{\frac{g}{2L}}$ **(d)** $\sqrt{\frac{g}{L}}$

100. A particle executes S.H.M with frequency 'f', the frequency with which its K.E oscillates is:

(a) f **(b)** 2f

(c) 3f **(d)** 4f

101. A body executes S.H.M with an amplitude x_o. Its energy is half kinetic and half potential when displacement is:

(a) $\frac{x_o}{2}$ **(b)** $\frac{x_o}{3}$

(c) $\frac{x_o}{\sqrt{2}}$ **(d)** $\frac{x_o}{2\sqrt{2}}$

102. Total energy of a particle executing S.H.M of amplitude A is proportional to:

(a) A^2 **(b)** A^{-2}

(c) A **(d)** A^{-1}

103. The velocity of a particle undergoing S.H.M is v at mean position. If its amplitude is doubled. The velocity at mean position will be:

(a) v **(b)** 2v

(c) $\sqrt{2}$ times **(d)** 4v

104. The maximum velocity of 1 kg mass attached to a spring constant of 1 Nm^{-1} upto the displacement of 5 cm is:

(a) 1 ms^{-1} **(b)** 0.01 ms^{-1}

(c) 5 ms^{-1} **(d)** 0.05 ms^{-1}

105. A simple pendulum is oscillating in a lift. If the lift starts moving upwards with a uniform acceleration, the period will:

(a) remains unaffected

(b) be shorter

(c) be longer

(d) may be shorter or longer depending on the magnitude of acceleration

106. A body of mass 5 kg in executing S.H.M with amplitude 10 cm. Its maximum velocity is 100 cms^{-1}. Its velocity will be 50 cms^{-1} at a displacement from the mean position equal to:

(a) 5 cm **(b)** $5\sqrt{3}$ cm

(c) 10 cm **(d)** $10\sqrt{3}$ cm

107. A particle of mass 200 g executes S.H.M. The restoring force is provided by a spring of force constant 80 Nm^{-1}. The time period of oscillation is:

(a) 0.31 sec. **(b)** 0.15 sec.

(c) 0.05 sec. **(d)** 0.02 sec.

108. A particle of mass 0.5 kg executes S.H.M its energy is 0.04 J. If time period is π-seconds its amplitude is:

(a) 10 cm **(b)** 15 cm

(c) 20 cm **(d)** 40 cm

109. The two spring mass system, shown in the figure oscillates with a period T. If one spring is used, the time period will be:



(a) $\frac{T}{\sqrt{2}}$ **(b)** $\frac{T}{2}$

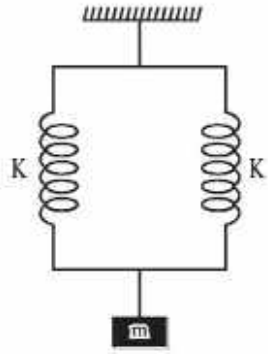
(c) $\sqrt{2}T$ **(d)** 2T

110. A particle executing S.H.M has an amplitude of 6 cm. Its acceleration at a distance of 2 cm from the mean position is 8 cms^{-2}. The maximum speed of the particle is:

(a) 8 cms^{-1} **(b)** 12 cms^{-1}

(c) 16 cms^{-1} **(d)** 24 cms^{-1}

111. The two spring mass system, shown in the figure, oscillates with a period T. If only one spring is used, the time period will be:



(a) $\frac{T}{\sqrt{2}}$ **(b)** $\frac{T}{2}$

(c) $\sqrt{2}T$ **(d)** 2T

112. The equation of displacement of a body executing S.H.M is $x = x_o \cos \omega t$. What is initial phase?

(a) 0° **(b)** 90°

(c) 180° **(d)** 270°

113. Which of the following is an example of damped oscillations?

(a) Mass attached to a spring **(b)** Bob of pendulum

(c) Shock absorber of a car **(d)** All of them

114. The sharpness of the resonance curve of a resonating curve depends on:

(a) Loss of K.E. **(b)** Loss of P.E.

(c) Frictional loss of energy **(d)** Loss of mechanic energy

115. The displacement of vibrating body executing SHM at quarter of the time period is:

(a) Zero **(b)** Maximum

(c) Half of the maximum **(d)** Quarter of the maximum

116. A body moves with SHM and makes a complete oscillations in n second. What is angular frequency?

(a) n rad s^{-1} **(b)** 1/n rad s^{-1}

(c) 2πn rad s^{-1} **(d)** 2π/n rad s^{-1}

117. The product of time period and frequency is equal to:

(a) 3 **(b)** 2

(c) 1 **(d)** Zero

118. Waves transmit from one place to another:

(a) Wavelength **(b)** Amplitude

(c) Mass **(d)** Energy

119. If water is disturbed in a ripple tank periodically, waves one after the other passing through a point are known as:

(a) Matter waves **(b)** Longitudinal waves

(c) Transverse period waves **(d)** Mechanical waves

120. When two identical travelling waves are superposed, the velocity of the resultant waves:

(a) Decreases **(b)** Increases

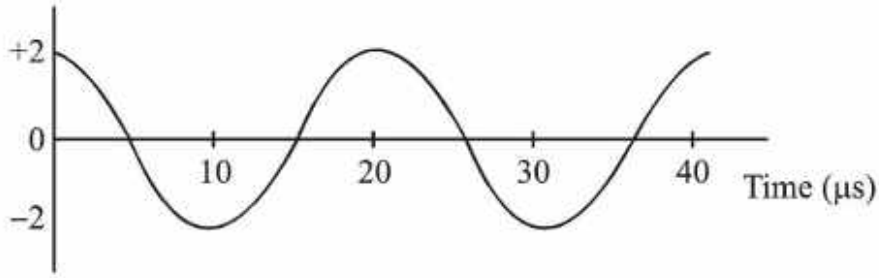
(c) Remains unchanged **(d)** Becomes zero

121. The distance between two consecutive nodes is:

(a) $\lambda/2$ **(b)** $\lambda/4$

(c) λ **(d)** 2λ

122. The diagram below represents the displacement of a particle caused by a progressive wave travelling at a speed 5.0 kms^{-1}.



When is the frequency of the vibration of the particle.

(a) 2.5 KHz **(b)** 5.0 KHz

(c) 25 KHz **(d)** 50 KHz

123. A particle performs simple harmonic motion of amplitude 0.020 m and frequency 2.5 Hz. What is its maximum speed?

(a) 0.008 ms^{-1} **(b)** 0.050 ms^{-1}

(c) 0.125 ms^{-1} **(d)** 0.314 ms^{-1}

124. Vibratory (or oscillatory) motion is always under:

(a) An applied force **(b)** An elastic restoring force and inertia

(c) Periodic force **(d)** Gravitational force

125. The maximum K.E. of the mass attached to an elastic spring is given by:

(a) $(K.E.)_{max} = \frac{kx_o}{2}$ **(b)** $(K.E.)_{max} = \frac{kx_o^2}{2}$

(c) $(K.E.)_{max} = \frac{kx}{2}$ **(d)** $(K.E.)_{max} = \frac{kx^2}{2}$

126. The length of second's pendulum is:

(a) 100 cm **(b)** 99 cm

(c) 99.2 cm **(d)** 98 cm

127. At what place motion of simple pendulum will be slowest:

(a) Poles **(b)** Equator

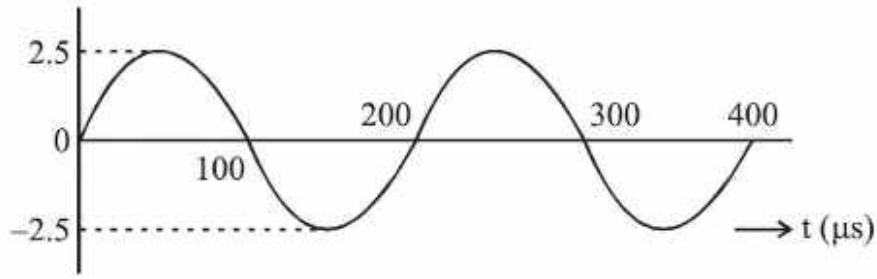
(c) On the surface of earth **(d)** At the centre of the earth

128. The frequency of the second pendulum is:

(a) 1 hertz **(b)** 0.5 hertz

(c) 1.5 hertz **(d)** 2 hertz

129. The diagram below represents the variation with time of pressure at a point in air through which a sound wave is travelling at 340 ms^{-1}.



What is the frequency of the wave?

(a) 1.7 Hz **(b)** 5.0×10^3 Hz

(c) 1.6×10^4 Hz **(d)** 3.1×10^4 Hz

130. A body performing simple harmonic motion has a displacement x given by the equation = 30 sin 50 t, where t is the time in seconds. What is the frequency of oscillations?

(a) 0.020 Hz **(b)** 0.13 Hz

(c) 8.0 Hz **(d)** 30 Hz

ANSWERS

1.	(d)	**2.**	(b)	**3.**	(a)	**4.**	(a)
5.	(b)	**6.**	(d)	**7.**	(b)	**8.**	(d)
9.	(b)	**10.**	(d)	**11.**	(b)	**12.**	(b)
13.	(d)	**14.**	(a)	**15.**	(b)	**16.**	(a)
17.	(a)	**18.**	(b)	**19.**	(a)	**20.**	(c)
21.	(b)	**22.**	(a)	**23.**	(c)	**24.**	(c)
25.	(c)	**26.**	(c)	**27.**	(c)	**28.**	(a)
29.	(c)	**30.**	(a)	**31.**	(d)	**32.**	(b)
33.	(a)	**34.**	(a)	**35.**	(b)	**36.**	(b)
37.	(c)	**38.**	(a)	**39.**	(a)	**40.**	(a)
41.	(a)	**42.**	(a)	**43.**	(b)	**44.**	(b)
45.	(a)	**46.**	(d)	**47.**	(b)	**48.**	(c)
49.	(b)	**50.**	(d)	**51.**	(a)	**52.**	(c)
53.	(a)	**54.**	(b)	**55.**	(b)	**56.**	(b)
57.	(a)	**58.**	(b)	**59.**	(b)	**60.**	(b)
61.	(b)	**62.**	(c)	**63.**	(b)	**64.**	(c)
65.	(c)	**66.**	(d)	**67.**	(a)	**68.**	(c)
69.	(c)	**70.**	(c)	**71.**	(b)	**72.**	(b)
73.	(a)	**74.**	(a)	**75.**	(c)	**76.**	(d)
77.	(a)	**78.**	(d)	**79.**	(d)	**80.**	(b)
81.	(b)	**82.**	(d)	**83.**	(a)	**84.**	(c)
85.	(a)	**86.**	(a)	**87.**	(a)	**88.**	(d)
89.	(c)	**90.**	(d)	**91.**	(b)	**92.**	(a)
93.	(c)	**94.**	(c)	**95.**	(b)	**96.**	(c)
97.	(a)	**98.**	(c)	**99.**	(b)	**100.**	(b)
101.	(c)	**102.**	(a)	**103.**	(b)	**104.**	(d)
105.	(a)	**106.**	(b)	**107.**	(a)	**108.**	(d)
109.	(c)	**110.**	(b)	**111.**	(a)	**112.**	(b)
113.	(d)	**114.**	(c)	**115.**	(b)	**116.**	(c)
117.	(c)	**118.**	(d)	**119.**	(d)	**120.**	(c)
121.	(a)	**122.**	(b)	**123.**	(d)	**124.**	(b)
125.	(b)	**126.**	(c)	**127.**	(d)	**128.**	(b)
129.	(b)	**130.**	(c)				

SHORT QUESTIONS

7.1 Name two characteristics of simple harmonic motion.

Ans. Two characteristics of simple Harmonic Motion are given as:

(i) Acceleration of a vibrating body is directly proportional to the displacement and is always directed towards the mean position i.e.,

$$a \propto -x$$

(ii) Total energy of the particle executing simple harmonic motion remains conserved.

$$E_{total} = K.E + P.E = \text{Constant}$$

(iii) Simple harmonic motion can be represented by function of sine or cosine in the form of equation i.e.,

$$x = x_o \sin(\omega t + \phi)$$

and $$x = x_o \cos(\omega t + \phi)$$

where ϕ is a measure of phase.

7.2 Does frequency depend on amplitude for harmonic oscillators?

Ans. No, the frequency of oscillator is independent of the amplitude of oscillator.

(i) In case of mass-spring system, the frequency of mass is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

According to this relation, the frequency of oscillator depends upon mass and spring constant but it does not depend upon the amplitude of oscillator.

(ii) In case of simple pendulum, the frequency of the harmonic oscillator is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

This relation shows that the frequency does not depend upon the amplitude but it depends upon the length of pendulum and acceleration due to gravity.

7.3 Can we realize an ideal simple pendulum?

Ans. No we cannot realize an ideal simple pendulum because ideal simple pendulum consists of heavy but small mass suspended from a frictionless support by means of an inextensible string. As there conditions are impossible to attain therefore we cannot realize an ideal simple pendulum.

7.4 What is the total distance traveled by an object moving with SHM in a time equal to its period, if its amplitude is A?

Ans. As we know that time period of a simple harmonic motion is the time required to complete one vibration. If A is the amplitude of vibration then the distance travelled by an object in a time equal to its period is 4A.

7.5 What happens to the period of simple pendulum if its length is doubled? What happens if the suspended mass is doubled?

Ans. We know that the time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Let T′ be the time period of simple pendulum when length becomes double.

i.e., $l' = 2l$

Then $$T' = 2\pi\sqrt{\frac{2l}{g}}$$

$$= 2\pi\sqrt{\frac{l}{g}\times\sqrt{2}}$$

$$T' = \sqrt{2}\,T$$

$$= 1.41\,T$$

If the length of pendulum is doubled then its time period increases by 1.41 times the original time period.

Mass: If mass of pendulum is doubled, there is no change in time period because it is independent of mass.

7.6 Does the acceleration of a simple harmonic oscillator remain constant during its motion? Is the acceleration ever zero? Explain.

Ans. No, the acceleration of a simple harmonic oscillator does not remain constant. The acceleration of harmonic oscillator varies with displacement because:

$$a = -\omega^2 x$$

where ω^2 is constant

So $$a = -\text{Constant} \times x$$

$$a \propto -x$$

This shows that acceleration is directly proportional to displacement.

As displacement is changing during motion, therefore acceleration is also changing.

The acceleration will be zero at the mean position i.e., $x = 0$.

So, $$a = -\omega^2(0)$$

$$a = 0$$

7.7 What is meant by phase angle? Does it define angle between maximum displacement and the driving force?

Ans. **Phase Angle:** The angle which specifies the displacement as well as the direction of motion of the point executing SHM is called phase angle.

The phase gives the information about the state of motion of the vibrating point. We can get the waveform of SHM by applying the concept of phase.

The phase angle does not define angle between maximum displacement and the driving force.

7.8 Under what conditions does the addition of two simple harmonic motions produce resultant, which is also simple harmonic?

Ans. In order to produce resultant SHM by the addition of two simple harmonic motions following conditions must be required:

(i) Two SHMs are parallel i.e., in same direction.

(ii) Two SHMs are in phase.

(iii) Two SHMs vibrate with same frequency.

7.9 Show that in SHM, the acceleration is zero when the velocity is greatest and the velocity is zero when the acceleration is greatest?

Ans. As the velocity and acceleration of a SHM are

$$a = -\omega^2 x \quad \text{and} \quad v = \omega\sqrt{x_o^2 - x^2}$$

At the mean position $x = 0$

Therefore;

$$a = -\omega^2(0) \quad \text{and} \quad v = \omega x_o = \text{Maximum value}$$

$$a = 0$$

At the extreme position $x = x_o$. Therefore

$$a = -\omega^2(x_o) \quad \text{and} \quad v = \omega\sqrt{x_o^2 - x_o^2}$$

$$a = -\omega^2 x_o \quad \text{and} \quad v = 0 = \text{Minimum value}$$

Thus it is clear that in SHM, acceleration is zero when then velocity is greatest and the velocity is zero when the acceleration is greatest.

7.10 In relation to SHM, explain the equations:

(i) $y = A \sin(\omega t + \varphi)$ (ii) $a = -\omega^2 x$

Ans. (i) $y = A \sin(\omega t + \phi)$

(ii) $a = -\omega^2 x$

Here, y = Instantaneous displacement

A = Amplitude

ϕ = Initial phase

ωt = Angle subtended in time 't'

This equation represents instantaneous the acceleration of an object executing SHM in which "a" represents acceleration, "ω" is the angular frequency and x represents its instantaneous displacement.

7.11 Explain the relation between total energy, potential energy and kinetic energy for body oscillating with SHM.

Ans. According to law of conservation of energy, the total energy of a body executing SHM remains constant. The K.E is maximum at the mean position and zero at the extreme position while the potential energy is maximum at the extreme position and zero at the mean position.

7.12 Describe some common phenomena in which resonance plays an important role.

Ans. The phenomenon resonance plays a very important role in:

(i) Musical instrument.

(ii) Producing electrical resonance in radio set with transmission of a particular radio frequency.

(iii) **In microwave oven:** The waves produced in this type of oven have a wavelength of 12 cm at a frequency of 2450 MHz.

7.13 If a mass spring system is hung vertically and set into oscillations, why does the motion eventually stop?

Ans. If a mass spring system is hung vertically and set into oscillations, the motion eventually stops due to air resistance and friction. Because of these frictional forces, energy is dissipated into heat and the system does not oscillate.

PROBLEMS WITH SOLUTIONS

PROBLEM 7.1

A 100.0 g body hung on a spring elongates the spring by 4.0 cm. When a certain object is hung on the spring and set vibrating, its period is 0.568 s. What is the mass of the object pulling the spring?

Data

Mass of body $= m_1 = 100$ g
$= 0.1$ kg

Extension in the string $= x = 4.0$ cm
$= 0.04$ m

Time period $= T = 0.568$ sec.

Amplitude $= x_o = 10$ cm
$= 0.1$ m

To Find

Mass of object pulling the spring $= m_2 = ?$

SOLUTION

By using the formula

$$T = 2\pi\sqrt{\frac{m_2}{K}}$$

But for the value of K

$$F = Kx \quad \text{and} \quad F = m_1 g$$

$$K = \frac{F}{x} = \frac{m_1 g}{x}$$

$$K = \frac{0.1 \times 9.8}{0.04}$$

$$= 24.5 \text{ N/m}$$

Therefore; $$T = 2\pi\sqrt{\frac{m_2}{K}}$$

Squaring

$$T^2 = 4\pi^2 \frac{m_2}{K}$$

$$\boxed{m_2 = \frac{T^2 \times K}{4\pi^2}}$$

Putting the values

$$m_2 = \frac{(0.568)^2 \times 24.5}{4(3.14)^2}$$

$$m_2 = 0.200 \text{ kg}$$

$$m_2 = 200 \text{ g}$$

Result

Mass of object pulling the spring = m_2 = 200 g

PROBLEM 7.2

A load of 15.0 g elongates a spring by 2.00 cm. If body of mass 294 g is attached to the spring and is set into vibration with an amplitude of 10.0 cm, what will be its (i) period (ii) spring constant (iii) maximum speed of its vibration.

Data

Mass elongates = m_1 = 15.0 g = 0.015 kg

Extension in the spring = x = 2.00 cm = 0.02 m

Mass attached with spring = m_2 = 294 g = 0.294 kg

To Find

(i) Time period = T = ?

(ii) Spring constant = K = ?

(iii) Maximum speed = v_o = ?

SOLUTION

(i) For time period

$$T = 2\pi\sqrt{\frac{m_2}{K}}$$

But $$K = \frac{m_1 g}{x}$$

$$= \frac{0.015 \times 9.8}{0.02}$$

$$= 7.35 \text{ N/m}$$

Therefore; $$T = 2(3.14)\sqrt{\frac{0.294}{7.35}}$$

$$T = 1.256 \text{ sec.}$$

(ii) Spring constant = K $= \frac{F}{x} = \frac{m_1 g}{x}$

$$K = \frac{0.015 \times 9.8}{0.02}$$

$$= 7.35 \text{ N/m}$$

(iii) For maximum speed

$$v_o = x_o \sqrt{\frac{K}{m_2}}$$

$$= 0.1 \sqrt{\frac{7.35}{0.294}}$$

$$= 0.5 \text{ m/s}$$

$$v_o = 50 \text{ cm/s}$$

Result

(i) Time period = T = 1.256 sec.

(ii) Spring constant = K = 7.35 N/m

(iii) Maximum speed = v_o = 50 cm/s

PROBLEM 7.3

An 8.0 kg body executes SHM with amplitude 30 cm. The restoring force is 60 N when the displacement is 30 cm. Find

(i) Period

(ii) Acceleration, speed, kinetic energy and potential energy when the displacement is 12 cm.

Data

Mass of body = m = 8.0 kg

Amplitude = x_o = 30 cm

= 0.3 m

Restoring force = F = 60 N

Displacement = x = 30 cm

= 0.3 m

To Find

(i) Time period = T = ?

(ii) Acceleration = a = ?

Speed = v = ?

Kinetic energy = K.E = ?

Potential energy = P.E = ?

When

Displacement = x_1 = 12 cm

= 0.12 m

SOLUTION

(i) For time period

$$T = 2\pi\sqrt{\frac{m}{K}}$$

As $K = \frac{F}{x}$

$$K = \frac{60}{0.3}$$

$$= 200 \text{ N/m}$$

So $$T = 2(3.14)\sqrt{\frac{8.0}{200}}$$

$$= 1.256 \text{ sec.}$$

$$T = 1.3 \text{ sec.}$$

(ii) For acceleration

$$a = -\frac{K}{m}x$$

$$= -\frac{200}{8.0} \times 0.12$$

$$a = 3 \text{ m/s}^2$$

For speed

$$v = \omega\sqrt{x_o^2 - x_1^2}$$

But $$\omega = \frac{2\pi}{T}$$

$$= \frac{2(3.14)}{1.3}$$

$$= 4.83 \text{ rad/s}$$

Therefore; $$v = 4.83\sqrt{(0.3)^2 - (0.12)^2}$$

$$v = 1.33 \text{ m/s}$$

For kinetic energy

$$K.E = \frac{1}{2}K(x_o^2 - x_1^2)$$

$$= \frac{1}{2} \times 200\,(0.3^2 - 0.12^2)$$

$$= 100(0.0756)$$

$$= 7.56 \text{ J}$$

For potential energy

$$P.E = \frac{1}{2}Kx_1^2$$

$$= \frac{1}{2} \times 200(0.12)^2$$

$$= 1.44 \text{ J}$$

Result

(i) Time period = T = 1.3 sec.

(ii) Acceleration = a = 3 m/s^2

Speed = v = 1.33 m/s

Kinetic energy = K.E = 7.56 J

Potential energy = P.E = 1.44 J

PROBLEM 7.4

A block of mass 4.0 kg is dropped from a height of 0.80 m on to a spring of spring constant K = 1960 Nm^{-1}. Find the maximum distance through which the spring will be compressed.

Data

Mass of block = m = 4.0 kg

Height = h = 0.80 m

Spring constant = K = 1960 N/m

To Find

Maximum distance = x_o = ?

SOLUTION

By formula

$$P.E = \frac{1}{2}Kx_o^2$$

$$\boxed{x_o^2 = \frac{2P.E}{K}} \qquad \ldots\ldots \text{(i)}$$

But $P.E = mgh$

$$= 4.0 \times 9.8 \times 0.80$$

$$= 31.36 \text{ J}$$

Putting in eq. (i)

So $$x_o^2 = \frac{2 \times 31.36}{1960}$$

$$x_o = 0.178$$

$$= 0.18 \text{ m}$$

Result

Maximum distance = x_o = 0.18 m

= 18 cm

PROBLEM 7.5

A simple pendulum is 50.0 cm long. What will be its frequency of vibration at a place where $g = 9.8\ ms^{-2}$?

Data

Length of simple pendulum = l = 50.0 cm
= 0.50 m

Value of g = $9.8\ m/s^2$

To Find

Frequency of vibration = f = ?

SOLUTION

By formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T = 2(3.14)\sqrt{\frac{0.5}{9.8}}$$

$$T = 1.41 \text{ sec.}$$

But $$f = \frac{1}{T}$$

$$= \frac{1}{1.41}$$

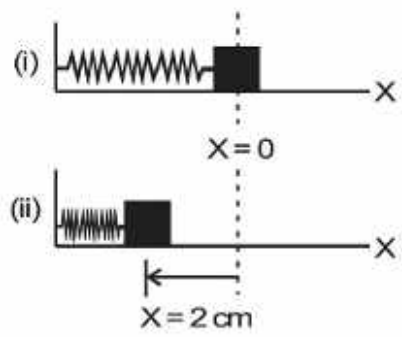
$$f = 0.70 \text{ Hz}$$

Result

Frequency of vibration = f = 0.70 Hz

PROBLEM 7.6

A block of mass 1.6 kg is attached to a spring with spring constant 1000 Nm^{-1}, as shown in Fig. The spring is compressed through a distance of 2.0 cm and the block is released from rest. Calculate the velocity of the block as it passes through the equilibrium position, x = 0, if the surface is frictionless.



Data

Mass of block = m = 1.6 kg

Spring constant = K = 1000 N/m

Maximum displacement = x_o = 2.0 cm
= 0.02 m

To Find

Velocity of the block = v_o = ?

SOLUTION

By formula

$$v_o = x_o\sqrt{\frac{K}{m}}$$

$$= 0.02\sqrt{\frac{100}{1.6}}$$

$$v_o = 0.50 \text{ m/s}$$

Result

Velocity of block $= v_o = 0.50$ m/s

PROBLEM 7.7

A car of mass 1300 kg is constructed using a frame supported by four springs. Each spring has a spring constant 20,000 Nm^{-1}. If two people riding in the car have a combined mass of 160 kg, find the frequency of vibration of the car, when it is driven over a pot hole in the road. Assume the weight is evenly distributed.

Data

Mass of car $= m_1 = 1300$ kg

Spring constant for each $= K = 20{,}000$ N/m

Mass of persons $= m_2 = 160$ kg

Spring constant for 4 springs $= 4 \times 20{,}000$

$= 80{,}000$ N/m

To Find

Frequency of vibration $= f = ?$

SOLUTION

By formula

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

But $m = m_1 + m_2$

$= 1300 + 160$

$= 1460$ kg

Therefore $f = \frac{1}{2(3.14)}\sqrt{\frac{80000}{1460}}$

$f = 1.18$ Hz

Result

Frequency of vibration $= f = 1.18$ Hz

PROBLEM 7.8

Find the amplitude, frequency and period of an object vibrating at the end of a spring, if the equation for its position, as a function of time, is

$$x = 0.25 \cos\left(\frac{\pi}{8}\right) t$$

What is the displacement of the object after 2.0 s?

Data

The given equation is

$$x = 0.25 \cos\left(\frac{\pi}{8}\right) t$$

Time $= t = 2.0$ sec.

To Find

Amplitude $= x_o = ?$

Frequency $= f = ?$

Time period $= T = ?$

SOLUTION

As we know that the equation for SHM is

$$x = x_o \cos \omega t \qquad \ldots\ldots \text{(i)}$$

and the given equation is

$$x = 0.25 \cos\left(\frac{\pi}{8}\right) t \qquad \ldots\ldots \text{(ii)}$$

Comparing the eq. (i) and (ii)

$$x_o = 0.25$$

So

Amplitude $= x_o = 0.25$ m

$$\omega = \frac{\pi}{8}$$

But $$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\frac{\pi}{8}}$$

$$T = 2\pi \times \frac{8}{\pi}$$

Time period $= T = 16$ sec.

$$\text{Frequency} = f = \frac{1}{T}$$

$$f = \frac{1}{16}\,\text{Hz}$$

For displacement after 2 sec. is

$$x = 0.25 \cos \frac{\pi}{8} t$$

$$= 0.25 \cos \left(\frac{\pi}{8} \times 2\right)$$

$$= 0.25 \cos \left(\frac{\pi}{4}\right)$$

$$= 0.25 \times 0.707$$

$$= 0.18\ \text{m}$$

Result

Amplitude $= x_o = 0.25$ m

Time period $= T = 16$ sec.

Frequency $= f = \frac{1}{16}$ Hz

Displacement $= x = 0.18$ m