

VECTORS AND EQUILIBRIUM

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

Understand the definition of scalars and vectors.

Understand and use rectangular coordinate system.

Understand the idea of unit vector, null vector and position vector.

Represent a vector as two perpendicular components (rectangular components).

Understand multiplication of vectors and solve problems.

Define the moment of force or torque.

Appreciate the use of the torque due to a force.

Appreciate the applications of the principle of moments.

Q.1 Define scalars and vectors. Give examples.

Ans. **SCALAR QUANTITIES**

Those physical quantities which are completely described by magnitude with proper units are called scalar quantities. e.g. time, current, speed etc. Scalars are added, subtracted, divided and multiplied by ordinary arithmetic rules.

VECTOR QUANTITIES

Those physical quantities which are completely described by magnitude with proper units as well as direction are called vector quantities. e.g. force, torque etc. Vectors are not added, subtracted, divided and multiplied by ordinary arithmetic rules but it can be used as vector addition, vector multiplication and vector subtraction.

Q.2 Describe how a vector quantity is represented?

Ans. **REPRESENTATION OF A VECTOR**

A vector is usually represented by a bold face letter that is **A** or by a letter with an arrow drawn above or below it that is $\vec{A}$ or $\underline{A}$.

The magnitude of a vector is denoted by $|\vec{A}|$ (modulus) or A.

(ii) By Graphically



A vector is represented graphically by a directed line segment with an arrow-head in the direction of the vector. The length of the line segment, according to the suitable scale, corresponds to the magnitude of the vector. The length of the line is called its magnitude and arrow head indicates the direction.

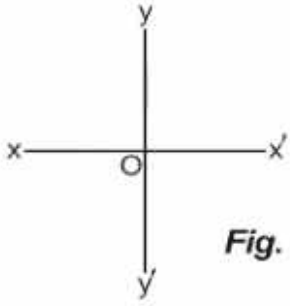


Fig. (i)

Q.3 Describe the rectangular coordinate system.

Ans. **RECTANGULAR COORDINATE SYSTEM**

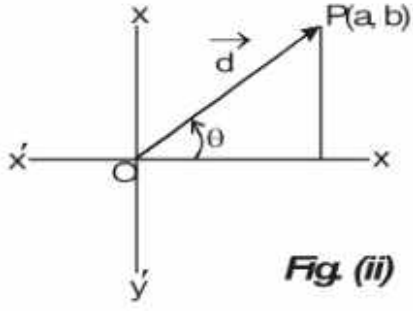
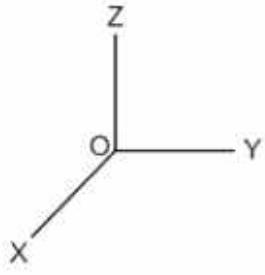


Fig. (ii)

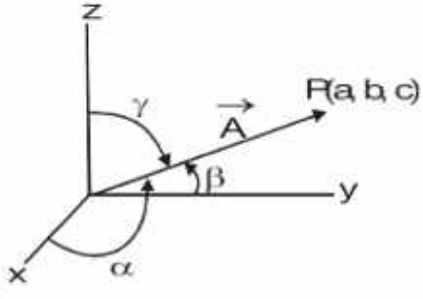
Two reference lines drawn at right angles as shown in figure. They are known as coordinate axes and their point of intersection is known as origin. This system of coordinate axes is called Cartesian or rectangular coordinate system.

One of the lines is named as x-axis, and the other the y-axis. Usually the x-axis is taken as the horizontal axis, with the positive direction to the right, and the y-axis as the vertical axis with the positive direction upward.



The direction of a vector in a plane is denoted by the angle which representative line of the vector makes with positive x-axis in the anti-clockwise direction as shown in Fig. (ii).

The direction of a vector in space requires another axis which is at right angle to both x and y axes, as shown in figure which is called z-axis.



The direction of a vector in space is specified by the three angles which the representative line of the vector makes with x, y and z axes respectively as shown in figure. The point P of a vector A is thus denoted by three coordinates (a, b, c).

Q.4 Explain the following terms:

(iii) Vector subtraction ***(iv) Multiplication of a vector***

(v) Unit vector ***(vi) Null vector***

(vii) Equal vector

Ans. ADDITION OF VECTORS

Consider two vectors $\vec{A}$ and $\vec{B}$ as shown in Fig. (i).

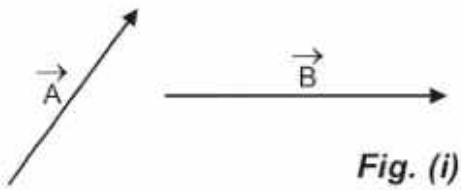


Fig. (i)

Their sum is obtained by drawing, their representative lines in such a way that tail of vector $\vec{B}$ coincides with the head of vector $\vec{A}$. Now if we join the tail of $\vec{A}$ to the head of $\vec{B}$ as shown in Fig. (ii).

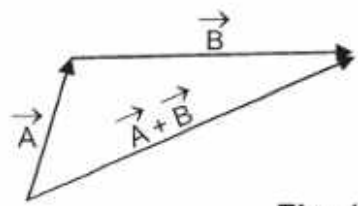


Fig. (ii)

If we join the tail of first vector with the head of last vector, it will represent the vector sum $\vec{A} + \vec{B}$ in magnitude and direction. This is known as head to tail rule of vector addition. Similarly the sum $\vec{B} + \vec{A}$ is represented by doted lines as shown in Fig. (iii). It is clear from Fig. (iii) that

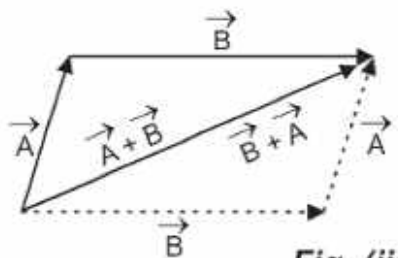


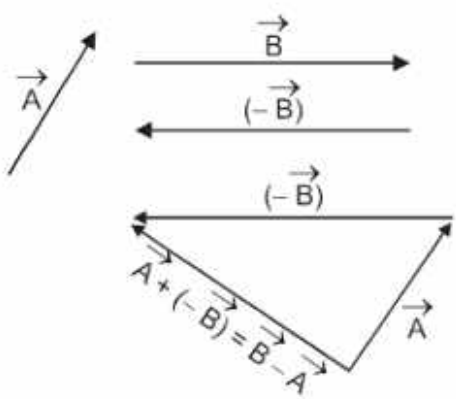
Fig. (iii)

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

So the vector addition is said to be commutative. It means that when vectors are added, the result is same for any order of addition.

RESULTANT VECTOR

The resultant of number of vectors is that single vector which would have the same effect as all the original vectors taken together.

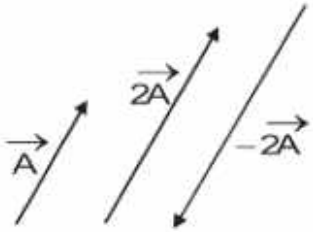


VECTOR SUBTRACTION

The subtraction of a vector is equal to the addition of the same vector with its direction reversed. To subtract vector $\vec{B}$ from vector $\vec{A}$ reverse the direction of $\vec{B}$ and add it to $\vec{A}$ as shown in figure.

MULTIPLICATION OF A VECTOR BY A SCALAR

The product of a vector $\vec{A}$ and a number $n > 0$ is defined to be a new vector $n\vec{A}$ having the same direction as $\vec{A}$ but a magnitude n times the magnitude of $\vec{A}$ as shown in figure.



If the vector is multiplied by a negative number, then its direction is reversed.

For Example

When velocity (vector) is multiplied by scalar i.e. mass m, the product is a new vector quantity called momentum having same dimensions as product of mass and velocity.

UNIT VECTOR

A unit vector in a given direction is a vector with magnitude one in that direction.

It is used to represent the direction of a vector.

A unit vector in the direction of $\vec{A}$ is written as $\hat{A}$.

As $\vec{A} = A\hat{A}$

$$\hat{A} = \frac{\vec{A}}{A}$$

The direction along x, y and z axes is represented by unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ respectively. Two of the more frequently used unit vectors are:

(i) The vector $\hat{r}$ which represents the direction of the vector r as shown.

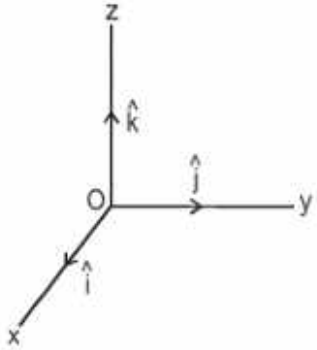
(ii) The vector $\hat{n}$ which represents the direction of a normal drawn on a surface as shown.

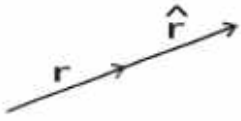
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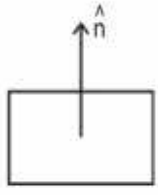
$$|\hat{A}| = \frac{|\vec{A}|}{A}$$

$$= \frac{A}{A}$$

$$= 1$$







NULL VECTORS

It is a vector of zero magnitude and arbitrary direction.

For example, sum of a vector and its negative vector is a null vector.

$$\vec{A} + (-\vec{A}) = \vec{0}$$

EQUAL VECTORS

Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal if they have the same magnitude and direction, regardless of the position of their initial points.

This means that parallel vectors of the same magnitude are equal to each other.

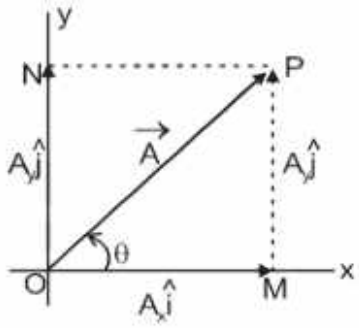
Q.5 Define rectangular components of a vector. How will you find resultant vector of the rectangular components are given?

Ans. RECTANGULAR COMPONENTS OF A VECTOR

The splitting up of a vector into its parts is called the resolution of a vector. Usually a vector can be resolved into two parts called the components of a vector. (OR) A component of a vector is its

Explanation

Consider a vector $\vec{A}$ represented by $\vec{OP}$ making an angle θ with x-axis. Draw projection OM of vector $\vec{OP}$ on x-axis and projection ON of vector OP on y-axis as shown in figure Projection OM being along x-direction is represented by $A_x \hat{i}$ and projection ON = MP along y-direction is represented by $A_y \hat{j}$.



By head to tail rule

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

Here $A_x\hat{i}$ and $A_y\hat{j}$ are the components of $\vec{A}$. Since these components are at right angle to each other, hence, are called rectangular components.

Considering the right angle triangle OPM

$$\frac{\text{Base}}{\text{Hypotenuse}} = \cos\theta \quad \Rightarrow \quad \cos\theta = \frac{OM}{OP}$$

$$\frac{A_x}{A} = \cos\theta$$

$$A_x = A\cos\theta$$

It is the magnitude of x-component of vector $\vec{A}$.

Now

$$\frac{\text{Perpendicular}}{\text{Hypotenuse}} = \sin\theta$$

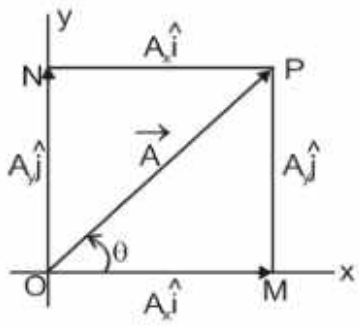
$$\frac{A_y}{A} = \sin\theta$$

$$A_y = A\sin\theta$$

It is the magnitude of y-component of $\vec{A}$.

Determination of a Vector from its Rectangular Components

If rectangular components $A_x \hat{i}$ and $A_y \hat{j}$ vectors are given. Then we can find the magnitude and direction of the vector.



Using Pythagorean theorem.

$$(H)^2 = (B)^2 + (P)^2$$

$$A^2 = A_x^2 + A_y^2$$

The magnitude of resultant

$$A = \sqrt{A_x^2 + A_y^2}$$

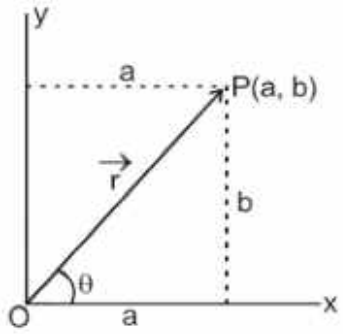
The direction of resultant

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Q.6 Define position vector.

Ans. **POSITION VECTOR**

It is a vector that describes the location of a particle with respect to the origin. It is represented by a straight line drawn in such a way that its tail coincides with the origin and the head with point P (a, b) as shown in figure. The projection of position vector r on the x and y-axes are the coordinates a and b and they are rectangular components of the vector r.



$$\vec{r} = a\hat{i} + b\hat{j}$$

and $$r = \sqrt{a^2 + b^2}$$

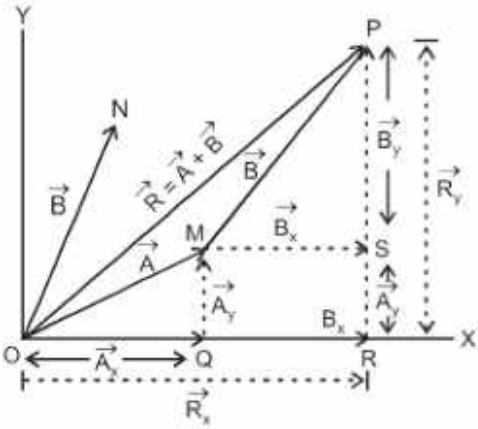
In three dimension $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$

$$r = \sqrt{a^2 + b^2 + c^2}$$

Q.7 Explain the addition of vectors by rectangular components.

Ans. **VECTOR ADDITION BY RECTANGULAR COMPONENTS**

Consider two vectors $\vec{A}$ and $\vec{B}$ which are represented by two directed lines OM and ON respectively. The $\vec{B}$ is added to $\vec{A}$ by head to tail rule as shown in figure. The resultant vector $\vec{R} = \vec{A} + \vec{B}$ is given in direction and magnitude by the $\overrightarrow{OP}$.



Resolving $\vec{A}$, $\vec{B}$ and $\vec{R}$ vector into their rectangular components.

In the figure, $\vec{A}_x$, $\vec{B}_x$ and $\vec{R}_x$ are the x–components of the vectors $\vec{A}$, $\vec{B}$, $\vec{R}$ and their magnitude are given by the lines OQ, MS, OR respectively.

From figure:

$$OR = OQ + QR$$

$$\therefore \quad QR = MS$$

$$OR = OQ + MS$$

$$\vec{R}_x = R_x \hat{i} = A_x \hat{i} + B_x \hat{i}$$

$$R_x \hat{i} = (A_x + B_x) \hat{i}$$

Equation (i) means that the sum of the magnitude of x–components of two vectors, which are to be added is equal to the x–component of the resultant.

Similarly $RP = RS + SP$

$\therefore$ $RS = QM$

$\therefore$ $RP = QM + SP$

$$R_y = A_y + B_y \quad \ldots\ldots\ldots \text{(ii)}$$

$$\vec{R}_y = R_y \hat{j} = (A_y + B_y) \hat{j}$$

Equation (ii) means that the sum of the magnitudes of y-components of two vectors is equal to the magnitude of y-component of the resultant.

Since $R_x \hat{i}$ and $R_y \hat{j}$ are rectangular components of the $\vec{R}$.

$$\vec{R} = R_x \hat{i} + R_y \hat{j} \quad \ldots\ldots\ldots \text{(iii)}$$

Putting the values of R_x and R_y.

$\therefore$ $$\vec{R} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}$$

Its magnitude is

$$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

For direction

$$\tan\theta = \frac{R_y}{R_x}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)$$

Do You Know?



The Chinese acrobats in this incredible balancing act are in equilibrium.

Similarly for any number of coplanar vectors $\vec{A}, \vec{B}, \vec{C}$ .....

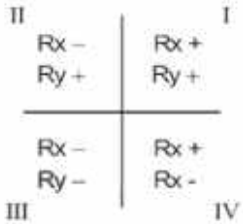
$$R = \sqrt{(A_x + B_x + C_x + \ldots)^2 + (A_y + B_y + C_y + \ldots)^2}$$

$$\theta = \tan^{-1}\left(\frac{A_y + B_y + C_y + \ldots}{A_x + B_x + C_x + \ldots}\right)$$

Important Steps for Vector Addition by Rectangular Components

(ii) Find x-components , R_x of the resultant vector by adding the x–components of all the vectors.

(iii) Find y-components , R_y of the resultant vector by adding the y–components of all the vectors.



(iv) Find magnitude of resultant vector $\vec{R}$ using

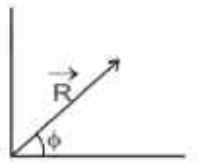
$$R = \sqrt{R_x^2 + R_y^2}$$

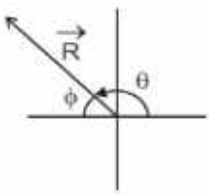
(v) Find the direction of resultant vector by using

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Where θ is the angle, which resultant vector makes with positive x-axis. Irrespective of sign of R_x and R_y determine the value of

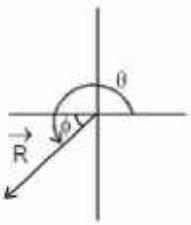
$$\phi = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$





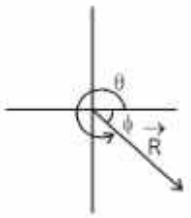
(i) If both R_x, R_y are + ve then $\vec{R}$ lies in first quadrant.

$\therefore \quad \theta = \phi$



(ii) If R_x is –ve and R_y is +ve then $\vec{R}$ lies in second quadrant.

$\therefore \quad \theta = 180^\circ - \phi$



(iii) If R_x, R_y both are –ve then $\vec{R}$ lies in third quadrant.

$\therefore \quad \theta = 180^\circ + \phi$

(iv) If R_x is +ve and R_y is –ve then $\vec{R}$ lies in fourth quadrant.

$\therefore \quad \theta = 360^\circ - \phi$

Point to Ponder

Why do you keep your legs far apart when you have to stand in the aisle of a bumpy-riding bus?

Ans. We should keep our legs apart so that when we bend, our centre of gravity will remain within live of action and we will remain in stable equilibrium.

Note: If θ is angle between $\vec{A}$ and $\vec{B}$, then their resultant is:

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

PRODUCT OF TWO VECTORS

There are two types of vector multiplication.

(i) Scalar Product

(ii) Vector Product

Ans. SCALAR OR DOT-PRODUCT

When the product of two vectors results into a scalar quantity then the product is called scalar product. The scalar product between two vectors can be expressed as by putting a dot (•) between the vectors and can be written as $\vec{A} \cdot \vec{B}$. So this product is also called dot product.

Examples

(i) $W = \vec{F} \cdot \vec{d}$, where work is a scalar quantity because it is the dot product of two vectors force and displacement.

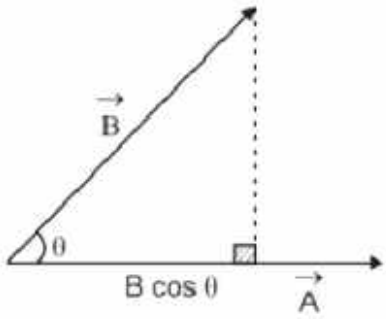
(ii) $P = \vec{F} \cdot \vec{V}$, where power is a vector because it is the dot product of two vectors force and velocity.

Explanation

Consider $\vec{A}$ and $\vec{B}$ are the two vectors having θ angle between them, then the scalar product can be written as:

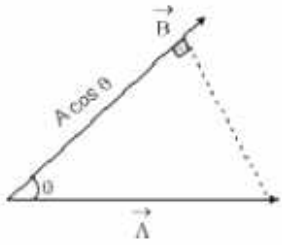
$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

Now draw perpendicular from head of $\vec{B}$ on $\vec{A}$. Also draw a perpendicular from head of $\vec{A}$ on $\vec{B}$.



$$\vec{A} \cdot \vec{B} = AB \cos\theta$$
$$= A (B \cos\theta)$$
$$= \text{Magnitude of } \vec{A} \text{ times } (\text{Projection of } \vec{B} \text{ on } \vec{A})$$

Similarly



$$\vec{B} \cdot \vec{A} = BA \cos\theta$$
$$= B (A \cos\theta)$$
$$= \text{Magnitude of } \vec{B} \text{ times } (\text{Projection of } \vec{A} \text{ on } \vec{B})$$
$$= \text{Magnitude of } \vec{B} (\text{Magnitude of component of } \vec{A} \text{ in direction of } \vec{B})$$

Characteristics of Scalar Product

Following are the characteristics of scalar product:

1. Scalar product is commutative (change of order of vectors has no effect) i.e., $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$.

Proof

$$\vec{A} \cdot \vec{B} = AB \cos\theta \quad \ldots\ldots\ldots (1)$$

$$\vec{B} \cdot \vec{A} = BA \cos\theta$$

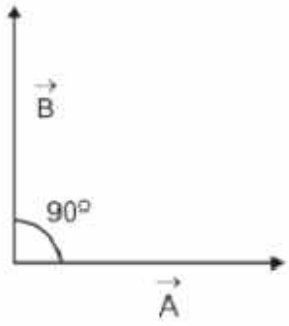
Since $AB = BA$

$$\therefore \quad \vec{B} \cdot \vec{A} = AB \cos\theta \quad \ldots\ldots\ldots (2)$$

From equations (1) and (2).

$$\therefore \quad \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

2. The scalar product of two mutually perpendicular vector is zero i.e., $\vec{A} \cdot \vec{B} = 0$.



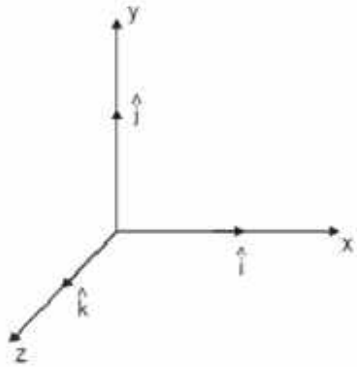
Now $\vec{A} \cdot \vec{B} = AB \cos\theta$

But $\theta = 90°$

$$\therefore \quad \vec{A} \cdot \vec{B} = AB \cos 90^\circ$$
$$= AB\,(0)$$
$$\vec{A} \cdot \vec{B} = 0$$

In case of unit vector

$$\hat{i} \cdot \hat{j} = 0$$
$$\hat{j} \cdot \hat{k} = 0$$
$$\hat{k} \cdot \hat{i} = 0$$

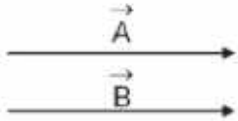


$$\hat{i} \cdot \hat{j} = |\hat{i}||\hat{j}| \cos 90^\circ$$
$$= 1 \times 1 \times 0$$
$$= 0$$

3. The scalar product of two parallel vectors is equal to the product of their magnitudes i.e., $\vec{A} \cdot \vec{B} = AB$.

Now $\vec{A} \cdot \vec{B} = AB \cos\theta$

But $\theta = 0°$ because the vectors are parallel



As $\vec{A} \cdot \vec{B} = AB \cos\theta$

$$\therefore \quad \vec{A} \cdot \vec{B} = AB \cos 0^\circ$$
$$= AB \times 1$$
$$= AB$$

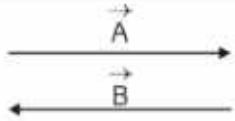
In the case of unit vectors

$$\hat{i} \cdot \hat{i} = 1, \ \hat{j} \cdot \hat{j} = 1, \ \hat{k} \cdot \hat{k} = 1.$$

$$\begin{aligned} \hat{i} \cdot \hat{i} &= |\hat{i}||\hat{i}| \cos 0^\circ \\ &= 1 \times 1 \times 1 \\ &= 1 \end{aligned}$$

For anti–parallel vectors

i.e. $\theta = 180^\circ$



As $\vec{A} \cdot \vec{B} = AB \cos 180^\circ$

$= AB(-1)$

$\therefore \quad \vec{A} \cdot \vec{B} = -AB$

4. The self scalar product of a $\vec{A}$ is equal to square of its magnitude i.e. $\vec{A} \cdot \vec{A} = A^2$.

As $\vec{A} \cdot \vec{B} = AB \cos \theta$

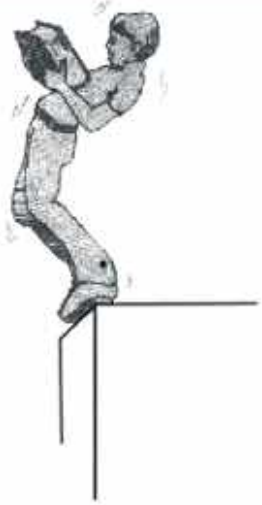
Put $\vec{B} = \vec{A}$

$\therefore \quad \vec{A} \cdot \vec{A} = AA \cos 0^\circ$

$= A^2 \times 1$

$= A^2$

What Should You Do?



You are falling off the edge. What should you do to avoid falling?

Ans. In order to avoid falling we should bend in backward direction so that our centre of gravity will remain within line of action and we will remain In stable equilibrium.

5. Scalar product of two vectors $\vec{A}$ and $\vec{B}$ in terms of their rectangular components.

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

Their scalar product can be written as

$$\vec{A} \cdot \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\begin{aligned} &= A_x B_x \hat{i} \cdot \hat{i} + A_x B_y \hat{i} \cdot \hat{j} + A_x B_z \hat{i} \cdot \hat{k} \\ &\quad + A_y B_x \hat{j} \cdot \hat{i} + A_y B_y \hat{j} \cdot \hat{j} + A_y B_z \hat{j} \cdot \hat{k} \\ &\quad + A_z B_x \hat{k} \cdot \hat{i} + A_z B_y \hat{k} \cdot \hat{j} + A_z B_z \hat{k} \cdot \hat{k} \\ &= A_x B_x (1) + A_x B_y (0) + A_x B_z (0) \\ &\quad + A_y B_x (0) + A_y B_y (1) + A_y B_z (0) \\ &\quad + A_z B_x (0) + A_z B_y (0) + A_z B_z (1) \end{aligned}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

6. Scalar product holds distributive law, i.e.,

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

$$= \vec{A} \cdot mn\,\vec{B}$$

$$= m\vec{A} \cdot n\vec{B}$$

Q.9 Define vector product. Also explain the characteristic of vector product.

Ans. **VECTOR OR CROSS PRODUCT**

When the product of two vectors results into a vector quantity, then the product is called vector product. The vector product between two vectors can be expressed as by putting a cross (×) and can be written as $\vec{A} \times \vec{B}$. So this product is also called cross product.

Examples

(i) $\vec{\tau} = \vec{r} \times \vec{F}$ where torque is a vector because it is a cross product of moment arm and force.

(ii) $\vec{L} = \vec{r} \times \vec{p}$ where angular momentum is a vector because it is the cross-product of perpendicular distance and momentum.

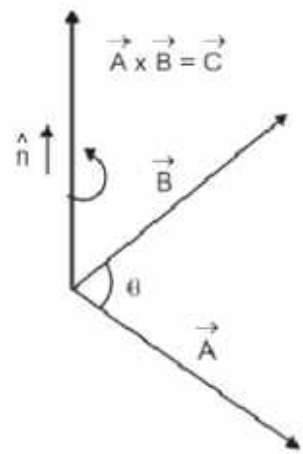
(iii) $\vec{F} = q\left(\vec{V} \times \vec{B}\right)$ where force is a vector because it is the cross-product of velocity and magnetic field.

Explanation

Consider $\vec{A}$ and $\vec{B}$ are the two vectors making an angle θ with each other, then their vector product is

$$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$$

Where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.



$$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$$

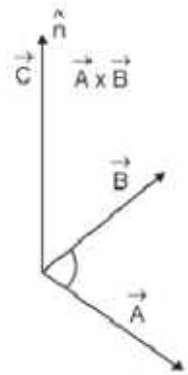
$$\left|\vec{A} \times \vec{B}\right| = AB \sin\theta \left|\hat{n}\right|$$

$$= AB \sin\theta \times 1$$

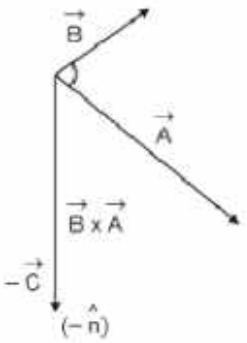
$$= AB \sin\theta$$

The direction of $\vec{A} \times \vec{B}$ is obtained by the right hand rule.

Right Hand Rule



Join the tails of two vectors $\vec{A}$ and $\vec{B}$ which determine the plane containing them. Rotate first vector in the plane, $(\vec{A})$ towards the second vector $(\vec{B})$ through the smaller of the two possible angles. Hold the right hand in such a way that thumb should be erect and fingers should be curl along the direction of rotation of $\vec{A}$, then thumb points toward the direction of $\vec{A} \times \vec{B}$.



Characteristics of Cross Product

Following are the characteristics of cross product:

1. Vector product is non-commutative i.e.

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

As by right hand-rule $\vec{A} \times \vec{B}$ is out of the paper and $\vec{B} \times \vec{A}$ is into the paper i.e., they have same magnitude but opposite in direction, therefore

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

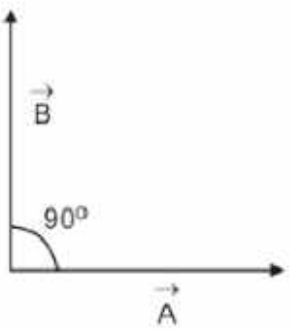
2. Cross product of two perpendicular vectors has maximum magnitude i.e.

$$\vec{A} \times \vec{B} = AB\,\hat{n}$$

As
$$\vec{A} \times \vec{B} = AB \sin \hat{n}$$

$$= AB \sin 90^\circ\, \hat{n}$$

$$= AB\,\hat{n}$$



In case of unit vector

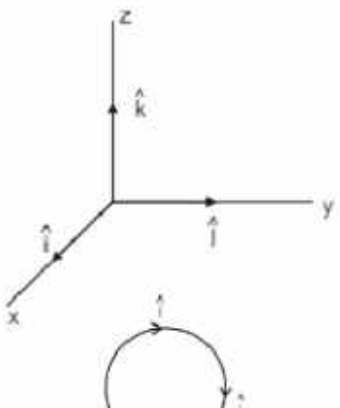
$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{k} \times \hat{i} = -\hat{j}$$



3. The cross product of two parallel vector is null vector i.e. $\vec{A} \times \vec{B} = \vec{0}$

As $\vec{A} \times \vec{B} = AB \sin\theta \hat{n}$

But $\theta = 0°$ because the vectors are parallel

As $\vec{A} \times \vec{B} = AB \sin\theta \hat{n}$



$\therefore \quad \vec{A} \times \vec{B} = AB \sin 0^\circ \hat{n}$

$= AB(0)\hat{n} = 0\hat{n}$

$\vec{A} \times \vec{B} = \vec{0}$

In case of unit vector

$\hat{i} \times \hat{i} = \vec{0}$

$\hat{j} \times \hat{j} = \vec{0}$

$\hat{k} \times \hat{k} = \vec{0}$

For anti-parallel vectors

$0 = 180^\circ$

$\therefore \quad \vec{A} \times \vec{B} = AB \sin 180^\circ \hat{n}$



$= AB(0)\hat{n}$

$= 0\hat{n}$

$\vec{A} \times \vec{B} = \vec{0}$

4. Self vector product is equal to a null vector.

$\vec{A} \times \vec{A} = AA \sin 0^\circ \hat{n}$

$= A^2(0)\hat{n}$

$= 0\hat{n}$

$\vec{A} \times \vec{A} = \vec{0}$

5. Cross product of two vectors $\vec{A}$ and $\vec{B}$ in terms of their rectangular components.

$$\vec{A} \times \vec{B} = \left(A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \right) \times \left(B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \right)$$

$$= A_x B_x \hat{i} \times \hat{i} + A_x B_y \hat{i} \times \hat{j} + A_x B_z \hat{i} \times \hat{k}$$
$$+ A_y B_x \hat{j} \times \hat{i} + A_y B_y \hat{j} \times \hat{j} + A_y B_z \hat{j} \times \hat{k}$$
$$+ A_z B_x \hat{k} \times \hat{i} + A_z B_y \hat{k} \times \hat{j} + A_z B_z \hat{k} \times \hat{k}$$

$$= A_x B_x (0) + A_x B_y \left(\hat{k} \right) + A_x B_z \left(-\hat{j} \right)$$
$$+ A_y B_x \left(-\hat{k} \right) + A_y B_y (0) + A_y B_z \left(\hat{i} \right)$$
$$+ A_z B_x \left(\hat{j} \right) + A_z B_y \left(-\hat{i} \right) + A_z B_z (0)$$

$$= A_x B_y \hat{k} - A_x B_z \hat{j} - A_y B_x \hat{k} + A_y B_z \hat{i} + A_z B_x \hat{j} - A_z B_y \hat{i}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

or
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \hat{i} (A_y B_z - A_z B_y) - \hat{j} (A_x B_z - A_z B_x) + \hat{k} (A_x B_y - A_y B_x)$$

$$= \hat{i} (A_y B_z - A_z B_y) - \hat{j} [-1(A_z B_x - A_x B_z)] + \hat{k} (A_x B_y - A_y B_x)$$

$$= \hat{i} (A_y B_z - A_z B_y) + \hat{j} (A_z B_x - A_x B_z) + \hat{k} (A_x B_y - A_y B_x)$$

6. The magnitude of $\vec{A} \times \vec{B}$ is equal to the area of the parallelogram formed with $\vec{A}$ and $\vec{B}$ as two adjacent sides i.e.

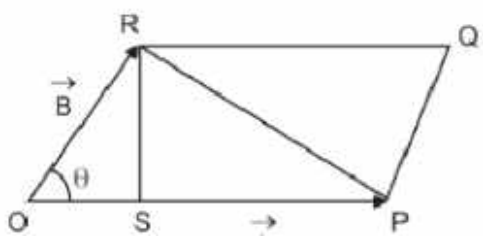
$$\left| \vec{A} \times \vec{B} \right| = \text{Area of parallelogram}$$

Solution

As $$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$$

$$\left| \vec{A} \times \vec{B} \right| = AB \sin\theta \quad \ldots\ldots\ldots \text{(i)}$$

Consider ΔORS



$$\frac{RS}{OR} = \sin\theta$$

$$RS = OR \sin\theta$$

$$RS = B \sin\theta$$

Multiply and divide by 2

$$\left| \vec{A} \times \vec{B} \right| = \frac{2}{2} (OP)(RS)$$

$$= 2\left(\frac{1}{2} (\text{Base})(\text{Height}) \right)$$

$$= 2\,[\text{Area of } \Delta OPR]$$

$$= \text{Area of } \Delta OPR + \text{Area of } \Delta PQR$$

$$\therefore \quad \left| \vec{A} \times \vec{B} \right| = \text{Area of parallelogram}$$

7. Vector product is distributive, i.e.,

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

8. Vector product is associative, i.e.,

$$mn(\vec{A} \times \vec{B}) = mn\, \vec{A} \times \vec{B}$$

$$= m\vec{A} \times n\vec{B}$$

$$= \vec{A} \times mn\, \vec{B}$$

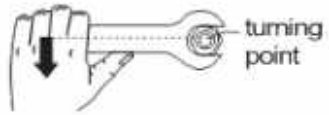
Q.10 Explain torque. Also calculate the torque due to a free $\vec{F}$ acting on the rigid body.

Ans. **TORQUE (MOMENT OF FORCE)**

The turning effect of a force is called its torque and is equal to the product of Force and the perpendicular distance from its line of action to the pivot which is the point around which the body rotates. This perpendicular distance between line of action of force and pivot is called moment arm.

Magnitude of the Torque is represented by "τ" is given by

$$\tau = lF$$



The nut is easy to turn with a spanner.

Torque depends upon two factors:

(i) Force

(ii) Moment arm



It is easier still if the spanner has a long handle.

When the line of action of the applied force passed through the pivot, then

Moment arm $l = 0$

$$\therefore \quad \tau = 0\,(F)$$

$$\tau = 0$$

SI unit of torque is "Nm".

Dimensions

$$[\tau] = Nm$$
$$= kg\ m / s^2 \times m$$
$$= kg\ m^2 / s^2$$
$$= ML^2 / T^2$$
$$= [ML^2 T^{-2}]$$

$F = ma$
$N = kg\ m/s^2$

Torque Due to a Force $\vec{F}$ Acting on a Rigid Body

Under the action of a force, if distance between the points of the body remains same such a body is called rigid body.

Let the force $\vec{F}$ acts on a rigid body at point P whose position vector relative to pivot O is $\vec{r}$. The force $\vec{F}$ can be resolved into two rectangular components i.e., F sin θ perpendicular to $\vec{r}$ and F cos θ in the direction of $\vec{r}$ as shown in Fig. (i). The torque due to F cos θ is zero as its line of action of force passes through pivot O. Therefore torque due to $\vec{F}$ is equal to the torque due to F sin θ. As

$$\tau = \text{Moment arm} \times F$$
$$\tau = r F \sin\theta$$

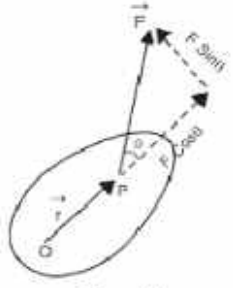


Fig. (i)

Alternatively the moment arm "*l*" is equal to the magnitude of the component of $\vec{r}$ perpendicular to the line of action $\vec{F}$ as shown in Fig. (ii).

As
$$\tau = l F$$
$$\tau = r \sin\theta\ F$$
$$\tau = r F \sin\theta$$

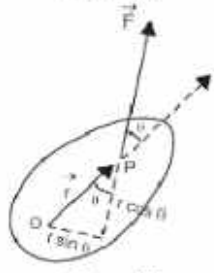


Fig. (ii)

Where θ is the angle between $\vec{r}$ and $\vec{F}$. From equations (1) and (2) torque can also be define as

Definition

The vector product of position vector $\vec{r}$ and the force $\vec{F}$ is also called torque.

$$\therefore \quad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = r F \sin\theta \, \hat{n}$$

where $r F \sin\theta$ is the magnitude of torque. The direction of torque represented by $\hat{n}$ is perpendicular to the plane containing $\vec{r}$ and $\vec{F}$ given by right hand rule.

Note: *Torque is a vector quantity.*

If $\vec{r}$ and $\vec{F}$ are in same direction, then torque will be minimum (0).

If $\vec{r}$ and $\vec{F}$ are in opposite direction, then torque will be minimum (0).

If $\vec{r}$ and $\vec{F}$ are perpendicular to each other, then torque will be maximum (rF).

Just as force determines the linear acceleration produced in a body, the torque acting on a body determines its angular acceleration. Torque is the analogous of force for rotational motion. If the body is at rest or rotating with uniform angular velocity, the angular acceleration will be zero. In this case the torque acting on the body will be zero.

Point to Ponder



Do you think the rider in the above figure is really in danger? What if people below were removed?

Ans. **There is no danger for the rider in the above figure because he is in stable equilibrium and no change if the people below were removed.**

Can You Do?



Stand with one arm and the side of one foot pressed against a wall. Can you raise the other leg side ways? If not, then why not?

Ans. **Yes.**

Q.11 Define equilibrium. Also state the conditions of equilibrium.

Ans. **EQUILIBRIUM**

If a body, under the action of a number of forces, is at rest or moving with uniform velocity, it is said to be in equilibrium.

1. Static Equilibrium

If a body is at rest, it is said to be in Static Equilibrium. For example book lying on the table.

2. Dynamic Equilibrium

If a body is moving with uniform velocity, or rotating with uniform angular velocity, it is said to be in Dynamic Equilibrium. For example jumping of paratrooper.

First Condition of Equilibrium

The vector sum of all the forces acting on a body must be equal to zero.

$$\Sigma \vec{F} = \vec{0}$$

In case of coplanar forces, the resultant force $\vec{F}_x$ is equal to sum of x–directed forces acting on the body. Therefore

$$\therefore \quad \Sigma \vec{F}_x = \vec{0}$$

Similarly for y–directed forces, the resultant $\vec{F}_y$ should be zero. Therefore

$$\therefore \quad \Sigma \vec{F}_y = \vec{0}$$

Second Condition of Equilibrium

Let two equal and opposite forces are acting on body as shown in figure.

Although, the first condition of equilibrium is satisfied, yet it may rotate having clockwise turning effect.

Thus for a body in equilibrium, the vector sum of all the torques acting on it about arbitrary axis should be zero. This is known as second condition of equilibrium. Mathematically

$$\Sigma \vec{\tau} = \vec{0}$$

Requirements for a body to be in complete equilibrium are

(i) $\Sigma \vec{F} = \vec{0}$

i.e. $\Sigma \vec{F}_x = \vec{0}$

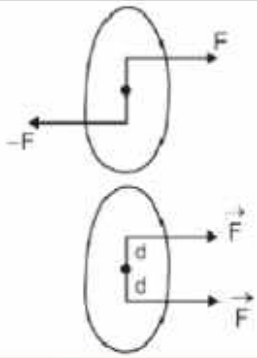
$\Sigma \vec{F}_y = \vec{0}$

and $\Sigma \vec{\tau} = \vec{0}$

When first condition is satisfied, there is no linear acceleration and body will be in translation equilibrium (linear equilibrium).

When second condition is satisfied that is no angular acceleration and the body will be in rotational a equilibrium.

For a body to be in complete equilibrium, both conditions should be satisfied, i.e., both linear acceleration and angular acceleration should be zero.



Interesting Application



A concurrent force system in equilibrium. The tension applied can be adjusted as desired.

Can You Do?



With your nose touching the end of the door, put your feet astride the door and try to rise up on your toes.

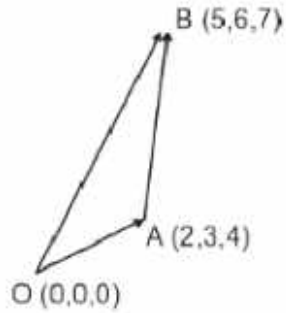
SOLVED EXAMPLES

EXAMPLE 2.1

The position of two aeroplanes at any instant are represented by points A (2, 3, 4) and B (5, 6, 7) from an origin 'O' in km as shown in the figure.

(a) What are their position vectors.

(b) Calculate the distance between two aeroplanes.



Data

Position of two acroplanes are

A(2, 3, 4) and B(5, 6, 7)

To Find

Distance between two aeroplanes AB = ?

SOLUTION

Position vector of aeroplane A is

$$\overrightarrow{OA} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

and that of B is

$$\overrightarrow{OB} = (5, 6, 7) - (0, 0, 0)$$

$$= 5\hat{i} + 6\hat{j} + 7\hat{k}$$

By head to tail rule

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

$$\therefore \quad \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= 5\hat{i} + 6\hat{j} + 7\hat{k} - 2\hat{i} - 3\hat{j} - 4\hat{k}$$

$$= 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$\therefore$ Distance between two aeroplanes is

$$AB = \sqrt{(3)^2 + (3)^2 + (3)^2}$$

$$= \sqrt{9 + 9 + 9}$$

$$= \sqrt{27}$$

Distance between two aeroplanes = AB = 5.2 km

EXAMPLE 2.2

Two forces of magnitudes 10N and 20N act on a body in directions making angles 30^o and 60^o with x-axis respectively. Find the resultant force.

Data

First force = F_1 = 10 N

Second force = F_2 = 20 N

First angle = $\theta_1 = 30^o$

Second angle = $\theta_2 = 60^o$

To Find

Resultant force = $\vec{F}$ = ?

SOLUTION

Resolving F_1 and F_2 into its components

$F_{1x} = F_1 \cos 30^o$

$= 10 \times 0.866$

$= 8.66$ N

$F_{1y} = F_1 \sin 30^o$

$= 10 \times 0.5$

$= 5$ N

$F_{2x} = F_2 \cos 60^o$

$= 20 \times 0.5$

$= 10$ N

$F_{2y} = F_2 \sin 60^o$

$= 20 \times 0.866$

$= 17.32$ N

Now $F_x = F_{1x} + F_{2x}$

$= 8.66 + 10$

$= 18.66$ N

Also $F_y = F_{1y} + F_{2y}$

$= 5 + 17.32$

$= 22.32$ N

$$= \sqrt{(18.66)^2 + (22.32)^2}$$

$$= \sqrt{348.19 + 498.18}$$

$$= \sqrt{84637}$$

$$F = 29 \text{ N}$$

For direction using

$$\phi = \tan^{-1} \frac{F_y}{F_x}$$

Putting values

$$\phi = \tan^{-1} \frac{22.32}{18.66}$$

$$= \tan^{-1} 1.196$$

$$\phi = 50^\circ$$

Since F_x and F_y are positive hence resultant lies in first quadrant.

$$\therefore \quad \theta = \phi$$

$$\therefore \quad \theta = 50^\circ$$

Result

Resultant force = F = 29 N

EXAMPLE 2.3

Find the angle between two forces of equal magnitude when the magnitude of their resultant is also equal to the magnitude of either of these forces.

Data

Angle between two forces F_1 and F_2 θ = ?

Such that

$$F_1 = F_2 = F$$

Also $R = F$

SOLUTION

Using $R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos \theta}$

$\because$ $F_1 = F_2 = R = F$

$\therefore$ $F = \sqrt{F^2 + F^2 + 2 F F \cos \theta}$

$F = \sqrt{2 F^2 + 2 F^2 \cos \theta}$

$$F^2 = 2F^2(1+\cos\theta)$$

$$1 = 2(1+\cos\theta)$$

$$\frac{1}{2} = 1+\cos\theta$$

$$\cos\theta = \frac{1}{2} - 1$$

$$= -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = 120^{\circ}$$

EXAMPLE 2.4

A force $\mathbf{F} = 2\hat{i} + 3\hat{j}$ units, has its point of application moved from point A(1, 3) to the point B(5, 7). Find the work done.

Data

$$\vec{F} = 2\hat{i} + 3\hat{j}$$

Distance covered from

A (1, 3) to B (5, 7)

To Find

Work done = W = ?

SOLUTION

Using $\vec{d} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$$= (5-1)\hat{i} + (7-3)\hat{j}$$

$$\vec{d} = 4\hat{i} + 4\hat{j}$$

Now using

$$W = \vec{F} \cdot \vec{d}$$

$$W = \left(2\hat{i} + 3\hat{j}\right)\left(4\hat{i} + 4\hat{j}\right)$$

Result

Work done = W = 20 Joule

EXAMPLE 2.5

Find the projection of vector $\vec{A} = 2\hat{i} - 8\hat{j} + \hat{k}$ in the direction of vector $\vec{B} = 3\hat{i} - 4\hat{j} - 12\hat{k}$.

Data

$$\vec{A} = 2\hat{i} - 8\hat{j} + \hat{k}$$

$$\vec{B} = 3\hat{i} - 4\hat{j} - 12\hat{k}$$

To Find

Projection of $\vec{A}$ along $\vec{B}$ = $A\cos\theta$ = ?

SOLUTION

Using $\vec{A} \cdot \vec{B} = AB\cos\theta$

$$\therefore \quad A\cos\theta = \frac{\vec{A} \cdot \vec{B}}{B}$$

$$= \frac{(2\hat{i} - 8\hat{j} + \hat{k}) \cdot (3\hat{i} - 4\hat{j} - 12\hat{k})}{\sqrt{(3)^2 + (-4)^2 + (-12)^2}}$$

$$= \frac{6 + 32 - 12}{\sqrt{9 + 16 + 144}}$$

$$= \frac{26}{\sqrt{169}}$$

$$= \frac{26}{13}$$

$$= 2$$

Result

Projection of $\vec{A}$ along $\vec{B}$ = $A\cos\theta$ = 2

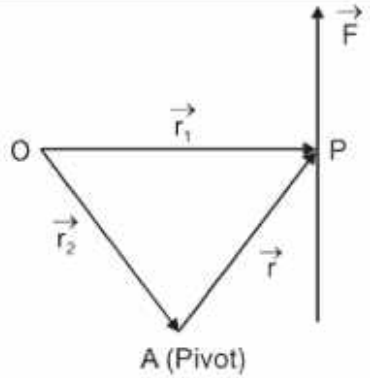
EXAMPLE 2.6

The line of action of a force $\vec{F}$ passes through a point P of a body whose position vector in metre is $\hat{i} - 2\hat{j} + \hat{k}$. If $\vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ (in Newton) determine the torque about the point 'A' whose position vector (in metre) is $2\hat{i} + \hat{j} + \hat{k}$.

Data

$$\vec{r}_1 = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k} \text{ (N)}$$



To Find

$\vec{\tau}$ about A = ?

SOLUTION

$$\vec{r}_2 = 2\hat{i} + \hat{j} + \hat{k}$$

By head to tail rule

$$\vec{r}_1 = \vec{r}_2 + \vec{r}$$

$$\therefore \quad \vec{r} = \vec{r}_1 - \vec{r}_2$$

Putting values

$$\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + \hat{j} + \hat{k})$$

$$= \hat{i} - 2\hat{j} + \hat{k} - 2\hat{i} - \hat{j} - \hat{k}$$

$$= -\hat{i} - 3\hat{j}$$

Now using

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 0 \\ 2 & -3 & 4 \end{vmatrix}$$

$$= \hat{i}(-12 + 0) - \hat{j}(-4 - 0) + \hat{k}(3 + 6)$$

$$= -12\hat{i} + 4\hat{j} + 9\hat{k} \text{ (Nm)}$$

Result

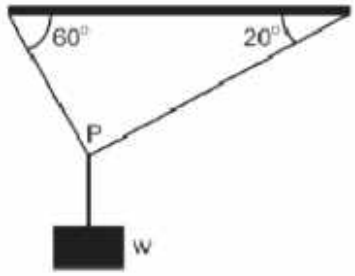
$\vec{\tau}$ about A $= -12\hat{i} + 4\hat{j} + 9\hat{k}$ (Nm)

EXAMPLE 2.7

A load is suspended by two cords as shown in the figure. Determine the maximum load that can be suspended at P if maximum breaking stress of the cord used is 50N

Data

Force = F = 50 N



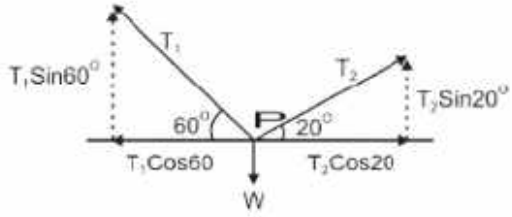
To Find

Maximum load = W = ?

SOLUTION

Resolving T_1 and T_2 into its components

Using 1st condition of equilibrium



Now using

$\sum F_y = 0$ and $\sum F_x = 0$

$T_1 \sin 60^\circ + T_2 \sin 20^\circ - W = 0$ $T_2 \cos 20^\circ - T_1 \cos 60^\circ = 0$

$50\ (0.866) + 26.6\ (0.34) = W$ $T_1 = 1.88\ T_2$

$W = 52$ N Since $T_1 > T_2$

$\therefore$ T_1 has maximum stress.

$T_1 = 50$ N

then $T_2 = 26.6$ N

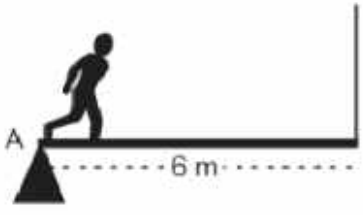
Result

Maximum load = W = 52 N

EXAMPLE 2.8

A uniform beam of 200N is supported horizontally as shown. If the breaking stress of the rope is 400N how far can the man of weight 400N walk from point A on the beam as shown in figure?

Data



Weight of beam = 200 N

Breaking stress of rope = 400N

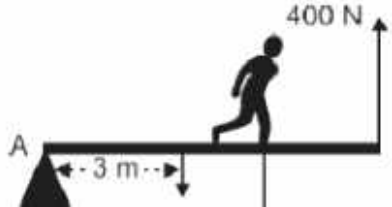
Distance of man from point A = d = ?

Weight of man = 400N

To Find

Distance = d = ?

SOLUTION



Taking point A as pivot

Using $\sum \tau = 0$

$400 \times 6 - 400 \times d - 200 \times 3 = 0$

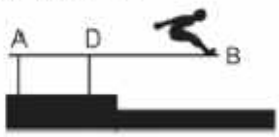
$2400 - 400\ d - 600 = 0$

Result

Distance = d = 4.5 m

EXAMPLE 2.9

A body weighing 300N is standing at the edge of a uniform diving board 4.0m in length. The weight of the board is 200N. Find the forces exerted by pedestals on the board.



Data

Weight of boy = 300N

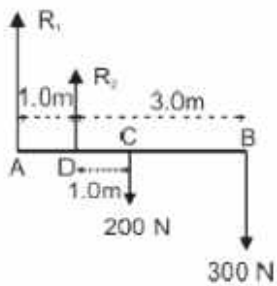
Length of board = 4m

Weight of board = 200N

To Find

Forces exerted by pedestals on board = ?

SOLUTION



Let R_1 and R_2 are the reaction forces exerted by the pedestals on the board. A little consideration will show that R_1 is in the wrong direction because the board must be actually pressed down in order to keep it in equilibrium. We will see that this assumption will be automatically corrected by calculations.

Now applying condition of equilibrium

$$\Sigma F_y = 0$$

$$R_1 + R_2 = 200 + 300$$

$$R_1 + R_2 = 500N \quad \ldots\ldots\ldots (1)$$

Now applying $\Sigma \tau = 0$ about point D.

$$-R_1 \times AD - 300 \times DB - 200 \times DC = 0$$

$$-R_1 \times 1 - 300 \times 3 - 200 \times 1 = 6$$

$$-R_1 - 900 - 200 = 0$$

$$-R_1 = 1100$$

$$R_1 = -1100\ N$$

Negative sign shows R_1 is directed downward.

$\Rightarrow$ Putting value of R_1 in equation (1).

$$-1100 + R_2 = 500$$

$$R_2 = 500 + 1100$$

$$= 1600\ N$$

$$\boxed{R_1 = -1100\ N = 1.1\ KN}$$

Force exerted by pedestals on board = R_1 = 1.1 KN

= R_2 = 1.6 KN

The negative sign tR_1 shows that it is directed downward.



VECTORS AND EQUILIBRIUM

Each question has four possible answers, encircled the correct answer:

1. A scalar quantity can be described by:

(a) Magnitude **(b)** Unit

(c) Magnitude and unit **(d)** Number

2. A vector quantity can be described by magnitude, unit and:

(a) Direction **(b)** Rotation

(c) Dimension **(d)** Unit vector

3. Which one of the following is a vector quantity:

(a) Energy **(b)** Power

(c) Work **(d)** Momentum

4. Which one of the following is a scalar quantity:

(a) Mass **(b)** Displacement

(c) Force **(d)** Torque

5. Two lines are drawn at right angle to each other are known as:

(a) Coordinate axis **(b)** xy-axis

(c) Components **(d)** Cartesian axis

6. A vector which gives the direction of a given vector is called:

(a) Unit vector **(b)** Position vector

(c) Null vector **(d)** Negative vector

7. When a vector is divided by its magnitude we get:

(a) Null vector **(b)** Unit vector

(c) Zero vector **(d)** Position vector

8. Pick out the scalar quantity among the following:

(a) Force **(b)** Torque

(c) Time **(d)** Velocity

9. Pick out the vector quantity among the following:

(a) Power **(b)** Energy

(c) Force **(d)** Mass

10. The magnitude of a null vector is:

(a) One **(b)** Zero

(c) Double **(d)** Negative

11. Null vector is a vector having zero magnitude and:

(a) Arbitrary direction **(b)** No direction

(c) Specific direction **(d)** Opposite direction

12. Unit vector of a vector $\vec{A}$ describes:

(a) Direction of a given vector **(b)** Magnitude of a given vector

(c) Shape of a given vector **(d)** All of above

13. The unit vector of among vector is determined:

(a) By multiplying the vector with its own magnitude

(b) By dividing the vector with its own magnitude

(c) Both (a) and (b)

(d) None of these

14. Unit vector is used to specify:

(a) Direction of a vector **(b)** Position of a vector

(c) Magnitude of a vector **(d)** Dimension of a vector

15. An example of a scalar quantity:

(a) Displacement **(b)** Acceleration

(c) Force **(d)** Speed

16. An example of a vector quantity:

(a) Speed **(b)** Work

(c) Acceleration **(d)** Mass

17. A vector which has magnitude one is called:

(a) Null vector **(b)** Unit vector

(c) Resultant vector **(d)** Position vector

18. A vector which has zero magnitude is called:

(a) Null vector **(b)** Unit vector

(c) Resultant vector **(d)** Position vector

19. The sum of two or more vectors is equal to a single vector which is called:

(a) Component of vector **(b)** Product vector

(c) Null vector **(d)** Resultant vector

20. When a vector $\vec{A}$ is multiplied by a number n, then its magnitude is given by:

(a) $n \times |\vec{A}|$ **(b)** $|n\vec{A}|$

(c) $n \times \vec{A}$ **(d)** None of these

21. When a vector $\vec{A}$ is multiplied by a negative number then its direction:

(a) Remains same **(b)** Changed by 180°

(c) Does not change **(d)** None of these

22. When a vector $\vec{A}$ is multiplied by a positive number then its direction:

(a) Remains same **(b)** Changed by 180°

(c) Does not change **(d)** None of these

23. The splitting up of a vector into its components is called:

(a) Sum of vector **(b)** Subtraction of a vector

(c) Resolution of a vector **(d)** None of these

24. The angle between two rectangular components is:

(a) 60° **(b)** 90°

(c) 180° **(d)** 270°

25. The resultant of two anti-parallel vectors $\vec{A}$ and $\vec{B}$ is:

(a) $\vec{A} + \vec{B}$ **(b)** $\vec{A} - \vec{B}$

(c) Zero **(d)** None of these

26. Two vectors having same magnitude and direction are called:

(a) Equal vectors **(b)** Unequal vectors

(c) Null vectors **(d)** None of these

27. The sum of two equal and opposite vectors is a vector called:

(a) Equal vector **(b)** Null vector

(c) Position vector **(d)** Unit vector

28. The magnitude of resultant of a vector $\vec{A}$ is given by:

(a) $\sqrt{A_x^2 + A_y^2}$ **(b)** $\sqrt{A_x + A_y}$

(c) $A_x^2 + A_y^2$ **(d)** None of these

29. What is the resultant of 3N and 4N forces acting at right angle to each other:

(a) 90 N **(b)** 5 N

(c) 7 N **(d)** 1 N

30. If a force of 10 N makes an angle of 30° with x-axis, its x-component is given by:

(a) 86.6 N **(b)** 0.866 N

(c) 8.66 N **(d)** None of these

31. Two forces of 10 N and 7 N respectively are acting to an object. The minimum value of their resultant is:

(a) 0 N **(b)** 10 N

(c) 7 N **(d)** 3 N

32. Two forces act together on a body, the magnitude of their resultant is greatest when the angle between the forces is:

(a) 45° **(b)** 60°

(c) 0° **(d)** 180°

33. The position vector in xy-plane is written as:

(a) $\vec{r} = x\hat{i} + y\hat{j}$ **(b)** $\vec{r} = y\hat{i} + z\hat{k}$

(c) $\vec{r} = y\hat{j} + z\hat{k}$ **(d)** None of these

34. The position vector in xz-plane is written as:

(a) $\vec{r} = x\hat{i} + y\hat{j}$ **(b)** $\vec{r} = y\hat{j} + z\hat{k}$

(c) $\vec{r} = x\hat{i} + z\hat{k}$ **(d)** None of these

35. The position vector in yz-plane is given by:

(a) $\vec{r} = x\hat{i} + z\hat{k}$ **(b)** $\vec{r} = x\hat{i} + y\hat{j}$

(c) $\vec{r} = y\hat{j} + z\hat{k}$ **(d)** $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

36. If a force of 50 N is acting along x-axis, then its component along y-axis will be:

(a) The same **(b)** Zero

(c) Half magnitude **(d)** None of these

37. A force of 10 N is acting along z-axis, its component along x-axis and y-axis is:

(a) 5 N, 8 N **(b)** 3 N, 4 N

(c) 5 N each **(d)** Zero

38. If two vectors of magnitude F_1 and F_2 act on a body at an angle θ, the magnitude of their resultant is:

(a) $\sqrt{F_1^2 + F_2^2}$ **(b)** $\sqrt{F_1^2 + F_2^2 + 2F_1F_2}$

(c) $\sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$ **(d)** $F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

39. The magnitude of a vector $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ is given by:

(a) $A_x + A_y + A_z$ **(b)** $A_x\cos\theta$

(c) $\sqrt{A_x^2 + A_y^2 + A_z^2}$ **(d)** None of these

40. If a vector $\vec{A}$ makes an angle θ with x-axis, the magnitude of its, x-component is:

(a) $A_y = A\sin\theta$ **(b)** $A_x = A\cos\theta$

(c) Both (a) and (b) **(d)** None of these

41. If a vector $\vec{A}$ makes an angle θ with x-axis, the magnitude of its y-component is:

(a) $A_y = A\sin\theta$ **(b)** $A_x = A\cos\theta$

(c) Both (a) and (b) **(d)** None of these

42. The reverse process of vector addition is called:

(a) Subtraction of a vector **(b)** Addition of a vector

(c) Negative of a vector **(d)** Resolution of a vector

43. The expression $\vec{r} = a\hat{i} + b\hat{j}$ is for:

(a) Unit vector **(b)** Position vector

(c) Null vector **(d)** Negative vector

44. The direction of a resultant vector $\vec{R}$ is given by:

(a) $\theta = \tan^{-1}\left(\frac{R_x}{R_y}\right)$ **(b)** $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

(c) $\theta = \sin^{-1}\left(\frac{R_y}{R_x}\right)$ **(d)** None of these

45. If both the components of a vector are negative then vector is in:

(a) 1^{st} quadrant **(b)** 2^{nd} quadrant

(c) 3^{rd} quadrant **(d)** 4^{th} quadrant

46. The scalar product is also known as:

(a) Vector product **(b)** Dot product

(c) Vector sum **(d)** Scalar sum

47. The scalar product of $\vec{A}$ and $\vec{B}$ is given by:

(a) $\vec{A} \times \vec{B}$ **(b)** $\vec{A} \cdot \vec{B}$

(c) $\vec{A} - \vec{B}$ **(d)** AB

48. The projection of vector $\vec{A}$ on $\vec{B}$ is given by:

(a) $\frac{\vec{A} \cdot \vec{B}}{|A|}$ **(b)** $\frac{\vec{A} \cdot \vec{B}}{|\vec{B}|}$

(c) $AB\cos\theta$ **(d)** $\frac{\vec{A} \cdot \vec{B}}{A\cos\theta}$

49. The self scalar product of $\vec{A}$ is given by:

(a) $\sqrt{A}$ **(b)** A^3

(c) A^2 **(d)** A

50. If $\vec{A}$ and $\vec{B}$ are anti-parallel then their scalar product is:

(a) $AB\cos\theta$ **(b)** $-AB$

(c) $-AB\cos\theta$ **(d)** Zero

51. The scalar product of two similar unit vector is:

(a) One **(b)** Zero

(c) Twice **(d)** Negative

52. If $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ this is called:

(a) Commutative law **(b)** Associative law

(c) Distributive law **(d)** None of these

53. If the multiplication of two vectors results into a vector quantity then the product is called:

(a) Dot product **(b)** Vector product

(c) Scalar product **(d)** None of these

54. If the multiplication of two vectors result into a scalar quantity then the product is called:

(a) Vector product **(b)** Cross product

(c) Scalar product **(d)** None of these

55. If $\vec{A} \times \vec{B}$ points along positive z-axis, then vector $\vec{A}$ and $\vec{B}$ will lie in:

(a) zx-plane **(b)** xy-plane

(c) yz-plane **(d)** None of these

56. If two vectors $\vec{A}$ and $\vec{B}$ are non-parallel vectors then the direction of $\vec{A} \times \vec{B}$ is along:

(a) y-axis **(b)** z-axis

(c) x-axis **(d)** None of these

57. Select the correct answer:

(a) $\hat{i} \cdot \hat{j} = \hat{k}$ **(b)** $\hat{i} \cdot \hat{j} = 0$

(c) $\hat{i} \cdot \hat{j} = -\hat{k}$ **(d)** $\hat{i} \cdot \hat{j} = 1$

58. Select the correct one:

(a) $\vec{A} \cdot \vec{B} = -\vec{B} \cdot \vec{A}$ **(b)** $\vec{A} \cdot \vec{B} = \frac{1}{2}\vec{B} \cdot \vec{A}$

(c) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ **(d)** None of these

59. Which of the following unit vectors represent the direction of normal drawn on a specific surface:

(a) $\hat{j}$ **(b)** $\hat{i}$

(c) $\hat{n}$ **(d)** $\hat{k}$

60. If $\hat{A} = 2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = -2\hat{i} + 2\hat{j} + \hat{k}$. What will be the value of $\vec{A} \cdot \vec{B}$:

(a) 9 **(b)** –9

(c) 5 **(d)** 10

61. If $\vec{A} = 2\hat{i} + 2\hat{j} + \hat{k}$ then the value of $\hat{B}$ is:

(a) $\frac{2\hat{i} + 2\hat{j} + \hat{k}}{5}$ **(b)** $\frac{2\hat{i} + 2\hat{j} + \hat{k}}{9}$

(c) $\frac{2\hat{i} + 2\hat{j} + \hat{k}}{3}$ **(d)** None of these

62. The scalar product of two vectors is zero when:

(a) They are equal vectors **(b)** They are in the same direction

(c) They are at right angle **(d)** None of these

63. If the vectors $\vec{A}$ and $\vec{B}$ are parallel to each other then:

(a) $\vec{A} \cdot \vec{B} = AB$ **(b)** $\vec{A} \cdot \vec{B} = \pm AB$

(c) $\vec{A} \cdot \vec{B} = 0$ **(d)** $\vec{A} \cdot \vec{B} = AB \cos\theta$

64. If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ then the value of $\vec{A} \cdot \vec{B}$ is:

(a) $A_xB_x + A_yB_z + A_zB_y$ **(b)** $A_xB_x + A_yB_y + A_zB_z$

(c) $A_yB_x + A_xB_y + A_zB_z$ **(d)** None of these

65. The scalar product of two vectors will be negative if:

(a) They are at right angle to each other **(b)** They are parallel

(c) They are anti-parallel **(d)** None of these

66. The dot product of $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k}$ is equal to:

(a) 0 **(b)** 1

(c) −1 **(d)** 2

67. The dot product of $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i}$ is equal to:

(a) 0 **(b)** 1

(c) −1 **(d)** 2

68. The vector product of two vectors $\vec{A}$ and $\vec{B}$ is given by:

(a) $AB \sin\theta$ **(b)** $AB \sin\theta\,\hat{n}$

(c) $\vec{AB} \sin\theta$ **(d)** $AB\,\hat{n}$

69. Vector product does not hold:

(a) Commutative law **(b)** Associative law

(c) Distributive law **(d)** None of these

70. The direction of vector product is:

(a) Parallel to plane **(b)** Perpendicular to plane

(c) Anti-parallel **(d)** Along the plane

71. Self cross-product of a vector is equal to:

(a) Zero **(b)** One

(c) Double **(d)** Negative

72. The cross product of unit vectors $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$ is:

(a) One **(b)** $\hat{i}$

(c) $\hat{k}$ **(d)** Zero

73. If $\vec{A} \times \vec{B} = 0$ then the angle between the vectors is:

(a) $60°$ **(b)** $90°$

(c) $270°$ **(d)** $180°$

74. The magnitude of $\vec{A} \times \vec{B}$ is equal to area of:

(a) Triangle **(b)** Circle

(c) Parallelogram **(d)** Rectangle

75. The cross product of two vectors will be negative when:

(a) They are anti-parallel **(b)** They are parallel

(c) They are rotated through an angle of $270°$ **(d)** None of these

76. The cross product of two parallel vectors $\vec{A}$ and $\vec{B}$ is equal to:

(a) $AB \sin\theta \hat{n}$ **(b)** $AB \sin\theta$

(c) AB **(d)** Zero

77. Select the correct one:

(a) $\vec{A} \cdot \vec{B} = -\vec{A} \cdot \vec{B}$ **(b)** $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

(c) $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ **(d)** None of these

78. The cross product of $\hat{i} \times \hat{j}$ is equal to:

(a) $\hat{k}$ **(b)** $-\hat{k}$

(c) $\vec{k}$ **(d)** $\hat{i}$

79. The cross product of $\hat{j} \times \hat{i}$ is equal to:

(a) $\hat{k}$ **(b)** $-\hat{k}$

(c) $\vec{k}$ **(d)** $\hat{i}$

80. Select the correct one:

(a) $\hat{j} \times \hat{k} = \hat{i}$ **(b)** $\hat{j} \times \hat{k} = -\hat{i}$

(c) $\hat{j} \times \hat{k} = \hat{j}$ **(d)** $\hat{j} \times \hat{k} = \hat{k}$

81. The turning effect of a force is called its moment or:

(a) Momentum **(b)** Inertia

(c) Torque **(d)** Impulse

82. The perpendicular distance from the line of action to the pivot is called:

(a) Displacement **(b)** Momentum

(c) Moment distance **(d)** Moment arm

83. The SI unit of torque is:

(a) $N \cdot m^2$ **(b)** $N \cdot m$

(c) N/m^2 **(d)** N^2m

84. The expression for torque is given by:

(a) $rF \cos\theta$ **(b)** $rF \sin\theta \hat{n}$

(c) $rF \sin\theta$ **(d)** $rF \cos\theta \hat{n}$

85. Torque acting on a body determines its:

(a) Velocity **(b)** Momentum

(c) Force **(d)** Angular momentum

86. When line of action of applied force passes through the pivot point then torque will be:

(a) Maximum **(b)** Constant

(c) Negative **(d)** Zero

87. The direction of torque $\vec{\tau} = \vec{r} \times \vec{F}$ is determined by:

(a) Head to tail rule **(b)** Right hand rule

(c) Left hand rule **(d)** None of these

88. Conventionally anti-clock wise torque is taken as:

(a) Zero **(b)** Negative

(c) Positive **(d)** None of these

89. Conventionally clockwise torque is taken as:

(a) Zero **(b)** Negative

(c) Positive **(d)** None of these

90. Torque is also called as:

(a) Moment of inertia **(b)** Moment arm

(c) Moment of force **(d)** Angular velocity

91. The dimension of torque are:

(a) $[ML^2T^{-2}]$ **(b)** $[MLT^{-1}]$

(c) $[ML^3T]$ **(d)** $[M^2LT^{-2}]$

92. Torque = ———— × Force:

(a) Velocity **(b)** Momentum

(c) Arm of the weight **(d)** Moment arm

93. Let torque $= \vec{\tau} = \vec{r} \times \vec{F}$ then direction of torque is:

(a) In the direction $\vec{F}$ **(b)** In the direction of $\vec{r}$

(c) Normal to the plane **(d)** None of these

94. Two equal and opposite forces acting on a body form a:

(a) Momentum **(b)** Torque

(c) Couple **(d)** None of these

95. The point at which the whole weight of the body acts is called:

(a) Torque **(b)** Centre of gravity

(c) Centre of mass **(d)** Centre of the body

96. The centre of gravity of a uniform body is:

(a) At the axis of rotation of the body **(b)** At its centre

(c) At its one end **(d)** None of these

97. The centre of gravity of a triangular plate is:

(a) At the axis of rotation of the body **(b)** At its centre

(c) At the intersections of medians **(d)** None of these

98. If a body is at rest or moving with uniform velocity then it is said to be in:

(a) Torque **(b)** Equilibrium

(c) Both (a) and (b) **(d)** None of these

99. Torque has zero value if angle between $\vec{r}$ and $\vec{F}$:

(a) 60° **(b)** 45°

(c) 90° **(d)** 0°

100. The torque has maximum value if angle between $\vec{r}$ and $\vec{F}$ is:

(a) 60° **(b)** 45°

(c) 90° **(d)** 0°

101. A body will be in translational equilibrium if:

(a) $\Sigma\vec{F} = 0$ **(b)** $\Sigma\vec{\tau} = 0$

(c) $\Sigma\vec{F_y} = 0$ **(d)** None of these

102. The condition of complete equilibrium is satisfied if:

(a) Vector sum of all the torques is zero **(b)** Vector sum of all forces and torques is zero

(c) Vector sum of all the forces is zero **(d)** None of these

103. $\hat{i} \cdot (\hat{j} \times \hat{k})$ is equal to:

(a) $\hat{k}$ **(b)** 2

(c) 1 **(d)** 0

104. If $|\bar{a} + \bar{b}| = |\bar{a} - \bar{b}|$ then angle between $\bar{a}$ and $\bar{b}$ is:

(a) 90° **(b)** 0°

(c) 180° **(d)** 45°

105. If $\vec{A} = 2\hat{i} + \hat{j} + 2\hat{k}$ then $|A|$ is:

(a) zero **(b)** 3

(c) 5 **(d)** 9

106. In rotational motion the analogue of force is:

(a) Moment arm **(b)** Torque

(c) Moment of inertia **(d)** None of these

107. The component of $9\hat{i} + 17\hat{j}$ along z-axis is:

(a) Zero **(b)** 18

(c) 26 **(d)** 11

108. If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, angle between $\vec{A}$ and $\vec{B}$ is:

(a) 0° **(b)** 90°

(c) 60° **(d)** 180°

109. If vectors $2\hat{i} + 4\hat{j} - 7\hat{k}$ and $2\hat{i} + 6\hat{j} + q\hat{k}$ are perpendiculars then value of q is:

(a) 4 **(b)** 7

(c) 8 **(d)** 10

110. The resultant of two vectors of magnitude 2 and 3 is 1. The angle between them is:

(a) 90° **(b)** 180°

(c) 0° **(d)** None of these

111. $|\hat{i} - \hat{j} - 3\hat{k}| =$

(a) $\sqrt{5}$ **(b)** $\sqrt{7}$

(c) $\sqrt{11}$ **(d)** $\sqrt{13}$

112. Resultant of two vectors of magnitude 24 and 7 is 25. The angle between them is:

(a) 90° **(b)** 180°

(c) 360° **(d)** 270°

113. If $|\vec{A} \times \vec{B}| = \sqrt{3}\,(\vec{A} \cdot \vec{B})$. Angle between $\vec{A}$ and $\vec{B}$ is:

(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ **(d)** $\frac{\pi}{3}$

114. If $\vec{P} = 3\hat{i} + 4\hat{j} - 2\hat{k}$, $\vec{Q} = 4\hat{i} - 3\hat{j} + 2\hat{k}$. Unit vector in the direction of $\vec{P} + \vec{Q}$ is:

(a) $7\hat{i} + \hat{j}$ **(b)** $\frac{7\hat{i} + \hat{j}}{5}$

(c) $\frac{1}{29}(2\hat{i} - 14\hat{j} - 25\hat{k})$ **(d)** None of these

115. Area of parallelogram =

(a) $\vec{A} \cdot \vec{B}$ **(b)** $\vec{A} \times \vec{B}$

(c) $|\vec{A} \times \vec{B}|$ **(d)** None of these

116. If the resultant of two vectors each of magnitude F is also of magnitude F, the angle between them is:

(a) 60° **(b)** 90°

(c) 180° **(d)** 120°

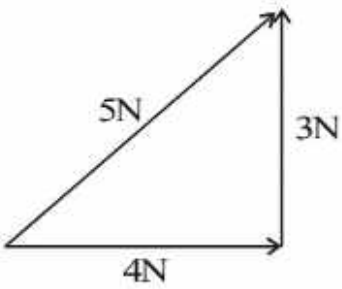
117. The resultant of two forces 3N and 4N making an angle 60° with each other is:

(a) 5 **(b)** 7

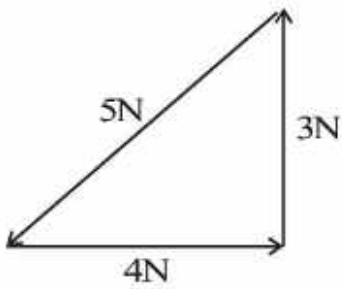
(c) 6.1 **(d)** 1

118. Which diagram correctly shows the addition of 4N and 3N vectors?

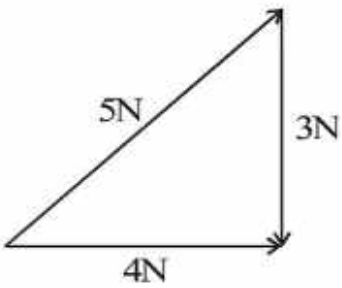
(a)



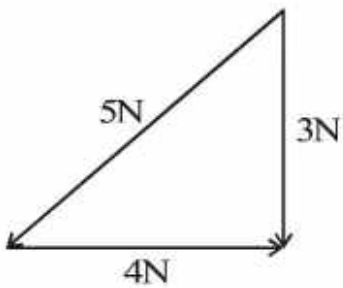
(b)



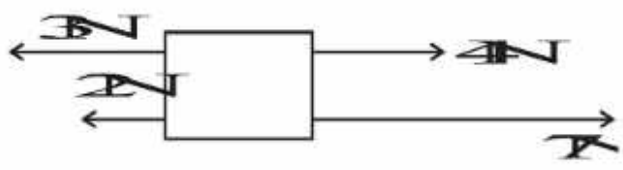
(c)



(d)



119. What is the resultant forces in diagram shown?



(a) Zero

(b) 6N to left

(c) 6N to right

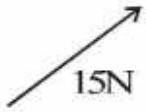
(d) 11N to right

120. A 9N force and a 12N force acting at right angles as shown in figure. Which of the following diagrams shows resultant force?



(a) 

(b) 

(c) 

(d) 

121. If position vector $\vec{r}$ and force $\vec{F}$ are in same direction then torque will be:

(a) Maximum **(b)** Minimum

(c) Same **(d)** None of these

122. If a vector is multiplied by a scalar then new quantity is:

(a) Scalar **(b)** Vector

(c) Both (a), (b) **(d)** None of these

123. If θ is angle between $\vec{A}$ and $\vec{B}$ then their resultant:

(a) $\sqrt{A^2 + B^2}$ **(b)** $\sqrt{A^2 + B^2 + 2AB\cos\theta}$

(c) $\sqrt{A^2 - B^2}$ **(d)** $\sqrt{A^2 + B^2 - AB\sin\theta}$

124. Scalar product of two vectors obey … law:

(a) Commutative **(b)** Distributive

(c) Associative **(d)** All

125. The angle between $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ is:

(a) 0° **(b)** 180°

(c) 90° **(d)** 45°

126. If $\vec{A}$ and $\vec{B}$ are parallel to each other then:

(a) $\vec{A} \cdot \vec{B} = 0$ **(b)** $\vec{A} \cdot \vec{B} = 1$

(c) $\vec{A} \cdot \vec{B} = AB$ **(d)** $\vec{A} \times \vec{B} = AB$

127. The magnitude of vector product of two vectors is $\sqrt{3}$ times then scalar product. Angle between vectors is:

(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ **(d)** $\frac{\pi}{4}$

128. The magnitude of the resultant of two forces is 10 N. One of the forces is of magnitude $10\sqrt{2}$ N. It makes an angle of 45° with resultant. The magnitude of other force is:

(a) 10 N **(b)** $10\sqrt{2}$ N

(c) 100 N **(d)** 10^9 N

129. A girl can throw a ball horizontally with a velocity 6 ms^{-1}. If she throws the ball at that speed while moving in a car at a speed of 8 ms^{-1} in a direction at right angles to the motion of the car, then the resultant velocity, in magnitude is:

(a) 2 ms^{-1} **(b)** 4 ms^{-1}

(c) 6 ms^{-1} **(d)** 10 ms^{-1}

130. Two forces of magnitudes 8 N and 15 N act at a point. If the resultant force is 17 N, then the angle between the forces has to be:

(a) 60° **(b)** 45°

(c) 90° **(d)** 30°

131. A vector $\vec{A}$ is added to the sum of two vectors $3\hat{i} - 2\hat{j} - 2\hat{k}$ and $2\hat{i} - \hat{j} + 3\hat{k}$ such that the resultant is a unit vector along z-axis. The value of $\vec{A}$ is:

(a) $-5\hat{i} + 3\hat{j}$ **(b)** $5\hat{i} - 3\hat{j}$

(c) $\hat{i} - \hat{k}$ **(d)** $\hat{k} + \hat{i} - \hat{j}$

132. A person travels 4 km east, then 4 km south and finally travels in such away that his journey terminates 8 km directly east of the starting point. What is the magnitude of the displacement during the third leg of the journey?

(a) 4 km **(b)** $\frac{4}{\sqrt{2}}$ km

(c) $4\sqrt{2}$ km **(d)** 16 km

133. The vector sum of N coplanar forces each of magnitude F, when each force is making an angle of $\frac{2\pi}{N}$ with that preceding it, is:

(a) F **(b)** $\frac{NF}{2}$

(c) NF **(d)** Zero

134. A vector $\vec{F_1}$ is along the positive direction of x-axis. Its vector product with another vector $\vec{F_2}$ is zero. Now, $\vec{F_2}$ is possibly equal to:

(a) $3\hat{j}$ **(b)** $-17.5(\hat{i} + \hat{j})$

(c) $11(\hat{j} + \hat{k})$ **(d)** $-2\hat{i}$

135. The resultant of three vectors whose magnitudes are 3 units in east, 12 units in north and 4 units vertically upwards is:

(a) $\sqrt{24}$ **(b)** 13

(c) $\sqrt{265}$ **(d)** 19

136. If the magnitudes of the vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 3, 4 and 5 units respectively and if $\vec{A} + \vec{B} = \vec{C}$, then the angle between $\vec{B}$ and $\vec{C}$ is:

(a) $\pi/2$ **(b)** arc cos (0.8)

(c) arc tan (0.75) **(d)** $\pi/4$

137. The point of application of the applied force $\vec{F} = 5\hat{i} - 3\hat{j} + 2\hat{k}$ is moved from $\vec{r_1} = 2\hat{i} + 7\hat{k} + 4\hat{k}$ to $\vec{r_2} = -5\hat{i} + 2\hat{j} + 3\hat{k}$. The work done by the applied force is:

(a) –22 units **(b)** 0 units

(c) –79.5 units **(d)** –9.8 units

138. If $0.6\hat{i} + 0.4\hat{j} + c\hat{k}$ represents a unit vector, then c is:

(a) 0.8 **(b)** $\sqrt{0.48}$

(c) $\sqrt{0.52}$ **(d)** Zero

139. Given $|\hat{a} \cdot \hat{b}|^2 - |\hat{a} \times \hat{b}|^2 = c$. What is value of c?

(a) ab sin θ **(b)** ab $\cos^2 \theta$

(c) sin 2θ **(d)** cos 2θ

140. If $\vec{P} \cdot \vec{Q} = |\vec{P} \times \vec{Q}|$. What is angle between $\vec{P}$ and $\vec{Q}$?

(a) 30° **(b)** 45°

(c) 60° **(d)** 90°

141. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{C} = \vec{C} \times \vec{A}$. Then:

(a) $\vec{A} = 0$ **(b)** $\vec{A} + \vec{B} = 0$

(c) $\vec{B} + \vec{C} = 0$ **(d)** $\vec{A} + \vec{B} + \vec{C} = 0$

ANSWERS

1.	(c)	**2.**	(a)	**3.**	(d)	**4.**	(a)
5.	(d)	**6.**	(a)	**7.**	(b)	**8.**	(c)
9.	(c)	**10.**	(b)	**11.**	(a)	**12.**	(d)
13.	(b)	**14.**	(a)	**15.**	(d)	**16.**	(c)
17.	(b)	**18.**	(a)	**19.**	(d)	**20.**	(a)
21.	(b)	**22.**	(c)	**23.**	(c)	**24.**	(b)
25.	(b)	**26.**	(a)	**27.**	(b)	**28.**	(a)
29.	(b)	**30.**	(c)	**31.**	(d)	**32.**	(c)
33.	(a)	**34.**	(c)	**35.**	(c)	**36.**	(b)
37.	(d)	**38.**	(c)	**39.**	(c)	**40.**	(b)
41.	(a)	**42.**	(d)	**43.**	(b)	**44.**	(b)
45.	(c)	**46.**	(b)	**47.**	(b)	**48.**	(b)
49.	(c)	**50.**	(b)	**51.**	(a)	**52.**	(a)
53.	(b)	**54.**	(c)	**55.**	(b)	**56.**	(b)
57.	(b)	**58.**	(c)	**59.**	(c)	**60.**	(a)
61.	(c)	**62.**	(c)	**63.**	(a)	**64.**	(b)
65.	(c)	**66.**	(b)	**67.**	(a)	**68.**	(b)
69.	(a)	**70.**	(b)	**71.**	(a)	**72.**	(d)
73.	(d)	**74.**	(c)	**75.**	(b)	**76.**	(d)
77.	(b)	**78.**	(a)	**79.**	(b)	**80.**	(a)
81.	(c)	**82.**	(d)	**83.**	(b)	**84.**	(b)
85.	(b)	**86.**	(d)	**87.**	(b)	**88.**	(c)
89.	(b)	**90.**	(c)	**91.**	(a)	**92.**	(d)
93.	(c)	**94.**	(c)	**95.**	(b)	**96.**	(b)
97.	(c)	**98.**	(b)	**99.**	(d)	**100.**	(c)
101.	(a)	**102.**	(b)	**103.**	(c)	**104.**	(a)
105.	(b)	**106.**	(b)	**107.**	(a)	**108.**	(b)
109.	(a)	**110.**	(b)	**111.**	(c)	**112.**	(a)
113.	(d)	**114.**	(b)	**115.**	(c)	**116.**	(d)
117.	(c)	**118.**	(a)	**119.**	(c)	**120.**	(c)
121.	(b)	**122.**	(b)	**123.**	(b)	**124.**	(d)
125.	(b)	**126.**	(c)	**127.**	(c)	**128.**	(a)
129.	(d)	**130.**	(c)	**131.**	(a)	**132.**	(c)
133.	(d)	**134.**	(d)	**135.**	(b)	**136.**	(b)
137.	(a)	**138.**	(b)	**139.**	(d)	**140.**	(b)
141.	(d)						

SHORT QUESTIONS

2.1 Define the terms (i) unit vector (ii) Position vector (iii) Components of a vector.

Ans. **(i) Unit Vector:** A vector whose magnitude is one called unit vector. It is used to find the direction of a vector. The formula for the unit vector is

$$\hat{A} = \frac{\vec{A}}{|A|}$$

(ii) Position Vector: It is a vector that describe the location of a particle with respect to origin. The position vector $\vec{r}$ of point P(a, b) in x-y plane is given by

$$\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$$

where $\hat{k}$ is the unit vector along z-axis. In three dimension, the position and $\vec{r}$ from origin will

$$\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along x, y and z-axis respectively.

(iii) Component of a Vector: The part of a vector effective in a particular direction is called the components of a vector. Usually a vector has two or more components, one along x-axis is called horizontal component and other along y-axis is called vertical component.

2.2 The vector sum of three vectors gives a zero resultant. What can be the orientation of the vectors?

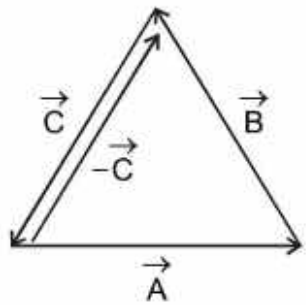
Ans. The resultant of three vectors of equal magnitudes is equal to zero if they are represented by the three adjacent sides of a triangle as shown. If we have three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$. By using head to tail rule where $-\vec{C}$ is the resultant of $\vec{A}$ and $\vec{B}$.

Hence

$$-\vec{C} = \vec{A} + \vec{B}$$

$$\vec{A} + \vec{B} + \vec{C} = 0$$

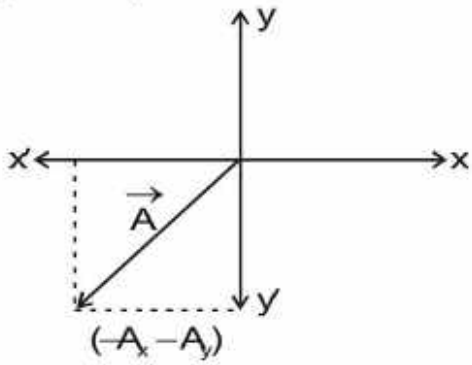
Thus the vector sum of three vectors is zero.

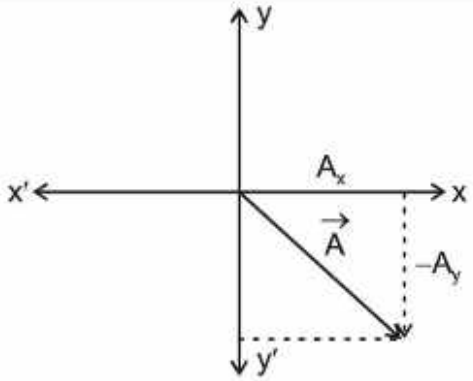


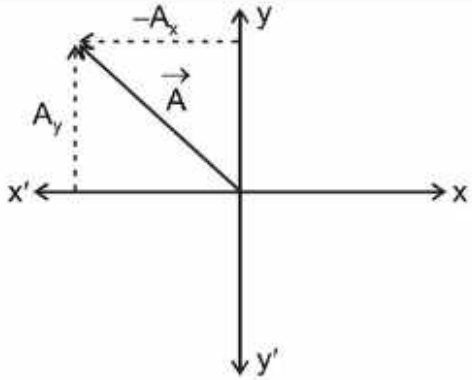
2.3 Vector A lies in the xy-plane. For what orientation will both of its rectangular components be negative. For what orientation will its components have opposite signs?

Ans. ***Case-I:*** If a vector $\vec{A}$ lies in third quadrant then both of its rectangular components A_x and A_y will be negative as shown.

Case-II: If a vector $\vec{A}$ lies in second and fourth quadrant then both of it, rectangular components A_x and A_y have in opposite sign as shown.







2.4 If one of the rectangular components of a vector is not zero, can its magnitude be zero? Explains.

Ans. No, its magnitude cannot be zero because, the magnitude of vector contains the sum of square of its components. So if one of the components of a vector is not zero and even if they have the opposite signs then the magnitude of a vector cannot be zero. According to formula

$$A = \sqrt{A_x^2 + A_y^2}$$

If $A_y = 0$

Then $A = \sqrt{A_x^2 + 0^2}$

$$A = \sqrt{A_x^2}$$

$$A = A_x$$

$\therefore$ $A \neq 0$

So if one of the rectangular components of a vector is not zero then its magnitude cannot be zero.

2.5 Can a vector have a component greater than the vector's magnitude?

Ans. (i) No, the magnitude of a vector cannot have a component greater than its magnitude because the components of a vector is always less in magnitude of resultant vector. Only in case of equilateral triangle, they are equal.

(ii) Yes, the statement is correct if we do not take the case of rectangular component. So a vector has a component greater than vector magnitude.

2.6 Can the magnitude of a vector have a negative value?

Ans. No, the magnitude of a vector cannot have a negative value. The magnitude of a vector always has a positive value. For example, if we have a vector $-3\vec{A}$, where 3 is the magnitude of a vector and the negative sign shows its direction.

(OR)

As magnitude of $\vec{A}$ is

$$A = \sqrt{A_x^2 + A_y^2}$$

Hence magnitude of a vector cannot have a negative value. e.g.,

If $A_x = -5$ and $A_y = 2$

then $A = \sqrt{(-5)^2 + (2)^2}$

$$= \sqrt{25 + 4}$$

$$= \sqrt{29}$$

2.7 If $\vec{A} + \vec{B} = \vec{0}$, what can you say about the components of the two vectors?

Ans. If $\vec{A} + \vec{B} = \vec{0}$

$$\vec{A} = -\vec{B}$$

In case of rectangular components

$$A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = -(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = -B_x\hat{i} - B_y\hat{j} - B_z\hat{k}$$

Comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$

$$A_x = -B_x$$

$$A_y = -B_y$$

$$A_z = -B_z$$

So it means that if the sum of the two vectors is zero then their rectangular components will be of the same magnitude but in opposite direction.

2.8 Under what circumstances would a vector have components that are equal in magnitude?

Ans. If θ be the angle which vector $\vec{A}$ makes with horizontal line having components A_x and A_y then

$$\tan\theta = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

According to question

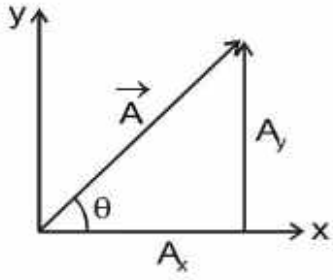
$$A_x = A_y$$

then

$$\theta = \tan^{-1}\left(\frac{A_y}{A_y}\right)$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 45°$$



So, if $\vec{A}$ makes an angle of 45° with x-axis then its both components will be equal in magnitude.

2.9 Is it possible to add a vector quantity to a scalar quantity? Explain.

Ans. No, a vector quantity cannot be added to a scalar quantity because scalar has only magnitude while vector has both the magnitude and direction. So they cannot be added to each other.

2.10 Can you add zero to a null vector?

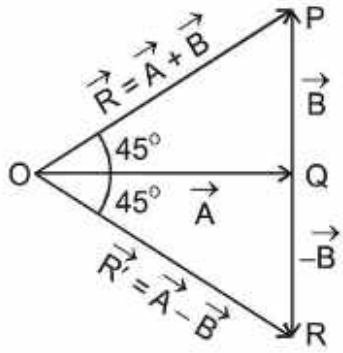
Ans. No, zero is not added to a null vector because zero is a scalar and null vector is a vector quantity.

2.11 Two vectors have unequal magnitudes. Can their sum be zero? Explain.

Ans. No, the sum of two vectors of unequal magnitude cannot be zero. It is only possible when two vectors have same magnitude and in opposite direction.

2.12 Show that the sum and difference of two perpendicular vectors of equal lengths are also perpendicular and of the same length.

Ans. Consider two vectors $\vec{A}$ and $\vec{B}$ having equal magnitudes as shown. From head to tail rule, $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ are the resultant having same magnitude because they are the hypotenuse of right angled triangle.



Since in ΔOPQ

$$\angle POQ = \angle QPO$$
$$= 45°$$

and magnitude of $\vec{R}$ will be

$$|\vec{R}| = \sqrt{A^2 + B^2}$$

and in ΔOQR

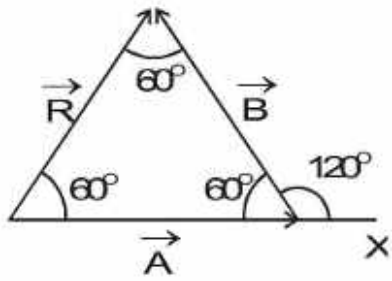
$$\angle ROQ = \angle ORQ = 45°$$

So $$\angle POQ + \angle ROQ = 45° + 45°$$
$$= 90°$$

Hence the resultants $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ are perpendicular to each other and they are in equal length.

2.13 How would the two vectors of the same magnitude have to be oriented, if they were to be combined to give a resultant equal to a vector of the same magnitude.

Ans. If two vectors $\vec{A}$ and $\vec{B}$ make an angle of 120°. Then their resultant would have the same magnitude as that of $\vec{A}$ and $\vec{B}$.



We know that:

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB + \cos\theta}$$

According to given condition $A = B = R$.

$$\because \quad R = \sqrt{R^2 + R^2 + 2R^2 \cos\theta}$$

$$R^2 = 2R^2 + 2R^2 \cos\theta$$

$$R^2 - 2R^2 = 2R^2 \cos\theta$$

$$\frac{-R^2}{2R^2} = \cos\theta \quad \Rightarrow \quad \cos\theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = 120°$$

2.14 The two vectors to be combined have magnitudes 60 N and 35 N. Pick the correct answer from those given below and tell why is it the only one of the three that is correct.

(i) 100N (ii) 70N (iii) 20N

Ans. When 60 N and 35 N forces are in same direction then maximum resultant is

$$= 60 + 35 = 95 \text{ N}$$

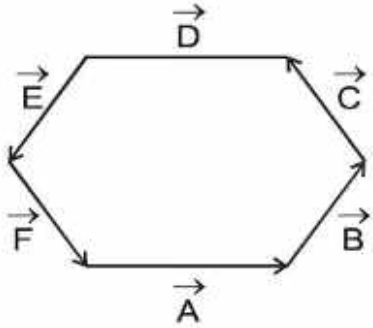
When 60 N and 35 N forces are in opposite direction then minimum resultant is

$$= 60 - 35 = 25 \text{ N}$$

Hence the range of the force is 25 N – 95 N then (ii) 70 N is correct.

2.15 Suppose the sides of a closed polygon represent vector arranged head to tail? What is the sum of these vectors?

Ans. We know that if the vectors are arranged by head to tail rule, which makes a closed polygon then its resultant is zero because there is no place to draw resultant. Consider $\vec{A}, \vec{B}, \vec{C}, \vec{D}, \vec{E}$ and $\vec{F}$ are the vectors which are arranged by head to tail rule then their resultant is zero i.e.,



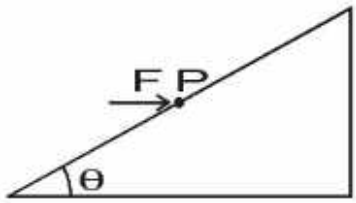
$$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} + \vec{F} = 0$$

2.16 Identify the correct answer:

(i) Two ships X and Y are traveling in different directions at equal speeds. The actual direction of motion of X is due north but to an observer on Y, the apparent direction of motion of X is north-east. The actual direction of motion of Y as observed from the shore will be.

(a) East (b) West (c) South-east (d) South-west

(ii) A horizontal force F is applied to a small object P of mass m at rest on a smooth plane inclined at an angle θ to the horizontal as shown in figure. The magnitude of the resultant force acting up and along the surface of the plane, on the object is:



(a) F cos θ – mg sin θ (b) F sin θ – mg cos θ

(c) F cos θ + mg cos θ (d) F sin θ + mg sin θ

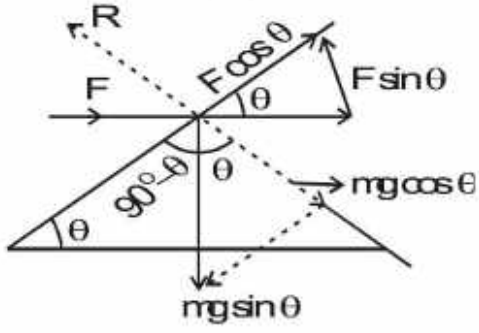
(e) mg tan θ

Ans. (i) We know that the ship x is moving towards North from shore and according to observer on ship y, the ship x is moving towards north-east direction so ship y is approaching towards the line of motion of ship x. Thus the motion of ship y is towards west so (b) is correct.

(ii) Now the horizontal force F and weight mg of the body can be resolved into its rectangular components as shown. The force acting up along the plane of the surface is

$$= F \cos \theta - mg \sin \theta$$

So (a) is correct.



2.17 If all the components of the vectors, $\vec{A}_1$ and $\vec{A}_2$ were reversed, how would this alter $\vec{A}_1 \times \vec{A}_2$?

Ans. The vectors $\vec{A}_1$ and $\vec{A}_2$ can be resolved into its rectangular components i.e.,

$$\vec{A}_1 = A_{1x}\hat{i} + A_{1y}\hat{j} + A_{1z}\hat{k}$$

$$\vec{A}_2 = A_{2x}\hat{i} + A_{2y}\hat{j} + A_{2z}\hat{k}$$

$$\vec{A}_1 \times \vec{A}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$$

On reversing the components

$$\vec{A}_1 = -A_{1x}\hat{i} - A_{1y}\hat{j} - A_{1z}\hat{k}$$

$$\vec{A}_2 = -A_{2x}\hat{i} - A_{2y}\hat{j} - A_{2z}\hat{k}$$

So $$\vec{A}_1 \times \vec{A}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -A_{1x} & -A_{1y} & -A_{1z} \\ -A_{2x} & -A_{2y} & -A_{2z} \end{vmatrix}$$

Taking (-1) as common from R_1 and R_2

$$\vec{A}_1 \times \vec{A}_2 = (-)(-)\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$$

$$\vec{A}_1 \times \vec{A}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$$

Hence if the components of the vectors are reversed then there is no effect on $\vec{A}_1 \times \vec{A}_2$.

2.18 Name the three different conditions that could make $\vec{A}_1 \times \vec{A}_2 = 0$.

Ans. We know that

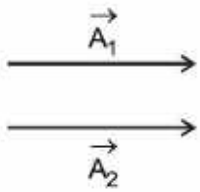
$$\vec{A}_1 \times \vec{A}_2 = A_1 A_2 \sin\theta\, \hat{n}$$

Therefore the required three conditions are

(i) Either $\vec{A}_1$ or $\vec{A}_2$ is null vector i.e., $|\vec{A}_1| = |\vec{A}_2| = 0$.

$$\vec{A}_1 \times \vec{A}_2 = 0$$

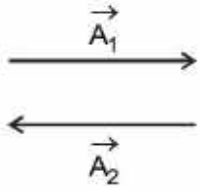
(ii) $\vec{A_1}$ and $\vec{A_2}$ are parallel i.e., $\theta = 0°$.



$$\vec{A_1} \times \vec{A_2} = A_1A_2 \sin 0° \hat{n} \qquad \because \sin 0° = 0$$

$$\vec{A_1} \times \vec{A_2} = 0$$

(iii) $\vec{A_1}$ and $\vec{A_2}$ are antiparallel i.e., $\theta = 180°$



$$\vec{A_1} \times \vec{A_2} = A_1A_2 \sin 180° \hat{n}$$

$$\Rightarrow \vec{A_1} \times \vec{A_2} = 0 \qquad \because \sin 180° = 0$$

2.19 Identify true or false statements and explain the reason:

(a) A body in equilibrium implies that it is not moving nor rotating.

(b) If coplanar forces acting on a body form a closed polygon, then the body is said to be in equilibrium.

Ans. (a) This statement is false because the body is said to be in equilibrium if it is moving with constant velocity or rotating with constant angular velocity.

(OR)

This statement is true because when a body is not moving nor rotating then it is in static equilibrium.

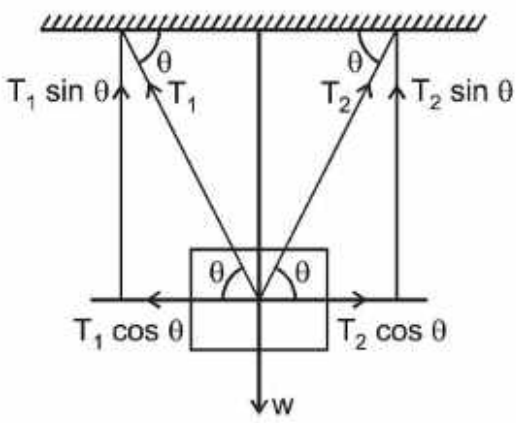
(b) This statement is true because when coplanar forces (vectors) acting on a body in the form of a closed polygon then $\sum \vec{F} = \vec{0}$ i.e., 1st condition of equilibrium, is satisfied so the body is in translational equilibrium.

(OR)

This statement is false because there may be any torque due to these forces i.e., 2nd condition of equilibrium is not satisfied so the body is not in complete equilibrium.

2.20 A picture is suspended from a wall by two strings. Show by diagram the configuration of the strings for which the tension in the settings will be minimum.

Ans. Let we suspend the picture from the wall by two strings as shown in figure.



Let T_1 and T_2 be the tension in string from figure.

Along x-axis

$$T_1 \cos \theta = T_2 \cos \theta$$

$$\Rightarrow \quad T_1 = T_2 = T$$

Along y-axis

$$T_1 \sin \theta + T_2 \sin \theta = w$$

$$T \sin \theta + T \sin \theta = w \qquad \because T_1 = T_2 = T_3$$

$$T = \frac{w}{2 \sin \theta}$$

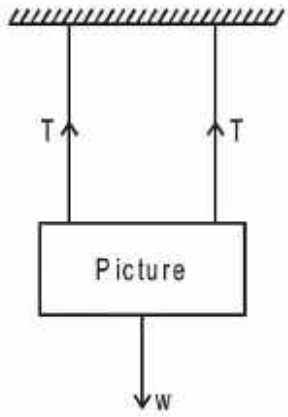
For minimum tension $\sin \theta$ should have max. value and max. value of $\sin \theta = 1$.

$$\Rightarrow \quad \sin \theta = 1$$

$$\sin \theta = \sin 90°$$

$$\theta = 90°$$

So for minimum tension θ should be 90° as shown in figure.



From figure

$$2T = w$$

$$T = \frac{w}{2}$$

So only in this case, the tension in both the string will be minimum.

2.21 Can a body rotate about its centre of gravity under the action of its weight?

Ans. No, a body cannot rotate about its centre of gravity under the action of weight because the line of action of force (weight) passes through its center of gravity (pivot) i.e., movement arm $r = 0$.

So $\tau = rF$

$$\tau = (0)\, F$$

$$\tau = 0$$

So the torque is zero.

PROBLEMS WITH SOLUTIONS

PROBLEM 2.1

Suppose, in a rectangular coordinates system, a vector $\vec{A}$ has its tail at the point P(–2, –3) and its tip at Q(3, 9). Determine the distance between these two points?

Data

The given points are

P(–2, –3) and Q(3, 9)

To Find

Distance between P and Q = r = ?

SOLUTION

Position vector of a $= \vec{r}_1 = -2\hat{i} - 3\hat{j}$

Position vector of b $= \vec{r}_2 = 3\hat{i} + 9\hat{j}$

$$\Rightarrow \quad \vec{A} = \vec{r}_2 - \vec{r}_1$$

$$= 3\hat{i} + 9\hat{j} - (-2\hat{i} - 3\hat{j})$$

$$= 3\hat{i} + 9\hat{j} + 2\hat{i} + 3\hat{j}$$

$$\vec{A} = 5\hat{i} + 12\hat{j} \qquad \ldots\ldots \text{(i)}$$

The magnitude of $\vec{A}$ is:

$$A = \sqrt{(5)^2 + (12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13 \text{ units}$$

Result

Distance between P and Q = r = 13 units

PROBLEM 2.2

A certain corner of a room is selected as the origin of the rectangular coordinate system. If an insect is crawling on an adjacent wall at a point having coordinates (2, 1), where the units are in meters? What is the distance of the insect from this corner of the room?

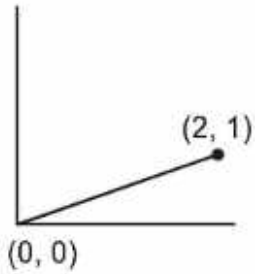
Data

In a coordinate system, the given point is (2, 1)

To Find

Distance of insect from the corner of room = r = ?

SOLUTION



Let the corner of the room whose coordinate are (0, 0)

The position vector is

$$\vec{r} = 2\hat{i} + 1\hat{j}$$

The magnitude of position vector $\vec{r}$ is

$$|\vec{r}| = \sqrt{(2)^2 + (1)^2}$$
$$= \sqrt{4+1}$$
$$= \sqrt{5}$$
$$r = 2.24 \text{ m}$$

Result

Distance of insect = r = 2.24 m

PROBLEM 2.3

What is the unit vector in the vector $\vec{A} = 4\hat{i} + 3\hat{j}$?

Data

The given vector is

$$\vec{A} = 4\hat{i} + 3\hat{j}$$

To Find

Unit vector = $\hat{A}$ = ?

SOLUTION

By formula

$$\hat{A} = \frac{\vec{A}}{|A|}$$
$$= \frac{4\hat{i} + 3\hat{j}}{\sqrt{4^2 + 3^2}}$$
$$= \frac{4\hat{i} + 3\hat{j}}{\sqrt{16+9}}$$

$$\hat{A} = \frac{4\hat{i} + 3\hat{j}}{5}$$

Result

$$\text{Unit vector} = \hat{A} = \frac{4\hat{i} + 3\hat{j}}{5}$$

PROBLEM 2.4

Two particles are located at $\vec{r}_1 = 3\hat{i} + 7\hat{j}$ and $\vec{r}_2 = -2\hat{i} + 3\hat{j}$ respectively. Find both the magnitude of vector $(\vec{r}_2 - \vec{r}_1)$ and its orientation with respect to the x-axis.

Data

The position vectors are

$$\vec{r}_1 = 3\hat{i} + 7\hat{j}$$

$$\vec{r}_2 = -2\hat{i} + 3\hat{j}$$

To Find

$$\text{Magnitude of vector} = \vec{r}_2 - \vec{r}_1 = ?$$

$$\text{Direction} = \theta = ?$$

SOLUTION

As we know that

$$\vec{r}_2 - \vec{r}_1 = -2\hat{i} + 3\hat{j} - (3\hat{i} + 7\hat{j})$$

$$= -2\hat{i} + 3\hat{j} - 3\hat{i} - 7\hat{j}$$

$$= -5\hat{i} - 4\hat{j}$$

$$|\vec{r}_2 - \vec{r}_1| = \sqrt{(-5)^2 + (-4)^2}$$

$$= \sqrt{25 + 16} = \sqrt{41}$$

$$|\vec{r}_2 - \vec{r}_1| = 6.4$$

Orientation w.r.t. x-axis:

$$\phi = \tan^{-1}\left(\frac{\text{y-component}}{\text{x-component}}\right)$$

$$= \tan^{-1}\left(\frac{4}{5}\right)$$

$$\phi = 39^\circ$$

As both components are –ive so $(\vec{r}_2 - \vec{r}_1)$ lies in 3rd quadrant.

$$\Rightarrow \quad \theta = 180 + \phi$$
$$= 180 + 39$$
$$\theta = 219^\circ$$

Result

Magnitude of vector $\vec{r}_2 - \vec{r}_1 = 6.4$

Orientation $= \theta = 219^\circ$

PROBLEM 2.5

If a $\vec{B}$ is added to vector $\vec{A}$, the result is $6\hat{i} + \hat{j}$. If $\vec{B}$ is subtracted from $\vec{A}$, the result is $-4\hat{i} + 7\hat{j}$. What is the magnitude of vector $\vec{A}$?

Data

If $\vec{B}$ is added to $\vec{A}$

$$\vec{A} + \vec{B} = 6\hat{i} + \hat{j}$$

If $\vec{B}$ is subtracted from $\vec{A}$

$$\vec{A} - \vec{B} = -4\hat{i} + 7\hat{j}$$

To Find

Magnitude of vector $\vec{A} = ?$

SOLUTION

Then $\vec{A} + \vec{B} = 6\hat{i} + \hat{j}$

$\vec{A} - \vec{B} = -4\hat{i} + 7\hat{j}$

$$2\vec{A} = 2\hat{i} + 8\hat{j}$$
$$2\vec{A} = 2(\hat{i} + 4\hat{j})$$
$$\vec{A} = \hat{i} + 4\hat{j}$$

Magnitude of vector $\vec{A} = \sqrt{(1)^2 + (4)^2}$
$$= \sqrt{1 + 16}$$
$$= \sqrt{17}$$
$$= 4.12$$

Result

Magnitude of vector $\vec{A} = 4.12$

PROBLEM 2.6

Given that $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} - 4\hat{j}$, find the magnitude and direction of

(a) $\vec{C} = \vec{A} + \vec{B}$ **(b)** $\vec{D} = 3\vec{A} - 2\vec{B}$

Data

The given vectors are

$$\vec{A} = 2\hat{i} + 3\hat{j}$$

$$\vec{B} = 3\hat{i} - 4\hat{j}$$

To Find

(a) Magnitude of $\vec{C} = \vec{A} + \vec{B} = ?$

Direction of $\vec{C} = ?$

(b) Magnitude of $\vec{D} = 3\vec{A} - 2\vec{B} = ?$

Direction of $\vec{D} = ?$

SOLUTION

(a) Now $\vec{C} = \vec{A} + \vec{B}$

Putting the values

$$\vec{C} = 2\hat{i} + 3\hat{j} + 3\hat{j} - 4\hat{j}$$

$$\vec{C} = 5\hat{i} - \hat{j}$$

Magnitude of $\vec{C} = \sqrt{(5)^2 + (-1)^2}$

$$= \sqrt{25 + 1}$$

$$= \sqrt{26}$$

$$= 5.1$$

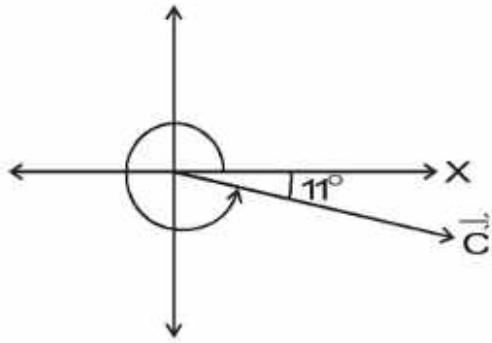
For direction, $\tan \phi = \dfrac{C_y}{C_x}$

$$\phi = \tan^{-1}\left(\frac{C_y}{C_x}\right) \qquad \text{where } C_y = -1,\ C_x = 5$$

So $\phi = \tan^{-1}\left(\dfrac{1}{5}\right)$

$$\phi = 11°$$

Since C_x is positive and C_y is negative.



So $\vec{C}$ lies in 4th quadrant.

$$\theta = 360° - 11°$$

$$\theta = 349°$$

(b) Now $\vec{D} = 3\vec{A} - 2\vec{B}$

$$= 3(2\hat{i} + 3\hat{j}) - 2(3\hat{i} - 4\hat{j})$$

$$= 5\hat{i} + 9\hat{j} - 6\hat{i} + 8\hat{j}$$

$$= 17\hat{j}$$

So the magnitude of $\vec{D} = \sqrt{(17)^2}$

$$= 17$$

And the direction of $\vec{D}$ is

$$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right) \quad \text{where } D_y = 17, \; D_x = 0$$

$$\theta = \tan^{-1}\left(\frac{17}{0}\right)$$

$$= \tan^{-1}(\infty)$$

$$\theta = 90°$$

Result

(a) Magnitude of vector $\vec{C} = 5.0$

Direction of vector $\vec{C} = \theta = 349°$

(b) Magnitude of vector $\vec{D} = 17$

Direction of vector $\vec{D} = \theta = 90°$

PROBLEM 2.7

Find the angle between two vectors $\vec{A} = 5\hat{i} + \hat{j}$ and $\vec{B} = 2\hat{i} + 4\hat{j}$.

Data

The given vectors are

$$\vec{A} = 5\hat{i} + \hat{j}$$

$$\vec{B} = 2\hat{i} + 4\hat{j}$$

To Find

Angle between the vectors $= \theta = ?$

SOLUTION

As we know that

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

$$\boxed{\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}} \qquad \ldots\ldots (i)$$

where $\vec{A} \cdot \vec{B} = (5\hat{i} + \hat{j}) \cdot (2\hat{i} + 4\hat{j})$

$$= 10(\hat{i} \cdot \hat{i}) + 20(\hat{i} \cdot \hat{j}) + 2(\hat{j} \cdot \hat{i}) + 4(\hat{j} \cdot \hat{j})$$

Since $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

and $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

So $\vec{A} \cdot \vec{B} = 10 + 4 = 14$

The magnitude of vector $\vec{A} = \sqrt{5^2 + 1^2}$

$$= \sqrt{25 + 1}$$

$$= \sqrt{26}$$

$$A = 5.1$$

The magnitude of vector $\vec{B} = \sqrt{2^2 + 4^2}$

$$= \sqrt{4 + 16}$$

$$= \sqrt{20}$$

$$B = 4.5$$

Putting the value in eq. (i)

$$\cos\theta = \frac{14}{(5.1)(4.5)}$$

$$\cos\theta = 0.61$$

$$\theta = \cos^{-1}(0.61)$$

$$\theta = 52^\circ$$

Result

Angle between vectors $= \theta = 52^\circ$

PROBLEM 2.8

Find the work done when the point of application of the force $3\hat{i} + 2\hat{j}$ moves in a straight line from the point (2, −1) to the point (6, 4).

Data

$$\text{Force} = \vec{F} = 3\hat{i} + 2\hat{j}$$

and the given points are A(2, −1), B(6, 4).

To Find

Work done = W = ?

SOLUTION

By formula

$$W = \vec{F} \cdot \vec{d}$$

Where $\vec{d}$ is distance between two given points

$$\begin{aligned} \text{So Coordinates } \vec{d} &= (6, 4) - (2, -1) \\ &= (6 - 2, 4 + 1) \\ &= (4, 5) \end{aligned}$$

$$\text{Now } \vec{d} = 4\hat{i} + 5\hat{j}$$

$$\begin{aligned} \text{Therefore } W &= (3\hat{i} + 2\hat{j}) \cdot (4\hat{i} + 5\hat{j}) \\ &= 12(\hat{i} \cdot \hat{i}) + 10(\hat{j} \cdot \hat{j}) \\ &= 12 + 10 \\ &= 22 \text{ Joule} \end{aligned}$$

Result

Work done = W = 22 J

PROBLEM 2.9

Show that the three vectors $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} - 3\hat{j} + \hat{k}$ and $4\hat{i} + \hat{j} - 5\hat{k}$ are mutually perpendicular.

Data

The given vectors are

$$\vec{A} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}$$

$$\vec{C} = 4\hat{i} + \hat{j} - 5\hat{k}$$

To Find

Are the vectors mutually perpendicular = ?

SOLUTION

If $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{C} = \vec{C} \cdot \vec{A} = 0$ so the vectors are mutual perpendicular.

Thus $\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j} + \hat{k}) \cdot (2\hat{i} - 3\hat{j} + \hat{k})$

$= 2(\hat{i} \cdot \hat{i}) - 3(\hat{j} \cdot \hat{j}) + 1(\hat{k} \cdot \hat{k})$

$= 2 - 3 + 1$

$\vec{A} \cdot \vec{B} = 0$

and $\vec{B} \cdot \vec{C} = (2\hat{i} - 3\hat{j} + \hat{k}) \cdot (4\hat{i} + \hat{j} - 5\hat{k})$

$= 8(\hat{i} \cdot \hat{i}) - 3(\hat{j} \cdot \hat{j}) - 5(\hat{k} \cdot \hat{k})$

$= 8 - 3 - 5$

$\vec{B} \cdot \vec{C} = 0$

$\vec{C} \cdot \vec{A} = (4\hat{i} + \hat{j} - 5\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$

$= 4(\hat{i} \cdot \hat{i}) + 1(\hat{j} \cdot \hat{j}) - 5(\hat{k} \cdot \hat{k})$

$= 4 + 1 - 5$

$\vec{C} \cdot \vec{A} = 0$

Hence $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} = \vec{C} \cdot \vec{A} = 0$

Result

So the given three vectors are mutually perpendicular.

PROBLEM 2.10

Given that $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} - 4\hat{k}$, find the length of the projection of $\vec{A}$ on $\vec{B}$.

Data

The given vectors are

$\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{B} = 3\hat{i} - 4\hat{k}$

To Find

Projection of $\vec{A}$ on $\vec{B}$ $= A \cos\theta$ = ?

SOLUTION

By formula

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\boxed{A\cos\theta = \frac{\vec{A} \cdot \vec{B}}{B}}$$

Now $\vec{A} \cdot \vec{B} = (\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} - 4\hat{k})$

$= 3(\hat{i} \cdot \hat{i}) - 12(\hat{k} \cdot \hat{k})$

$= 3 - 12$

$= -9$

And magnitude of vector $\vec{B}$

$$B = \sqrt{(3)^2 + (-4)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5$$

Putting in above equation

$$A\cos\theta = \frac{-9}{5}$$

Result

Projection of $\vec{A}$ on $\vec{B}$ $= A\cos\theta = -\frac{9}{5}$

PROBLEM 2.11

Vector $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 4 unit north, 3 unit west and 8 unit east, respectively. Describe carefully (a) $\vec{A} \times \vec{B}$ (b) $\vec{A} \times \vec{C}$ (c) $\vec{B} \times \vec{C}$.

Data

The given vectors are

$\vec{A}$ = 4 units north

$\vec{B}$ = 3 units west

$\vec{C}$ = 8 units east

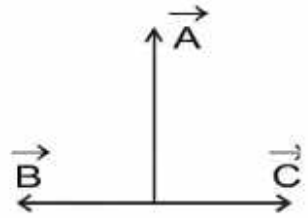
To Find

(a) $\vec{A} \times \vec{B} = ?$

(b) $\vec{A} \times \vec{C} = ?$

(c) $\vec{B} \times \vec{C} = ?$

SOLUTION



(a) As $\vec{A} \times \vec{B} = AB \sin\theta \hat{n}$

$\vec{A} \times \vec{B} = (4)(3) \sin 90° \hat{n}$

$= 12$ units vertically upward according to right hand rule

(b) $\vec{A} \times \vec{C} = AC \sin\theta \hat{n}$

$= (4)(8) \sin 90° \hat{n}$

$= 32$ units vertically downward

(c) $\vec{B} \times \vec{C} = BC \sin\theta \hat{n}$

$= (3)(8) \sin 180° \hat{n}$

$= 24(0) \hat{n}$

$\vec{B} \times \vec{C} = \vec{0}$

Result

(a) $\vec{A} \times \vec{B} = 12$ units vertically upward

(b) $\vec{A} \times \vec{C} = 32$ units vertically downward

(c) $\vec{B} \times \vec{C} = \vec{0}$ (Null vector)

PROBLEM 2.12

The torque or turning effect of force about a given point is given by $\vec{r} \times \vec{F}$ where $\vec{r}$ is the vector from the given point to the point of application of $\vec{F}$. Consider a force $\vec{F} = -3\hat{i} + \hat{j} + \hat{k}$ acting on the point $7\hat{i} + 3\hat{j} + \hat{k}$ (m). What is the torque in Nm about the origin?

Data

Force $= \vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$

Position vector $= \vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$

To Find

Torque acting $= \vec{\tau} = \vec{r} \times \vec{F} = ?$

SOLUTION

As $\vec{\tau} = \vec{r} \times \vec{F}$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} - \hat{j} \begin{vmatrix} 7 & 1 \\ -3 & 5 \end{vmatrix} + \hat{k} \begin{vmatrix} 7 & 3 \\ -3 & 1 \end{vmatrix}$$

$$= \hat{i}(15-1) - \hat{j}(35+3) + \hat{k}(7+9)$$

$$= 14\hat{i} - 38\hat{j} + 16\hat{k}$$

Result

Torque acting $= \vec{\tau} = 14\hat{i} - 38\hat{j} + 16\hat{k}$ (Nm)

PROBLEM 2.13

The line of action of force $\vec{F} = \hat{i} - 2\hat{j}$, passes through the point whose position vector is $(-\hat{j} + \hat{k})$. Find (a) the moment of F about the origin (b) the moment of $\vec{F}$ about the point of which the position vector is $\hat{i} + \hat{k}$.

Data

Force $= \vec{F} = \hat{i} - 2\hat{j}$

Position vector $= \vec{r} = -\hat{j} + \hat{k}$

To Find

(a) Moment of force about origin $= \vec{\tau} = ?$

(b) Moment of force about $\hat{i} + \hat{k} = \vec{\tau}' = ?$

SOLUTION

(a) First for the moment of force about origin is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & 1 \\ 1 & -2 & 0 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} -1 & 1 \\ -2 & 0 \end{vmatrix} - \hat{j} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \hat{k} \begin{vmatrix} 0 & -1 \\ 1 & -2 \end{vmatrix}$$

$$= \hat{i}(0+2) - \hat{j}(0-1) + \hat{k}(0+1)$$

$$\vec{\tau} = 2\hat{i} + \hat{j} + \hat{k}$$

(b) The moment of force about $\hat{i} + \hat{k}$ is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

So $$\vec{r'} = (-\hat{j} + \hat{k}) - (\hat{i} + \hat{k})$$

$$= -\hat{j} + \hat{k} - \hat{i} - \hat{k}$$

$$= -\hat{i} - \hat{j}$$

So $$\vec{\tau'} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 0 \\ 1 & -2 & 0 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} -1 & 0 \\ -2 & 0 \end{vmatrix} - \hat{j} \begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix} + \hat{k} \begin{vmatrix} -1 & -1 \\ 1 & -2 \end{vmatrix}$$

$$= \hat{i}(0) - \hat{j}(0) + \hat{k}(2+1)$$

$$= 0\hat{i} - 0\hat{j} + 3\hat{k}$$

$$\vec{\tau'} = 3\hat{k}$$

Result

(a) Moment of force about origin $= \vec{\tau} = 2\hat{i} + \hat{j} + \hat{k}$

(b) Moment of force about $\hat{i} + \hat{k} = \vec{\tau'} = 3\hat{k}$

PROBLEM 2.14

The magnitude of dot and cross products of two vectors are $6\sqrt{3}$ and 6 respectively. Find the angle between the vector.

Data

Magnitude of dot product $= \vec{A} \cdot \vec{B} = 6\sqrt{3}$

Magnitude of vector product $= |\vec{A} \times \vec{B}| = 6$

To Find

Angle between the vectors $= \theta = ?$

SOLUTION

As $\vec{A} \cdot \vec{B} = AB \cos\theta$ ..... (i)

$|\vec{A} \times \vec{B}| = AB \sin\theta$ ..... (ii)

Divide eq. (ii) by (i)

$$\frac{|\vec{A} \times \vec{B}|}{\vec{A} \cdot \vec{B}} = \frac{AB \sin\theta}{AB \cos\theta}$$

$$\frac{6}{6\sqrt{3}} = \tan\theta$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\theta = 30°$$

Result

Angle between vectors $= \theta = 30°$

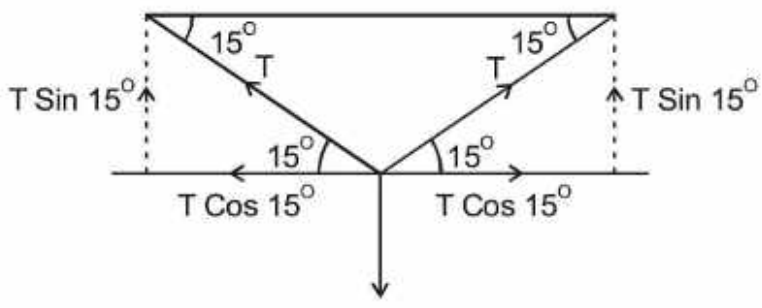
PROBLEM 2.15

A load of 10N is suspended from a clothes line. This distorts the line so that it makes an angle of 15° with the horizontal at each end. Find the tension in the clothes line.

Data

Load $= W = 10$ N

Angle with horizontal $= \theta = 15°$



To Find

Tension in the string $= T = ?$

SOLUTION

According to 1st condition of equilibrium.

Now using $\Sigma F_y = 0$ and $\Sigma F_x = 0$

So $T_1 \sin\theta + T_2 \sin\theta - \text{Load} = 0$ $T_2 \cos 15° - T_1 \cos 15° = 0$

$2T \sin\theta = \text{Load}$ $T_2 \cos 15° = T_1 \cos 15°$

$T = \dfrac{\text{Load}}{2 \sin\theta}$ $T_1 = T_2$

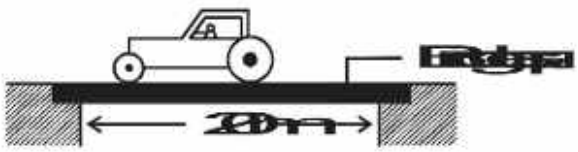
$= \dfrac{5}{0.2588}$ $\therefore$ $T_1 = T_2 = T$

$T = 19.3$ N

Result

Tension in string $= T = 19.3$ N

PROBLEM 2.16



A tractor of weight 15000N crosses a single span bridge of weight 8000 N and of length 21m. The span bridge is supported half a metre from either end. The bridge is supported half a metre from either end. The tractors front wheels take 1/3 of the total weight of the tractor, and the rear wheels are 3m behind the front wheels. Calculate the force on the bridge supports when the rear wheels are at the middle of the bridge span.

Data

Weight of tractor $= W_1 = 15000$ N

Weight of bridge $= W_2 = 8000$ N

Length of bridge $= l_1 = 21$ m

Length of tractor $= l_2 = 3.0$ m

Weight of front wheels $= W_3 = \dfrac{1}{3} \times 15000$

$= 5000$ N

Weight of rear wheels $= W_4 = 15000 - 5000$

$= 10000$ N

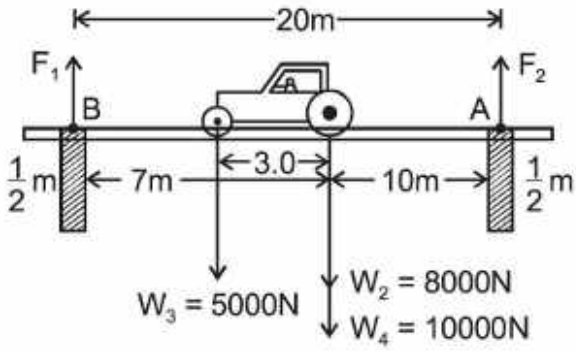
To Find

Forces on the bridge supports

$F_1 = ?$

and $F_2 = ?$

SOLUTION



By using 1st condition of equilibrium

$$\Sigma F_y = 0$$

$$F_1 + F_2 - 5000 - 8000 - 10000 = 0$$

$$F_1 + F_2 = 23000 \text{ N} \quad \ldots\ldots \text{(i)}$$

According to 2nd condition of equilibrium

$$\Sigma \tau = 0$$

Suppose the point B is the point of rotation therefore

Torque of Force F_1 $= \tau_1 = F_1 \times 0$

$= 0$

Torque of Force F_2 $= \tau_2 = F_2 \times 20$

Torque of Force W_2 and W_4 $= \tau_3 = -18000 \times 10$

Torque of Force W_3 $= \tau_4 = -5000 \times 7$

Then $\tau_1 + \tau_2 + \tau_3 + \tau_4 = 0$

$0 + F_2 \times 20 - 180000 - 35000 = 0$

$20F_2 = 215000$

$F_2 = \dfrac{215000}{20}$

$F_2 = 10750 \text{ N}$

$F_2 = 10.750 \times 10^3 \text{ N}$

$F_2 = 10.75 \text{ KN}$

Putting in eq. (i)

$F_1 + 10750 = 23000$

$F_1 = 23000 - 10750$

$= 12250 \text{ N}$

$F_1 = 12.25 \times 10^3 \text{ N}$

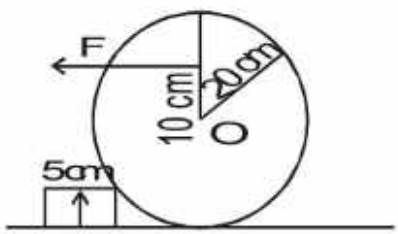
$= 12.25 \text{ KN}$

Result

Force on the bridge supports

$F_1 = 12.25 \text{ KN}$ and $F_2 = 10.75 \text{ KN}$

PROBLEM 2.17



A spherical ball of weight 50N is to be lifted over the step as shown in the figure.

Calculate the minimum force needed just to lift it above the floor.

Data

Weight of the spherical ball = W = 50 N

Radius of the spherical ball = r = 20 cm

= 0.2 m

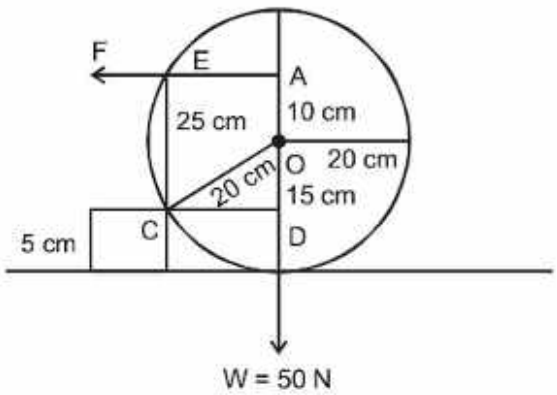
Height of the step = h = 5 cm

= 0.05 m

To Find

Minimum force required to lift the ball = F = ?

SOLUTION



Now consider the triangle ΔOCD is right angled triangle, by Pythagorean theorem

$$(OC)^2 = (OD)^2 + (CD)^2$$

$$(CD)^2 = (OC)^2 - (OD)^2$$

$$= (20)^2 - (15)^2$$

$$\sqrt{(CD)^2} = \sqrt{1750}$$

$$= 13.2 \text{ cm}$$

$$CD = 0.13 \text{ m}$$

By using 2nd condition of equilibrium

$$\Sigma\tau = 0$$

So $$-W \times CD + F \times CB = 0$$

$$-50 \times 0.13 + F \times 0.25 = 0$$

$$0.25\,F = 6.5$$

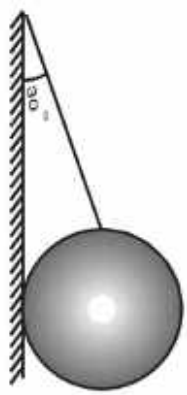
$$F = \frac{6.5}{0.25}$$

$$F = 26 \text{ N}$$

Result

Minimum force required to left ball = F = 26 N

PROBLEM 2.18



A uniform sphere of weight 10 N is held by a string attached to a frictionless wall so that the string makes an angle of 30° with the wall. Find the tension in the string and the force exerted on the sphere by the wall.

Data

Weight of the sphere = W = 10.0 N

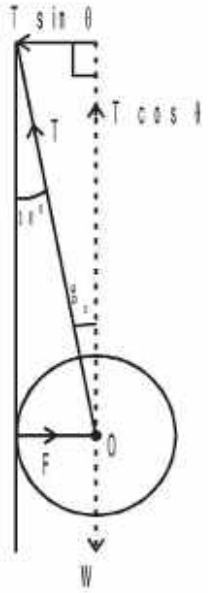
Angle of string with wall = θ = 30°

To Find

(a) Tension in the string = T = ?

(b) Force exerted by the wall = F = ?

SOLUTION



(a) Let F be the force exerted on sphere by wall, T is the tension in string and W is the weight of the sphere.

By using 1st condition of equilibrium

$$\Sigma F_y = 0 \quad \text{and} \quad \Sigma F_x = 0$$

So $F - T \sin\theta = 0$

$$F = T \sin\theta$$

and $T \cos\theta - W = 0$

$$T \cos\theta = W$$

$$T = \frac{W}{\cos\theta}$$

$$= \frac{10.0}{\cos 30} = \frac{10.0}{0.866}$$

$$T = 11.55 \text{ N}$$

and $F = T \sin\theta$

$$= 11.55 \sin 30°$$

$$F = 5.8 \text{ N}$$

Result

(a) Tension in the string = T = 11.55 N

(b) Force exerted by the wall = F = 5.8 N