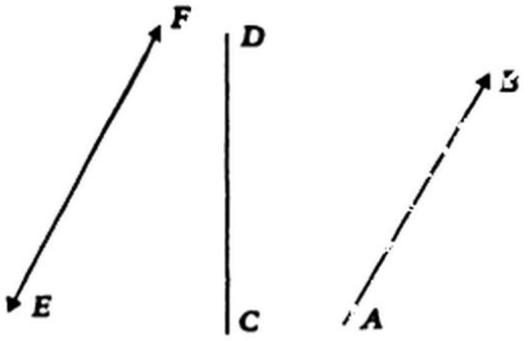
Chapter # 10

GEOMETRY

EXERCISE 10.1

Q.1 *In figure (1) which set of point is a (a) line, (b) line-segment and (c) ray?*



Solution:

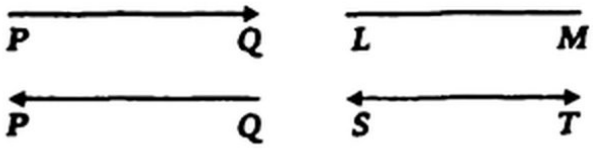
$\overrightarrow{AB}$ is a ray.

CD is a line-segment.

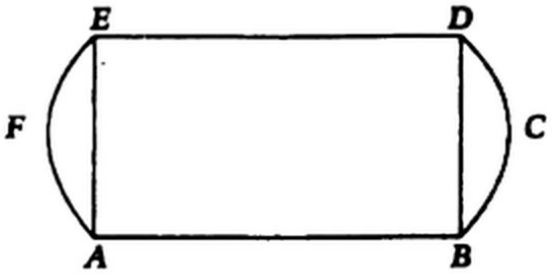
EF is a line.

Q.2 *Draw a figure to represent* $\overrightarrow{QP}$, $\overrightarrow{ST}$, $\overrightarrow{PQ}$, $\overrightarrow{LM}$.

Solution:



Q.3 ***Name the line-segment contained in figure (2).***

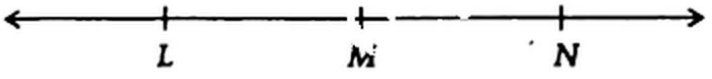


Solution:

The given figure contain following line-segments.

$\overline{AB}$, $\overline{BD}$, $\overline{DE}$, and $\overline{EA}$,

Q.4 ***Name the rays whose end points are L. M and N in Fig (3).***



Figure

Solution:

The rays name are following:

$\overrightarrow{ML}$, $\overrightarrow{NL}$, $\overrightarrow{MN}$, $\overrightarrow{LN}$,

Q.5 ***If $\overrightarrow{AB}$ and $\overrightarrow{AC}$ be opposite rays, Which of the points A, B, C is between the other two and which point is beyond which point?***

Solution:

Suppose the given rays from the following figures.

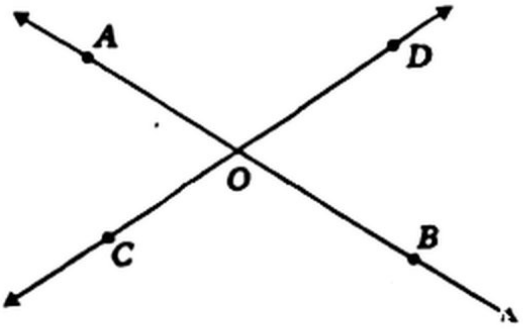


The point A is between points B and C on line

BA. Point C is beyond point A on line CA, point B is beyond point A.

Q.6 ***Lines AB, CD intersect in a point O***

Name the two pairs of opposite rays so formed.



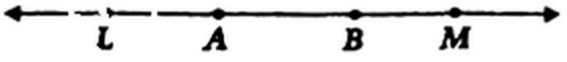
Solution:

The two pairs of opposite rays are.

(1) $\overrightarrow{OC}$ and $\overrightarrow{OD}$

(2) $\overrightarrow{OA}$ and $\overrightarrow{OB}$

Q.7 ***Figure (5) contains some collinear rays. Give reasons in support of the following statements.***



(i) $\overrightarrow{AL}$ ***and*** $\overrightarrow{AL}$ ***are opposite rays.***

(ii) $\overrightarrow{BM}$ ***and*** $\overrightarrow{AL}$ ***are not opposite rays.***

(iii) $\overrightarrow{AM}$ ***and*** $\overrightarrow{BL}$ ***are not opposite rays.***

(iv) $\overrightarrow{LA}$ ***and*** $\overrightarrow{LM}$ ***are the same rays.***

Solution:

Two rays are said to be opposite rays if and only if:

(i) They are collinear.

(ii) They have a common end-point.

(iii) No other point is common to them.

Hence:

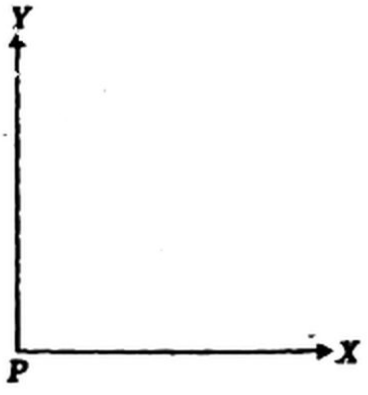
(i) A is common point between $\overrightarrow{AB}$ and $\overrightarrow{AL}$, the two rays are co-linear and there is no other common point except A. So the statement is right.

(ii) There is no common point between $\overrightarrow{AL}$ & $\overrightarrow{BM}$, so these are not opposite rays.

(iii) $\overrightarrow{AM}$ and $\overrightarrow{BL}$ are co-linear rays but there is no common point between them. There are many common points in the co-linear. So $\overrightarrow{AM}$ and $\overrightarrow{BL}$ are not opposite rays.

(iv) $\overrightarrow{LA}$ and $\overrightarrow{LM}$ is the the same ray which starts from point *L* and *A*, *M* are two point on it.

Q.8 ***In figure (6) $\overrightarrow{PX}$ and $\overrightarrow{PY}$ are co-terminous rays but they are not opposite rays. Why?***



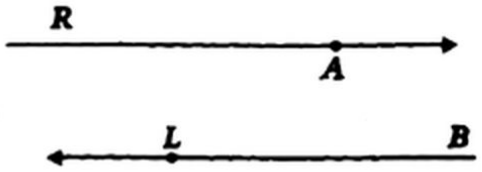
Solution:

Opposite rays have always a common end-point and are always co-linear.

Two rays $\overrightarrow{PY}$ and $\overrightarrow{PX}$ have a common end-point

but they are not co-linear, so these are not opposite rays.

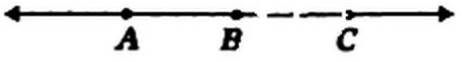
Q.9 Are $\overrightarrow{RA}$ and $\overrightarrow{BQ}$ (figure 7) opposite rays? Give reasons for your answer.



Solution:

Opposite rays are always co-linear and have a common end-point. So $\overrightarrow{RA}$ and $\overrightarrow{BQ}$ are not opposite rays.

Q.10 $\overrightarrow{BA}$ and $\overrightarrow{BC}$ (figure 8) are opposite rays. Is ABC an angle? If not, why?



Solution:

ABC is not an angle, because an angle is a union of such two non-colinear rays which have a common end-point. Here *BC* and *BA* have a common point but they are co-linear. Here *ABC* is not an angle end.

(iii) The complement of $\angle ACL$ is $\angle LAC$ or $\angle ABL$.

(iv) The supplement of $\angle ACL$ is $\angle ACD$.

(v) Adjacent supplement angles are:

(a) $\angle ACD$ and $\angle ACL$.

(b) $\angle ALC$ and $\angle BLA$.

(vi) Pair of angles which are supplementary but not adjacent is $\angle ACD$ and $\angle BAL$

(vii) In adjacent supplementary angles one arm is common and their interiors are disjoint and their outer arms are two coterminous rays.

Q.2 ***Is each of two complementary angles an acute angle?***

Solution:

Yes, as the sum of two complementary angles is 80°. So each of them will be less than 90° so an acute one.

Q.3 ***Can both of two supplementary agnles be;***

(i) Acute (ii) Obtuse

Solution:

Since the sum of two supplementary angles is always 180°. Hence either the two supplementary angles will be of 90° each or one will be acute and the other obtuse. Both the angles cannot be either obtuse or acute at the same time.

Q.4 ***If two supplementary angles are congruent (equal in measure), what will be the measure of each?***

Solution:

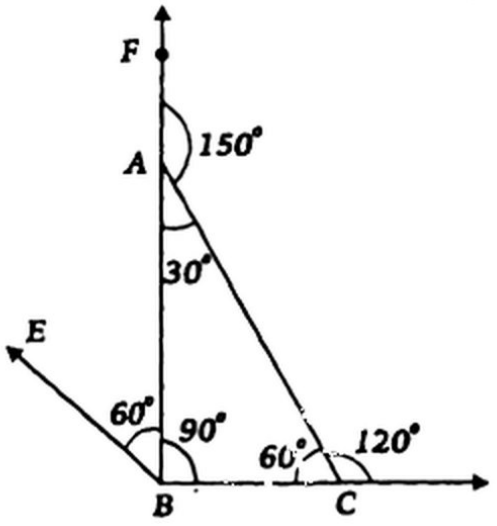
Each of them will be 90°.

Q.5 ***Two complementary angles are congruent, What is be the measure of each?***

Solution:

Each of them will be of 45°.

Q.6 ***With reference to Figure (2);***



(i) Name two pairs of angles which are adjacent and supplementary. Are their outer arms collinear?

(ii) Name a pair of angles which are adjacent but not supplementary. Are the outer arms of these angle co-linear?

(iii) What conclusion can you draw from the answers to Q. (i) and (ii)?

Solution:

(i) (a) $\angle BAC$ and $\angle FAC$

(b) $\angle ACD$ and $\angle ACB$

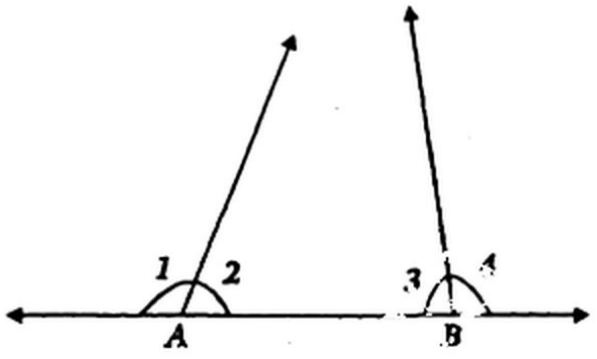
Yes, their outer arms are colinear.

(ii) $\angle CBA$ and $\angle ABE$

The outer arms of these angles are not co-linear.

(iii) We conclude from the above that if two adjacent angles are supplementary then their outer arms are co-linear and if two angles are adjacent but not supplementary then their outer arms are not co-linear.

Q.7 *In (figure 3).*



(i) Measure angles $\angle 1$, $\angle 2$ and find $m<1$ Are these supplementary angles?

(ii) Measure angles $\angle 3$, $\angle 4$. Are these supplementary angles?

Solution:

(i) $m\angle 1 = 73^\circ$

$m\angle 2 = 107^\circ$

$m\angle 1 + m\angle 2 = 73^\circ + 107^\circ = 180^\circ$

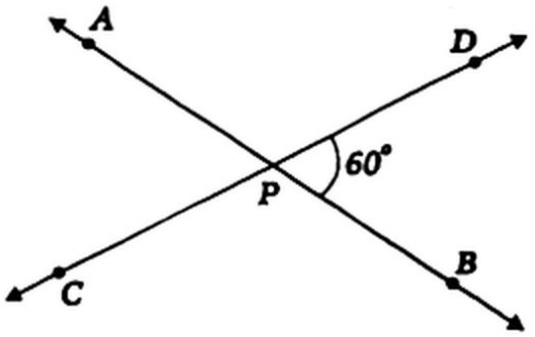
Hence these are supplementary angles.

(ii) $m\angle 3 = 72^\circ$

$m\angle 4 = 180^\circ$

$m\angle 3 + m\angle 4 = 72^\circ + 108^\circ = 180^\circ$

Hence these angles are supplementary.



Q.8 $\overrightarrow{AB}$ ***and*** $\overrightarrow{CD}$ ***intersect at a point P (figure 4).***

(i) Name two pairs of vertical angles:

(ii) Name two pairs of adjacent angles:

(iii) Measure the remaining angles and verify that vertical angles are congruent.

Solution:

(i) Two pairs of vertical angles are:

$\angle DPB$ and APC

$\angle APD$ and CPB

(ii) Two pairs of adjacent angles are:

$\angle DPB$ and $\angle DPA$

$\angle APC$ and $\angle BPC$

(ii) $\angle BPD = 60^\circ$

$\angle APD = 120^\circ$

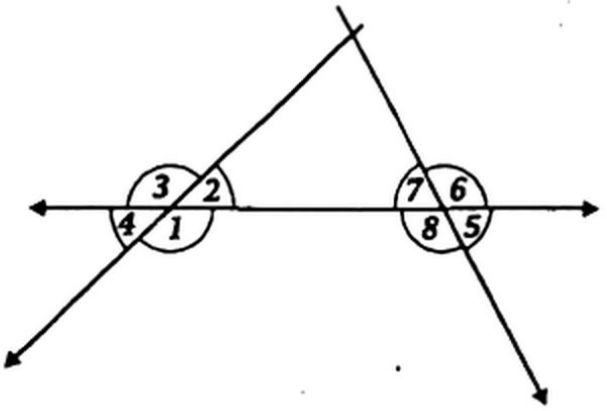
$\angle APC = 60^\circ$

$\angle CPB = 120^\circ$

Hence, Vertical angles $\angle BPD = \angle APC$

Vertical angles $\angle APD = \angle CPB$

Q.9 ***Measure all the angles from $\angle 1$ to $\angle 8$ in figure (5) and find out.***



(i) Which pairs of angles are congruent?

(ii) Which pairs are supplementary?

(iii) What conclusion do you draw about vertical angles.

Solution:

$\angle 1 = 115^\circ$

$\angle 2 = 65^\circ$

$\angle 3 = 115^\circ$

$\angle 4 = 65^\circ$

$\angle 5 = 50^\circ$

$\angle 6 = 130^\circ$

$\angle 7 = 50^\circ$

$\angle 8 = 130^\circ$

(i) The following pairs of angles are congruent.

$\angle 1$ and $\angle 3$

$\angle 2$ and $\angle 4$

$\angle 5$ and $\angle 7$

$\angle 6$ and $\angle 8$

(ii) Following pairs are supplementary:

$\angle 1$ and $\angle 2$

$\angle 2$ and $\angle 3$

$\angle 3$ and $\angle 4$

$\angle 5$ and $\angle 6$

$\angle 6$ and $\angle 7$

$\angle 7$ and $\angle 8$

$\angle 4$ and $\angle 1$

$\angle 8$ and $\angle 5$

(iii) (a) We conclude that the vertical angles are always equal.

(b) Vertical angles have a common vertex.

(c) Their arms are opposite rays.

EXERCISE 10.3

Q.1 ***What conclusions can you draw from the following statements:***

(i) ***$\angle ABC$ and $\angle ABD$ are adjacent angles and the sum of their measures is $180°$.***

(ii) ***Point B belongs to a line AC and lies between A and C and BD is a ray which does not lie on $\overrightarrow{AB}$.***

(iii) ***Lines l and m intersect at a point "O" and thus form four angles.***

Solution:

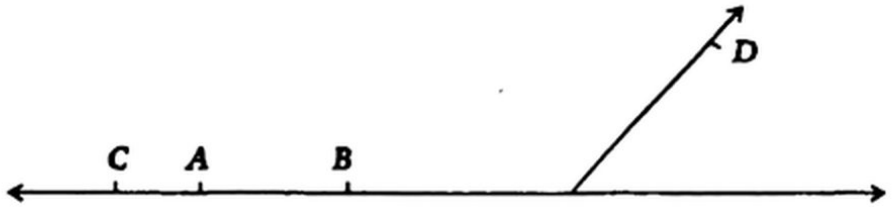
(i) The two adjacent angles which with their sum as $180°$ are called supplementary angles and their external arms are two opposite rays.

(ii) *AC* is a;

(a) Straight line of indefinite length.

(b) *BD* lies somewhere between *A* and *C*.

(c) *BD* is a line which does not lie on *AC*.



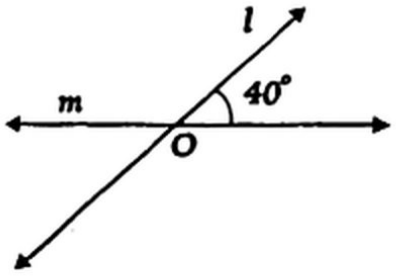
(iii) If two lines intersect at a point making four angles then.

(a) The two lines are straight lines.

(b) The vertical angles are equal.

(c) Four pairs of supplementary angles are formed.

Q.2 ***Find the measures of the remaining angles in figure (1).***



Solution:

$\angle 1 = 40^\circ$ (given)

$\angle 1 = \angle 3$ (vertical angles)

$\therefore \angle 3 = 40^\circ$

$\angle 1 + \angle 2 = 180^\circ$ (supp. and adjacent angles)

$\therefore \angle 2 = 180^\circ - 40^\circ = 140^\circ$

$\angle 2 = \angle 4$ (vertical angles)

$\therefore \angle 4 = 140^\circ$

Ans:

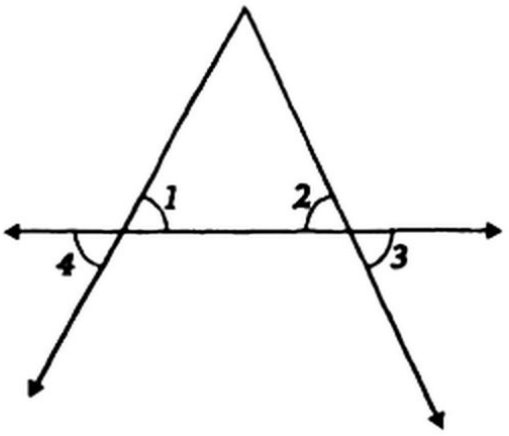
$\angle 1 = 40^\circ$

$\angle 2 = 140^\circ$

$\angle 3 = 40^\circ$

$\angle 4 = 140^\circ$

Q.3 ***In figure (2) angles 1 and 2 are congruent. Are angles 3 and 4 also congruent?***



Solution:

$\angle 1 = \angle 2$ (given)

$\angle 1 = \angle 4$ (vertical angles)

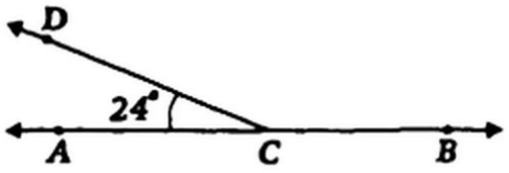
$\therefore \quad \angle 2 = \angle 4$

$\angle 2 = \angle 3$ (vertical angles)

$\angle 2 = \angle 4$ (Proved)

$\therefore \quad \angle 4 = \angle 3$ ***Ans.***

Q.4 ***In figure (3) $m\angle ACD = 24^\circ$, Find in $m\angle DCB$.***



Solution:

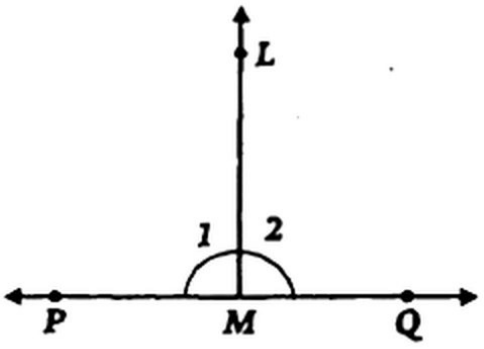
$m\angle BCD = 24^\circ$ (given)

$m\angle ACD + m\angle BCD = 180^\circ$ (supp. angles)

$\therefore \quad m\angle ACD = 180^\circ - \angle BCD$ (given)

or $\quad m\angle ACD = 180^\circ - 24^\circ = 156^\circ$ ***Ans***

Q.5 ***In figure (4) $m\angle 1 = m\angle 2$. Prove that $\overrightarrow{ML}$ to $\overrightarrow{PQ}$***



Solution:

$m\angle 1 = m\angle 2$ (given)

$m\angle 1 + m\angle 2 = 180°$ (supplementary angles)

But $m\angle 1 = 180/2 = 90°$

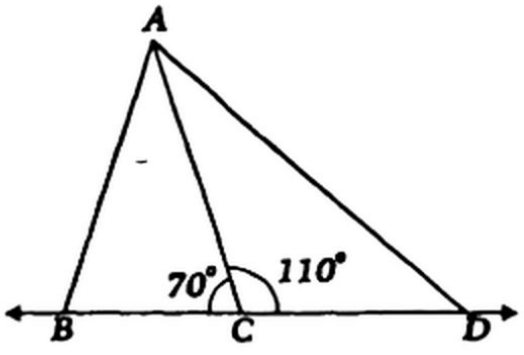
So $m\angle 1 + m\angle 1 = 180°$

or $2m(\angle 1) = 180°$

or $m\angle 1 = m\angle 2$ (given)

or $\overrightarrow{LM} \perp \overrightarrow{PQ}$ ***(Proved)***

Q.6 ***In figure (5) $m\angle ACB = 70°$ and $m\angle ACD = 110°$. Prove that $\overrightarrow{CB}$ and $\overrightarrow{CD}$ and colinear rays.***



Solution:

$$\left.\begin{matrix} m\angle ACB = 70 \\ m\angle ACD = 110 \end{matrix}\right\} \text{ given}$$

$$m\angle ACB + m\angle ACD = 70^\circ + 110^\circ = 180^\circ$$

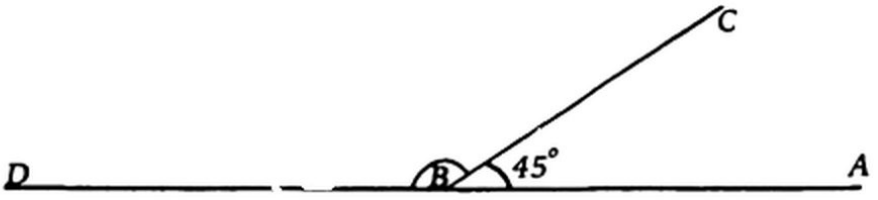
Hence $m\angle ACB$ and $\angle ACD$ are adjacent supplementary angle.

$\therefore$ $\overleftrightarrow{CB}$ and $\overleftrightarrow{CD}$ being outer arms of two adjacent supplement angles are collinear rays.

Q.7 A rod is folded in such a way that it makes an angle of 45°. In order to make it straight. What will be the measure of the angle?

Solution:

Suppose when the rod is folded is makes a figure give here.



To make it straight we will have to unfold it according to $m\angle CBD$.

Now $m\angle ABC = 45^\circ$ (given)

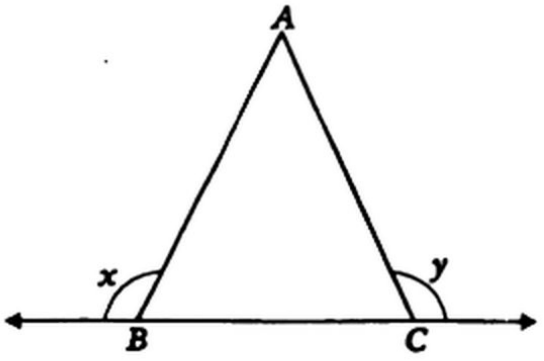
$m\angle ABC + m\angle CBD = 180^\circ$ (Supp. Adj. angles)

$\therefore m\angle CBD = 180^\circ - \angle ABC$

or $m\angle CBD = 180^\circ - 45^\circ$

or $m\angle CBD = 135^\circ$ ***Ans.***

Q.8 ***In figure (6) $\angle ABC = \angle CACB$. Prove that $m\angle x = B\angle y$.***



Solution:

$m\angle ACB + m\angle x = 180^\circ$ (supplementary angles)

$\therefore$ $m\angle x = 180^\circ - m\angle ABC$

or $m\angle x = 180^\circ - mAACB$ ($m\angle ABC = \angle ACB$ (given)

again $m\angle ACB + m\angle y = 180^\circ$ (supplementary angles)

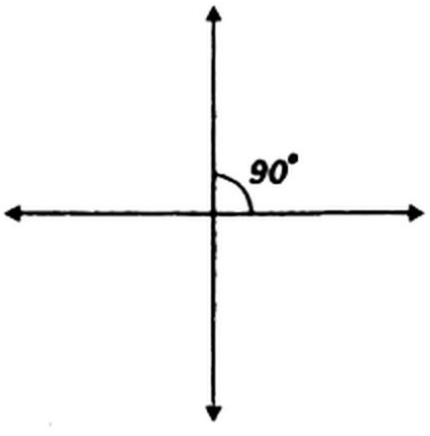
or $m\angle \angle y = 180^\circ - m\angle ACB$

Now $m\angle x = 180^\circ - m\angle ACB$ (Proved)

$m\angle y = 180^\circ - m\angle ACB$ (Proved)

So $m\angle x = m\angle y$

Q.9 ***In figure (7) one angle is right angle, prove that the remaining angles are also right angle:***



Solution:

Let $m\angle 1 = 90°$

$m\angle 1 + \angle 2 = 180°$ (supplementary angles)

$m\angle 2 = 180° - m\angle 1$

or $m\angle 2 = 180° - 90°$

or $m\angle 2 = 90°$

$m\angle 3 = m\angle 1$ (vertical angle)

But $m\angle 1 = 90°$ (given)

Hence $m\angle 3 = 90°$

$m\angle 2 = m\angle 4$ (vertical angle)

But $m\angle 2 = 90°$ ***(Prove)***

$m\angle 4 = 90°$

Ans:

$m\angle 1 = 90°$ (given)

$m\angle 2 = 90°$ {Prove (i)}

$m\angle 3 = 90°$ {Prove (ii)}

$m\angle 4 = 90°$ {Prove (iii)}

EXERCISE 10.4

Q.1 What is the criterion for two lines to be parallel?

Solution:

Parallel lines are coplanar and do not intersect.

Q.2 What is the criterion for two line-segments to be parallel?

Solution:

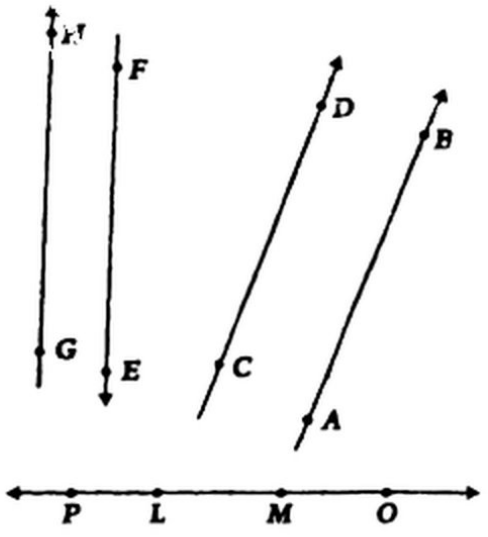
Two parallel line-segments lie on two parallel lines.

Q.3 Are opposite rays always oppositely directed.

Solution:

Yes, opposite rays are always oppositely directed.

Q.4 Look at the adjoining figure and tell which of the pairs of rays are similarly directed, oppositely directed or opposite rays?



(i) $\overrightarrow{AB}$, $\overrightarrow{CD}$; (ii) $\overrightarrow{EF}$, $\overrightarrow{GH}$;

(iii) $\overrightarrow{LP}$, $\overrightarrow{MO}$; (iv) $\overrightarrow{MP}$, $\overrightarrow{LP}$;

(v) $\overrightarrow{MO}$, $\overrightarrow{LO}$; (vi) $\overrightarrow{MO}$, $\overrightarrow{LO}$;

Solution:

(i) $\overrightarrow{AB}$, $\overrightarrow{CD}$; Similarly directed

(ii) $\overrightarrow{EF}$, $\overrightarrow{GH}$; Opssitely directed

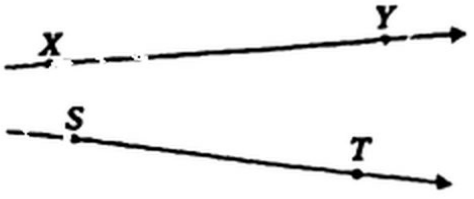
(iii) $\overrightarrow{LP}$, $\overrightarrow{MO}$; Oppositely directed

(iv) $\overrightarrow{MP}$, $\overrightarrow{LP}$; Similarly directed

(v) $\overrightarrow{MO}$, $\overrightarrow{LO}$; Oppositely directed

(vi) $\overrightarrow{MO}$, $\overrightarrow{MP}$; Oppositely directed

Q.5 Tell whether the rays $\overrightarrow{XY}$ and $\overrightarrow{ST}$ in figure (2) are similarly directed. Justify your answer with reasons.



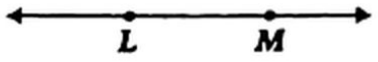
Solution:

Similarly directed rays are:

(i) Parallel,

(ii) They lie on the same side of the line passing through their end-point, Since $\overrightarrow{XY}$ and $\overrightarrow{ST}$ are not parallel, so they are not similarly directed rays.

Q.6 ***$\overrightarrow{LM}$ and $\overrightarrow{ML}$ (figure 3) are not opposite rays. Why? Are they oppositely directed rays?***



Solution:

Opposite rays have their common end-points. In this case there are many common points. So these are not opposite rays. These may, however, be called oppositely directed rays.

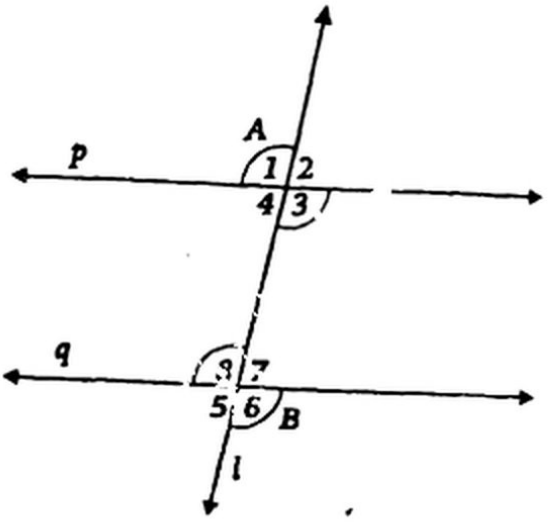
EXERCISE 10.5

Q.1 ***In how many points will a transversal meet two lines?***

Solution:

A transversal will meet two lines in two points.

Q.2 ***A line l intersects p, q in points A and B and makes 8 angles. Look at the figure (1) and Name the following pairs of angles.***



(i) ***Interior alternate angles.***

(ii) ***Exterior alternate angles.***

(iii) ***Corresponding angles.***

(iv) ***A pair of adjacent supplementary angles.***

(v) ***A pair of vertical angles.***

Solution:

(i) $m\angle 4, m\angle 6$ and $\angle 3, \angle 5$

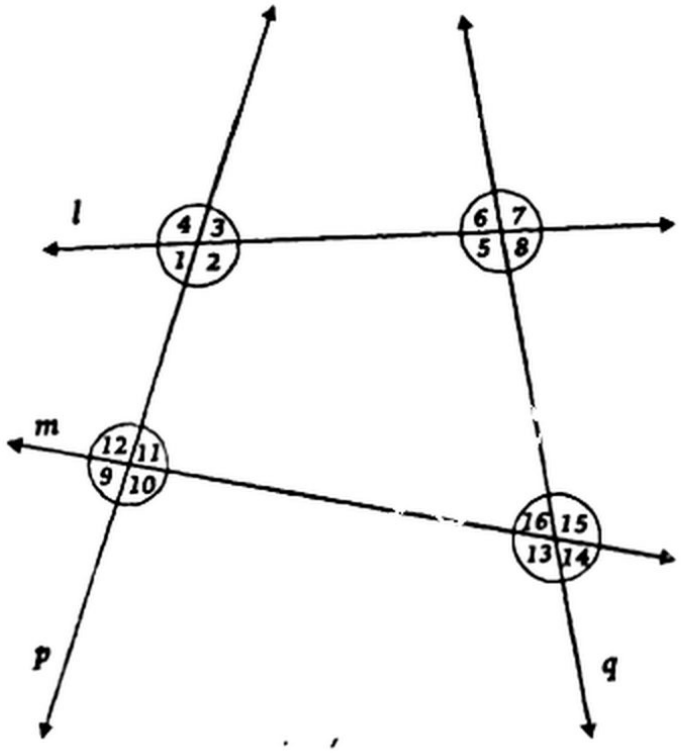
(ii) $m\angle 2, m\angle 8$ and $\angle 1, \angle 7$

(iii) $m\angle 2, m\angle 6, \angle 3, \angle 7, \angle 5, \angle 4, \angle 8,$

(iv) $m\angle 1, m\angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7, \angle 8,$

(v) $m\angle 1, m\angle 3, \angle 2, \angle 4, \angle 5, \angle 7, \angle 6, \angle 8,$

Q.3 Look at figure (2) and answer the following questions.



(i) The line p is a transversal of which line?

(ii) Name four interior and four exterior angles formed by the lines p, l and m.

(iii) Line m is transversal of which line?

(iv) Name one pair of corresponding angles, one pair of interior alternate angles and one pair of exterior alternate angles formed by the lines m p and q.

Solution:

(i) Line p is a transversal to lines l and m.

(ii) 4 interior angles are: $\angle 1$, $\angle 2$, $\angle 11$ and $\angle 12$.

4 exterior angles are: $\angle 3$, $\angle 4$, $\angle 9$ and $\angle 10$.

(iii) The line m is transversal to lines p and q.

(iv) A pair of corresponding angles $\angle 9$, and $\angle 13$.

A pair of internal alternate angles $\angle 12$, $\angle 16$.

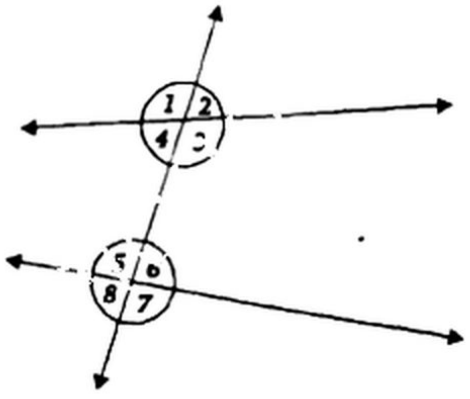
A pair of external alternate angles $\angle 12$, $\angle 14$.

Q.4 With reference to figure (3) state the kinds of the following pairs of angles:

(i) $\angle 1, \angle 3$; ***(ii)*** $\angle 3, \angle 4$;

(iii) $\angle 3, \angle 7$; ***(iv)*** $\angle 4, \angle 6$;

(v) $\angle 1, \angle 7$



Solution:

(i) $\angle 1$, $\angle 3$ Pair of vertical angles.

(i) $\angle 3$, $\angle 4$ Pair of adjacent supplement angles.

(iii) $\angle 3$, $\angle 7$ Pair of corresponding angles.

(iv) $\angle 4$, $\angle 6$ Pair of internal alternate angles.

(v) $\angle 1$, $\angle 7$ Pair of external alternate angles.

EXERCISE 10.6

Q.1 ***What conclusion can you draw from each of the following statemens:***

(i) ***A line AB is parallel to a line CD and a line L intersects the line AB.***

(ii) ***A line AB is parallel to a line CD and the line CD is parallel to line EF.***

(iii) ***A line AB is parallel to line CD and $\overleftrightarrow{EF}$ intersects them.***

(iv) ***A lines l intersects a line AB and CD such that a pair of alternate angles are congruent.***

Solution:

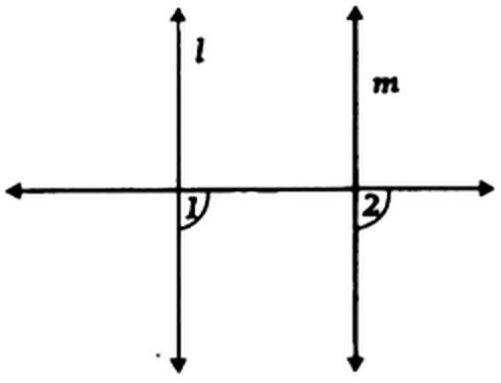
(i) The statement is self-explanatory. The conclusion is clear.

(ii) Line EF is parallel to a line *AB* also.

(iii) Alternate and corresponding angles are congruent. Also the interior angles on the same side are supplementary.

(iv) Lines *AB* and *CD* are parallel.

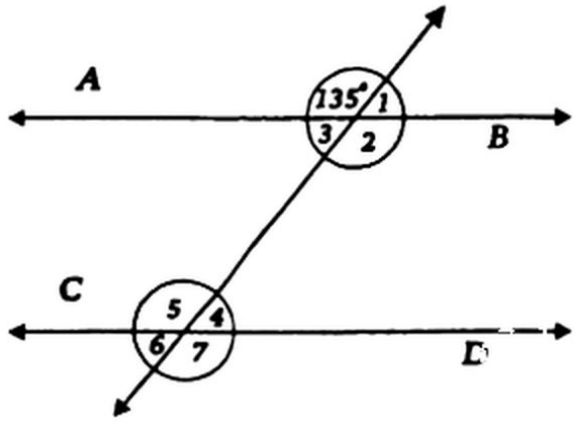
Q.2 ***Lines l, m are parallel. What can you say about ∠1 and ∠2? (Figure 1)***



Solution:

$\angle 1 \cong \angle 2$ being corresponding angles.

Q.3 ***Line AB and CD are parallel (Figure 2). One of the angles, as shown, is of 135°. Find the measure of the remaining angles.***



Solution:

$\angle 4 + \angle 1 = 180°$ (suppl.y. adjacent angles)

$\therefore \quad \angle 1 = 180° - 135°$

$\therefore \quad \angle 1 = 180 - 135$

or $\angle 1 = 45°$ ..... (1)

$\angle 4 = 2$ (vertical angles)

$\angle 4 = 135°$ (given)

$\therefore \quad \angle 2 = 135°$ ..... (2)

$\angle 1 = 45°$ (Proved)

$\angle 1 = \angle 3$ (vertical angles)

$\therefore \quad \angle 3 = 45°$ ..... (3)

$\angle 1 = 135°$ (given)

$\angle 1 = \angle 5$ (corresponding angles)

$\therefore \quad \angle 5 = 135°$ ..... (4)

$\angle 3 = 45°$ (Proved)

$\angle 3 = \angle 6$ (corresponding angles)

$\therefore \quad \angle 6 = 45°$ ..... (5)

$\angle 5 = 135°$ (Proved)

$\angle 5 = \angle 7$ (vertical angles)

$\therefore$ $\angle 7 = 135^\circ$ ..... (6)

$\angle 1 = 45^\circ$ (Proved)

$\angle 1 = \angle 8$ (corresponding angles)

$\therefore$ $\angle 8 = 45^\circ$ ..... (7)

$\angle 1 = 45^\circ$ (1)

$\angle 2 = 135^\circ$ (2)

$\angle 3 = 45^\circ$ (3)

$\angle 4 = 135^\circ$ (given)

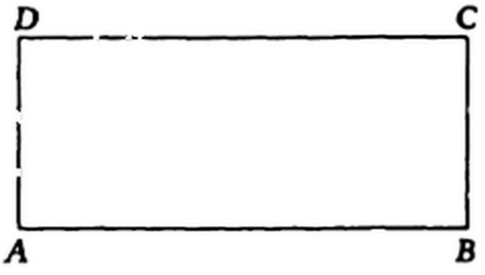
$\angle 5 = 135^\circ$ (4)

$\angle 6 = 45^\circ$ (5)

$\angle 7 = 135^\circ$ (6)

$\angle 8 = 45^\circ$ (7)

Q.4 ***In a parallelogram, ABCD one angle is a right angle. Prove that its remaining angles are also right angles.***



Given: *ABCD* a parallelogram

Let $\angle 1$ be a right angle

Construction:

Extend *AB* in the direction of *B* forming angle 5 and *DC* in the direction of C forming $\angle 6$.

Now $\angle 1 = 90^\circ$ (given)

$\angle 1 + \angle 5 = 180^\circ$ (adj. supp. angles)

$\therefore$ $\angle 5 - 180^\circ - \angle 1$ ..... (7)

or $\angle 5 = 180^\circ - 90^\circ$

or $\angle 5 = 90^\circ$

Now AD is parallel to angle BC, BA intersects it at B and A.

$\angle 5 = \angle 4$ (corresponding angles)

$\therefore$ $\angle 4 - 90^\circ$

Again $\overline{AB} \| \overline{DF}$ while BC intersects this

$\angle 1 + \angle 2 = 180^\circ$

$\therefore$ $\angle 2 = 180^\circ - \angle 1$

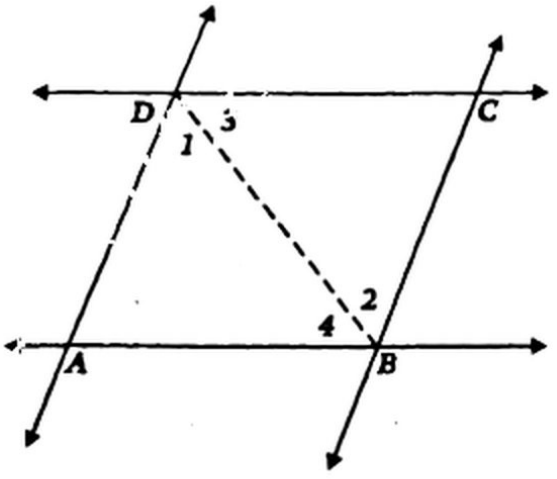
or $\angle 2 = 180^\circ - 90^\circ$ (Since $\angle 1 = 90^\circ$ given)

or $\angle 2 = 90^\circ$

Similarly we can prove that $\angle 3 = 90^\circ$

Q.5 $\overline{AB} \| \overline{CD}$ *and* $\overline{AD} \| \overline{BC}$. *prove that*

$m\angle 1 = m\angle 2$, $m\angle 3 = m\angle 4$



Solution:

$\overline{AB} \| \overline{CD}$

$\overline{AD} \| \overline{BC}$

To prove that

$\angle 1 = \angle 3$

$\angle 3 = \angle 4$

Proof

$\overline{AB} \| \overline{CD}$ (given)

$\overleftrightarrow{BD}$ intersects them

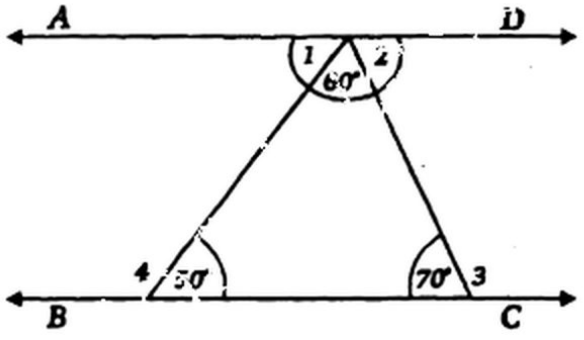
$\angle 3 = \angle 4$ (internal alternate angles)

Similarly:

$\overline{AD} \| \overline{BC}$ and $\overline{BD}$ intersects them.

$\therefore$ $\angle 1 = \angle 2$ (internal alternate angle)

Q.6 ***In figure (5) $\overline{AD} \| \overline{BC}$, find the measures of $\angle 1, \angle 2, \angle 3, \angle 4$***



Solution:

$\overline{AD} \| \overline{BC}$ and $\overline{AB}$ intersects them

$\therefore$ $m\angle 1 = m\angle ABC$

$m\angle ABC = 50°$ (given)

$m\angle 1 = 50°$

Similarly we can prove that

$m\angle 2 = 70°$

Again, $m\angle 4 + m\angle ABC = 180°$ (adj. supp. angle)

$m\angle 4 = 180° - \angle ABC$

$m\angle 4 = 180° - 50°$

$m\angle 4 = 130^\circ$

Similarly $m\angle 3 + m\angle ACB = 180^\circ$

(adj. supp. angles)

or $\angle 3 = 180^\circ - m\angle ABC$

or $\angle 3 = 180^\circ - 70^\circ$

or $\angle 3 = 110^\circ$

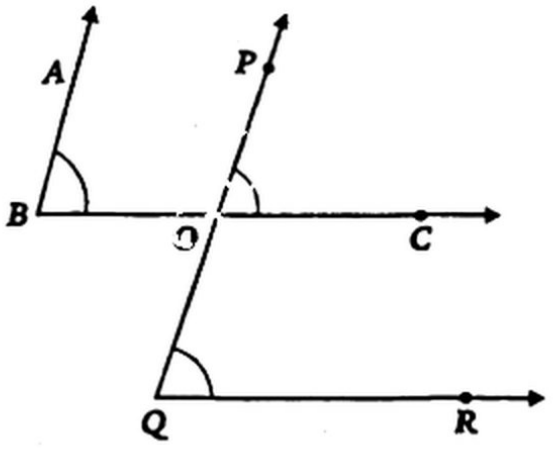
$\angle 1 = 50^\circ$

$\angle 2 = 70^\circ$

$\angle 3 = 110^\circ$

$\angle 4 = 130^\circ$ **Ans**

Q.7 ***The arms of angles ABC and PQR are two pairs of similarly directed rays. Show that the angles are congruent.***



Solution:

Given: Arm of $\angle ABC$ and $\angle PQR$ are similarly directed rays.

$\therefore$ $\overline{AB} \| \overline{PQ}$

$\overline{BC} \| \overline{QR}$

Proof: $\overline{BC} \| \overline{QR}$ and $\overleftrightarrow{PQ}$ intersects them.

$\therefore$ $m\angle PQR = m\angle x$ (corresponding angles)

Similarly $\overline{AB} \| \overline{PQ}$ and $\overline{BC}$ intersects them.

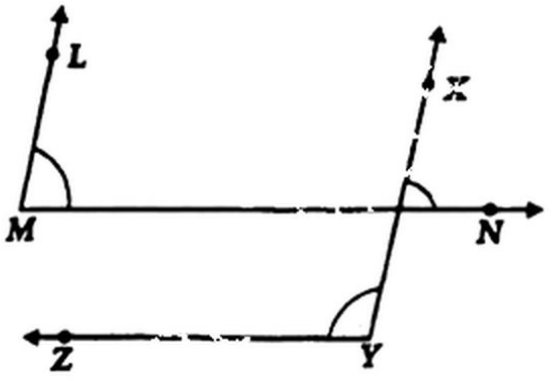
$m\angle x = m\angle ABC$ (corresponding angle)

Now $m\angle PQR = m\angle x$ (Proved)

$m\angle ABC = m\angle x$ (Proved)

$\therefore$ $m\angle PQR = m\angle ABC$ (Proved)

Q.8 ***One pair of arm of angles LHN and XYZ are similarly directed rays and the other pairs are opposite directed rays. Show that the angles are supplementary.***



Solution:

Given: Arms of angles *LHN* and *XYZ*.

$\overrightarrow{ML}$ and $\overrightarrow{XY}$ (similarly directed rays).

$\overrightarrow{MN}$ and $\overrightarrow{YZ}$ (opposite directed rays).

Hence: $\overline{LM} \| \overline{XY}$ and $\overline{MN}$ intersects them.

$\therefore$ $m\angle LMN = \angle 1$ (corresponding angles)

$\overrightarrow{MN} = \overrightarrow{YZ}$ and $\overline{XY}$ intersects them.

$\therefore$ $m\angle 2 = m\angle XYZ$ (internal alternate angles)

Now $\angle 1 + \angle 2 = 180^\circ$ (Adjacent supp. angles)

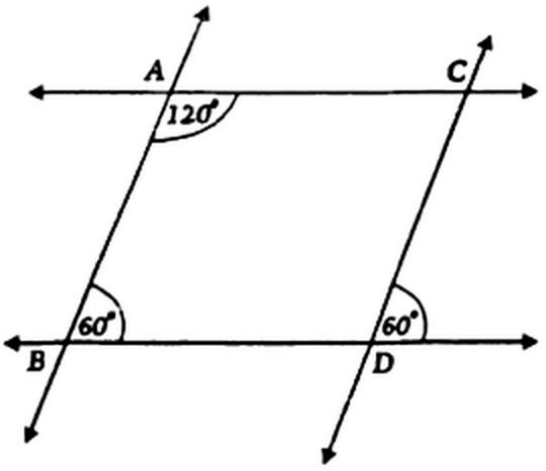
But $m\angle 1 = m\angle LMN$ (Proved)

and $m\angle 2 = m\angle XYZ$ (Proved)

$\therefore$ $m\angle LMN + m\angle XYZ = 180^o$

$\therefore$ $\angle LMN$ and $\angle XYZ$ are supplementary

Q.9 ***In figure (8) prove that;*** $\overline{AB} \| \overline{CD}$ ***and*** $\overline{AC} \| \overline{DB}$



Solution:

Given: $m\angle CAB = 120^o$

$m\angle ABC = 60^o$

$m\angle CDE = 60^o$

Now $\overrightarrow{AB}$ intersects $\overrightarrow{AC}$ and $\overrightarrow{BD}$

$m\angle ABD = 60^o$

$m\angle CAB = 120^o$

$\therefore$ $m\angle ABD + m\angle CAB = 60^o + 120^o = 180^o$

$\therefore$ $\angle ABD$ and $\angle CAB$ (supplementary and side of intersecting line).

$\overline{AC} \| \overline{BD}$

Again $m\angle ABC = 60^o$ (given)

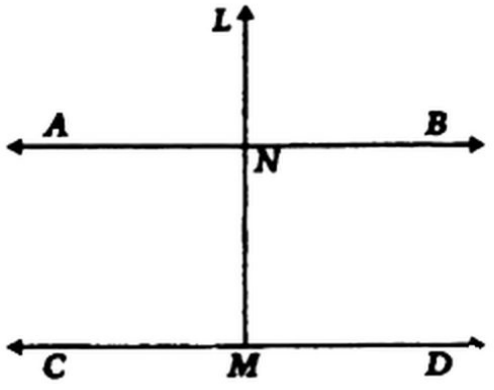
$m\angle CDE = 60^o$ (given)

$\therefore$ $m\angle ABC = m\angle CDE = 60^o$

$m\angle ABC = m\angle CDE$ are compulsory angles

$\therefore$ $\overrightarrow{AB} \| \overrightarrow{CD}$

Q.10 $\overline{LM}$ ***is perpendicular to both the lines AB and CD shows that*** $\overline{AB} \| \overline{CD}$.



Solution:

$\overline{LN}$ is perpendicular to AB.

$\therefore \quad m\angle 1 = 90^\circ$

Again, $\overline{LM}$ is perpendicular to CD

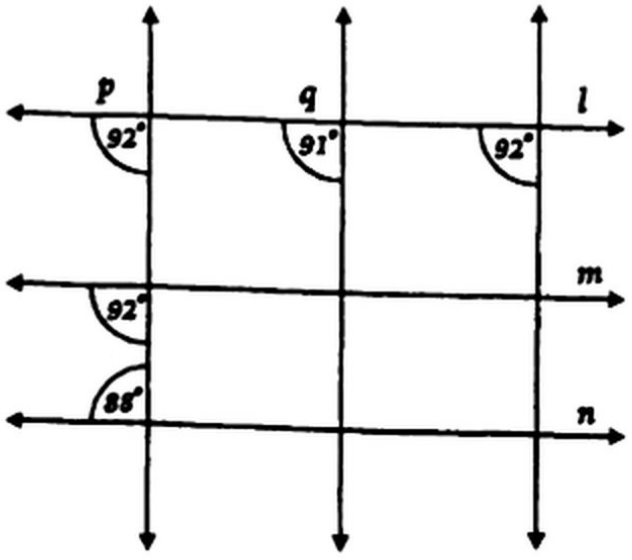
$\therefore \quad m\angle 2 = 90^\circ$

$\therefore \quad m\angle 1 = m\angle 2$

Now $\angle 1$ and $-\angle 2$ are corresponding angles and are equal.

Hence $\overline{AB} \| \overline{CD}$

Q.11 In figure (10) tell which lines are parallel and which are non-parallel. Give reasons for your answer.



Solution:

Lines p, q and r intersect lines l, m and n.

Similarly line l, m and n intersect lines p, q and r.

$m\angle 1 = m\angle 2 = m\angle 3 = 92^\circ$ (given)

But these angles are corresponding $\angle 8$

Hence p, q and r are parallel lines.

Again $m\angle 1 = m\angle 4 = 92^\circ$ (given)

But these angles are corresponding angles.

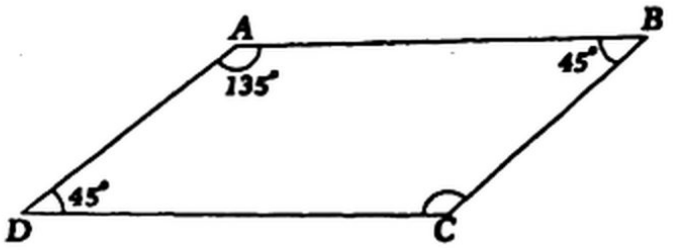
Hence l and m are parallel lines.

$m\angle 5$ is 88° which is $\neq 92^\circ$

$\therefore$ $m\angle 5$ is $\neq m\angle 1$ or $m\angle 4$

Hence line n is not parallel to line l or m.

Q.12 In a quadrilateral ABCD measures of those angles are as shown in figure. Show that the quadrilateral is a parallelogram.



Solution:

Given: $m\angle DAB = 135^\circ$

$m\angle ADC = 45^\circ$

$m\angle ABC = 45^\circ$

Construction:

Extend $\overleftrightarrow{BA}$ towards A upto E.

Now $\angle 1 + \angle DAB = 180^\circ$

$\therefore\ m\angle 1 = 180^\circ - m\angle DAB$

$\therefore\ m\angle 1 = 180^\circ - 135^\circ$ (Since $\angle DAB = 135^\circ$ given)

$\therefore\ m\angle 1 = 45^\circ$

Now $\overleftrightarrow{EA}$ intersects $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$.

$m\angle ABC = 45^\circ$ (given)

$m\angle 1 = 45^\circ$ (Proved)

$\therefore\ m\angle 1 = \angle ABC$

But $\angle 1$ and $\angle ABC$ are corresponding angles.

$\therefore\ \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$

Similarly $\overleftrightarrow{AD}$ intersects $\overleftrightarrow{EA}$ and $\overleftrightarrow{DC}$.

$m\angle 1 = 45^\circ$ (Proved)

$m\angle ADC = 45^\circ$ (Given)

$\therefore\ m\angle 1 = m\angle ADC$

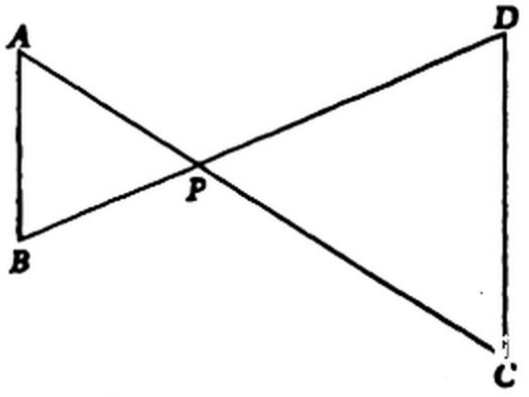
But $\angle 1$ and $\angle ADC$ are internal angles.

$\therefore\ \overleftrightarrow{BE} \parallel \overleftrightarrow{CD}$

or $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$

EXERCISE 10.7

Q.1 ***In the correspondence $\Delta PAB \longleftrightarrow \Delta PCD$ name the pair of corresponding angles and sides.***



Solution:

Corresponding angles are:

$$\angle APB \longleftrightarrow \angle DPC$$

$$\angle PAB \longleftrightarrow \angle PDC$$

$$\angle PBA \longleftrightarrow \angle PCD$$

Corresponding sides are:

$$\overline{AP} \longleftrightarrow \overline{PD}$$

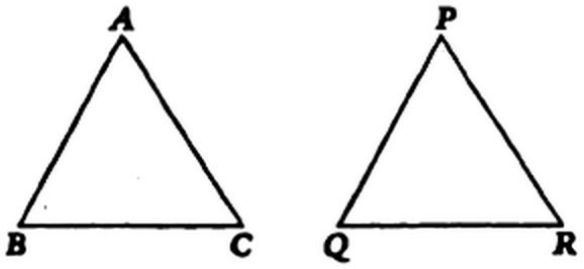
$$\overline{BP} \longleftrightarrow \overline{PC}$$

$$\overline{AB} \longleftrightarrow \overline{CD}$$

Q.2 ***ΔABC and ΔPQR are two equilateral triangles & $\overline{AB} = \overline{PQ}$. Which correspondence of the triangle are congruences.***

Solution:

In correspondence with $\Delta ABC \longleftrightarrow \Delta PQR$



$\overline{AB} = \overline{PQ}$

$\overline{AC} = \overline{PR}$

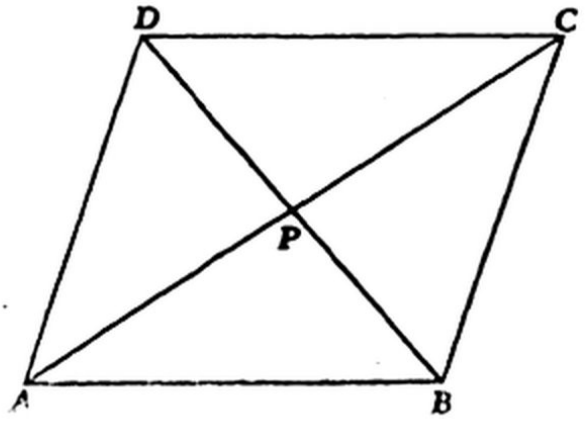
$\overline{BC} = \overline{QR}$

$m\angle A = m\angle P$

$m\angle B = m\angle Q$

$m\angle C = m\angle R$

Q.3 ***In the adjoining figure (3) are shown a quadrilateral ABCD are four triangles PAB, PBC, PCD, PDA. Which pairs of triangles appear to be congruent? Also name the relevant correspondence in which a pair is congruent.***



Solution:

ΔDPC appear to be congruent to ΔAPB.

Correspondence:

$$\overline{DC} \longleftrightarrow \overline{AB}$$

$$\overline{DP} \longleftrightarrow \overline{AP}$$

$$\overline{CP} \longleftrightarrow \overline{BP}$$

$$m\angle CDP \longleftrightarrow m\angle CAB$$

$$m\angle DCP \longleftrightarrow m\angle PBA$$

$$m\angle DP \longleftrightarrow m\angle APB$$

Similarly ΔDPA appears to be congruent to ΔCPC.

Correspondence:

$$\overline{AP} \longleftrightarrow \overline{BC}$$

$$\overline{AP} \longleftrightarrow \overline{BP}$$

$$\overline{DP} \longleftrightarrow \overline{CP}$$

$$m\angle DPA \longleftrightarrow m\angle CPB$$

$$m\angle PDA \longleftrightarrow mPCB$$

$$m\angle PAD \longleftrightarrow mPBA$$

Q.4 $\Delta ABC \cong \Delta ADC$ ***(figure 4). Name the pairs of congruent angles and congruent sides.***

Solution:

$\Delta ABC \cong \Delta ADC$ (given)

$$\overline{AB} \cong \overline{AD}$$

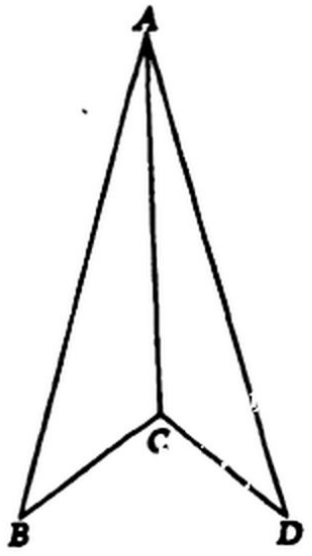
$$\overline{BC} \cong \overline{DC}$$

$\overline{AC}$ is common in both the Δs.

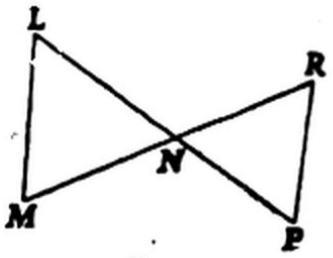
$m\angle CAB \cong m\angle CAD$

$m\angle ABC \cong m\angle ADC$

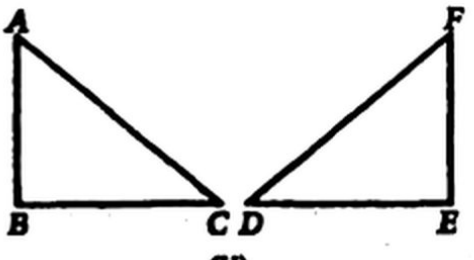
$m\angle ACB \cong m\angle ACD$



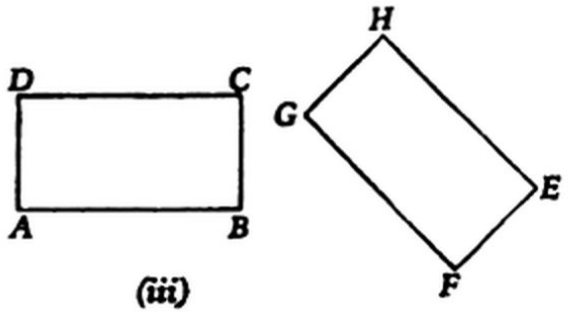
Q.5 ***Below are given a number of pairs of figures. Which of the pairs of figures are congruent? Specify the relevant correspondence in case of a congruent figure.***

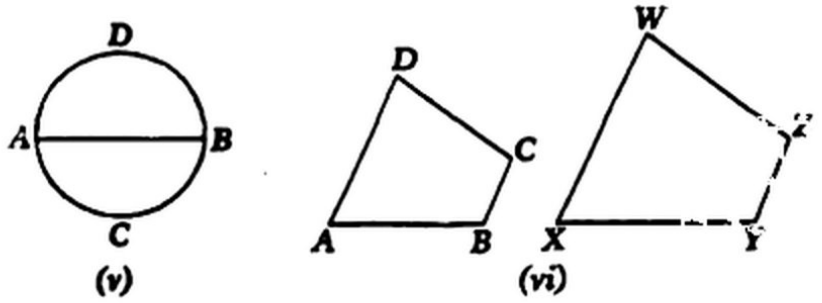


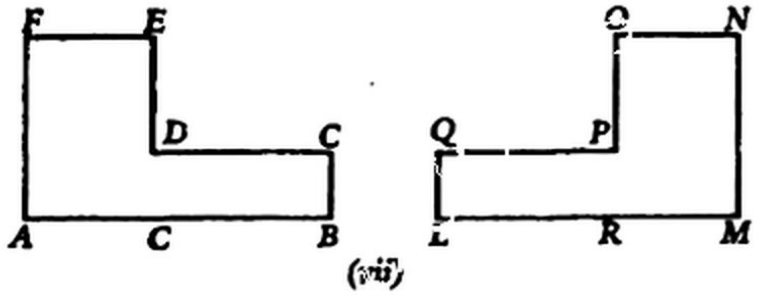
(i)



(ii)







Solution:

(i) $\Delta ABC \cong DEF$

Corres. $\overline{AB} \longleftrightarrow \overline{EF}$

$\overline{BC} \longleftrightarrow \overline{ED}$

$m\angle A \longleftrightarrow m\angle F$

$m\angle B \longleftrightarrow m\angle E$

$m\angle C \longleftrightarrow m\angle D$

(ii) Figures not congruent.

(iii) Figures not congruent.

(iv) $\Delta ABC \cong \Delta DCA$.

Correspondence:

$$\overline{AD} \longleftrightarrow \overline{BC}$$
$$\overline{DC} \longleftrightarrow \overline{AB}$$
$$\overline{AC} \longleftrightarrow \overline{AC}$$
$$m\angle D \longleftrightarrow m\angle B$$
$$m\angle DAC \longleftrightarrow m\angle ACB$$
$$m\angle DCA \longleftrightarrow m\angle CAB$$

(v) Congruent.

Correspondence:

$$\overline{AB} \longleftrightarrow \overrightarrow{AB}$$
$$\overset{\frown}{ADB} \longleftrightarrow \overset{\frown}{ACB}$$

(vi) Figures not congruent.

(vii) Figures congruent.

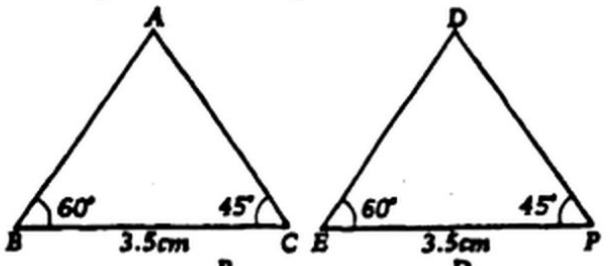
Correspondence:

$$\overline{NM} \longleftrightarrow \overline{PA}$$
$$\overline{NO} \longleftrightarrow \overline{FE}$$
$$\overline{ED} \longleftrightarrow \overline{OP}$$
$$\overline{DC} \longleftrightarrow \overline{PQ}$$
$$\overline{CB} \longleftrightarrow \overline{QL}$$
$$\overline{AC} \longleftrightarrow \overline{LM}$$
$$m\angle F \longleftrightarrow m\angle N$$
$$m\angle A \longleftrightarrow m\angle M$$
$$m\angle B \longleftrightarrow m\angle L$$
$$m\angle C \longleftrightarrow m\angle Q$$
$$m\angle E \longleftrightarrow m\angle E$$

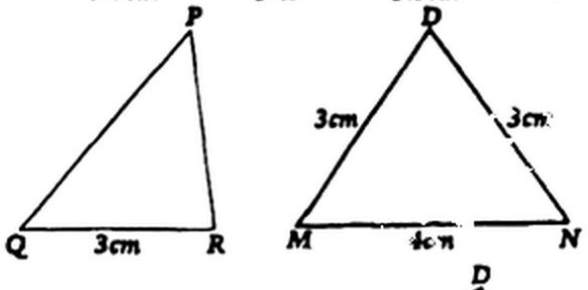
EXERCISE 10.8

Q.1 ***Pairs of congruent triangles are given below. Which theorems will you cite for proving their congruence.***

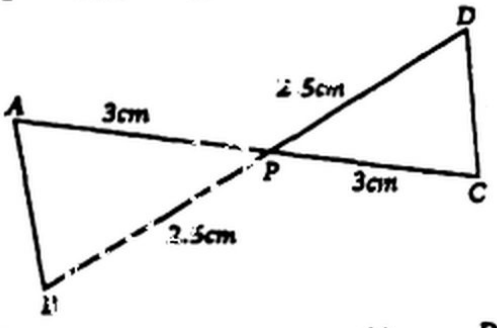
(i)



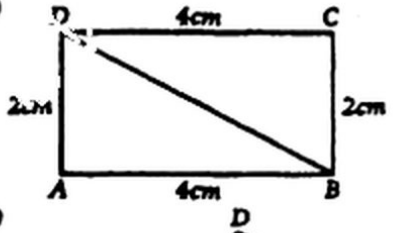
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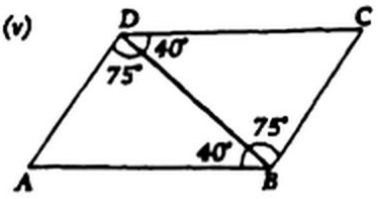
(iii)



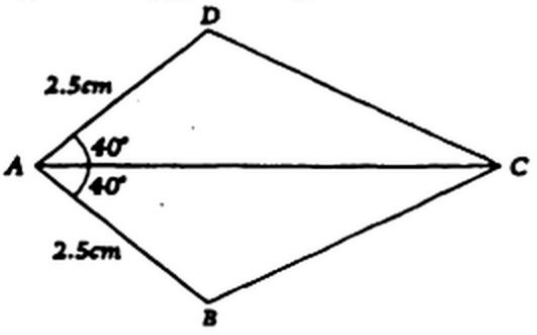
(iv)



(v)



(vi)



Solution:

(i) Theorem 1st, i.e, if two angles and included sides of one triangles is congruent to the corresponding two angles and included sides of other triangle then the triangle are congruent.

$(ASA \cong ASA)$

(ii) Theorem 4th, i.e, if one leg and hypotenuse of one right-angled triangle is congruent to the corresponding leg and hypotenuse of the other triangles then the triangles are congruent.

$(HL \cong HL)$

(iii) Theorem 2nd, i.e, if two sides and included angles of one triangle is congruent to the corresponding two sides and included angle of the other triangle then the triangles are congruent.

$(SAS \cong SAS)$

(iv) Theorem 3rd, i.e, if three sides of a triangle are congruent to the corresponding sides of the other triangle then the triangles are congruent.

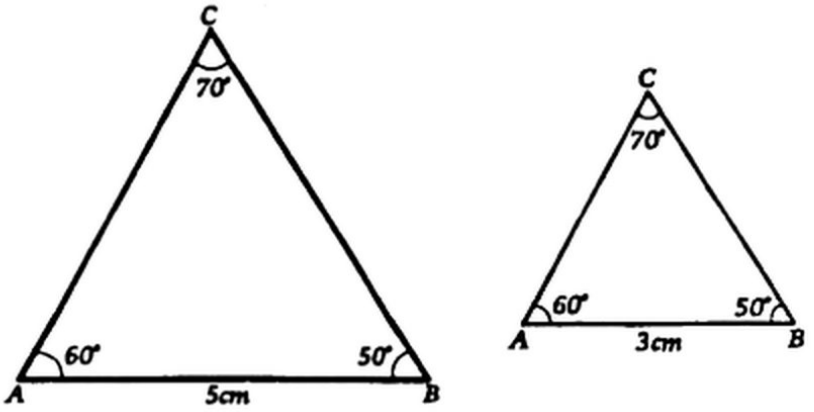
$(SSS \cong SSS)$

(v) Theorem 1st, i.e, $(ASA \cong ASA)$.

(vi) Theorem 2nd, i.e, $(SAS \cong SAS)$.

Q.2 ***If in a given correspondence of two triangles the three angles of one triangle are congruent to the corresponding angle of the other, will the triangle be congruent? Verify by drawings figure.***

Solution:

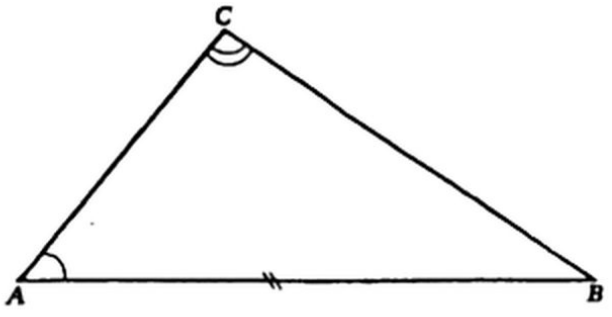


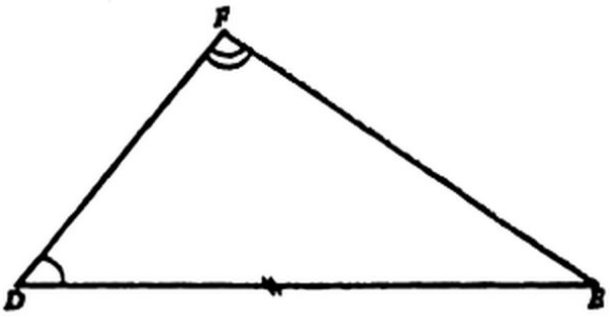
In given triangles *ABC* and *DEF*.

$$m\angle A \cong m\angle D$$
$$m\angle B \cong m\angle E$$
$$m\angle C \cong m\angle F$$

But the triangle do not appear to be congruent.

Q.3 ***If in a given correspondence of two triangles two angles and a non-included side of one triangle are congruent to the corresponding, two angles and a non-included side of the other triangle, will the triangle be congruent. (What about the magnitudes of the third triangles of the triangles? What form does the theorem take now?).***





Solution:

In triangle *ABC* and *DEF*.

Given $\overline{BC} \cong \overline{EF}$

$m\angle A \cong m\angle D$

$m\angle C \cong m\angle F$

The sum of three angles of a triangle is 180°.

$\therefore \quad m\angle A + m\angle B + m\angle C = 180^\circ$

and $m\angle D + m\angle F + m\angle E = 180^\circ$

or $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$

But $m\angle A + m\angle C = m\angle D + m\angle F$

Since $m\angle A\ m\angle D)$

$m\angle C + m\angle F)$ Given

or $m\angle B + m\angle E)$

Now two sides of a triangle and their included side is congruent to the side corresponding two angles and included side of the other.

Hence according to *ASA* $\cong$ *ASA*

$\Delta ABC \cong \Delta DEF$. (Proved)

Q.4 ***Prove that the diagonal of a rectangle divides it into two congruent triangles.***

Solution:

Given: *ABCD* is a rectangle

$\therefore \quad \overline{AB} \cong \overline{DC}$

$\overline{AD} \cong \overline{BC}$

$\overline{AC}$ is a diagonal

To prove tha $\Delta ABC \cong \Delta DAC$

Proof: ΔABC and ΔDAC

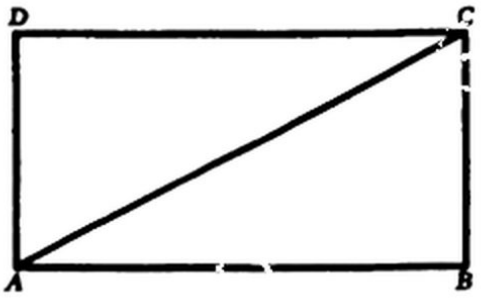
$\therefore \quad \overline{AB} \cong \overline{DC}$ (given)

$\overline{AD} \cong \overline{BC}$ (given)

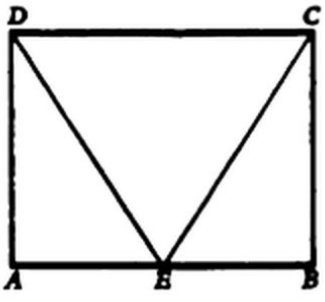
$\overline{AC} \cong \overline{AC}$ (common)

Hence $S.S.S \cong S.S.S$

$\therefore$ The triangles are congruent. (Proved).



Q.5 ***In the adjoining figure is shown a square ABCD. Prove that the correspondence $\Delta DAE \leftrightarrow \Delta CBE$ is a congruence.***



Solution:

The congruencies can only be proved when E is taken as middle point of $\overline{AB}$.

Given: Square $ABCD$.

To prove that $\Delta ADE \cong \Delta BCE$

Proof In ΔADE and ΔBCE

$AD = BC$ (opposite sides of a square)

$m\angle A = m\angle B$

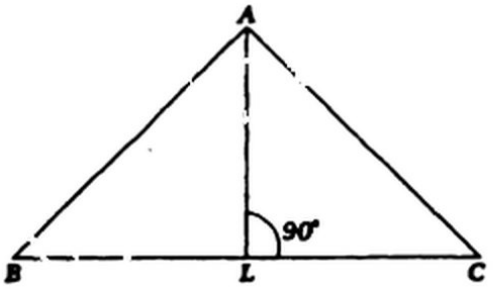
(right angles as the figure is a square)

$\overline{AE} = \overline{BE}$ (E is mid-point of AB)

$\because$ $SAS \cong SAS$

$\because$ $\Delta ADE \cong \Delta BCE$, (Proved)

Q.6 ***The bisector $\overline{AL}$ of angle A of ΔABC is perpendicular to $\overline{BC}$. Prove that the triangle is isoseles.***



Solution:

Given: $m\angle BAL \cong m\angle CAL$

$m\angle ALC \cong m\angle ALB$

To prove that $\overline{AB} \cong \overline{AC}$

Proof: In triangle ABL and ACL

$m\angle BAL \cong m\angle CAL$ (given)

$m\angle ALB \cong m\angle ALC$ (given)

$\overline{AL} \cong \overline{AL}$ (common)

$\because$ $ASA \cong ASA$

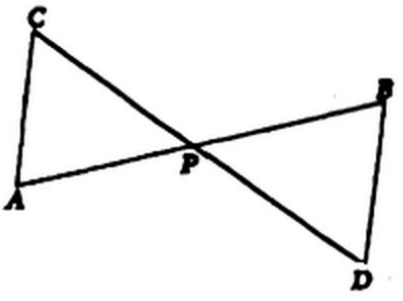
$\because \quad \Delta ABL \cong \Delta ACL$

$\because \quad \overline{AB} \cong \overline{AC}$

$\because$ ΔABC is isoceles (Proved)

Q.7 ***Line-segments $\overline{AB}$ and $\overline{CD}$ bisect each other at a point P. Prove that:***

$$\Delta PAC \cong \Delta PBD$$



Solution:

Given: $\overline{AB}$ and $\overline{CD}$ bisect each other at P.

Hence: $\overline{AP} = \overline{PB}$

$\overline{CP} = \overline{PD}$

To prove that:

$$\Delta PAC \cong \Delta PBD$$

Proof. In ΔPAC and ΔPBD

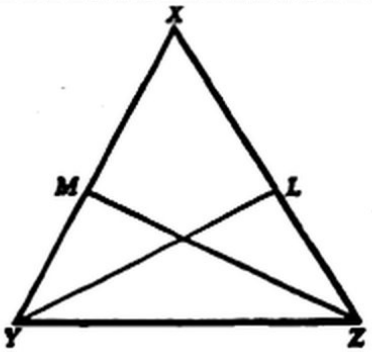
$\overline{PA} = \overline{PB}$ (given)

$\overline{CP} = \overline{PD}$ (given)

$\because \quad S.A.S \cong S.A.S$

$\because \quad \Delta P.A.C \cong \Delta P.B.D$ (Proved)

Q.8 ***In ΔXYZ, $\overline{XY} \cong \overline{YZ}$ and $\overline{YL}$, ZM are the perpendiculars from the ends of the base to the congruent sides. Prove that $\overline{YL} \cong \overline{ZM}$.***



Solution:

Given: (i) XYZ is an iscoceles Δ.

(ii) $\overline{YL}$ and $\overline{ZM}$ are perpendiculars to $\overline{XZ}$ and $\overline{XY}$ respectively.

To prove that: $\overline{YL} = \overline{ZM}$

Proof: In triangle XYL and XZM.

$m\angle XMZ \cong m\angle XLY$ (right angle)

$m\overline{LX} \cong m\overline{LX}$ (common)

$\because$ $m\angle XZM \cong m\angle XYL$ (If two angles of a triangle are congruent to the angles of the other Δ then the third angle is also congruent).

Now in ΔXYL and ΔXZM.

$\overline{XY} = \overline{XZ}$ (given)

$m\overline{LX} \cong m\overline{LX}$ (given)

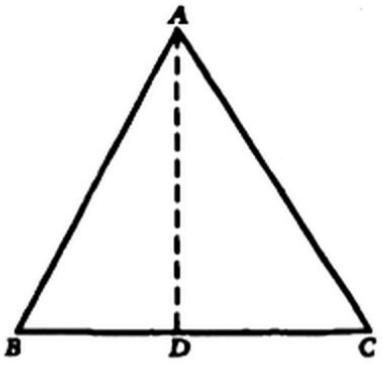
$m\angle XYL \cong m\angle XZM$ (Proved)

Hence $ASA \cong ASA$

$\because$ $\Delta XYL \cong \Delta XZM$

$\because$ $YL = ZM$ (Proved)

Q.9 *In ΔABC, $\overline{AB} \cong \overline{AC}$. Prove that $\angle B \cong \angle C$.*



Solution:

Given: ΔABC

$$\overline{AB} = \overline{AC}$$

To prove that: $\angle B = \angle C$

Construction: Let $\overline{AD}$ be the bisector of $\angle A$ and meet at D.

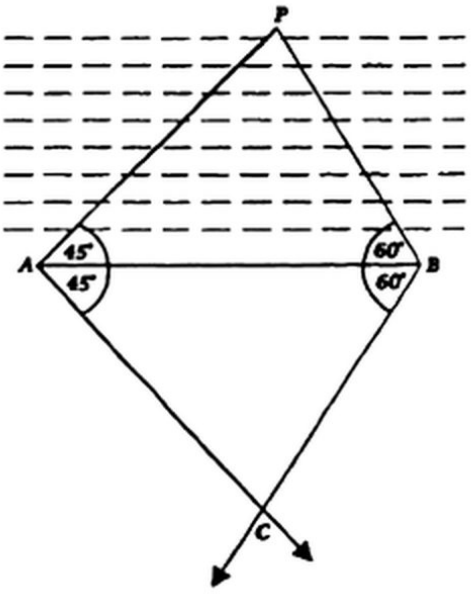
Proof: In ΔABD and ΔACD.

$\overline{AD} = \overline{AD}$ (common)

$\because \quad \Delta ABD \cong ACD$

$\therefore \quad m\angle B \cong m\angle C$ (Proved)

Q.10 *A ship P was observed from two points. A, B on the shore & it was found that $\overrightarrow{PA}$ and $\overrightarrow{PB}$ make angles of 45° and 60° with the shore. A scout who wanted to measure the distance of the ship from A, drew $\overrightarrow{AC}$ and $\overrightarrow{BC}$ on the other side making angles of 45° and 60° the shore, and compieted the triangle ACB. The scout claimed that the distance AP was equal to the distance AC. Justify his claim.*



Solution:

In triangle PAB and CAB.

$\overline{AB} = \overline{AB}$ (common)

$m\angle PAB \cong m\angle CAB$. (given)

$m\angle PBA \cong m\angle CBA$. (given)

Hence $ASA \cong ASA$

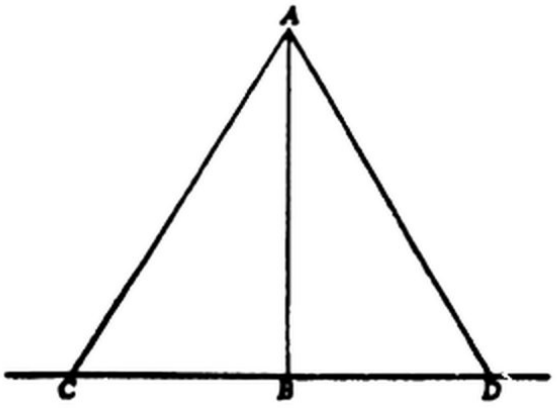
$\because \quad \Delta PAB \cong \Delta CAP$

$\because \quad \overline{AP} \cong \overline{AC}$ *(Proved)*

Q.11 For hoisting a flag a pole $\overline{AB}$ had to be erected vertically on the edge of a roof. For this purpose, wires $\overline{AC}$ and $\overline{AD}$ were attached to A and fixed at points C, D so that the wires in stretched position the $\overline{BC}$ and $\overline{BD}$ were congruent. How will you prove that the pole was actually in vertical position?

Solution:

(*N.B* – This can not proved until and unless the wires $\overline{AC}$ and $\overline{AD}$ are supported to be equal in lengths. Hence it is an omission in questions.



Proof: In ΔAB $\overline{AC} = \overline{AD}$ (given)

$\overline{AB} = \overline{AB}$ (common)

Hence $S\overline{BS} \cong \overline{BD}S$(given)

$\because$ $\Delta ABC \cong \Delta ABD$

$\because$ $m\angle ABC \cong m\angle ABD$

But $\angle ABC$ and $\angle ABD$ are adjacent supplement angles.

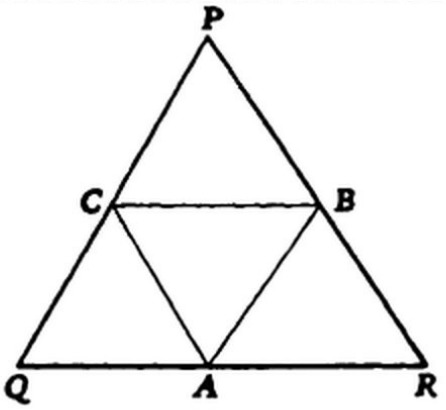
$\because$ $m\angle ABC = 90^o$

$m\angle ABD = 90^o$

$\therefore$ $\overline{AB}$ is perpendicular to CD.

$\therefore$ Pole has been vertically erected. (Proved)

Q.12 PQR is an equilateral triangle A, B, C are the middle points of the sides. Prove that ΔABC is also equilateral.



Solution:

Given: $\overline{PQ} = \overline{PR} = \overline{QR}$ and C, B and A are their middle points respectively.

To prove that $CA = CB = AB$.

Proof: $m\Delta PCB$ and ΔQCA

$\overline{PC} = \overline{QC}$ (given)

$\overline{PB} = \overline{QA}$ (halves of equal lines)

$m\angle P \cong m\angle Q$ (angles of equilateral Δ)

$\because$ $SAS \cong SAS$

$\because$ $\Delta PCB \cong \Delta QCA$

$\overline{CA} = \overline{CB}$

Similarly we can prove that;

$\Delta ABR \cong \Delta QCA$

$\because$ $\overline{CA} = \overline{AB}$

$\because$ $\overline{AB} = \overline{BC} = \overline{CA}$

$\because$ ΔABC is equilateral. (Proved)