

### Exercise 6.11

With the help of the given matrices answer the questions from 1 to 3.

$$A = \begin{bmatrix} 2 & -2 \\ -5 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & -2 \\ 0 & 4 \end{bmatrix}, C = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix},$$

$$D = \begin{bmatrix} -3 & 2 & 0 \\ 0 & 1 & 5 \\ 4 & -2 & 2 \end{bmatrix}, E = [-3 \ 2 \ 0], F = \begin{bmatrix} -3 & 4 \\ 0 & 5 \\ 3 & -1 \end{bmatrix}$$

- 1- What are the orders of matrices A, C and F?
- 2- What are the orders of matrices B, D and E?
- 3- What element is in the second row and third column of matrix D?

*Answers:*

1. (i) (R) Number of rows in matrix A = 2  
(C) Number of columns in matrix B = 2  
Order of matrix A (R × C) = 2-by-2
- (ii) (R) Number of rows in matrix C = 3  
(C) Number of columns in matrix C = 1  
Order of matrix C (R × C) = 3-by-1
- (iii) (R) Number of rows in matrix F = 3  
(C) Number of columns in matrix F = 2

Order of matrix  $F(R \times C) = 3\text{-by-}2$

2.(i) (R) Number of rows in matrix  $B = 2$

(C) Number of columns in matrix  $B = 2$

Order of matrix  $B(R \times C) = 2\text{-by-}2$

(ii) (R) Number of rows in matrix  $D = 3$

(C) Number of columns in matrix  $D = 3$

Order of matrix  $D(R \times C) = 3\text{-by-}3$

(iii) (R) Number of rows in matrix  $E = 1$

(C) Number of columns in matrix  $E = 3$

Order of matrix  $E(R \times C) = 1\text{-by-}3$

3. The element is in the second row and third column of matrix  $D$  is 5.

**Q.4.** Which of the following matrices are equal and which of them are not?

$$A = [4], B = [1 \quad 2], C = \begin{bmatrix} 6 \\ 9 \end{bmatrix}, D = [2 + 2],$$

$$E = \begin{bmatrix} 3 + 3 \\ 8 + 1 \end{bmatrix}, F = \begin{bmatrix} 5 & 4 \\ 5 & 2 \end{bmatrix}, G = \begin{bmatrix} 1 & 3 \\ 6 & 8 \end{bmatrix},$$

$$H = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & 4 \\ 2 & 6 & 3 \end{bmatrix}, I = \begin{bmatrix} 1 & 3 \\ 6 & 7 \end{bmatrix}, J = \begin{bmatrix} 1 & 3 \\ 6 & \frac{16}{2} \end{bmatrix}$$

$$K = \begin{bmatrix} 1 & 2 & 3+2 \\ 0 & 3 & 4 \\ 2 & 4+2 & 3 \end{bmatrix}, L = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 3 & 4 \\ 2 & 6 & 3 \end{bmatrix}$$

**Answers:**

(i)  $B = F, G = J, H = K, C = E, A = D$

(ii)  $I$  and  $L$  are not equal to any matrix.

### **TYPES OF MATRICES**

(i) **Row Matrix:**

A matrix with only one row is called a row matrix.

For example:  $B = [2 \ 3 \ 4]$  is of order 1 - by - 3.

(ii) **Column Matrix:**

A matrix with only one column is called a column matrix.

For example:  $D = \begin{bmatrix} 3 \\ 2 \\ 7 \end{bmatrix}$  is of order 3 - by - 1.

(iii) **Rectangular Matrix:**

If in a matrix, the number of rows and the number of columns are not equal, then the matrix is called a rectangular matrix.

For example:  $C = \begin{bmatrix} 1 & 2 & 7 \\ 3 & 4 & 5 \end{bmatrix}$

(iv) **Square Matrix:**

If a matrix has equal number of rows and columns, it is called a square matrix.

For example:  $Q = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$

(v) **Zero or Null Matrix:**

If all the elements in a matrix are zeros, it is called a zero matrix or null matrix. A null matrix is denoted by the letter  $O$ .

For example:  $O = [0]$  is of order 1 - by - 1.

$O = [0 \ 0]$  is of order 1 - by - 2.

(vi) **Diagonal Matrix:**

A square matrix in which all the elements except at least the one element in the diagonal are zeros is called a diagonal matrix.

For example:  $A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}$

(vii) **Scalar Matrix:**

A diagonal matrix having equal elements is called a scalar matrix.

For example:  $B = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$

(viii) **Unit Matrix or Identity Matrix:**

A scalar matrix having each element equal to 1 is called a unit or identity matrix. Identity or unit matrix is generally denoted by  $I$ .

For example:  $I = [1]$   $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(ix) **Transpose of a Matrix:**

If  $A$  is a matrix of order ( $m$  - by -  $n$ ), then a matrix ( $n$  - by -  $m$ ) obtained by interchanging the rows and columns of  $A$  is called the transpose of  $A$ . It is denoted by  $A'$ .

For example:  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , then  $A' = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

**(x) Symmetric Matrix:**

A square matrix  $A$  is called symmetric if  $A' = A$

For example:  $A = \begin{bmatrix} p & q \\ q & r \end{bmatrix}$ , and  $A' = \begin{bmatrix} p & q \\ q & r \end{bmatrix}$

**(xi) Skew-Symmetric Matrix:**

A square matrix  $A$  is called skew symmetric (or anti-symmetric) if  $A' = -A$

For example:  $A = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix}$

$$A' = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -4 \\ 3 & 4 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix} = -A$$

$A' = -A$  Hence  $A$  is skew symmetric.

**Exercise 6.2**

- 1- Identify row matrices, column matrices, square matrices, and rectangular matrices in the following matrices.

$$A = [3 \ 1 \ 1 \ 1], B = \begin{bmatrix} 5+2 & 4 \\ 2 & 6 \end{bmatrix}, C = \begin{bmatrix} a+x \\ b+y \end{bmatrix},$$

$$D = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}, E = \begin{bmatrix} x & -2 \\ b & 5 \end{bmatrix}, F = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 5 \\ 1 & -5 & 0 \end{bmatrix},$$

$$G = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 7 & 8 \end{bmatrix}, H = [0]$$

**Solution:**

$$A = \begin{bmatrix} 3 & 1 & 1 & 1 \end{bmatrix} \quad \text{Row Matrix}$$

$$C = \begin{bmatrix} (a+x) \\ (b+y) \end{bmatrix} \quad \text{Column Matrix}$$

$$B = \begin{bmatrix} (5+2) & 4 \\ 2 & 6 \end{bmatrix}, D = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \quad \text{Square Matrix}$$

$$E = \begin{bmatrix} x & -2 \\ b & 5 \end{bmatrix}, F = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 5 \\ 1 & -5 & 0 \end{bmatrix}, H = [0]$$

$$C = \begin{bmatrix} (a+x) \\ (b+y) \end{bmatrix}, G = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 7 & 8 \end{bmatrix} \quad \text{Rectangle Matrix}$$

2. Identify, diagonal matrices, scalar matrices, identity matrices.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, F = \begin{bmatrix} 8 & 0 \\ 0 & 0 \end{bmatrix},$$

$$G = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

**Solution:**

$$G = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \quad \text{Diagonal Matrix}$$

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}, F = \begin{bmatrix} 8 & 0 \\ 0 & 0 \end{bmatrix}$$

Scalar Matrix

$$B = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}, E = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, G = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Identity Matrix

3. Find transpose of the following matrices.

$$A = \begin{bmatrix} 3 & 4 \\ -1 & 4 \end{bmatrix}, B = \begin{bmatrix} -3 & -2 \\ -1 & 4 \end{bmatrix},$$

$$C = \begin{bmatrix} a & -b \\ c & d \end{bmatrix}, D = \begin{bmatrix} l & m & n \\ p & q & r \\ a & b & c \end{bmatrix}$$

**Solution:**

$$A = \begin{bmatrix} 3 & 4 \\ -1 & 4 \end{bmatrix}, A' = \begin{bmatrix} 3 & -1 \\ 4 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & -2 \\ -1 & 4 \end{bmatrix}, B' = \begin{bmatrix} -3 & -1 \\ -2 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} a & -b \\ c & d \end{bmatrix}, C' = \begin{bmatrix} a & c \\ -b & d \end{bmatrix}$$

$$D = \begin{bmatrix} l & m & n \\ p & q & r \\ a & b & c \end{bmatrix}, D' = \begin{bmatrix} l & p & a \\ m & q & b \\ n & r & c \end{bmatrix}$$

4. Identify all row matrices, if:

$$A = [3 \ 4 \ 5], B = \begin{bmatrix} -1 & 3 \\ 4 & 6 \end{bmatrix}, C = [e \ f \ g],$$

$$D = \begin{bmatrix} 3 & 7 & 5 \\ 4 & 6 & 2 \\ 1 & 9 & 8 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 & 6 \\ 3 & 7 & 3 \end{bmatrix}$$

**Solution:** Row Matrix

$$A = [3 \ 4 \ 5], C = [e \ f \ g]$$

5. Identify all column matrices, if:

$$A = \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 6 & 5 \\ 4 & 7 \end{bmatrix}, C = \begin{bmatrix} a \\ b \\ c \end{bmatrix},$$

$$D = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & 5 \\ -2 & 3 & 4 \end{bmatrix}, E = \begin{bmatrix} 5 \\ 7 \\ -4 \end{bmatrix}, F = [9 \ 7 \ 1]$$

**Solution:** Column Matrix

$$A = \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix}, C = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, E = \begin{bmatrix} 5 \\ 7 \\ -4 \end{bmatrix}$$

6. Identify all column matrices, if:

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 6 & 5 \\ 7 & 3 & 4 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix},$$

$$D = \begin{bmatrix} 7 & 8 \\ 6 & 5 \end{bmatrix}, E = [3 \ 7 \ 5], F = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

**Solution:** Column Matrix

$$C = \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$$

7. Identify all 3-by-3 square matrices, if:

$$A = \begin{bmatrix} 2 & -3 & 6 \\ 1 & 5 & 4 \\ 3 & 6 & -3 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}, C = [7 \ 3 \ 4]$$

**Solution:**

$$A = \begin{bmatrix} 2 & -3 & 6 \\ 1 & 5 & 4 \\ 3 & 6 & -3 \end{bmatrix}$$

### *A Scalar Multiplication*

Any element from the set of real numbers is also called a scalar. We define the product of a matrix  $A$  and a scalar  $k$ , denoted by  $kA$ , to be the matrix formed by multiplying each element of  $A$  by  $k$ .

For example:  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , then  $kA = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$

## Laws of Addition of Matrices

### Commutative Law:

For any two matrices A and B of the same order

$$A + B = B + A$$

This law is called commutative law of matrices with respect to addition.

### Associative Law:

For three matrices A, B and C of same order,

$$(A + B) + C = A + (B + C)$$

This law is called associative law of matrices with respect to addition.

### Additive Identity of Matrices

$$A + O = O + A = A$$

### Additive Inverse of a Matrix

If two matrices A and B are such that their sum  $(A + B)$  is a zero matrix, then A and B are called **additive inverse** of each other.

For example:  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , and  $B = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$

## Exercise 6.3

1- If  $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix}$   $B = \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$

Find

- |                |               |               |
|----------------|---------------|---------------|
| (i) $A + B$    | (ii) $A - B$  | (iii) $B - A$ |
| (iv) $2A + 3B$ | (v) $3A - 4B$ | (vi) $A - 2B$ |

**Solutions:**

(i)  $A + B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 2+0 & 3+1 & 4+5 \\ 1+2 & 5+3 & 5+6 \\ 4+1 & 9+4 & 3-2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 9 \\ 3 & 8 & 11 \\ 5 & 13 & 1 \end{bmatrix}$$

(ii)

$$A-B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 3-1 & 4-5 \\ 1-2 & 5-3 & 5-6 \\ 4-1 & 9-4 & 3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & -1 \\ 3 & 5 & 5 \end{bmatrix}$$

(iii)

$$B-A = \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0-2 & 1-3 & 5-4 \\ 2-1 & 3-5 & 6-5 \\ 1-4 & 4-9 & -2-3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 & 1 \\ 1 & -2 & 1 \\ -3 & -5 & -5 \end{bmatrix}$$

(iv)

$$\begin{aligned}
 2A + 3B &= 2 \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} + 3 \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 4 & 6 & 8 \\ 2 & 10 & 10 \\ 8 & 18 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 3 & 15 \\ 6 & 9 & 18 \\ 3 & 12 & -6 \end{bmatrix} \\
 &= \begin{bmatrix} 4+0 & 6+3 & 8+15 \\ 2+6 & 10+9 & 10+18 \\ 8+3 & 18+12 & 6-6 \end{bmatrix} \\
 &= \begin{bmatrix} 4 & 9 & 23 \\ 8 & 19 & 28 \\ 11 & 30 & 0 \end{bmatrix}
 \end{aligned}$$

(v)

$$\begin{aligned}
 3A - 4B &= 3 \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} - 4 \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 6 & 9 & 12 \\ 3 & 15 & 15 \\ 12 & 27 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 4 & 20 \\ 8 & 12 & 24 \\ 4 & 16 & -8 \end{bmatrix} \\
 &= \begin{bmatrix} 6-0 & 9-4 & 12-20 \\ 3-8 & 15-12 & 15-24 \\ 12-4 & 27-16 & 9+8 \end{bmatrix} \\
 &= \begin{bmatrix} 6 & 5 & -8 \\ -5 & 3 & -9 \\ 8 & 11 & 17 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad A-2B &= \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} - 2 \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 10 \\ 4 & 6 & 12 \\ 2 & 8 & -4 \end{bmatrix} \\
 &= \begin{bmatrix} 2-0 & 3-2 & 4-10 \\ 1-4 & 5-6 & 5-12 \\ 4-2 & 9-8 & 3+4 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 1 & -6 \\ -3 & -1 & -7 \\ 2 & 1 & 7 \end{bmatrix}
 \end{aligned}$$

2- Find the additive inverse of the following matrices.

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 6 \end{bmatrix}, B = \begin{bmatrix} \sqrt{2} & 3 \\ 4 & \sqrt{3} \end{bmatrix}, C = \begin{bmatrix} 1 \\ -7 \\ 4 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & 4 \\ 2 & -1 & -3 \end{bmatrix}, E = [2 \ 5 \ -3]$$

Sol:

| Matrix                                              | Additive Inverse                                         |
|-----------------------------------------------------|----------------------------------------------------------|
| $A = \begin{bmatrix} 4 & 3 \\ 2 & 6 \end{bmatrix},$ | $-A = \begin{bmatrix} -4 & -3 \\ -2 & -6 \end{bmatrix},$ |

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$$B = \begin{bmatrix} \sqrt{2} & 3 \\ 4 & \sqrt{3} \end{bmatrix}, \quad -B = \begin{bmatrix} -\sqrt{2} & -3 \\ -4 & -\sqrt{3} \end{bmatrix},$$

$$C = \begin{bmatrix} 1 \\ -7 \\ 4 \end{bmatrix}, \quad -C = \begin{bmatrix} -1 \\ 7 \\ -4 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & 4 \\ 2 & -1 & -3 \end{bmatrix}, \quad -D = \begin{bmatrix} -1 & 0 & 2 \\ 0 & -3 & -4 \\ -2 & 1 & 3 \end{bmatrix},$$

$$E = [2 \ 5 \ -3] \quad -E = [-2 \ -5 \ 3]$$

3- If  $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 7 \\ 4 & 6 \end{bmatrix}$  then show that

(i)  $4A - 3A = A$       (ii)  $3B - 3A = 3(B - A)$

Sol:

$$\begin{aligned} \text{(i)} \quad 4A - 3A &= 4 \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} - 3 \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 12 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix} \\ &= \begin{bmatrix} 8-6 & 12-9 \\ 4-3 & 20-15 \end{bmatrix} \\ 4A - 3A &= \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = A \quad (\text{Proved}) \end{aligned}$$

(ii)  $3B - 3A = 3(B - A)$

$$\begin{aligned}
 \text{L.H.S. } 3B - 3A &= 3 \begin{bmatrix} 1 & 7 \\ 4 & 6 \end{bmatrix} - 3 \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \\
 &= \begin{bmatrix} 3 & 21 \\ 12 & 18 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix} \\
 &= \begin{bmatrix} 3-6 & 21-9 \\ 12-3 & 18-15 \end{bmatrix} \\
 &= \begin{bmatrix} -3 & 12 \\ 9 & 3 \end{bmatrix} \dots\dots\dots(i)
 \end{aligned}$$

$$\begin{aligned}
 \text{R.H.S. } = 3(B - A) &= 3 \left( \begin{bmatrix} 1 & 7 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \right) \\
 &= 3 \left( \begin{bmatrix} 1-2 & 7-3 \\ 4-1 & 6-5 \end{bmatrix} \right) \\
 &= 3 \begin{bmatrix} -1 & 4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 12 \\ 9 & 3 \end{bmatrix} \quad \text{(ii)}
 \end{aligned}$$

$$3B - 3A = 3(B - A) \text{ From (i) and (ii)}$$

4- Find  $x$  and  $y$  if  $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

Sol:

$$x + 3 = 2$$

$$x = 2 - 3$$

$$\boxed{x = -1}$$

and  $3y - 4 = 2$

$$3y = 2 + 4$$

$$3y = 6$$

$$y = \frac{6}{3}$$

$$\boxed{y = 2}$$

5. If  $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix}$ ,  $C = \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix}$  then

prove that,

(i)  $A + B = B + A$  (ii)  $A + (B + C) = (A + B) + C$

(i)  $A + B = B + A$

Sol:

$$A + B = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & 3+7 \\ 4+6 & 5+5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 \\ 10 & 10 \end{bmatrix} \dots\dots (i)$$

and  $B + A = \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 4+1 & 7+3 \\ 6+4 & 5+5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 \\ 10 & 10 \end{bmatrix} \dots\dots (ii)$$

From (i) and (ii)  $A + B = B + A$

(ii)  $A + (B + C) = (A + B) + C$

$$\begin{aligned}
 A + (B + C) &= \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} + \left( \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix} \right) \\
 &= \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 4+2 & 7+6 \\ 6+3 & 5-2 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 6 & 13 \\ 9 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 1+6 & 3+13 \\ 4+9 & 5+3 \end{bmatrix} = \begin{bmatrix} 7 & 16 \\ 13 & 8 \end{bmatrix} \dots\dots (i)
 \end{aligned}$$

and

$$\begin{aligned}
 (A + B) + C &= \left( \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix} \right) + \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix} \\
 &= \left( \begin{bmatrix} 1+4 & 3+7 \\ 4+6 & 5+5 \end{bmatrix} \right) + \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 5 & 10 \\ 10 & 10 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 5+2 & 10+6 \\ 10+3 & 10-2 \end{bmatrix} = \begin{bmatrix} 7 & 16 \\ 13 & 8 \end{bmatrix} \dots\dots (ii)
 \end{aligned}$$

From (i) and (ii)

$$A + (B + C) = (A + B) + C$$

6- Solve the matrix equation for X.

$$3X - 2A = B \text{ if } A = \begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$$

Solution:

$$3X - 2A = B$$

$$3X = B + 2A$$

$$3X = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix} + 2 \begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ -8 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4 & -3+6 \\ 4-8 & 4+2 \end{bmatrix}$$

$$3X = \begin{bmatrix} 6 & 3 \\ -4 & 6 \end{bmatrix}$$

$$X = \frac{1}{3} \begin{bmatrix} 6 & 3 \\ -4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \times \frac{1}{3} & 3 \times \frac{1}{3} \\ -4 \times \frac{1}{3} & 6 \times \frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ -\frac{4}{3} & 2 \end{bmatrix}$$

7. Find  $a, b, c, d, e$  and  $f$  such that

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - \begin{bmatrix} 3 & -2 & 1 \\ 5 & 0 & -4 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 4 & 6 \end{bmatrix}$$

$$\text{Sol: } \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} + \begin{bmatrix} 3 & -2 & 1 \\ 5 & 0 & -4 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} a+3 & b-2 & c+1 \\ d+5 & e-0 & f-4 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 4 & 6 \end{bmatrix}$$

Hence

$$a+3 = -1, \quad b-2 = -2, \quad c+1 = 3$$

$$a = -1 - 3 \quad b = -2 + 2 \quad c = 3 - 1$$

$$\boxed{a = -4}$$

$$\boxed{b = 0}$$

$$\boxed{c = 2}$$

and

$$d+5 = -2, \quad e-0 = 4, \quad f-4 = 6$$

$$d = -2 - 5 \quad \boxed{e = 4} \quad f = 6 + 4$$

$$\boxed{d = -7}$$

$$\boxed{f = 10}$$

8. Find  $w, x, y, z$  such that

$$\begin{bmatrix} w & x \\ y & z \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 6 & -3 \end{bmatrix}$$

$$\text{Sol: } \begin{bmatrix} w+3 & x+0 \\ y-1 & z+5 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 6 & -3 \end{bmatrix}$$

$$\text{Hence } w+3 = 2, \quad x+0 = 1$$

$$w = 2 - 3 \quad \boxed{x = 1}$$

$$\boxed{w = -1}$$

$$y-1 = 6, \quad z+5 = -3$$

$$y = 6 + 1 \quad z = -3 - 5$$

$$\boxed{y = 7} \quad \boxed{z = -8}$$

$$\boxed{y = 7}$$

$$\boxed{z = -8}$$

9. If  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  then what is the additive inverse of  $A$ ?

Sol:  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$-A = -\begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad (\text{Additive Inverse of } A)$$

$$= \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$$

10. Given that  $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$  verify that  $A^2 - 4A + 5I = 0$ .

Sol:  $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$$\text{L.H.S} = A^2 - 4A + 5I$$

$$= \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} - 4 \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1-3 \\ 2+6 & -2+9 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4 & -4+4 \\ 8-8 & 7-12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$\begin{aligned}
 &= \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \\
 &= \begin{bmatrix} -5+5 & 0+0 \\ 0+0 & -5+5 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0
 \end{aligned}$$

Hence proved

$$A^2 - 4A + 5I = 0$$

11- If  $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$ , then verify that

$$(A + B)' = A' + B'$$

Sol:  $A + B = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$

$$= \begin{bmatrix} 2+3 & 4-2 \\ 1+4 & 5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 5 & 11 \end{bmatrix}$$

$$(A + B)' = \begin{bmatrix} 5 & 5 \\ 2 & 11 \end{bmatrix} \dots\dots (i)$$

Now  $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$

$$A' = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} \dots\dots (ii)$$

and

$$B = \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$$

$$B' = \begin{bmatrix} 3 & 4 \\ -2 & 6 \end{bmatrix} \dots\dots (iii)$$

$$A' + B' = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ -2 & 6 \end{bmatrix} \quad \text{From (i) and (ii)}$$

$$= \begin{bmatrix} 2+3 & 1+4 \\ 4-2 & 5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 5 \\ 2 & 11 \end{bmatrix} \dots\dots (iv)$$

$$(A + B)' = A' + B' \quad \text{From (i) and (iv)}$$

12- If  $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & -7 \\ 5 & 8 \end{bmatrix}$ ,  $C = \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$  then

show that  $A + B - C = \begin{bmatrix} 2 & -10 \\ 8 & 2 \end{bmatrix}$

**Solution:**

$$A + B - C = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 2 & -7 \\ 5 & 8 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2-7 \\ 3+5 & -4+8 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -5 \\ 8 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-1 & -5-5 \\ 8-0 & 4-2 \end{bmatrix} = \begin{bmatrix} 2 & -10 \\ 8 & 2 \end{bmatrix}$$

Hence proved

$$A + B - C = \begin{bmatrix} 2 & -10 \\ 8 & 2 \end{bmatrix}$$

## MULTIPLICATION OF MATRICES

Two matrices  $A$  and  $B$  are said to be conformable for the product  $AB$ , if the number of columns in  $A$  is equal to the number of rows in  $B$ .

### **Remember that:**

For multiplication  $AB$  of two matrices  $A$  and  $B$  the following points should be kept in mind.

- (i) The number of columns in  $A$  = number of rows in  $B$ .
- (ii) The product of matrices  $A$  and  $B$  is denoted by  $A \times B$  or  $AB$ .
- (iii) If  $A$  is a  $m$ -by- $p$  matrix and  $B$  is a  $p$ -by- $n$  matrix then  $AB$  is  $m$ -by- $n$  matrix.

### **Associative Law of Matrices with respect to Multiplication**

If three matrices  $A$ ,  $B$  and  $C$  are conformable for multiplication, then

$$A(BC) = (AB)C$$

is called associative law with respect to multiplication.

### **Distributive Laws:**

If the matrices  $A$ ,  $B$  and  $C$  are conformable for addition and multiplication, then

- (i)  $A(B + C) = AB + AC$  (left distributive law for matrices)
  - (ii)  $(A + B)C = AC + BC$  (right distributive law for matrices)
- (i) and (ii) are called distributive laws.

### Exercise 6.4

*In Problems 1 to 8 Verify Each Statement, Using*

$$A = \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}, C = \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$$

1.  $(AB)C = A(BC)$

$$\text{L.H.S} = (AB)C$$

$$\begin{aligned} (AB)C &= \left( \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \right) \times \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \\ &= \left( \begin{bmatrix} 8-4 & 4+8 \\ 0+0 & 0+0 \end{bmatrix} \right) \times \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 12 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -4+48 & 8+24 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 44 & 32 \\ 0 & 0 \end{bmatrix} \dots\dots (i) \end{aligned}$$

$$\text{R.H.S} = A(BC)$$

$$\begin{aligned} A(BC) &= \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \times \left( \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \right) \\ &= \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} -2+4 & 4+2 \\ 2+16 & -4+8 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 6 \\ 18 & 4 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 8 + 36 & 24 + 8 \\ 0 + 0 & 0 + 0 \end{bmatrix} = \begin{bmatrix} 44 & 32 \\ 0 & 0 \end{bmatrix} \dots\dots (ii)$$

$$(AB)C = A(BC) \quad \text{from (i) and (ii)}$$

2.  $AB \neq BA$

Sol:  $AB = \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 8 - 4 & 4 + 8 \\ 0 + 0 & 0 + 0 \end{bmatrix} = \begin{bmatrix} 4 & 12 \\ 0 & 0 \end{bmatrix} \dots\dots (i)$$

$$BA = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \times \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 8 + 0 & 4 + 0 \\ -8 + 0 & -4 + 0 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ -8 & -4 \end{bmatrix} \dots\dots (ii)$$

$$AB \neq BA \quad \text{from (i) and (ii)}$$

3.  $A(B + C) = AB + AC$

Sol: L.H.S = A(B + C)

Putting values of A, B, C

$$A(B + C) = \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \left( \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \left( \begin{bmatrix} 2 - 1 & 1 + 2 \\ -2 + 4 & 4 + 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & 12+12 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 8 & 24 \\ 0 & 0 \end{bmatrix} \dots\dots (i)$$

$$\text{R.H.S} = AB + AC$$

Putting values of A, B, C

$$\begin{aligned} AB + AC &= \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 8-4 & 4+8 \\ 0+0 & 0+0 \end{bmatrix} + \begin{bmatrix} -4+8 & 8+4 \\ 0+0 & 0+0 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 12 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 12 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 4+4 & 12+12 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 8 & 24 \\ 0 & 0 \end{bmatrix} \dots\dots (ii) \end{aligned}$$

from (i) and (ii)

$$A(B + C) = AB + AC$$

4.  $(B + C)A = BA + CA$

Sol: L.H.S =  $(B + C)A$

$$\begin{aligned} (B+C)A &= \left( \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \right) \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \\ &= \left( \begin{bmatrix} 2-1 & 1+2 \\ -2+4 & 4+2 \end{bmatrix} \right) \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 4+0 & 2+0 \\ 8+0 & 4+0 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix} \dots\dots (i) \end{aligned}$$

$$\text{R.H.S} = BA + CA$$

$$BA + CA$$

$$\begin{aligned} BA + CA &= \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 8+0 & 4+0 \\ -8+0 & -4+0 \end{bmatrix} + \begin{bmatrix} -4+0 & -2+0 \\ 16+0 & 8+0 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 4 \\ -8 & -4 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 16 & 8 \end{bmatrix} \\ &= \begin{bmatrix} 8-4 & 4-2 \\ -8+16 & -4+8 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix} \dots\dots (ii) \end{aligned}$$

from (i) and (ii)

$$(B + C)A = BA + CA$$

5.  $(B + C)(B - C) \neq B^2 - C^2$

Sol: L.H.S =  $(B + C)(B - C)$

Putting values of B, C

$$\begin{aligned} (B + C)(B - C) &= \left( \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \right) \left( \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \right) \\ &= \begin{bmatrix} 2-1 & 1+2 \\ -2+4 & 4+2 \end{bmatrix} \begin{bmatrix} 2+1 & 1-2 \\ -2-4 & 4-2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -6 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 3-18 & -1+6 \\ 6-36 & -2+12 \end{bmatrix} \\ &= \begin{bmatrix} -15 & 5 \\ -30 & 10 \end{bmatrix} \dots\dots (i) \end{aligned}$$

$$\text{R.H.S } B^2 - C^2 = B \times B - C \times C$$

$$\begin{aligned}
 &= \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 4-2 & 2+4 \\ -4-8 & -2+16 \end{bmatrix} - \begin{bmatrix} 1+8 & -2+4 \\ -4+8 & 8+4 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 6 \\ -12 & 14 \end{bmatrix} - \begin{bmatrix} 9 & 2 \\ 4 & 12 \end{bmatrix} \\
 &= \begin{bmatrix} 2-9 & 6-2 \\ -12-4 & 14-12 \end{bmatrix} \\
 &= \begin{bmatrix} -7 & 4 \\ -16 & 2 \end{bmatrix} \dots\dots (ii)
 \end{aligned}$$

$$(B + C)(B - C) \neq B^2 - C^2 \quad \text{from (i) and (ii)}$$

$$6. \quad (BC)' = C' B'$$

$$\text{Sol: L.H.S} = (BC)'$$

$$\begin{aligned}
 BC &= \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} -2+4 & 4+2 \\ 2+16 & -4+8 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 6 \\ 18 & 4 \end{bmatrix}
 \end{aligned}$$

$$(BC)' = \begin{bmatrix} 2 & 18 \\ 6 & 4 \end{bmatrix} \dots\dots (i) \quad \text{and}$$

$$\text{Now R.H.S} = C' B'$$

$$C = \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$$

$$C' = \begin{bmatrix} -1 & 4 \\ 2 & 2 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$$

$$B' = \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$$

Now  $C'B' = \begin{bmatrix} -1 & 4 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$

$$= \begin{bmatrix} -2 + 4 & 2 + 16 \\ 4 + 2 & -4 + 8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 18 \\ 6 & 4 \end{bmatrix} \dots\dots (ii)$$

$$(BC)' = C'B'$$

from (i) and (ii)

7.  $BI = B$

Sol: L.H.S =  $BI$

$$BI = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BI = \begin{bmatrix} 2+0 & 0+1 \\ -2+0 & 0+4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} = B$$

8.  $BC \neq CB$

Sol:  $BC = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -2 + 4 & 4 + 2 \\ 2 + 16 & -4 + 8 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 18 & 4 \end{bmatrix} \dots\dots\dots (i)$$

$$CB = \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 - 4 & -1 + 8 \\ 8 - 4 & 4 + 8 \end{bmatrix} = \begin{bmatrix} -6 & 7 \\ 4 & 12 \end{bmatrix} \dots\dots\dots (ii)$$

from (i) and (ii)

$BC \neq CB$

**Find the Matrix Products.**

9.  $\begin{bmatrix} 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Sol:  $= \begin{bmatrix} 2(1) + 5(2) & 2(-1) + (5)(3) \end{bmatrix}$

$$= \begin{bmatrix} 2 + 10 & -2 + 15 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 13 \end{bmatrix}$$

10.  $\begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Sol:  $= \begin{bmatrix} 3(-1) + 4(2) \\ -1(-1) + (-2)(2) \end{bmatrix}$

$$= \begin{bmatrix} -3 + 8 \\ +1 - 4 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$11. \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix}$$

$$\text{Sol:} = \begin{bmatrix} 2(1) + (-3)(0) & 2(-1) + (-3)(-2) \\ 1(1) + 2(0) & 1(-1) + (2)(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 0 & -2 + 6 \\ 1 + 0 & -1 - 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 1 & -5 \end{bmatrix}$$

$$12. \begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} -1 & 5 \\ -1 & 3 \end{bmatrix}$$

$$\text{Sol:} = \begin{bmatrix} -3(-1) + (2)(-1) & -3(5) + (2)(3) \\ 4(-1) + (-1)(-1) & 4(5) + (-1)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 3 - 2 & -15 + 6 \\ -4 + 1 & 20 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -9 \\ -3 & 17 \end{bmatrix}$$

$$13. = \begin{bmatrix} -5 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$$

$$\text{Sol:} = \begin{bmatrix} (-5)(-2) + (-2)(0) & (-5)(1) + (-2)(-3) \\ 1(-2) + (-3)(0) & (1)(1) + (-3)(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 10 + 0 & -5 + 6 \\ -2 + 0 & 1 + 9 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 1 \\ -2 & 10 \end{bmatrix}$$

14.  $\begin{bmatrix} -2 & 4 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} -5 & -5 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -0 \end{bmatrix}$

Sol: 
$$\begin{aligned} &= \left( \begin{bmatrix} -2 & 4 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} -5 & -5 \\ 1 & -3 \end{bmatrix} \right) \begin{bmatrix} 1 & -1 \\ 2 & -0 \end{bmatrix} \\ &= \begin{bmatrix} (-2)(-5) + (4)(1) & (-2)(-5) + (4)(-3) \\ (0)(-5) + (-3)(1) & (0)(-5) + (-3)(-3) \end{bmatrix} \\ &= \begin{bmatrix} 10 + 4 & 10 - 12 \\ 0 - 3 & 0 + 9 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 14 & -2 \\ -3 & 9 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 14(1) + (-2)(2) & 14(-1) + (-2)(0) \\ -3(1) + 9(2) & -3(-1) + 9(0) \end{bmatrix} \\ &= \begin{bmatrix} 14 - 4 & -14 + 0 \\ -3 + 18 & 3 + 0 \end{bmatrix} \\ &= \begin{bmatrix} 10 & -14 \\ 15 & 3 \end{bmatrix} \end{aligned}$$

15. If  $\begin{bmatrix} 1 & 5 \\ 3 & a \end{bmatrix} \begin{bmatrix} b \\ 7 \end{bmatrix} = \begin{bmatrix} 35 \\ 10 \end{bmatrix}$ , then find the values of  $a$  and  $b$ .

Sol: 
$$\begin{bmatrix} 1 & 5 \\ 3 & a \end{bmatrix} \begin{bmatrix} b \\ 7 \end{bmatrix} = \begin{bmatrix} 35 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 1(b) + 5(7) \\ 3(b) + (a)(7) \end{bmatrix} = \begin{bmatrix} 35 \\ 10 \end{bmatrix}$$


---

$$\begin{bmatrix} b + 35 \\ 3b + 7a \end{bmatrix} = \begin{bmatrix} 35 \\ 10 \end{bmatrix}$$

Now  $b + 35 = 35$

Hence  $\boxed{b = 0}$

and  $3b + 7a = 10$

$$3(0) + 7a = 10 \quad (\text{Putting values of } b)$$

$$7a = 10$$

$$\boxed{a = \frac{10}{7}}$$

16. If  $A = \begin{bmatrix} 2 & 6 \\ 7 & 8 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$ , then verify  
 $(AB)' = B' A'$

Sol:

$$\text{L.H.S} = (AB)'$$

$$\begin{aligned} AB &= \begin{bmatrix} 2 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 2(-1) + 6(2) & 2(-3) + 6(0) \\ 7(-1) + 8(2) & 7(-3) + 8(0) \end{bmatrix} \\ &= \begin{bmatrix} -2 + 12 & -6 + 0 \\ -7 + 16 & -21 + 0 \end{bmatrix} \\ &= \begin{bmatrix} 10 & -6 \\ 9 & -21 \end{bmatrix} \end{aligned}$$

$$(AB)' = \begin{bmatrix} 10 & 9 \\ -6 & -21 \end{bmatrix} \dots\dots (i)$$

$$\text{R.H.S} = B' A'$$

$$B = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$$

$$B' = \begin{bmatrix} -1 & 2 \\ -3 & 0 \end{bmatrix}$$

and

$$A = \begin{bmatrix} 2 & 6 \\ 7 & 8 \end{bmatrix}$$

$$A' = \begin{bmatrix} 2 & 7 \\ 6 & 8 \end{bmatrix}$$

$$\begin{aligned} \text{Now } B' A' &= \begin{bmatrix} -1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ 6 & 8 \end{bmatrix} \\ &= \begin{bmatrix} -1(2) + 2(6) & -1(7) + 2(8) \\ -3(2) + (0)(6) & -3(7) + 0(8) \end{bmatrix} \\ &= \begin{bmatrix} -2 + 12 & -7 + 16 \\ -6 + 0 & -21 + 0 \end{bmatrix} \\ &= \begin{bmatrix} 10 & 9 \\ -6 & -21 \end{bmatrix} \dots\dots (ii) \end{aligned}$$

$$(AB)' = B' A' \quad \text{from (i) and (ii)}$$

## MULTIPLICATIVE INVERSE OF A MATRIX

### Determinant Function

If  $A$  is a square matrix, then  $\det A$  or  $|A|$  read "The determinant of  $A$ " is used to denote the unique real number.

**Singular Matrix:**

A square matrix  $A$  is called a singular matrix. If  $\det A = 0$

$$\text{Example : } A = \begin{bmatrix} 12 & 6 \\ 6 & 3 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 12 & 6 \\ 6 & 3 \end{vmatrix} = 36 - 36$$

$\det A = 0$ . Hence matrix  $A$  is singular.

**Non-Singular Matrix:**

A square matrix  $A$  is called non-singular matrix, if  $\det A \neq 0$ .

**Example**

$$\text{If } A = \begin{bmatrix} 2 & 5 \\ 6 & 8 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 2 & 5 \\ 6 & 8 \end{vmatrix} = 16 - 30$$

$\det A = -14$ . Hence matrix  $A$  is non-singular.

**Adjoint of a Matrix:**

Let  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  be a square matrix of order 2-by-2.

Then the matrix obtained by interchanging the elements of the diagonal (i.e  $a$  and  $d$ ) and by changing the signs of the other elements  $b$  and  $c$  is called the adjoint of the matrix  $A$ .

**Multiplicative Inverse**

In the set of real numbers, we know that for each real

number  $a$  (except zero) there exists a real number  $a^{-1}$  such that  $aa^{-1} = 1$ .

The number  $a^{-1}$  is called the multiplicative inverse of  $a$ .

$$A^{-1} = \frac{\text{adj } A}{|A|}, \quad |A| \neq 0$$

If  $A$  is a singular matrix then the multiplicative inverse of  $A$  does not exist.

**Remember that:**

- (i) Inverse of square matrix  $A$  is denoted by  $A^{-1}$ .
- (ii) Only non-singular matrices have inverses.
- (iii) Inverse of square matrix  $A$  is always unique.
- (iv) Non-square matrices cannot possess inverses.
- (v)  $A^{-1} = \frac{\text{adj } A}{|A|}$

### Exercise 6.5

1- Find the determinants of the following matrices.

(i)  $\begin{bmatrix} u & v \\ x & y \end{bmatrix}$

(ii)  $\begin{bmatrix} -2 & 5 \\ 1 & 4 \end{bmatrix}$

(iii)  $\begin{bmatrix} -8 & -4 \\ -4 & -2 \end{bmatrix}$

(iv)  $\begin{bmatrix} 1 & 3 \\ 1 & 8 \\ 1 & 1 \\ 8 & 4 \end{bmatrix}$

$$(i). \quad A = \begin{bmatrix} u & v \\ x & y \end{bmatrix}$$

$$\text{Sol:} \quad \text{Let } |A| = \begin{vmatrix} u & v \\ x & y \end{vmatrix} = uy - vx$$

$$(ii) \quad P = \begin{bmatrix} -2 & 5 \\ 1 & 4 \end{bmatrix}$$

$$\begin{aligned} \text{Sol:} \quad \text{Let } |P| &= \begin{vmatrix} -2 & 5 \\ 1 & 4 \end{vmatrix} \\ &= (-2)(4) - (5)(1) \\ &= -8 - 5 = -13 \end{aligned}$$

$$(iii) \quad B = \begin{bmatrix} -8 & -4 \\ -4 & -2 \end{bmatrix}$$

$$\begin{aligned} \text{Sol:} \quad \text{Let } |B| &= \begin{vmatrix} -8 & -4 \\ -4 & -2 \end{vmatrix} \\ &= (-8)(-2) - (-4)(-4) \\ &= 16 - 16 = 0 \end{aligned}$$

$$(iv) \quad A = \begin{bmatrix} \frac{1}{1} & \frac{3}{8} \\ \frac{1}{1} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{4} \end{bmatrix}$$

$$\text{Sol:} \quad \text{Let } |A| = \begin{vmatrix} \frac{1}{1} & \frac{3}{8} \\ \frac{1}{1} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{4} \end{vmatrix}$$

$$\begin{aligned}
 &= (1) \left( \frac{1}{4} \right) - \left( \frac{3}{8} \right) \left( \frac{1}{8} \right) \\
 &= \frac{1}{4} - \frac{3}{64} \\
 &= \frac{16-3}{64} = \frac{13}{64}
 \end{aligned}$$

2. Identify the singular and non-singular matrices.

(i) 
$$\begin{bmatrix} -1 & 3 \\ 1 & -3 \end{bmatrix}$$

Sol: Let  $A = \begin{bmatrix} -1 & 3 \\ 1 & -3 \end{bmatrix}$

$$\begin{aligned}
 |A| &= \begin{vmatrix} -1 & 3 \\ 1 & -3 \end{vmatrix} \\
 &= (-1)(-3) - (3)(1) \\
 &= 3 - 3 = 0
 \end{aligned}$$

Therefore,  $\begin{bmatrix} -1 & 3 \\ 1 & -3 \end{bmatrix}$  is singular matrix

(ii) 
$$\begin{bmatrix} 3 & 8 \\ 4 & 9 \end{bmatrix}$$

Sol: Let  $B = \begin{bmatrix} 3 & 8 \\ 4 & 9 \end{bmatrix}$

$$\begin{aligned}
 |B| &= \begin{vmatrix} 3 & 8 \\ 4 & 9 \end{vmatrix} \\
 &= (3)(9) - (8)(4)
 \end{aligned}$$

$$= 27 - 32 = -5 \neq 0$$

Therefore,  $\begin{bmatrix} 3 & 8 \\ 4 & 9 \end{bmatrix}$  is non-singular matrix.

(iii)  $\begin{bmatrix} -a & b \\ a & b \end{bmatrix}$

Sol Let  $P = \begin{bmatrix} -a & b \\ a & b \end{bmatrix}$

$$\begin{aligned} |P| &= \begin{vmatrix} -a & b \\ a & b \end{vmatrix} \\ &= (-a)(b) - (a)(b) \\ &= -ab - ab \\ &= -2ab \neq 0 \end{aligned}$$

Therefore,  $\begin{bmatrix} -a & b \\ a & b \end{bmatrix}$  is non-singular matrix.

3. Find the inverse of each matrix A and show that  $A^{-1}A = I$ . If the inverse does not exist, give reason.

(i)  $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

Sol:  $|A| = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$

$$\begin{aligned} &= (1)(3) - (2)(1) \\ &= 3 - 2 = 1 \end{aligned}$$

$$\begin{aligned}
 A^{-1} &= \frac{\text{adj } A}{|A|} \\
 &= \frac{\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}}{1} \\
 &= \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}
 \end{aligned}$$

Now  $A^{-1}A = ?$

$$\begin{aligned}
 A^{-1}A &= \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 3(1) + (-2)(1) & 3(2) + (-2)(3) \\ -1(1) + (1)(1) & -1(2) + (1)(3) \end{bmatrix} \\
 &= \begin{bmatrix} 3-2 & 6-6 \\ -1+1 & -2+3 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I
 \end{aligned}$$

$$A^{-1}A = I \quad (\text{proved})$$

$$(ii) \quad A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

$$\text{Sol: } |A| = \begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix} = (2)(3) - (1)(5) = 6 - 5 = 1$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}}{1}$$

$$= \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$\begin{aligned} \text{and } A^{-1}A &= \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 3(2) + (-1)(5) & 3(1) + (-1)(3) \\ -5(2) + (2)(5) & -5(1) + (2)(3) \end{bmatrix} \\ &= \begin{bmatrix} 6-5 & 3-3 \\ -10+10 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

$$A^{-1}A = I \quad (\text{proved})$$

$$(iii) \quad A = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$$

$$\text{Sol: } |A| = (2)(3) - (-1)(0)$$

$$|A| = \begin{vmatrix} 2 & 0 \\ -1 & 3 \end{vmatrix} = (2)(3) - (-1)(0)$$

$$= 6 - 0 = 6$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$A^{-1} = \frac{\begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}}{6}$$

$$= \frac{1}{6} \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$$

$$\begin{aligned} \text{Now } A^{-1}A &= \frac{1}{6} \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix} \\ &= \frac{1}{6} \begin{bmatrix} (3)(2) + (0)(-1) & (3)(0) + (0)(3) \\ (1)(2) + (2)(-1) & (1)(0) + (2)(3) \end{bmatrix} \\ &= \frac{1}{6} \begin{bmatrix} 6 + 0 & 0 + 0 \\ 2 - 2 & 0 + 6 \end{bmatrix} \\ &= \frac{1}{6} \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \\ &= \begin{bmatrix} \frac{6}{6} & \frac{0}{6} \\ \frac{0}{6} & \frac{6}{6} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

$$A^{-1}A = I \quad (\text{proved})$$

$$(iv) \quad A = \begin{bmatrix} -6 & 4 \\ 3 & -2 \end{bmatrix}$$

$$\begin{aligned} \text{Sol: } |A| &= \begin{vmatrix} -6 & 4 \\ 3 & -2 \end{vmatrix} \\ &= (-6)(-2) - (4)(3) \\ &= 12 - 12 = 0 \end{aligned}$$

It is a singular matrix, its inverse is not possible.

$$(v) \quad A = \begin{bmatrix} 1 & 3 \\ 2 & 8 \end{bmatrix}$$

$$\begin{aligned} \text{Sol: } |A| &= \begin{vmatrix} 1 & 3 \\ 2 & 8 \end{vmatrix} \\ &= (1)(8) - (3)(2) \\ &= 8 - 6 = 2 \end{aligned}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} 8 & -3 \\ -2 & 1 \end{bmatrix}}{2}$$

$$= \frac{1}{2} \begin{bmatrix} 8 & -3 \\ -2 & 1 \end{bmatrix}$$

$$\begin{aligned} \text{Now } A^{-1}A &= \frac{1}{2} \begin{bmatrix} 8 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 8 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 8(1) + (-3)(2) & 8(3) + (-3)(8) \\ -2(1) + 1(2) & -2(3) + (1)(8) \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 8 - 6 & 24 - 24 \\ -2 + 2 & -6 + 8 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{2}{2} & \frac{0}{2} \\ \frac{0}{2} & \frac{2}{2} \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{-1}A = I \quad (\text{proved})$$

$$(vi) \quad A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{aligned} \text{Sol: } |A| &= \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} \\ &= (-1)(-1) - (0)(0) \\ &= 1 - 0 = 1 \end{aligned}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}}{1}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\text{Now } A^{-1}A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} (-1)(-1) + (0)(0) & (-1)(0) + (0)(-1) \\ 0(-1) + (-1)(0) & (0)(0) + (-1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{-1}A = I \quad (\text{proved})$$

$$(vii) \quad A = \begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

$$\text{Sol:} \quad |A| = \begin{vmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{vmatrix}$$

$$= \left( \frac{3}{5} \right) \left( \frac{3}{5} \right) - \left( -\frac{4}{5} \right) \left( \frac{4}{5} \right)$$

$$= \frac{9}{25} + \frac{16}{25}$$

$$= \frac{9+16}{25}$$

$$= \frac{25}{25} = 1$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix}}{1}$$

$$= \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

$$\begin{aligned}
 \text{Now } A^{-1}A &= \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix} \\
 &= \begin{bmatrix} \left(\frac{3}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{4}{5}\right) & \left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right) \\ \left(-\frac{4}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{3}{5}\right)\left(\frac{4}{5}\right) & \left(-\frac{4}{5}\right)\left(-\frac{4}{5}\right) + \left(\frac{3}{5}\right)\left(\frac{3}{5}\right) \end{bmatrix} \\
 &= \begin{bmatrix} \frac{9}{25} + \frac{16}{25} & -\frac{12}{25} + \frac{12}{25} \\ -\frac{12}{25} + \frac{12}{25} & \frac{16}{25} + \frac{9}{25} \end{bmatrix} \\
 A^{-1}A &= \begin{bmatrix} \frac{25}{25} & \frac{0}{25} \\ \frac{0}{25} & \frac{25}{25} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I
 \end{aligned}$$

$$A^{-1}A = I \quad \text{Hence proved}$$

4. Let  $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(a) Find  $M^{-1}$

(b) Verify that  $M^{-1}M = MM^{-1}$

Sol:  $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$|M| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$$= (1)(4) - (2)(3)$$

$$= 4 - 6 = -2$$

$$M^{-1} = \frac{\text{adj } M}{|M|}$$

$$= \frac{\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}}{-2}$$

$$= \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 4\left(-\frac{1}{2}\right) & (-2)\left(-\frac{1}{2}\right) \\ -3\left(-\frac{1}{2}\right) & (1)\left(-\frac{1}{2}\right) \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

$$M^{-1}M = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} (-2)(1) + (1)(3) & (-2)(2) + (1)(4) \\ \left(\frac{3}{2}\right)(1) + \left(-\frac{1}{2}\right)(3) & \left(\frac{3}{2}\right)(2) + \left(-\frac{1}{2}\right)(4) \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 3 & -4 + 4 \\ \frac{3}{2} - \frac{3}{2} & 3 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \dots\dots(i)$$

$$\begin{aligned}
 \text{and now } MM^{-1} &= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -2 & \\ \frac{3}{2} & \\ 2 & \end{bmatrix} \\
 &= \begin{bmatrix} (1)(-2) + (2)\left(\frac{3}{2}\right) & (1)(1) + (2)\left(-\frac{1}{2}\right) \\ (3)(-2) + (4)\left(\frac{3}{2}\right) & (3)(1) + (4)\left(-\frac{1}{2}\right) \end{bmatrix} \\
 &= \begin{bmatrix} -2 + 3 & 1 - 1 \\ -6 + 6 & 3 - 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \dots\dots (ii)
 \end{aligned}$$

$$M^{-1}M = MM^{-1}$$

from (i) and (ii)

5. If  $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$ , verify that

$$(AB)^{-1} = B^{-1}A^{-1}$$

**Sol:** L.H.S =  $(AB)^{-1}$

$$\begin{aligned}
 AB &= \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} (5)(4) + (2)(3) & (5)(2) + (2)(-1) \\ (2)(4) + (1)(3) & (2)(2) + (1)(-1) \end{bmatrix} \\
 &= \begin{bmatrix} 20 + 6 & 10 - 2 \\ 8 + 3 & 4 - 1 \end{bmatrix} \\
 &= \begin{bmatrix} 26 & 8 \\ 11 & 3 \end{bmatrix}
 \end{aligned}$$

$$\text{Now } |AB| = \begin{vmatrix} 26 & 8 \\ 11 & 3 \end{vmatrix}$$

$$= (26)(3) - (11)(8)$$

$$= 78 - 88$$

$$= -10$$

$$(AB)^{-1} = \frac{\text{adj}(AB)}{|AB|}$$

$$= \frac{\begin{bmatrix} 3 & -8 \\ -11 & 26 \end{bmatrix}}{-10} = -\frac{1}{10} \begin{bmatrix} 3 & -8 \\ -11 & 26 \end{bmatrix}$$

$$= \begin{bmatrix} (3)\left(-\frac{1}{10}\right) & (-8)\left(-\frac{1}{10}\right) \\ (-11)\left(-\frac{1}{10}\right) & (26)\left(-\frac{1}{10}\right) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{3}{10} & \frac{4}{5} \\ \frac{11}{10} & -\frac{13}{5} \end{bmatrix} \dots\dots\dots(i)$$

$$\text{R.H.S} = B^{-1}A^{-1}$$

$$B^{-1}A^{-1} = ?$$

$$B = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (2)(3)$$

$$= -4 - 6$$

$$= -10$$

$$B^{-1} = \frac{\text{adj } B}{|B|}$$

$$B^{-1} = \frac{\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}}{-10}$$

$$= -\frac{1}{10} \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)\left(-\frac{1}{10}\right) & (-2)\left(-\frac{1}{10}\right) \\ (-3)\left(-\frac{1}{10}\right) & (4)\left(-\frac{1}{10}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{10} & \frac{2}{10} \\ \frac{3}{10} & -\frac{4}{10} \end{bmatrix}$$

$$\text{and } A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= (5)(1) - (2)(2)$$

$$= 5 - 4$$

$$= 1$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}}{1} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$$

$$\begin{aligned}
 \text{Now } B^{-1}A^{-1} &= \begin{bmatrix} \frac{1}{10} & \frac{1}{5} \\ \frac{3}{10} & -\frac{2}{5} \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} \\
 &= \begin{bmatrix} \left(\frac{1}{10}\right)(1) + \left(\frac{1}{5}\right)(-2) & \left(\frac{1}{10}\right)(-2) + \left(\frac{1}{5}\right)(5) \\ \left(\frac{3}{10}\right)(1) + \left(-\frac{2}{5}\right)(-2) & \left(\frac{3}{10}\right)(-2) + \left(-\frac{2}{5}\right)(5) \end{bmatrix} \\
 &= \begin{bmatrix} \frac{1}{10} - \frac{2}{5} & -\frac{1}{5} + 1 \\ \frac{3}{10} + \frac{4}{5} & -\frac{3}{5} - 2 \end{bmatrix} \\
 &= \begin{bmatrix} \frac{1-4}{10} & \frac{-1+5}{5} \\ \frac{3+8}{10} & \frac{-3-10}{5} \end{bmatrix} \\
 &= \begin{bmatrix} -\frac{3}{10} & \frac{4}{5} \\ \frac{11}{10} & -\frac{13}{5} \end{bmatrix} \dots\dots(ii)
 \end{aligned}$$

from (i) and (ii)

$$(AB)^{-1} = B^{-1}A^{-1}$$

### **SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS**

To determine the value of two variables, we need a pair of equations. Such a pair of equations is called a system of simultaneous linear equations.



- 1- Write the equation  $2x + ky = 7$  and  $4x - 9y = 4$  in matrix form. Also find the value of  $k$  if the matrix of the coefficients is singular.

Sol:

$$2x + ky = 7$$

$$4x - 9y = 4$$

In matrix form

$$\begin{bmatrix} 2 & k \\ 4 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$$A \cdot X = B$$

$$A = \begin{bmatrix} 2 & k \\ 4 & -9 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$$|A| = 0$$

Now, if  $A$  is singular matrix.

Therefore,

$$\begin{bmatrix} 2 & k \\ 4 & -9 \end{bmatrix} = 0$$

$$(2)(-9) - (k)(4) = 0$$

$$-18 - 4k = 0$$

$$-4k = 18$$

$$k = \frac{18}{-4}$$

$$= -\frac{9}{2}$$

2. Solve the simultaneous equations by the matrix inversion method where possible. Where there is no solution, explain why this is so.

$$(i) \quad \begin{aligned} 2x - 5y &= 1 \\ 3x - 7y &= 2 \end{aligned}$$

Sol. In matrix form

$$\begin{bmatrix} 2 & -5 \\ 3 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$A \cdot X = B$$

$$A = \begin{bmatrix} 2 & -5 \\ 3 & -7 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\text{and} \quad A = \begin{bmatrix} 2 & -5 \\ 3 & -7 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -5 \\ 3 & -7 \end{vmatrix}$$

$$= (2)(-7) - (-5)(3)$$

$$= -14 + 15 = 1 \neq 0$$

A is non-singular matrix. Therefore, equations can be solve

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} -7 & 5 \\ -3 & 2 \end{bmatrix}}{1}$$

$$= \begin{bmatrix} -7 & 5 \\ -3 & 2 \end{bmatrix}$$

because  $AX = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 & 5 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7(1) + (5)(2) \\ -3(1) + (2)(2) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 + 10 \\ -3 + 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Hence,  $x = 3$

$y = 1$

S.S =  $\{(3, 1)\}$

(ii)  $3x + 2y = 10$

$2y - 3x = -4$

Sol: Write equations again

$3x + 2y = 10$

$-3x + 2y = -4$

In matrix form

$$\begin{bmatrix} 3 & 2 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ -4 \end{bmatrix}$$

$AX = B$

$$A = \begin{bmatrix} 3 & 2 \\ -3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 10 \\ -4 \end{bmatrix}$$

$$V = A^{-1}B$$

Now

$$A = \begin{bmatrix} 3 & 2 \\ -3 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ -3 & 2 \end{vmatrix}$$

$$= (3)(2) - (2)(-3)$$

$$= 6 + 6 = 12 \neq 0$$

A is non-singular matrix, therefore equations can be solve

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} 2 & -2 \\ 3 & 3 \end{bmatrix}}{12}$$

$$= \frac{1}{12} \begin{bmatrix} 2 & -2 \\ 3 & 3 \end{bmatrix}$$

Now

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{12} \begin{bmatrix} 2 & -2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{12} \begin{bmatrix} 2(10) + (-2)(-4) \\ 3(10) + (3)(-4) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{12} \begin{bmatrix} 20 + 8 \\ 30 - 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{12} \begin{bmatrix} 28 \\ 18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{28}{12} \\ \frac{18}{12} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{3} \\ \frac{3}{2} \end{bmatrix}$$

Hence,  $x = \frac{7}{3}$

$$y = \frac{3}{2}$$

$$S.S = \left\{ \left( \frac{7}{3}, \frac{3}{2} \right) \right\}$$

(iii)  $4x + 5y = 0$

$$2x + 5y = 1$$

Sol:  $\begin{bmatrix} 4 & 5 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

In matrix form

Let  $A X = B$

Now  $A^{-1} A X = A^{-1} B$

$$X = A^{-1} B$$

Now  $A = \begin{bmatrix} 4 & 5 \\ 2 & 5 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & 5 \\ 2 & 5 \end{vmatrix}$$

$$\begin{aligned}
 |A| &= (4)(5) - (5)(2) \\
 &= 20 - 10 = 10 \neq 0
 \end{aligned}$$

A is non-singular matrix, therefore equations can be solve.

$$|A| = 10$$

$$\begin{aligned}
 A^{-1} &= \frac{\text{adj } A}{|A|} \\
 &= \frac{\begin{bmatrix} 5 & -5 \\ -2 & 4 \end{bmatrix}}{10} = \frac{1}{10} \begin{bmatrix} 5 & -5 \\ -2 & 4 \end{bmatrix}
 \end{aligned}$$

Now

$$\begin{aligned}
 A^{-1}B &= \frac{1}{10} \begin{bmatrix} 5 & -5 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
 &= \frac{1}{10} \begin{bmatrix} (5)(0) + (-5)(1) \\ (-2)(0) + (4)(1) \end{bmatrix} \\
 &= \frac{1}{10} \begin{bmatrix} 0 - 5 \\ 0 + 4 \end{bmatrix} \\
 &= \frac{1}{10} \begin{bmatrix} -5 \\ 4 \end{bmatrix} \\
 &= \begin{bmatrix} \frac{-5}{10} \\ \frac{4}{10} \end{bmatrix}
 \end{aligned}$$

$$A^{-1}B = \begin{bmatrix} -\frac{1}{2} \\ \frac{2}{5} \end{bmatrix}$$

But  $X = A^{-1}B$

Thus  $X = \begin{bmatrix} -\frac{1}{2} \\ \frac{2}{5} \end{bmatrix}$

or  $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{2}{5} \end{bmatrix}$

Hence,  $x = -\frac{1}{2}$

$$y = \frac{2}{5}$$

$$\text{S.S} = \left\{ \left( -\frac{1}{2}, \frac{2}{5} \right) \right\}$$

(iv)  $5x + 6y = 25$

$$3x + 4y = 17$$

Sol:  $\begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 25 \\ 17 \end{bmatrix}$

In matrix form

Let  $A X = B$

$$A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B$$

Now  $A = \begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}$

$$|A| = \begin{vmatrix} 5 & 6 \\ 3 & 4 \end{vmatrix}$$

$$= (5)(4) - (6)(3)$$

$$= 20 - 18$$

$$|A| = 2 \neq 0$$

A is non-singular matrix. Therefore equations can be solve.

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix}}{2}$$

$$= \frac{1}{2} \begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix}$$

Now  $A^{-1}B = \frac{1}{2} \begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 25 \\ 17 \end{bmatrix}$

$$= \frac{1}{2} \begin{bmatrix} (4)(25) + (-6)(17) \\ (-3)(25) + (5)(17) \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 100 - 102 \\ -75 + 85 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -2 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-2}{2} \\ \frac{10}{2} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

But  $X = A^{-1}B$

Thus  $X = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

or  $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

Hence,  $x = -1$

$$y = 5$$

$$\text{S.S} = \{(-1, 5)\}$$

(v)  $x + y = 2$

$$y = 2 + x$$

**Sol:** Write equations again

$$x + y = 2$$

$$-x + y = 2$$

In matrix form

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Let  $A X = B$

$$A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B$$

Now  $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$$

$$= (1)(1) - (1)(-1)$$

$$= 1 + 1$$

$$|A| = 2 \neq 0$$

A is non-singular matrix. Therefore equations can be solve.

$$\begin{aligned} A^{-1} &= \frac{\text{adj } A}{|A|} \\ &= \frac{\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}}{2} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \end{aligned}$$

and  $A^{-1}B = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

Now

$$\begin{aligned} &= \frac{1}{2} \begin{bmatrix} (1)(2) + (-1)(2) \\ (1)(2) + (1)(2) \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 2 - 2 \\ 2 + 2 \end{bmatrix} \end{aligned}$$

$$= \frac{1}{2} \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

while  $X = A^{-1}B$

$$X = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

or  $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

Hence,  $x = 0$

$$y = 2$$

$$\text{s.s} = \{(0, 2)\}$$

$$(vi) \quad \frac{x}{2} + \frac{y}{3} = 1 \quad (i)$$

$$-4x + y = 14 \quad (ii)$$

**Sol:** Multiply (i) by 6.

$$6\left(\frac{x}{2}\right) + 6\left(\frac{y}{3}\right) = 6$$

$$3x + 2y = 6$$

(2nd condition)

$$-4x + y = 14$$

$$\begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \end{bmatrix} \quad (\text{In matrix form})$$

Let  $A X = B$

$$A^{-1} A X = A^{-1} B$$

$$X = A^{-1} B$$

Now  $A = \begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & 2 \\ -4 & 1 \end{vmatrix}$$

$$= (3)(1) - (2)(-4)$$

$$= 3 + 8$$

$$= 11 \neq 0$$

A is non-singular matrix. Therefore equations can be solve.

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}}{11} = \frac{1}{11} \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$$

and now  $A^{-1} B = \frac{1}{11} \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 6 \\ 14 \end{bmatrix}$

$$= \frac{1}{11} \begin{bmatrix} (1)(6) + (-2)(14) \\ (4)(6) + (3)(14) \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} 6 - 28 \\ 24 + 42 \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} -22 \\ 66 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-22}{11} \\ \frac{66}{11} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

while  $X = A^{-1}B$

$$X = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

or  $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$

Hence,  $x = -2$   
 $y = 6$

$$S.S = \{(-2, 6)\}$$

3. Solve, using matrix inversion method

$$3x - y = 10$$

$$2x + 3y = 3$$

In matrix form

$$\begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

Let  $AX = B$

and  $A^{-1}AX = A^{-1}B$

$$X = A^{-1}B$$

Now  $A = \begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix}$$

$$= (3)(3) - (-1)(2)$$

$$= 9 + 2$$

$$= 11 \neq 0$$

A is non-singular matrix. Therefore equations can be solve.

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}}{11} = \frac{1}{11} \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$$

and

now  $A^{-1}B = \frac{1}{11} \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 3 \end{bmatrix}$

$$= \frac{1}{11} \begin{bmatrix} (3)(10) + (1)(3) \\ (-2)(10) + (3)(3) \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} 30 + 3 \\ -20 + 9 \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} 33 \\ -11 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{33}{11} \\ \frac{-11}{11} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

while  $X = A^{-1}B$

$$X = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

or  $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

Hence,  $x = 3$

$$y = -1$$

$$\text{S.S} = \{(3, -1)\}$$

4. Use Cramer's rule to solve the simultaneous equations. Give the reason where solution is not possible.

(i)  $x + 2y = 3$   
 $x + 3y = 5$

**Remember that:**

**Cramer's Rule**

If  $a_1x + a_2y = b_1$

$$a_3x + a_4y = b_2$$

then

$$x = \frac{\begin{vmatrix} b_1 & a_2 \\ b_2 & a_4 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix}} = \frac{|D_1|}{|A|}$$

$$y = \frac{\begin{vmatrix} a_1 & b_1 \\ a_3 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix}} = \frac{|D_2|}{|A|}$$

**Sol:**

$$x + 2y = 3$$

$$x + 3y = 5$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \quad \text{In matrix form}$$

$$AX = B$$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = (1)(3) - (1)(2) = 3 - 2 = 1$$

$$|D_1| = \begin{vmatrix} 3 & 2 \\ 5 & 3 \end{vmatrix} = (3)(3) - (2)(5) = 9 - 10 = -1$$

$$|D_2| = \begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix} = (1)(5) - (3)(1) = 5 - 3 = 2$$

$$x = \frac{|D_1|}{|A|} = \frac{-1}{1} = -1$$

$$y = \frac{|D_2|}{|A|} = \frac{2}{1} = 2$$

$$S.S = \{(-1, 2)\}$$

(ii)  $2x + y = 1$

$$5x + 3y = 2$$

Sol:  $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

In matrix form

$$AX = B$$

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix}$$

$$= (2)(3) - (1)(5)$$

$$= 6 - 5 = 1$$

$$|D_1| = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$

$$= (1)(3) - (1)(2)$$

$$= 3 - 2 = 1$$

$$|D_2| = \begin{vmatrix} 2 & 1 \\ 5 & 2 \end{vmatrix} = (2)(2) - (1)(5)$$

$$= 4 - 5$$

$$= -1$$

$$x = \frac{|D_1|}{|A|} = \frac{1}{1} = 1$$

$$y = \frac{|D_2|}{|A|} = \frac{-1}{1} = -1$$

$$\text{S.S} = \{(1, -1)\}$$

(iii)  $x + 3y = 1$

$$2x + 8y = 0$$

Sol:  $\begin{bmatrix} 1 & 3 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

In matrix form

$$AX = B$$

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 8 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 \\ 2 & 8 \end{vmatrix} = (1)(8) - (3)(2)$$

$$= 8 - 6 = 2$$

$$|D_1| = \begin{vmatrix} 1 & 3 \\ 0 & 8 \end{vmatrix} = (1)(8) - (3)(0)$$

$$= 8 - 0$$

$$= 8$$

$$|D_2| = \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = (1)(0) - (1)(2)$$

$$= 0 - 2$$

$$= -2$$

$$x = \frac{|D_1|}{|A|} = \frac{8}{2} = 4$$

$$y = \frac{|D_2|}{|A|} = \frac{-2}{2} = -1$$

$$\text{S.S} = \{(4, -1)\}$$

(iv)  $-2x + 6y = 5$

$$x - 3y = -7$$

Sol:  $\begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \end{bmatrix}$  In matrix form

$$AX = B$$

$$A = \begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -2 & 6 \\ 1 & -3 \end{vmatrix}$$

$$= (-2)(-3) - (6)(1)$$

$$= 6 - 6 = 0$$

$$A = \begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix} \text{ is singular matrix, therefore S.S}$$

is not possible.

(v)  $x - 3y = 5$

$$2x - 5y = 9$$

Sol:  $\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$  In matrix form

$$AX = B$$

$$A = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -3 \\ 2 & -5 \end{vmatrix}$$

$$= (1)(-5) - (-3)(2)$$

$$= -5 + 6 = 1$$

$$|D_1| = \begin{vmatrix} 5 & -3 \\ 9 & -5 \end{vmatrix} = (5)(-5) - (-3)(9)$$

$$= -25 + 27$$

$$= 2$$

$$|D_2| = \begin{vmatrix} 1 & 5 \\ 2 & 9 \end{vmatrix}$$

$$= (1)(9) - (5)(2)$$

$$= 9 - 10 = -1$$

$$x = \frac{|D_1|}{|A|} = \frac{2}{1} = 2$$

$$y = \frac{|D_2|}{|A|} = \frac{-1}{1} = -1$$

$$\text{s.s} = \{(2, -1)\}$$

$$(vi) \quad 5x + 2y = 13$$

$$2x + 5y = 17$$

$$\text{Sol:} \quad \begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 17 \end{bmatrix} \quad \text{In matrix form}$$

$$AX = B$$

$$A = \begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 2 \\ 2 & 5 \end{vmatrix} = (5)(5) - (2)(2)$$

$$= 25 - 4$$

$$= 21$$

$$|D_1| = \begin{vmatrix} 13 & 2 \\ 17 & 5 \end{vmatrix} = (13)(5) - (2)(17)$$

$$= 65 - 34$$

$$= 31$$

$$|D_2| = \begin{vmatrix} 5 & 13 \\ 2 & 17 \end{vmatrix} = (5)(17) - (13)(2)$$

$$= 85 - 26 = 59$$

$$x = \frac{|D_1|}{|A|} = \frac{31}{21}$$

$$y = \frac{|D_2|}{|A|} = \frac{59}{21}$$

$$\text{s.s} = \left\{ \left( \frac{31}{21}, \frac{59}{21} \right) \right\}$$

5. Write the following matrices in the form of linear equations.

$$(i) \begin{bmatrix} 2 & -1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\text{Sol: } \begin{bmatrix} (2)(x) + (-1)(y) \\ (5)(x) + (2)(y) \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\text{then } \begin{bmatrix} 2x - y \\ 5x + 2y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\text{and } 2x - y = 2$$

$$5x + 2y = 4$$

$$(ii) \begin{bmatrix} -5 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\text{Sol: } \begin{bmatrix} (-5)(x) + (2)(y) \\ (2)(x) + (-3)(y) \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -5x + 2y \\ 2x - 3y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\text{thus, } -5x + 2y = 2$$

$$\text{and } 2x - 3y = -1$$

$$(iii) \begin{bmatrix} -4 & 1 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\text{Sol: } \begin{bmatrix} -4(x) + (1)(y) \\ 5(x) + (4)(y) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -4x + y \\ 5x + 4y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

thus,  $-4x + y = 1$

and  $5x + 4y = -1$

(iv) 
$$\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Sol: 
$$\begin{bmatrix} (0.8)(x) + (-0.6)(y) \\ (0.6)(x) + (0.8)(y) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 0.8x - 0.6y \\ 0.6x + 0.8y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

thus,  $0.8x - 0.6y = 1$

and  $0.6x + 0.8y = 2$