



-I Solve by using factrization method.

Q.1. $x^2 - 4x - 12 = 0$

Sol. $x^2 - 6x + 2x - 12 = 0$

$$(x^2 - 6x) + (2x - 12) = 0$$

$$x(x - 6) + 2(x - 6) = 0$$

$$(x - 6)(x + 2) = 0$$

If $x - 6 = 0$

then $x = 6$

and if $x + 2 = 0$

then $x = -2$

$$S. S. = \{6, -2\}$$

Q.2. $x^2 - 6x + 5 = 0$

Sol. $x^2 - x - 5x + 5 = 0$

$$(x^2 - x) - (5x - 5) = 0$$

$$x(x - 1) - 5(x - 1) = 0$$

$$(x - 1)(x - 5) = 0$$

If $x - 1 = 0$

then $x = 1$

and if $x - 5 = 0$

then $x = 5$

$$S.S. = \{1, 5\}$$

Q.3.

$$x^2 = 8 - 7x$$

Sol.

$$x^2 + 7x - 8 = 0$$

$$x^2 + 8x - x - 8 = 0$$

$$(x^2 + 8x) - (x + 8) = 0$$

$$x(x + 8) - 1(x + 8) = 0$$

$$(x + 8)(x - 1) = 0$$

If $x + 8 = 0$

then $x = -8$

and if $x - 1 = 0$

then $x = 1$

$$\text{Solution set} = \{-8, 1\}$$

Q.4.

$$5x = x^2 + 6$$

Sol.

$$0 = x^2 + 6 - 5x$$

$$x^2 + 6 - 5x = 0$$

$$x^2 - 5x + 6 = 0$$

$$x^2 - 2x - 3x + 6 = 0$$

$$(x^2 - 2x) - (3x - 6) = 0$$

$$x(x - 2) - 3(x - 2) = 0$$

$$(x - 2)(x - 3) = 0$$

If $x - 2 = 0$

then $x = 2$

and if $x - 3 = 0$

then $\boxed{x = 3}$

Solution set = $\{2, 3\}$

Q.5. $3x^2 - 10x + 8 = 0$

Sol. $3x^2 - 6x - 4x + 8 = 0$

$$(3x^2 - 6x) - (4x - 8) = 0$$

$$3x(x - 2) - 4(x - 2) = 0$$

$$(x - 2)(3x - 4) = 0$$

If $x - 2 = 0$

then $\boxed{x = 2}$

and if $3x - 4 = 0$

$$3x = 4$$

then $\boxed{x = \frac{4}{3}}$

Solution set = $\left\{2, \frac{4}{3}\right\}$

Q.6. $2x^2 + 15x - 8 = 0$

Sol. $2x^2 - x + 16x - 8 = 0$

$$(2x^2 - x) + (16x - 8) = 0$$

$$x(2x - 1) + 8(2x - 1) = 0$$

$$(2x - 1)(x + 8) = 0$$

If $2x - 1 = 0$

$$2x = 1$$

then

$$x = \frac{1}{2}$$

and if

$$x + 8 = 0$$

then

$$x = -8$$

$$\text{Solution set} = \left\{ \frac{1}{2}, -8 \right\}$$

Q.7.

$$\frac{x}{4}(x+1) = 3$$

Sol.

$$4\left(\frac{x}{4}\right)(x+1) = 3 \times 4 \quad (\text{Multiply by } 4)$$

$$x(x+1) = 12$$

$$x^2 + x = 12$$

$$x^2 + x - 12 = 0$$

$$x^2 + 4x - 3x - 12 = 0$$

$$(x^2 + 4x) - (3x + 12) = 0$$

$$x(x+4) - 3(x+4) = 0$$

$$(x+4)(x-3) = 0$$

$$\text{If } x + 4 = 0$$

$$\text{then } x = -4$$

$$\text{and if } x - 3 = 0$$

$$\text{then } x = 3$$

$$\text{Solution set} = \{-4, 3\}$$

Q.8. $3x^2 - 8x - 3 = 0$

Sol. $3x^2 - 9x + x - 3 = 0$

$$(3x^2 - 9x) + (x - 3) = 0$$

$$3x(x - 3) + 1(x - 3) = 0$$

$$(x - 3)(3x + 1) = 0$$

If $x - 3 = 0$

then $\boxed{x = 3}$

and if $3x + 1 = 0$

$$3x = -1$$

then $\boxed{x = -\frac{1}{3}}$

$$\text{Solution set} = \left\{ 3, -\frac{1}{3} \right\}$$

Q.9. $2x = \frac{2}{x} + 3$

Sol. (Multiply by x)

$$x(2x) = x\left(\frac{2}{x}\right) + 3(x)$$

$$2x^2 = 2 + 3x$$

$$2x^2 - 3x - 2 = 0$$

$$2x^2 - 4x + x - 2 = 0$$

$$(2x^2 - 4x) + (x - 2) = 0$$

$$2x(x - 2) + 1(x - 2) = 0$$

$$(x - 2)(2x + 1) = 0$$

If $x - 2 = 0$

then $\boxed{x = 2}$

and if $2x + 1 = 0$

$$2x = -1$$

then $\boxed{x = -\frac{1}{2}}$

Solution set = $\left\{2, -\frac{1}{2}\right\}$

Q.10. $5x^2 - 6x - 8 = 0$

Sol. $5x^2 - 10x + 4x - 8 = 0$

$$(5x^2 - 10x) + (4x - 8) = 0$$

$$5x(x - 2) + 4(x - 2) = 0$$

$$(x - 2)(5x + 4) = 0$$

If $x - 2 = 0$

then $\boxed{x = 2}$

and if $5x + 4 = 0$

$$5x = -4$$

then $\boxed{x = -\frac{4}{5}}$

Solution set = $\left\{2, -\frac{4}{5}\right\}$

Q.11. $(2x + 3)(x - 2) = 0$

Sol. If $x - 2 = 0$

then $\boxed{x = 2}$

and if $2x + 3 = 0$

$$2x = -3$$

then $x = -\frac{3}{2}$

$$\text{Solution set} = \left\{ 2, -\frac{3}{2} \right\}.$$

Q.12.

$$(2x + 1)(5x - 4) = 0$$

Sol.

If $2x + 1 = 0$

$$2x = -1$$

then

$$x = -\frac{1}{2}$$

and if $5x - 4 = 0$

$$5x = 4$$

then

$$x = \frac{4}{5}$$

$$\text{Solution set} = \left\{ -\frac{1}{2}, \frac{4}{5} \right\}$$

Q.13.

$$4x(3x - 1) - 2 = (2x - 1)(5x + 1)$$

Sol.

$$12x^2 - 4x - 2 = 10x^2 - 3x - 1$$

$$12x^2 - 10x^2 - 4x + 3x - 2 + 1 = 0$$

$$2x^2 - x - 1 = 0$$

$$2x^2 - 2x + x - 1 = 0$$

$$(2x^2 - 2x) + (x - 1) = 0$$

$$2x(x - 1) + 1(x - 1) = 0$$

$$(x - 1)(2x + 1) = 0$$

If $x - 1 = 0$

then $\boxed{x = 1}$

and if $2x + 1 = 0$

$$2x = -1$$

then $\boxed{x = -\frac{1}{2}}$

$$\text{Solution set} = \left\{1, -\frac{1}{2}\right\}$$

II- Solve by completing the square method.

Q.14. $x^2 - 10x - 3 = 0$

Sol. $x^2 - 10x = 3$

$\frac{10}{2} = 5$ then add $(5)^2$ on both sides

$$x^2 - 10x + (5)^2 = 3 + (5)^2$$

$$(x - 5)^2 = 3 + 25$$

$$(x - 5)^2 = 28$$

(By taking square root on both sides)

$$x - 5 = \sqrt{28}$$

$$x - 5 = \pm 2\sqrt{7}$$

$$x = 5 \pm 2\sqrt{7}$$

Q.15. $x^2 - 6x - 3 = 0$

Sol. $x^2 - 6x = 3$

$\frac{6}{2} = 3$ then add $(3)^2$ on both sides

$$x^2 - 6x + (3)^2 = 3 + (3)^2$$

$$(x-3)^2 = 3+9$$

$$(x-3)^2 = 12$$

(By taking square root on both sides)

$$x-3 = \pm\sqrt{12}$$

$$x = \pm 2\sqrt{3}$$

$$x = 3 \pm 2\sqrt{3}$$

Q.16

$$x^2 + x - 1 = 0$$

Sol.

$$x^2 + x = 1$$

Adding $\left(\frac{1}{2}\right)^2$ on both sides

$$x^2 + x + \left(\frac{1}{2}\right)^2 = 1 + \left(\frac{1}{2}\right)^2$$

$$\left(x + \frac{1}{2}\right)^2 = 1 + \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{4+1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{5}{4}$$

$$x + \frac{1}{2} = \pm\sqrt{\frac{5}{4}} \text{ (Taking square root)}$$

$$x = -\frac{1}{2} \pm \sqrt{\frac{5}{4}}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

Q.17. $x^2 + 6x - 3 = 0$

Sol. $x^2 + 6x = 3$

$\frac{6}{2} = 3$, then $(3)^2$ adding on both sides

$$x^2 + 6x + (3)^2 = 3 + (3)^2$$

$$(x + 3)^2 = 3 + 9$$

$$(x + 3)^2 = 12$$

$$x + 3 = \pm\sqrt{12} \text{ (Taking square root)}$$

$$x + 3 = \pm 2\sqrt{3}$$

$$x = -3 \pm 2\sqrt{3}$$

Q.18. $2x^2 - 4x + 1 = 0$

Sol. $2x^2 - 4x = -1$

Divided by co-efficient 2 of x .

$$x^2 - 2x = -\frac{1}{2}$$

$\frac{2}{2} = 1$, add $(1)^2$ on both sides

$$x^2 - 2x + (1)^2 = -\frac{1}{2} + (1)^2$$

$$(x - 1)^2 = -\frac{1}{2} + 1$$

$$(x - 1)^2 = \frac{-1 + 2}{2}$$

$$(x - 1)^2 = \frac{1}{2}$$

$$x - 1 = \pm\sqrt{\frac{1}{2}} \text{ (Taking square root)}$$

thus, $x = 1 \pm \sqrt{\frac{1}{2}}$

Q.19. $2x^2 - 6x + 3 = 0$

Sol. $2x^2 - 6x = -3$

Divided by co-efficient 2 of x .

$$x^2 - 3x = -\frac{3}{2}$$

Adding $\left(\frac{3}{2}\right)^2$ on both sides

$$x^2 - 3x + \left(\frac{3}{2}\right)^2 = -\frac{3}{2} + \left(\frac{3}{2}\right)^2$$

$$\left(x - \frac{3}{2}\right)^2 = -\frac{3}{2} + \frac{9}{2}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{-3+9}{2}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{6}{2}$$

$$\left(x - \frac{3}{2}\right)^2 = 3$$

$$x - \frac{3}{2} = \pm\sqrt{3} \text{ (Taking square root)}$$

$$x = \frac{3}{2} \pm \sqrt{3}$$

$$x = \frac{3 \pm 2\sqrt{3}}{2}$$

Q.20. $3x^2 + 5x - 4 = 0$

Sol. $3x^2 + 5x = 4$

Divided by 3 co-efficient of x .

$$x^2 + \frac{5}{3}x = \frac{4}{3}$$

$$\frac{1}{2}\left(\frac{5}{3}\right) = \frac{5}{6}, \text{ Adding } \left(\frac{5}{6}\right)^2 \text{ on both sides}$$

$$x^2 + \frac{5}{3}x + \left(\frac{5}{6}\right)^2 = \left(\frac{5}{6}\right)^2 + \frac{4}{3}$$

$$\left(x + \frac{5}{6}\right)^2 = \frac{25}{36} + \frac{4}{3}$$

$$\left(x + \frac{5}{6}\right)^2 = \frac{25 + 48}{36}$$

$$\left(x + \frac{5}{6}\right)^2 = \frac{73}{36}$$

$$x + \frac{5}{6} = \pm \frac{\sqrt{73}}{6} \text{ (Taking square root)}$$

$$x = -\frac{5}{6} \pm \frac{\sqrt{73}}{6}$$

$$x = \frac{-5 \pm \sqrt{73}}{6}$$

Q21.

$$x^2 + mx + n = 0$$

Sol.

$$x^2 + mx = -n$$

Adding $\left(\frac{m}{2}\right)^2$ on both sides.

$$x^2 + mx + \left(\frac{m}{2}\right)^2 = \left(\frac{m}{2}\right)^2 - n$$

$$\left(x + \frac{m}{2}\right)^2 = \frac{m^2}{4} - n$$

$$\left(x + \frac{m}{2}\right)^2 = \frac{m^2 - 4n}{4}$$

$$x + \frac{m}{2} = \pm \sqrt{\frac{m^2 - 4n}{4}} \text{ (Taking square root)}$$

$$x = -\frac{m}{2} \pm \sqrt{\frac{m^2 - 4n}{4}}$$

$$x = -\frac{m}{2} \pm \frac{\sqrt{m^2 - 4n}}{2}$$

$$x = \frac{-m \pm \sqrt{m^2 - 4n}}{2}$$

Q22

$$11x^2 = 6x + 21$$

Sol.

$$11x^2 - 6x = 21$$

$$x^2 - \frac{6}{11}x = \frac{21}{11} \quad (\text{Divided by } 11)$$

$$\left(\frac{6}{11} \times \frac{1}{2}\right)^2 = \frac{3}{11} \text{ adding } \left(\frac{3}{11}\right)^2 \text{ on both sides}$$

$$x^2 - \frac{6}{11}x + \left(\frac{3}{11}\right)^2 = \left(\frac{3}{11}\right)^2 + \frac{21}{11}$$

$$\left(x - \frac{3}{11}\right)^2 = \frac{9}{121} + \frac{21}{11}$$

$$\left(x - \frac{3}{11}\right)^2 = \frac{9 + 231}{121}$$

$$\left(x - \frac{3}{11}\right)^2 = \frac{240}{121}$$

$$x - \frac{3}{11} = \pm \sqrt{\frac{240}{121}} \quad (\text{Taking square root})$$

$$x - \frac{3}{11} = \pm \sqrt{\frac{16 \times 15}{11 \times 11}}$$

$$x - \frac{3}{11} = \pm \frac{4\sqrt{15}}{11}$$

$$x = \frac{3}{11} \pm \frac{4\sqrt{15}}{11}$$

$$x = \frac{3 \pm 4\sqrt{15}}{11}$$

Q23. $2x^2 + 8x - 26 = 0$

Sol. $2x^2 + 8x = 26$

$$x^2 + 4x = 13 \text{ (Dividing by 2)}$$

$$\frac{4}{2} = 2 \text{ adding } (2)^2 \text{ on both sides.}$$

$$x^2 + 4x + (2)^2 = (2)^2 + 13$$

$$(x+2)^2 = 4 + 13$$

$$(x+2)^2 = 17$$

$$x+2 = \pm\sqrt{17} \text{ (Taking square root)}$$

$$x = -2 \pm \sqrt{17}$$

Q24. $5x^2 - 20x - 28 = 0$

Sol. $5x^2 - 20x = 28$

$$x^2 - 4x = \frac{28}{5} \text{ (Divided by 5)}$$

$$\frac{4}{2} = 2, \text{ Adding } (2)^2 \text{ on both sides.}$$

$$x^2 - 4x + (2)^2 = (2)^2 + \frac{28}{5}$$

$$(x-2)^2 = 4 + \frac{28}{5}$$

$$(x-2)^2 = \frac{20+28}{5}$$

$$(x-2)^2 = \frac{48}{5}$$

$$x - 2 = \pm \sqrt{\frac{48}{5}} \quad (\text{Taking square root})$$

$$x - 2 = \pm \frac{4\sqrt{3}}{\sqrt{5}}$$

$$x = 2 \pm \frac{4\sqrt{3}}{\sqrt{5}} \quad (\text{Rationalize})$$

$$x = 2 \pm 4 \frac{\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$x = 2 \pm \frac{4\sqrt{15}}{5}$$

$$x = \frac{10 \pm 4\sqrt{15}}{5} \quad \text{Ans.}$$

Q25

$$x^2 - 11x - 26 = 0$$

Sol.

$$x^2 - 11x = 26$$

Adding $\left(\frac{11}{2}\right)^2$ on both sides.

$$x^2 - 11x + \left(\frac{11}{2}\right)^2 = \left(\frac{11}{2}\right)^2 + 26$$

$$\left(x - \frac{11}{2}\right)^2 = \frac{121}{4} + 26$$

$$\left(x - \frac{11}{2}\right)^2 = \frac{121 + 104}{4}$$

$$\left(x - \frac{11}{2}\right)^2 = \frac{225}{4}$$

$$x - \frac{11}{2} = \pm \sqrt{\frac{225}{4}} \quad (\text{Taking square root})$$

$$x - \frac{11}{2} = \pm \frac{15}{2}$$

$$x = \frac{11}{2} \pm \frac{15}{2}$$

$$x = \frac{11 \pm 15}{2}$$

$$x = \frac{11+15}{2}, \frac{11-15}{2}$$

$$x = \frac{26}{2}, \frac{-4}{2}$$

$$x = 13, -2$$

$$\text{S.S.} = \{13, -2\}$$

5.3 The Quadratic Formula

If $ax^2 + bx + c = 0$ $a \neq 0$

then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



Solve using quadratic formula:

Q.1. $x^2 - 5x + 6 = 0$

Sol. Here $a = 1$

$$b = -5$$

$$c = 6$$

(Formula) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Putting values of a, b, c

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - 24}}{2}$$

$$x = \frac{5 \pm \sqrt{1}}{2}$$

$$x = \frac{5 \pm 1}{2}$$

$$x = \frac{5+1}{2}, \frac{5-1}{2}$$

$$x = \frac{6}{2}, \frac{4}{2}$$

$$x = 3, 2$$

Solutin set = {3, 2}

Q.2.

$$(3 - 4x) = (4x - 3)^2$$

Sol.

$$3 - 4x = 16x^2 - 24x + 9$$

$$0 = 16x^2 - 24x + 9 - 3 + 4x$$

$$0 = 16x^2 - 20x + 6$$

$$\text{or } 16x^2 - 20x + 6 = 0$$

$$8x^2 - 10x + 3 = 0 \quad (\text{Divided by 2})$$

Here

$$a = 8$$

$$b = -10$$

$$c = 3$$

(Formula)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Putting the values of a, b, c

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(8)(3)}}{2(8)}$$

$$x = \frac{10 \pm \sqrt{100 - 96}}{16}$$

$$x = \frac{10 \pm \sqrt{4}}{16}$$

$$x = \frac{10 \pm 2}{16}$$

$$x = \frac{10+2}{16}, \frac{10-2}{16}$$

$$x = \frac{12}{16}, \frac{8}{16}$$

$$x = \frac{3}{4}, \frac{1}{2}$$

$$\text{Solution set} = \left\{ \frac{3}{4}, \frac{1}{2} \right\}$$

Q.3. $3x^2 + x - 2 = 0$

Sol. Here $a = 3$

$$b = 1$$

$$c = -2$$

(Formula) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{-1 \pm \sqrt{1+24}}{6}$$

$$x = \frac{-1 \pm \sqrt{25}}{6}$$

$$x = \frac{-1 \pm 5}{6}$$

$$x = \frac{-1+5}{6}, \frac{-1-5}{6}$$

$$x = \frac{4}{6}, \frac{-6}{6}$$

$$x = \frac{2}{3}, -1$$

$$\text{Solution set} = \left\{ \frac{2}{3}, -1 \right\}$$

Q.4. $10x^2 - 5x = 15$

Sol. $10x^2 - 5x - 15 = 0$

$$2x^2 - x - 3 = 0 \quad (\text{Divided by } 5)$$

Here $a = 2$

$$b = -1$$

$$c = -3$$

Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{1+24}}{4}$$

$$x = \frac{1 \pm \sqrt{25}}{4}$$

$$x = \frac{1 \pm 5}{4}$$

$$x = \frac{1+5}{4}, \frac{1-5}{4}$$

$$x = \frac{6}{4}, \frac{-4}{4}$$

$$x = \frac{3}{2}, -1$$

$$\text{Solution set} = \left\{ \frac{3}{2}, -1 \right\}$$

Q.5. $(x-1)(x+3) - 12 = 0$

Sol. $x^2 + 2x - 3 - 12 = 0$

$$x^2 + 2x - 15 = 0$$

Here $a = 1$

$$b = 2$$

$$c = -15$$

(Formula) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-15)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4+60}}{2}$$

$$x = \frac{-2 \pm \sqrt{64}}{2}$$

$$x = \frac{-2 \pm 8}{2}$$

$$x = \frac{-2+8}{2}, \frac{-2-8}{2}$$

$$x = \frac{6}{2}, \frac{-10}{2}$$

$$x = 3, -5$$

$$\text{Solution set} = \{3, -5\}$$

Q.6. $x(2x + 7) - 3(2x + 7) = 0$

Sol. $2x^2 + 7x - 6x - 21 = 0$

$$2x^2 + x - 21 = 0$$

Here $a = 2$

$$b = 1$$

$$c = -21$$

(Formula)
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

By putting the values of a, b, c

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-21)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{1 + 168}}{4}$$

$$x = \frac{-1 \pm \sqrt{169}}{4}$$

$$x = \frac{-1 \pm 13}{4}$$

$$x = \frac{-1 + 13}{4}, \frac{-1 - 13}{4}$$

$$x = \frac{12}{4}, \frac{-14}{4}$$

$$x = 3, -\frac{7}{2}$$

$$\text{Solution set} = \left\{3, -\frac{7}{2}\right\}$$

Q.7. $\frac{x+1}{x+4} = \frac{2x-1}{x+6}$, where $x \neq -4, -6$

Sol. $(x+1)(x+6) = (2x-1)(x+4)$

$$x^2 + 7x + 6 = 2x^2 + 7x - 4$$

$$x^2 - 2x^2 + 7x - 7x + 6 + 4 = 0$$

$$-x^2 + 10 = 0$$

$$x^2 - 10 = 0$$

Here $a = 1$

$$b = 0$$

$$c = -10$$

(Formula) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-(0) \pm \sqrt{(0)^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{0 \pm \sqrt{0 + 40}}{2}$$

$$x = \frac{0 \pm \sqrt{40}}{2}$$

$$x = \frac{0 \pm 2\sqrt{10}}{2}$$

$$x = \pm \frac{2\sqrt{10}}{2}$$

$$x = \pm \sqrt{10}$$

Solution set = $\{\pm\sqrt{10}\}$

Q.8. $\frac{x}{6} + \frac{6}{x} = \frac{4}{x} + \frac{x}{4}$, where $x \neq 0$

Sol. Multiplying by L.C.M. $12x$

$$12x \left(\frac{x}{6} \right) + 12x \left(\frac{6}{x} \right) = 12x \left(\frac{4}{x} \right) + 12x \left(\frac{x}{4} \right)$$

$$2x^2 + 72 = 48 + 3x^2$$

$$2x^2 - 3x^2 + 72 - 48 = 0$$

$$-x^2 + 24 = 0$$

$$x^2 - 24 = 0$$

Here $a = 1$

$$b = 0$$

$$c = -24$$

(Formula) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-(0) \pm \sqrt{(0)^2 - 4(1)(-24)}}{2(1)}$$

$$x = \frac{\pm \sqrt{0 + 96}}{2}$$

$$x = \frac{\pm \sqrt{96}}{2}$$

$$x = \frac{\pm \sqrt{16 \times 6}}{2}$$

$$x = \frac{\pm 4\sqrt{6}}{2}$$

$$x = \pm 2\sqrt{6}$$

Solution set = $\{\pm 2\sqrt{6}\}$

Q.9. $\frac{x+4}{x-4} + \frac{x-4}{x+4} = \frac{10}{3}$ where $x \neq -4$

Sol. Multiplying by $3(x-4)(x+4)$

$$3(x+4)(x+4) + 3(x-4)(x-4) = 10(x-4)(x+4)$$

$$3(x^2 + 8x + 16) + 3(x^2 - 8x + 16) = 10(x^2 - 16)$$

$$3x^2 + 24x + 48 + 3x^2 - 24x + 48 = 10x^2 - 160$$

$$3x^2 + 3x^2 - 10x^2 + 24x - 24x + 48 + 48 + 160 = 0$$

$$-4x^2 + 256 = 0$$

$$4x^2 - 256 = 0$$

$$x^2 - 64 = 0 \quad (\text{Divided by 4})$$

Here $a = 1$

$$b = 0$$

$$c = -64$$

(Formula)
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

By putting the values of a, b, c

$$x = \frac{-0 \pm \sqrt{(0)^2 - 4(1)(-64)}}{2(1)}$$

$$x = \frac{\pm \sqrt{0 + 256}}{2}$$

$$x = \frac{\pm \sqrt{256}}{2}$$

$$x = \frac{\pm 16}{2}$$

$$x = \pm 8$$

Solution set = $\{8, -8\}$

Q.10. $\frac{1}{x-1} + \frac{1}{x-2} = \frac{2}{x-3}$ where $x \neq 1, 2, 3$

Sol. Multiplying by $(x-1)(x-2)(x-3)$

$$(x-1)(x-2)(x-3) \times \frac{1}{(x-1)} + (x-1)(x-2)(x-3) \times \frac{1}{(x-2)} = (x-1)(x-2)(x-3) \times \frac{2}{(x-3)}$$

$$(x-2)(x-3) + (x-1)(x-3) = (x-1)(x-2)(2)$$

$$x^2 - 5x + 6 + x^2 - 4x + 3 = 2(x^2 - 3x + 2)$$

$$2x^2 - 9x + 9 = 2x^2 - 6x + 4$$

$$-9x + 6x = 4 - 9$$

$$-3x = -5$$

$$x = \frac{-5}{-3}$$

$$x = \frac{5}{3}$$

Q.11. $(x+4)(x-1) + (x+5)(x+2) = 6$

Sol. $x^2 + 3x - 4 + x^2 + 7x + 10 = 6$

$$2x^2 + 10x - 4 + 10 - 6 = 0$$

$$2x^2 + 10x = 0$$

Here $a = 2$

$$b = 10$$

$$c = 0$$

(Formula) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(2)(0)}}{2(2)}$$

$$x = \frac{-10 \pm \sqrt{100}}{4}$$

$$x = \frac{-10 \pm 10}{4}$$

$$x = \frac{-10+10}{4}, \frac{-10-10}{4}$$

$$x = \frac{0}{4}, \frac{-20}{4}$$

$$x = 0, -5$$

Solution set = $\{0, -5\}$

Q.12 $(2x+4)^2 - (4x-6)^2 = 0$

Sol.

$$(4x^2 + 16x + 16) - (16x^2 - 48x + 36) = 0$$

$$-12x^2 + 64x - 20 = 0$$

(Dividing by -4)

$$3x^2 - 16x + 5 = 0$$

Here $a = 3$

$$b = -16$$

$$c = 5$$

$$\text{(Formula)} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

By putting the values of a, b, c

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(3)(5)}}{2(3)}$$

$$x = \frac{16 \pm \sqrt{256 - 60}}{6}$$

$$x = \frac{16 \pm \sqrt{196}}{6}$$

$$x = \frac{16 \pm 14}{6}$$

$$x = \frac{16+14}{6}, \frac{16-14}{6}$$

$$x = \frac{30}{6}, \frac{2}{6}$$

$$x = 5, \frac{1}{3}$$

$$\text{Solution set} = \left\{ 5, \frac{1}{3} \right\}$$

Exercise 5.3

Q.1. Find two consecutive positive odd numbers such that the sum of their squares is 74.

Sol. Let 1st odd number = $2x + 1$

2nd odd number = $2x + 3$

According to statement

$$(2x+1)^2 + (2x+3)^2 = 74$$

$$4x^2 + 4x + 1 + 4x^2 + 12x + 9 = 74$$

$$8x^2 + 16x + 10 - 74 = 0$$

$$8x^2 + 16x - 64 = 0$$

$$x^2 + 2x - 8 = 0 \text{ (Divided by 8)}$$

$$x^2 + 4x - 2x - 8 = 0$$

$$(x^2 + 4x) - (2x + 8) = 0$$

$$x(x + 4) - 2(x + 4) = 0$$

$$(x + 4)(x - 2) = 0$$

$$\text{If } x - 2 = 0$$

$$\text{then } \boxed{x = 2}$$

$$\text{and if } x + 4 = 0$$

$$\text{then } \boxed{x = -4}$$

$$\text{when } x = 2 \text{ then 1}^{\text{st}} \text{ number} = 2x + 1$$

$$= 2(2) + 1$$

$$= 4 + 1$$

$$= 5$$

$$\text{and } 2^{\text{nd}} \text{ number} = 2x + 3$$

$$= 2(2) + 3$$

$$= 4 + 3$$

$$= 7$$

$$\text{Required odd number} = 5, 7$$

Q.2. Find two consecutive positive even numbers such that the sum of their squares is 164.

$$\text{Sol } \text{Let } 1^{\text{st}} \text{ number} = 2x$$

$$2^{\text{nd}} \text{ number} = 2x + 2$$

According to the statement

$$(2x)^2 + (2x + 2)^2 = 164$$

$$4x^2 + 4x^2 + 8x + 4 = 164$$

$$8x^2 + 8x + 4 - 164 = 0$$

$$8x^2 + 8x - 160 = 0$$

$$x^2 + x - 20 = 0 \quad (\text{Divided by } 8)$$

$$x^2 + 5x - 4x - 20 = 0$$

$$(x^2 + 5x) - (4x + 20) = 0$$

$$x(x + 5) - 4(x + 5) = 0$$

$$(x + 5)(x - 4) = 0$$

If $x - 4 = 0$

then $x = 4$

and if $x + 5 = 0$

then $x = -5$

when $x = 4$, then 1st number = $2x$

$$= 2(4) = 8$$

And 2nd number = $2x + 2$

$$= 2(4) + 2 = 8 + 2 = 10$$

Required even number = 8, 10

Q.3. The difference of two numbers is 9 and the product of the numbers is 162. Find the two numbers.

Sol. Let 1st number = x

$$2^{\text{nd}} \text{ number} = x + 9$$

According to the statement

$$x^2 + 9x = 162$$

$$x^2 + 9x - 162 = 0$$

$$x^2 + 18x - 9x - 162 = 0$$

$$(x^2 + 18x) - (9x + 162) = 0$$

$$x(x + 18) - 9(x + 18) = 0$$

$$(x - 9)(x + 18) = 0$$

$$\text{If } x - 9 = 0$$

$$\text{then } \boxed{x = 9}$$

$$\text{and if } x + 18 = 0$$

$$\boxed{x = -18}$$

$$\text{If } x = 9, \text{ then 1st number} = x = 9$$

$$\text{2nd number} = x + 9$$

$$= 9 + 9 = 18$$

$$\text{Required numbers} = \boxed{9, 18}$$

$$\text{If } x = -18$$

$$\text{1st number} = x = -18$$

$$\text{2nd number} = x + 9$$

$$= -18 + 9 = -9$$

$$\text{Required numbers} = \boxed{-18, -9}$$

Q.4. The base and height of a triangle are $(x + 3)\text{cm}$ and $(2x - 5)\text{cm}$ respectively. If the area of the triangle is 20cm^2 , find x .

Sol. Length of base = $x + 3\text{cm}$
 Length of altitude = $2x - 5\text{cm}$

$$\text{Area of } \Delta = 20\text{cm}^2$$

According to the statement

$$\frac{(x + 3)(2x - 5)}{2} = 20$$

$$\text{Area of } \Delta = \frac{B \times A}{2}$$

$$\frac{2x^2 + x - 15}{2} = 20$$

$$2x^2 + x - 15 = 2 \times 20$$

$$2x^2 + x - 15 = 40$$

$$2x^2 + x - 15 - 40 = 0$$

$$2x^2 + x - 55 = 0$$

$$2x^2 + 11x - 10x - 55 = 0$$

$$(2x^2 + 11x) - (10x + 55) = 0$$

$$x(2x + 11) - 5(2x + 11) = 0$$

$$(2x + 11)(x - 5) = 0$$

$$\text{If } x - 5 = 0$$

then

$$\boxed{x = 5}$$

Therefore, Length of base = $x + 3$
 $= 5 + 3$

$$= 8\text{cm}$$

$$\text{Length of altitude} = 2x - 5$$

$$= 2(5) - 5$$

$$= 10 - 5$$

$$= 5\text{cm}$$

$$\text{and if } 2x + 11 = 0$$

$$2x = -11$$

then

$$x = -\frac{11}{2}$$

Cancelling due to negativity.

Q.5. The perimeter and area of a rectangle are 22cm and 30cm^2 respectively. Find the length and breadth of the rectangle.

$$\text{Sol. Rectangle's perimeter} = 22\text{cm}$$

$$\text{Rectangle's area} = 30\text{cm}^2$$

$$\text{Let, Length of rectangle} = x \text{ cm}$$

$$\text{Breadth of rectangle} = \frac{30}{x} \text{ cm}$$

$$\therefore \text{Perimeter} = (\text{length} + \text{breadth}) \times 2$$

$$22 = \left(\frac{30}{x} + x \right) \times 2$$

$$11 = x + \frac{30}{x} \quad (\text{Divided by 2})$$

$$11x = 30 + x^2 \quad (\text{Multiplying by } x)$$

$$0 = 30 + x^2 - 11x$$

$$x^2 - 11x + 30 = 0$$

$$x^2 - 5x - 6x + 30 = 0$$

$$(x^2 - 5x) - (6x - 30) = 0$$

$$x(x - 5) - 6(x - 5) = 0$$

$$(x - 5)(x - 6) = 0$$

$$\text{If } x - 5 = 0$$

$$\text{then } \boxed{x = 5}$$

$$\text{Length of rectangle} = x$$

$$= 5\text{cm}$$

$$\text{and Breadth of rectangle} = \frac{30}{x}$$

$$= \frac{30}{5}$$

$$= 6\text{ cm}$$

$$\text{Length of rectangle} = 5\text{cm}$$

$$\text{Breadth of rectangle} = 6\text{cm}$$

$$\text{and if } x - 6 = 0$$

$$\text{then } \boxed{x = 6}$$

$$\text{Length of rectangle} = x$$

$$= 6\text{cm}$$

$$\text{and Breadth of rectangle} = \frac{30}{x}$$

$$= \frac{30}{6}$$

$$= 5\text{cm}$$

Length of rectangle = 6cm

So, Breadth of rectangle = 5cm

Q.6. The product of two consecutive positive numbers is 156. Find the numbers.

Sol. Let, 1st number = x

2nd number = $x + 1$

According to the statement

$$(x)(x + 1) = 156$$

$$x^2 + x = 156$$

$$x^2 + x - 156 = 0$$

$$x^2 + 13x - 12x - 156 = 0$$

$$(x^2 + 13x) - (12x + 156) = 0$$

$$x(x + 13) - 12(x + 13) = 0$$

$$(x - 12)(x + 13) = 0$$

If $x - 12 = 0$

then $x = 12$

Required numbers = $x, x + 1$

$$= 12, 12 + 1$$

$$= 12, 13$$

and if $x + 13 = 0$

then $x = -13$

Required numbers = $x, x + 1$

$$= -13, -13 + 1$$

$$= \boxed{-13, -12}$$

Q.7. Find two consecutive positive odd numbers given that the difference between their reciprocals is $\frac{2}{63}$.

Sol. Suppose that 1st number = $2x + 1$

$$2^{\text{nd}} \text{ number} = 2x + 3$$

According to given condition

$$\frac{1}{2x+1} - \frac{1}{2x+3} = \frac{2}{63}$$

Multiplying by $(2x+1)(2x+3)(63)$

$$\begin{aligned} 63(\cancel{2x+1})(2x+3) \times \frac{1}{\cancel{(2x+1)}} - 63(2x+1)(\cancel{2x+3}) \times \frac{1}{\cancel{(2x+3)}} \\ = \frac{2}{63} \times 63(2x+1)(2x+3) \end{aligned}$$

$$63(2x+3) - 63(2x+1) = 2(2x+1)(2x+3)$$

$$\cancel{126x} + 189 - \cancel{126x} - 63 = 2(4x^2 + 8x + 3)$$

$$126 = 8x^2 + 16x + 6$$

$$8x^2 + 16x + 6 - 126 = 0$$

$$8x^2 + 16x - 120 = 0$$

$$x^2 + 2x - 15 = 0 \quad (\text{Dividing by 8})$$

$$x^2 + 5x - 3x - 15 = 0$$

$$(x^2 + 5x) - (3x + 15) = 0$$

$$x(x+5) - 3(x+5) = 0$$

$$(x + 5)(x - 3) = 0$$

If $x - 3 = 0$

then $x = 3$

Required numbers: 1st number = $2x + 1$ required numbers

$$= 2(3) + 1$$

$$= 6 + 1$$

$$\text{1st number} = 7$$

$$\text{2nd number} = 2x + 3$$

$$= 2(3) + 3$$

$$= 6 + 3$$

$$\text{2nd number} = 9$$

and if $x + 5 = 0$

then $x = -5$

Cancelling due to negativity.

Q.8. The sum of the two positive number is 12 and the sum of whose squares is 80. Find the numbers.

Sol. Suppose that 1st number = x

$$\text{2nd number} = 12 - x$$

According to given condition

$$(x)^2 + (12 - x)^2 = 80$$

$$x^2 + 144 + x^2 - 24x = 80$$

$$2x^2 - 24x + 144 - 80 = 0$$

$$2x^2 - 24x + 64 = 0$$

$$x^2 - 12x + 32 = 0 \quad (\text{Dividing by 2})$$

$$x^2 - 4x - 8x + 32 = 0$$

$$x(x - 4) - 8(x - 4) = 0$$

$$(x - 4)(x - 8) = 0$$

$$\text{If } x - 4 = 0$$

$$\text{then } \boxed{x = 4}$$

$$\text{Required numbers: 1st number} = \boxed{x = 4}$$

$$\text{2nd number} = 12 - x$$

$$\boxed{\text{2nd number} = 12 - 4 = 8}$$

$$\text{and if } x - 8 = 0$$

$$x = 8$$

$$\text{Hence } \boxed{\text{1st number} = 8}$$

$$\text{2nd number} = 12 - x$$

$$= 12 - 8$$

$$\boxed{\text{2nd number} = 4}$$

$$\text{Required numbers} = 8, 4$$

$$\text{or Required numbers} = 4, 8$$