



UNIT 3

Algebraic Manipulation



- ▶ H.C.F and L.C.M
- ▶ Basic Operations on Algebraic Fractions
- ▶ Square Roots of Algebraic Fractions

After completion of this unit, the students will be able to:

- ▶ find highest common factor (HCF) and least common multiple (LCM) of algebraic expressions.
- ▶ use factor or division method to determine HCF and LCM.
- ▶ know the relationship between HCF and LCM.
- ▶ use HCF and LCM to reduce fractional expressions involving $+$, $-$, $\times$, $\div$.
- ▶ find square root of an algebraic expression by factorization and division.

Exercise 3.1

Find H.C.F by factorization.

Q.1 $abxy, a^2bc$.

Sol.

$$\text{Factorization of } abxy = \boxed{a} \times \boxed{b} \times x \times y$$

$$\text{Factorization of } a^2bc = \boxed{a} \times \boxed{a} \times \boxed{b} \times c$$

$$\text{common factors} = a, b$$

$$\begin{aligned} \text{Thus, H.C.F} &= a \times b \\ &= ab \end{aligned}$$

Q.2 $6pqr, 15qrs$

Sol.

$$\text{Factorization of } 6pqr = 2 \times \boxed{3} \times p \times \boxed{q} \times \boxed{r}$$

$$\text{Factorization of } 15qrs = 5 \times \boxed{3} \times \boxed{q} \times \boxed{r} \times s$$

$$\text{Common factors} = 3, q, r$$

$$\begin{aligned} \text{Thus, H.C.F} &= 3 \times q \times r \\ &= 3qr \end{aligned}$$

Q.3 $8xy^2z^3, 12x^2y^2z^2$

Sol.

$$\text{Factorization of } 8xy^2z^3 = \boxed{2} \times \boxed{2} \times 2 \times \boxed{x} \times \boxed{y} \times \boxed{y} \times \boxed{z} \times \boxed{z} \times \boxed{z}$$

$$\text{Factorization of } 12x^2y^2z^2 = \boxed{2} \times \boxed{2} \times 3 \times \boxed{x} \times \boxed{x} \times \boxed{y} \times \boxed{y} \times \boxed{z} \times \boxed{z}$$

$$\text{Common factors} = 2, 2, x, y, y, z, z$$

$$\begin{aligned} \text{Thus, H.C.F} &= 2 \times 2 \times x \times y \times y \times z \times z \\ &= 4xy^2z^2 \end{aligned}$$

Q.4 $14a^2bc, 21ab^2$

Sol.

$$\text{Factorization of } 14a^2bc = 2 \times \boxed{7} \times \boxed{a} \times a \times \boxed{b} \times c$$

$$\text{Factorization of } 21ab^2 = 3 \times \boxed{7} \times \boxed{a} \times \boxed{b} \times b$$

$$\text{Common factors} = 7, a, b$$

$$\begin{aligned} \text{Thus, H.C.F} &= 7 \times a \times b \\ &= 7ab \end{aligned}$$

Q.5 $3x^5y^2, 12x^2y^4, 15x^3y^2$

Sol.

$$\text{Factorization of } 3x^5y^2 = \boxed{3} \times x \times x \times \boxed{x} \times \boxed{x} \times x \times \boxed{y} \times \boxed{y}$$

$$\text{Factorization of } 12x^2y^4 = \boxed{3} \times 2 \times 2 \times \boxed{x} \times \boxed{x} \times y \times \boxed{y} \times \boxed{y} \times \boxed{y}$$

$$\text{Factorization of } 15x^3y^2 = \boxed{3} \times 5 \times \boxed{x} \times \boxed{x} \times x \times \boxed{y} \times \boxed{y}$$

$$\text{Common factors} = 3, x, x, y, y$$

$$\begin{aligned} \text{Thus, H.C.F} &= 3 \times x \times x \times y \times y \\ &= 3x^2y^2 \end{aligned}$$

Q.6 $4abc^3, 8a^3bc, 6ab^3c$

Sol.

$$\text{Factorization of } 4abc^3 = 2 \times 2 \times a \times b \times c \times c \times c$$

$$\text{Factorization of } 8a^3bc = 2 \times 2 \times 2 \times a \times a \times a \times b \times c$$

$$\text{Factorization of } 6ab^3c = 2 \times 3 \times a \times b \times b \times b \times c$$

$$\text{Common factors} = 2, a, b, c$$

$$\begin{aligned} \text{Thus, H.C.F} &= 2 \times a \times b \times c \\ &= 2abc \end{aligned}$$

Q.7 $x^3 + 64, x^2 - 16$

Sol.

$$\begin{aligned} \text{Factorization of } x^3 + 64 &= (x)^3 + (4)^3 \\ &= (x + 4)[(x)^2 - (x)(4) + (4)^2] \\ &= (x + 4)(x^2 - 4x + 16) \dots\dots\dots (i) \end{aligned}$$

$$\begin{aligned} \text{Factorization of } x^2 - 16 &= (x)^2 - (4)^2 \\ &= (x + 4)(x - 4) \dots\dots\dots (ii) \end{aligned}$$

Common factors are: $(x + 4)$

$$\text{Thus, H.C.F} = x + 4$$

Q.8 $x^2 - y^2, x^4 - y^4, x^6 - y^6$

$$\text{Factorization of } x^2 - y^2 = (x + y)(x - y) \dots\dots\dots (i)$$

$$\begin{aligned} \text{Factorization of } x^4 - y^4 &= (x^2)^2 - (y^2)^2 \\ &= (x^2 + y^2)(x^2 - y^2) \\ &= (x^2 + y^2)(x + y)(x - y) \dots\dots\dots (ii) \end{aligned}$$

$$\begin{aligned} \text{Factorization of } x^6 - y^6 &= (x^3)^2 - (y^3)^2 \\ &= (x^3 - y^3)(x^3 + y^3) \\ &= (x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2) \dots\dots (iii) \end{aligned}$$

Common factors are : $(x + y), (x - y)$

$$\begin{aligned} \text{Thus, H.C.F} &= (x + y)(x - y) \\ &= x^2 - y^2 \end{aligned}$$

Q.9 $t^2 - 9, (t + 3)^2, t^2 + t - 6$

Sol.

Factorization of $t^2 - 9 = (t)^2 - (3)^2$
 $= (t + 3)(t - 3) \dots \dots \dots (i)$

Factorization of $(t + 3)^2 = (t + 3)(t + 3) \dots \dots \dots (ii)$

Factorization of $t^2 + t - 6 = t^2 + 3t - 2t - 6$
 $= t(t + 3) - 2(t + 3)$
 $= (t + 3)(t - 2) \dots \dots \dots (iii)$

Common factor is : $(t + 3)$

Thus, H.C.F = $(t + 3)$

Q.10 $x^2 - x - 2, x^2 + x - 6, x^2 - 3x + 2$

Sol.

Factorization of $x^2 - x - 2 = x^2 - 2x + x - 2$
 $= (x^2 - 2x) + (x - 2)$
 $= x(x - 2) + 1(x - 2)$
 $= (x - 2)(x + 1) \dots \dots \dots (i)$

Factorization of $x^2 + x - 6 = x^2 + 3x - 2x - 6$
 $= (x^2 + 3x) - (2x + 6)$
 $= x(x + 3) - 2(x + 3)$
 $= (x + 3)(x - 2) \dots \dots \dots (ii)$

Factorization of $x^2 - 3x + 2 = x^2 - x - 2x + 2$
 $= (x^2 - x) - (2x - 2)$
 $= x(x - 1) - 2(x - 1)$
 $= (x - 1)(x - 2) \dots \dots \dots (iii)$

Common factor is : $(x - 2)$

Thus, H.C.F = $(x - 2)$

Q.11 $1 - x^2, x^3 + 1, 1 - x - 2x^2$

Sol.

Factorization of $1 - x^2 = (1)^2 - (x)^2$

$$= (1 + x)(1 - x) \dots\dots\dots (i)$$

Factorization of $x^3 + 1 = (x)^3 + (1)^3$

$$= (x + 1)(x^2 - x + 1) \dots\dots\dots (ii)$$

Factorization of $1 - x - 2x^2 = 1 - 2x + x - 2x^2$

$$= (1 - 2x) + (x - 2x^2)$$

$$= 1(1 - 2x) + x(1 - 2x)$$

$$= (1 - 2x)(1 + x)$$

Common factor is : $1 + x$

Thus, H.C.F = $1 + x$

Q.12 $x^3 - 8, x^2 - 7x + 10$

Sol:

Factorization of $x^3 - 8 = (x)^3 - (2)^3$

$$= (x - 2)[(x)^2 + (x)(2) + (2)^2]$$

$$= (x - 2)(x^2 + 2x + 4) \dots\dots\dots (i)$$

Factorization of $x^2 - 7x + 10 = x^2 - 2x - 5x + 10$

$$= (x^2 - 2x) - (5x - 10)$$

$$= x(x - 2) - 5(x - 2)$$

$$= (x - 2)(x - 5) \dots\dots\dots (ii)$$

Common factors are: $x - 2$

Thus, H.C.F = $x - 2$

Q.13 $x^2 + 3x + 2, x^2 + 4x + 3, x^2 + 5x + 4$

Sol:

Factorization of $x^2 + 3x + 2 = x^2 + x + 2x + 2$

$$= (x^2 + x) + (2x + 2)$$

$$= x(x + 1) + 2(x + 1)$$

$$= (x + 1)(x + 2) \dots\dots\dots (i)$$

Factorization of $x^2 + 4x + 3 = x^2 + x + 3x + 3$

$$= (x^2 + x) + (3x + 3)$$

$$= x(x + 1) + 3(x + 1)$$

$$= (x + 1)(x + 3) \dots\dots\dots (ii)$$

Factorization of $x^2 + 5x + 4 = x^2 + x + 4x + 4$

$$\begin{aligned}
 &= (x^2 + x) + (4x + 4) \\
 &= x(x + 1) + 4(x + 1) \\
 &= (x + 1)(x + 4) \dots\dots\dots (iii)
 \end{aligned}$$

Common factors are: $(x + 1)$

Thus, H.C.F = $x + 1$

Q.14: $x^4 + x^3 - 6x^2$, $x^4 - 9x^2$, $x^3 + x^2 - 6x$

Sol:

Factorization of $x^4 + x^3 - 6x^2 = x^2(x^2 + x - 6)$

$$\begin{aligned}
 &= x^2(x^2 + 3x - 2x - 6) \\
 &= x^2[(x^2 + 3x) - (2x + 6)] \\
 &= x^2[x(x + 3) - 2(x + 3)] \\
 &= x^2(x + 3)(x - 2) \\
 &= x \times x(x + 3)(x - 2) \dots\dots\dots (i)
 \end{aligned}$$

Factorization of $x^4 - 9x^2 = x^2(x^2 - 9)$

$$\begin{aligned}
 &= x^2[(x)^2 - (3)^2] \\
 &= x^2(x - 3)(x + 3) \\
 &= x \times x(x - 3)(x + 3) \dots\dots\dots (ii)
 \end{aligned}$$

Factorization of $x^3 + x^2 - 6x = x(x^2 + x - 6)$

$$\begin{aligned}
 &= x(x^2 + 3x - 2x - 6) \\
 &= x[(x^2 + 3x) - (2x + 6)] \\
 &= x[x(x + 3) - 2(x + 3)] \\
 &= x(x + 3)(x - 2) \dots\dots\dots (iii)
 \end{aligned}$$

Common factors are: $x, x + 3$

Thus, H.C.F. = $x(x + 3)$

$$= x^2 + 3x$$

Q.15 $35a^2c^3b$, $45a^3cb^2$, $30ac^2b^3$

Sol:

Factorization of $35a^2c^3b = 5 \times 7 \times \boxed{a} \times a \times c \times \boxed{c} \times c \times \boxed{b}$

Factorization of $45a^3cb^2 = 5 \times 3 \times 3 \times \boxed{a} \times a \times a \times \boxed{c} \times \boxed{b} \times b$

Factorization of $30ac^2b^3 = 5 \times 2 \times 3 \times \boxed{a} \times \boxed{c} \times c \times \boxed{b} \times b \times b$

Common factors = $5, a, b, c$

Thus, H.C.F. = $5 \times a \times b \times c$

$$= 5abc$$



Find the H.C.F by Division Method.

Q.1 $x^4 + x^2 + 1$, $x^4 + x^3 + x + 1$

Sol:

$$x^4 + x^2 + 1 \overline{) \begin{array}{r} x^4 + x^3 + x + 1 \\ \underline{+ x^4} \quad \underline{+ x^2} \quad \underline{+ 1} \end{array}}$$

Take common x $\begin{array}{r} x^3 - x^2 + x \\ x^2 - x + 1 \overline{) \begin{array}{r} x^4 \quad + x^2 \quad + 1 \\ \underline{- x^4} \quad \underline{+ x^3} \quad \underline{+ x^2} \end{array}} \end{array}$

$$\begin{array}{r} x^3 \quad + 1 \\ \underline{- x^3} \quad \underline{+ x^2} \quad \underline{+ x} \\ x^2 - x + 1 \\ \underline{+ x^2} \quad \underline{+ x} \quad \underline{+ 1} \\ 0 \end{array}$$

$$\text{H.C.F} = x^2 - x + 1$$

Q.2. $6x^3 + 7x^2 - 9x + 2, 8x^4 + 6x^3 - 15x^2 + 9x - 2$

Sol:

$$\begin{array}{r}
 4x + 5 \\
 6x^3 + 7x^2 - 9x + 2 \overline{) 8x^4 + 6x^3 - 15x^2 + 9x - 2} \quad \text{multiply by 3} \\
 \underline{24x^4 + 18x^3 - 45x^2 + 27x - 6} \\
 -24x^4 + 28x^3 - 36x^2 + 8x \\
 \hline
 -10x^3 - 9x^2 + 19x - 6 \\
 \times(-3) \\
 \hline
 30x^3 + 27x^2 - 57x + 18 \\
 \pm 30x^3 \pm 35x^2 \pm 45x \pm 10 \\
 \hline
 -4 \overline{) -8x^2 - 12x + 8}
 \end{array}$$

$$\begin{array}{r}
 3x - 1 \\
 2x^2 + 3x - 2 \overline{) 6x^3 + 7x^2 - 9x + 2} \\
 \underline{+ 6x^3 + 9x^2 - 6x} \\
 -2x^2 - 3x + 2 \\
 \underline{+ 2x^2 + 3x + 2} \\
 0
 \end{array}$$

$$\text{H.C.F} = 2x^2 + 3x - 2$$

Q.3 $4x^3 + 2x^2 - 6x, 4x^3 - 8x + 4$

Sol: $2 \overline{) 4x^3 + 2x^2 - 6x} \quad 4 \overline{) 4x^3 - 8x + 4}$
 $\quad 2x^3 + x^2 - 3x \quad \quad x^3 - 2x + 1$

Now we find H.C.F of $x^3 - 2x + 1$ and $2x^3 + x^2 - 3x$ and

H.C.F of 2, 4 is 2.

$$\begin{array}{r}
 2 \\
 x^3 - 2x + 1 \overline{) \begin{array}{l} 2x^3 + x^2 - 3x \\ \underline{+ 2x^3} \\ -x^2 + 4x + 2 \end{array}} \\
 \phantom{x^3 - 2x + 1 \overline{)} \begin{array}{l} x^2 + x - 2 \\ \underline{+ x^2} \\ -x + 2 \\ \underline{+ x - 2} \\ 0 \end{array} \left(\begin{array}{l} x^3 - 2x + 1 \\ \underline{+ x^3} \\ -x^2 + 2x + 1 \\ \underline{+ x^2} \\ -x + 1 \\ \underline{+ x - 1} \\ 0 \end{array} \right)
 \end{array}$$

$$\begin{array}{r}
 -1 \\
 -x^2 + 1 \overline{) \begin{array}{l} x^2 + x - 2 \\ \underline{+ x^2} \\ -x - 2 \\ \underline{+ x + 1} \\ -x - 1 \\ \underline{+ x + 1} \\ 0 \end{array}} \\
 \phantom{-x^2 + 1 \overline{)} \begin{array}{l} x - 1 \\ \underline{+ x - 1} \\ 0 \end{array} \left(\begin{array}{l} -x^2 + 1 \\ \underline{+ x^2} \\ -x + 1 \\ \underline{+ x - 1} \\ 0 \end{array} \right)
 \end{array}$$

Required H.C.F = $2(x - 1)$

Q.4 $x^3 + 7x^2 + 12x$, $x^3 - 2x^2 - 15x$

Sol:

$$\begin{array}{r}
 x \overline{) x^3 + 7x^2 + 12x} \\
 \hline
 x^3 - 2x^2 - 15x \\
 \hline
 x^2 + 7x + 12 \\
 \hline
 x^2 - 2x - 15 \\
 \hline
 \hline
 \hline
 \end{array}$$

H.C.F of x and x is x .

Now we find H.C.F of $x^2 - 2x - 15$ and $x^2 + 7x + 12$.

$$\begin{array}{r}
 x^3 + 7x + 12 \overline{) \begin{array}{l} x^3 - 2x - 15 \\ + x^3 + 7x + 12 \\ \hline -9 \quad -9x - 27 \end{array}} \\
 \begin{array}{l} x+3 \overline{) \begin{array}{l} + x^3 + 7x + 12 \\ \pm x^3 \pm 3x \\ \hline +4x + 12 \\ +4x + 12 \\ \hline 0 \end{array}} \end{array}
 \end{array}$$

Required H.C.F = $x(x+3)$

Q.5 $x^3 - x^2 - x + 1$, $x^4 - 2x^3 + 2x - 1$

Sol.

$$\begin{array}{r}
 x^3 - x^2 - x + 1 \overline{) \begin{array}{l} x^4 - 2x^3 + 2x - 1 \\ \pm x^4 \mp x^3 \mp x^2 \pm x \\ \hline -x^3 + x^2 + x - 1 \\ \mp x^3 \pm x^2 \pm x \mp 1 \\ \hline 0 \end{array}}
 \end{array}$$

Required H.C.F = $x^3 - x^2 - x + 1$

$$= x^2(x-1) - 1(x-1)$$

$$= (x^2 - 1)(x-1)$$

$$= [(x)^2 - (1)^2](x-1)$$

$$= (x-1)(x+1)(x-1)$$

$$= (x+1)(x-1)^2$$

Q.6 $x^3 - x^2 - x - 2$, $x^3 + 3x^2 - 6x - 8$

Sol:

$$\begin{array}{r}
 x^3 - x^2 - x - 2 \overline{) x^3 + 3x^2 - 6x - 8} \\
 \underline{+x^3 \mp x^2 \mp x \mp 2} \\
 4x^2 - 5x - 6 \overline{) x^3 - x^2 - x - 2} \\
 \underline{\times 4} \\
 \downarrow \\
 4x^2 - 5x - 6 \overline{) 4x^3 - 4x^2 - 4x - 8} \quad x \\
 \underline{+4x^3 \mp 5x^2 \mp 6x} \\
 x^2 + 2x - 8 \overline{) 4x^2 - 5x - 6} \quad 4 \\
 \underline{+4x^2 \pm 8x \mp 32} \\
 -13 \overline{) -13x + 26} \\
 \underline{x - 2} \\
 \downarrow \\
 x - 2 \overline{) x^2 + 2x - 8} \\
 \underline{+x^2 \mp 2x} \\
 4x - 8 \\
 \underline{+4x \mp 8} \\
 0
 \end{array}$$

Required H.C.F = $x - 2$

Q.7 $x^2 + 3x - 4$, $x^3 - 2x^2 - 2x + 3$

Sol:

$$\begin{array}{r}
 x^2 + 3x - 4 \overline{) x^3 - 2x^2 - 2x + 3} \quad x - 5 \\
 \underline{+x^3 \pm 3x^2 \mp 4x} \\
 -5x^2 + 2x + 3 \\
 \underline{+5x^2 \mp 15x \pm 20} \\
 17 \overline{) 17x - 17} \\
 x - 1 \overline{) x^2 + 3x - 4} \quad x + 4 \\
 \underline{+x^2 \mp x} \\
 4x - 4 \\
 \underline{+4x \mp 4} \\
 0
 \end{array}$$

Required H.C.F = $x - 1$

Q.8 $3x^3 - 14x^2 + 9x + 10, 15x^3 - 34x^2 + 21x - 10$

Sol.

$$\begin{array}{r}
 \begin{array}{r}
 5 \\
 3x^3 - 14x^2 + 9x + 10 \overline{) 15x^3 - 34x^2 + 21x - 10} \\
 \underline{\pm 15x^3 \mp 70x^2 \pm 45x \pm 50} \\
 12 \mid 36x^2 - 24x - 60
 \end{array} \\
 \begin{array}{r}
 3x^2 - 2x - 5 \overline{) 3x^3 - 14x^2 + 9x + 10} \mid x \cdot 4 \\
 \underline{\pm 3x^3 \mp 2x^2 \mp 5x} \\
 -12x^2 + 14x + 10 \\
 \mp 12x^2 \pm 8x \pm 20 \\
 \hline
 2 \mid 6x - 10 \\
 \hline
 3x - 5
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \downarrow \\
 x + 1
 \end{array}$$

$$\begin{array}{r}
 3x - 5 \overline{) 3x^2 - 2x - 5} \\
 \underline{\pm 3x^2 \mp 5x} \\
 3x - 5 \\
 \underline{\pm 3x \mp 5} \\
 0
 \end{array}$$

Required H.C.F = $3x - 5$

Q.9 $2x^4 + x^3 + 4x + 2$, $6x^3 + 5x^2 + x$, $2x^4 + 3x^3 + x^2 + 2x + 1$

Sol: First we will find the H.C.F of $2x^4 + x^3 + 4x + 2$ and $2x^4 + 3x^3 + x^2 + 2x + 1$

$$\begin{array}{r}
 1 \\
 2x^4 + x^3 + 4x + 2 \overline{) 2x^4 + 3x^3 + x^2 + 2x + 1} \\
 \underline{\pm 2x^4 \pm x^3 \qquad \pm 4x \pm 2} \qquad \qquad \qquad x \\
 2x^3 + x^2 - 2x - 1 \overline{) 2x^4 + x^3 + 4x + 2} \\
 \underline{\pm 2x^4 \pm x^3 \mp 2x^2 \mp x} \\
 2x^2 + 5x + 2
 \end{array}$$

↓

$$\begin{array}{r}
 x - 2 \\
 2x^2 + 5x + 2 \overline{) 2x^3 + x^2 - 2x - 1} \\
 \underline{\pm 2x^3 \pm 5x^2 \pm 2x} \\
 -4x^2 - 4x - 1 \\
 \underline{\mp 4x^2 \mp 10x \mp 4} \\
 3 \overline{) 6x + 3} \\
 \underline{3} \quad 3x + 3 \\
 \text{H.C.F} \quad 2x + 1 \overline{) 2x^2 + 5x + 2} \\
 \underline{\pm 2x^2 \pm x} \\
 4x + 2 \\
 \underline{\pm 4x \pm 2} \\
 0
 \end{array}$$

Now we will find H.C.F of $2x + 1$ and $6x^3 + 5x^2 + x$

$$\begin{array}{r}
 3x^2 + x \\
 2x + 1 \overline{) 6x^3 + 5x^2 + x} \\
 \underline{\pm 6x^3 \pm 3x^2} \\
 2x^2 + x \\
 \underline{\pm 2x^2 \pm x} \\
 0
 \end{array}$$

Required H.C.F = $2x + 1$

Q.10 $x^3 + x^2 - 5x + 3$, $x^3 - 7x + 6$, $x^3 + 2x^2 - 2x + 3$

Sol First we will find the H.C.F of $x^3 - 7x + 6$ and $x^3 + x^2 - 5x + 3$.

$$\begin{array}{r}
 1 \\
 x^3 - 7x + 6 \quad \overline{) \quad x^3 + x^2 - 5x + 3} \\
 \underline{\pm x^3 \quad \mp 7x \pm 6} \qquad \qquad \qquad x - 2 \\
 x^2 + 2x - 3 \quad \overline{) \quad x^3 \quad - 7x + 6} \\
 \underline{\pm x^3 \pm 2x^2 \mp 3x} \\
 - 2x^2 - 4x + 6 \\
 \underline{\mp 2x^2 \mp 4x \pm 6} \\
 0
 \end{array}$$

Now we will find H.C.F of $x^2 + 2x - 3$ and $x^3 + 2x^2 - 2x + 3$.

$$\begin{array}{r}
 x \\
 x^2 + 2x - 3 \quad \overline{) \quad x^3 + 2x^2 - 2x + 3} \\
 \underline{\pm x^3 \pm 2x^2 \mp 3x} \qquad \qquad \qquad x - 1 \\
 x + 3 \quad \overline{) \quad x^2 + 2x - 3} \\
 \underline{\pm x^2 \pm 3x} \\
 - x - 3 \\
 \underline{\mp x \mp 3} \\
 0
 \end{array}$$

H.C.F $x + 3$

Least Common Multiple (LCM)

The least common multiple of two or more algebraic expressions is the expression of lowest degree which is divisible by each of them without remainder.

The abbreviation of the words *least common multiple* is *L.C.M.*



Find L.C.M by Factorization.

Q.1 $21a^4x^3y, 35a^2x^4y, 28a^3xy^4$

Sol:

Factorization of $21a^4x^3y = 3 \times 7 \times a \times a \times a \times a \times x \times x \times x \times y$
 Factorization of $35a^2x^4y = 5 \times 7 \times a \times a \times x \times x \times x \times x \times y$
 Factorization of $28a^3xy^4 = 2 \times 2 \times 7 \times a \times a \times a \times x \times y \times y \times y \times y$

Product of common factors $= 7 \times a \times a \times a \times x \times y \times x \times x$
 $= 7a^3x^3y \dots\dots\dots (i)$

Product of uncommon factors $= 3 \times 5 \times 2 \times 2 \times a \times x \times y \times y \times y$
 $= 60axy^3 \dots\dots\dots (ii)$

L.C.M $= (i) \times (ii)$
 $= 7a^3x^3y \times 60axy^3$
 $= 420a^4x^4y^4$

Q.2 $3a^4b^2c^3, 5a^2b^3c^5$

Sol:

Fac of $3a^4b^2c^3 = 3 \times a \times a \times a \times a \times b \times b \times c \times c \times c$
 Fac of $5a^2b^3c^5 = 5 \times a \times a \times b \times b \times b \times c \times c \times c \times c \times c$

Product of common factors $= a \times a \times b \times b \times c \times c \times c$
 $= a^2b^2c^3 \dots\dots\dots (i)$

Product of uncommon factors $= 3 \times 5 \times a \times a \times b \times c \times c$
 $= 15a^2bc^2 \dots\dots\dots (ii)$

L.C.M $= (i) \times (ii)$
 $= (a^2b^2c^3)(15a^2bc^2)$
 $= 15a^4b^3c^5$

Q.3 $2ab, 3ab, 4ca$

Sol:

$$\begin{aligned}\text{Factorization of } 2ab &= \boxed{2} \times \boxed{a} \times \boxed{b} \\ \text{Factorization of } 3ab &= 3 \times \boxed{a} \times \boxed{b} \\ \text{Factorization of } 4ca &= 2 \times \boxed{2} \times \boxed{a} \times \boxed{c}\end{aligned}$$

$$\text{Product of common factors} = 2 \times a \times b = 2ab \dots\dots\dots (i)$$

$$\text{Product of uncommon factors} = 3 \times 2 \times c = 6c \dots\dots\dots (ii)$$

$$\begin{aligned}\text{L.C.M} &= (i) \times (ii) \\ &= (2ab)(6c) \\ &= 12abc\end{aligned}$$

Q.4 x^2yz, xy^2z, xyz^2

Sol:

$$\begin{aligned}\text{Factorization of } x^2yz &= \boxed{x} \times x \times \boxed{y} \times \boxed{z} \\ \text{Factorization of } xy^2z &= \boxed{x} \times \boxed{y} \times \boxed{y} \times \boxed{z} \\ \text{Factorization of } xyz^2 &= \boxed{x} \times \boxed{y} \times \boxed{z} \times \boxed{z}\end{aligned}$$

$$\text{Product of common factors} = x \times y \times y \times z = xy^2z \dots\dots\dots (i)$$

$$\text{Product of uncommon factors} = x \times z = xz \dots\dots\dots (ii)$$

$$\begin{aligned}\text{L.C.M} &= (i) \times (ii) \\ &= xy^2z \times xz \\ &= x^2y^2z^2\end{aligned}$$

Q.5 $p^3q - pq^3, p^5q^2 - p^2q^5$

Sol:

$$\begin{aligned}\text{Factorization of } p^3q - pq^3 &= pq(p^2 - q^2) \\ &= pq(p - q)(p + q) \dots\dots\dots (i)\end{aligned}$$

$$\text{Factorization of } p^5q^2 - p^2q^5 = p^2q^2(p^3 - q^3)$$

$$= ppqq(p - q)(p^2 + pq + q^2) \dots\dots\dots (ii)$$

In (i) and (ii)

$$\text{Product of common factors} = pq(p - q) \dots\dots\dots (iii)$$

$$\text{Product of uncommon factors} = pq(p + q)(p^2 + pq + q^2) \dots\dots\dots (iv)$$

$$\begin{aligned} \text{L.C.M} &= (iii) \times (iv) \\ &= [pq(p - q)] [pq(p + q)(p^2 + pq + q^2)] \\ &= p^2q^2(p - q)(p + q)(p^2 + pq + q^2) \end{aligned}$$

Q. $x^3 + 64, x^2 - 16$

Sol:

$$\begin{aligned} \text{Factorization of } x^3 + 64 &= (x)^3 + (4)^3 \\ &= (x + 4) [(x)^2 - (x)(4) + (4)^2] \\ &= (x + 4)(x^2 - 4x + 16) \dots\dots\dots (i) \end{aligned}$$

$$\begin{aligned} \text{Factorization of } x^2 - 16 &= (x)^2 - (4)^2 \\ &= (x + 4)(x - 4) \dots\dots\dots (ii) \end{aligned}$$

In (i) and (ii)

$$\text{Product of common factors} = (x + 4) \dots\dots\dots (iii)$$

$$\text{Product of uncommon factors} = (x - 4)(x^2 - 4x + 16) \dots\dots\dots (iv)$$

$$\begin{aligned} \text{L.C.M} &= (iii) \times (iv) \\ &= (x + 4)(x - 4)(x^2 - 4x + 16) \end{aligned}$$

Q. $x^2 - x - 2, x^2 + x - 6, x^2 - 3x + 2$

Sol:

$$\begin{aligned} \text{Factorization of } x^2 - x - 2 &= x^2 - 2x + x - 2 \\ &= (x^2 - 2x) + (x - 2) \\ &= x(x - 2) + 1(x - 2) \\ &= (x - 2)(x + 1) \dots\dots\dots (i) \end{aligned}$$

$$\begin{aligned} \text{Factorization of } x^2 + x - 6 &= x^2 + 3x - 2x - 6 \\ &= (x^2 + 3x) - (2x + 6) \end{aligned}$$

$$= x(x + 3) - 2(x + 3)$$

$$= (x + 3)(x - 2) \dots \dots \dots (ii)$$

$$\text{Factorization of } x^2 - 3x + 2 = x^2 - x - 2x + 2$$

$$= (x^2 - x) - (2x + 2)$$

$$= x(x - 1) - 2(x - 1)$$

$$= (x - 1)(x - 2) \dots \dots \dots (iii)$$

In (i), (ii) and (iii)

$$\text{Product of common factors} = (x - 2) \dots \dots \dots (iv)$$

$$\text{Product of uncommon factors} = (x + 1)(x + 3)(x - 1) \dots \dots \dots (v)$$

$$\text{L.C.M} = (iv) \times (v)$$

$$= (x - 2)(x + 1)(x + 3)(x - 1)$$

$$\text{Q.8 } y^2 - 9, (y + 3)^2, y^2 + y - 6$$

Sol:

$$\text{Factorization of } y^2 - 9 = (y)^2 - (3)^2$$

$$= (y + 3)(y - 3) \dots \dots \dots (i)$$

$$\text{Factorization of } (y + 3)^2 = (y + 3)(y + 3) \dots \dots \dots (ii)$$

$$\text{Factorization of } y^2 + y - 6 = y^2 + 3y - 2y - 6$$

$$= (y^2 + 3y) - (2y + 6)$$

$$= y(y + 3) - 2(y + 3)$$

$$= (y + 3)(y - 2) \dots \dots \dots (iii)$$

In (i), (ii) and (iii)

$$\text{Product of common factors} = (y + 3) \dots \dots \dots (iv)$$

$$\text{Product of uncommon factors} = (y - 3)(y + 3)(y - 2) \dots \dots \dots (v)$$

$$\text{L.C.M} = (iv) \times (v)$$

$$= (y + 3)(y - 3)(y + 3)(y - 2)$$

$$\text{Q.9 } 1 - x^2, x^3 + 1, 1 - x - 2x^2$$

Sol:

$$\text{Factorization of } 1 - x^2 = (1)^2 - (x)^2$$

$$= (1 + x)(1 - x) \dots (i)$$

Factorization of $x^3 + 1 = (x)^3 + (1)^3$
 $= (x + 1)(x^2 - x + 1) \dots (ii)$

Factorization of $1 - x - 2x^2 = 1 - 2x + x - 2x^2$
 $= (1 - 2x) + (x - 2x^2)$
 $= 1(1 - 2x) + x(1 - 2x)$
 $= (1 - 2x)(1 + x) \dots (iii)$

In (i), (ii) and (iii)

Product of common factors $= (1 + y) \dots (iv)$

Product of **uncommon** factors $= (1 - y)(1 - 2y)(y^2 - y + 1) \dots (v)$

$\text{C}_y \text{ (8)} \quad \text{L.C.M} = (iv) \times (v)$
 $= (1 + y)(1 - y)(1 - 2y)(y^2 - y + 1)$

Q.10 $x^2 - y^2, x^4 - y^4, x^6 - y^6$

Sol:

Factorization of $x^2 - y^2 = (x + y)(x - y) \dots (i)$
 $= (x)^2 - (y)^2$

Factorization of $x^4 - y^4 = (x^2)^2 - (y^2)^2$
 $= (x^2 + y^2)(x^2 - y^2)$
 $= (x^2 + y^2)[(x)^2 - (y)^2]$
 $= (x^2 + y^2)(x + y)(x - y) \dots (ii)$

Factorization of $x^6 - y^6 = (x^3)^2 - (y^3)^2$
 $= (x^3 + y^3)(x^3 - y^3)$
 $= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2) \dots (iii)$

In (i), (ii) and (iii)

Product of common factors $= (x + y)(x - y) \dots (iv)$

Product of uncommon factors $= (x^2 + y^2)(x^2 - xy + y^2)(x^2 + xy + y^2) \dots (v)$

$\text{L.C.M} = (iv) \times (v)$

$= (x + y)(x - y)(x^2 + y^2)$

$$\begin{aligned}
 & (x^2 - xy + y^2)(x^2 + xy + y^2) \\
 &= (x + y)(x - y)(x^2 + y^2) \\
 & (x^4 + x^2y^2 + y^4)
 \end{aligned}$$

Q.11 $x^3 + 1, x^4 + x^2 + 1, (x^2 + x + 1)^2$

Sol:

Factorization of $x^3 + 1 = (x)^3 + (1)^3$
 $= (x + 1)(x^2 - x + 1) \dots \dots \dots (i)$

Factorization of $x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2$ (completing square)
 $= (x^2 + 1)^2 - (x)^2$
 $= (x^2 + 1 + x)(x^2 + 1 - x)$
 $= (x^2 + x + 1)(x^2 - x + 1) \dots \dots \dots (ii)$

Factorization of $(x^2 + x + 1)^2 = (x^2 + x + 1)(x^2 + x + 1) \dots \dots \dots (iii)$

In (i), (ii) and (iii)

Product of common factors $= (x^2 + x + 1)(x^2 - x + 1) \dots (iv)$

Product of uncommon factors $= (x + 1)(x^2 + x + 1) \dots \dots \dots (v)$

L.C.M $= (iv) \times (v)$

$$\begin{aligned}
 &= (x^2 + x + 1)(x^2 - x + 1)(x + 1)(x^2 + x + 1) \\
 &= (x + 1)(x^2 - x + 1)(x^2 + x + 1)^2
 \end{aligned}$$

Q.12 $x^3 + y^3, x^4 - y^4, x^6 + y^6$

Sol:

Factorization of $x^3 + y^3 = (x + y)(x^2 - xy + y^2) \dots \dots \dots (i)$

Factorization of $x^4 - y^4 = (x^2)^2 - (y^2)^2$
 $= (x^2 + y^2)(x^2 - y^2)$
 $= (x^2 + y^2)[(x^2) - (y^2)]$
 $= (x^2 + y^2)(x + y)(x - y) \dots \dots \dots (ii)$

Factorization of $x^6 + y^6 = (x^2)^3 + (y^2)^3$
 $= (x^2 + y^2)[(x^2)^2 - (x^2)(y^2) + (y^2)^2]$
 $= (x^2 + y^2)(x^4 - x^2y^2 + y^4) \dots \dots \dots (iii)$

In (i), (ii) and (iii)

$$\text{Product of common factors} = (x + y)(x^2 + y^2) \dots \text{(iv)}$$

$$\text{Product of uncommon factors} = (x^2 - xy + y^2)(x - y)(x^4 - x^2y^2 + y^4) \dots \text{(v)}$$

$$\begin{aligned} \text{L.C.M} &= (\text{iv}) \times (\text{v}) \\ &= (x + y)(x^2 + y^2)(x^2 - x + y^2)(x - y) \\ &\quad (x^4 - x^2y^2 + y^4) \\ &= (x + y)(x - y)(x^2 + y^2) \\ &\quad (x^2 - xy + y^2)(x^4 - x^2y^2 + y^4) \end{aligned}$$

Q.13 $2x^2 + 5x + 3$, $x^2 + 2x + 1$, $2x^2 + 9x + 9$

Sol:

$$\begin{aligned} \text{Factorization of } 2x^2 + 5x + 3 &= 2x^2 + 2x + 3x + 3 \\ &= (2x^2 + 2x) + (3x + 3) \\ &= 2x(x + 1) + 3(x + 1) \\ &= (x + 1)(2x + 3) \dots \text{(i)} \end{aligned}$$

$$\begin{aligned} \text{Factorization of } x^2 + 2x + 1 &= x^2 + x + x + 1 \\ &= (x^2 + x) + (x + 1) \\ &= x(x + 1) + 1(x + 1) \\ &= (x + 1)(x + 1) \dots \text{(ii)} \end{aligned}$$

$$\begin{aligned} \text{Factorization of } 2x^2 + 9x + 9 &= 2x^2 + 3x + 6x + 9 \\ &= (2x^2 + 3x) + (6x + 9) \\ &= x(2x + 3) + 3(2x + 3) \\ &= (2x + 3)(x + 3) \dots \text{(iii)} \end{aligned}$$

In (i), (ii) and (iii)

$$\text{Product of common factors} = (x + 1)(2x + 3) \dots \text{(iv)}$$

$$\text{Product of uncommon factors} = (x + 1)(x + 3) \dots \text{(v)}$$

$$\begin{aligned} \text{L.C.M} &= (\text{iv}) \times (\text{v}) \\ &= (x + 1)(2x + 3)(x + 1)(x + 3) \\ &= (x + 1)^2(2x + 3)(x + 3) \end{aligned}$$

Q.14 $x^4 + x^3 - 6x^2$, $x^4 - 9x^2$, $x^3 + x^2 - 6x$

Sol:

$$\begin{aligned}\text{Factorization of } x^4 + x^3 - 6x^2 &= x^2(x^2 + x - 6) \\ &= x^2(x^2 + 3x - 2x - 6) \\ &= x^2[(x^2 + 3x) - (2x + 6)] \\ &= x^2[x(x + 3) - 2(x + 3)] \\ &= x^2(x + 3)(x - 2) \dots \dots \dots (i)\end{aligned}$$

$$\begin{aligned}\text{Factorization of } x^4 - 9x^2 &= x^2(x^2 - 9) \\ &= x^2[(x^2 - 3^2)] \\ &= x^2(x + 3)(x - 3) \dots \dots \dots (ii)\end{aligned}$$

$$\begin{aligned}\text{Factorization of } x^3 + x^2 - 6x &= x(x^2 + x - 6) \\ &= x(x^2 + 3x - 2x - 6) \\ &= x[x(x + 3) - 2(x + 3)] \\ &= x(x + 3)(x - 2) \dots \dots \dots (iii)\end{aligned}$$

In (i), (ii) and (iii)

$$\text{Product of common factors} = x^2(x + 3)(x - 2) \dots \dots \dots (iv)$$

$$\text{Product of uncommon factors} = (x - 3) \dots \dots \dots (v)$$

$$\begin{aligned}\text{L.C.M} &= (iv) \times (v) \\ &= x^2(x + 3)(x - 2)(x - 3)\end{aligned}$$

Q.15 $x^2 + 4xy + 4y^2$, $x^2 + 3xy + 2y^2$, $x^2 + 2xy + y^2$

Sol:

$$\begin{aligned}\text{Factorization of } x^2 + 4xy + 4y^2 &= x^2 + 2xy + 2xy + 4y^2 \\ &= (x^2 + 2xy) + (2xy + 4y^2) \\ &= x(x + 2y) + 2y(x + 2y) \\ &= (x + 2y)(x + 2y) \dots \dots \dots (i)\end{aligned}$$

$$\begin{aligned}\text{Factorization of } x^2 + 3xy + 2y^2 &= x^2 + xy + 2xy + 2y^2 \\ &= (x^2 + xy) + (2xy + 2y^2) \\ &= x(x + y) + 2y(x + y) \\ &= (x + y)(x + 2y) \dots \dots \dots (ii)\end{aligned}$$

$$\begin{aligned}
 \text{Factorization of } x^2 + 2xy + y^2 &= x^2 + xy + xy + y^2 \\
 &= (x^2 + xy) + (xy + y^2) \\
 &= x(x + y) + y(x + y) \\
 &= (x + y)(x + y) \dots \dots \dots (iii)
 \end{aligned}$$

In (i), (ii) and (iii)

$$\text{Product of common factors} = (x + 2y)(x + y) \dots \dots \dots (iv)$$

$$\text{Product of uncommon factors} = (x + 2y)(x + y) \dots \dots \dots (v)$$

$$\begin{aligned}
 \text{L.C.M} &= (iv) \times (v) \\
 &= (x + 2y)(x + y)(x + 2y)(x + y) \\
 &= (x + y)(x + y)(x + 2y)(x + 2y) \\
 &= (x + y)^2(x + 2y)^2
 \end{aligned}$$

Relationship between HCF and LCM

If A and B are two algebraic expressions and *H.C.F* and *L.C.M* of these is represented by H and L respectively, then the relation among them can be expressed as:

$$\boxed{A \times B = H \times L}$$

It is called a formula between *L.C.M* and *H.C.F*.

PROOF: Suppose that

$$\frac{A}{H} = x \quad \text{and} \quad \frac{B}{H} = y$$

$$A = Hx \quad \dots \dots \dots (i)$$

$$B = Hy \quad \dots \dots \dots (ii)$$

Since there is no common factor between x and y.

$$\text{Therefore } L = H \cdot x \cdot y$$

$$\begin{aligned}
 HL &= H(Hx \cdot y) \text{ (multiplying both the sides by H)} \\
 &= (Hx) \cdot (Hy)
 \end{aligned}$$

$$HL = A \cdot B$$

$$(i) \quad L = \frac{A \times B}{H}$$

$$(ii) \quad H = \frac{A \times B}{L}$$

$$(iii) \quad A = \frac{H \times L}{B}$$

Exercise 3.4

Find the H.C.F and L.C.M of the following.

Q.1 $x^3 + x^2 + x + 1, x^3 - x^2 + x - 1$

Sol:

First we find H.C.F

$$\begin{array}{r}
 \begin{array}{r}
 1 \\
 x^3 + x^2 + x + 1 \overline{) x^3 - x^2 + x - 1} \\
 \underline{\pm x^3 \pm x^2 \pm x \pm 1} \\
 -2x^2 - 2 \quad x + 1 \\
 -2 \overline{) -2x^2 - 2} \quad x + 1 \\
 \hline
 x^2 + 1 \overline{) x^3 + x^2 + x + 1} \\
 \underline{\pm x^3 \quad \pm x} \\
 x^2 \quad + 1 \\
 \underline{\pm x^2 \quad \pm 1} \\
 0
 \end{array}
 \end{array}$$

$$\text{H.C.F} = x^2 + 1$$

Now we will find L.C.M

$$\begin{aligned}
 \text{L.C.M} &= \frac{(x^3 + x^2 + x + 1)(x^3 - x^2 + x - 1)}{(x^2 + 1)} \\
 &= (x^3 + x^2 + x + 1)(x - 1) \\
 &= x^4 + x^3 + x^2 + x - x^3 - x^2 - x - 1 \\
 \text{L.C.M} &= x^4 - 1
 \end{aligned}$$

Working
 $x - 1$

$$\begin{array}{r}
 x^2 + 1 \overline{) x^3 - x^2 + x - 1} \\
 \underline{\pm x^3 \quad \pm x} \\
 -x^2 \quad -1 \\
 \underline{+x^2 \quad +1} \\
 0
 \end{array}$$

Q.2 $x^3 - 3x^2 - 4x + 12, x^3 - x^2 - 4x + 4$

Sol: First we find H.C.F

$$\begin{array}{r}
 \quad 1 \\
 x^3 - 3x^2 - 4x + 12 \quad \left| \begin{array}{l} x^3 - x^2 - 4x + 4 \\ \pm x^3 \mp 3x^2 \mp 4x \pm 12 \end{array} \right. \\
 \hline
 2 \quad \left| \begin{array}{l} 2x^2 - 8 \\ x - 3 \end{array} \right. \\
 \hline
 \begin{array}{r|l} x^2 - 4 & x^3 - 3x^2 - 4x + 12 \\ \hline & \pm x^3 \quad \mp 4x \\ \hline & - 3x^2 \quad + 12 \\ & \mp 3x^2 \quad \pm 12 \\ \hline & 0 \end{array}
 \end{array}$$

H.C.F = $x^2 - 4$

Now we will find L.C.M

$$\begin{array}{r}
 \quad x - 3 \\
 \text{L.C.M} = \frac{(x^3 - 3x^2 - 4x + 12)(x^3 - x^2 - 4x + 4)}{(x^2 - 4)}
 \end{array}$$

$$= (x - 3)(x^3 - x^2 - 4x + 4)$$

$$= x^4 - x^3 - 4x^2 + 4x - 3x^3 + 3x^2 + 12x - 12$$

$$= x^4 - 4x^3 - x^2 + 16x - 12$$

Q.3 $2x^3 + 2x^2 + x + 1, 2x^3 - 2x^2 + x - 1$

Sol: First we find H.C.F

$$\begin{array}{r}
 \quad 1 \\
 2x^3 + 2x^2 + x + 1 \quad \left| \begin{array}{l} 2x^3 - 2x^2 + x - 1 \\ \pm 2x^3 \pm 2x^2 \pm x \pm 1 \end{array} \right. \\
 \hline
 -2 \quad \left| \begin{array}{l} -4x^2 - 2 \\ x + 1 \end{array} \right. \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 2x^2 + 1 & 2x^3 + 2x^2 + x + 1 \\
 & \pm 2x^3 \qquad \pm x \\
 \hline
 & 2x^2 \qquad + 1 \\
 & \pm 2x^2 \qquad \pm 1 \\
 \hline
 & 0
 \end{array}$$

$$\text{H.C.F} = 2x^2 + 1$$

Now we will find L.C.M

$$x + 1$$

$$\text{L.C.M} = \frac{(2x^3 + 2x^2 + x + 1)(2x^3 - 2x^2 + x - 1)}{(2x^2 + 1)}$$

$$= (x + 1)(2x^3 - 2x^2 + x - 1)$$

$$= 2x^4 - 2x^3 + x^2 - x + 2x^3 - 2x^2 + x - 1$$

$$= 2x^4 - x^2 - 1$$

Q.4 $6x^3 + 7x^2 - 9x + 2, 8x^4 + 6x^3 - 15x^2 + 9x - 2$

Sol:

First we find H.C.F

$$4x + 5$$

$$\begin{array}{r|l}
 6x^3 + 7x^2 - 9x + 2 & 8x^4 + 6x^3 - 15x^2 + 9x - 2 \\
 & \times 3 \\
 \hline
 & 24x^4 + 18x^3 - 45x^2 + 27x - 6 \\
 & \pm 24x^4 \pm 28x^3 \mp 36x^2 \pm 8x \\
 \hline
 & -10x^3 - 9x^2 + 19x - 6 \\
 & -1 \mid \\
 \hline
 & 10x^3 + 9x^2 - 19x + 6 \\
 & \times 3 \\
 \hline
 & 30x^3 + 27x^2 - 57x + 18 \\
 & \pm 30x^3 \pm 35x^2 \mp 45x \pm 10 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 -1 \overline{) -8x^2 - 12x + 8} \\
 4 \overline{) 8x^2 + 12x - 8} \quad 3x - 1 \\
 \hline
 2x^2 + 3x - 2 \overline{) 6x^3 + 7x^2 - 9x + 2} \\
 \quad \underline{\pm 6x^3 \pm 9x^2 \mp 6x} \\
 \quad \quad -2x^2 - 3x + 2 \\
 \quad \quad \underline{\mp 2x^2 \mp 3x \pm 2} \\
 \quad \quad \quad 0
 \end{array}$$

$$\text{H.C.F} = 2x^2 + 3x - 2$$

Now we will find L.C.M

$$3x - 1$$

$$\text{L.C.M} = \frac{(6x^3 + 7x^2 - 9x + 2)(8x^4 + 6x^3 - 15x^2 + 9x - 2)}{(2x^2 + 3x - 2)}$$

$$= (3x - 1)(8x^4 + 6x^3 - 15x^2 + 9x - 2)$$

Q.5 $3x^4 + 17x^3 + 27x^2 + 7x - 6$, $6x^4 + 7x^3 - 27x^2 + 17x - 3$

Sol: First we find H.C.F

2

$$\begin{array}{r}
 3x^4 + 17x^3 + 27x^2 + 7x - 6 \overline{) 6x^4 + 7x^3 - 27x^2 + 17x - 3} \\
 \quad \underline{\pm 6x^4 \pm 34x^3 \pm 54x^2 \pm 14x \mp 12} \\
 \quad \quad -3 \overline{) -27x^3 - 81x^2 + 3x + 9} \\
 \quad \quad \quad \underline{9x^3 + 27x^2 - x - 3}
 \end{array}$$



$$x + 2 + 2$$

$$\begin{array}{r}
 9x^3 + 27x^2 - x - 3 \overline{) 3x^4 + 17x^3 + 27x^2 + 7x - 6} \\
 \quad \times 3 \\
 \quad \underline{9x^4 + 51x^3 + 81x^2 + 21x - 18} \\
 \quad \quad \pm 9x^4 \pm 27x^3 \mp x^2 \mp 3x
 \end{array}$$

$$\begin{array}{r}
 24x^3 + 82x^2 + 24x - 18 \\
 \pm 18x^3 \pm 54x^2 \mp 2x \mp 6 \\
 \hline
 6x^3 + 28x^2 + 26x - 12 \\
 \times 3 \\
 \hline
 18x^3 + 84x^2 + 78x - 36 \\
 \pm 18x^3 \pm 54x^2 \mp 2x \mp 6 \\
 \hline
 2 \mid 30x^2 + 80x - 30 \\
 \hline
 5 \mid 15x^2 + 40x - 15 \qquad 3x + 1 \\
 \hline
 3x^2 + 8x - 3 \mid 9x^3 + 27x^2 - x - 3 \\
 \mid \pm 9x^3 \pm 24x^2 \mp 9x \\
 \hline
 3x^2 + 8x - 3 \\
 \pm 3x^2 \pm 8x \mp 3 \\
 \hline
 0
 \end{array}$$

$$\text{H.C.F} = 3x^2 + 8x - 3$$

Now we will find L.C.M

$$\begin{aligned}
 \text{L.C.M} &= \frac{(3x^4 + 17x^3 + 27x^2 + 7x - 6)(\cancel{6x^4 + 7x^3 - 27x^2 + 17x - 3})}{(\cancel{3x^2 + 8x - 3})} \\
 &= (2x^2 - 3x + 1)(3x^4 + 17x^3 + 27x^2 + 7x - 6)
 \end{aligned}$$

Working

$$\begin{array}{r}
 2x^2 - 3x + 1 \\
 3x^2 + 8x - 3 \left[\begin{array}{r} 6x^4 + 7x^3 - 27x^2 + 17x - 3 \\ \pm 6x^4 \pm 16x^3 \mp 6x^2 \\ \hline -9x^3 - 21x^2 + 17x - 3 \\ \mp 9x^3 \mp 24x^2 \pm 9x \\ \hline 3x^2 + 8x - 3 \\ \pm 3x^2 \pm 8x \mp 3 \\ \hline 0 \end{array} \right.
 \end{array}$$

Q.6 $2x^4 + 3x^3 - 13x^2 - 7x + 15, 2x^4 + x^3 - 20x^2 - 7x + 24$

Sol: First we find H.C.F

1

$$\begin{array}{r}
 2x^4 + 3x^3 - 13x^2 - 7x + 15 \quad \overline{) \quad 2x^4 + x^3 - 20x^2 - 7x + 24} \\
 \underline{\pm 2x^4 \pm 3x^3 \mp 13x^2 \mp 7x \pm 15} \\
 -1 \overline{) \quad -2x^3 - 7x^2 + 9} \quad x - 2 \\
 \underline{2x^3 + 7x^2 - 9} \quad \overline{) \quad 2x^4 + 3x^3 - 13x^2 - 7x + 15} \\
 \underline{\pm 2x^4 \pm 7x^3 \mp 9x} \\
 -4x^3 - 13x^2 + 2x + 15 \\
 \underline{\mp 4x^3 \mp 14x^2 \pm 18} \\
 x^2 + 2x - 3
 \end{array}$$

$$\begin{array}{r}
 \downarrow \\
 \quad \quad \quad 2x^2 - x - 5 \\
 x^2 + 2x - 3 \quad \overline{) \quad 2x^4 + 3x^3 - 13x^2 - 7x + 15} \\
 \underline{\pm 2x^4 \pm 4x^3 \mp 6x^2} \\
 -x^3 - 7x^2 - 7x + 15 \\
 \underline{\mp x^3 \mp 2x^2 \pm 3x} \\
 -5x^2 - 10x + 15 \\
 \underline{\mp 5x^2 \mp 10x \pm 15} \\
 0
 \end{array}$$

$$\text{H.C.F} = x^2 + 2x - 3$$

Now we will find L.C.M

$$2x^2 - x - 5$$

$$\text{L.C.M} = \frac{(2x^4 + 3x^3 - 13x^2 - 7x + 15)(2x^4 + x^3 - 20x^2 - 7x + 24)}{x^2 + 2x - 3}$$

$$= (2x^2 - x - 5)(2x^4 + x^3 - 20x^2 - 7x + 24)$$

Q.7 $x^4 - x^3 - x + 1, x^4 + x^3 - x - 1$

Sol: First we find H.C.F

$$\begin{array}{r}
 \begin{array}{r}
 1 \\
 x^4 - x^3 - x + 1 \overline{) x^4 + x^3 - x - 1} \\
 \underline{\pm x^4 \mp x^3 \mp x \pm 1} \\
 2x^3 - 2
 \end{array}
 \quad
 \begin{array}{r}
 x - 1 \\
 2 \overline{) 2x^3 - 2} \\
 \underline{2x^3 - 2} \\
 0
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 x^3 - 1 \overline{) x^4 - x^3 - x + 1} \\
 \underline{\pm x^4 \mp x} \\
 -x^3 + 1 \\
 \underline{-x^3 + 1} \\
 0
 \end{array}$$

H.C.F = $x^3 - 1$

Now we will find L.C.M

$$\begin{aligned}
 \text{L.C.M} &= \frac{(x^4 - x^3 - x + 1)(x^4 + x^3 - x - 1)}{(x^3 - 1)} \\
 &= (x - 1)(x^4 + x^3 - x - 1)
 \end{aligned}$$

Q.8 $x^4 + x^3 + x + 1, x^4 + x^3 - x - 1$

Sol: First we find H.C.F

$$\begin{array}{r}
 \begin{array}{r}
 1 \\
 x^4 + x^3 + x + 1 \overline{) x^4 + x^3 - x - 1} \\
 \underline{\pm x^4 \pm x^3 \pm x \pm 1} \\
 -2x - 2
 \end{array}
 \quad
 \begin{array}{r}
 x^3 + 1 \\
 -2 \overline{) -2x - 2} \\
 \underline{-2x - 2} \\
 0
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 x + 1 \overline{) x^4 + x^3 + x + 1} \\
 \underline{\pm x^4 \pm x^3} \\
 x + 1 \\
 \underline{\pm x \pm 1} \\
 0
 \end{array}$$

H.C.F = $x + 1$

Now we will find L.C.M.

$$\begin{aligned}
 & (x^3 + 1) \\
 \text{L.C.M} &= \frac{(x^4 + x^3 + x + 1)(x^4 + x^3 - x - 1)}{(x + 1)} \\
 &= (x^3 + 1)(x^4 + x^3 - x + 1)
 \end{aligned}$$

Find the Required Polynomial.

Q.9. $A = x^2 - 5x - 14$, $H = x - 7$, $L = x^3 - 10x^2 + 11x + 70$, $B = ?$

Sol: Formula: $B = \frac{H \times L}{A}$

$$\begin{aligned}
 & (x - 5) \\
 &= \frac{(x - 7)(x^3 - 10x^2 + 11x + 70)}{(x^2 - 5x - 14)} \\
 &= (x - 7)(x - 5) \\
 &= x^2 - 12x + 35
 \end{aligned}$$

Working
 $x - 5$

$$\begin{array}{r}
 x^2 - 5x - 14 \overline{) \begin{array}{l} x^3 - 10x^2 + 11x + 70 \\ \pm x^3 \mp 5x^2 \mp 14x \dots \\ \hline -5x^2 + 25x + 70 \\ \mp 5x^2 \pm 25x \pm 70 \\ \hline 0 \end{array}}
 \end{array}$$

Q.10. $B = 3x^2 + 14x + 8$, $H = 3x + 2$, $L = 6x^3 + 25x^2 + 2x - 8$,
 $A = ?$

Sol: Formula: $A = \frac{H \times L}{B}$

$$\begin{aligned}
 & (2x - 1) \\
 &= \frac{(3x + 2)(6x^3 + 25x^2 + 2x - 8)}{(3x^2 + 14x + 8)}
 \end{aligned}$$

$$= (3x + 2)(2x - 1)$$

$$= 6x^2 + x - 2$$

$$\begin{array}{r} \text{Working} \\ 2x - 1 \end{array}$$

$$\begin{array}{r} 3x^2 + 14x + 18 \quad \left| \begin{array}{l} 6x^3 + 25x^2 + 2x - 8 \\ \pm 6x^3 \pm 28x^2 \pm 16x \\ \hline -3x^2 - 14x - 8 \\ +3x^2 + 14x + 8 \\ \hline 0 \end{array} \right. \end{array}$$

Q.11. The product of two polynomials and their L.C.M. are $x^4 + 6x^3 - 3x^2 - 56x - 48$ and $x^3 + 2x^2 - 11x - 12$ respectively. Find their H.C.F.

Sol:

$$A \times B = x^4 + 6x^3 - 3x^2 - 56x - 48$$

$$H = x^3 + 2x^2 - 11x - 12$$

$$L = ?$$

$$\text{Formula: } L = \frac{A \times B}{H}$$

$$\begin{aligned} &= \frac{x^4 + 6x^3 - 3x^2 - 56x - 48}{x^3 + 2x^2 - 11x - 12} \\ &= x + 4 \end{aligned}$$

$$\begin{array}{r} \text{Working} \\ x + 4 \end{array}$$

$$\begin{array}{r} x^3 + 2x^2 - 11x - 12 \quad \left| \begin{array}{l} x^4 + 6x^3 - 3x^2 - 56x - 48 \\ \pm x^4 \pm 2x^3 \mp 11x^2 \mp 12x \\ \hline +4x^3 + 8x^2 - 44x - 48 \\ \pm 4x^3 \pm 8x^2 \mp 44x \mp 48 \\ \hline 0 \end{array} \right. \end{array}$$

Q.12. The product of two polynomials and their L.C.M. are $x^4 + 5x^3 - x^2 - 17x + 12$ and $x^3 + 6x^2 + 5x - 12$ respectively.

Find their H.C.F.

Sol: $A \times B = x^4 + 5x^3 - x^2 - 17x + 12$

$L = x^3 + 6x^2 + 5x - 12$

$H = ?$

Formula $H = \frac{A \times B}{L}$

$$= \frac{x^4 + 5x^3 - x^2 - 17x + 12}{x^3 + 6x^2 + 5x - 12}$$

$H = x - 1$

Working
 $x - 1$

$x^3 + 6x^2 + 5x - 12$

$ \begin{array}{r} x^4 + 5x^3 - x^2 - 17x + 12 \\ \pm x^4 \pm 6x^3 \pm 5x^2 \mp 12x \\ \hline -x^3 - 6x^2 - 5x + 12 \\ + x^3 + 6x^2 + 5x \pm 12 \\ \hline 0 \end{array} $

Q.13. The product of two polynomials and their H.C.F. are $x^4 - 12x^3 + 53x^2 - 102x + 72$ and $x - 3$ respectively. Find L.C.M.

Sol:

$A \times B = x^4 - 12x^3 + 53x^2 - 102x + 72$

$L = x - 3$

$H = ?$

Formula: $H = \frac{A \times B}{L}$

$$= \frac{x^4 - 12x^3 + 53x^2 - 102x + 72}{(x - 3)}$$

$$H = x^3 - 9x^2 + 26x - 24$$

Working

$$\begin{array}{r}
 x^3 - 9x^2 + 26x - 24 \\
 x - 3 \overline{) \begin{array}{l} x^4 - 12x^3 + 53x^2 - 102x + 72 \\ \underline{+ x^4 + 3x^3} \\ - 9x^3 + 53x^2 - 102x + 72 \\ \underline{+ 9x^3 + 27x^2} \\ 26x^2 - 102x + 72 \\ \underline{+ 26x^3 + 78x} \\ - 24x + 72 \\ \underline{+ 24x + 72} \\ 0 \end{array}}
 \end{array}$$

Q.14. The product of two polynomials and their H.C.F. is $x^4 - 5x^3 + 2x^2 + 20x - 24$ and $x + 2$ respectively. Find their L.C.M.

Sol:

$$A \times B = x^4 - 5x^3 + 2x^2 + 20x - 24$$

$$H = x + 2$$

$$L = ?$$

$$\text{Formula } L = \frac{A \times B}{H}$$

$$= \frac{x^4 - 5x^3 + 2x^2 + 20x - 24}{(x + 2)}$$

$$H = x^3 - 7x^2 + 16x - 12$$

Working

$$\begin{array}{r}
 x^3 - 7x^2 + 16x - 12 \\
 x + 2 \overline{) \begin{array}{l} x^4 - 5x^3 + 2x^2 + 20x - 24 \\ \underline{+ x^4 + 2x^3} \\ - 7x^3 + 2x^2 + 20x - 24 \end{array}}
 \end{array}$$

$$\mp 7x^3 \mp 14x^2$$

$$+ 16x^2 + 20x - 24$$

$$\pm 16x^3 \pm 32x$$

$$- 12x - 24$$

$$\mp 12x \mp 24$$

$$0$$

Q.15. One algebraic expression is $x^3 + 3x^2 - 4x - 12$ and $x^3 + 5x^2 - 4x - 20$ other one is . Their H.C.F is $x^2 - 4$. Find their L.C.M.

Sol:

$$A = x^3 + 3x^2 - 4x - 12$$

$$B = x^3 + 5x^2 - 4x - 20$$

$$H = x^2 - 4$$

$$L = ?$$

$$\text{Formula: } L = \frac{A \times B}{H}$$

$$= \frac{(x^3 + 3x^2 - 4x - 12)(x^3 + 5x^2 - 4x - 20)}{(x^2 - 4)}$$

$$= (x + 3)(x^3 + 5x^2 - 4x - 20)$$

$$= x^4 + 8x^3 + 11x^2 - 32x - 60$$

Working

$$x + 3$$

$$\begin{array}{r} x^2 - 4 \overline{) \begin{array}{l} x^3 + 3x^2 - 4x - 12 \\ \underline{\pm x^3} \quad \mp 4x \\ 3x^2 \quad - 12 \\ \underline{\pm 3x^2} \quad \mp 12 \\ 0 \end{array}} \end{array}$$

Q.16. One algebraic expression is $x^3 - x^2 + 2x - 2$ and other one is $x^3 - x^2 - 2x + 2$. Their H.C.F is $x - 1$. Find their L.C.M.

Sol:

$$A = x^3 - x^2 + 2x - 2$$

$$B = x^3 - x^2 - 2x + 2$$

$$H = x - 1$$

$$L = ?$$

$$\text{Formula: } L = \frac{A \times B}{H}$$

$$\begin{aligned} & (x^2 + 2) \\ &= \frac{(x^3 - x^2 + 2x - 2)(x^3 - x^2 - 2x + 2)}{(x - 1)} \\ &= (x^2 + 2)(x^3 - x^2 - 2x + 2) \\ &= x^5 - x^4 - 2x + 4 \end{aligned}$$

Working

$$x^2 + 2$$

$$\begin{array}{r} x-1 \overline{) \begin{array}{l} x^3 - x^2 + 2x - 2 \\ \underline{+ x^3 - x^2} \\ 2x^2 - 2 \\ \underline{+ 2x^2 - 2} \\ 0 \end{array}} \end{array}$$

Q.17. Prove that $H^3 + L^3 = A^3 + B^3$ where $H + L = A + B$ 'H' and 'L' stand for H.C.F and L.C.M respectively and 'A, B' represent two polynomials.

Sol:

Proof: We know that: _____

$$H + L = A + B$$

$$(H + L)^3 = (A + B)^3 \quad \text{Taking cube}$$

$$H^3 + L^3 + 3HL(H + L) = A^3 + B^3 + 3AB(A + B)$$

$$H^3 + L^3 = A^3 + B^3 + 3AB(A + B) - 3HL(H + L) \quad (i)$$

Now $H + L = A + B$

and $H \times L = A \times B$

Putting values in (i) $H + L$ and HL

$$H^3 + L^3 = A^3 + B^3 + 3AB(A + B) - 3AB(A + B)$$

$$H^3 + L^3 = A^3 + B^3 \quad \text{Proved}$$

Exercise 3.5

Simplify

Q.1 $\frac{1}{a} + \frac{2}{a+1} - \frac{3}{a+2}$

Sol: $= \frac{1(a+1)(a+2) + 2(a)(a+2) - 3(a)(a+1)}{(a)(a+1)(a+2)}$

$$= \frac{a^2 + 3a + 2 + 2a^2 + 4a - 3a^2 - 3a}{(a)(a+1)(a+2)}$$

$$= \frac{4a + 2}{(a)(a+1)(a+2)}$$

$$= \frac{2(2a + 1)}{(a)(a+1)(a+2)}$$

Q.2 $\frac{2a}{(x-2a)} - \frac{x-a}{x^2-5ax+6a^2} + \frac{2}{x-3a}$

Sol: $= \frac{2a}{(x-2a)} - \frac{x-a}{x^2-2ax-3ax+6a^2} + \frac{2}{x-3a}$

$$\begin{aligned}
&= \frac{2a}{(x-2a)} - \frac{x-a}{x(x-2a)-3a(ax-2a)} + \frac{2}{x-3a} \\
&= \frac{2a}{(x-2a)} - \frac{x-a}{(x-2a)(x-3a)} + \frac{2}{x-3a} \\
&= \frac{2a(x-3a) - (x-a) + 2(x-2a)}{(x-2a)(x-3a)} \\
&= \frac{2ax - 6a^2 - x + a + 2x - 4a}{(x-2a)(x-3a)} \\
&= \frac{2ax + x - 3a - 6a^2}{(x-2a)(x-3a)}
\end{aligned}$$

Q.3 $\frac{1}{a^2+1} - \frac{a^4}{a^2+1} + \frac{a^6}{a^2-1} - \frac{1}{a^2-1}$

Sol: $= \frac{1(a^2-1) - a^4(a^2-1) + a^6(a^2+1) - 1(a^2+1)}{(a^2+1)(a^2-1)}$

$$= \frac{a^2 - 1 - a^6 + a^4 + a^8 + a^6 - a^2 - 1}{(a^2+1)(a^2-1)}$$

$$= \frac{a^8 + a^4 - 2}{(a^2+1)(a^2-1)}$$

$$= \frac{a^8 + 2a^4 - a^4 - 2}{(a^2+1)(a^2-1)}$$

$$= a^4(a^4+2) - 1(a^4+2)$$

$$= \frac{(a^4+2)(a^4-1)}{(a^2-1)}$$

$$= a^4 + 2$$

Q.4 $\frac{1}{x^2+x+1} - \frac{1}{x^2-x+1} + \frac{2x+1}{x^4+x^2+1}$

Sol:
$$= \frac{1}{x^2+x+1} - \frac{1}{x^2-x+1} + \frac{2x+1}{(x^2+x+1)(x^2-x+1)}$$
$$= \frac{1(x^2-x+1) - 1(x^2+x+1) + 2x+1}{(x^2+x+1)(x^2-x+1)}$$
$$= \frac{x^2-x+1-x^2-x-1+2x+1}{(x^2+x+1)(x^2-x+1)}$$
$$= \frac{1}{(x^2+x+1)(x^2-x+1)}$$
$$= \frac{1}{(x^4+x^2+1)}$$

Q.5 $\frac{a^2(b-c)}{(a+b)(a+c)} - \frac{b^2(c-a)}{(b+c)(b+a)} + \frac{c^2(a-b)}{(c+a)(c+b)}$

Sol:
$$= \frac{a^2(b-c)(b+c) - b^2(c-a)(c+a) + c^2(a-b)(a+b)}{(a+b)(a+c)(b+c)}$$
$$= \frac{a^2(b^2-c^2) - b^2(c^2-a^2) + c^2(a^2-b^2)}{(a+b)(a+c)(b+c)}$$
$$= \frac{a^2b^2 - a^2c^2 - b^2c^2 + a^2b^2 + a^2c^2 - b^2c^2}{(a+b)(a+c)(b+c)}$$
$$= \frac{2a^2b^2 - 2b^2c^2}{(a+b)(a+c)(b+c)}$$
$$= \frac{2b^2(a^2-c^2)}{(a+b)(a+c)(b+c)}$$
$$= \frac{2b^2(a+c)(a-c)}{(a+b)(a+c)(b+c)}$$
$$= \frac{2b^2(a-c)}{(a+b)(b+c)}$$

Q.6 $\frac{1}{x-1} + \frac{1}{x+1} - \frac{x+2}{x^2+x+1} - \frac{x-2}{x^2-x+1}$

Sol: Changing order

$$\begin{aligned}
 &= \left(\frac{1}{x-1} - \frac{x+2}{x^2+x+1} \right) + \left(\frac{1}{x+1} - \frac{x-2}{x^2-x+1} \right) \\
 &= \frac{(x^2+x+1) - (x-1)(x+2)}{(x-1)(x^2+x+1)} + \frac{(x^2-x+1) - (x-2)(x+1)}{(x+1)(x^2-x+1)} \\
 &= \frac{(x^2+x+1) - (x^2+x-2)}{x^3-1} + \frac{(x^2-x+1) - (x^2-x-2)}{x^3+1} \\
 &= \frac{x^2+x+1-x^2-x+2}{x^3-1} + \frac{x^2-x+1-x^2+x+2}{x^3+1} \\
 &= \frac{3}{x^3-1} + \frac{3}{x^3+1} \\
 &= \frac{3(x^3+1) + 3(x^3-1)}{(x^3-1)(x^3+1)} \\
 &= \frac{3x^3+3+3x^3-3}{x^6-1} \\
 &= \frac{6x^3}{x^6-1}
 \end{aligned}$$

Q.7 $\frac{a^2+ab+b^2}{a+b} + \frac{a^2-ab+b^2}{a-b}$

Sol:
$$\begin{aligned}
 &= \frac{(a-b)(a^2+ab+b^2) + (a+b)(a^2-ab+b^2)}{(a+b)(a-b)} \\
 &= \frac{(a^3-b^3) + (a^3+b^3)}{a^2-b^2} \\
 &= \frac{a^3-b^3+a^3+b^3}{a^2-b^2} \\
 &= \frac{2a^3}{a^2-b^2}
 \end{aligned}$$

$$\text{Q.8} \quad \frac{x^4 - y^4}{x^2 - 2xy + y^2} \times \frac{x - y}{x(x + y)} \div \frac{x^2 + y^2}{x}$$

$$\begin{aligned} \text{Sol:} &= \frac{(x^2 + y^2)(x^2 - y^2)}{(x - y)^2} \times \frac{x - y}{x(x + y)} \times \frac{x}{(x^2 + y^2)} \\ &= \frac{(x + y)(x - y)}{(x - y)(x - y)} \times \frac{(x - y)}{x(x + y)} \times x \\ &= 1 \end{aligned}$$

$$\text{Q.9} \quad \frac{x^2 - 1}{x^2 + x - 2} \times \frac{x^3 + 8}{x^2 + 4x^2 + 16} \div \frac{x^2 + x}{x^3 + 2x^2 + 4x}$$

$$\begin{aligned} \text{Sol:} &= \frac{(x + 1)(x - 1)}{x^2 + 2x - x - 2} \times \frac{(x)^3 + (2)^3}{x^4 + 4x^2 + 16} \times \frac{x^3 + 2x^2 + 4x}{x^2 + x} \\ &= \frac{(x + 1)(x - 1)}{x(x + 2) - 1(x + 2)} \times \frac{(x + 2)(x^2 - 2x + 4)}{x^4 + 8x^2 + 16 - 4x^2} \times \frac{x(x^2 + 2x + 4)}{x(x + 1)} \end{aligned}$$

(Completing square)

$$\begin{aligned} &= \frac{(x + 1)(x - 1)}{(x + 2)(x - 1)} \times \frac{(x + 2)(x^2 - 2x + 4)}{(x^2 + 4)^2 - (2x)^2} \times \frac{x^2 + 2x + 4}{x + 1} \\ &= \frac{(x^2 - 2x + 4)(x^2 + 2x + 4)}{(x^2 + 4 - 2x)(x^2 + 4 + 2x)} \\ &= \frac{(x^2 - 2x + 4)(x^2 + 2x + 4)}{(x^2 - 2x + 4)(x^2 + 2x + 4)} \\ &= 1 \end{aligned}$$

$$\text{Q.10} \quad \frac{a^3 + 64b^3}{a^2 + 20ab + 64b^2} \div \frac{a^2 - 4ab + 16b^2}{a^2 + 4ab + 16b^2} \times \frac{a^2 + 12ab - 64b^2}{a^3 - 64b^3}$$

Sol:

$$\begin{aligned} &= \frac{(a)^3 + (4b)^3}{a^2 + 4ab + 16ab + 64b^2} \times \frac{a^2 + 4ab + 16b^2}{a^2 - 4ab + 16b^2} \times \frac{a^2 + 12ab - 64b^2}{(a)^3 - (4b)^3} \\ &= \frac{(a + 4b)(a^2 - 4ab + 16b^2)}{a(a + 4b) + 16b(a + 4b)} \times \frac{a^2 + 4ab + 16b^2}{a^2 - 4ab + 16b^2} \times \frac{a(a + 16b) - 4b(a + 16b)}{(a - 4b)(a^2 + 4ab + 16b^2)} \\ &= \frac{(a + 4b)(a^2 - 4ab + 16b^2)}{(a + 4b)(a + 16b)} \times \frac{a^2 + 4ab + 16b^2}{a^2 - 4ab + 16b^2} \times \frac{(a + 16b)(a - 4b)}{(a - 4b)(a^2 + 4ab + 16b^2)} \\ &= 1 \end{aligned}$$

$$\text{Q.11} \quad \frac{a}{(a+b)^2 - 2ab} \times \frac{a^4 - b^4}{(a+b)^3 - 3ab(a+b)} + \frac{(a+b)^2 - 4ab}{(a+b)^2 - 3ab}$$

$$\text{Sol:} = \frac{a}{a^2 + b^2 + 2ab - 2ab} \times \frac{(a^2)^2 - (b^2)^2}{a^3 + b^3 + 3ab(a+b) - 3ab(a+b)} \times \frac{(a+b)^2 - 3ab}{(a+b)^2 - 4ab}$$

$$= \frac{a}{(a^2 + b^2)} \times \frac{(a^2 + b^2)(a^2 - b^2)}{a^3 + b^3} \times \frac{a^2 + b^2 + 2ab - 3ab}{a^2 + b^2 + 2ab - 4ab}$$

$$= a \times \frac{(a+b)(a-b)}{(a+b)(a^2 - ab + b^2)} \times \frac{(a^2 + b^2 - ab)}{a^2 + b^2 - 2ab}$$

$$= \frac{a \times (a-b)}{a^2 - 2ab + b^2} = \frac{a(a-b)}{(a-b)^2}$$

$$= \frac{a(a-b)}{(a-b)(a-b)}$$

$$= \frac{a}{a-b}$$

$$\text{Q.12} \quad \frac{a^2 - 1}{a^2 - a - 2} + \frac{a^2 + 5a + 6}{a^2 - 5a + 6} + \frac{a^2 - 4a + 3}{a^2 + 4a + 3}$$

$$\text{Sol:} = \frac{(a)^2 - (1)^2}{a^2 - 2a + a - 2} \times \frac{a^2 - 5a + 6}{a^2 + 5a + 6} \times \frac{a^2 + 4a + 3}{a^2 - 4a + 3}$$

$$= \frac{(a-1)(a+1)}{a(a-2)+1(a-2)} \times \frac{a^2 - 2a - 3a + 6}{a^2 + 2a + 3a + 6} \times \frac{a^2 + a + 3a + 3}{a^2 - a - 3a + 3}$$

$$= \frac{(a-1)(a+1)}{(a-2)(a+1)} \times \frac{a(a-2)-3(a-2)}{a(a+2)+3(a+2)} \times \frac{a(a+1)+3(a+1)}{a(a-1)-3(a-1)}$$

$$= \frac{(a-1)(a+1)}{(a-2)(a+1)} \times \frac{(a-2)(a-3)}{(a+2)(a+3)} \times \frac{(a+1)(a+3)}{(a-1)(a-3)}$$

$$= \frac{a+1}{a+2}$$

Exercise 3.6

Find the Square Root of the Following.

Q.1 $16x^2 + 24xy + 9y^2$

Sol:
$$\begin{aligned} &= 16x^2 + 12xy + 12xy + 9y^2 \\ &= (16x^2 + 12xy) + (12xy + 9y^2) \\ &= 4x(4x + 3y) + 3y(4x + 3y) \\ &= (4x + 3y)(4x + 3y) \\ &= (4x + 3y)^2 \end{aligned}$$

$$\begin{aligned} \sqrt{16x^2 + 24xy + 9y^2} &= \sqrt{(4x + 3y)^2} \\ &= \pm(4x + 3y) \end{aligned}$$

Q.2 $(x^2 - 7x + 12)(x^2 - 9x + 20)(x^2 - 8x + 15)$

Sol:
$$\begin{aligned} &= [x^2 - 3x - 4x + 12][x^2 - 4x - 5x + 20][x^2 - 3x - 5x + 15] \\ &= [(x^2 - 3x) - (4x - 12)][(x^2 - 4x) - (5x - 20)][(x^2 - 3x) - (5x - 15)] \\ &= [x(x - 3) - 4(x - 3)][x(x - 4) - 5(x - 4)][x(x - 3) - 5(x - 5)] \\ &= (x - 3)(x - 4)(x - 4)(x - 5)(x - 3)(x - 5) \\ &= (x - 3)^2(x - 4)^2(x - 5)^2 \end{aligned}$$

Now we will take square root.

$$\begin{aligned} &= \sqrt{(x - 3)^2(x - 4)^2(x - 5)^2} \\ &= \pm(x - 3)(x - 4)(x - 5) \end{aligned}$$

Q.3 $(x^2 + 8x + 7)(2x^2 - x - 3)(2x^2 + 11x - 21)$

Sol:
$$\begin{aligned} &= (x^2 + x + 7x + 7)(2x^2 - 3x + 2x - 3)(2x^2 + 14x - 3x - 21) \\ &= [(x^2 + x) + (7x + 7)][(2x^2 - 3x) + (2x - 3)][(2x^2 + 14x) - (3x + 21)] \\ &= [x(x + 1) + 7(x + 1)][x(2x - 3) + 1(2x - 3)][2x(x + 7) - 3(x + 7)] \\ &= [(x + 1)(x + 7)][(2x - 3)(x + 1)][(x + 7)(2x - 3)] \\ &= (x + 1)^2(x + 7)^2(2x - 3)^2 \\ &= \sqrt{(x + 1)^2(x + 7)^2(2x - 3)^2} \quad (\text{Taking square root}) \\ &= \pm(x + 1)(x + 7)(2x - 3) \end{aligned}$$

Q.4 $x(x+2)(x+4)(x+6)+16$

Sol: $= x(x+6)(x+2)(x+4)+16$ (Rearranging)
 $= [x(x+6)][(x+2)(x+4)]+16$
 $= (x^2+6x)(x^2+6x-8)+16$(i)

Let $x^2+6x=y$

$= (y)(y+8)+16$ from (i)
 $= y^2+8y+16$
 $= y^2+4y+4y+16$
 $= (y^2+4y)+(4y+16)$
 $= y(y+4)+4(y+4)$
 $= (y+4)(y+4)$
 $= (y+4)^2$

Putting values of y in x^2+6x

$= (x^2+6x+4)^2$
 $= \sqrt{(x^2+6x+4)^2}$ (Taking square root)
 $= \pm(x^2+6x+4)$

Q.5 $(2x+1)(2x+3)(2x+5)(2x+7)+16$

Sol: (Rearranging)
 $= (2x+1)(2x+7)(2x+3)(2x+5)+16$
 $= [(2x+1)(2x+7)][(2x+3)(2x+5)]+16$
 $= [4x^2+16x+7][(4x^2+16x+15)]+16$

Let $4x^2+16x=y$

$= (y+7)(y+15)+16$
 $= y^2+22y+105+16$
 $= y^2+22y+121$
 $= y^2+11y+11y+121$
 $= (y^2+11y)+(11y+121)$

$$= y(y + 11) + 11(y + 11)$$

$$= (y + 11)(y + 11)$$

$$= (y + 11)^2$$

Putting value of y

$$= (4x^2 + 16x + 11)^2$$

$$= \sqrt{(4x^2 + 16x + 11)^2} \quad (\text{Taking square root})$$

$$= \pm(4x^2 + 16x + 11)$$

Q.6 $\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 27 ; x \neq 0$

Sol: $\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 27 \quad (i)$

Let $x + \frac{1}{x} = y$

$$x^2 + \frac{1}{x^2} + 2 = y^2 \quad (\text{Squaring both sides.})$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

Putting values $x + \frac{1}{x} = y$ and $x^2 + \frac{1}{x^2} = y^2 - 2$ in (i).

$$= y^2 - 2 - 10y + 27$$

$$= y^2 - 10y + 25$$

$$= y^2 - 5y - 5y + 25$$

$$= y(y - 5) - 5(y - 5)$$

$$= (y - 5)(y - 5)$$

$$= (y - 5)^2$$

$$= \left(x + \frac{1}{x} - 5\right)^2$$

$$= \sqrt{\left(x + \frac{1}{x} - 5\right)^2} \quad (\text{Taking square root})$$

$$= \pm \left(x + \frac{1}{x} - 5 \right)$$

Q.7 $\left(t - \frac{1}{t} \right)^2 - 4 \left(t + \frac{1}{t} \right) + 8, (t \neq 0)$

Sol: Let $t + \frac{1}{t} = y$

We know that

$$\begin{aligned} \left(t - \frac{1}{t} \right)^2 &= \left(t + \frac{1}{t} \right)^2 - 4t \times \frac{1}{t} \\ &= y^2 - 4 \end{aligned}$$

Now $t + \frac{1}{t} = y$ and $\left(t + \frac{1}{t} \right)^2 = y^2 - 4$ putting in given values

$$\begin{aligned} &= y^2 - 4 - 4y + 8 \\ &= y^2 - 4y + 4 \\ &= y^2 - 2y - 2y + 4 \\ &= y(y - 2) - 2(y - 2) \\ &= (y - 2)(y - 2) \\ &= (y - 2)^2 \end{aligned}$$

Putting value of y .

$$\begin{aligned} &= \left(t + \frac{1}{t} - 2 \right)^2 \\ &= \sqrt{\left(t + \frac{1}{t} - 2 \right)^2} \quad (\text{Taking square root}) \\ &= \pm \left(t + \frac{1}{t} - 2 \right) \end{aligned}$$

Q.8 $\left(x^2 + \frac{1}{x^2} \right)^2 - 4 \left(x + \frac{1}{x} \right)^2 + 12; x \neq 0$

Sol: Let $x + \frac{1}{x} = y$

$$x + \frac{1}{x} = y$$

Squaring both sides

$$\left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2 \dots\dots\dots (i)$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

Putting $x + \frac{1}{x} = y$ and $x^2 + \frac{1}{x^2} = y^2 - 2$ in the given statement.

$$= (y^2 - 2)^2 - 4y^2 + 12$$

$$= y^4 - 4y^2 + 4 - 4y^2 + 12$$

$$= y^4 - 8y^2 + 16$$

$$= y^4 - 4y^2 - 4y^2 + 16$$

$$= y^2(y^2 - 4) - 4(y^2 - 4)$$

$$= (y^2 - 4)(y^2 - 4)$$

$$= (y^2 - 4)^2$$

From (i), putting values $y^2 = x^2 + \frac{1}{x^2} + 2$ •

$$= \left(x^2 + \frac{1}{x^2} + 2 - 4\right)^2$$

$$= \left(x^2 + \frac{1}{x^2} - 2\right)^2$$

$$= \sqrt{\left(x^2 + \frac{1}{x^2} - 2\right)^2} \quad \text{(Taking square root)}$$

$$= \pm \left(x^2 + \frac{1}{x^2} - 2\right)$$

Q.9 $4x^4 + 12x^3 + 25x^2 + 24x + 16$

Sol: $2x^2 + 3x + 4$

$2x^2$	$4x^4 + 12x^3 + 25x^2 + 24x + 16$ $\pm 4x^4$
$4x^2 + 3x$	$12x^3 + 25x^2 + 24x + 16$ $\pm 12x^3 \pm 9x^2$
$4x^2 + 6x + 4$	$+ 16x^2 + 24x + 16$ $\pm 16x^2 \pm 24x \pm 16$
	0

Square root $= \pm(2x^2 + 3x + 4)$

Q.10 $\frac{9x^2}{4y^2} - \frac{3x}{2y} - \frac{7}{4} + \frac{2y}{3x} + \frac{4y^2}{9x^2} \quad (x \neq 0, y \neq 0)$

Sol:

$$\text{Square root} = \pm \left(\frac{3x}{2y} - \frac{1}{2} - \frac{2y}{3x} \right)$$

Q.11. For what value of x , $x^4 + 4x^2 + x + \frac{8}{x^2} + \frac{4}{x^4}$ is a complete square, where $x \neq 0$

Sol: First we will find square root

$$\begin{array}{r}
 x^2 + 2 + \frac{2}{x^2} \\
 \hline
 x^2 \left| \begin{array}{l} x^4 + 4x^2 + x + \frac{8}{x^2} + \frac{4}{x^4} \\ \pm x^4 \end{array} \right. \\
 \hline
 2x^2 + 2 \left| \begin{array}{l} 4x^2 + x + \frac{8}{x^2} + \frac{4}{x^4} \\ \pm 4x^2 \pm 4 \end{array} \right. \\
 \hline
 2x^2 + 4 + \frac{2}{x^2} \left| \begin{array}{l} -4 + x + \frac{8}{x^2} + \frac{4}{x^4} \\ \pm 4 \quad \pm \frac{8}{x^2} \pm \frac{4}{x^4} \end{array} \right. \\
 \hline
 \phantom{2x^2 + 4 + \frac{2}{x^2}} \left| \begin{array}{l} -8 + x \end{array} \right.
 \end{array}$$

For completing square remainder must be zero.

Therefore $-8 + x = 0$

$\Rightarrow x = 8$

Q.12. If $x^4 + lx^3 + mx^2 + 12x + 9$ is a complete square then find the values of l and m .

Sol:

$$\begin{array}{r}
 x^2 + 2x + 3 \\
 \hline
 x^2 \left| \begin{array}{l} x^4 + lx^3 + mx^2 + 12x + 9 \\ \pm x^4 \end{array} \right. \\
 \hline
 2x^2 + 2x \left| \begin{array}{l} lx^3 + mx^2 + 12x + 9 \\ \pm 4x^3 \pm 4x^2 \end{array} \right. \\
 \hline
 2x^2 + 4x + 3 \left| \begin{array}{l} (l-4)x^3 + mx^2 - 4x^2 + 12x + 9 \\ \pm 6x^2 \pm 12x \pm 9 \end{array} \right. \\
 \hline
 \left| \begin{array}{l} (l-4)x^3 + mx^2 - 10x^2 \end{array} \right.
 \end{array}$$

For completing square remainder must be zero

$$(l - 4)x^3 + (m - 10)x^2 = 0$$

Therefore, $l - 4 = 0$ and $m - 10 = 0$

$$l = 4, m = 10$$
