

Exercise 2.1

Factorize:

Q.1 $3a(x + y) - 7b(x + y)$

Sol: $= (x + y)(3a - 7b)$

Q.2 $ax + ay - x^2 - xy$

Sol: $= (ax + ay) - (x^2 + xy)$

$$= a(x + y) - x(x + y)$$

$$= (x + y)(a - x)$$

Q.3 $a^3 + a - 3a^2 - 3$

Sol: $= (a^3 + a) - (3a^2 + 3)$

$$= a(a^2 + 1) - 3(a^2 + 1)$$

$$= (a^2 + 1)(a - 3)$$

Q.4 $x^3 + y - xy - x$

Sol: $= x^3 - x - xy + y$ (writing in order)

$$= x(x^2 - 1) - y(x - 1)$$

$$= x(x - 1)(x + 1) - y(x - 1)$$

$$= (x - 1)[x(x + 1) - y]$$

$$= (x - 1)(x^2 + x - y)$$

Q.5 $3ax + 6ay - 8bx - 4by$

Sol: $= 3ax - 4bx + 6ay - 8by$ (writing in order)

$$= x(3a - 4b) + 2y(3a - 4b)$$

$$= (3a - 4b)(x + 2y)$$

Q.6 $2a^2 - bc - 2ab + ac$

Sol: $= 2a^2 + ac - 2ab - bc$ (writing in order)

$$= a(2a + c) - b(2a + c)$$

$$= (2a + c)(a - b)$$

$$= (a - b)(2a + c)$$

Q.7 $a(a - b + c) - bc$

Sol: $= a^2 - ab + ca - bc$

$$= a(a - b) + c(a - b)$$

$$= (a - b)(a + c)$$

Q.8 $8 - 4a - 2a^3 + a^4$

Sol: $= 4(2 - a) - a^3(2 - a)$
 $= (2 - a)(4 - a^3)$
 $= (4 - a^3)(2 - a)$

Q.9 $16x^2 - 24xa + 9a^2$

Sol: $= 16x^2 - 12xa - 12xa + 9a^2$
 $= 4x(4x - 3a) - 3a(4x - 3a)$
 $= (4x - 3a)(4x - 3a)$
 $= (4x - 3a)^2$

Q.10 $1 - 14x + 49x^2$

Sol: $= (1)^2 - 2(1)(7x) + (7x)^2$
 $= (1 - 7x)^2$

Q.11 $20x^2 + 5 - 20x$

Sol: $= 20x^2 - 20x + 5$
 $= 5[4x^2 - 4x + 1]$
 $= 5[(2x)^2 - 2(2x)(1) + (1)^2]$
 $= 5(2x - 1)^2$

Q.12 $2a^3b + 2ab^3 - 4a^2b^2$

Sol: $= 2ab[a^2 + b^2 - 2ab]$
 $= 2ab[a^2 - 2ab + b^2]$
 $= 2ab[(a)^2 - 2(a)(b) + (b)^2]$
 $= 2ab(a - b)^2$

Q.13 $x^2 + x + \frac{1}{4}$

Sol: $= (x)^2 + 2(x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2$
 $= \left(x + \frac{1}{2}\right)^2$

Q.14 $x^2 + \frac{1}{x^2} - 2$

Sol:
$$= x^2 - 2 + \frac{1}{x^2}$$

$$= (x)^2 - 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2$$

$$= \left(x - \frac{1}{x}\right)^2$$

Q.15 $5x^3 - 30x^2 + 45x$

Sol:
$$= 5x[x^2 - 6x + 9]$$

$$= 5x[(x)^2 - 2(x)(3) + (3)^2]$$

$$= 5x(x - 3)^2$$

Q.16 $a^2 + b^2 + 2ab + 2bc + 2ac$

Sol:
$$= (a^2 + 2ab + b^2) + 2ac + 2bc$$

$$= [(a)^2 + 2(a)(b) + (b)^2] + 2c(a + b)$$

$$= (a + b)^2 + 2c(a + b)$$

$$= (a + b)[(a + b) + 2c]$$

$$= (a + b)(a + b + 2c)$$

Exercise 2.2

Resolve into factors.

Q.1 $x^2 + 2xy + y^2 - a^2$

Sol:
$$= (x^2 + 2xy + y^2) - a^2$$

$$= [(x)^2 + 2(x)(y) + (y)^2] - (a)^2$$

$$= (x + y)^2 - (a)^2$$

$$= (x + y + a)(x + y - a)$$

Q.2 $4a^2 - 4ab + b^2 - 9c^2$

Sol:
$$= (4a^2 - 4ab + b^2) - 9c^2$$

$$= [(2a)^2 - 2(2a)(b) + (b)^2] - (3c)^2$$

$$= (2a - b)^2 - (3c)^2$$

$$= (2a - b + 3c)(2a - b - 3c)$$

Q.3 $x^2 + 6ax + 9a^2 - 16b^2$

Sol: $= (x^2 + 6ax + 9a^2) - 16b^2$

$$= [(x)^2 + 2(x)(3a) + (3a)^2] - (4b)^2$$

$$= (x + 3a)^2 - (4b)^2$$

$$= (x + 3a + 4b)(x + 3a - 4b)$$

Q.4 $y^2 - c^2 + 2cx - x^2$

Sol: $= y^2 - (c^2 - 2cx + x^2)$

$$= (y)^2 - [(c)^2 - 2(c)(x) + (x)^2]$$

$$= (y)^2 - (c - x)^2 \quad \text{Formula: } [a^2 - b^2 = (a + b)(a - b)]$$

$$= [y + (c - x)][y - (c - x)]$$

$$= (y + c - x)(y - c + x)$$

$$= (y - x + c)(y + x - c)$$

Q.5 $x^2 + y^2 + 2xy - 4x^2y^2$

Sol: $= (x^2 + 2xy + y^2) - 4x^2y^2$

$$= [(x)^2 + 2(x)(y) + (y)^2] - (2xy)^2$$

$$= (x + y)^2 - (2xy)^2$$

$$= (x + y + 2xy)(x + y - 2xy)$$

Q.6 $a^2 - 4ab + 4b^2 - 9a^2c^2$

Sol: $= (a^2 - 4ab + 4b^2) - (9a^2c^2)$

$$= [(a)^2 - 2(a)(2b) + (2b)^2] - [3ac]^2$$

$$= (a - 2b)^2 - (3ac)^2$$

$$= (a - 2b + 3ac)(a - 2b - 3ac)$$

Q.7 $x^2 - 2xy + y^2 - a^2 + 2ab - b^2$

Sol: $= (x^2 - 2xy + y^2) - (a^2 - 2ab + b^2)$

$$= [(x)^2 - 2(x)(y) + (y)^2] [(a)^2 - 2(a)(b) + (b)^2]$$

$$= (x - y)^2 - (a - b)^2$$

$$= [(x - y) + (a - b)][(x - y) - (a - b)]$$

$$= (x - y + a - b)(x - y - a + b)$$

Q.8 $y^4 + 4$

Sol: $= (y^2)^2 + (2)^2 + (2)(y^2)(2) - 2(y^2)(2)(\text{completing square root})$

$$\begin{aligned}
 &= [(y^2)^2 + (2)^2 + 4y^2] - 4y^2 \\
 &= (y^2 + 2)^2 - (2y)^2 \\
 &= (y^2 + 2 + 2y)(y^2 + 2 - 2y) \\
 &= (y^2 + 2y + 2)(y^2 - 2y + 2)
 \end{aligned}$$

Q.9 $z^4 + 64y^4$

Sol:
$$\begin{aligned}
 &= (z^2)^2 + (8y^2)^2 + 2(z^2)(8y^2) - 2(z^2)(8y^2) \text{ (completing square root)} \\
 &= (z^2 + 8y^2)^2 - 16z^2y^2 \\
 &= (z^2 + 8y^2)^2 - (4zy)^2 \\
 &= (z^2 + 8y^2 - 4zy)(z^2 + 8y^2 + 4zy)
 \end{aligned}$$

Q.10 $x^4 + 324$

Sol:
$$\begin{aligned}
 &= (x^2)^2 + (18)^2 + 2(x^2)(18) - 2(x^2)(18) \text{ (completing square root)} \\
 &= (x^2 + 18)^2 - 36x^2 \\
 &= (x^2 + 18)^2 - (6x)^2 \\
 &= (x^2 + 18 + 6x)(x^2 + 18 - 6x) \\
 &= (x^2 + 6x + 18)(x^2 - 6x + 18)
 \end{aligned}$$

Q.11 $z^4 - z^2 + 16$

Sol:
$$\begin{aligned}
 &= (z^2)^2 + (4)^2 + 2(z^2)(4) - 9z^2 \text{ (completing square root)} \\
 &= (z^2 + 4)^2 - (3z)^2 \\
 &= (z^2 + 4 - 3z)(z^2 + 4 + 3z) \\
 &= (z^2 - 3z + 4)(z^2 + 3z + 4)
 \end{aligned}$$

Q.12 $4x^4 - 5x^2y^2 + y^4$

Sol:
$$\begin{aligned}
 &= (2x^2)^2 - 5x^2y^2 + (y^2)^2 \text{ (completing square root)} \\
 &= (2x^2)^2 + 2(2x^2)(y^2) + (y^2)^2 - 9x^2y^2 \text{ (completing square root)} \\
 &= (2x^2 + y^2)^2 - (3xy)^2 \\
 &= (2x^2 + y^2 - 3xy)(2x^2 + y^2 + 3xy)
 \end{aligned}$$

Exercise 2.3

Factorize:

Q.1 $x^2 + 9x + 20$

Sol:
$$\begin{aligned}
 &= x^2 + 4x + 5x + 20 \\
 &= (x^2 + 4x) + (5x + 20)
 \end{aligned}$$

$$= x(x + 4) + 5(x + 4)$$

$$= (x + 4)(x + 5)$$

Q.2 $x^2 + 5x - 14$

Sol: $= x^2 + 7x - 2x - 14$

$$= (x^2 + 7x) - (2x + 14)$$

$$= x(x + 7) - 2(x + 7)$$

$$= (x + 7)(x - 2)$$

Q.3 $x^2 + 5x - 6$

Sol: $= x^2 + 6x - x - 6$

$$= (x^2 + 6x) - (x + 6)$$

$$= x(x + 6) - 1(x + 6)$$

$$= (x + 6)(x - 1)$$

Q.4 $x^2 - 7x + 12$

Sol: $= x^2 - 3x - 4x + 12$

$$= (x^2 - 3x) - (4x - 12)$$

$$= x(x - 3) - 4(x - 3)$$

$$= (x - 3)(x - 4)$$

Q.5 $x^2 - x - 156$

Sol: $= x^2 - 13x + 12x - 156$

$$= (x^2 - 13x) + (12x - 156)$$

$$= x(x - 13) + 12(x - 13)$$

$$= (x - 13)(x + 12)$$

Q.6 $x^2 - x - 2$

Sol: $= x^2 - 2x + x - 2$

$$= (x^2 - 2x) + (x - 2)$$

$$= x(x - 2) + 1(x - 2)$$

$$= (x - 2)(x + 1)$$

Q.7 $x^2 - 9x - 90$

Sol: $= x^2 - 15x + 6x - 90$

$$= (x^2 - 15x) + (6x - 90)$$

$$= x(x - 15) + 6(x - 15)$$

$$= (x - 15)(x + 6)$$

Q.8 $a^2 - 12a - 85$

Sol: $= a^2 - 17a + 5a - 85$
 $= (a^2 - 17a) + (5a - 85)$
 $= a(a - 17) + 5(a - 17)$
 $= (a - 17)(a + 5)$

Q.9 $98 - 7x - x^2$

Sol: $= 98 - 14x + 7x - x^2$
 $= (98 - 14x) + (7x - x^2)$
 $= 14(7 - x) + x(7 - x)$
 $= (7 - x)(14 + x)$

Q.10 $y^2 - 11y - 152$

Sol: $= y^2 - 19y + 8y - 152$
 $= (y^2 - 19y) + (8y - 152)$
 $= y(y - 19) + 8(y - 19)$
 $= (y - 19)(y + 8)$

Q.11 $2x^2 + 3x + 1$

Sol: $= 2x^2 + 2x + x + 1$
 $= (2x^2 + 2x) + (x + 1)$
 $= 2x(x + 1) + 1(x + 1)$
 $= (x + 1)(2x + 1)$

Q.12 $3x^2 + 5x + 2$

Sol: $= 3x^2 + 3x + 2x + 2$
 $= (3x^2 + 3x) + (2x + 2)$
 $= 3x(x + 1) + 2(x + 1)$
 $= (x + 1)(3x + 2)$

Q.13 $2x^2 - x - 1$

Sol: $= 2x^2 - 2x + x - 1$
 $= (2x^2 - 2x) + (x - 1)$
 $= 2x(x - 1) + 1(x - 1)$
 $= (x - 1)(2x + 1)$

Q.14 $6x^2 + 7x - 3$

$$\begin{aligned}\text{Sol: } &= 6x^2 + 9x - 2x - 3 \\ &= (6x^2 + 9x) - (2x + 3) \\ &= 3x(2x + 3) - 1(2x + 3) \\ &= (2x + 3)(3x - 1)\end{aligned}$$

Q.15 $2 - 3x - 2x^2$

$$\begin{aligned}\text{Sol: } &= 2 - 4x + x - 2x^2 \\ &= (2 - 4x) + (x - 2x^2) \\ &= 2(1 - 2x) + x(1 - 2x) \\ &= (1 - 2x)(2 + x)\end{aligned}$$

Q.16 $8 + 6x - 5x^2$

$$\begin{aligned}\text{Sol: } &= 8 + 10x - 4x - 5x^2 \\ &= (8 + 10x) - (4x + 5x^2) \\ &= 2(4 + 5x) - x(4 + 5x) \\ &= (4 + 5x)(2 - x)\end{aligned}$$

Q.17 $3u^2 - 10u + 8$

$$\begin{aligned}\text{Sol: } &= 3u^2 - 6u - 4u + 8 \\ &= (3u^2 - 6u) - (4u - 8) \\ &= 3u(u - 2) - 4(u - 2) \\ &= (u - 2)(3u - 4)\end{aligned}$$

Q.18 $10x^2 - 7x - 12$

$$\begin{aligned}\text{Sol: } &= 10x^2 - 15x + 8x - 12 \\ &= (10x^2 - 15x) + (8x - 12) \\ &= 5x(2x - 3) + 4(2x - 3) \\ &= (2x - 3)(5x + 4)\end{aligned}$$

Q.19 $5x^2 - 32x + 12$

$$\begin{aligned}\text{Sol: } &= 5x^2 - 30x - 2x + 12 \\ &= (5x^2 - 30x) - (2x - 12) \\ &= 5x(x - 6) - 2(x - 6) \\ &= (x - 6)(5x - 2)\end{aligned}$$

Q.20 $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

Sol: $4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$
 $(4\sqrt{3}x^2 + 8x) - (3x + 2\sqrt{3})$
 $4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$
 $= (\sqrt{3}x + 2)(4x - \sqrt{3})$

Formulae

- (i) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
 (ii) $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
 (iii) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
 (iv) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Exercise 2.4

Factorize:

Q.1 $8x^3 - y^3$

Sol: $(2x)^3 - (y)^3$
 $(2x - y)[(2x)^2 + (2x)(y) + (y)^2]$
 $(2x - y)(4x^2 + 2xy + y^2)$

Q.2 $27x^3 + 1$

Sol: $=(3x)^3 + (1)^3$
 $=(3x + 1)[(3x)^2 - (3x)(1) + (1)^2]$
 $=(3x + 1)(9x^2 - 3x + 1)$

Q.3 $1 - 343x^3$

Sol: $=(1)^3 - (7x)^3$
 $=(1 - 7x)[(1)^2 + 1(7x) + (7x)^2]$
 $=(1 - 7x)(1 + 7x + 49x^2)$

Q.4 $a^3b^3 + 512$

Sol: $=(ab)^3 + (8)^3$
 $(ab + 8)[(ab)^2 - (ab)(8) + (8)^2]$
 $=(ab + 8)(a^2b^2 - 8ab + 64)$

Q.5 $27 - 1000y^3$

Sol:
$$\begin{aligned} &= (3)^3 - (10y)^3 \\ &= (3 - 10y)[(3)^2 + (3)(10y) + (10y)^2] \\ &= (3 - 10y)(9 + 30y + 100y^2) \end{aligned}$$

Q.6 $27x^3 - 64y^3$

Sol:
$$\begin{aligned} &= (3x)^3 - (4y)^3 \\ &= (3x - 4y)[(3x)^2 + (3x)(4y) + (4y)^2] \\ &= (3x - 4y)(9x^2 + 12xy + 16y^2) \end{aligned}$$

Q.7 $x^3y^3 + z^3$

Sol:
$$\begin{aligned} &= (xy)^3 + (z)^3 \\ &= (xy + z)[(xy)^2 - (xy)(z) + (z)^2] \\ &= (xy + z)(x^2y^2 - xyz + z^2) \end{aligned}$$

Q.8 $216p^3 - 343$

Sol:
$$\begin{aligned} &= (6p)^3 - (7)^3 \\ &= (6p - 7)[(6p)^2 + (6p)(7) + (7)^2] \\ &= (6p - 7)(36p^2 + 42p + 49) \end{aligned}$$

Q.9 $8x^3 - \frac{1}{27}$

Sol:
$$\begin{aligned} &= (2x)^3 - \left(\frac{1}{3}\right)^3 \\ &= \left(2x - \frac{1}{3}\right) \left[(2x)^2 + (2x)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 \right] \\ &= \left(2x - \frac{1}{3}\right) \left(4x^2 + \frac{2}{3}x + \frac{1}{9} \right) \end{aligned}$$

Q.10 $a^3 + b^3 + a + b$

Sol:
$$\begin{aligned} &= (a^3 + b^3) + (a + b) \\ &= (a + b)(a^2 - ab + b^2) + (a + b) \\ &= (a + b)[(a^2 - ab + b^2 + 1)] \\ &= (a + b)(a^2 - ab + b^2 + 1) \end{aligned}$$

Q.11 $a - b - a^3 + b^3$

Sol:
$$\begin{aligned} &= (a - b) - (a^3 - b^3) \\ &= (a - b) - [(a - b)(a^2 + ab + b^2)] \\ &= (a - b) [1 - (a^2 + ab + b^2)] \\ &= (a - b)(1 - a^2 - ab - b^2) \end{aligned}$$

Q.12 $x - 8xy^3$

Sol:
$$\begin{aligned} &= x(1 - 8y^3) \\ &= x[(1)^3 - (2y)^3] \\ &= x[(1 - 2y)\{(1)^2 + (1)(2y) + (2y)^2\}] \\ &= x[(1 - 2y)(1 + 2y + 4y^2)] \\ &= x(1 - 2y)(1 + 2y + 4y^2) \end{aligned}$$

Q.13 $x^{12} - y^{12}$

Sol:
$$\begin{aligned} &= (x^6)^2 - (y^6)^2 \\ &= (x^6 - y^6)(x^6 + y^6) \\ &= \{(x^3)^2 - (y^3)^2\} \{(x^2)^3 + (y^2)^3\} \\ &= (x^3 - y^3)(x^3 + y^3)(x^2 + y^2)(x^4 - x^2y^2 + y^4) \\ &= (x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)(x^2 + y^2) \\ &\quad (x^4 - x^2y^2 + y^4) \\ &= (x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2) \\ &\quad (x^4 - x^2y^2 + y^4) \end{aligned}$$

Q.14 $1 - \frac{64p^3}{q^3}$

Sol:
$$\begin{aligned} &= (1)^3 - \left(\frac{4p}{q}\right)^3 \\ &= \left(1 - \frac{4p}{q}\right) \left[(1)^2 + (1)\left(\frac{4p}{q}\right) + \left(\frac{4p}{q}\right)^2\right] \\ &= \left(1 - \frac{4p}{q}\right) \left(1 + \frac{4p}{q} + \frac{16p^2}{q^2}\right) \end{aligned}$$

Q.15 $1 + 64u^3$

Sol: $= (1)^3 + (4u)^3$
 $= (1 + 4u)[(1)^2 - (1)(4u) + (4u)^2]$
 $= (1 + 4u)(1 - 4u + 16u^2)$

Q.16 $8x^3 - 6x - 9y + 27y^3$

Sol: $= 8x^3 + 27y^3 - 6x - 9y$
 $= (8x^3 + 27y^3) - (6x + 9y)$
 $= [(2x)^3 + (3y)^3] - 3(2x + 3y)$
 $= (2x + 3y)[(2x)^2 - (2x)(3y) + (3y)^2] - 3(2x + 3y)$
 $= (2x + 3y)(4x^2 - 6xy + 9y^2) - 3(2x + 3y)$
 $= (2x + 3y)(4x^2 - 6xy + 9y^2 - 3)$

Q.17 $z^3 + 125$

Sol: $= (z)^3 + (5)^3$
 $= (z + 5)[(z)^2 - (z)(5) + (5)^2]$
 $= (z + 5)(z^2 - 5z + 25)$

Q.18 $x^9 + y^9$

Sol: $= (x^3)^3 + (y^3)^3$
 $= (x^3 + y^3)[(x^3)^2 - (x^3)(y^3) + (y^3)^2]$
 $= (x^3 + y^3)(x^6 - x^3y^3 + y^6)$
 $= (x + y)(x^2 - xy + y^2)(x^6 - x^3y^3 + y^6)$

Q.19 $m^6 - n^6$

Sol: $= (m^3)^2 - (n^3)^2$
 $= (m^3 + n^3)(m^3 - n^3)$
 $= (m + n)(m^2 - mn + n^2)(m - n)(m^2 + mn + n^2)$
 $= (m + n)(m - n)(m^2 - mn + n^2)(m^2 + mn + n^2)$

Q.20 $64x^7 - xa^6$

Sol: $= x(64x^6 - a^6)$
 $= x\{(8x^3)^2 - (a^3)^2\}$
 $= x(8x^3 + a^3)(8x^3 - a^3)$

$$\begin{aligned}
 & x[(2x)^3 + (a)^3] \{ (2x)^3 - (a)^3 \} \\
 &= x(2x + a)(4x^2 - 2ax + a^2)(2x - a)(4x^2 + 2ax + a^2) \\
 &= x(2x + a)(2x - a)(4x^2 - 2ax + a^2)(4x^2 + 2ax + a^2)
 \end{aligned}$$

Q.21 $x^3 - 27a^3$

Sol $= (x)^3 - (3a)^3$

$$\begin{aligned}
 &= (x - 3a)[(x)^2 + (x)(3a) + (3a)^2] \\
 &= (x - 3a)(x^2 + 3ax + 9a^2)
 \end{aligned}$$

Q.22 $x^3 + 27a^3$

Sol $= (x)^3 + (3a)^3$

$$\begin{aligned}
 &= (x + 3a)\{(x)^2 - (x)(3a) + (3a)^2\} \\
 &= (x + 3a)(x^2 - 3ax + 9a^2)
 \end{aligned}$$



I. Evaluate each of the polynomials for the value indicated.

Q.1 $P(x) = 2x^3 - 5x^2 + 7x - 7; P(2)$

Sol: $P(x) = 2x^3 - 5x^2 + 7x - 7$

$$\begin{aligned}
 P(2) &= 2(2)^3 - 5(2)^2 + 7(2) - 7 \\
 &= 2 \times 8 - 5 \times 4 + 7 \times 2 - 7 \\
 &= 16 - 20 + 14 - 7 \\
 &= 3
 \end{aligned}$$

Q.2 $P(x) = x^4 - 10x^2 + 25x - 2; P(-4)$

Sol: $P(x) = x^4 - 10x^2 + 25x - 2$

$$\begin{aligned}
 P(-4) &= (-4)^4 - 10(-4)^2 + 25(-4) - 2 \\
 &= 256 - 160 - 100 - 2 \\
 &= -6
 \end{aligned}$$

Q.3 $P(x) = x^4 + 5x^3 - 13x^2 - 30; P(-1)$

Sol: $P(x) = x^4 + 5x^3 - 13x^2 - 30$

$$\begin{aligned}
 P(-1) &= (-1)^4 + 5(-1)^3 - 13(-1)^2 - 30 \\
 &= 1 - 5 - 13 - 30 \\
 &= -47
 \end{aligned}$$

Q.4 $P(x) = x^5 - 10x^3 + 7x + 6; P(3)$

Sol: $P(x) = x^5 - 10x^3 + 7x + 6$

$$\begin{aligned}
 P(3) &= (3)^5 - 10(3)^3 + 7(3) + 6 \\
 &= 243 - 270 + 21 + 6 \\
 &= 0
 \end{aligned}$$

Q.5 $P(x) = x^4 + 4x^3 - 9x^2 + 19x + 6; P(-2)$

Sol: $P(x) = x^4 + 4x^3 - 9x^2 + 19x + 6$

$$\begin{aligned}
 P(-2) &= (-2)^4 + 4(-2)^3 - 9(-2)^2 + 19(-2) + 6 \\
 &= 16 + 4(-8) - 9(4) - 38 + 6 \\
 &= 16 - 32 - 36 - 38 + 6 \\
 &= -106 + 22 \\
 &= -84
 \end{aligned}$$

II. *Determine whether the second polynomial is a factor of the first polynomial without dividing (Hint: evaluate directly and use the factor theorem).*

Q.6 $x^{18} - 1; x + 1$

Sol: Let $P(x) = x^{18} - 1$

Here $x - a = x + 1$

Thus $a = -1$

Now $P(-1) = (-1)^{18} - 1$

$$= 1 - 1 = 0$$

Since $P(-1) = 0$

Then by factor theorem $x + 1$ is a factor of $x^{18} - 1$.

Q.7 $x^{18} - 1; x - 1$

Sol: Let $P(x) = x^{18} - 1$

Here $x - a = x - 1$

Thus $a = 1$

$$\begin{aligned}\text{Now } P(-1) &= (1)^{18} - 1 \\ &= 1 - 1 = 0\end{aligned}$$

$$\text{Since } P(-1) = 0$$

Then by factor theorem, $x - 1$ is a factor of $x^{18} - 1$.

$$\text{Q.8 } x^9 - 2^9; x + 2$$

$$\text{Sol: Let } P(x) = x^9 - 2^9$$

$$\text{Here } x - a = x + 2$$

$$\text{Thus } a = -2$$

$$\begin{aligned}\text{Now } P(-2) &= (-2)^9 - (2)^9 \\ &= -(2)^9 - (2)^9 \\ &= -ve \neq 0\end{aligned}$$

$$\text{Since } P(-2) \neq 0$$

Therefore, $x + 2$ is not a factor of $x^9 - 2^9$.

$$\text{Q.9 } x^9 + 2^9; x - 2$$

$$\text{Sol: Let } P(x) = x^9 + 2^9$$

$$\text{Here } x - a = x - 2$$

$$\text{Thus } a = 2$$

$$\begin{aligned}\text{Now } P(2) &= (2)^9 + (2)^9 \\ &\neq 0\end{aligned}$$

$$\text{Since } P(2) \neq 0$$

Therefore, $x - 2$ is not a factor of $x^9 + 2^9$.

$$\text{Q.10 } 3x^4 - 2x^3 + 5x - 6; x - 1$$

$$\text{Sol: Let } P(x) = 3x^4 - 2x^3 + 5x - 6$$

$$\text{Here } x - a = x - 1$$

$$\text{Thus } a = 1$$

$$\begin{aligned}\text{Now } P(1) &= 3(1)^4 - 2(1)^3 + 5(1) - 6 \\ &= 3 - 2 + 5 - 6 \\ &= 8 - 8 = 0\end{aligned}$$

$$\text{Since } P(1) = 0$$

Then by factor theorem, $x - 1$ is a factor of $3x^4 - 2x^3 + 5x - 6$.

Q.11 $5x^6 - 7x^3 - 6x + x ; x - 1$

Sol: Let $P(x) = 5x^6 - 7x^3 - 6x + x$

Here $x - a = x - 1$

Thus $a = 1$

$$\begin{aligned}\text{Now } P(1) &= 5(1)^6 - 7(1)^3 - 6(1) + (1) \\ &= 5 - 7 - 6 + 1 \\ &= 6 - 13 \\ &= -7 \neq 0\end{aligned}$$

Since $P(1) \neq 0$

Then by factor theorem, $x - 1$ is not a factor of $5x^6 - 7x^3 - 6x + x$.

Q.12 $3x^3 - 7x^2 - 8x + 2 ; x + 1$

Sol: Let $P(x) = 3x^3 - 7x^2 - 8x + 2$

Here $x - a = x + 1$

therefore $a = -1$

$$\begin{aligned}\text{Now } P(-1) &= 3(-1)^3 - 7(-1)^2 - 8(-1) + 2 \\ &= -3 - 7 + 8 + 2 \\ &= 0\end{aligned}$$

Since $P(-1) = 0$

Then by factor theorem, $x + 1$ is a factor of $3x^3 - 7x^2 - 8x + 2$.

Q.13 $5x^8 - 2x^5 + 3x^3 + 6x + 2 ; x + 1$

Sol: Let $P(x) = 5x^8 - 2x^5 + 3x^3 + 6x + 2$

Here $x - a = x + 1$

therefore $a = -1$

$$\begin{aligned}\text{Now } P(-1) &= 5(-1)^8 - 2(-1)^5 + 3(-1)^3 + 6(-1) + 2 \\ &= 5 + 2 - 3 - 6 + 2 \\ &= 9 - 9 = 0\end{aligned}$$

Since $P(-1) = 0$

Then by factor theorem, $x + 1$ is a factor of $5x^8 - 2x^5 + 3x^3 + 6x + 2$.

Q.14 $6x^3 + 2x^2 - x + 9 ; x - 1$

Sol: Let $P(x) = 6x^3 + 2x^2 - x + 9$

Here $x = a = x - 1$

therefore $a = 1$

$$\begin{aligned}\text{Now } P(1) &= 6(1)^3 + 2(1)^2 - (1) + 9 \\ &= 6 + 2 - 1 + 9 \\ &= 17 - 1 = 16 \neq 0\end{aligned}$$

Since $P(1) \neq 0$

Then by factor theorem, $x - 1$ is not a factor of $6x^3 + 2x^2 - x + 9$.

Q.15 $4x^3 - 3x^2 - 8x + 4 ; x - 2$

Sol: Let $P(x) = 4x^3 - 3x^2 - 8x + 4$

Here $x = a = x - 2$

therefore $a = 2$

$$\begin{aligned}\text{Now } P(2) &= 4(2)^3 - 3(2)^2 - 8(2) + 4 \\ &= 4(8) - 3(4) - 8(2) + 4 \\ &= 32 - 12 - 16 + 4 \\ &= 36 - 28 = 8 \neq 0\end{aligned}$$

Since $P(2) \neq 0$

Then by factor theorem, $x - 2$ is not a factor of $4x^3 - 3x^2 - 8x + 4$.

Q.16 $5x^3 + 3x^2 - x + 1 ; x + 1$

Sol: Let $P(x) = 5x^3 + 3x^2 - x + 1$

Here $x = a = x + 1$

therefore $a = -1$

$$\begin{aligned}\text{Now } P(-1) &= 5(-1)^3 + 3(-1)^2 - (-1) + 1 \\ &= 5(-1) + 3(1) + 1 + 1 \\ &= -5 + 3 + 1 + 1 \\ &= -5 + 5 \\ &= 0\end{aligned}$$

Since $P(-1) = 0$

Then by factor theorem, $x+1$ is a factor of $5x^3 + 3x^2 - x + 1$.

Q.17 $2y^3 - 8y^2 + y - 4 ; y - 4$

Sol: Let $P(y) = 2y^3 - 8y^2 + y - 4$

Here $y - a = y - 4$

therefore $a = 4$

$$\begin{aligned}\text{Now } P(4) &= 2(4)^3 - 8(4)^2 + (4) - 4 \\ &= 2(64) - 8(16) + 4 - 4 \\ &= 128 - 128 + 4 - 4 \\ &= 132 - 132 \\ &= 0\end{aligned}$$

Since $P(4) = 0$

Then by factor theorem, $y - 4$ is not a factor of $2y^3 - 8y^2 + y - 4$.

Q.18 $z^3 - 5z^2 - 4z - 4 ; z + 2$

Sol: Let $P(z) = z^3 - 5z^2 - 4z - 4$

Here $z - a = z + 2$

therefore $a = -2$

$$\begin{aligned}\text{Now } P(-2) &= (-2)^3 - 5(-2)^2 - 4(-2) - 4 \\ &= -8 - 5(4) + 8 - 4 \\ &= -8 - 20 + 8 - 4 \\ &= -32 + 8 \\ &= -24 \neq 0\end{aligned}$$

Since $P(-2) \neq 0$

Then by factor theorem, $z+2$ is not a factor of $z^3 - 5z^2 - 4z - 4$.

III. Solve

Q.19. If $P(x) = x^3 - kx^2 + 3x + 5$ is divided by $x - 1$, find k , if remainder is 8.

Sol: Let $P(x) = x^3 - kx^2 + 3x + 5$

Here $x - a = x - 1$

therefore $a = 1$

$$\begin{aligned}\text{Now } P(1) &= (1)^3 - k(1)^2 + 3(1) + 5 \\ &= 1 - k + 3 + 5 \\ &= 9 - k = \text{Remainder}\end{aligned}$$

$$\begin{aligned}\text{But } 9 - k &= 8 \\ -k &= 8 - 9 \\ -k &= -1 \\ k &= 1\end{aligned}$$

Q.20. If $P(x) = 3x^3 + kx - 26$ is divided by $x - 2$, find k , if remainder is 0.

Sol: Lets $P(x) = 3x^3 + kx - 26$

Here $x - a = x - 2$

therefore $a = 2$

$$\begin{aligned}\text{Now } P(2) &= 3(2)^3 + k(2) - 26 \\ &= 3(8) + 2k - 26 \\ &= 24 + 2k - 26 \\ &= 2k - 2 = \text{Remainder}\end{aligned}$$

But Remainder = 0

Thus, $2k - 2 = 0$

$$2k = 2$$

$$k = \frac{2}{2}$$

$$k = 1$$
