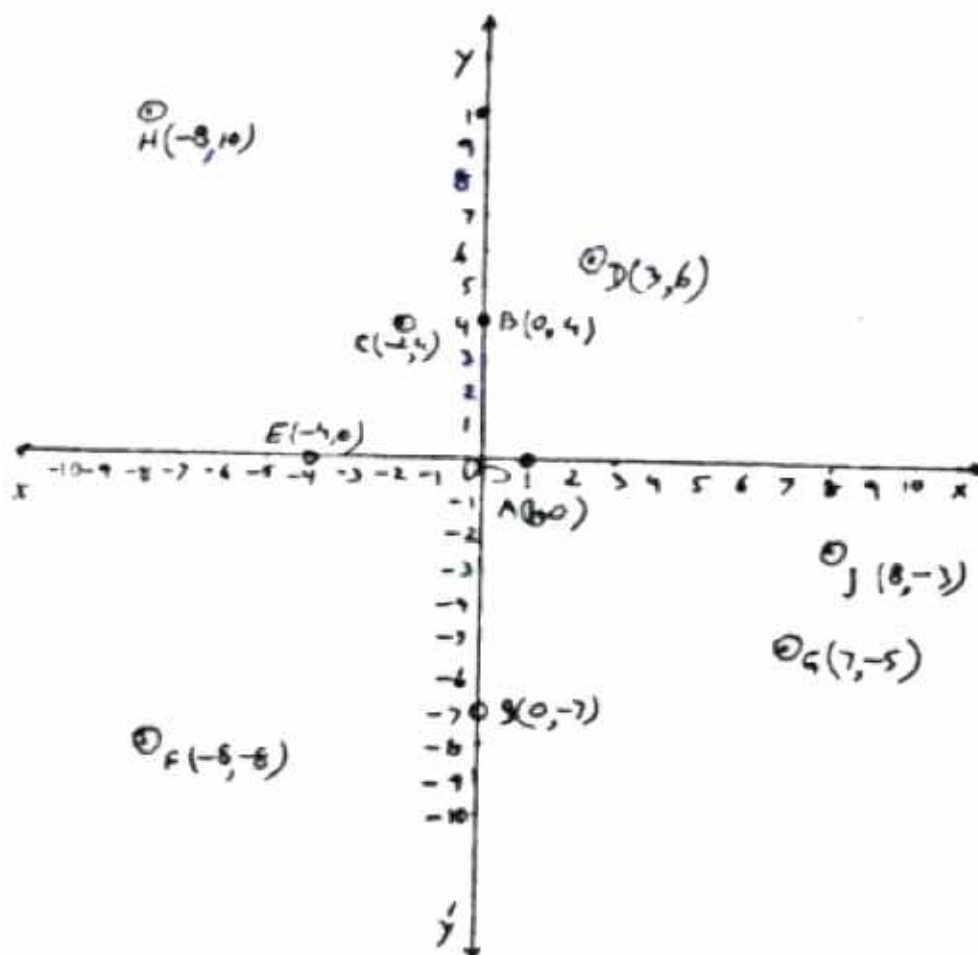


Exercise 10.1

Q.1. Describe the location of these points on the number plane.

(i)	A(1, 0)	(ii)	B(0, 4)	(iii)	C(-2, 4)	(iv)	D(3, 6)
(v)	E(-4, 0)	(vi)	F(-8, -8)	(vii)	G(7, -5)	(viii)	H(-8, 10)
(ix)	I(0, -7)	(x)	J(8, -3)				

Solution:



The points are represented on the graph paper.

Q.2. Find the distance between the following pairs of points.

(i)	$(2, 1), (-4, 3),$	(ii)	$(-1, 3), (-2, -1)$
(iii)	$(7, -2), (-2, 3)$	(iv)	$(a, -b), (b, -a)$

(i) Let $P(x_1, y_1) = (2, 1)$:J

and $Q(x_2, y_2) = (-4, 3)$

distance formula $m\overline{PQ} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$= \sqrt{(2 + 4)^2 + (1 - 3)^2}$$

$$= \sqrt{(6)^2 + (-2)^2}$$

$$= \sqrt{36 + 4}$$

$$= \sqrt{40}$$

$$= \sqrt{2 \times 2 \times 10}$$

$$= 2\sqrt{10} \text{ units}$$

(ii) Let $P(x_1, y_1) = (-1, 3)$

and $Q(x_2, y_2) = (-2, -1)$

distance formula $m\overline{PQ} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$= \sqrt{(-1 + 2)^2 + (3 + 1)^2}$$

$$= \sqrt{(1)^2 + (4)^2}$$

$$= \sqrt{1 + 16}$$

$$= \sqrt{17} \text{ units}$$

(iii) Let $P(x_1, y_1) = (7, -2)$

and $Q(x_2, y_2) = (-2, 3)$

distance formula $m\overline{PQ} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$\begin{aligned} &= \sqrt{(7 + 2)^2 + (-2 - 3)^2} \\ &= \sqrt{(9)^2 + (-5)^2} \\ &= \sqrt{81 + 25} \\ &= \sqrt{106} \text{ units} \end{aligned}$$

(iv) Let $P(x_1, y_1) = (a, -b)$

and $Q(x_2, y_2) = (b, -a)$

$$\begin{aligned} m\overline{PQ} &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(a - b)^2 + (-b + a)^2} \\ &= \sqrt{(a - b)^2 + (a - b)^2} \\ &= \sqrt{2(a - b)^2} = (a - b) \sqrt{2} \text{ units} \end{aligned}$$

Q.3. Express by an equation, the fact, that the point $P(x, y)$ is equidistant from $A(2, 4)$ and $B(6, 8)$.

Sol. $P(x, y) ; A(2, 4)$

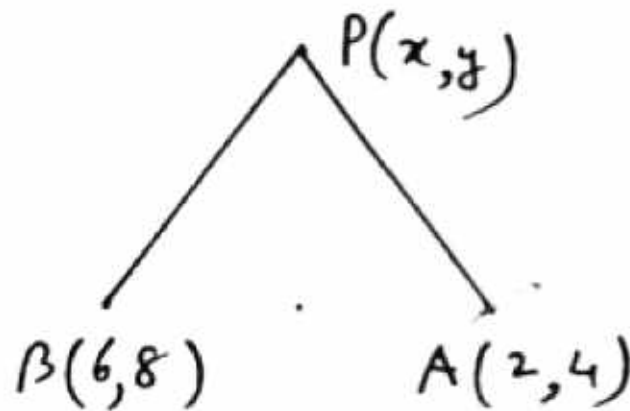
Applying the distance formula

$$m\overline{PA} = \sqrt{(x - 2)^2 + (y - 4)^2}$$

and $P(x, y) ; B(6, 8)$

Applying the distance formula

$$m\overline{PB} = \sqrt{(x-6)^2 + (y-8)^2}$$



According to the statement

$$m\overline{PA} = m\overline{PB}$$

Therefore

$$\sqrt{(x-2)^2 + (y-4)^2} = \sqrt{(x-6)^2 + (y-8)^2}$$

By taking square

$$(x-2)^2 + (y-4)^2 = (x-6)^2 + (y-8)^2$$

$$x^2 - 4x + 4 + y^2 - 8y + 16 = x^2 - 12x + 36 + y^2 - 16y + 64$$

$$-4x + 12x - 8y + 16y + 4 + 16 - 36 - 64 = 0$$

(Divided by 8)

$$8x + 8y - 80 = 0$$

or

$$x + y - 10 = 0$$

Q.4. Show that the points A(5, 4), B(4, -3), C(-2, 5) are equidistant from D(1, 1).

Sol. A(5, 4), B(4, -3), C(-2, 5), D(1, 1)

(i) D(1, 1); A(5, 4)

Let find $m\overline{DA}$

Suppose that D(x_1, y_1); A(x_2, y_2)

Distance formula

$$\begin{aligned}m\overline{DA} &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\&= \sqrt{(1 - 5)^2 + (1 - 4)^2} \\&= \sqrt{(-4)^2 + (-3)^2} \\&= \sqrt{(16 + 9)} = \sqrt{25} = 5 \quad (P)\end{aligned}$$

(ii) Now find $m\overline{DB}$

$$D(1,1); B(4,-3)$$

Let

$$D(x_1, y_1); B(x_2, y_2)$$

Distance formula

$$\begin{aligned}m\overline{DB} &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\&= \sqrt{(1 - 4)^2 + (1 + 3)^2} \\&= \sqrt{(-3)^2 + (4)^2} \\&= \sqrt{(9 + 16)} = \sqrt{25} = 5 \quad (Q)\end{aligned}$$

(iii) Now find $m\overline{DC}$

$$D(1,1); C(-2,5)$$

Suppose that:

$$D(x_1, y_1); C(x_2, y_2)$$

Distance formula:

$$\begin{aligned}m\overline{DC} &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\&= \sqrt{(1 \times 2)^2 + (1 - 5)^2}\end{aligned}$$

$$\begin{aligned}
 &= \sqrt{(3)^2 + (-4)^2} \\
 &= \sqrt{(9 + 16)} = \sqrt{25} = 5 \quad (R)
 \end{aligned}$$

From R, Q, P

Points A, B, C are equidistant from "D".

Q.5. Find the point on the x -axis which is equidistant from (2, 4) and (6, 8).

(Hint: call the point $(x, 0)$. Find x .)

Sol. Y ordinate is "0" on x -axis.

Let we suppose the required point is $P(x, 0)$.

Let we have supposed that points A (2, 4), B(6, 8)

Now we find $m\overline{PA}$ and $m\overline{PB}$.

Distance formula:

$$\begin{aligned}
 m\overline{PA} &= \sqrt{(x-2)^2 + (0-4)^2} \\
 &= \sqrt{x^2 - 4x + 4 + 16} \\
 &= \sqrt{x^2 - 4x + 20}
 \end{aligned}$$

$$\begin{aligned}
 m\overline{PB} &= \sqrt{(x-6)^2 + (0-8)^2} && \text{and} \\
 &= \sqrt{x^2 - 12x + 36 + 64}
 \end{aligned}$$

$$m\overline{PB} = \sqrt{x^2 - 12x + 100}$$

$$m\overline{PA} = m\overline{PB}$$

Now, according to the statement

$$\sqrt{x^2 - 4x + 20} = \sqrt{x^2 - 12x + 100}$$

$$x^2 - 4x + 20 = x^2 - 12x + 100$$

By taking square

$$-4x + 12x = 100 - 20$$

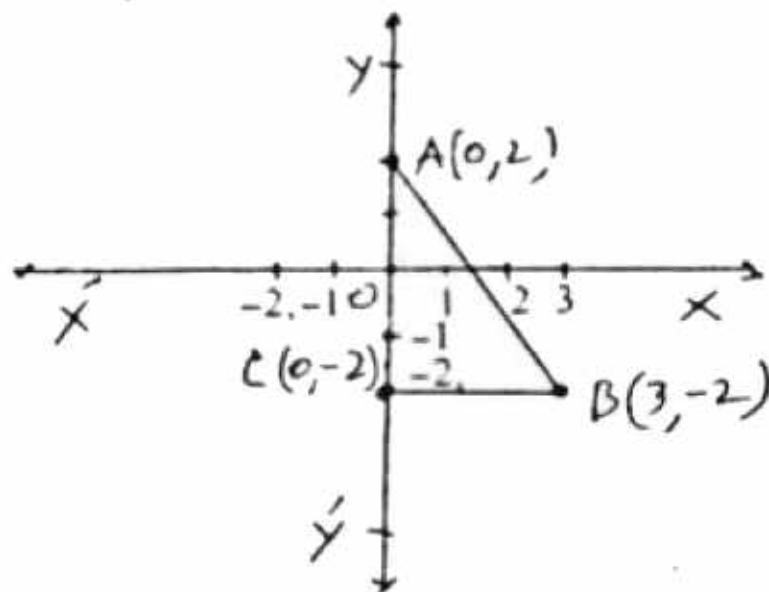
$$8x = 80$$

$$x = \frac{80}{8} = 10$$

Required points (10, 0)

Q.6. Show that the points A(0, 2), B(3, -2) and C(0, -2) are vertices of a right triangle.

Sol. Find the lengths of sides.



Distance formula:

A(0, 2), B(3, -2)

$$\begin{aligned} m_{AB} &= \sqrt{(0-3)^2 + (2+2)^2} \\ &= \sqrt{(-3)^2 + (4)^2} \end{aligned}$$

$$= \sqrt{9+16}$$

$$= \sqrt{25}$$

$$= 5 \quad \text{units}$$

$$B(3, -2) \text{ and } C(0, -2)$$

$$m\overline{BC} = \sqrt{(3-0)^2 + (-2+2)^2}$$

$$= \sqrt{(3)^2 + (0)^2}$$

$$= \sqrt{9}$$

$$= 3 \quad \text{units}$$

$$C(0, -2) \text{ and } A(0, 2)$$

$$m\overline{CA} = \sqrt{(0-0)^2 + (-2-2)^2}$$

$$= \sqrt{0+(-4)^2}$$

$$= \sqrt{16}$$

$$= 4 \quad \text{units}$$

$$(m\overline{BC})^2 + (m\overline{CA})^2 = (3)^2 + (4)^2$$

$$= 9 + 16$$

$$= 25 \dots\dots\dots(i)$$

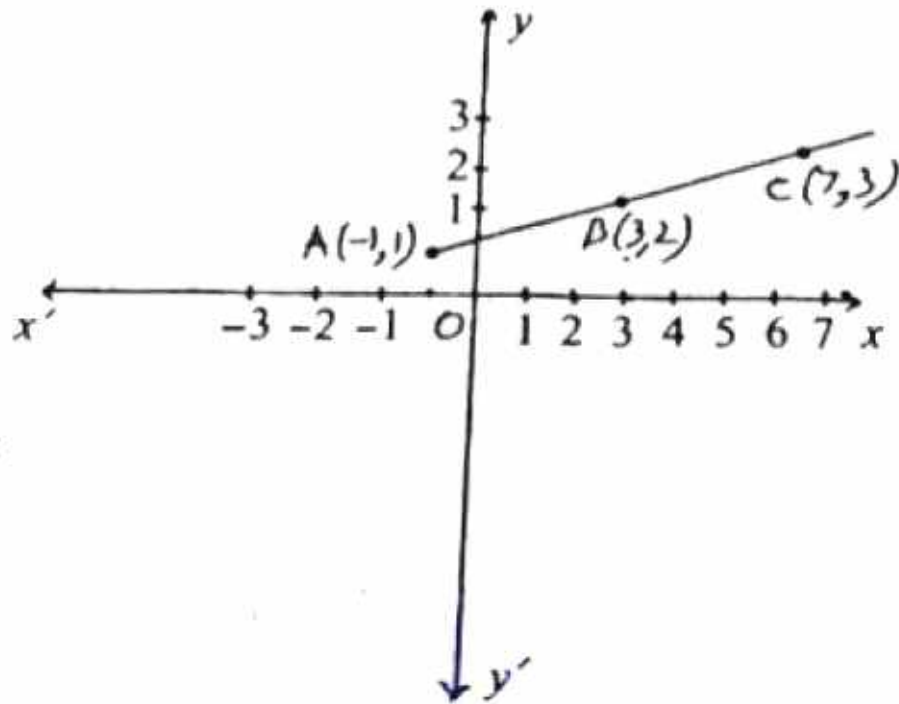
$$(m\overline{AB})^2 = (5)^2 \text{ and}$$

$$= 25 \dots\dots\dots(ii)$$

From (i) and (ii); It is proved that the points A , B and C are the vertices of a right angle triangle.

Q.7. Show that the points $A(-1, 1)$, $B(3, 2)$, $C(7, 3)$ are collinear.

Sol.



Let we find the mid distance of points $A(-1, 1)$, $B(3, 2)$, $C(7, 3)$

$A(-1, 1)$ and $B(3, 2)$

$$\begin{aligned} m\overline{AB} &= \sqrt{(-1-3)^2 + (1-2)^2} \\ &= \sqrt{(-4)^2 + (-1)^2} \\ &= \sqrt{16+1} = \sqrt{17} \end{aligned}$$

$B(3, 2)$ and $C(7, 3)$

$$\begin{aligned} m\overline{BC} &= \sqrt{(3-7)^2 + (2-3)^2} \\ &= \sqrt{(-4)^2 + (-1)^2} \end{aligned}$$

$$m\overline{BC} = \sqrt{16+1} = \sqrt{17}$$

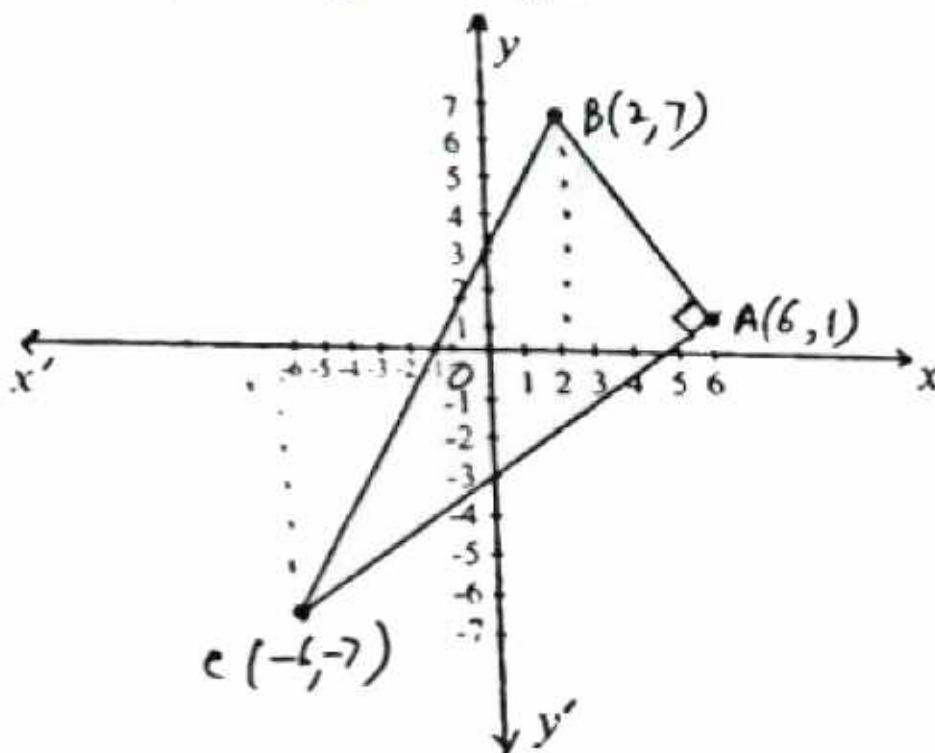
$A(-1, 1)$ and $C(7, 3)$

$$\begin{aligned} m\overline{AC} &= \sqrt{(-1-7)^2 + (1-3)^2} \\ &= \sqrt{(-8)^2 + (-2)^2} \\ &= \sqrt{64+4} \\ &= \sqrt{68} \\ &= \sqrt{4 \times 17} = 2\sqrt{17} \end{aligned}$$

$$\begin{aligned} m\overline{AB} + m\overline{BC} &= \sqrt{17} + \sqrt{17} \quad \text{Now} \\ &= 2\sqrt{17} = m\overline{AC} \end{aligned}$$

Points A, B and C are collinear.

Q.8. Show that the points $A(6, 1)$, $B(2, 7)$ and $C(-6, -7)$ are vertices of a right triangle.



Let we find the square of mid distances $A(6, 1)$, $B(2, 7)$,

$C(-6, -7)$

Using the distance formula

$A(6, 1)$, $B(2, 7)$

$$(m\overline{AB})^2 = (6-2)^2 + (1-7)^2$$

$$= (4)^2 + (-6)^2$$

$$= 16 + 36 = 52$$

$$B(2, 7), C(-6, -7)$$

$$(m\overline{BC})^2 = (2+6)^2 + (7+7)^2$$

$$= (8)^2 + (14)^2$$

$$= 64 + 196 = 260$$

$$A(6, 1), C(-6, -7)$$

$$(m\overline{CA})^2 = (-6-6)^2 + (-7-1)^2$$

$$= (-12)^2 + (-8)^2$$

$$= 144 + 64 = 208$$

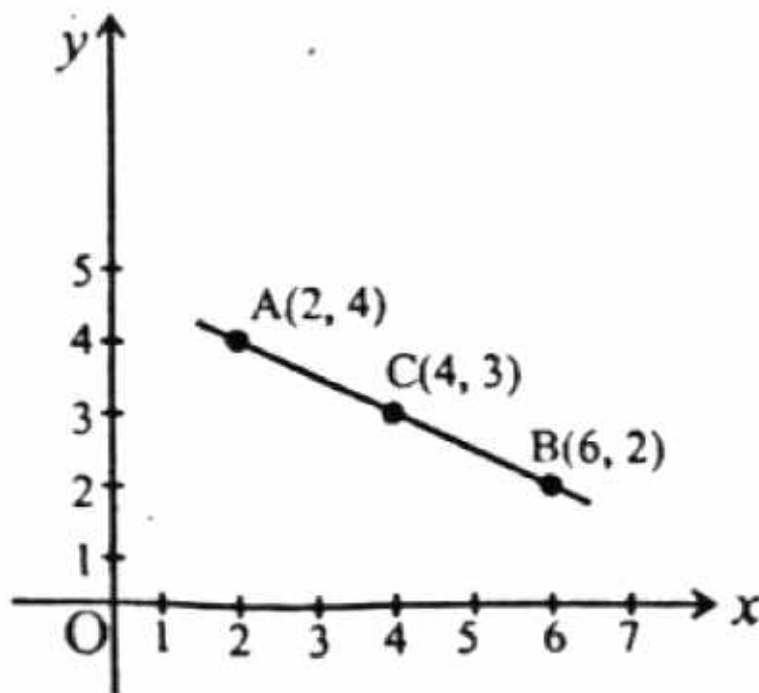
$$\text{Now } (m\overline{AB})^2 + (m\overline{CA})^2 = 52 + 208$$

$$= 260 = (m\overline{BC})^2$$

The sum of two lengths is equal to the length of third side.
Points A, B, C are vertices of right angled.

Q.9. Show that the points A(2, 4), B(6, 2), C(4, 3) are collinear.

Sol:



Let we find the mid distance of A(2, 4), B(6, 2), C(4, 3)

A(2, 4), B(6, 2)

$$\begin{aligned}m\overline{AB} &= \sqrt{(2-6)^2 + (4-2)^2} \\&= \sqrt{(-4)^2 + (2)^2} \\&= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5} \quad \text{units}\end{aligned}$$

A(2, 4), C(4, 3)

$$\begin{aligned}m\overline{AC} &= \sqrt{(2-4)^2 + (4-3)^2} \\&= \sqrt{(-2)^2 + (1)^2} \\&= \sqrt{4+1} \\&= \sqrt{5} \quad \text{units}\end{aligned}$$

C(4, 3), B(6, 2)

$$\begin{aligned}m\overline{CB} &= \sqrt{(4-6)^2 + (3-2)^2} \\&= \sqrt{(-2)^2 + (1)^2} \\&= \sqrt{4+1} \\&= \sqrt{5} \quad \text{units}\end{aligned}$$

$$\begin{aligned}m\overline{AC} + m\overline{CB} &= \sqrt{5} + \sqrt{5} \\&= 2\sqrt{5} \quad \text{units} \\&= m\overline{AB}\end{aligned}$$

Thus, A, B, C are collinear.

$$\begin{aligned}
 &= \sqrt{16 + 196} \\
 &= \sqrt{212} \\
 &= 2\sqrt{53} \quad \text{units}
 \end{aligned}$$

$$B(-6, -3), C(4, -9)$$

$$\begin{aligned}
 m\overline{BC} &= \sqrt{(-6 - 4)^2 + (-3 + 9)^2} \\
 &= \sqrt{(-10)^2 + (6)^2} \\
 &= \sqrt{100 + 36} \\
 &= \sqrt{136} \\
 &= \sqrt{4 \times 34} \\
 &= 2\sqrt{34} \quad \text{units}
 \end{aligned}$$

$$C(4, -9), A(-2, 11)$$

$$\begin{aligned}
 m\overline{CA} &= \sqrt{(4 + 2)^2 + (-9 - 11)^2} \\
 &= \sqrt{(6)^2 + (-20)^2} \\
 &= \sqrt{36 + 400} \\
 &= \sqrt{436} \\
 &= \sqrt{4 \times 109} \\
 &= 2\sqrt{109} \quad \text{units}
 \end{aligned}$$

From (i), (ii), (iii)

$$m\overline{AB} = 2\sqrt{53}$$

$$m\overline{BC} = 2\sqrt{34}$$

$$m\overline{C'A} = 2\sqrt{109}$$

$$\text{Now } m\overline{AB} \neq m\overline{BC'} \neq m\overline{C'A}$$

The points A(-2, 11) B(-6, -3) and C(4, -9) are vertices of scalene.

Q.12. Show that the points A(6, 1), B(2, 7) and C(-6, 7) are of a scalene triangle.

Sol. Let we find the mid distance of A(6, 1), B(2, 7), C(-6, 7)

$$A(6, 1), B(2, 7)$$

$$\begin{aligned} m\overline{AB} &= \sqrt{(6-2)^2 + (1-7)^2} \\ &= \sqrt{(4)^2 + (-6)^2} \\ &= \sqrt{16+36} \\ &= \sqrt{52} \\ &= \sqrt{4 \times 13} \\ &= 2\sqrt{13} \dots\dots\dots(i) \end{aligned}$$

$$B(2, 7), C(-6, 7)$$

$$\begin{aligned} m\overline{BC'} &= \sqrt{(2+6)^2 + (7-7)^2} \\ &= \sqrt{(8)^2 + (0)^2} \\ &= \sqrt{64} \\ &= 8 \dots\dots\dots(ii) \end{aligned}$$

$$C(-6, 7), A(6, 1)$$

$$m\overline{C'A} = \sqrt{(-6-6)^2 + (7-1)^2}$$

$$\begin{aligned}
 &= \sqrt{(-12)^2 + (6)^2} \\
 m\overline{CA} &= \sqrt{144 + 36} \dots\dots (iii) \\
 &= \sqrt{180} \\
 &= \sqrt{9 \times 4 \times 5} \\
 &= 3 \times 2\sqrt{5} \\
 &= 6\sqrt{5} \quad \text{units}
 \end{aligned}$$

From (i), (ii), (iii)

$$m\overline{AB} \neq m\overline{BC} \neq m\overline{CA}$$

Thus points $A(6, 1)$, $B(2, 7)$ and $C(-6, 7)$ are vertices of scalene triangle.

13. Show that the points $A(2, -5)$, $B(-4, -3)$ and $C(-1, 5)$ are of a scalene triangle.

sol. We find the distance between the points $A(2, -5)$, $B(-4, -3)$, $C(-1, 5)$

$$\begin{aligned}
 &A(2, -5), B(-4, -3) \\
 m\overline{AB} &= \sqrt{(2+4)^2 + (-5+3)^2} \\
 &= \sqrt{(6)^2 + (-2)^2} \\
 &= \sqrt{36+4} \\
 &= \sqrt{40} = 2\sqrt{10}
 \end{aligned}$$

$$\begin{aligned}
 &B(-4, -3), C(-1, 5) \\
 m\overline{BC} &= \sqrt{(-4+1)^2 + (-3-5)^2} \\
 &= \sqrt{(-3)^2 + (-8)^2}
 \end{aligned}$$

$$= \sqrt{9 + 64}$$

$$= \sqrt{73} \dots\dots\dots (ii)$$

$$C(-1, 5), A(2, -5)$$

$$m\overline{CA} = \sqrt{(-1-2)^2 + (5+5)^2}$$

$$= \sqrt{(-3)^2 + (10)^2}$$

$$= \sqrt{9 + 100}$$

$$= \sqrt{109} \dots\dots\dots (iii)$$

$$m\overline{AB} = 2\sqrt{10}$$

$$m\overline{BC} = \sqrt{73}$$

$$m\overline{CA} = \sqrt{109}$$

$$m\overline{AB} \neq m\overline{BC} \neq m\overline{CA}$$

Thus, A, B, C are vertices of scalene triangle.