

EXERCISES

Solve:

- 1- If $P(x) = x^4 + 3x^2 - 5x + 9$, then find $P(x)$, for $x = 0, x = 1$.

$$\begin{aligned}\text{Sol. } P(x) &= x^4 + 3x^2 - 5x + 9 \\ P(0) &= (0)^4 + 3(0)^2 - 5(0) + 9 \\ &= 0 + 0 - 0 + 9 \\ &= 9\end{aligned}$$

$$\begin{aligned}\text{and } P(1) &= (1)^4 + 3(1)^2 - 5(1) + 9 \\ &= 1 + 3 - 5 + 9 \\ &= 13 - 5 \\ &= 8\end{aligned}$$

- 2- If $P(x) = 2x^3 + 2x^2 + x - 1$, then find $P(-2)$

$$\begin{aligned}\text{Sol. } P(x) &= 2x^3 + 2x^2 + x - 1 \\ \text{Therefore, } P(-2) &= 2(-2)^3 + 2(-2)^2 + (-2) - 1 \\ &= 2(-8) + 2(4) - 2 - 1 \\ &= -16 + 8 - 2 - 1 \\ &= 8 - 16 - 2 - 1 \\ &= 8 - 19 \\ &= -11\end{aligned}$$

- 3- If $P(y) = 3y^2 + \frac{y}{4} + 9$, then find $P(0)$.

$$\begin{aligned}\text{Sol. } P(y) &= 3y^2 + \frac{y}{4} + 9 \\ P(0) &= 3(0)^2 + \frac{0}{4} + 9 \\ &= 3(0) + 0 + 9 \\ &= 9\end{aligned}$$

- 4- If $P(x) = 9x^3 - 2x^2 + 3x + 1$, then find $P(1)$ and $P(2)$.

$$\begin{aligned}\text{Sol. } P(x) &= 9x^3 - 2x^2 + 3x + 1 \\ P(1) &= 9(1)^3 - 2(1)^2 + 3(1) + 1\end{aligned}$$

$$\begin{aligned}
 &= 9(1) - 2(1) + 3 + 1 \\
 &= 9 - 2 + 3 + 1 \\
 &= 13 - 2 \\
 &= 11
 \end{aligned}$$

and $P(2) = 9(2)^3 - 2(2)^2 + 3(2) + 1$

$$\begin{aligned}
 &= 9(8) - 2(4) + 6 + 1 \\
 &= 72 - 8 + 6 + 1 \\
 &= 79 - 8 \\
 &= 71
 \end{aligned}$$

5- If $P(x) = \frac{x^2 - 5x + 6}{x + 1}$, then find $P(1)$ and $P(2)$.

Sol. $P(x) = \frac{x^2 - 5x + 6}{x + 1}$

$$\begin{aligned}
 P(1) &= \frac{(1)^2 - 5(1) + 6}{1 + 1} \\
 &= \frac{1 - 5 + 6}{1 + 1} \\
 &= \frac{2}{2} \\
 &= 1
 \end{aligned}$$

and $P(2) = \frac{(2)^2 - 5(2) + 6}{2 + 1}$

$$\begin{aligned}
 &= \frac{4 - 10 + 6}{3} \\
 &= \frac{0}{3} \\
 &= 0
 \end{aligned}$$

6- If $P(r) = 2\pi r$, then find $P(r)$, for $r = 3$ and $\pi = \frac{22}{7}$.

Sol. $P(r) = 2\pi r$

By putting the values of r and π

$$\begin{aligned}
 P(r) &= 2 \times \frac{22}{7} \times 3 \\
 &= \frac{132}{7} \\
 &= 18.9 \text{ (approximately)}
 \end{aligned}$$

7- If $P(r) = 4\pi r^2$, then find $P(r)$, for $r = 8$ and $\pi = \frac{22}{7}$.

Sol. $P(r) = 4\pi r^2$

By putting the values of r and π

$$\begin{aligned}
 P(r) &= 4 \times \frac{22}{7} \times (8)^2 \\
 &= 4 \times \frac{22}{7} \times 64 \\
 &= \frac{5632}{7} \\
 &= 804.57
 \end{aligned}$$

8- If $P(y) = y^4 + \frac{3y^3}{2} - y^2 + 1$, then find $P(y)$, for $y = 2$ and $y = -2$.

Sol. $P(y) = y^4 + \frac{3y^3}{2} - y^2 + 1$

for $y = 2$

$$\begin{aligned}
 P(2) &= (2)^4 + \frac{3(2)^3}{2} - (2)^2 + 1 \\
 &= 16 + \frac{3 \times 8}{2} - 4 + 1 \\
 &= 16 + 12 - 4 + 1 \\
 &= 25
 \end{aligned}$$

$$P(-2) = (-2)^4 + \frac{3(-2)^3}{2} - (-2)^2 + 1 \quad \text{for } y = -2$$

$$= 16 + \frac{3(-8)}{2} - 4 + 1$$

$$= 16 - 12 - 4 + 1$$

$$= 1$$

Reduce the given rational expressions to lowest terms.

Q.9 $\frac{8x^2y^2}{12x^4y}$

Sol. $= \frac{2y^{2-1}}{3x^{4-2}}$

$$= \frac{2y}{3x^2}$$

Q.10 $\frac{25a^3b^2}{14a^2b^4}$

Sol. $= \frac{25a^{3-2}}{14b^{4-2}}$

$$= \frac{25a}{14b^2}$$

Q.11 $\frac{16a^6b^7}{12a^3b^5 + 20a^5b^4}$

Sol. $= \frac{16a^6b^7}{4a^3b^4(3b + 5a^2)}$

$$= \frac{4a^{6-3}b^{7-4}}{(5a^2 + 3b)}$$

$$= \frac{4a^3b^3}{5a^2 + 3b}$$

Q.12 $\frac{18m^5x^3}{27m^4x^8 - 36m^6x^6}$

Sol. $= \frac{18m^5x^3}{9m^4x^6(3x^2 - 4m^2)}$

$$= \frac{2m^{5-4}}{x^{6-3}(3x^2 - 4m^2)}$$

$$= \frac{2m}{x^3(3x^2 - 4m^2)}$$

$$= \frac{2m}{3x^5 - 4m^2x^3}$$

Q.13 $\frac{5c-5d}{c^2-d^2}$

Sol. $= \frac{5(c-d)}{(c-d)(c+d)}$

$$= \frac{5}{(c+d)}$$

Q.14 $\frac{x^2-y^2}{3y-3x}$

Sol. $= \frac{(x-y)(x+y)}{3(y-x)}$

$$= \frac{(x-y)(x+y)}{-3(x-y)}$$

$$= \frac{x+y}{-3}$$

Simplify:

Q.15 $\frac{x}{x-y} + \frac{x^2}{x^2+y^2}$

Sol. $= \frac{x(x^2+y^2) + x^2(x-y)}{(x-y)(x^2+y^2)}$

$$= \frac{x^3 + xy^2 + x^3 - x^2y}{x^3 + xy^2 - x^2y - y^3}$$

$$= \frac{2x^3 - x^2y + xy^2}{x^3 - x^2y + xy^2 - y^3}$$

Q.16 $\frac{x^2+2x}{x^2+x-2} + \frac{3x}{x+1}$

Sol. $= \frac{(x^2+2x)(x+1) + 3x(x^2+x-2)}{(x^2+x-2)(x+1)}$

$$= \frac{x^3 + x^2 + 2x^2 + 2x + 3x^3 + 3x^2 - 6x}{x^3 + x^2 - 2x + x^2 + x - 2}$$

$$= \frac{4x^3 + 6x^2 - 4x}{x^3 + 2x^2 - x - 2}$$

Q.17 $\frac{x+2}{x^2+3x+2} - \frac{x-5}{x^2-x-6}$

Sol. $= \frac{x+2}{x^2+x+2x+2} - \frac{x-5}{x^2-x-6}$

$$= \frac{x+2}{x(x+1)+2(x+1)} - \frac{x-5}{x^2-x-6}$$

$$= \frac{x+2}{(x+2)(x+1)} - \frac{x-5}{x^2-x-6}$$

$$= \frac{1}{x+1} - \frac{x-5}{x^2-x-6}$$

$$= \frac{1(x^2-x-6) - (x+1)(x-5)}{(x+1)(x^2-x-6)}$$

$$= \frac{x^2-x-6 - (x^2-5x+x-5)}{(x+1)(x^2-x-6)}$$

$$= \frac{x^2-x-6-x^2+5x-x+5}{x^3-x^2-6x+x^2-x-6}$$

$$= \frac{3x-1}{x^3-7x-6}$$

Q.18 $\frac{8x^2+18y^2}{4x^2-9y^2} - \frac{2x+3y}{2x-3y}$

Sol. $= \frac{8x^2+18y^2}{(2x)^2-(3y)^2} - \frac{2x+3y}{2x-3y}$

$$= \frac{8x^2+18y^2}{(2x+3y)(2x-3y)} - \frac{2x+3y}{2x-3y}$$

$$= \frac{8x^2+18y^2 - (2x+3y)(2x+3y)}{(2x+3y)(2x-3y)}$$

$$\begin{aligned}
&= \frac{8x^2 + 18y^2 - (4x^2 + 12xy + 9y^2)}{(2x + 3y)(2x - 3y)} \\
&= \frac{8x^2 + 18y^2 - 4x^2 - 12xy - 9y^2}{(2x + 3y)(2x - 3y)} \\
&= \frac{4x^2 - 12xy + 9y^2}{(2x + 3y)(2x - 3y)} \\
&= \frac{(2x)^2 - 2(2x)(3y) + (3y)^2}{(2x + 3y)(2x - 3y)} \\
&= \frac{(2x - 3y)^2}{(2x + 3y)(2x - 3y)} \\
&= \frac{(2x - 3y)(\cancel{2x - 3y})}{(2x + 3y)(\cancel{2x - 3y})} \\
&= \frac{2x - 3y}{2x + 3y}
\end{aligned}$$

Q.19 $\frac{x}{x^2 + xy} - \frac{y}{x^2 - y^2}$

Sol.
$$\begin{aligned}
&= \frac{x}{x(x + y)} - \frac{y}{(x + y)(x - y)} \\
&= \frac{1}{x + y} - \frac{y}{(x + y)(x - y)} \\
&= \frac{1(x - y) - y}{(x + y)(x - y)} \\
&= \frac{x - y - y}{(x + y)(x - y)} \\
&= \frac{x - 2y}{x^2 - y^2}
\end{aligned}$$

Q.20 $\frac{x + y}{xy + y^2} - \frac{x}{x^2 - xy}$

Sol.
$$= \frac{x + y}{y(x + y)} - \frac{x}{x(x - y)}$$

$$\begin{aligned}
 &= \frac{1}{y} - \frac{1}{x-y} \\
 &= \frac{1(x-y) - 1(y)}{(y)(x-y)} \\
 &= \frac{x-y-y}{y(x-y)} \\
 &= \frac{x-2y}{xy-y^2}
 \end{aligned}$$

Q.21 $\frac{(x+1)^2}{x^2-1} - \frac{x^2+1}{x^2+1}$

Sol.
$$\begin{aligned}
 &= \frac{(x+1)(x+1)}{(x+1)(x-1)} - \frac{x^2+1}{x^2+1} \\
 &= \frac{x+1}{x-1} - 1 \\
 &= \frac{x+1-1(x-1)}{x-1} \\
 &= \frac{x+1-x+1}{x-1} \\
 &= \frac{2}{x-1}
 \end{aligned}$$

Q.22 $\frac{5x}{x-9} + \frac{x^2-2x+1}{x^2-12x+27} - \frac{6x}{x-3}$

Sol.
$$\begin{aligned}
 &= \frac{5x}{x-9} + \frac{x^2-2x+1}{x^2-9x-3x+27} - \frac{6x}{x-3} \\
 &= \frac{5x}{x-9} + \frac{x^2-2x+1}{x(x-9)-3(x-9)} - \frac{6x}{x-3} \\
 &= \frac{5x}{x-9} + \frac{x^2-2x+1}{(x-9)(x-3)} - \frac{6x}{x-3} \\
 &= \frac{5x(x-3) + x^2-2x+1 - 6x(x-9)}{(x-9)(x-3)}
 \end{aligned}$$

$$= \frac{5x^2 - 15x + x^2 - 2x + 1 - 6x^2 + 54x}{(x-9)(x-3)}$$

$$= \frac{37x+1}{x^2-12x+27}$$

Q.23 $\frac{x^2-4x+4}{x^2-4} \div \frac{x}{x-2}$

Sol.
$$= \frac{x^2-2x-2x+4}{(x)^2-(2)^2} \times \frac{x-2}{x}$$

$$= \frac{x(x-2)-2(x-2)}{(x+2)(x-2)} \times \frac{x-2}{x}$$

$$= \frac{(x-2)(x-2)}{x+2} \times \frac{1}{x}$$

$$= \frac{x^2-4x+4}{x^2+2x}$$

Q.24 $\frac{x^2-36}{x^2-1} \div \frac{x-6}{1-x}$

Sol.
$$= \frac{(x)^2-(6)^2}{(x)^2-(1)^2} \times \frac{1-x}{x-6}$$

$$= \frac{(x-6)(x+6)}{(x-1)(x+1)} \times \frac{-(x-1)}{x-6}$$

$$= \frac{x+6}{x+1} \times (-1)$$

$$= -\frac{x+6}{x+1}$$

Q.25 $\frac{x^2-5x}{x-1} \div \frac{x^2-25}{x^2+x+20}$

Sol.
$$= \frac{x(x-5)}{x-1} \times \frac{x^2+x+20}{x^2-25}$$

$$= \frac{x(x-5)}{x-1} \times \frac{x^2+x+20}{(x)^2-(5)^2}$$

$$= \frac{x(x-5)}{x-1} \times \frac{x^2+x+20}{(x-5)(x+5)}$$

$$= \frac{x(x^2+x+20)}{(x-1)(x+5)}$$

$$= \frac{x^3+x^2+20x}{x^2+4x-5}$$

Q.26 $\frac{2x^2-5x-12}{4x^2+4x-3} \div \frac{2x^2-7x-4}{6x^2+5x-4}$

Sol. $= \frac{2x^2-8x+3x-12}{4x^2+6x-2x-3} \times \frac{6x^2+5x-4}{2x^2-7x-4}$

$$= \frac{2x(x-4)+3(x-4)}{2x(2x+3)-1(2x+3)} \times \frac{6x^2+8x-3x-4}{2x^2-8x+x-4}$$

$$= \frac{(x-4)(2x+3)}{(2x+3)(2x-1)} \times \frac{2x(3x+4)-1(3x+4)}{2x(x-4)+1(x-4)}$$

$$= \frac{x-4}{2x-1} \times \frac{(3x+4)(2x-1)}{(x-4)(2x+1)}$$

$$= \frac{3x+4}{2x+1}$$

Q.27 $\frac{x(2x-1)^2}{2x^2-1} + \frac{4x^2-1}{4x^2+4x+1}$

Sol. $= \frac{x(2x-1)(2x-1)}{2x^2-1} \times \frac{4x^2+4x+1}{4x^2-1}$

$$= \frac{x(2x-1)(2x-1)}{(2x^2-1)} \times \frac{4x^2+2x+2x+1}{(2x)^2-(1)^2}$$

$$= \frac{x(2x-1)(2x-1)}{(2x^2-1)} \times \frac{2x(2x+1)+1(2x+1)}{(2x+1)(2x-1)}$$

$$= \frac{x(2x-1)(2x-1)}{(2x^2-1)} \times \frac{(2x+1)(2x+1)}{(2x+1)(2x-1)}$$

$$= \frac{x(2x-1)(2x+1)}{2x^2-1}$$

$$= \frac{x(4x^2 - 1)}{2x^2 - 1}$$

$$= \frac{4x^3 - x}{2x^2 - 1}$$

Q.28 $\frac{x^2 + x}{x^2 - 1} \times \frac{x + 1}{x^3 + 1}$

Sol. $= \frac{x(x+1)}{(x+1)(x-1)} \times \frac{(x+1)}{(x+1)(x^2 - x + 1)}$

$$= \frac{x}{(x-1)(x^2 - x + 1)}$$

$$= \frac{x}{x^3 - x^2 + x - x^2 + x - 1}$$

$$= \frac{x}{x^3 - 2x^2 + 2x - 1}$$

Q.29 $\frac{x^2 - 9}{x^2 - 6x + 9} \times \frac{x}{3x + 9}$

Sol. $= \frac{(x)^2 - (3)^2}{x^2 - 3x - 3x + 9} \times \frac{x}{3(x+3)}$

$$= \frac{(x+3)(x-3)}{x(x-3) - 3(x-3)} \times \frac{x}{3(x+3)}$$

$$= \frac{(x+3)(x-3)}{(x-3)(x-3)} \times \frac{x}{3(x+3)}$$

$$= \frac{x}{3(x-3)}$$

$$= \frac{x}{3x - 9}$$

Q.30 $\frac{x+5}{x^2 + 6x} \times \frac{x^3 + 6x^2}{x+5}$

Sol. $= \frac{(x+5)}{x(x+6)} \times \frac{x^2(x+6)}{(x+5)}$

$$\begin{aligned}
 &= \frac{x^2}{x} \\
 &= x^{2-1} \\
 &= x
 \end{aligned}$$

Q.31 $\frac{x^2 - 2x + 1}{x^2 - 1} \times \frac{x+1}{x-1}$

Sol.
$$\begin{aligned}
 &= \frac{x^2 - x - x + 1}{(x)^2 - (1)^2} \times \frac{x+1}{x-1} \\
 &= \frac{x(x-1) - 1(x-1)}{(x+1)(x-1)} \times \frac{(x+1)}{(x-1)} \\
 &= \frac{(x-1)(x-1)}{(x+1)(x-1)} \times \frac{(x+1)}{(x-1)} \\
 &= 1
 \end{aligned}$$

Q.32 $\frac{x^2 + 4x + 3}{x+3} \times \frac{x^2 - 2x + 1}{x^2 - 1}$

Sol.
$$\begin{aligned}
 &= \frac{x^2 + x + 3x + 3}{x+3} \times \frac{x^2 - x - x + 1}{(x)^2 - (1)^2} \\
 &= \frac{x(x+1) + 3(x+1)}{(x+3)} \times \frac{x(x-1) - 1(x-1)}{(x-1)(x+1)} \\
 &= \frac{(x+1)(x+3)}{(x+3)} \times \frac{(x-1)(x-1)}{(x-1)(x+1)} \\
 &= x - 1
 \end{aligned}$$

Formulae

- (i) $(a + b)^2 = a^2 + 2ab + b^2$
- (ii) $(a - b)^2 = a^2 - 2ab + b^2$
- (iii) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
- (iv) $(a + b)^2 - (a - b)^2 = 4ab$
- (v) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
 $= a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- (vi) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$\begin{aligned}
 &= a^3 + b^3 + 3ab(a + b) \\
 \text{(vii)} \quad (a - b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3 \\
 &= a^3 - b^3 - 3ab(a - b) \\
 \text{(viii)} \quad a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\
 \text{(ix)} \quad a^3 - b^3 &= (a - b)(a^2 + ab + b^2)
 \end{aligned}$$

EXERCISES

Solve the following questions using formulas.

Q.1 $(x + 2y)^2 + (x - 2y)^2$

Sol. $= 2[(x)^2 + (2y)^2]$ [Formula: $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$]
 $= 2[x^2 + 4y^2]$
 $= 2x^2 + 8y^2$

Q.2 $(5x + 3y)^2 + (5x - 3y)^2$

Sol. $= 2[(5x)^2 + (3y)^2]$ [Formula: $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$]
 $= 2[25x^2 + 9y^2]$
 $= 50x^2 + 18y^2$

Q.3 $(3l + 2m)^2 - (3l - 2m)^2$

Sol. $= 4(3l)(2m)$ [Formula: $(a + b)^2 - (a - b)^2 = 4ab$]
 $= 24lm$

Q.4 $(l + m)(l - m)(l^2 + m^2)(l^4 + m^4)$

Sol. $= [(l + m)(l - m)](l^2 + m^2)(l^4 + m^4)$
[Formula: $(a + b)(a - b) = a^2 - b^2$]
 $= (l^2 - m^2)(l^2 + m^2)(l^4 + m^4)$
 $= [(l^2)^2 - (m^2)^2](l^4 + m^4)$
 $= (l^4 - m^4)(l^4 + m^4)$
 $= (l^4)^2 - (m^4)^2$
 $= l^8 - m^8$

Q.5 $\left(ab - \frac{1}{ab}\right)^3$

Sol. [Formula: $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$]

$$= (ab)^3 - 3(ab)^2 \left(\frac{1}{ab}\right) + 3ab \left(\frac{1}{ab}\right)^2 - \left(\frac{1}{ab}\right)^3$$

$$= a^3b^3 - 3ab + \frac{3}{ab} - \frac{1}{a^3b^3}$$

Q.6 $(2x + 3y + 2)^2$

Sol. [(a + b + c)² = a² + b² + c² + 2ab + 2bc + 2ca]

$$= (2x)^2 + (3y)^2 + (2)^2 + 2(2x)(3y) + 2(3y)(2) + 2(2)(2x)$$

$$= 4x^2 + 9y^2 + 4 + 12xy + 12y + 8x$$

Q.7 $(2p + q)^3$

Sol. [Formula: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]

$$= (2p)^3 + 3(2p)^2(q) + 3(2p)(q)^2 + (q)^3$$

$$= 8p^3 + 3(4p^2)(q) + 6pq^2 + q^3$$

$$= 8p^3 + 12p^2q + 6pq^2 + q^3$$

Q.8 $(3p + q + r)^2$

Sol. [Formula: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$]

$$= (3p)^2 + (q)^2 + (r)^2 + 2(3p)(q) + 2(q)(r) + 2(r)(3p)$$

$$= 9p^2 + q^2 + r^2 + 6pq + 2qr + 6rp$$

Q.9 $(2x + 3y)^3$

Sol. [Formula: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]

$$= (2x)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2 + (3y)^3$$

$$= 8x^3 + 3(4x^2)(3y) + 3(2x)(9y^2) + 9y^3$$

$$= 8x^3 + 36x^2y + 54xy^2 + 9y^3$$

Q.10 $(x + y)^3 - 1$

Sol.
$$= (x + y)^3 - (1)^3$$

[Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$$= (x + y - 1)[(x + y)^2 + (x + y)(1) + (1)^2]$$

$$= (x + y - 1)[x^2 + y^2 + 2xy + x + y + 1]$$

Q.11 $(x - y)^3 + 64$

Sol. $= (x - y)^3 + (4)^3$

$$\begin{aligned} & \text{[Formula: } a^3 + b^3 = (a + b)(a^2 - ab + b^2)\text{]} \\ &= (x - y + 4)[(x - y)^2 - (x - y)(4) + (4)^2] \\ &= (x - y + 4)(x^2 + y^2 + 2xy - 4x + 4y + 16) \end{aligned}$$

Q.12 $8x^3 + 27y^3$

Sol. $= (2x)^3 + (3y)^3$

$$\begin{aligned} & \text{[Formula: } a^3 + b^3 = (a + b)(a^2 - ab + b^2)\text{]} \\ &= (2x + 3y)[(2x)^2 - (2x)(3y) + (3y)^2] \\ &= (2x + 3y)(4x^2 - 6xy + 9y^2) \end{aligned}$$

Q.13 $x^6 - 729y^6$

Sol. $= (x^3)^2 - (27y^3)^2$ [Formula: $a^2 - b^2 = (a + b)(a - b)$]

$$\begin{aligned} &= (x^3 + 27y^3)(x^3 - 27y^3) \\ &= [(x)^3 + (3y)^3][(x)^3 - (3y)^3] \end{aligned}$$

(Formulas)

$$\begin{aligned} a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\ a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \\ &= (x + 3y)(x^2 - 3xy + 9y^2)(x - 3y)(x^2 + 3xy + 9y^2) \\ &= (x + 3y)(x - 3y)(x^2 - 3xy + 9y^2)(x^2 + 3xy + 9y^2) \end{aligned}$$

Q.14 $64a^6 - b^6$

Sol. $= (8a^3)^2 - (b^3)^2$

$$\begin{aligned} &= (8a^3 - b^3)(8a^3 + b^3) \quad \text{[Formula: } a^2 - b^2 = (a - b)(a + b)\text{]} \\ &= [(2a)^3 - (b)^3][(2a)^3 + (b)^3] \\ & \quad a^3 + b^3 = (a + b)(a^2 - ab + b^2) \end{aligned}$$

(Formulas)

$$\begin{aligned} a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \\ &= (2a - b)(4a^2 + 2ab + b^2)(2a + b)(4a^2 - 2ab + b^2) \\ &= (2a - b)(2a + b)(4a^2 + 2ab + b^2)(4a^2 - 2ab + b^2) \end{aligned}$$

Q.15 Find the value of $a^3 - b^3$ when $a - b = 4$ and $ab = -5$.

Sol. $a - b = 4$

$ab = -5$

$a^3 - b^3 = ?$

$a - b = 4$

$(a - b)^3 = (4)^3$

Taking cube

$a^3 - b^3 - 3(-5)(4) = 64$

$a^3 - b^3 + 60 = 64$

$a^3 - b^3 = 64 - 60$

$a^3 - b^3 = 4$

Q.16 Show that $\left(z + \frac{1}{z}\right)^2 - \left(z - \frac{1}{z}\right)^2 = 4$

L.H.S $= \left(z + \frac{1}{z}\right)^2 - \left(z - \frac{1}{z}\right)^2$

$= \left(z^2 + 2(z)\left(\frac{1}{z}\right) + \frac{1}{z^2}\right) - \left(z^2 - 2(z)\left(\frac{1}{z}\right) + \frac{1}{z^2}\right)$

$= \left(z^2 + 2 + \frac{1}{z^2}\right) - \left(z^2 - 2 + \frac{1}{z^2}\right)$

$= z^2 + 2 + \frac{1}{z^2} - z^2 + 2 - \frac{1}{z^2}$

$= 4 = \text{R.H.S}$

Q.17 Find the value of $a^2 + b^2$ and ab when $a + b = 5$ and

$a - b = 3$.

Sol. $a + b = 5$

$a - b = 3$

$a^2 + b^2 = ?$

$ab = ?$

Formula

$$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

$$(5)^2 + (3)^2 = 2(a^2 + b^2)$$

$$25 + 9 = 2(a^2 + b^2)$$

$$34 = 2(a^2 + b^2)$$

$$\text{Thus, } a^2 + b^2 = \frac{34}{2} = 17$$

$$\text{Formula: } (a + b)^2 - (a - b)^2 = 4ab$$

$$(5)^2 - (3)^2 = 4ab$$

$$25 - 9 = 4ab$$

$$4ab = 16$$

$$ab = \frac{16}{4}$$

$$ab = 4$$

Q.18 Find the value of $a^2 + b^2 + c^2$ if $ab + bc + ca = 11$ and

$$a + b + c = 6$$

Sol. $a + b + c = 6$

$$ab + bc + ca = 11$$

$$a^2 + b^2 + c^2 = ?$$

$$a + b + c = 6$$

$$(a + b + c)^2 = (6)^2 \quad \text{Taking square root}$$

$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 36$$

$$\text{By putting values } ab + bc + ca = 11$$

$$a^2 + b^2 + c^2 + 2(11) = 36$$

$$a^2 + b^2 + c^2 + 22 = 36$$

$$a^2 + b^2 + c^2 = 36 - 22$$

$$a^2 + b^2 + c^2 = 14$$

Q.19 Find the value of $x^3 + y^3$ if $xy = 10$ and $x + y = 7$.

Sol. $x + y = 7$

$$xy = 10$$

$$x^3 + y^3 = ?$$

$$x + y = 7$$

$$(x + y)^3 = (7)^3 \quad \text{Taking cube}$$

$$x^3 + y^3 + 3xy(x + y) = 343$$

$$x^3 + y^3 + 3(10)(7) = 343$$

$$x^3 + y^3 + 210 = 343$$

$$x^3 + y^3 = 343 - 210$$

$$x^3 + y^3 = 133$$

Q.20 Find the value of $(x - y)^2$ if $x^2 + y^2 = 86$ and $xy = -16$.

Sol. $xy = -16 \quad \text{(i)}$

$$x^2 + y^2 = 86 \quad \text{(ii)}$$

$$(x - y)^2 = ?$$

$$(x - y)^2 = x^2 + y^2 - 2xy$$

$$= (x^2 + y^2) - 2(xy)$$

$$= 86 - 2(-16) \quad \text{From (i) and (ii)}$$

$$= 86 + 32$$

$$= 118$$

Q.21 Find the value of $ab + bc + ca$ when the values of

$$a^2 + b^2 + c^2 = 81, a + b + c = 11$$

Sol. $a + b + c = 11 \dots\dots\dots \text{(i)}$

$$a^2 + b^2 + c^2 = 81 \dots\dots\dots \text{(ii)}$$

$$ab + bc + ca = ?$$

$$a + b + c = 11$$

$$(a + b + c)^2 = (11)^2 \quad \text{Taking square root}$$

$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 121$$

$$81 + 2(ab + bc + ca) = 121 \quad \text{from (ii)}$$

$$2(ab + bc + ca) = 121 - 81$$

$$2(ab + bc + ca) = 40$$

$$\begin{aligned} ab + bc + ca &= \frac{40}{2} \\ ab + bc + ca &= 20 \end{aligned}$$

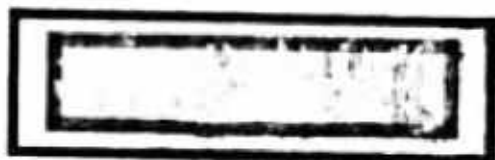
Q.22 Find the value of $(a + b + c)^2$ when the values of $a^2 + b^2 + c^2 = 32$ and $ab + bc + ca = 7$.

Sol.

$$\begin{aligned} ab + bc + ca &= 7 & (i) \\ a^2 + b^2 + c^2 &= 32 & (ii) \\ (a + b + c)^2 &= ? \\ (a + b + c)^2 &= a^2 + b^2 + c^2 + 2(ab + bc + ca) \\ &= 32 + 2(7) & \text{From (i) and (ii)} \\ &= 32 + 14 \\ (a + b + c)^2 &= 46 \end{aligned}$$

Laws of Radicals

$$\begin{aligned} (i) \quad (\sqrt[n]{a})^n &= a & (ii) \quad \sqrt[n]{ab} &= \sqrt[n]{a} \sqrt[n]{b} \\ (iii) \quad \sqrt[n]{\frac{a}{b}} &= \frac{\sqrt[n]{a}}{\sqrt[n]{b}} & (iv) \quad (\sqrt[n]{a})^m &= \sqrt[n]{a^m} \end{aligned}$$



Q.1 Remove the radical sign from the denominator:

(i) $\frac{1}{\sqrt{5}}$

Sol.

$$\begin{aligned} &= \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{\sqrt{5}}{(\sqrt{5})^2} \\ &= \frac{\sqrt{5}}{5} \end{aligned}$$

$$(ii) \quad \frac{2}{\sqrt{2}} \cdot \frac{7}{\sqrt{3}}$$

Sol.

$$\begin{aligned} &= \frac{2 \times 7}{\sqrt{2 \times 3}} \\ &= \frac{14}{\sqrt{6}} \\ &= \frac{14}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} \\ &= \frac{14 \times \sqrt{6}}{6} = \frac{7\sqrt{6}}{3} \end{aligned}$$

$$(iii) \quad \frac{\sqrt{6}}{\sqrt{7}}$$

Sol.

$$\begin{aligned} &= \frac{\sqrt{6}}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \\ &= \frac{\sqrt{6 \times 7}}{\sqrt{7 \times 7}} \\ &= \frac{\sqrt{42}}{7} \end{aligned}$$

Q.2 Simplify these expressions:

$$(i) \quad \sqrt{2} + \sqrt{8}$$

Sol.

$$\begin{aligned} &= \sqrt{2} + \sqrt{2 \times 2 \times 2} \\ &= \sqrt{2} + 2\sqrt{2} \\ &= \sqrt{2}(1 + 2) \\ &= 3\sqrt{2} \end{aligned}$$

(ii) $4\sqrt{50} + \sqrt{200} + \sqrt{50}$

Sol.

$$\begin{aligned} &= 4\sqrt{5 \times 5 \times 2} + \sqrt{5 \times 5 \times 2 \times 2 \times 2} + \sqrt{5 \times 5 \times 2} \\ &= 4 \times 5\sqrt{2} + 5 \times 2\sqrt{2} + 5\sqrt{2} \\ &= 20\sqrt{2} + 10\sqrt{2} + 5\sqrt{2} \\ &= \sqrt{2}(20 + 10 + 5) \\ &= \sqrt{2}(35) \\ &= 35\sqrt{2} \end{aligned}$$

(iii) $(\sqrt{12} - \sqrt{2})(\sqrt{20} - 3\sqrt{2})$

Sol.

$$\begin{aligned} &= \sqrt{12} \times \sqrt{20} - \sqrt{12} \times 3\sqrt{2} - \sqrt{2} \times \sqrt{20} + \sqrt{2} \times 3\sqrt{2} \\ &= \sqrt{2 \times 2 \times 3} \times \sqrt{2 \times 2 \times 5} - \sqrt{2 \times 2 \times 3} \times 3\sqrt{2} \\ &\quad - \sqrt{2} \times \sqrt{2 \times 2 \times 5} + 3\sqrt{2 \times 2} \\ &= 2\sqrt{3} \times 2\sqrt{5} - 2\sqrt{3} \times 3\sqrt{2} - \sqrt{2} \times 2\sqrt{5} + 3 \times 2 \\ &= 4\sqrt{3 \times 5} - 6\sqrt{3 \times 2} - 2\sqrt{2 \times 5} + 6 \\ &= 4\sqrt{15} - 6\sqrt{6} - 2\sqrt{10} + 6 \end{aligned}$$

(iv) $(6 + \sqrt{2})(5 - \sqrt{5})$

Sol.

$$\begin{aligned} &= 6(5 - \sqrt{5}) + \sqrt{2}(5 - \sqrt{5}) \\ &= 30 - 6\sqrt{5} + 5\sqrt{2} - \sqrt{2 \times 5} \\ &= 30 - 6\sqrt{5} + 5\sqrt{2} - \sqrt{10} \end{aligned}$$

(v) $(\sqrt{3} - 2)(5 - \sqrt{5})$

Sol.

$$\begin{aligned} &= \sqrt{3}(5 - \sqrt{5}) - 2(5 - \sqrt{5}) \\ &= 5\sqrt{3} - \sqrt{3 \times 5} - 10 + 2\sqrt{5} \\ &= 5\sqrt{3} - \sqrt{15} - 10 + 2\sqrt{5} \end{aligned}$$

(vi) $(7 + \sqrt{3})(5 + \sqrt{2})$

Sol.

$$\begin{aligned} &= 7(5 + \sqrt{2}) + \sqrt{3}(5 + \sqrt{2}) \\ &= 35 + 7\sqrt{2} + 5\sqrt{3} + \sqrt{3} \times 2 \\ &= 35 + 7\sqrt{2} + 5\sqrt{3} + \sqrt{6} \end{aligned}$$

Q.3 Rationalize the denominators of the following:

(i) $\frac{1}{\sqrt{3} + 2}$

Sol.

$$\begin{aligned} &= \frac{1}{\sqrt{3} + 2} \times \frac{\sqrt{3} - 2}{\sqrt{3} - 2} \\ &= \frac{\sqrt{3} - 2}{(\sqrt{3})^2 - (2)^2} \\ &= \frac{\sqrt{3} - 2}{3 - 4} \\ &= \frac{\sqrt{3} - 2}{-1} \\ &= 2 - \sqrt{3} \end{aligned}$$

(ii) $\frac{1}{4 - \sqrt{5}}$

Sol.

$$\begin{aligned} &= \frac{1}{4 - \sqrt{5}} \times \frac{4 + \sqrt{5}}{4 + \sqrt{5}} \\ &= \frac{4 + \sqrt{5}}{(4)^2 - (\sqrt{5})^2} \end{aligned}$$

$$= \frac{4 + \sqrt{5}}{16 - 5}$$

$$= \frac{4 + \sqrt{5}}{11}$$

$$(iii) \quad \frac{4\sqrt{3}}{\sqrt{7} + \sqrt{5}}$$

Sol.

$$= \frac{4\sqrt{3}}{\sqrt{7} + \sqrt{5}} \times \frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} - \sqrt{5}}$$

$$= \frac{(4\sqrt{3})(\sqrt{7} - \sqrt{5})}{(\sqrt{7})^2 - (\sqrt{5})^2}$$

$$= \frac{(4\sqrt{3})(\sqrt{7} - \sqrt{5})}{7 - 5}$$

$$= \frac{4\sqrt{3}(\sqrt{7} - \sqrt{5})}{2}$$

$$2\sqrt{3}(\sqrt{7} - \sqrt{5})$$

$$(iv) \quad \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}$$

Sol.

$$= \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}} \times \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} - \sqrt{y}}$$

$$= \frac{(\sqrt{x} - \sqrt{y})^2}{(\sqrt{x})^2 - (\sqrt{y})^2}$$

$$= \frac{x + y - 2\sqrt{xy}}{x - y}$$

$$(v) \quad \frac{5\sqrt{7}}{2+3\sqrt{7}}$$

Sol.

$$\begin{aligned} &= \frac{5\sqrt{7}}{2+3\sqrt{7}} \times \frac{2-3\sqrt{7}}{2-3\sqrt{7}} \\ &= \frac{5\sqrt{7}(2-3\sqrt{7})}{(2)^2 - (3\sqrt{7})^2} \\ &= \frac{(5\sqrt{7})(2) - (5\sqrt{7})(3\sqrt{7})}{4 - 9 \times 7} \\ &= \frac{10\sqrt{7} - 105}{4 - 63} \\ &= \frac{10\sqrt{7} - 105}{-59} \\ &= \frac{105 - 10\sqrt{7}}{59} \end{aligned}$$

$$(vi) \quad \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

Sol.

$$\begin{aligned} &= \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} \\ &= \frac{(\sqrt{3} + \sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2} \\ &= \frac{3 + 2 + 2\sqrt{3} \times 2}{3 - 2} \\ &= \frac{5 + 2\sqrt{6}}{1} \\ &= 5 + 2\sqrt{6} \end{aligned}$$

$$(vii) \quad \frac{29}{11+3\sqrt{5}}$$

Sol.

$$\begin{aligned} &= \frac{29}{11+3\sqrt{5}} \times \frac{11-3\sqrt{5}}{11-3\sqrt{5}} \\ &= \frac{29(11-3\sqrt{5})}{(11)^2 - (3\sqrt{5})^2} \\ &= \frac{29(11-3\sqrt{5})}{121-9 \times 5} \\ &= \frac{29(11-3\sqrt{5})}{121-45} \\ &= \frac{29(11-3\sqrt{5})}{76} \end{aligned}$$

$$(viii) \quad \frac{17}{3\sqrt{7}+2\sqrt{3}}$$

Sol.

$$\begin{aligned} &= \frac{17}{3\sqrt{7}+2\sqrt{3}} \times \frac{3\sqrt{7}-2\sqrt{3}}{3\sqrt{7}-2\sqrt{3}} \\ &= \frac{17(3\sqrt{7}-2\sqrt{3})}{(3\sqrt{7})^2 - (2\sqrt{3})^2} \\ &= \frac{17(3\sqrt{7}-2\sqrt{3})}{9 \times 7 - 4 \times 3} \\ &= \frac{17(3\sqrt{7}-2\sqrt{3})}{63-12} \\ &= \frac{17(3\sqrt{7}-2\sqrt{3})}{51} \\ &= \frac{3\sqrt{7}-2\sqrt{3}}{3} \end{aligned}$$

Q.4 If $x = \sqrt{5} + 2$, then find the values of (i) $x + \frac{1}{x}$ and

(ii) $x^2 + \frac{1}{x^2}$.

Sol. (i) $x = \sqrt{5} + 2$(1)

$$\frac{1}{x} = \frac{1}{\sqrt{5} + 2}$$

$$= \frac{1}{\sqrt{5} + 2} \times \frac{\sqrt{5} - 2}{\sqrt{5} - 2}$$

$$= \frac{\sqrt{5} - 2}{(\sqrt{5})^2 - (2)^2}$$

$$= \frac{\sqrt{5} - 2}{5 - 4}$$

$$\frac{1}{x} = \sqrt{5} - 2$$
.....(2)

$$x + \frac{1}{x} = \sqrt{5} + 2 + \sqrt{5} - 2$$

from (1) + (2)

Thus

$$\boxed{x + \frac{1}{x} = 2\sqrt{5}}$$

(ii) $\left(x + \frac{1}{x}\right)^2 = (2\sqrt{5})^2$

Taking square root

$$x^2 + \frac{1}{x^2} + 2 = 4 \times 5$$

$$x^2 + \frac{1}{x^2} + 2 = 20$$

$$x^2 + \frac{1}{x^2} = 20 - 2$$

Hence,

$$\boxed{x^2 + \frac{1}{x^2} = 18}$$

Q.5 If $x = 2 + \sqrt{3}$, then find the values of (i) $x - \frac{1}{x}$ and

(ii) $x^2 + \frac{1}{x^2}$.

Sol. (i) $x = 2 + \sqrt{3}$ (1)

$$\frac{1}{x} = \frac{1}{2 + \sqrt{3}}$$

$$= \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} \text{ (Rationalize the denominators)}$$

$$= \frac{2 - \sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$= \frac{2 - \sqrt{3}}{4 - 3} = \frac{2 - \sqrt{3}}{1} = 2 - \sqrt{3} \text{(2)}$$

$$x - \frac{1}{x} = (2 + \sqrt{3}) - (2 - \sqrt{3}) \quad \text{from (1), (2)}$$

$$x - \frac{1}{x} = 2 + \sqrt{3} - 2 + \sqrt{3}$$

Hence,

$$\boxed{x - \frac{1}{x} = 2\sqrt{3}}$$

(ii) $\left(x - \frac{1}{x}\right)^2 = (2\sqrt{3})^2$ Taking square root

$$x^2 + \frac{1}{x^2} - 2 = 4 \times 3$$

$$x^2 + \frac{1}{x^2} - 2 = 12$$

Hence,

$$\boxed{x^2 + \frac{1}{x^2} = 12 + 2 = 14}$$

Q.6 If $x = \sqrt{3} - \sqrt{2}$, then find the values of (i) $x - \frac{1}{x}$ and

(ii) $x^2 + \frac{1}{x^2}$.

Sol. (i) $x = \sqrt{3} - \sqrt{2}$ (1)

$$\begin{aligned}\frac{1}{x} &= \frac{1}{\sqrt{3} - \sqrt{2}} \\ &= \frac{1}{(\sqrt{3} - \sqrt{2})} \times \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} + \sqrt{2})} \\ &= \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3})^2 - (\sqrt{2})^2} \\ &= \frac{(\sqrt{3} + \sqrt{2})}{3 - 2}\end{aligned}$$

$$\frac{1}{x} = \sqrt{3} + \sqrt{2} \text{(2)}$$

$$\begin{aligned}x - \frac{1}{x} &= (\sqrt{3} - \sqrt{2}) - (\sqrt{3} + \sqrt{2}) \quad \text{from (1), (2)} \\ &= \sqrt{3} - \sqrt{2} - \sqrt{3} - \sqrt{2}\end{aligned}$$

Hence, $\boxed{x - \frac{1}{x} = -2\sqrt{2}}$

(ii) $\left(x - \frac{1}{x}\right)^2 = (-2\sqrt{2})^2$ Taking square root

$$x^2 + \frac{1}{x^2} - 2 = 4 \times 2$$

$$x^2 + \frac{1}{x^2} - 2 = 8$$

$$x^2 + \frac{1}{x^2} = 8 + 2$$

Hence, $\boxed{x^2 + \frac{1}{x^2} = 10}$

Q.7 If $\frac{1}{x} = 3 - \sqrt{2}$, then evaluate (i) $x + \frac{1}{x}$ and (ii) $x - \frac{1}{x}$.

Sol. (i) $\frac{1}{x} = 3 - \sqrt{2}$ (1)

$$\begin{aligned}
 x &= \frac{1}{3-\sqrt{2}} \\
 &= \frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} \\
 &= \frac{3+\sqrt{2}}{(3)^2 - (\sqrt{2})^2} \\
 &= \frac{3+\sqrt{2}}{9-2} \\
 x &= \frac{3+\sqrt{2}}{7} \dots\dots\dots(2)
 \end{aligned}$$

$$\begin{aligned}
 x + \frac{1}{x} &= \frac{3+\sqrt{2}}{7} + 3-\sqrt{2} \quad \text{Now from (1) + (2)} \\
 &= \frac{3+\sqrt{2} + 7(3-\sqrt{2})}{7} \\
 &= \frac{3+\sqrt{2} + 21-7\sqrt{2}}{7} \\
 &= \frac{24+\sqrt{2}-7\sqrt{2}}{7} \\
 &= \frac{24+\sqrt{2}(1-7)}{7} \\
 &= \frac{24+\sqrt{2}(-6)}{7}
 \end{aligned}$$

$$\text{Hence, } \boxed{x + \frac{1}{x} = \frac{24-6\sqrt{2}}{7}}$$

$$\begin{aligned}
 \text{(ii)} \quad x - \frac{1}{x} &= \frac{3+\sqrt{2}}{7} - (3-\sqrt{2}) \quad \text{from (1),(2)} \\
 &= \frac{3+\sqrt{2}}{7} - 3 + \sqrt{2}
 \end{aligned}$$

$$= \frac{3 + \sqrt{2} - 21 + 7\sqrt{2}}{7}$$

$$= \frac{-18 + \sqrt{2} + 7\sqrt{2}}{7}$$

$$= \frac{-18 + \sqrt{2}(1 + 7)}{7}$$

Hence,

$$\boxed{x - \frac{1}{x} = \frac{-18 + 8\sqrt{2}}{7}}$$

Q.8 If $\frac{1}{p} = \sqrt{10} + 3$, then evaluate (i) $\left(p + \frac{1}{p}\right)^2$ and

(ii) $\left(p - \frac{1}{p}\right)^2$

Sol.

(i) $\frac{1}{p} = \sqrt{10} + 3 \dots\dots\dots(1)$

$$p = \frac{1}{\sqrt{10} + 3}$$

$$p = \frac{1}{\sqrt{10} + 3} \times \frac{\sqrt{10} - 3}{\sqrt{10} - 3} \text{ (Rationalize the denominators)}$$

$$= \frac{\sqrt{10} - 3}{(\sqrt{10})^2 - (3)^2}$$

$$= \frac{\sqrt{10} - 3}{10 - 9}$$

$$= \frac{\sqrt{10} - 3}{1}$$

$$= \sqrt{10} - 3 \dots\dots\dots(2)$$

$$p + \frac{1}{p} = (\sqrt{10} - 3) + (\sqrt{10} + 3) \quad \text{from (1)+(2)}$$

$$= \sqrt{10} - 3 + \sqrt{10} + 3$$

$$p + \frac{1}{p} = 2\sqrt{10}$$

$$\left(p + \frac{1}{p}\right)^2 = (2\sqrt{10})^2 \quad \text{Taking square root}$$

$$= 4 \times 10$$

$$\text{Hence, } \boxed{\left(p + \frac{1}{p}\right)^2 = 40}$$

$$(ii) \quad p - \frac{1}{p} = (\sqrt{10} - 3) - (\sqrt{10} + 3) \quad \text{Now from (1) - (2)}$$

$$= \sqrt{10} - 3 - \sqrt{10} - 3$$

$$p - \frac{1}{p} = -6$$

$$\left(p - \frac{1}{p}\right)^2 = (-6)^2 \quad \text{Taking square root}$$

$$\text{Hence, } \boxed{\left(p - \frac{1}{p}\right)^2 = 36}$$

Q.9 Rationalize

$$(i) \quad \frac{b + \sqrt{b^2 - a^2}}{b - \sqrt{b^2 - a^2}}$$

$$\text{Sol.} \quad = \frac{b + \sqrt{b^2 - a^2}}{b - \sqrt{b^2 - a^2}} \times \frac{b + \sqrt{b^2 - a^2}}{b + \sqrt{b^2 - a^2}}$$

$$\begin{aligned}
&= \frac{(b + \sqrt{b^2 - a^2})^2}{(b)^2 - (\sqrt{b^2 - a^2})^2} \\
&= \frac{b^2 + b^2 - a^2 + 2b\sqrt{b^2 - a^2}}{b^2 - (b^2 - a^2)} \\
&= \frac{2b^2 - a^2 + 2b\sqrt{b^2 - a^2}}{b^2 - b^2 + a^2} \\
&= \frac{2b^2 - a^2 + 2b\sqrt{b^2 - a^2}}{a^2}
\end{aligned}$$

$$(ii) \quad \frac{\sqrt{a+3} - \sqrt{a-3}}{\sqrt{a+3} + \sqrt{a-3}}$$

Sol.

$$\begin{aligned}
&= \frac{\sqrt{a+3} - \sqrt{a-3}}{\sqrt{a+3} + \sqrt{a-3}} \times \frac{\sqrt{a+3} - \sqrt{a-3}}{\sqrt{a+3} - \sqrt{a-3}} \\
&= \frac{(\sqrt{a+3} - \sqrt{a-3})^2}{(\sqrt{a+3})^2 - (\sqrt{a-3})^2} \\
&= \frac{(a+3) + (a-3) - 2\sqrt{(a+3)(a-3)}}{(a+3) - (a-3)} \\
&= \frac{a+3+a-3-2\sqrt{(a)^2 - (3)^2}}{a+3-a+3} \\
&= \frac{2a-2\sqrt{a^2-9}}{6} \\
&= \frac{2(a-\sqrt{a^2-9})}{6} \\
&= \frac{a-\sqrt{a^2-9}}{3}
\end{aligned}$$