

Exercise 9.1

1. Prove that, only the diameters of a circle are the intersecting chords which bisect each other.

Given: A circle having diameters $\overline{AC}$ and $\overline{BD}$ which passes through centre O.



To Prove: Diameters $\overline{AC}$ and $\overline{BD}$ bisect each other.

Proof:

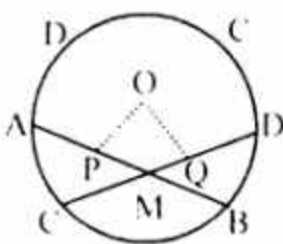
Statements	Reasons
$\overline{OA} \cong \overline{OC}$ (i)	Common
Similarly, $\overline{OB} \cong \overline{OD}$ (ii)	
$m\overline{OA} = m\overline{OD}$ (iii)	Radii of the same circle
From (i), (ii) and (iii), we have $m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD}$	

Hence AC and BD are intersecting chords which bisect each other.

2. Two chords of a circle do not pass through the center. Prove that they cannot bisect each other.

Given:

A circle with center O having two chords $\overline{AB}$ and $\overline{CD}$.



To Prove:

M is not the mid-point of chords $\overline{AB}$ and $\overline{CD}$.

Construction:

Join O to P and Q such that $\overline{OP} \perp \overline{AB}$ and $\overline{OQ} \perp \overline{CD}$.

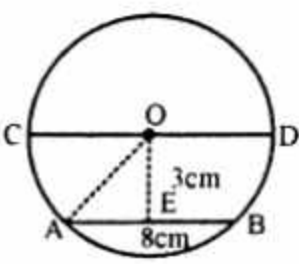
Proof:

Statements	Reasons
O is the center of the circle with $\overline{OP} \perp \overline{AB}$. Thus $\overline{OP} \perp \overline{AB}$ Now point M lies between P and B. Therefore, M is not the midpoint of AB. Hence $\overline{AB}$ and $\overline{CD}$ cannot bisect each other.	Construction

3. If the length of the chord AB = 8 cm. Its distance from the center is 3 cm, then measure the diameter of such circle.

Given:

$mAB = 8\text{cm}$, $mOE = 3\text{cm}$



Required:

to find the length of diameter
i.e., $mCD = ?$

Construction:

Join O to A and E.

Proof:

Statements	Reasons
In $\triangle AEO$ $(AO)^2 = \overline{AE}^2 + \overline{EO}^2$ $= \left[\frac{1}{2}(\overline{AB})\right]^2 + (3)^2$ $= \left[\frac{1}{2} \times 8\right]^2 + 9$ $= (4)^2 + 9$ $= 16 + 9 = 25\text{cm}$ $\Rightarrow \overline{AO} = \sqrt{25} = 5\text{cm}$	

$m\overline{AO} = m\overline{OC} = m\overline{OD} = 5\text{cm}$

$\Rightarrow \overline{CD} = \overline{CO} + m\overline{OD}$
 $= 5\text{cm} + 5\text{cm}$
 $= 10\text{cm}$

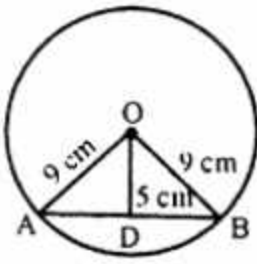
Hence

Diameter = 10cm

4. Calculate the length of a chord which stands at a distance 5cm from the center of a circle whose radius is 9cm.

Given:

$m\overline{OA} = m\overline{OB} = 9\text{cm}$,



1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.

$m\overline{OD} = 5\text{cm}$

Required:

$m\overline{AB} = ?$

Proof:

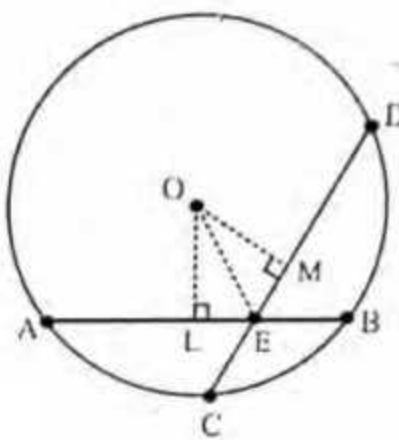
| Statements | Reasons |
|---|---|
| In $\triangle OAD$
$m\overline{OA}^2 = m\overline{OD}^2 + m\overline{AD}^2$
$m\overline{OA}^2 - m\overline{OD}^2 = m\overline{AD}^2$
$9^2 - 5^2 = \left[\frac{1}{2}m(\overline{AB})\right]^2$
$\left[\frac{1}{2}m(\overline{AB})\right]^2 = 81 - 25$
$\frac{1}{4}m(\overline{AB})^2 = 56$
$\Rightarrow m\overline{AB}^2 = 56 \times 4 = 224$
$AB = 14.97\text{cm}$ | $\left[\because AD = \frac{1}{2}\overline{AB}\right]$ |

Exercise 9.2

1. Two equal chords of a circle intersect, show that the segments of the one are equal corresponding to the segments of the other.

Given:

A circle with center "O". Two equal chords $\overline{AB}$ and $\overline{CD}$ (i.e. $m\overline{AB} = m\overline{CD}$) intersect each other at point E.



To Prove:

$m\overline{AE} = m\overline{ED}$ and $m\overline{EB} = m\overline{EC}$

Construction:

Draw perpendiculars $\overline{OL}$ and $\overline{OM}$ from the center "O" to the chords $\overline{AB}$ and $\overline{CD}$ respectively. L and M are the midpoints of $\overline{AB}$ and $\overline{CD}$ respectively.

Proof:

| Statements | Reasons |
|--|--|
| In $\triangle OLE \leftrightarrow \triangle OME$
$\overline{OL} \cong \overline{OM}$

$m\angle OLE = m\angle OME = 90^\circ$
$m\overline{OE} \cong m\overline{OE}$
$\therefore \triangle OLE \cong \triangle OME$ | Two equal chords of a circle are equidistant from the center.
$\overline{OL} \perp \overline{AB}$ and $\overline{OM} \perp \overline{CD}$
Common side
H.S $\cong$ H.S |

| | |
|--|--|
| $\overline{LE} \cong \overline{ME} \dots\dots (i)$

$m\overline{AL} = \frac{1}{2}m\overline{AB}$

$m\overline{DM} = \frac{1}{2}m\overline{CD}$

$m\overline{AL} = \overline{DM} \dots\dots (ii)$
$m\overline{AL} + m\overline{LE} = m\overline{DM} + m\overline{ME}$

$m\overline{AE} = m\overline{DE} \dots\dots (iii)$
Now, $m\overline{AB} = m\overline{CD}$

$m\overline{AE} + m\overline{EB} = m\overline{DE} + m\overline{EC}$

$m\overline{AE} + m\overline{EB} = m\overline{AE} + m\overline{EC}$

$m\overline{EB} = m\overline{EC}$ | Corresponding sides of congruent triangles

Both are half of equal chords.
Adding (i) and (ii).

Given

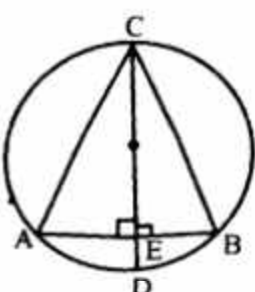
From (iii)
By cancellation property. |
|--|--|

2. AB is the chord of a circle and the diameter CD is perpendicular bisector of AB . Prove that $m\overline{AC} = m\overline{BC}$.

Given:

In a circle.

$AB \perp CD$ and $AE \cong EB$



To Prove:

$m\overline{AC} = m\overline{BC}$

Construction:

Join C to A and B.

Proof:

| Statements | Reasons |
|---|--|
| In $\triangle ACE \leftrightarrow \triangle ECB$

$AE \cong EB$

$m\angle AEC = m\angle CEB$

$CE \cong CE$

$\therefore \triangle AEC \cong \triangle ECB$

$\Rightarrow \overline{AC} \cong \overline{BC}$

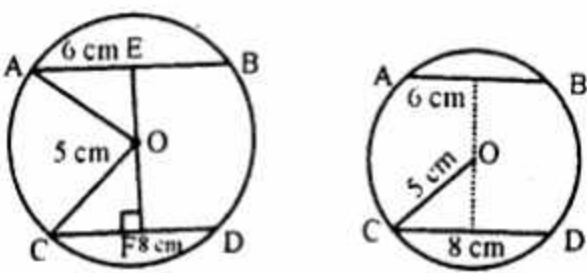
$m\overline{AC} = m\overline{BC}$ | A diameter $CD \perp$ on chord AB bisect it.
Given
Common side
S.A.S $\cong$ S.A.S
Corresponding sides of congruent triangles. |

3. As shown in the figure, find the distance between two parallel chords $\overline{AB}$ and $\overline{CD}$.

Given:

$m\overline{AB} = 6\text{cm}$ and $m\overline{CD} = 8\text{cm}$

$m\overline{OC} = 5\text{cm}$



Required:

$m\overline{EF} = ?$

In $\square OCF$

$m\overline{OC}^2 = \overline{OF}^2 + \overline{FC}^2$

$5^2 = \overline{OF}^2 + 4^2$
 $\Rightarrow \overline{OF}^2 = 25 - 16 = 9$
 $\overline{OF} = \sqrt{9} = 3\text{cm}$
In $\square OAE$
 $\overline{OA}^2 = \overline{OE}^2 + \overline{EA}^2$
 $5^2 = \overline{OE}^2 + 3^2$
 $\Rightarrow \overline{OE}^2 = 25 - 9 = 16$
 $\overline{OE} = \sqrt{16} = 4$
 $\therefore \overline{EF} = \overline{OE} + \overline{OF} = 4 + 3 = 7\text{cm}$

THEOREM 1

9.1 (I) One and only one circle can pass through three non-collinear points.

Given:

A, B and C are three non collinear points in a plane.

To prove:

One and only one circle can pass through three non-collinear points A, B and C.

Construction:

Join A with B and B with C. AB and BC are distinct and non collinear lines. Draw $\overline{DF} \perp$ bisector to $\overline{AB}$ and $\overline{HK} \perp$ bisector to BC.

So $\overline{DF}$ and $\overline{HK}$ are not parallel rather they meet each other at some point O. Also join A, B and C with point O.

Proof:

| Statements | Reasons |
|---|--|
| Every point on $\overline{DF}$ is equidistant from A and B. | $DF \perp$ bisector to AB (construction) |
| In particular $m \overline{OA} = m \overline{OB}$ (i) | By Locus Theorem |
| Similarly, every point on $\overline{HK}$ is equidistant from to B and C. | $\overline{HK}$ is $\perp$ bisector to BC (construction) |
| In particular $m \overline{OB} = m \overline{OC}$ (ii) | By Locus theorem 1 |
| Now O is the only point common to $\overline{DF}$ and $\overline{HK}$ which is equidistant from A, B and C. | |

| | |
|--|----------------------|
| i.e., $mOA = mOB = mOC$
However, there is no such other point expect O. | Using (i) and (ii) ' |
|--|----------------------|

Hence a circle with center O and radius $\overline{OA}$ will pass through A, B and C. Ultimately there is only one circle which passes through three given points A, B and C.

Example:

Show that only one circle can be drawn to pass through the vertices of any rectangle.

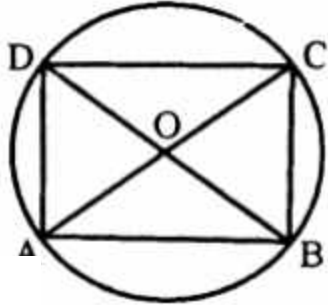
Given:

ABCD is an inscribed rectangle w. r. t. a circle with center O.

To Prove:

Only one circle can be drawn through the vertices of the rectangle ABCD.

Construction: Diagonals AC and BD of the rectangle meet each other at point O.



Proof:

| Statements | Reasons |
|--|---|
| ABCD is a rectangle.
$m \overline{AC} = m \overline{BD}$ (i) | Given
Diagonals of a rectangle are equal. |
| $\overline{AC}$ and $\overline{BD}$ meet each other at O
$\overline{OA} = \overline{OC}$ and $\overline{OB} = \overline{OD}$ (ii) | Construction
Diagonals of rectangle bisect each either |
| $\Rightarrow \overline{OA} = \overline{OB} = \overline{OC} = \overline{OD}$ (iii) | Using (i) and (ii) |

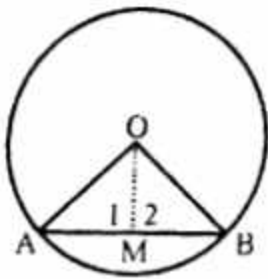
| | |
|--|--|
| i.e., point O is equidistant from all vertices of the rectangle ABCD.
Hence $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, and $\overline{OD}$ are the radii of the same circle with center O. | |
|--|--|

THEOREM 2

A straight line, drawn from the center of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord.

Given:

M is the mid-point of any chord $\overline{AB}$ of a circle with center at O.
Where chord $\overline{AB}$ is not the diameter of the circle.



To prove:

$\overline{OM} \perp$ the chord $\overline{AB}$

Construction:

Join A and B with center O.
Write $\angle 1$ and $\angle 2$ as shown in the Figure.

Proof:

| Statements | Reasons |
|--|---|
| In $\triangle OAM \leftrightarrow \triangle OBM$ | |
| $m \overline{OA} = m \overline{OB}$ | Radii of the same circle |
| $m \overline{AM} = m \overline{BM}$ | Given |
| $m \overline{OM} = m \overline{OM}$ | Common |
| $\triangle OAM \cong \triangle OBM$ | S.S.S $\cong$ S.S.S |
| $\Rightarrow m\angle 1 = m\angle 2$ (i) | Corresponding angles of congruent triangles |
| i.e., $m\angle 1 + m\angle 2 = m\angle AMB = 180^\circ$ (ii) | Adjacent supplementary angles |
| $m\angle 1 = m\angle 2 = 90^\circ$ | From (i) and (ii) |

| | |
|---|--|
| i.e., $\overline{OM} \perp \overline{AB}$ | |
|---|--|

THEOREM 3

9.1 (iii) Perpendicular from the center of a circle on a chord bisects it.

Given:

$\overline{AB}$ is the chord of a circle with center at O so that $\overline{OM} \perp$ chord $\overline{AB}$

To prove:

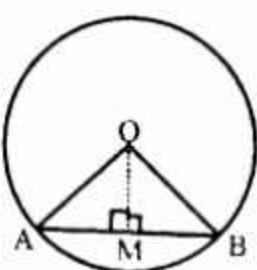
M is the midpoint of chord $\overline{AB}$

i.e., $m\overline{AM} = m\overline{BM}$

Construction:

Join A and B with center O.

Proof:



| Statements | Reasons |
|---|--|
| In $\angle t \triangle OAM \leftrightarrow \triangle OBM$ | |
| $m\angle OMA = m\angle OMB = 90^\circ$ | Given |
| hyp. $m\overline{OA} = \text{hyp. } m\overline{OB}$ | Radii of the same circle |
| $m\overline{OM} = m\overline{OM}$ | Common |
| $\triangle OAM \cong \triangle OBM$ | In $\angle t \triangle s$ H.S $\cong$ H.S |
| Hence, $m\overline{AM} = m\overline{BM}$ | Corresponding sides of congruent triangles |
| $\overline{OM}$ bisects the chord AB. | |

Corollary 1: $\perp$ bisector of the chord of a circle passes through the center of a circle.

Corollary 2: The diameter of a circle passes through the mid points of two parallel chords of a circle.

Example:

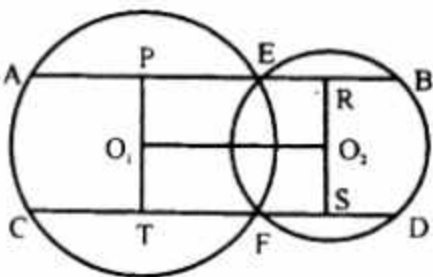
Parallel lines passing through the points of in intercepted by them are equal.

Given:

Two circles have centers O_1 and O_2 .

They intersect each other at points E and F.

Line segment $\overline{AB} \parallel$ Line segment $\overline{CD}$



To Prove:

$m\overline{AB} = m\overline{CD}$

Construction:

Draw PT and RS $\perp$ both AB and CD and join the centers O_1 and O_2 .

Proof:

| Statements | Reasons |
|---|--|
| PRST is a rectangle. | Construction |
| $m\overline{PR} = m\overline{TS}$ (i) | |
| Now $m\overline{PR} = m\overline{PE} + m\overline{ER}$ | |
| $= \frac{1}{2}m\overline{AE} + \frac{1}{2}m\overline{EB}$ | By Theorem 3 |
| $= \frac{1}{2}(m\overline{AE} + m\overline{EB})$ | |
| $m\overline{PR} = \frac{1}{2}(m\overline{AB})$ (ii) | $m\overline{AE} + m\overline{EB} = m\overline{AB}$ |

| | |
|---|---------------------------|
| Similarly, $m\overline{TS} = \frac{1}{2}m\overline{CD}$ (iii) | |
| $\Rightarrow \frac{1}{2}m\overline{AB} = \frac{1}{2}m\overline{CD}$ | Using (i), (ii) and (iii) |
| i.e., $m\overline{AB} = m\overline{CD}$ | |

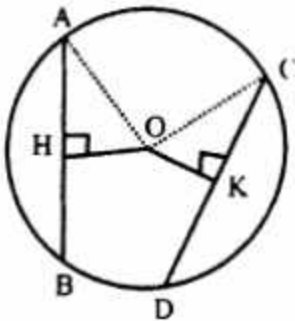
THEOREM 4

9.1 (iv) If two chords of a circle are congruent then they will be equidistant from the center.

Given:

$\overline{AB}$ and $\overline{CD}$ are two equal chords of a circle with center at O.

So that $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$.



To prove:

$m\overline{OH} = m\overline{OK}$

Construction:

Join O with A and O with C. So that we have $\triangle OAH$ and $\triangle OCK$.

Proof:

| Statements | Reasons |
|--|---|
| $\overline{OH}$ bisects chord $\overline{AB}$ | $\overline{OH} \perp \overline{AB}$ (By Theorem 3) |
| i.e., $m\overline{AH} = \frac{1}{2}m\overline{AB}$ (i) | |
| Similarly, $\overline{OK}$ bisects chord $\overline{CD}$ | $\overline{OK} \perp \overline{CD}$ (By Theorem 3) |
| i.e., $m\overline{CK} = \frac{1}{2}m\overline{CD}$ (ii) | |
| But $m\overline{AB} = m\overline{CD}$ (iii) | Given |
| Hence, $m\overline{AH} = m\overline{CK}$ (iv) | Using (i),(ii) and (iii) |
| Now in $\triangle OAH \leftrightarrow \triangle OCK$ | Given $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$ |
| hyp $\overline{OA} = \text{hyp } \overline{OC}$ | Radii of the same circle |

| | |
|---|------------------------|
| $m\overline{AH} = m\overline{CK}$ | Already proved in (iv) |
| $\triangle OAH \cong \triangle OCK$ | H.S postulate |
| $\Rightarrow m\overline{OH} = m\overline{OK}$ | |

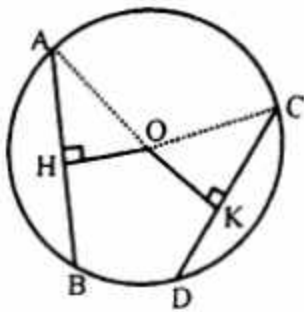
THEOREM 5

9.1 (v) Two chords of a circle which are equidistant from the center, are congruent.

Given:

$\overline{AB}$ and $\overline{CD}$ are two chords of a circle with center at O.

$\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$, so that $m\overline{OH} = m\overline{OK}$



To prove:

$m\overline{AB} = m\overline{CD}$

Construction:

Join A and C with O. So that we can form $\triangle OAH$ and $\triangle OCK$.

Proof:

| Statements | Reasons |
|---|---|
| In $\triangle OAH \leftrightarrow \triangle OCK$ | |
| hyp $\overline{OA} = \text{hyp } \overline{OC}$ | Radii of the same circle |
| $m\overline{OH} = m\overline{OK}$ | Given |
| $\triangle OAH \cong \triangle OCK$ | H.S postulate |
| So $m\overline{AH} = m\overline{CK}$ (i) | Corresponding sides of congruent triangles |
| But $m\overline{AH} = \frac{1}{2}m\overline{AB}$ (ii) | $\overline{OH} \perp \overline{AB}$ (Given) |
| Similarly, $m\overline{CK} = \frac{1}{2}m\overline{CD}$ (iii) | $\overline{OK} \perp \overline{CD}$ (Given) |
| Since $m\overline{AH} = m\overline{CK}$ | Already proved in (i) |
| | Using (ii) and (iii) |

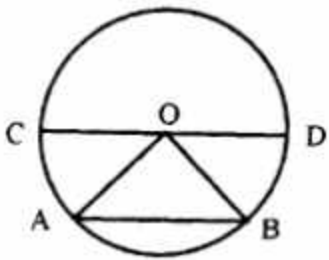
| | |
|---|--|
| $\frac{1}{2}m\overline{AB} = \frac{1}{2}m\overline{CD}$ | |
| Or $m\overline{AB} = m\overline{CD}$ | |

Example:

Prove that the largest chord in a circle is the diameter.

Given:

$\overline{AB}$ is a chord and $\overline{CD}$ is the diameter of a circle with center point O.



To prove:

If $\overline{AB}$ and $\overline{CD}$ are distinct, then $m\overline{CD} > m\overline{AB}$.

Construction:

Join O with A and O with B then form a $\triangle OAB$.

Proof:

Sum of two sides of a triangle is greater than its third side.

In $\triangle OAB \Rightarrow m\overline{OA} + m\overline{OB} > m\overline{AB}$..(i)

But $\overline{OA}$ and $\overline{OB}$ are the radii of the same circle with center O.

So that $m\overline{OA} + m\overline{OB} = m\overline{CD}$..(ii)

$\Rightarrow \text{Diameter } \overline{CD} > \text{chord } \overline{AB}$ using (i) & (ii).

Hence, diameter CD is greater than any other chord drawn in the circle.

Miscellaneous Exercise 9

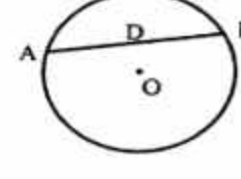
Q1. Multiple Choice Questions:

Four possible answers are given for the following questions.

Tick (✓) the correct answer.

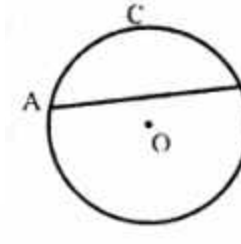
(i) In the circular figure, ADB is called

- (a) an arc (b) a secant
(c) a chord (d) a diameter



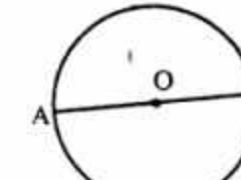
(ii) In the circular figure, $\widehat{ACB}$ is called

- (a) an arc (b) a secant
(c) a chord (d) a diameter



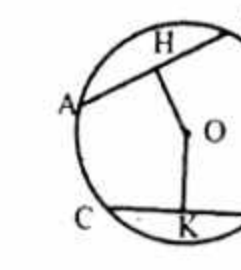
(iii) In the circular figure, AOB is called

- (a) an arc (b) a secant
(c) a chord (d) a diameter



(iii) In a circular figure, two chords AB and CD are equidistant from the center. They will be:

- (a) parallel (b) non congruent
(c) congruent (d) perpendicular



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(v) Radii of a circle are

- (a) all equal (b) double of the diameter
(c) all unequal (d) half of any chord

(vi) A chord passing through the center of a circle is called:

- (a) radius (b) diameter
(c) circumference (d) secant

(vii) Right bisector of the chord of a circle always passes through the

- (a) radius (b) circumference
(c) center (d) diameter

(viii) The circular region bounded by two radii and the corresponding arc is called

- (a) circumference of a circle (b) sector of a circle
(c) diameter of a circle (d) segment of a circle

(ix) The distance of any point of the circle to its center is called

- (a) radius (b) diameter
(c) a chord (d) an arc

(x) Line segment joining any point of the circle to the center is called

- (a) circumference (b) diameter

2

- (c) radial segment (d) perimeter

(xi) Locus of a point in a plane equidistant from a fixed point is called

- (a) radius (b) circle
(c) circumference (d) diameter

(xii) The symbol for a triangle is denoted by

- (a) $\angle$ (b) Δ
(c) $\perp$ (d) $\odot$

(xiii) A complete circle is divided into

- (a) 90 degrees (b) 180 degrees
(c) 270 degrees (d) 360 degrees

(xiv) Through how many non collinear points, a circle can pass?

- (a) one (b) two
(c) three (d) none

Answers:

| | | | | | | | | | |
|------|---|-------|---|--------|---|-------|---|-----|---|
| (i) | c | (ii) | a | (iii) | d | (iv) | c | (v) | a |
| (vi) | b | (vii) | c | (viii) | b | (ix) | a | (x) | c |
| (xi) | b | (xii) | b | (xiii) | d | (xiv) | c | | |

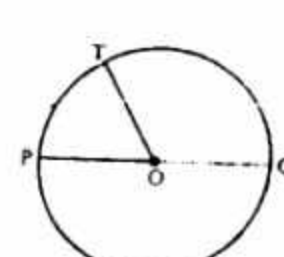
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Q2. Differentiate between the following terms and illustrate them by diagrams.

(i) A circle and a circumference.

Ans: Circle and circumference

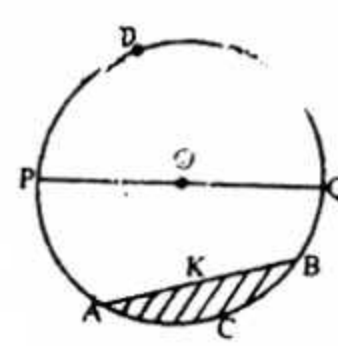
A circle is the locus of a moving point P in a plane which is always equidistant from some fixed-point O. The fixed-point O not lying on the circle is called the center. Whereas the boundary traced by moving point P is called circumference of the circle.



(ii) A chord and the diameter of a circle.

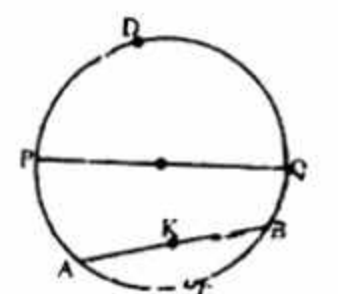
Ans: Chord and the diameter of a circle

A chord AKB of a circle is a straight line joining any two points on the circumference of a circle. Whereas diameter POQ is the chord passing through the center of a circle.



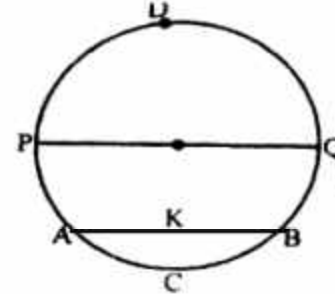
(iii) A chord and an arc of a circle.

Ans: An arc ACB of a circle is any portion of its circumference. A chord AKB of a circle is a straight line joining any two points on the circumference of a circle.



(iv) Minor arc and major arc of a circle.

Ans: If the above figure, the smaller arc $\widehat{ACB}$ is minor arc, $\widehat{ADB}$ is major arc of a circle.

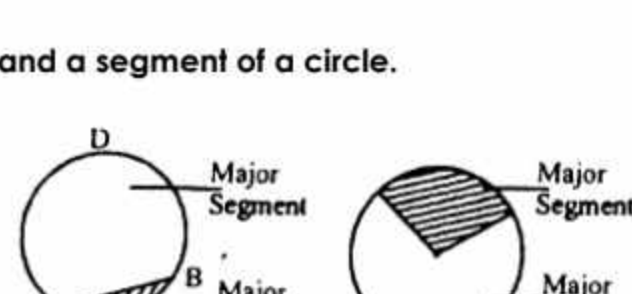


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(v) Interior and exterior of a circle.

Ans: The internal area or part of a circle is called interior of a circle and external part or area of a circle is called exterior of a circle.

(vi) A sector and a segment of a circle.



Ans: A sector of a circle is the center in the plane figure bounded by two radii and the intercepted between them. Any pair of radii divides a circle into two sectors. Segments is the portion of a circle bounded by an arc and a corresponding chord. Any chord divides a circle into two segments.

Summary

- ✓ $2\pi r$ is the circumference of a circle with radius r .
- ✓ πr^2 is the area of a circle with radius r .
- ✓ Three or more points lying on the same line are called collinear points otherwise they are non-collinear points.
- ✓ The circle passing through the vertices of a triangle is called its circumcircle whereas $\perp$ bisectors of sides of the triangle provides the center.
- ✓ One and only one circle can pass through three non-collinear points.

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- ✓ A straight line, drawn from the center of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord.
- ✓ Perpendicular from the center of a circle on a chord bisects it.
- ✓ If two chords of a circle are congruent, then they will be equidistant from the center.
- ✓ Two chords of a circle which are equidistant from the center are congruent.

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