Exercise 7.1

1. Locate the following angles:

(i) 30°



(ii) 22 1/2°



(iii) 135°



(iv) 225°



(v) −4 R

(vi) −120°

(vii) −150°

(viii) −225°

(ix) −180°

(x) −90°

2. *Express the following sexagesimal measures of angles in decimal form.

(i) 40° 30'

Solution:

$$40°\,30' = 40° + \left(\frac{30}{60}\right)° = 40° + 0.5° = 40.5°$$

(ii) 40° 30' 36"

Solution:

$$60'\,36'' = 60' + \left(\frac{36}{60}\right)' = 60' + \left(\frac{36}{60\times60}\right)°$$
$$= 60' + \frac{36}{3600}° = 60' + 0.01°$$
$$= 40° + 0.5° + 0.01° = 40.51°$$

$$= 40.508125°$$

(iii) 125° 23' 56"

Solution:

$$125°\,23'\,56'' = 125° + \left(\frac{23}{60}\right)° + \left(\frac{56}{60\times60}\right)°$$
$$= 125° + \frac{23}{60}° + \frac{56}{3600}°$$
$$= 125° + 0.3833° + 0.0156°$$
$$= 125.3856°$$

3. Express the following into 0° M' S", form.

(i) 47.34°

Solution:

$$47.34° = 47° + 0.34°$$
$$= 47° + \left(\frac{34}{100}\right)°$$
$$= 47° + \frac{34}{100}°$$
$$= 47° + \left(\frac{3}{5}\times60\right)'$$
$$= 47° + 20.4'$$
$$= 47° + 2' + (0.4\times60)''$$
$$= 47° + 2' + 24''$$
$$= 47°\,21'\,36''$$

(ii) 125.45°

Solution:

$$125.45° = 125° + 0.45°$$
$$= 125° + \left(\frac{45}{100}\right)°$$
$$= 125° + \frac{45}{100}°$$
$$= 125° + \left(\frac{9}{20}\times60\right)'$$
$$= 125° + 27'$$
$$= 125°\,27'$$

(iii) 225.75°

Solution:

$$225.75° = 225° + 0.75°$$
$$= 225° + \left(\frac{75}{100}\right)°$$
$$= 225° + \frac{75}{100}°$$
$$= 225° + \left(\frac{3}{4}\times60\right)'$$
$$= 225°\,45'$$
$$= 225°\,45'$$

(iv) -22.5°

Solution:

$$-22.5° = -22° - 0.5°$$
$$= -22° - \left(\frac{5}{10}\right)°$$
$$= -22° - \frac{5}{10}°$$
$$= -22° - \left(\frac{1}{2}\times60\right)'$$
$$= -22° - 30'$$
$$= -22°\,30'$$

(v) -47.58°

Solution:

$$-47.58° = -47° - 0.58°$$
$$= -47° - \left(\frac{58}{100}\right)°$$
$$= -47° - \frac{58}{100}°$$
$$= -47° - \left(\frac{29}{50}\times60\right)'$$
$$= -47° - 34.8'$$
$$= -47° - 34' - (0.8\times60)''$$
$$= -47°\,-34'\,-48''$$
$$= -47°34'48''$$

(vi) 315.18°

Solution:

$$315.18° = 315° + 0.18°$$
$$= 315° + \left(\frac{18}{100}\right)°$$
$$= 315° + \frac{18}{100}°$$
$$= 315° + \left(\frac{9}{50}\times60\right)'$$
$$= 315° + 10.8'$$
$$= 315° + 10' + (0.8\times60)''$$
$$= 315°\,+10'\,+48''$$
$$= 315°10'48''$$

4. Express the following angles into radians.

(i) 30°

Solution:

$$30° = 30\times1° = 30\times\left(\frac{\pi}{180}\text{ radians}\right)$$
$$= \frac{\pi}{6}\text{ radians}$$

(ii) 45°

Solution:

$$60° = 60\times1° = 60\times\left(\frac{\pi}{180}\text{ radians}\right)$$
$$= \frac{\pi}{3}\text{ radians}$$

(iii) 135°

Solution:

$$135° = 135\times1° = 135\times\left(\frac{\pi}{180}\text{ radians}\right)$$
$$= \frac{3\pi}{4}\text{ radians}$$

(iv) 225°

Solution:

$$225° = 225\times1° = 225\times\left(\frac{\pi}{180}\text{ radians}\right)$$
$$= \frac{5\pi}{4}\text{ radians}$$

(v) -150°

Solution:

$$-150° = -150\times1° = -150\times\left(\frac{\pi}{180}\text{ radians}\right)$$
$$= -\frac{5\pi}{6}\text{ radians}$$

(vi) - 225°

Solution:

$$-225° = -225\times1° = -225\times\left(\frac{\pi}{180}\text{ radians}\right)$$
$$= -\frac{5\pi}{4}\text{ radians}$$

(vii) 300°

Solution:

$$300° = 300\times1° = 300\times\left(\frac{\pi}{180}\text{ radians}\right)$$
$$= \frac{5\pi}{3}\text{ radians}$$

(viii) 315°

Solution:

$$315° = 315\times1° = 315\times\left(\frac{\pi}{180}\text{ radians}\right)$$
$$= \frac{7\pi}{4}\text{ radians}$$

5. Convert each of following to degrees.

(i) $\frac{3\pi}{4}$

Solution:

$$\frac{3\pi}{4} = \frac{3\pi}{4}\text{ radians} = \frac{3\pi}{4}\times1\text{ radian}$$
$$= \frac{3\pi}{4}\times\frac{180}{\pi}° = 135°$$

(ii) $\frac{7\pi}{8}$

Solution:

$$\frac{7\pi}{8} = \frac{7\pi}{8}\text{ radians} = \frac{7\pi}{8}\times1\text{ radian}$$
$$= \frac{7\pi}{8}\times\frac{180}{\pi}° = 157.5°$$

(iv) $\frac{13\pi}{18}$

Solution:

$$\frac{13\pi}{18} = \frac{13\pi}{18}\text{ radians} = \frac{13\pi}{18}\times1\text{ radian}$$
$$= \frac{13\pi}{18}\times\frac{180}{\pi}° = 146.25°$$

(v) 3

Solution:

$$3 = 3\text{ radians} \quad 3\times1\text{ radian}$$
$$3\times\frac{180}{\pi}° \quad 3\times57.295779$$
$$=171.89°$$

(vi) 4.5

Solution:

$$4.5 = 4.5\text{ radians} \qquad 4.5\times1\text{ radian}$$
$$4.5\times\frac{180}{\pi}° \qquad 4.5\times57.295779$$
$$=257.81°$$

(vii) $\frac{17\pi}{9}$

Solution:

$$\frac{17\pi}{9} = \frac{17\pi}{9}\text{ radians} = \frac{17\pi}{9}\times1\text{ radian}$$
$$= \frac{17\pi}{9}\times\frac{180}{\pi}° = 157.5°$$

(viii) $\frac{13\pi}{6}$

Solution:

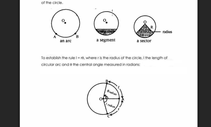
$$\frac{13\pi}{6} = \frac{13\pi}{6}\text{ radians} = \frac{13\pi}{6}\times1\text{ radian}$$
$$= \frac{13\pi}{6}\times\frac{180}{\pi}° = 146.25$$

Sector of a Circle

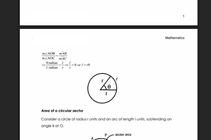
(i) A part of the circumference of a circle is called an arc.

(ii) A part of the circle bounded by an arc and a chord is called segment of a circle.

(iii) A part of the circle bounded by the two radii and an arc is called sector of the circle.

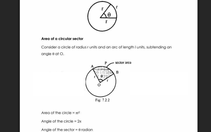


To establish the rule $l = r\theta$, where r is the radius of the circle, l the length of circular arc and θ the central angle measured in radians.



Let an arc AB denoted by l subtends an angle θ radian at the center of the circle. It is fact of plane geometry that measure of central angles of the arcs of a circle are proportional to the lengths of their arcs.

$$\frac{m\angle AOB}{m\angle ACB} = \frac{arcAB}{arcAC}$$
$$\frac{\theta}{2\pi\text{ radian}} = \frac{l}{2\pi r} \Rightarrow \theta = \frac{l}{r} \Rightarrow l = r\theta$$



Area of a circular sector

Consider a circle of radius r units and an arc of length l units, subtending an angle θ at O.



Area of the circle = πr^2

Angle of the circle = 2π

Angle of the sector = θ radian

Then by elementary geometry, we can use the proportion,

$$\frac{\text{Area of sector AOBP}}{\text{area of circle}} = \frac{\text{angle of the sector}}{\text{angle of the circle}}$$
$$\text{or, } \frac{\text{area of sector AOBP}}{\pi r^2} = \frac{\theta}{2\pi}$$
$$\Rightarrow \text{area of sector AOBP} = \frac{\theta}{2\pi} \times \pi r^2 = \frac{1}{2} r^2\theta$$

EXERCISE 7.2

1. Find θ, when

(i) r = 8.2m , l = 3.5m

Solution:

We know that

$$l = r\theta \qquad \Rightarrow \theta = \frac{l}{r}$$
$$\theta = \frac{3.5}{8.2} \qquad \Rightarrow \theta = 0.37 \text{ radians}$$

(ii) r = 4.5m , l = 2.5m

Solution:

We know that

$$l = r\theta \qquad \Rightarrow \theta = \frac{l}{r}$$
$$\theta = \frac{2.5}{4.5} \qquad \Rightarrow \theta = 1.8 \text{ radians}$$

2. Find l, when

(i) θ =180° , r = 4.9 cm

Solution:

$$\theta = 180 = 180 \times 1^c = 180 \times \frac{\pi}{180} = 3.1415 \text{ radians}$$

We know that

$$l = r\theta \qquad \Rightarrow l = (4.9)(3.1415)$$
$$l = 15.4cm$$

(ii) θ =45° 30' , r = 10mm

Solution:

$$\theta = 45°30' = 45° + \left(\frac{30}{60}\right)° = 45° + 0.5° = 45.5°$$
$$= 45.5 \times 1^c = 45.5 \times \frac{\pi}{180} = 1.856 \text{ radians}$$

We know that

$$l = r\theta \qquad \Rightarrow l = (10)(1.856)$$
$$l = 13.56mm$$

3. Find l, when

(i) r = 4cm , θ = $\frac{\pi}{6}$ radian

Solution:

We know that

$$l = r\theta \qquad \Rightarrow l = \frac{\pi}{6}$$
$$l = \frac{\pi}{1.6} \qquad \Rightarrow l = 0.19cm$$

(ii) r = 32cm , θ = 45°

Solution:

We know that

$$\theta = 45° = 45 \times 1^c = 45 \times \frac{\pi}{180} = \frac{\pi}{4} \text{ radians}$$
$$= 0.7854 \text{ radians}$$

We know that

$$l = r\theta \qquad \Rightarrow l = \frac{\pi}{4} r$$
$$l = \frac{\pi}{4} \times 32cm \qquad \Rightarrow l = 80.2cm$$

4. In a circle of radius 12m, find the length of an arc which subtends a central angle θ = 1.5 radian.

Solution:

In (1) (r), θ = 1.5 radians, r =**?**

We know that

$$l = r\theta \quad \text{i.e } l = (12)(1.5) \Rightarrow l = 18 \text{ m}$$

5. In a circle of radius 10m, find the distance travelled by a point moving on this circle if the point makes 2.5 revolution. (2.5 revolution = ?θ)

Solution:

$$\text{i.e } (2.5)\text{rev}, \theta = 2.5 \text{ revolutions}, r = \text{?}$$

We know that

$$l = r\theta \quad \text{i.e } l = (2)(2.5) \Rightarrow l = 15 \text{ m}$$

6. In a circle of radius 15m, how long an arc subtends a central angle of 84°.

Solution:

$$r = 12cm \qquad l = ?$$
$$\theta = 84 = 84 \times 1^c = 84 \times \frac{\pi}{180} \text{ radians}$$
$$= 1.4661 \text{ radians}$$

We know that

$$l = r\theta$$
$$l = (12)\left(\frac{22}{7}\right)(84) = 17.6 \text{ cm}$$

7. What is the length of the arc APB?

Solution:

$$\text{arc } \overset{\frown}{APB} = ?, \quad r = 8cm$$
$$\theta = 90 = 90 \times 1^c = 90 \times \frac{\pi}{180} \text{ radians}$$
$$= \frac{\pi}{2} \text{ radians} = 1.57 \text{radian}$$

We know that

$$l = r\theta$$
$$l = (8)(1.57) = 12.57 \text{ cm}$$

8. In a circle of radius 12cm, how long an arc subtends a central angle of 84°.

Solution:

$$r = 12cm \qquad l = ?$$
$$\theta = 84 = 84 \times 1^c = 84 \times \frac{\pi}{180} \text{ radians}$$
$$= 1.4661 \text{ radians}$$

We know that

$$l = r\theta$$
$$l = (12)(1.466) = 17.6 \text{ cm}$$

9. Find the area of the sectors OPB.

Solution:

$$(a)\text{Here, } r = 8cm, \theta = 60° = 60 \times 1^c = 60 \times \frac{\pi}{180} = \frac{\pi}{3} \text{ rad.}$$

We know that

$$\text{Area of the sector} = \frac{1}{2} r^2\theta$$
$$= \frac{1}{2}(8)^2\left(\frac{\pi}{3}\right)$$
$$= \frac{1}{2}(64)\left(\frac{\pi}{3}\right)$$
$$= 33.5$$
$$= 33.5cm^2$$

$$(b)\text{Here } r = 20cm, \theta = 45° = 45 \times 1^c = 45 \times \frac{\pi}{180} = \frac{\pi}{4} \text{ rad.}$$

We know that

$$\text{Area of the sector} = \frac{1}{2} r^2\theta$$
$$= \frac{1}{2}(20)^2\left(\frac{\pi}{4}\right)$$
$$= \frac{1}{2}(400)\left(\frac{\pi}{4}\right)$$
$$= 50\pi$$
$$= 50\left(\frac{22}{7}\right)$$

$$= 157.14cm^2$$

10. Find area of the sector inside a central angle of 20° in a circle of radius 3m.

Solution:

$$Area = ?, r = 3m, \theta = 20° = 20 \times \frac{\pi}{180} \text{ rad.}$$

We know that

$$\text{Area of the sector} = \frac{1}{2} r^2\theta$$
$$= \frac{1}{2}(9)\left(\frac{\pi}{9}\right)$$
$$= \frac{9\pi}{18}$$
$$= \frac{\pi}{2}$$
$$= \frac{22}{14}$$
$$= 1.57m^2$$

11. Sahar is making a skirt. Each panel of this skirt is of the shape shown shaded in the diagram. How much material (cloth) is required for each panel?

Solution:

$$r = 1m, \theta = 120cm = 60cm$$

$$\theta = 60 = 60 \times 1^c = 60 \times \frac{\pi}{180} = \frac{\pi}{3} = \frac{\pi}{3} \times \frac{180}{\pi} = 60 \text{ degree}$$

We know that

$$\text{Area of the sector} = \frac{1}{2} r^2\theta$$
$$= \frac{1}{2}(60)\left(\frac{\pi}{3}\right)$$
$$= \frac{1}{2}(20\pi)\left(\frac{22}{7}\right)$$
$$= \frac{440}{14}$$
$$= 31.42cm^2$$

12. Find the area of the sector with central angle of $\frac{\pi}{3}$ radian in a circle of radius 10cm.

Solution:

$$\theta = \frac{\pi}{3} \text{ rad} \qquad r = 10cm, \qquad Area = ?$$

We know that

$$\text{Area of the sector} = \frac{1}{2} r^2\theta$$
$$= \frac{1}{2}(100)\left(\frac{\pi}{3}\right)$$
$$= \frac{1}{2}(100)\left(\frac{22}{21}\right)$$
$$= 52.38cm^2$$

13. The area of the sector with a central angle θ in a circle of radius 2m is 10 square meters. Find θ in radians.

Solution:

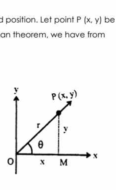
$$r = 2m, \quad Area = 10m^2 \qquad \theta = ?$$

We know that

$$\text{Area of the sector} = \frac{1}{2} r^2\theta$$
$$10 = \frac{1}{2}(2)^2\theta$$
$$10 = 2\theta$$
$$\theta = \frac{10}{2}$$
$$\theta = 5 \text{ radians}$$

Trigonometric Identities:

Consider an angle ∠MOP = θ radian in standard position. Let point P (x, y) be on the terminal side of the angle. By Pythagorean theorem, we have from right triangle OMP.



$OM^2 + MP^2 = OP^2$

$x^2 + y^2 = r^2 \quad .....(i)$

Dividing both sides by r^2 we get

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$

$\Rightarrow \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$

$\Rightarrow (\cos\theta)^2 + (\sin\theta)^2 = 1$

$\therefore \boxed{\cos^2\theta + \sin^2\theta = 1} \quad (1)$

Dividing (i) by x^2, we have

$\frac{x^2}{x^2} + \frac{y^2}{x^2} = \frac{r^2}{x^2}$

$\Rightarrow \quad 1 + \left(\frac{y}{x}\right)^2 + \left(\frac{r}{x}\right)^2 \qquad \because \tan\theta = \frac{y}{x} \text{ and } \sec\theta = \frac{r}{x}$

$\Rightarrow \quad 1 + (\tan\theta)^2 + (\sec\theta)^2 = 1$

$\therefore \quad \boxed{1 + \tan^2\theta = \sec^2\theta} \text{ or } \sec^2\theta - \tan^2\theta = 1 \quad (2)$

Again dividing both sides of (i) by y^2, we get

$\frac{x^2}{y^2} + \frac{y^2}{y^2} = \frac{r^2}{y^2}$

$\left(\frac{x}{y}\right)^2 + 1 = \left(\frac{r}{y}\right)^2 \qquad \because \cot\theta = \frac{x}{y} \text{ and } \mathrm{cosec}\theta \frac{r}{y}$

$\Rightarrow \quad (\cot\theta)^2 + 1 = (\mathrm{cosec}\theta)^2$

$\therefore \quad \boxed{1 + \cot^2\theta = \mathrm{cosec}^2\theta} \quad \text{or} \quad \mathrm{cosec}^2\theta - \cot^2\theta = 1 \quad (3)$

The identities (1), (2) and (3) are also known as Pythagorean Identities.

The fundamental identities are used to simplify expressions involving trigonometric functions.

Example 1:

Verify that cotθ secθ = cosecθ

Solution:

Expressing left hand side in terms of sine and cosine, we have

$L.H.S = \cot\theta\sec\theta = \frac{\cos\theta}{\sin\theta}.\frac{1}{\cos\theta}$

$= \frac{1}{\sin\theta} = \mathrm{cosec}\theta$

$= R.H.S$

Example 2

Verify that $\tan^4\theta + \tan^2\theta = \tan^2\theta \sec^2\theta$

Solution:

$L.H.S = \tan^4\theta + \tan^2\theta = \tan^2\theta(\tan^2\theta + 1) \qquad \because \tan^2\theta + 1 = \sec^2\theta$

$= \tan^2\theta\sec^2\theta$

$= R.H.S$

Example 3

Show that $\frac{\cot^2\alpha}{\mathrm{cosec}\alpha - 1} = \mathrm{cosec}\alpha + 1$

Solution:

$\frac{\cot^2\alpha}{\mathrm{cosec}\alpha - 1} \qquad \begin{pmatrix} \because \mathrm{cosec}^2\theta - \cot\theta = 1 \\ \cot^2\theta = \mathrm{cosec}^2\theta - 1 \end{pmatrix}$

$= \frac{(\mathrm{cosec}^2\alpha - 1)}{\mathrm{cosec}\alpha - 1} = \frac{(\mathrm{cosec}\alpha - 1)(\mathrm{cosec}\alpha + 1)}{(\mathrm{cosec}\alpha - 1)} = \mathrm{cosec}\alpha + 1 = R.H.S$

Example 4

Express the trigonometric functions in terms of tan θ.

Solution:

By using reciprocal identity, we can express cotθ in terms of tanθ.

i.e., $\cot\theta = \frac{1}{\tan\theta}$

By solving the identity $1 + \tan^2\theta = \sec^2\theta$

We have expressed secθ in terms of tanθ.

$\sec\theta = \pm\sqrt{\tan^2\theta + 1} \because$

$\cos\theta = \frac{1}{\sec\theta} \Rightarrow \cos\theta = \frac{1}{\pm\sqrt{\tan^2\theta + 1}}$

Because $\sin\theta = \tan\theta\cos\theta$, we have

$\sin\theta = \tan\theta\left(\frac{1}{\pm\sqrt{\tan^2\theta + 1}}\right) = \frac{\tan\theta}{\pm\sqrt{\tan^2\theta + 1}}$

$\mathrm{cosec}\theta = \frac{1}{\sin\theta} = \frac{\pm\sqrt{\tan^2\theta + 1}}{\tan\theta}$

Exercise 7.4

In Problems 1- 6, simplify each expression to a single trigonometric function.

1. $\frac{\sin^2 x}{\cos^2 x}$

Solution:

$\frac{\sin^2 x}{\cos^2 x} = \left(\frac{\sin x}{\cos x}\right)^2$

$= (\tan x)^2 \qquad \because \frac{\sin x}{\cos x} = \tan x$

$= \tan^2 x$

2. tanx sinx secx

Solution:

$\tan x \sin x \sec x = \frac{\sin x}{\cos x}.\sin x.\frac{1}{\cos x}$

$= \frac{\sin x.\sin x}{\cos x.\cos x}$

$= \frac{\sin^2 x}{\cos^2 x} = \left(\frac{\sin x}{\cos x}\right)^2$

$= (\tan x)^2 \qquad \because \frac{\sin x}{\cos x} = \tan x$

$= \tan^2 x$

3. $\frac{\tan x}{\sec x}$

Solution:

$\frac{\tan x}{\sec x} = \frac{\sin x / \cos x}{1 / \cos x}$

$= \frac{\sin x}{\cos x}.\frac{\cos x}{1}$

$= \sin x$

4. $1 - \cos^2 x$

Solution:

$1 - \cos^2 x = 1 - (1 - \sin^2 x) \qquad \because \cos^2 x = 1 - \sin^2 x$

$= 1 - 1 + \sin^2 x$

$= \sin^2 x$

5. $\sec^2 x - 1$

Solution:

$\sec^2 x - 1 = 1 + \tan^2 x - 1 \qquad \because \sec^2 x = 1 + \tan^2 x$

$= \tan^2 x$

6. $\sin^2 x.\cot^2 x$

Solution:

$\sin^2 x.\cot^2 x = \sin^2 x.\frac{\cos^2 x}{\sin^2 x} \qquad \because \cot x = \frac{\cos x}{\sin x}$

$= \cos^2 x$

In problem 7 - 24, verify the identities.

7. $(1 - \sin\theta)(1 + \sin\theta) = \cos^2\theta$

Solution:

$L.H.S = (1 - \sin\theta)(1 + \sin\theta)$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

$= R.H.S$

Hence proved.

8. $\frac{\sin\theta + \cos\theta}{\cos\theta} = 1 + \tan\theta$

Solution:

$L.H.S = \frac{\sin\theta + \cos\theta}{\cos\theta}$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}$

$= \tan\theta + 1 \qquad \because \frac{\sin\theta}{\cos\theta} = 1$

$= R.H.S$

Hence proved.

9. $(\tan\theta + \cot\theta)\tan\theta = \sec^2\theta$

Solution:

$L.H.S = (\tan\theta + \cot\theta)\tan\theta$

$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right).\frac{\sin\theta}{\cos\theta}$

$= \left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right).\frac{\sin\theta}{\cos\theta}$

$= \left(\frac{1}{\sin\theta\cos\theta}\right)\frac{\sin\theta}{\cos\theta} \qquad \because \sin^2\theta + \cos^2\theta = 1$

$= \frac{1}{\cos^2\theta}$

$= \sec^2\theta$

$= R.H.S$

Hence proved.

10. $(\cot\theta + \mathrm{cosec}\theta)(\tan\theta - \sin\theta) = \sec\theta - \cos\theta$

Solution:

$L.H.S = (\cot\theta + \mathrm{cosec}\theta)(\tan\theta - \sin\theta)$

$= \left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)\left(\frac{\sin\theta}{\cos\theta} - \sin\theta\right)$

$= \left(\frac{\cos\theta + 1}{\sin\theta}\right)\left(\frac{\sin\theta - \sin\theta\cos\theta}{\cos\theta}\right)$

$= \left(\frac{1 + \cos\theta}{\sin\theta}\right)\left(\frac{\sin\theta(1 - \cos\theta)}{\cos\theta}\right)$

$= \frac{(1 + \cos\theta)(1 - \cos\theta)}{\cos\theta}$

$= \frac{1 - \cos^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} - \cos\theta$

$= \sec\theta - \cos\theta$

$= R.H.S$

Hence proved.

11. $\frac{\sin\theta + \cos\theta}{\tan^2\theta - 1} = \frac{\cos^2\theta}{\sin\theta - \cos\theta}$

Solution:

$L.H.S = \frac{\sin\theta + \cos\theta}{\tan^2\theta - 1}$

$= \frac{\sin\theta + \cos\theta}{\frac{\sin^2\theta}{\cos^2\theta} - 1}$

$= \frac{\sin\theta + \cos\theta}{\frac{\sin^2\theta - \cos^2\theta}{\cos^2\theta}}$

$= \frac{\cos^2\theta(\sin\theta + \cos\theta)}{\sin^2\theta - \cos^2\theta}$

$= \frac{\cos^2\theta(\sin\theta + \cos\theta)}{(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)}$

$= \frac{\cos^2\theta}{\sin\theta - \cos\theta}$

$= R.H.S$

Hence proved.

12. $\frac{\cos^2\theta}{\sin\theta} + \sin\theta = \mathrm{cosec}\theta$

Solution:

$L.H.S = \frac{\cos^2\theta}{\sin\theta} + \sin\theta$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}$

$= \mathrm{cosec}\theta$

$= R.H.S$

Hence proved.

13. $\sec\theta - \cos\theta = \tan\theta\sin\theta$

Solution:

$L.H.S = \sec\theta - \cos\theta$

$= \frac{1}{\cos\theta} - \cos\theta$

$= \frac{1 - \cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\sin\theta}{\cos\theta}.\sin\theta$

$= \tan\theta\sin\theta$

$= R.H.S$

Hence proved.

14. $\frac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta$

Solution:

$L.H.S = \frac{\sin^2\theta}{\cos\theta} + \cos\theta$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta}$

$= \sec\theta$

$= R.H.S$

Hence proved.

15. $\tan\theta + \cot\theta = \sec\theta\mathrm{cosec}\theta$

Solution:

$L.H.S = \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta}.\frac{1}{\cos\theta}$

$= \mathrm{cosec}\theta\sec\theta$

$= R.H.S$

Hence proved.

16. $(\tan\theta + \cot\theta)(\cos\theta + \sin\theta) = \sec\theta + \mathrm{cosec}\theta$

Solution:

$L.H.S = (\tan\theta + \cot\theta)(\cos\theta + \sin\theta)$

$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)(\cos\theta + \sin\theta)$

$= \left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)(\cos\theta + \sin\theta)$

$= \left(\frac{1}{\sin\theta\cos\theta}\right)(\cos\theta + \sin\theta)$

$= \frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}$

$= \frac{\cos\theta}{\sin\theta\cos\theta} + \frac{\sin\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta} + \frac{1}{\cos\theta}$

$= \mathrm{cosec}\theta + \sec\theta$

$= R.H.S$

Hence proved.

17. $\sin\theta(\tan\theta + \cot\theta) = \sec\theta$

Solution:

$L.H.S = \sin\theta(\tan\theta + \cot\theta)$

$= \sin\theta\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= \sin\theta\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)$

$= \sin\theta\left(\frac{1}{\cos\theta\sin\theta}\right)$

$= \frac{1}{\cos\theta}$

$= \sec\theta$

$= R.H.S$

Hence proved.

18. $\frac{1 + \cos\theta}{\sin\theta} + \frac{\sin\theta}{1 + \cos\theta} = 2\mathrm{cosec}\theta$

Solution:

$L.H.S = \frac{1 + \cos\theta}{\sin\theta} + \frac{\sin\theta}{1 + \cos\theta}$

$= \frac{(1 + \cos\theta)^2 + (\sin\theta)^2}{\sin\theta(1 + \cos\theta)}$

$= \frac{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1 + \cos\theta)}$

$= \frac{1 + 2\cos\theta + 1}{\sin\theta(1 + \cos\theta)}$

$= \frac{2 + 2\cos\theta}{\sin\theta(1 + \cos\theta)}$

$= \frac{2(1 + \cos\theta)}{\sin\theta(1 + \cos\theta)}$

$= \frac{2}{\sin\theta}$

$= 2\mathrm{cosec}\theta$

$= R.H.S$

Hence proved.

19. $\frac{1}{1 - \cos\theta} + \frac{1}{1 + \cos\theta} = 2\mathrm{cosec}^2\theta$

Solution:

$L.H.S = \frac{1}{1 - \cos\theta} + \frac{1}{1 + \cos\theta}$

$= \frac{1 + \cos\theta + 1 - \cos\theta}{(1 - \cos\theta)(1 + \cos\theta)}$

$= \frac{2}{1 - \cos^2\theta}$

$= \frac{2}{\sin^2\theta}$

$= 2\mathrm{cosec}^2\theta$

$= R.H.S$

Hence proved.

20. $\frac{1 + \sin\theta}{1 - \sin\theta} - \frac{1 - \sin\theta}{1 + \sin\theta} = 4\tan\theta\sec\theta$

Solution:

$L.H.S = \frac{1 + \sin\theta}{1 - \sin\theta} - \frac{1 - \sin\theta}{1 + \sin\theta}$

$= \frac{(1 + \sin\theta)^2 - (1 - \sin\theta)^2}{(1 - \sin\theta)(1 + \sin\theta)}$

$= \frac{(1 + 2\sin\theta + \sin^2\theta) - (1 - 2\sin\theta + \sin^2\theta)}{1 - \sin^2\theta}$

$= \frac{1 + 2\sin\theta + \sin^2\theta - 1 + 2\sin\theta - \sin^2\theta}{\cos^2\theta}$

$= \frac{4\sin\theta}{\cos^2\theta}$

$= 4\frac{\sin\theta}{\cos\theta}.\frac{1}{\cos\theta}$

$= 4\tan\theta\sec\theta$

$= R.H.S$

Hence proved

21. $\sin^3\theta = \sin\theta - \sin\theta\cos^2\theta$

Solution:

$R.H.S = \sin\theta - \sin\theta\cos^2\theta$

$= \sin\theta(1 - \cos^2\theta)$

$= \sin\theta.\sin^2\theta$

$= \sin^3\theta$

$= L.H.S$

Hence proved.

22. $\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)$

Solution:

$L.H.S = \cos^4\theta - \sin^4\theta$

$= (\cos^2\theta)^2 - (\sin^2\theta)^2$

$= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= (\cos^2\theta - \sin^2\theta)(1)$

$= \cos^2\theta - \sin^2\theta$

$= R.H.S$

Hence proved.

23. $\sqrt{\frac{1 + \cos\theta}{1 - \cos\theta}} = \frac{\sin\theta}{1 - \cos\theta}$

Solution:

$L.H.S = \sqrt{\frac{1 + \cos\theta}{1 - \cos\theta}}$

$= \sqrt{\frac{1 + \cos\theta}{1 - \cos\theta} \times \frac{1 - \cos\theta}{1 - \cos\theta}}$

$= \sqrt{\frac{(1 + \cos\theta)(1 - \cos\theta)}{(1 - \cos\theta)^2}}$

$= \sqrt{\frac{1 - \cos^2\theta}{(1 - \cos\theta)^2}}$

$= \sqrt{\frac{\sin^2\theta}{(1 - \cos\theta)^2}}$

$= \frac{\sqrt{\sin^2\theta}}{\sqrt{(1 - \cos\theta)^2}}$

$= \frac{\sin\theta}{1 - \cos\theta}$

$= R.H.S$

Hence proved.

24. $\sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = \frac{\sec\theta + 1}{\tan\theta}$

Solution:

$L.H.S = \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$

$= \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1} \times \frac{\sec\theta + 1}{\sec\theta + 1}}$

$= \sqrt{\frac{(\sec\theta + 1)^2}{(\sec\theta - 1)(\sec\theta + 1)}}$

$= \sqrt{\frac{(\sec\theta + 1)^2}{\sec^2\theta - 1}}$

$= \frac{\sqrt{(\sec\theta + 1)^2}}{\sqrt{\tan^2\theta}}$

$= \frac{\sec\theta + 1}{\tan\theta}$

$= R.H.S$

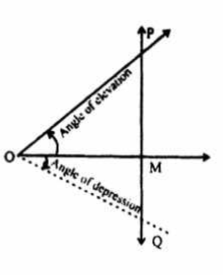
Hence proved.

Angle of Elevation and Angle of Depression:

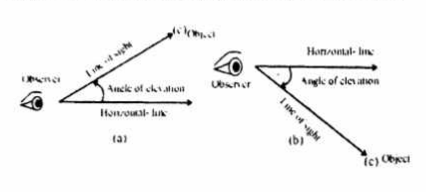
One of the objects of trigonometry is to find the distances between points or the heights of objects, without actually measuring these distances or heights.

Angle of elevation: Suppose O, P and Q are three points, P being at a higher level of O and Q being at lower level than O. Let a horizontal line drawn through O meet in M, the vertical line drawn through P and Q.

The angle MOP is called the angle of elevation of point P as seen from O. For looking at Q below the horizontal line we have to lower our eyes and ∠MOQ is called the angle of depression.



We measure an angle of elevation from a horizontal line up to an object or an angle of depression from a horizontal line down to an object,

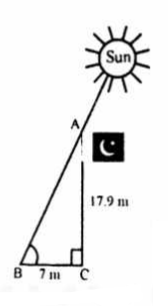


Find angle of elevation and angle of depression.

For finding distances height and angles by the use of trigonometric functions, consider the following examples:

Example 1

A flagpole 17.9-meter-high casts a 7-meter shadow. Find the angle of elevation of the sun.



Solution:

From the figure, we observe that is the angle of elevation.

Using the fact that

$$\tan\alpha = \frac{AC}{BC} = \frac{17.9}{7} \approx 2.55714$$

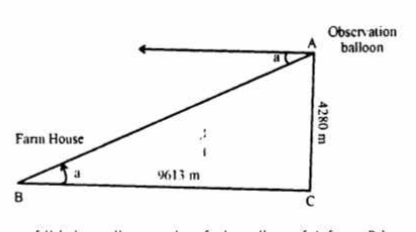
Solving for α gives us

$$\alpha = \tan^{-1}(2.55714)$$
$$= (68.6666)^\circ \approx 68^\circ 40'$$
$$\Rightarrow \quad \alpha \approx 68^\circ 40'$$

Example 2

An observation balloon is 4280 meter above the ground and 9613 meters away from a farmhouse. Find the angle of depressor of the farmhouse as observed from the observation balloon.

Solution:



For problems of this type the angle of elevation of A from B is considered equal to the

angle of depression of B from A, as shown in the diagram.

$$\tan\alpha = \frac{AC}{BC} = \frac{4280}{9613} = 0.44523$$
$$\alpha = \tan^{-1}(0.44523) = 24^\circ$$

So, angle of depression is 24°.

Exercise 7.5

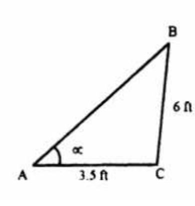
1. Find the angle of elevation of the sun if a 6 feet man casts a 3.5 feet shadow.

Solution:

From the figure, we observe that ∝ is the angle of elevation.

Using the fact that

$$\tan\propto = \frac{\overline{BC}}{\overline{AC}}$$
$$\tan\propto = \frac{6}{3.5}$$
$$\tan\propto = 1.7143$$
$$\propto = \tan^{-1}(1.7143)$$
$$\propto = 59.74^\circ$$



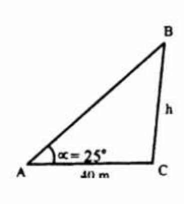
2. A tree casts a 40-meter shadow when the angle of elevation of the sun is 25°. Find the height of the tree.

Solution:

From the figure, we observe that ∝ is the angle of elevation.

Using the fact that

$$\tan\propto = \frac{\overline{BC}}{\overline{AC}}$$
$$\tan 25^\circ = \frac{h}{40}$$
$$h = 40 \times \tan 25^\circ$$
$$= 40(0.44663)$$
$$= 18.652m$$



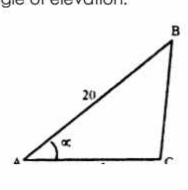
3. A 20 feet long ladder is leaning against a wall. The bottom of the ladder is 5 feet from the base of the wall. Find the acute angle (angle of elevation) the ladder makes with the ground.

Solution:

From the figure, we observe that ∝ is the angle of elevation.

Using the fact that

$$\cos\propto = \frac{\overline{AC}}{\overline{AB}}$$
$$\cos\propto = \frac{5}{20}$$
$$\propto = \cos^{-1}(0.25)$$
$$\propto = 75.5^\circ$$
$$or \propto = 75^\circ 30'$$



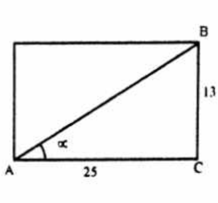
4. The base of a rectangle is 25 feet and the night of the rectangle is 13 feet. Find the angle that the diagonal of the rectangle makes with the base.

Solution:

From the figure, we observe that ∝ is the angle of elevation.

Using the fact that

$$\tan\propto = \frac{\overline{BC}}{\overline{AC}}$$
$$\tan\propto = \frac{13}{25}$$
$$\propto = \tan^{-1}(0.52)$$
$$\propto = 27.47^\circ$$



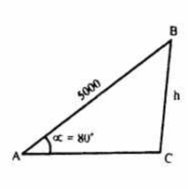
5. A rocket is launched and climbs at a constant angle of 80°. Find the altitude of the rocket after it travels 5000 meters.

Solution:

From the figure, we observe that ∝ is the angle of elevation.

Using the fact that

$$\sin\propto = \frac{\overline{BC}}{\overline{AB}}$$
$$\sin 80^\circ = \frac{h}{5000}$$
$$h = 5000 \times \sin 80^\circ$$
$$h = 5000(0.9848)$$
$$h = 4924.04m$$



6. An aero plane pilot flying at an altitude of 4000m wishes to make an approach to an airport at an angle of 50⁰ with the horizontal. How far from the airport will the plane be when the pilot begins to descend?

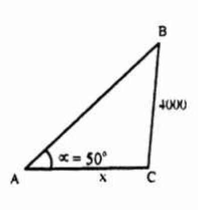
Solution:

From the figure, we observe that ∝ is the angle of elevation.

Using the fact that

$$\tan\propto = \frac{\overline{BC}}{\overline{AC}}$$

$$\tan 50^\circ = \frac{4000}{x}$$
$$x = \frac{4000}{\tan 50^\circ}$$
$$x = \frac{4000}{1.1918}$$
$$x = 3356.4m$$



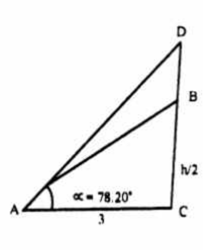
7. A guy wire (supporting wire) runs from the middle of a utility pole to the ground. The wire makes an angle of 78.2° with the ground and touch the ground 3 meters from the base of the pole. Find the height of the pole.

Solution:

From the figure, we observe that ∝ is the angle of elevation.

Using the fact that

$$\tan\propto = \frac{\overline{BC}}{\overline{AC}}$$
$$\tan 78.2^\circ = \frac{h/2}{3}$$
$$\frac{h}{2} = 3 \times \tan 78.2^\circ$$
$$\frac{h}{2} = 3(4.7867)$$
$$h = 6(407867)$$
$$h = 28.72m$$



8. A road is inclined at an angle 5.7°. Suppose that we drive 2 miles up this road starting from sea level. How high above sea level are we?

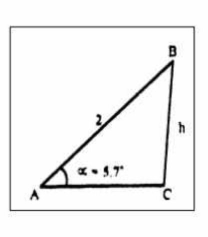
Solution:

From the figure, we observe that ∝ is the angle of elevation.

Using the fact that

$$\sin\propto = \frac{\overline{BC}}{\overline{AB}}$$

$$\sin 5.7^\circ = \frac{h}{2}$$
$$h = 2\cos 5.7^\circ$$
$$= 2(0.099)$$
$$= 0.198 \text{ miles}$$



9. A television antenna of 8 feet height is located on the top of a house. From a point on the ground the angle of elevation to the top of the house is 17° and the angle of elevation to the top of the antenna is 21.8°. Find the height of the house.

Solution:

In □ABC

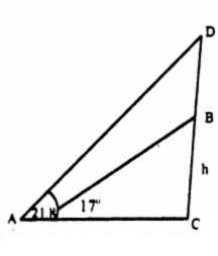
$$\tan 17^\circ = \frac{\overline{BC}}{\overline{AC}}$$
$$0.3057 = \frac{h}{AC}$$
$$AC = \frac{h}{0.3057} \quad (1)$$

In □ADC

$$\tan 21.8^\circ = \frac{\overline{CD}}{\overline{AC}}$$
$$0.4 = \frac{\overline{BC} + \overline{BD}}{AC}$$
$$0.4 = \frac{h+8}{AC}$$
$$AC = \frac{h+8}{0.4} \quad (2)$$

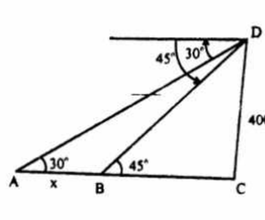
Comparing equation (1) and (2), we have

$$\frac{h}{0.3057} = \frac{h+8}{0.4}$$
$$0.4h = 0.3057(h+8)$$
$$0.4h = 0.3057h + 2.4456$$
$$0.4h - 0.3057h = 2.4456$$
$$0.0943h = 2.4456$$
$$h = \frac{2.4456}{0.0943}$$
$$h = 25.94 \text{ feet}$$



10. From an observation point, the angles of depression of two boats in line with this: point are found to 30° and 45°. Find the distance between the two boats if the point of observation is 4000 feet high.

Solution:



In □BCD

$$\tan 45^\circ = \frac{\overline{CD}}{\overline{BC}}$$
$$1 = \frac{4000}{\overline{BC}}$$
$$BC = 4000$$

In □ACD

$$\tan 30^\circ = \frac{\overline{CD}}{\overline{AC}}$$
$$\frac{1}{\sqrt{3}} = \frac{4000}{\overline{AB} + \overline{BC}}$$
$$\frac{1}{\sqrt{3}} = \frac{4000}{x + 4000}$$
$$x + 4000 = 4000\sqrt{3}$$
$$x = 4000\sqrt{3} - 4000$$
$$x = 6928.2 - 4000$$
$$x = 2928.2 \text{ feet}$$

11. Two ships, which are in line with the base of a vertical cliff, are 120 meters apart. The angles of depression from the top of the cliff to the ships are 30° and 45°, as show in the diagram.

(a) Calculate the distance $\overline{BC}$

(b) Calculate the height $\overline{CD}$, of the cliff.

Solution:

In □BCD

$$\tan 45^\circ = \frac{\overline{CD}}{\overline{BC}}$$
$$1 = \frac{\overline{CD}}{\overline{BC}}$$

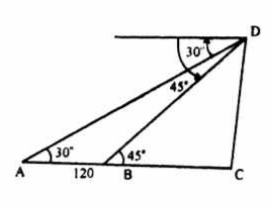
$$\overline{BC} = \overline{CD} \quad \text{........}(1)$$

In □ACD

$$\tan 30^\circ = \frac{\overline{CD}}{\overline{AC}}$$
$$\frac{1}{\sqrt{3}} = \frac{\overline{CD}}{\overline{AB} + \overline{BC}}$$
$$\frac{1}{\sqrt{3}} = \frac{\overline{BC}}{\overline{AB} + \overline{BC}}$$
$$\frac{1}{\sqrt{3}} = \frac{\overline{BC}}{120 + \overline{BC}} \quad \because From(1)\overline{BC} = \overline{CD}$$
$$\sqrt{3}\,\overline{BC} = 120 + \overline{BC}$$
$$\sqrt{3}\,\overline{BC} - \overline{BC} = 120$$
$$(\sqrt{3} - 1)\overline{BC} = 120$$
$$0.732\overline{BC} = 120$$
$$\overline{BC} = \frac{120}{0.732}$$
$$\overline{BC} = 164m$$

Put $\overline{BC} = 164$ in eq(1), we get

$$\overline{CD} = 164m$$



12. Suppose that we are standing on a bridge 30 feet above a river watching a log (piece of wood) floating toward we. If the angle with the horizontal to the front of the log is 16.7° and angle with the horizontal to the back of the log is 14°, how long is the log?

Solution:

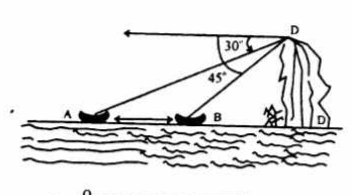
In □BOC

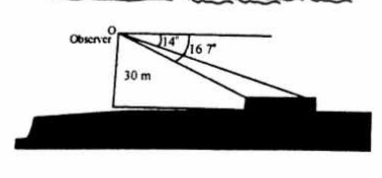
$$\tan 16.7^\circ = \frac{\overline{OC}}{\overline{BC}}$$
$$0.3 = \frac{30}{\overline{BC}}$$

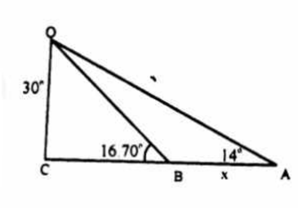
$$\overline{BC} = \frac{30}{0.3}$$
$$\overline{BC} = 100m$$

In □AOC

$$\tan 14^\circ = \frac{\overline{OC}}{\overline{AC}}$$
$$0.2493 = \frac{30}{\overline{AB} + \overline{BC}}$$
$$0.2493 = \frac{30}{x + 100}$$
$$0.2493(x + 100) = 30$$
$$0.2493x + 24.93 = 30$$
$$0.2493x = 30 - 24.93$$
$$0.2493x = 5.07$$
$$x = \frac{5.07}{0.2493}$$
$$x = 20.33m$$







Miscellaneous Exercise – 7

Q1. Multiple Choice Question

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) The union of two non-collinear rays, which have common end point is called

(a) an angle **(b)** a degree

(c) a minute **(d)** a radian

(ii) The system of measurement in which the angle is measured in radians is called

(a) CGS system **(b)** sexagesimal system

(c) MKS system **(d)** circular system

(iii) 20°

(a) 360′ **(b)** 630′

(c) 1200′ **(d)** 3600′

(iv) $\frac{3\pi}{4}$ **radians =**

(a) 115° **(b)** 135°

(c) 150° **(d)** 30°

(v) If tanθ = $\sqrt{3}$ **, then θ is equal to**

(a) 90° **(b)** 45°

(c) 60° **(d)** 30°

(vi) sec²θ =

(a) 1 - sin²θ **(b)** 1 + tan²θ

(c) 1 + cos²θ **(d)** 1 - tan²θ

(vii) $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$

(a) 2 sec²θ **(b)** 2 cos²θ

(c) sec²θ **(d)** cosθ

(viii) $\frac{1}{2}$ **cosec45°**

(a) $\frac{1}{\sqrt{2}}$ **(b)** $\frac{1}{\sqrt{3}}$

(c) $\sqrt{2}$ **(d)** $\frac{\sqrt{3}}{2}$

(ix) secθ cotθ =

(a) sinθ **(b)** $\frac{1}{\cos\theta}$

(c) $\frac{1}{\sin\theta}$ **(d)** $\frac{\cos\theta}{\sin\theta}$

(x) cosec²θ - cot²θ =

(a) -1 **(b)** 1

(c) 0 **(d)** tanθ

Answer:

(i)	a	**(ii)**	d	**(iii)**	c	**(iv)**	b	**(v)**	c
(vi)	b	**(vii)**	a	**(viii)**	b	**(ix)**	c	**(x)**	b

Q2. Write short answers of the following questions.

(i) Define an angle.

Ans:

An angle is defined as the union of two non-collinear rays with some common endpoint.

(ii) What is the sexagesimal system of measurement of angles?

Ans:

In sexagesimal system of measurement of angles, we find angles in degree, minutes and seconds.

(iii) How many minutes are in two right angles?

Ans:

In two right angles, we have 10800 minutes.

(iv) Define radian measure of an angle.

Ans:

The angle subtended at the center of the circle by an arc, whose length is equal to the radius of the circle, is called one radian.

(v) Convert $\frac{\pi}{4}$ **radian to degree measure.**

Ans:

$\frac{\pi}{4}$ c = $\frac{\pi}{4}$ radian = $\frac{\pi}{4}\times\frac{180^\circ}{\pi}=45^\circ$

(vi) Convert 15° **to radians.**

Ans:

$15^\circ = 15\times 1^\circ = 15\times\frac{\pi}{180^\circ}=\frac{\pi}{12}$ radians.

(vii) What is radian measure of the central angle of an arc 50m long on the circle of radius 25m.

Ans:

θ = ? , l = 50m, r = 25m

We know that

$l = r\theta$

$\theta = \frac{l}{r}=\frac{50}{25}=2$ radian

(viii) Find r when l = 56 cm and θ = 45°

Ans:

r = ? , l = 56 cm,

θ = 45° = 45 x 1° = $45\times\frac{\pi}{180}=\frac{\pi}{4}=0.7854$ radians

We know that

$l = r\theta$

$r=\frac{l}{\theta}=\frac{56}{0.7854}$

$= 71.30cm$

(ix) Find tanθ when cosθ = $\frac{9}{41}$ **and θ terminal side of the angle θ is in fourth quadrant.**

Ans:

$\cos\theta=\frac{9}{41}$

$\Rightarrow x = 9$ and $r = 41$

By Pythagoras theorem, we have

$r^2 = x^2 + y^2$

$y^2 = r^2 - x^2$

$y = \sqrt{r^2 - x^2}$

$=\sqrt{(41)^2-(9)^2}$

$=\sqrt{1681-81}$

$=\sqrt{1600}$

$= 40$

Now: $\tan\theta=\frac{y}{x}=-\frac{40}{9}$

(x) Prove that (1 - sin² θ)) (1 + tan² θ) = 1:

L.H.S = $\left(1-\sin^2\theta\right)\left(1+\tan^2\theta\right)$

$=\left(1-\sin^2\theta\right)\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)$

$=\left(\cos^2\theta\right)\left(\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}\right)$

$=\left(\cos^2\theta\right)\left(\frac{1}{\cos^2\theta}\right)$

$= 1$

$= R.H.S$

Hence proved.

Q3. Fill in the blanks

(i) π radians = _____degree.

(ii) The terminal side of angle 230° lies in _____quadrant.

(iii) Terminal side of the angle -30° lies in _____quadrant.

(iv) Area of a circular sector is _____.

(v) If r = 2 cm and θ = 3 radian, then area of the circular sector is _____.

(vi) The general form of the angle 480° is _____.

(vii) If sinθ = $\frac{1}{2}$, then θ = _____.

(viii) If θ = 300°, then sec (-300)° = _____.

(ix) 1 + cot²θ = _____.

(x) Sec²θ — tan²θ = _____.

Answer

(i)	180	**(ii)**	III	**(iii)**	IV
(iv)	$\frac{1}{2}r^2\theta$	**(v)**	6cm²	**(vi)**	2kπ + 120°, where k = 1
(vii)	θ=30° or $\frac{\pi}{6}$ rad	**(viii)**	2	**(ix)**	cosec²θ
(x)	$\frac{1-\sin\theta}{\cos\theta}$				

Summary

✓ If we divide the circumference of a circle into 360 equal arcs, then the angle subtended at the center of the circle by one arc is called one degree and is denoted by 1°.

✓ The angle subtended at the center of the circle by an arc, whose length is equal to the radius of the circle, is called one radian.

✓ Relationship between radian and degree measure: $1^\circ=\frac{\pi}{180}$radian = 0.0175 radian and 1 radian = $\left(\frac{180}{\pi}\right)^\circ\approx 57.295$ degrees

✓ Relation between central angle and arc length of a circle: $l = r\theta$

✓ Area of a circular sector, $A=\frac{1}{2}r^2\theta$

✓ Two or more than two angles with the same initial and terminal sides are called coterminal angles.

✓ An angle is called a quadrantal angle, if its terminal side lies on the x-axis or y-axis.

✓ A general angle is said to be in standard position if its vertex is at the origin and its initial side is directed along the positive direction of the x-axis of a rectangular coordinate system.

✓There are six fundamental trigonometric ratios (functions) known as sine, cosine, tangent, cotangent, secant and cosecant.

✓ Trigonometric identities:

(a) cos²θ + sin²θ =1 **(b)** 1+ tan²θ = sec²θ **(c)** 1 + cot²θ = cosec²θ