Exercise 5.1

1. If X = {1, 4, 7, 9} and Y = {2, 4, 5, 9}

then find

(i) X ∪ Y

(ii) X ∩ Y

(iii) Y ∪ X

(iv) Y ∩ X

Solution:

(i) X ∪ Y = {1, 4, 7, 9} ∪ {2, 4, 5, 9}

= {1, 2, 4, 7, 9}

(ii) X ∩ Y = {1, 4, 7, 9} ∩ {2, 4, 5, 9}

= {4, 9}

(iii) Y ∪ X = {2, 4, 5, 9} ∪ {1, 4, 7, 9}

= {1, 2, 4, 5, 7, 9}

(iv) Y ∪ X = {2, 4, 5, 9} ∩ {1, 4, 7, 9}

= {4, 9}

2. If P be all prime numbers less than or equal to 17 and Set of first 12 natural numbers then find the following:

(i) P ∩ Y **(ii)** Y ∪ P **(iii)** Y ∩ Y **(iv)** Y ∩ Φ

Solution:

Here set {2, 3, 5, 7, 11, 13, 17} = P & {1, 2, 3, 4, ..., 12} = Y

(i) P ∩ Y = {2, 3, 5, 7, 11, 13, 17} ∩ {1, 2, 3, 4, ..., 12}

= {1, 2, 3, 4, ..., 12, 13, 17}

= {1, 2, 3, 4, ..., 12} ∪ {13, 17}

= Y ∪ {13, 17}

(ii) Y ∪ P = {1, 2, 3, 4, ..., 12} ∪ {2, 3, 5, 7, 11, 13, 17}

= {1, 2, 3, 4, ..., 12, 13, 17}

= Y ∪ {13, 17}

(iii) Y ∩ Y = {1, 2, 3, 4, ..., 12} ∩ {1, 2, 3, 4, ..., 12}

= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

(iv) Y ∩ Φ = {1, 2, 3, 4, ..., 12} ∩ {}

= {}

3. If X = {1, 4, 7, 9} , Y = {1, 2, 4, 5, 7, 9, 11, 13, 17}

then find (i) X ∪ Y (ii) X ∩ Y (iii) Y ∪ Y (iv) X ∩ Y

(v) X ∩ Y (vi) Y ∩ Y

Solution:

X = {1, 4, 7, 9} ∩ Y = {1, 2, 4, 5, 7, 9, 11, 13, 17}

(i) X ∪ Y = {1, 4, 7, 9} ∪ {1, 2, 4, 5, 7, 9, 11, 13, 17}

= {1, 2, 4, 5, 7, 9, 11, 13, 17}

(ii) X ∩ Y = {1, 4, 7, 9} ∩ {1, 2, 4, 5, 7, 9, 11, 13, 17}

= {1, 4, 7, 9}

(iii) Y ∪ Y = {1, 2, 4, 5, 7, 9, 11, 13, 17} ∪ {1, 2, 4, 5, 7, 9, 11, 13, 17}

= {1, 2, 4, 5, 7, 9, 11, 13, 17}

(iv) X ∩ Y = {1, 4, 7, 9} ∩ {1, 2, 4, 5, 7, 9, 11, 13, 17}

= {1, 4, 7, 9}

(v) X ∩ Y = {1, 4, 7, 9} ∩ {1, 2, 4, 5, 7, 9, 11, 13, 17}

= {1, 4, 7, 9}

(vi) Y ∩ Y = {1, 2, 4, 5, 7, 9, 11, 13, 17} ∩ {1, 2, 4, 5, 7, 9, 11, 13, 17}

= {1, 2, 4, 5, 7, 9, 11, 13, 17}

4. If Z = {x | x is N ? 3 ≤ x ≤ 20}, X = {x | x is prime 10 < x < 20} and Y = {x | x ∈ N ? 4 ≤ x ≤ 17}. Find the value of:

(i) (X ∪ Y)' (ii) (X ∩ Y)' (iii) (X ∩ Y)' (iv) X' ∪ Y'

Solution:

Z = {4, 5, 6, 7, ..., 20}

X = {11, 13, 17, 19, 23}

Y = {4, 5, 6, 7, ..., 16, 17}

(i) (X ∪ Y)'= U - (X ∪ Y)

Now

X ∪ Y = {11, 13, 17, 19, 23} ∪ {4, 5, 6, 7, ..., 16, 17}

= {4, 5, 6, 7, ..., 16, 17, 19, 23}

(X ∪ Y)' = {4, 5, 6, 7, ..., 24, 25} - {11, 13, 17, 19, 23}

= {18, 20, 21, 22, 24, 25}

(ii) (X ∩ Y)'

Now X' ∩ U - X

= {4, 5, 6, 7, ..., 24, 25} - {11, 13, 17, 19, 23}

= {4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25}

Y' ∩ U - Y

= {4, 5, 6, 7, ..., 24, 25} - {4, 5, 6, 7, ..., 16, 17}

= {18, 19, 20, ..., 24, 25}

X' ∩ Y' = {4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25}

= {18, 19, 20, ..., 24, 25}

= {18, 20, 21, 22, 24, 25}

(iii) (X ∩ Y)' = U - (X ∩ Y)

Now

X ∩ Y = {11, 13, 17, 19, 23} ∩ {4, 5, 6, 7, ..., 16, 17}

= {11, 13, 17}

(X ∩ Y)' = {4, 5, 6, 7, ..., 24, 25} - {11, 13, 17}

= {4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 19, 20, 21, 22, 24, 25}

Y' ∩ U - Y

= {4, 5, 6, 7, ..., 24, 25} - {4, 5, 6, 7, ..., 16, 17}

= {18, 19, 20, ..., 24, 25}

(iv) X' ∪ Y'

Now X' = U - X

= {4, 5, 6, 7, ..., 24, 25} - {11, 13, 17, 19, 23}

= {4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25}

Y' = U - Y

= {4, 5, 6, ..., 15, 12, 14, 15, 16, 18, ..., 25} ∪ = { 18, 19, 20, ..., 24, 25}

= {4, 5, ..., 10, 12, 14, 16, 18, ..., 25}

5. If X = {2, 4, ..., 20} and Y = {2, 3, 12, ..., 24}. Then find the following:

(i) X ∩ Y (ii) Y - X

Solution:

X = {2, 4, ..., 20} ∩ Y = {2, 3, 12, ..., 24}

= {2, 4, 6, 8, 10, 12, 14, 16, ..., 20}

(ii) Y - X = {2, 3, 12, ..., 24} - {2, 4, 6, ..., 20}

= {3}

6. If A = N and B = W. Then find the value of

(i) A ∩ B (ii) B ∩ A

Solution:

A ∩ B = {N} ∩ B = W

(i) A ∩ B = N ∩ W

= N

(ii) B ∩ A = W ∩ N

= {0, 1, 2, ...} ∩ {1, 2, ...}

= {1}

Properties of Union and Intersection:

(a) Commutative property of union.

For any two sets A and B, prove that A ∪ B = B ∪ A.

Proof:

Let x ∈ A ∪ B

x ∈ A ∪ B ⇒ x ∈ A or x ∈ B (by definition of union of sets)

⇒ x ∈ B or x ∈ A

⇒ x ∈ B ∪ A

∴ A ∪ B ⊆ B ∪ A ... (i)

Now let y ∈ B ∪ A

y ∈ B ∪ A ⇒ y ∈ B or y ∈ A (by definition of union of sets)

⇒ y ∈ A or y ∈ B

⇒ y ∈ A ∪ B

∴ B ∪ A ⊆ A ∪ B ... (ii)

From (i) and (ii), we have A ∪ B = B ∪ A. (by definition of equal sets)

(b) Commutative property of intersection

For any two sets A and B, prove that A ∩ B = B ∩ A.

Proof: Let x ∈ A ∩ B

x ∈ A ∩ B ⇒ x ∈ A and x ∈ B (by definition of intersection of sets)

⇒ x ∈ B and x ∈ A

⇒ x ∈ B ∩ A

∴ A ∩ B ⊆ B ∩ A ... (i)

Now let y ∈ B ∩ A

y ∈ B ∩ A ⇒ y ∈ A and y ∈ B (by definition of intersection of sets)

⇒ y ∈ A and y ∈ B

⇒ y ∈ A ∩ B

∴ B ∩ A ⊆ A ∩ B ... (ii)

From (i) and (ii), we have A ∩ B = B ∩ A. (by definition of equal sets)

From (i) and (ii), we have A ∩ B = B ∩ A. (by definition of equal sets)**(c) Associative property of union**

For any three sets A, B and C, prove that (A ∪ B) ∪ C = A ∪ (B ∪ C)

Proof: Let x ∈ (A ∪ B) ∪ C

⇒ x ∈ (A ∪ B) or x ∈ C ... (i)

⇒ x ∈ A or x ∈ B or x ∈ C

⇒ x ∈ A or x ∈ B ∪ C

⇒ x ∈ A ∪ (B ∪ C) ... (ii)

∴ (A ∪ B) ∪ C ⊆ A ∪ (B ∪ C) ... (ii)

Similarly, A ∪ (B ∪ C) ⊆ (A ∪ B) ∪ C ... (iii)

From (ii) and (iii), we have

(A ∪ B) ∪ C = A ∪ (B ∪ C)

(c) Associative property of intersection

For any three sets A, B and C, prove that (A ∩ B) ∩ C = A ∩ (B ∩ C)

Proof: Let x ∈ (A ∩ B) ∩ C

⇒ x ∈ (A ∩ B) and x ∈ C

⇒ (x ∈ A and x ∈ B) and x ∈ C

⇒ x ∈ A and (x ∈ B and x ∈ C)

⇒ x ∈ A ∩ (B ∩ C)

∴ (A ∩ B) ∩ C ⊆ A ∩ (B ∩ C) ... (i)

Similarly, A ∩ (B ∩ C) ⊆ (A ∩ B) ∩ C ... (ii)

From (i) and (ii), we have

(A ∩ B) ∩ C = A ∩ (B ∩ C)

(d) Distributive property of union over intersection

For any three sets A, B and C, prove that A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

Proof: Let x ∈ A ∪ (B ∩ C)

⇒ x ∈ A or x ∈ B ∩ C

⇒ x ∈ A or (x ∈ B and x ∈ C)

⇒ (x ∈ A or x ∈ B) and (x ∈ A or x ∈ C)

⇒ x ∈ A ∪ B and x ∈ A ∪ C

⇒ x ∈ (A ∪ B) ∩ (A ∪ C)

Therefore, A ∪ (B ∩ C) ⊆ (A ∪ B) ∩ (A ∪ C) ... (i)

Similarly, now let y ∈ (A ∪ B) ∩ (A ∪ C)

⇒ y ∈ (A ∪ B) and y ∈ (A ∪ C)

⇒ (y ∈ A or y ∈ B) and (y ∈ A or y ∈ C)

⇒ y ∈ A or (y ∈ B and y ∈ C)

⇒ y ∈ A or y ∈ B ∩ C

⇒ y ∈ y ∈ A ∪ (B ∩ C)

∴ (A ∪ B) ∩ (A ∪ C) ⊆ A ∪ (B ∩ C) ... (ii)

From (i) and (ii), we have A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

∴ (A ∩ B) ∩ C ⊆ A ∩ (B ∩ C) ... (i)

Similarly, A ∩ (B ∩ C) ⊆ (A ∩ B) ∩ C ... (ii)

From (i) and (ii), we have

(A ∩ B) ∩ C = A ∩ (B ∩ C)

(e) Distributive property of union over intersection

For any three sets A, B and C, prove that A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

Proof: Let x ∈ A ∪ (B ∩ C)

⇒ x ∈ A or x ∈ B ∩ C

⇒ x ∈ A or (x ∈ B and x ∈ C)

⇒ (x ∈ A or x ∈ B) and (x ∈ A or x ∈ C)

⇒ x ∈ A ∪ B and x ∈ A ∪ C

⇒ x ∈ (A ∪ B) ∩ (A ∪ C)

Therefore, A ∪ (B ∩ C) ⊆ (A ∪ B) ∩ (A ∪ C) ... (i)

Similarly, now let y ∈ (A ∪ B) ∩ (A ∪ C)

⇒ y ∈ (A ∪ B) and y ∈ (A ∪ C)

⇒ (y ∈ A or y ∈ B) and (y ∈ A or y ∈ C)

⇒ y ∈ A or (y ∈ B and y ∈ C)

⇒ y ∈ A or y ∈ B ∩ C

⇒ y ∈ y ∈ A ∪ (B ∩ C)

∴ (A ∪ B) ∩ (A ∪ C) ⊆ A ∪ (B ∩ C) ... (ii)

From (i) and (ii), we have A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

(f) Distributive property of intersection over union

For any three sets A, B and C, prove that A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

Proof: Let x ∈ A ∩ (B ∪ C)

⇒ x ∈ A and x ∈ B ∪ C

⇒ x ∈ A and (x ∈ B or x ∈ C)

⇒ (x ∈ A and x ∈ B) or (x ∈ A and x ∈ C)

⇒ (x ∈ A ∩ B) ∪ (x ∈ A ∩ C)

⇒ x ∈ (A ∩ B) ∪ (A ∩ C)

Hence by def. of subsets

A ∩ (B ∪ C) ⊆ (A ∩ B) ∪ (A ∩ C) ... (i)

Similarly (A ∩ B) ∪ (A ∩ C) ⊆ A ∩ (B ∪ C) ... (ii)

From (i) and (ii), we have A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

(g) De Morgan's laws

For any two sets A and B, prove that

(i) (A ∪ B)' = A' ∩ B'

Proof: Let x ∈ (A ∪ B)'

⇒ x ∉ A ∪ B (by definition of complement of set)

⇒ x ∉ A or x ∉ B

⇒ x ∈ A' and x ∈ B' (by definition of intersection of sets)

⇒ (A ∪ B)' ⊆ A' ∩ B' ... (i)

Similarly, A' ∩ B' ⊆ (A ∪ B)' ... (ii)

Using (i) and (ii), we have (A ∪ B)' = A' ∩ B'

(ii) Let x ∈ (A ∩ B)'

⇒ x ∉ A ∩ B

⇒ x ∉ A or x ∉ B

⇒ x ∈ A' or x ∈ B'

⇒ x ∈ A' ∪ B'

∴ (A ∩ B)' ⊆ A' ∪ B' ... (iii)

Let y ∈ A' ∪ B'

⇒ y ∈ A' or y ∈ B'

⇒ y ∉ A or y ∉ B

⇒ y ∉ A ∩ B

⇒ y ∈ (A ∩ B)'

⇒ A' ∪ B' ⊆ (A ∩ B)' ... (iv)

From (iii) and (iv) we have proved that

(A ∩ B)' = A' ∪ B'

Exercise 5.2

1. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4\}$ & $A \times B$, $(B \times A)^c$ is equal to (A) 3, 4, 5, 6, 7, 8, (B) 1, 2, 3, 4, (C) $\{(3, 3), (4, 4)\}$, (D) None of the above. Solution

Solution

$A \times B = \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4), (4,3), (4,4)\}$

$B \times A = \{(3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

$(B \times A)^c = \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4), (4,3), (4,4)\}$

(B) 3 Y'(x) 2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(B) Y=2+3

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(C) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(D) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(A) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

2. If $N = \{1, 2, 3, 4, 5, 6\}$, $P = \{2, 4, 6\}$, $Q = \{3, 6\}$, then which one of the following is true? (A) $n(P \cup Q) = n(N)$ (B) $n(P \cap Q) = n(N)$ (C) $n(P \cup Q) = n(P)$ (D) $n(P \cap Q) = n(P)$ Solution

Solution

$N = \{1, 2, 3, 4, 5, 6\}$

$P = \{2, 4, 6\}$

$Q = \{3, 6\}$

$P \cup Q = \{2, 3, 4, 6\}$

$P \cap Q = \{6\}$

$n(P \cup Q) = 4$

$n(P \cap Q) = 1$

$n(N) = 6$

$n(P) = 3$

$n(P \cup Q) \neq n(N)$

$n(P \cap Q) \neq n(N)$

$n(P \cup Q) \neq n(P)$

$n(P \cap Q) = n(P)$

(A) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(B) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(C) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(D) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

3. If $N(A) = 32$, $N(B) = 42$ & $N(A \cap B) = 10$, then $N(A \cup B)$ is equal to (A) 74, (B) 64, (C) 84, (D) None of these. Solution

Solution

$N(A) = 32$

$N(B) = 42$

$N(A \cap B) = 10$

$N(A \cup B) = N(A) + N(B) - N(A \cap B)$

$N(A \cup B) = 32 + 42 - 10$

$N(A \cup B) = 64$

(A) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(B) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(C) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(D) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

4. If $N(U) = 53$, $N(A) = 28$, $N(B) = 32$ & $N(A' \cap B') = 17$, then $N(A \cap B)$ is equal to (A) 24, (B) 20, (C) 12, (D) None of these. Solution

Solution

$N(U) = 53$

$N(A) = 28$

$N(B) = 32$

$N(A' \cap B') = 17$

$N(A \cup B) = N(U) - N(A' \cap B')$

$N(A \cup B) = 53 - 17$

$N(A \cup B) = 36$

$N(A \cap B) = N(A) + N(B) - N(A \cup B)$

$N(A \cap B) = 28 + 32 - 36$

$N(A \cap B) = 24$

(A) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(B) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(C) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(D) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

5. If $N(A) = 28$, $N(B) = 32$ & $N(A \cap B) = 24$, then $N(A \cup B)$ is equal to (A) 36, (B) 32, (C) 24, (D) None of these. Solution

Solution

$N(A) = 28$

$N(B) = 32$

$N(A \cap B) = 24$

$N(A \cup B) = N(A) + N(B) - N(A \cap B)$

$N(A \cup B) = 28 + 32 - 24$

$N(A \cup B) = 36$

(A) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(B) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(C) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(D) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

6. If $N(A) = 28$, $N(B) = 32$ & $N(A \cup B) = 36$, then $N(A \cap B)$ is equal to (A) 24, (B) 20, (C) 12, (D) None of these. Solution

Solution

$N(A) = 28$

$N(B) = 32$

$N(A \cup B) = 36$

$N(A \cap B) = N(A) + N(B) - N(A \cup B)$

$N(A \cap B) = 28 + 32 - 36$

$N(A \cap B) = 24$

(A) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(B) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(C) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(D) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

7. If $N(A) = 28$, $N(B) = 32$ & $N(A \cap B) = 24$, then $N(A \cup B)$ is equal to (A) 36, (B) 32, (C) 24, (D) None of these. Solution

Solution

$N(A) = 28$

$N(B) = 32$

$N(A \cap B) = 24$

$N(A \cup B) = N(A) + N(B) - N(A \cap B)$

$N(A \cup B) = 28 + 32 - 24$

$N(A \cup B) = 36$

(A) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(B) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(C) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(D) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

8. If $N(A) = 28$, $N(B) = 32$ & $N(A \cup B) = 36$, then $N(A \cap B)$ is equal to (A) 24, (B) 20, (C) 12, (D) None of these. Solution

Solution

$N(A) = 28$

$N(B) = 32$

$N(A \cup B) = 36$

$N(A \cap B) = N(A) + N(B) - N(A \cup B)$

$N(A \cap B) = 28 + 32 - 36$

$N(A \cap B) = 24$

(A) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(B) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(C) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(D) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

9. If $N(A) = 28$, $N(B) = 32$ & $N(A \cap B) = 24$, then $N(A \cup B)$ is equal to (A) 36, (B) 32, (C) 24, (D) None of these. Solution

Solution

$N(A) = 28$

$N(B) = 32$

$N(A \cap B) = 24$

$N(A \cup B) = N(A) + N(B) - N(A \cap B)$

$N(A \cup B) = 28 + 32 - 24$

$N(A \cup B) = 36$

(A) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(B) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(C) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(D) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

10. If $N(A) = 28$, $N(B) = 32$ & $N(A \cup B) = 36$, then $N(A \cap B)$ is equal to (A) 24, (B) 20, (C) 12, (D) None of these. Solution

Solution

$N(A) = 28$

$N(B) = 32$

$N(A \cup B) = 36$

$N(A \cap B) = N(A) + N(B) - N(A \cup B)$

$N(A \cap B) = 28 + 32 - 36$

$N(A \cap B) = 24$

(A) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(B) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(C) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

(D) Y=3x+2

Solution

$Y'(x) = 3$

$Y(x) = 3x + C$

$Y(0) = 3(0) + C = 2$

$C = 2$

$Y(x) = 3x + 2$

Verify the fundamental properties for given sets

(a) A and B are two subsets of U. Prove $A \cap B \subseteq A \subseteq A \cup B$ (commutative law).

Solution

$A \cap B \subseteq A$ Subset Of (A, B, U)

Proof: $A \cap B = \{x : x \in A \text{ and } x \in B\} \subseteq A$

and $A \subseteq \{x : x \in A \text{ or } x \in B\} = A \cup B$

Hence, verified that $A \cap B \subseteq A \subseteq A \cup B$

(b) Commutative property of intersection

For example A = {1, 2, 3, 4} and B = {2, 3, 5}

Then $A \cap B = \{1, 2, 3, 4\} \cap \{2, 3, 5\} = \{2, 3\}$

and $B \cap A = \{2, 3, 5\} \cap \{1, 2, 3, 4\} = \{2, 3\}$

Hence, verified that $A \cap B = B \cap A$.

(c) Commutative property of Union

For example A = {1, 2, 3, 4} and B = {2, 3, 5}

Then $A \cup B = \{1, 2, 3, 4\} \cup \{2, 3, 5\} = \{1, 2, 3, 4, 5\}$

and $B \cup A = \{2, 3, 5\} \cup \{1, 2, 3, 4\} = \{1, 2, 3, 4, 5\}$

Hence, verified that $A \cup B = B \cup A$.

(d) $A \cap (B \cap C) = (A \cap B) \cap C$ (Associative law)

Suppose, $A = \{1, 2, 3\}$ $B = \{2, 3, 4\}$ and $C = \{3, 4, 5\}$.

Now, $B \cap C = \{2, 3, 4\} \cap \{3, 4, 5\}$

$= \{3, 4\}$

$A \cap (B \cap C) = \{1, 2, 3\} \cap \{3, 4\}$

$= \{3\}$ ... (i)

Also, $A \cap B = \{1, 2, 3\} \cap \{2, 3, 4\}$

$= \{2, 3\}$

$(A \cap B) \cap C = \{2, 3\} \cap \{3, 4, 5\}$

$= \{3\}$ ... (ii)

From (i) and (ii), we have

$A \cap (B \cap C) = (A \cap B) \cap C$

Hence, verified the property of set's Associativity.

Distributive laws

(a) Union is distributive over intersection of sets

Suppose, $A = \{1, 2, 3, 4\}$ $B = \{2, 3, 4\}$ and $C = \{3, 4, 5\}$ Let us prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Suppose $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4\}$ and $C = \{3, 4, 5\}$

Now $B \cap C = \{2, 3, 4\} \cap \{3, 4, 5\}$

$= \{3, 4\}$

$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup \{3, 4\}$

$= \{1, 2, 3, 4\}$ ... (i)

$A \cup B = \{1, 2, 3, 4\} \cup \{2, 3, 4\}$

$= \{1, 2, 3, 4\}$

$A \cup C = \{1, 2, 3, 4\} \cup \{3, 4, 5\}$

$= \{1, 2, 3, 4, 5\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4\} \cap \{1, 2, 3, 4, 5\}$

$= \{1, 2, 3, 4\}$ ... (ii)

From (i) and (ii), we have

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Hence, verified that Union is distributive over intersection.

(b) Intersection is distributive over union of sets

Suppose $A = \{1, 2, 3, 4\}$ $B = \{2, 3, 4\}$ and $C = \{3, 4, 5\}$

Suppose $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4\}$ and $C = \{3, 4, 5\}$

Now $B \cup C = \{2, 3, 4\} \cup \{3, 4, 5\}$

$= \{2, 3, 4, 5\}$

$A \cap (B \cup C) = \{1, 2, 3, 4\} \cap \{2, 3, 4, 5\}$

$= \{2, 3, 4\}$ ... (i) $A \cap B = \{1, 2, 3, 4\} \cap \{2, 3, 4\}$

$= \{2, 3, 4\}$ $A \cap C = \{1, 2, 3, 4\} \cap \{3, 4, 5\}$

$= \{3, 4\}$ $(A \cap B) \cup (A \cap C) = \{2, 3, 4\} \cup \{3, 4\}$

$= \{2, 3, 4\}$ ... (ii)

From (i) and (ii), we have

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Hence, verified that Intersection is distributive over union of sets.

(c) De Morgan's laws and $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$

Suppose $U = \{1, 2, 3, 4, ... 10\}$

$A = \{1, 2, 3, 4\}$ $A' = \{5, 6, 7, 8, 9, 10\}$

$B = \{2, 3, 4, 5\}$ $B' = \{1, 6, 7, 8, 9, 10\}$

Now $A \cup B = \{1, 2, 3, 4\} \cup \{2, 3, 4, 5\}$

$= \{1, 2, 3, 4, 5\}$

$(A \cup B)' = \{6, 7, 8, 9, 10\}$ ... (i)

$A' \cap B' = \{5, 6, 7, 8, 9, 10\} \cap \{1, 6, 7, 8, 9, 10\}$

$= \{6, 7, 8, 9, 10\}$ ... (ii)

From (i) and (ii), we have $(A \cup B)' = A' \cap B'$

Now $A \cap B = \{1, 2, 3, 4\} \cap \{2, 3, 4, 5\}$

$= \{2, 3, 4\}$

$(A \cap B)' = \{1, 5, 6, 7, 8, 9, 10\}$ ... (iii)

$A' \cup B' = \{5, 6, 7, 8, 9, 10\} \cup \{1, 6, 7, 8, 9, 10\}$

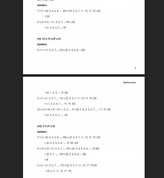
$= \{1, 5, 6, 7, 8, 9, 10\}$ ... (iv)

From (iii) and (iv), we have $(A \cap B)' = A' \cup B'$



(d) Cardinality of Union of sets

(e) Complement laws for union and intersection of sets.



Use Venn diagram to verify

(a) Commutative law for union and intersection of sets.



(b) De Morgan's law

Fig 1.1 : $(A \cup B)'$ is shown by shaded region

Fig 1.2 : $A' \cap B'$ is shown by shaded region

Fig 1.3 : $(A \cap B)'$ is shown by shaded region

Fig 1.4 : $A' \cup B'$ is shown by shaded region

Proof of $(A \cup B)' = A' \cap B'$ by Venn diagram

Fig 1.1 shows $(A \cup B)'$ and Fig 1.2 shows $A' \cap B'$. As the shaded regions are identical, so $(A \cup B)' = A' \cap B'$.

Proof of $(A \cap B)' = A' \cup B'$ by Venn diagram

Fig 1.3 shows $(A \cap B)'$ and Fig 1.4 shows $A' \cup B'$. As the shaded regions are identical, so $(A \cap B)' = A' \cup B'$.

Fig 1.5 : $A \cup (B \cap C)$ is shown by shaded region

Fig 1.6 : $(A \cup B) \cap (A \cup C)$ is shown by shaded region

Fig 1.7 : $A \cap (B \cup C)$ is shown by shaded region

Fig 1.8 : $(A \cap B) \cup (A \cap C)$ is shown by shaded region

Proof of $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ by Venn diagram

Fig 1.5 shows $A \cup (B \cap C)$ and Fig 1.6 shows $(A \cup B) \cap (A \cup C)$. As the shaded regions are identical, so $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Proof of $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by Venn diagram

Fig 1.7 shows $A \cap (B \cup C)$ and Fig 1.8 shows $(A \cap B) \cup (A \cap C)$. As the shaded regions are identical, so $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Fig 1.9 : $A \cup A' = U$ is shown by shaded region

Fig 1.10 : $A \cap A' = \emptyset$ is shown by shaded region

Proof of $A \cup A' = U$ by Venn diagram

Fig 1.9 shows $A \cup A' = U$.

Proof of $A \cap A' = \emptyset$ by Venn diagram

Fig 1.10 shows $A \cap A' = \emptyset$.

Fig 1.11 : $U' = \emptyset$ is shown by shaded region

Fig 1.12 : $\emptyset' = U$ is shown by shaded region

Proof of $U' = \emptyset$ by Venn diagram

Fig 1.11 shows $U' = \emptyset$.

Proof of $\emptyset' = U$ by Venn diagram

Fig 1.12 shows $\emptyset' = U$.

Fig 1.13 : $(A')'$ is shown by shaded region

Proof of $(A')' = A$ by Venn diagram

Fig 1.13 shows $(A')' = A$.

Use Venn diagram to represent

(a) Union and intersection of sets

Fig 1.14 : Union of sets is shown by shaded region

Fig 1.15 : Intersection of sets is shown by shaded region

Use Venn diagram to verify

(a) Commutative law for union and intersection of sets.

Fig 1.16 : $A \cup B$ is shown by shaded region

Fig 1.17 : $B \cup A$ is shown by shaded region

Proof of $A \cup B = B \cup A$ by Venn diagram

Fig 1.16 shows $A \cup B$ and Fig 1.17 shows $B \cup A$. As the shaded regions are identical, so $A \cup B = B \cup A$.

Fig 1.18 : $A \cap B$ is shown by shaded region

Fig 1.19 : $B \cap A$ is shown by shaded region

Proof of $A \cap B = B \cap A$ by Venn diagram

Fig 1.18 shows $A \cap B$ and Fig 1.19 shows $B \cap A$. As the shaded regions are identical, so $A \cap B = B \cap A$.

Fig 1.20 : $A \cup (B \cap C)$ is shown by shaded region

Fig 1.21 : $(A \cup B) \cap (A \cup C)$ is shown by shaded region

Proof of $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ by Venn diagram

Fig 1.20 shows $A \cup (B \cap C)$ and Fig 1.21 shows $(A \cup B) \cap (A \cup C)$. As the shaded regions are identical, so $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Fig 1.22 : $A \cap (B \cup C)$ is shown by shaded region

Fig 1.23 : $(A \cap B) \cup (A \cap C)$ is shown by shaded region

Proof of $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by Venn diagram

Fig 1.22 shows $A \cap (B \cup C)$ and Fig 1.23 shows $(A \cap B) \cup (A \cap C)$. As the shaded regions are identical, so $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Exercise 5.3

1. If $U = \{1, 2, 3, 4, ..., 10)$

$A = \{1, 3, 5, 7, 9)$

$B = (1, 4, 7, 10)$ then verify the following questions,

(i) $A - B = A \cap B'$

L.H.S. $= A - B$

$= \{1, 3, 5, 7, 9) — \{1, 4, 7, 10\}$

$= \{3, 5, 9\}$ .....(i)

R.H.S. $= A \cap B'$

$= A \cap (U - B)$

$= \{1, 3, 5, 7, 9\} \cap (\{1, 2, 3, 4, ..., 10\} - \{1, 4, 7, 10\})$

$= (1, 3, 5, 7, 9) \cap (2, 3, 5, 6, 8, 9)$

$= (3, 5, 9)$ .....(ii)

From (1) and (ii), we have L.H.S. = R.H.S.

Hence Proved

(ii) $B - A = B \cap A'$

L.H.S. $= B - A$

$= (1, 4, 7, 10) - (1, 3, 5, 7, 9)$

$= (4, 10)$ ....(i)

R.H.S. $= B \cap A'$

$= B \cap (U - A)$

$= (1, 4, 7, 10) \cap \{1, 2, 3, 4, ..., 10\} - (1, 3, 4, 5, 7, 9))$

$= (1, 4, 7, 10) \cap (2, 4, 6, 8, 10)$

$= (4, 10)$ ....(ii)

From (i) and (ii), we have L.H.S. = R.H.S.

Hence Proved

(iii) $(A \cup B)' = A' \cap B'$

L.H.S. $= (A \cup B)'$

$= U - (A \cup B)$

$= \{1, 2, 3, 4, ..., 10\} - (\{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\})$

$= (1, 2, 3, 4, ..., 10) - (1, 3, 4, 5, 7, 9, 10)$

$= (2, 6, 8)$ .....(i)

R.H.S. $= A' \cap B'$

$= (U - A) \cap (U - B)$

$= (\{1, 2, 3, 4, ..., 10\} - \{1, 3, 5, 7, 9\}) \cap (1, 2, 3, 4, ..., 10) - \{1, 4, 7, 10\})$

$= (2, 4, 6, 8, 10\} \cap (2, 3, 5, 6, 8, 9)$

$= (2, 6, 8)$ ......ii)

From (i) and (ii), we have L.H.S. = R.H.S.

Hence Proved

(iv) $(A \cap B)' = A' \cup B'$

L.H.S. $= (A \cup B)'$

$= U - (A \cap B)$

$= (1, 2, 3, 4, ..., 10) - (\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\})$

$= (1, 2, 3, 4, ...10) - \{1, 7\}$

$= (2, 3, 4, 5, 6, 8, 9, 10)$ ....(i)

R.H.S. $= A' \cup B'$

$= (U - A) \cup (U - B)$

$= (\{1, 2, 3, 4, ..., 10\} - \{1, 3, 5, 7, 9\}) \cup (\{1, 2, 3, 4, ..., 10) - \{1, 4, 7, 10))$

$= \{2, 4, 6, 8, 10) \cup (2, 3, 5, 6, 8, 9)$

$= \{2, 3, 4, 5, 6, 8, 9, 10)$ .....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S. Hence Proved

(v) $(A - B)' = A' \cup B$

L.H.S. $= (A - B)'$

$= U - (A - B)$

$= \{1, 2, 3, 4, ..., 10) - (\{1, 3, 5, 7, 9) - \{1, 4, 7, 10\})$

$= \{1, 2, 3, 4, ... 10\} - \{3, 5, 9)$

$= \{1, 2, 3, 6, 7, 8, 10)$ .....(i)

R.H.S. $= A' \cup B'$

$= (U - A) \cup B$

$= (\{1, 2, 3, 4, ..., 10) - \{1, 3, 5, 7, 9\}) \cup \{1, 4, 7, 10)$

$= \{2, 4, 6, 8, 10) \cup \{1, 4, 7, 10)$

$= \{1, 2, 4, 6, 7, 8, 10)$ ....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S. Hence Proved

(VI) $(B - A)' = B' \cup A$

L.H.S. $= (B \cup A)'$

$= U - (B - A)$

$= \{l, 2, 3, 4, ..., 10\} - \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\})$

$= \{1, 2, 3, 4, ..., 10) - \{4, 10)$

$= \{l, 2, 3, 5, 6, 7, 8, 9\}$ .....(i)

R.H.S. $= B' \cup A$

$= (U - B) \cup A$

$= (\{1, 2, 3, 4, ..., 10\} - \{1, 3, 5, 7, 9\}) \cup \{1, 3, 5, 7, 9)$

$= \{2, 3, 5, 6, 8, 9\} \cup \{1, 3, 5, 7, 9)$

$= \{1, 2, 3, 5, 6, 7, 8, 9\}$ .....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

2. If $U = (1, 2, 3, 4, -, 10)$; $A = (1, 3, 5, 7, 9)$; $B = (1, 4, 7, 10)$; $C = \{1, 5, 8, 10)$ then verify the following:

Solution:

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

L.H.S. $= (A \cup B) \cup C$

$= (\{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\}) \cup \{l, 5, 8, 10\}$

$= \{1, 3, 4, 5, 7, 9, 10\} \cup \{1, 5, 8, 10\}$

$= \{1, 3, 4, 5, 7, 8, 9, 10\}$ ....(i)

R.H.S. $= A \cup (B \cup C)$

$= \{1, 3, 5, 7, 9) \cup (\{1, 4, 7, 10) \cup \{1, 5, 8, 10))$

$= \{l, 3, 5, 7, 9\} \cup \{1, 4, 5, 7, 8, 10)$

$= \{1, 3, 4, 5, 7, 8, 9, 10)$ ....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

L.H.S. $= (A \cap B) \cap C$

$= (\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}) \cap \{1, 5, 8, 10\}$

$= \{1, 7\} \cap \{1, 5, 8, 10)$

$= \{1\}$ ......(i)

R.H.S. $= A \cap (B \cap C)$

$= \{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\} \cap \{1, 5, 8, 10\})$

$= (1, 3, 5, 7, 9) \cap \{1, 10\}$

$= \{1)$ ......(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

(iii) $A \cup (B \cup C) = (A \cup B) \cap (A \cup C)$

L.H.S. $= A \cup (B \cap C)$

$= \{1, 3, 5, 7, 9) \cup \{1, 4, 7, 10)) \cap \{1, 5, 8, 10)$

$= \{1, 3, 5, 7, 9) \cup \{1, 10)$

$= \{1, 3, 5, 7, 9, 10)$ ......(i)

R.H.S. $= (A \cup B \cap (A \cup C)$

$= (\{1, 3, 5, 7, 9\} \cup (\{1, 4, 7, 10\}) \cap (\{1, 3, 5, 7\} \cup \{1, 5, 8, 10))$

$= \{1, 3, 4, 5, 7, 9, 10\} \cap \{1, 3, 5, 7, 8, 9, 10\}$

$= \{1, 3, 5, 7, 9, 10)$ ....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S. $= A \cap (B \cup C)$

$= \{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\} \cup \{1, 5, 8, 10\})$

$= \{1, 3, 5, 7, 9\} \cap \{1, 4, 5, 7, 8, 10)$

$= \{1, 5, 7)$ ....(i)

R.H.S. $= (A \cap B \cup (A \cap C)$

$= (\{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\}) \cup (\{1, 3, 5, 7, 9\} \cap \{1, 5, 8, 10\})$

$= \{1, 7) \cup \{1, 5)$

$= \{1, 5, 7)$ ....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

3. If $U = N$; then verify De-Morgan's laws by using $A = \emptyset$ and $B = P$. Solution:

Solution:

$U = N$, $A = \emptyset$, $B = P$

(i) $(A \cap B)' = A' \cup B'$

L.H.S. $= (A \cap B)'$

$= U - (A \cap B)$

$= N - (\emptyset \cap P)$

$= N - \emptyset$

$= N$ ....(i)

R.H.S. $= A' = B'$

$= (U - A) \cup (U - B)$

$= (N - \emptyset) \cup (N - P)$

$= N \cup (N - P)$

$= N$ ....(ii)

From (i) and (ii), we have

L. H.S. = R.H.S

Hence Proved.

(ii) $(A \cup B)' = A' \cap B'$

L.H.S. $= (A \cup B)'$

$= U - (A \cup B)$

$= N - (\emptyset \cup P)$

$= N - \emptyset$

$= N$ .....(i)

R.H.S. $= A' \cap B'$

$= (U - A) \cap (U - B)$

$= (N - \emptyset) \cap (N - P)$

$= N \cap (N - P)$

$= N - P$ ......(ii)

From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.

4. If $U = \{1, 2, 3, 4, ... 10)$, $A - \{1, 3, 5, 7, 9)$ and $B = \{2, 3, 4, 5, 8)$ then prove the following questions by Venn diagram:

(i) $A - B = A \cap B'$

Solution:

$U = \{1, 2, 3, 4, ..., 10\}$ $A = \{1, 3, 5, 7, 9\}$

$B = \{2, 3, 4, 5, 8\}$

(i) $A - B = A \cap B'$

L.H.S. $= A - B$

$= \{1, 3, 5, 7, 9\} - \{2, 3, 4, 5, 8\}$

$= \{1, 7, 9\}$ ......(i)

R.H.S. $= A \cap B'$

$= A \cap (U - B)$

$= \{1, 3, 5, 7, 9\} \cap (\{l, 2, 3, 4, ..., 10\} - \{2, 3, 4, 5, 8\})$

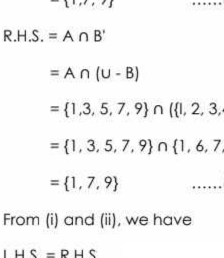
$= \{1, 3, 5, 7, 9\} \cap \{1, 6, 7, 9, 10\}$

$= \{1, 7, 9\}$ ......(ii)

From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.



A B

$A \cap B'$

(ii) $B - A = B \cap A'$

Solution:

L.H.S $= B - A$

$= \{2, 3, 4, 5, 8\} - \{l, 3, 5, 7, 9\}$

$= \{2, 4, 8\}$ ....(i)

R.H.S. $= B \cap A'$

$= B \cap (U - A)$

$= \{2, 3, 4, 5, 8\} \cap (\{1, 2, 3, 4, ..., 10\} - \{1, 3, 5, 7, 9\})$

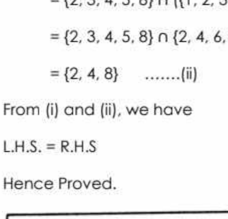
$= \{2, 3, 4, 5, 8\} \cap \{2, 4, 6, 8\}$

$= \{2, 4, 8\}$ ....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.



B - A

$B \cap A'$

(iii) $(A \cup B)' = A' \cap B'$

Solution:

L.H.S. $= (A \cup B)'$

$= \{1, 2, 3, 4, ..., 10\} - (\{1, 3, 5, 7, 9\} \cup \{2, 3, 4, 5, 8\})$

$= \{1, 2, 3, 4, ..., 10\} - \{1, 2, 3, 4, 5, 7, 8, 9\}$

$= \{6, 10\}$ .....(i)

R.H.S. $= A' \cap B'$

$= (U - A) \cap (U - B)$

$= (\{1, 2, 3, 4, ..., 10\} - \{1, 3, 5, 7, 9\}) \cap (\{1, 2, 3, 4, ..., 10\} - \{2, 3, 4, 5, 8\})$

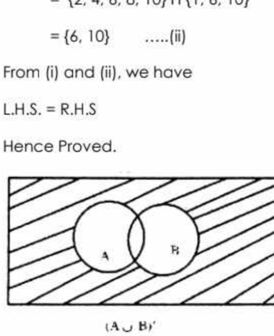
$= \{2, 4, 6, 8, 10\} \cap \{1, 6, 10\}$

$= \{6, 10\}$ .....(ii)

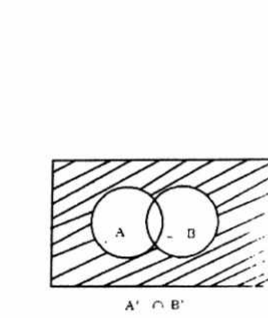
From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.



$(A \cup B)'$



$A' \cap B'$

(iv) $(A \cap B)' = A \cup B'$

Solution:

L.H.S $= (A \cap B)'$

$= U - (A \cap B)$

$= \{1, 2, 3, 4, ..., 10\} - (\{1, 3, 5, 7, 9\} \cap \{2, 3, 4, 5, 8\})$

$= \{1, 2, 3, 4, ..., 10\} - \{3, 5\}$

$= \{1, 2, 4, 6, 7, 8, 9, 10\}$ .....(i)

R.H.S. $= A' \cup B'$

$= (U - A) \cup (U - B)$

$= (\{l, 2, 3, 4, , 10\} - \{1, 3, 5, 7, 9\} \cup (\{1, 2, 3, 4, ..., 10\} - \{2, 3, 4, 5, 8\})$

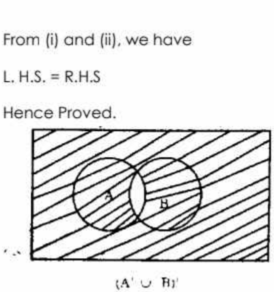
$= \{2, 4, 6, 8, 10\} - \{1, 6, 7, 9, 10\}$

$= \{1, 2, 4, 6, 7, 8, 9, 10\}$ ....(i)

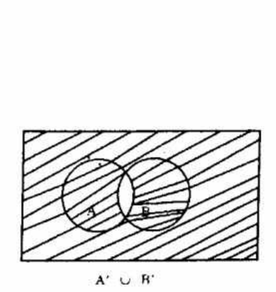
From (i) and (ii), we have

L. H.S. = R.H.S

Hence Proved.



$(A' \cup B)'$



$A' \cup B'$

(v) $(A - B)' = A' \cup B$

Solution:

L.H.S. $= (A - B)'$

$= U - (A - B)$

$= \{1, 2, 3, 4, ..., 10\} - (\{1, 3, 5, 7, 9\} - \{2, 3, 4, 5, 8\})$

$= \{1, 2, 3, 4, ..., 10\} - \{1, 7, 9\}$

$= \{2, 3, 4, 5, 6, 8, 10\}$ ......(i)

R.H.S. $= A' \cup B$

$= (U - A) \cup B$

$= \{1, 2, 3, 4, ..., 10\} - (\{1, 3, 5, 7, 9\}) \cup \{2, 3, 4, 5, 8\}$

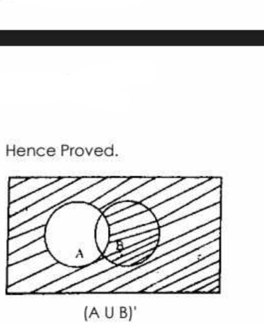
$= \{2, 4, 6, 8, 10\} \cup \{2, 3, 4, 5, 8\}$

$= \{2, 3, 4, 5, 6, 8, 10\}$ ....(ii)

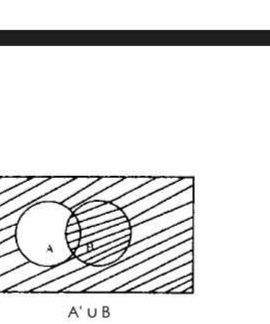
From (i) and (ii), we have

L. H.S. = R.H.S

Hence Proved.



$(A \cup B)'$



$A' \cup B$

(vi) $(B - A)' = B' \cup A$

Solution:

L.H.S. $= (B - A)'$

$= U - (B - A)$

$= \{1, 2, 3, 4, ..., 10\} - (\{2, 3, 4, 5, 8\} - \{l, 3, 5, 7, 9\})$

$= \{1, 2, 3, 4, ..., 10\} - \{2, 4, 8\}$

$= \{1, 3, 4, 5, 6, 7, 9, 10\}$ .....(i)

R.H.S. $= B' \cap A$

$= (U - B) \cup A$

$= (\{1, 2, 3, 4, ..., 10\} - \{2, 3, 4, 5, 8\}) \cup \{1, 3, 5, 7, 9\}$

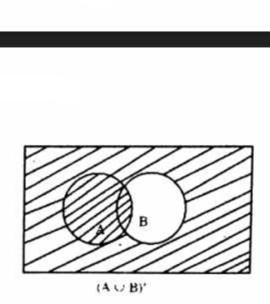
$= \{1, 3, 5, 7, 9, 10\} \cup \{1, 3, 5, 7, 9\}$

$= \{1, 3, 5, 7, 9, 10\}$ ....(ii)

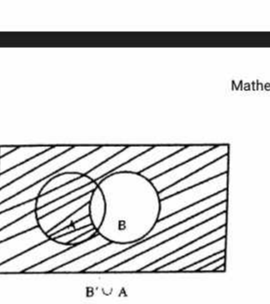
From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.



$(A \cup B)'$



$B' \cup A$

5.1.4 (viii) Ordered pairs and Cartesian product:

5.1.4 (a) Ordered pairs:

Any two numbers x and y, written in the form (x, y] is called an ordered pair. In an ordered pair (x, y), the order of numbers is important, that is, x is the first co-ordinate and y is the second co-ordinate. For example, (3, 2) is different from (2, 3).

It is obvious that $(x, y) \neq (y, x)$ unless $x = y$.

Note that $(x, y) = (s, t)$, if $x = s$ and $y = t$.

5.1.4(b) Cartesian product:

Cartesian product of two non-empty sets A and B denoted by $A \times B$ consists of all ordered pairs (x, y) such that $x \in A$ and $y \in B$.

Example: If $A = \{1, 2, 3\}$ and $B = \{2, 5\}$, then find $A \times B$ and $B \times A$.

Solution: $A \times B = \{(1, 2), (1, 5), (2, 2), (2, 5), (3, 2), (3, 5)\}$

Since set A has 3 elements and set B has 2 elements.

Hence product set $A \times B$ has $3 \times 2 = 6$ ordered pairs.

We note that $B \times A - \{(2, 1), (2, 2), (2, 3), (5, 1), (5, 2), (5, 3)\}$

Evidently $A \times B \neq B \times A$.

Exercise 5.4

1. If A = {a, b} and B = {c, d}. then find A × B and B × A.

Solution:

A = {a, b} and B = {c, d}

A × B = {a, b} × {c, d}

= {(a,c), (a,d), (b,c), (b,d)}

B × A = {c, d} × {a, b}

= {(c, a), (c, b), (d, a), (d, b)}

2. If A = {1, 2, 4}, B = {-1, 3}, then find A × B, B × A, A × A, B × B.

Solution:

A = {1, 2, 4} and B = {-1, 3}

A × B = {1, 2, 4} × {-1, 3}

= {(1, -1), (1, 3), (2, -1), (2, 3), (4, -1), (4, 3)}

B × A = {-1, 3} × {1, 2, 4}

= {(-1, 1), (-1, 2), (-1, 4), (3, 1), (3, 2), (3, 4)}

A × A = {1, 2, 4} × {1, 2, 4}

= {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (2, 4), (4, 1), (4, 2), (4, 4)}

B × B = {-1, 3} × {-1, 3}

= {(-1, -1), (-1, 3), (3, -1), (3, 3)}

3. Find x and y, if

(i) (x - 4, 4 -2) = (2, 1)

Solution:

⇒ (x - 4 = 2 and (y - 2 = 1

x = 2 + 4 y = 1 + 2

x = 6 y = 3

(ii) (2x + 3, 3) = (7, 5 - 4y) Solution:

⇒ 2x + 3 = 7 and 3 = 5 - 4

2x = 7 - 3 x = 4 + 3

2x = 12 y = 7

$x = \frac{12}{2}$

x = 6

(iii) (3 - 2x, 5 - 1) - (x - 7, 2y + 5)

Solution:

⇒ 3 - 2x = x - 7 and 5 - 1 = 2y + 5

- 2x - x = - 7 - 3 5 - 2y = 1 + 5

- 3x = - 10 - 2y = 6

⇒ 3x =10 y = - 4

$x = \frac{10}{3}$

4. Find the sets - X and Y. If X × Y = {(a, a), (b, a), (c, a), (d, a)}

Solution:

X × Y = {(a, a), (b, a), (c, a), (d, a)}

⇒ X = {a, b, c, d} and Y = {a}

5. If A × (a, b, c) and Y= (d, e), then find the number of elements in

(i) X × Y

Solution:

Since set X has 3 elements and set y has 2 elements.

Hence, product X × Y has 3 × 2 = 6 elements.

(ii) Y - X

Solution:

Since set y has 2 elements and set X has 3 elements,

(iii) X ×X

Solution:

Since set X has 3 elements. Hence, product X × X has 9 elements.

Binary relation:

If A and B are two non-empty sets, then a subset R ⊆ A × B is called binary relation from set A into set B, because there exists some relationship between first and second element of each ordered pair in R.

Domain of relation denoted by Dom R is the set consisting of all the first elements of each ordered pair in the relation.

Range of relation denoted by Rang R is the set consisting of all the second elements of each ordered pair in the relation.

Function or Mapping:

Suppose A and B are two non-empty sets, then relation f: A → B is called a function.

If (i) Dom f = A (ii) every x ∈ A appears in one and only one ordered pair in f.

Alternate Definition:

Suppose A and B are two non-empty sets, then relation f: A → B is called a function if (i) Dom f = A (ii) ∀ x ∈ A we can associate some unique image element y = f (x) ∈ B.

Domain, Co-domain and Range of function:

If f: A → B is a function, then A is called the domain of f and B is called the co-domain of f.

Domain f is the set consisting of all first elements of each ordered pair in f and range f is the set consisting of all second elements of each ordered pair in f

Example:

Suppose A = {1, 2, 3} and B = {1, 2, 3, 4, 5}

Define a function f: A → B

f = {(x, y) | y = x + 1 ∀ x ∈ A, y ∈ B} f = {(0, 1), (1, 2), (2, 3), (3, 4)}

Dom f = {0, 1, 2, 3} = A

Rang f = {1, 2, 3, 4} ⊆ = B



The following are the examples of relations but not functions.

g is not a function, because an element a ∈ A has two images in set B and A is not a function because an element d ∈ D. A has no image in set B.



Demonstrate the following:

(a) Into function:

A function f: A → B is called an into function, if at least one element in B is not an image of some element of set A i.e.,

Range of f ⊂ set B.

For example, we define a function f: A → S such that

f = {(0, 1), (1, 1), (2, 3), (3, 3)}

where A = {0, 1, 2, 3} and B = {1, 2, 3, 4} f is an into function.

(b) One -one function:

A function f: A → B is called one-one function, if all distinct elements of A, have distinct images in B, i. e., f (x1) ≠ f (x2) ⇒ x1 ≠ x2 in A or ∀ x1, x2 ∈ A, x1 ≠ x2 ⇒ f (x1) ≠ f (x2)

For example, if A = {0, 1, 2, 3}

and B = {1, 2, 3, 4, 5}, then we define a function f: A → B such that

f = {(x, y) | y = x + 1, ∀ x ∈ A, y ∈ B}.

= {(0,1), (1, 2), (2, 3), (3,4)} f is one-one function.

(c) Into and one-one function: (Injective function)

The function f discussed in (b) is also an into function. Thus f is an into and one-one function.

(d) An onto or surjective function:

A function f: A → B is called an onto function, if every element of set B is an image

of at least one element of set A i.e. Range of f = B.

For example, if A = {0, 1, 2, 3}

and B = {1, 2, 3}, then f: A → B such

that f = {(0, 1), (1, 2), (2, 3), (3, 3)}.

Here Rang f = {1, 2, 3} = B.

Thus, f is defined is an onto function.

(e) Bijective function or one to one correspondence:

A function f: A → B is called bijective function if f function f is one-one and onto.

For example, if A = {1, 2, 3, 4} and B = {2, 3, 4, 5}

We define a function f: A → B such that f = {(x, y) | y = x + 1, ∀ x ∈ A, y ∈ B}

Then f = {(1, 2), (2, 3), (3, 4), (4, 5)}

Evidently this function is one-one because distinct elements of A have distinct images in B. This is an onto function also because every element of B is the image of at least one element of A.

Note: (1) Every function is a relation but converse may not be true.

(2) Every function may not be one one

(3) Every function may not be onto.

Example: Suppose A = {1, 2, 3}

B = {1, 2, 3, 4, 5, 6}

We define a function f: A → B = {(x, y) | y = 2x, ∀ x ∈ A , y ∈ B

Then f = {1, 2), (2, 4), (3, 6)}.

Evidently this function is one-one but not an onto.

Examine whether g given relation is a function:

A relation in which each x in its domain, has a unique image in its range, is a function.

Differentiate between one-to-one correspondence and one one function:

A function f from set A to set B is one-one if distinct elements of A has distinct images in B. The domain of/s A and its range is contained in B.

In one-to-one correspondence between two sets A and B, each element of either set is assigned with exactly one element of the other set. If the sets A and B are finite, then these sets have the same number of elements, that is, n (A) = n (B).

Exercise 5.5

1. If L = {a, b, c}, M = {3, 4}, then Find two binary relations of L x M and M x L.

Solution:

L = {a, b, c}, {3, 4}

L x M = {a, b, c} × {3,4}

= {(a, 3), (a, 4),(b, 3), (b, 4), (c, 3), (c, 4)}

Then R_1 = {(a, 3), (b, 4),(c, 3)}

R_2 = {(a, 4), (b, 3),(c, 4)}

= {(3, a), (3, b), (3, c), (4, a), (4, b), (4, c)}

Here R_1 = {(3, a), (4, a),(4, c)}

R_2 = {(3, b), (4, c)}

2. If Y = {-2, 1, 2}, then make two binary relations for Y x Y. Also find their domain and range.

Solution:

Y = {-2, 1, 2}

YxY = {-2, 1, 2} × {-2, 1, 2}

= {(-2, -2)}, (-2,1), (-2, 2), (1, -2), (1, 1), (1, 2), (2, -2), (2, 1), (2, 2)}

R_1 = {(-2, -2)}, (-2, 1), (1, 2), (2, 2)

Dom R_1 = {-2,1 , 2}

Dom R_1 = {-2,1,2}

Range R_1 = {-2, 1, 2}

and R_2 = {(-2, 1), (1, (),(-2, 2)}

Dom R_2 = {-2, 1}

Range R_2 = {1 , 2}

3. If L = {a, b, c} and M = {d, e, f, g}, then find two binary relations in each:

(i) L x L (ii) L × M (iii) M × M

Solution

(i) L x L

L = {a, b, c}

Now LxL = {a, b, c} × {a, b, c}

= {(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)}

R_1 = {(a, a)}

R_2 = {(a, b), (b, b), (c, b)}

(ii) L x M

L = {a, b, c}

M = {d, e, f, g} =a, b, c}

Now L x M = {a, b, c}, {d, e, f, g} =

= {(a, d), (a, e), (a, f), (a, g), (b, d), (b, e), (b, f), (b, g), (c, d), (c, e), (c, f), (c, g)}

Here R_1 = {(a, d), (b, f)}

R_2 = {(b, e), (b, g), (c, d), (c, e)}

(iii) M x M

M = {d, e, f, g}

Now M × M = {d, e, f, g} × {d, e, f, g}

= {(d, d), (d, e), (d, f), (d, g), (e, d), (e, e), (e, f), (e, g), (f, d), (f, e), (f, f), (f, g), (g, d), (g, e), (g, f), (g, g)}

Here R_1 = {(d, e), (d, f), (f, f)}

R_2 = {(d, f), (e, d), (e, e), (g, g)}

4. If set M has 5 elements, then find the number of binary relations in M.

Solution:

Number of elements in M = 5

Number of elements in M = 5

Number of elements in M × M = $2^{5×5} = 2^{25}$

5. If L = {x : x ∈ N ^ x ≤ 5}, M = {y | y ∈ P ^ y < 10}, then make the following relations from L to M.

(i) R_1 = {(x, y) | y < x} (ii) R_2 = {(x, y) | y = x}

(iii) R_3 = {(x, y)| x + y = 6} (iv) R_4 = {(x, y) | y - x = 2}

Also write the domain and range of each relation.

(i) R_1 = {(x, y) | Y < x} •

Solution:

R_1 = {x | x ∈ N ^ x ≤ 5},

Thus, L = {1, 2, 3, 4, 5}

and M = {y | y ∈ P ^ y < 10}

Thus, M = {2, 3, 5, 7}

Now L x M = {1, 2, 3, 4, 5}, {2, 3, 5, 7}

= {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7),(5, 2), (5, 3), (5, 5), (5, 7)

R_1 = {(x, y) | y < x}

= {(3, 2) (4, 2), (4, 3),(,5, 2), (5, 3)}

DomR_1 = {3, 4, 5}

Range R_1 = {2, 3}

R_2 = {(x, y)| y = x}

(ii) R_2 = {(x, y) | y = x}

R_2 = {(2, 2), (3, 3), (5, 5)}

Dom R_2 = {2, 3, 5}

Range R_2 = {2, 3, 5}

(iii) R_3 = {(x, y)| x + y = 6}

R_3 = {(1, 5), (3, 3), (4, 2)}

Dom R_3 = {1, 3, 4}

Range $R__$ = {5, 3, 2}

(iv) R_4 = {(x, y) | y - x = 2}

R_4 = {(1, 3), (3, 5), (5, 7)}

Dom R_4 = {1 , 3, 5}

Range R_4 = {3, 5, 7}

Also write the domain and range of each relation.

6. Indicate relations, into function, one-one function, onto function and bijective function from the following. Also find their domain and the range.

Solution:

(i) R_1 = {(1, 1), (2, 2), (3, 3), (4, 4)}

Bijective

Dom R_1 = {1, 2, 3, 4}

Range R_1 = {1, 2, 3, 4}

(ii) R_2 = {(1, 2), (2, 1), (3, 4), (3, 5)}

Relation

Dom R_2 = {1, 2, 3}

Range R_2 = {1, 2, 4, 5}

(iii) R_3 = {(b, a), (c, a), (d, a)}

Function

Dom R_3 = {b, c, d}

Range R_3 = {a}

(iv) R_4= {(1, 1), (2, 3), (3, 4), (5, 4)}

On to function

Dom R_4 = {1, 2, 3, 4, 5}

Range R_4 = {1, 3, 4}

(v) R_5 = {(a, b), (b, a), (c, d), (d, e)}

One-one function

Dom R_5 = {a, b, c, d}

Dom R_5 = {a, b, d, e}

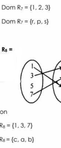
(vi) R_6 = {(1, 2), (2, 3), (1, 3), (3, 4)}

Relation

Dom R_6 = {1 , 2, 3}

Dom R_6 = {2, 3, 4}

(vii) R_7 =



one-one function

Dom R_7 = {1, 2, 3}

Dom R_7 = {r, p, s}

(viii) R_8 =

Relation

Dom R_8 = {1, 3, 7}

Dom R_8 = {c, a, b}

Miscellaneous Exercise 5

Q1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick mark (√) the correct answer.

(i) A collection of well-defined distinct objects is called

(a) subset **(b)** power set
(c) set **(d)** none of these

(ii) A set $G = \left\{\frac{p}{q} : a, b \in Z^+, b \neq 0\right\}$ is called a set of

(a) Whole numbers **(b)** Natural numbers
(c) Irrational numbers **(d)** Rational numbers

(iii) The different number of ways to describe a set are

(a) 1 **(b)** 2
(c) 3 **(d)** 4

(iv) A set with no element is called

(a) Subset **(b)** Empty set
(c) Singleton set **(d)** Super set

(v) The set $\{x : x \notin W \wedge x \leq 101\}$ is

(a) Infinite set **(b)** A subset
(c) Null set **(d)** Finite set

(vi) The set having only one element is called

(a) Null set **(b)** Power set
(c) Singleton set **(d)** Subset

(vii) Power set of an empty set is

(a) ∅ **(b)** {∅}
(c) {∅, {∅}} **(d)** { ∅ }

(viii) The number of elements in power set $\{1, 2, 3\}$ is

(a) 4 **(b)** 6
(c) 8 **(d)** 7

(ix) If $A \subseteq B$, then $A \cup B$ is equal to

(a) A **(b)** B
(c) ∅ **(d)** none of these

(x) If $A \subseteq B$, then $A \cap B$ is equal to

(a) A **(b)** B
(c) ∅ **(d)** none of these

(xi) If $A \subseteq B$, then $A - B$ is equal to

(a) A **(b)** B
(c) ∅ **(d)** $B - A$

(xii) $(A \cup B) \cup C$ is equal to

(a) $A \cap (B \cup C)$ **(b)** $(A \cup B) \cap C$
(c) $A \cup (B \cup C)$ **(d)** $A \cap (B \cap C)$

(xiii) $A \cup (B \cap C)$ is equal to

(a) $(A \cap B) \cup (A \cup C)$ **(b)** $A \cap (B \cap C)$
(c) $(A \cap B) \cup (A \cap C)$ **(d)** $A \cup (B \cup C)$

(xiv) If A and B are disjoint sets, then $A \cup B$ is equal to

(a) A **(b)** B
(c) ∅ **(d)** $A \cap A$

(xv) If number of elements in set A is 3 and in set B is 4, then number of elements in $A \times B$ is

(a) 3 **(b)** 4
(c) 12 **(d)** 7

(xvi) If number of elements in set A is 2 and in set B is 2, then number of binary relations in $A \times B$ is

(a) 2^2 **(b)** 2^4
(c) 2^8 **(d)** 2^6

(xvii) The domain of $f = \{(0, 2), (2, 2), (3, 2), (5, 2)\}$ is

(a) {0, 2, 4} **(b)** {2, 2, 2, 2}
(c) {0, 2, 4} **(d)** {2, 3, 4}

(xviii) The range of $R = \{(1, 3), (2, 2), (3, 1), (4, 4)\}$ is

(a) {1, 2, 4} **(b)** {3, 2, 4}
(c) {1, 2, 3, 4} **(d)** {1, 3, 4}

(xix) Point $(-1, 4)$ lies in the quadrant

(a) I **(b)** II
(c) III **(d)** IV

(xx) The relation $\{(1, 3), (2, 3), (3, 3), (3, 4)\}$ is

(a) onto function **(b)** into function
(c) not a function **(d)** one-one function

Answers.

(i)	c	**(ii)**	d	**(iii)**	c	**(iv)**	b	**(v)**	d
(vi)	c	**(vii)**	d	**(viii)**	c	**(ix)**	b	**(x)**	a
(xi)	c	**(xii)**	c	**(xiii)**	a	**(xiv)**	d	**(xv)**	c
(xvi)	b	**(xvii)**	b	**(xviii)**	c	**(xix)**	b	**(xx)**	c

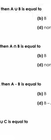
Q2. Write short answers of the following questions.

(i) Define a subset and give one example.
Ans: Set A is said to be subset of a set B, denoted by $A \subseteq B$, if and only if each element of A is an element of B.
For example, if $A = \{2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5\}$ then A is subset of B.

(ii) Write all the subsets of the set $\{a, b\}$.
Ans: Let $A = \{a, b\}$

Subsets of A are: ∅, $\{a\}$, $\{b\}$, $\{a, b\}$

(iii) Show $A \cap B$ by Venn diagram. When $A \subseteq B$.
Ans:



$A \cap B$

(iv) Show by Venn diagram $A \cap (B \cup C)$.
Ans:



$A \cap (B \cup C)$

Double shaded region shows $A \cap (B \cup C)$.

(v) Define intersection of two sets.
Ans: The intersection of two sets A and B written as $A \cap B$ is the set consisting of all the common elements of a and B.

(vi) Define a function.
Ans: Suppose A and B are two non-empty sets, then relation of $A \to B$ is called one-one function, if all distinct elements of A have distinct images in B i.e.

(vii) Define one-one function.
Ans:

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2, x \neq$$
or
$$\forall x_1, x_2 \in A \Rightarrow f(x_1) \neq f(x_2)$$

(viii) Define an onto function.
Ans: A function $f: A \to B$ is called an onto function, if every element of set B is an image of at least one element of set A i.e. Range $f = B$.

(ix)Define a bijective function.
Ans: A function $f: A \to B$ is called bijective function if function f is one-one and onto.
$f = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$.
This function is one-one because distinct element of A has distinct images in B. This
is an onto function also because every element of B is the image of at least one element of A.

(x) Write De Morgan's laws.
Ans:

(i) $(A \cap B)' = A' \cup B'$
(ii) $(A \cup B)' = A' \cap B'$

Q3. Fill in the blanks

(i)	If $A \subseteq B$, then $A \cup B =$ _____.						
(ii)	If $A \cap B = \emptyset$ then A and B are _____.						
(iii)	If $A \subseteq$ Bonn B $\subseteq A$ then _____.						
(iv)	$A \cap (B \cap C) =$ _____.						
(v)	$A \cup (B \cap C) =$ _____.						
(vi)	The complement of U is _____.						
(vii)	The complement of ∅ is _____.						
(viii)	$A \cap A' =$ _____.						
(ix)	$A \cup A' =$ _____.						
(x)	The set $\{x \mid x \in A$ and $x \notin B\} =$ _____.						
(xi)	The point $(-5, -7)$ lies in _____ quadrant.						
(xii)	The point $(4, -6)$ lies in _____ quadrant.						
(xiii)	The y co-ordinate of every point is _____ on x-axis.						
(xiv)	The x co-ordinate of every point is _____ on y-axis.						
(xv)	The domain of $\{(a, b), (b, c)\}$ is _____.						
(xvi)	The range of $\{(a, a), (b, b), (c, c)\}$ is _____.						
(xvii)	Venn diagram was first used by _____.						
(xviii)	A subset of A x A is called the _____ in A.						
(xix)	If $A \to B$ and range of f = B, then f is an _____ function.						
(xx)	The relation $\{(a, b), (a, c), (a, d)\}$ is _____ a function.						

Answer

(i)	B	**(ii)**	Disjoint	**(iii)**	$A = B$		
(iv)	$(A \cap B) \cap C$	**(v)**	$A \cup (B \cup C)$	**(vi)**	U		
(vii)	∅	**(viii)**	∅	**(ix)**	U		
(x)	$A \setminus B$	**(xi)**	III-quad	**(xii)**	IV quad		

(xiii)	zero	**(xiv)**	0	**(xv)**	$\{a, b, c\}$		
(xvi)	$\{a, b, c\}$	**(xvii)**	John Venn	**(xviii)**	Binary relation		
(xix)	Onto	**(xx)**	Not				

Summary

✓ A set is the well-defined collection of distinct objects with some common properties.

✓ Union of two sets A and B denoted by $A \cup B$ is the set containing elements which either belong to A or to B or to both.

✓ Intersection of two sets A and B denoted by $A \cap B$ is the set of common elements of both A and B. In symbols $A \cap B = \{x \mid x \in A$ and $x \in B\}$.

✓ The set difference of B and A denoted by $A - B$ is the set of all those elements of B which do not belong to A.

✓ Complement of a set A w.r.t. universal set U denoted by $A' = A^c = U - A$ contains all those elements of U which do not belong to A.

✓ British mathematician John Venn (1834 - 1923) introduced rectangle for a universal set U and to subsets A and B as closed figures inside this rectangle.

✓ An ordered pair of elements is written according to a specific order for which the order of elements is strictly maintained.

✓ Cartesian product of two non-empty sets A and B denoted by $A \times B$ consists of all ordered pairs (a, b) such that $a \in A, a \in B$.

✓ If A and B are any two non-empty sets, then a non-empty subset R ⊆ $A \times B$ is called binary relation from set A into set B.

✓ If A and B are two non-empty sets, then relation $f: A \to B$ is called a function if it

Don $t \in$ set A, (i) every $a \in A$ appears in one and only one ordered pair $\in f$.

✓ Dom f is the set consisting of all first elements of each ordered pair $\in f$ and Rang of f,

is the set consisting of all second elements of each ordered pair $\in f$.

✓ A function $f: A \to B$ is called an into function if at least one element in B is not an image of some element of set A i.e., range of $f \subset B$.

✓ A function $f: A \to B$ is called an onto function if every element of set B is an image of at least one element of set A i.e., range of $f = B$.

✓ A function $f: A \to B$ is called one-one function if all distinct elements of A have distinct images in B.

✓ A function $f: A \to B$ is called bijective function if f function f is one-one and onto.