

Exercise 12.1

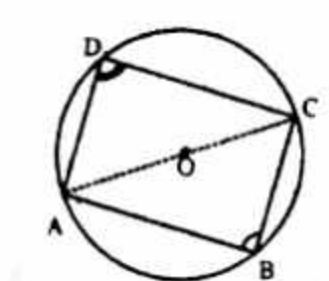
1. Prove that in a given cyclic quadrilateral, sum of opposite angles is two right angles and conversely.

Solution:

Given:

A quadrilateral ABCD.

In scribed in a circle whose centre is O.



To Prove:

$$\angle A + \angle C = 2\text{ rts.}$$

$$\text{And } \angle B + \angle D = 2\text{ rts.}$$

Construction:

Join A with O and O with C

Proof:

Arc ABC subtends $\angle AOC$ at the centre O and,

$\angle ADC$ at a point D on the remaining part of the circumference.

$$m\angle ADC = \frac{1}{2} \angle AOC \quad \dots\dots(i)$$

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Similarly, arc ADC subtends reflex $\angle AOC$ at the centre and $\angle ABC$ on the circumferences.

$$m\angle ABC = \frac{1}{2} \angle AOC \quad \dots\dots(ii)$$

By Adding (i) & (ii)

$$m\angle ADC + m\angle ABC = \frac{1}{2} [m\angle AOC + m\angle AOC]$$

$$m\angle D + m\angle B = \frac{1}{2} [4\text{ rts}]$$

$$m\angle D + m\angle B = 2\text{ rts.} \quad \text{Proved.}$$

Similarly, by Joining B with O and O with D it can be proved that

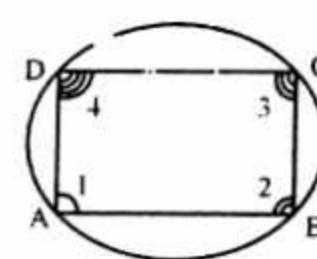
$$\angle A + \angle C = 2\text{ rts.}$$

2. Show that paralleloaram inscribed in a circle will be a rectangle.

Solution:

Given:

ABCD is a parallelogram inscribed in a circle with centre "O".



To Prove:

ABCD is a rectangle.

Proof

$$m\angle 1 + m\angle 3 = 180^\circ \quad \text{————(i)}$$

[cyclic quadrilateral]

$$\text{But } m\angle 1 + m\angle 3 = 180^\circ \quad \text{————(ii)}$$

[opp. $\angle$ s of a parallelogram]

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From (i) & (ii) we get

$$2m\angle 1 = 180^\circ$$

$$\Rightarrow m\angle 1 = 90^\circ$$

$$\therefore m\angle 1 = m\angle 3 = 90^\circ.$$

By this

$$m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = 90^\circ.$$

Hence, the parallelogram ABCD is a rectangle.

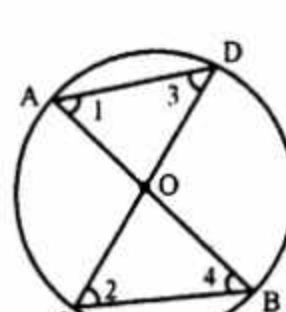
3. AOB and COD are two intersecting chords of a circle. Show that $\triangle AOD$ and $\triangle BOC$ are equiangular.

Solution:

Given:

Two chords AOB & COD

Intersecting each other at O.



To Prove:

$\triangle AOD$ and $\triangle BOC$ are equiangular.

Proof:

$\triangle AOB$ and $\triangle COD$

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$$\angle 1 = \angle 2.$$

$$\angle 3 = \angle 4$$

[angles in the same segment of a circle].

$$\therefore \angle AOB = \angle COB.$$

[Vertical angles].

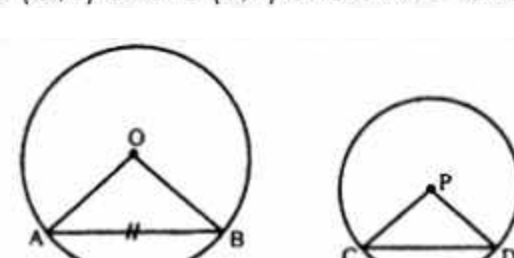
$\triangle AOD$ and $\triangle COD$ are equiangular triangles.

4. $\overline{AD}$, and $\overline{BC}$ are two parallel chords of a circle prove that arc $\overline{AB} \cong$ arc $\overline{CD}$ and arc $\overline{AC} \cong$ arc $\overline{BD}$.

Solution:

Given:

Two circles, C (O, r) and C (P, r) chord AB = chord CD.



To Prove:

$$\overline{AB} = \overline{CD}$$

Construction:

Join OA, OB and PC, PD.

Proof:

In $\triangle AOB$ & $\triangle CPD$

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$$OA \cong PC$$

$$\text{and } OB \cong PD \quad [\text{radii of equal circles}]$$

$$AB = CD \quad [\text{given}]$$

$$\therefore \triangle AOB \cong \triangle CPD \quad \text{S.S.S.}$$

$$\therefore \angle AOB \cong \angle CPD$$

There are the angles subtended by the AB and CD at the centre of equal circle

$$\overline{AB} = \overline{CD}$$

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THEOREM 1

12.1 (i) The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

Given:

AC is an arc of a circle with centre O.

Whereas $\angle AOC$ is the central angle

and $\angle ABC$ is circum angle.

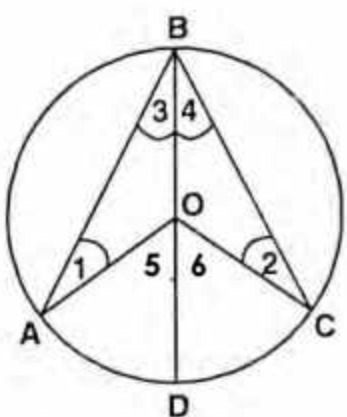
To prove:

$m\angle AOC = 2m\angle ABC$

Construction:

Join B with O and produce it to meet the circle at D.

Write angles $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ and $\angle 6$ as shown in the figure.



Proof

Statements	Reasons
As $m\angle 1 = m\angle 3$ (i)	Angles opposite to equal sides in $\triangle OAB$
And $m\angle 2 = m\angle 4$ (ii)	Angles opposite to equal sides in $\triangle OBC$
Now $m\angle 5 = m\angle 1 + m\angle 3$ (iii)	External angle is the sum of internal opposite angles.
Similarly $m\angle 6 = m\angle 2 + m\angle 4$ (iv)	
Again $m\angle 5 = m\angle 3 + m\angle 3 = 2m\angle 3$ (v)	Using (i) and (iii)
and $m\angle 6 = m\angle 4 + m\angle 4 = 2m\angle 4$ (vi)	Using (ii) and (iv)
Then from figure $\Rightarrow m\angle 5 + m\angle 6 = 2m\angle 3 + 2m\angle 4$	Adding (v) and (vi)

$\Rightarrow m\angle AOC = 2(m\angle 3 + m\angle 4) = 2m\angle ABC$	
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Example:

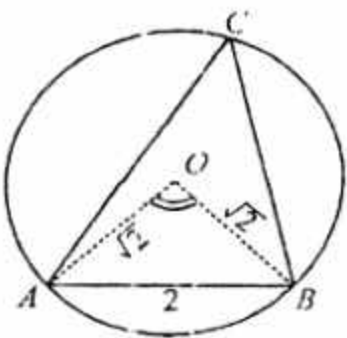
The radius of a circle is $\sqrt{2}$ cm. A chord 2 cm in length divides the circle into two segments. Prove that the angle of large segment is 45° .

Given:

In a circle with centre O and radius $mOA = mOB = \sqrt{2}$ cm,

The length of chord AB = 2 cm divides the circle into two

segments with ACB as larger one.



To prove:

$m\angle ACB = 45^\circ$

Construction:

Join O with A and O with B.

Proof:

Statements	Reasons
In $\triangle OAB$ $(\overline{OA})^2 + (\overline{OB})^2 = (\sqrt{2})^2 + (\sqrt{2})^2$ $= 2 + 2 = 4$ $= (2)^2 = (\overline{AB})^2$ $\triangle OAB$ is right angled triangle With $m\angle AOB = 90^\circ$	$m\overline{OA} = m\overline{OB} = \sqrt{2}$ cm Given $mAB = 2$ cm Which being a central angle standing on an arc AB

Then $m\angle ACB = \frac{1}{2} m\angle AOB$ $= \frac{1}{2} (90^\circ) = 45^\circ$	By theorem 1 Circum angle is half of the central angle.
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THEOREM 2

12.1 (ii) Any two angles in the same segment of a circle are equal.

Given:

$\angle ACB$ and $\angle ADB$ are the circum angles in the same segment of a circle with centre O .

To prove:

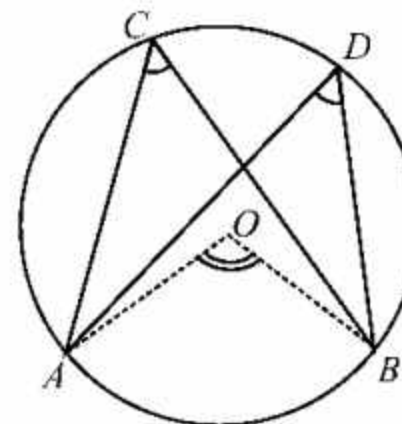
$$m\angle ACB = m\angle ADB$$

Construction:

Join O with A and O with B .

So that $\angle AOB$ is the central angle.

Proof:

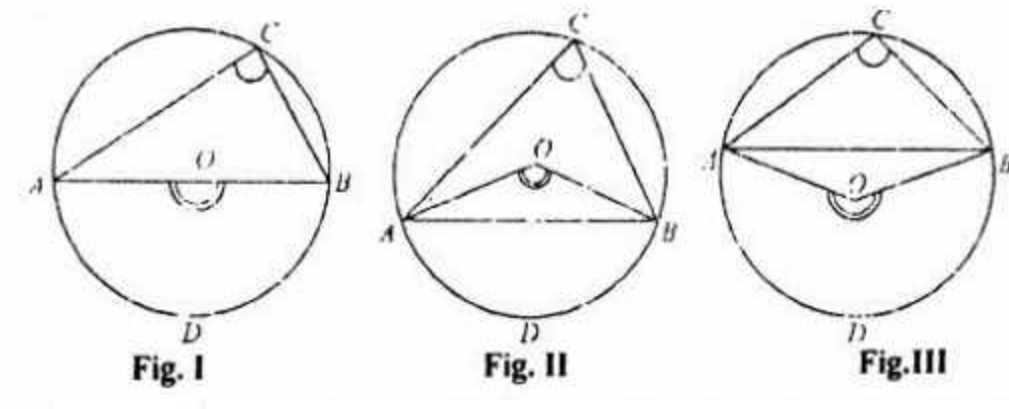


Statements	Reasons
Standing on the same arc AB of a circle. $\angle AOB$ is the central angle whereas $\angle ACB$ and $\angle ADB$ are circum angles	Construction Given
$m\angle AOB = 2m\angle ACB$ (i)	By theorem 1
and $m\angle AOB = 2m\angle ADB$ (ii)	By theorem 1
$\Rightarrow 2\angle ACB = 2\angle ADB$	Using (i) and (ii)
Hence, $m\angle ACB = m\angle ADB$	

THEOREM 3

12.1 (iii) The angle

- in a semi-circle is a right angle,
- in a segment greater than a semi-circle is less than a right angle,
- in a segment less than a semi-circle is greater than a right angle;



Given:

AB is the chord corresponding to an arc ADB

Whereas $\angle AOB$ is a central angle and $\angle ACB$ is

a circum angle of a circle with centre O.

To prove:

In fig. (I) If sector ACB is a semi-circle then $m\angle ACB = 1\text{rt}$ In fig (II) If sector ACB is greater than a semi-circle then $m\angle ACB < 1\text{rt}$ In fig (III) If sector ACB is less than a semi-circle then $m\angle ACB > 1\text{rt}$

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Proof:

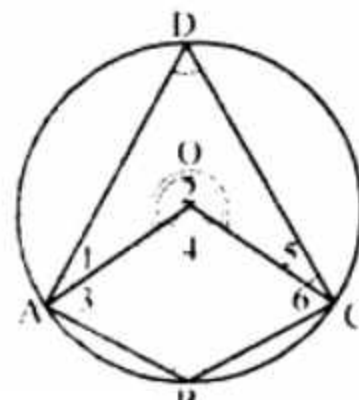
Statements	Reasons
In each figure, $\overline{AB}$ is the chord of a circle with centre O.	Given
$\angle AOB$ is the central angle standing on an arc ADB.	
Whereas $\angle ACB$ is the circum angle	Given
Such that $m\angle AOB = 2m\angle ACB$ (i)	By theorem 1
Now in fig (I) $m\angle AOB = 180^\circ$	A straight angle
$\therefore m\angle AOB = 2\text{rt}$ (ii)	
$\Rightarrow m\angle ACB = 1\text{rt}$	Using (i) and (ii)
In fig (II) $m\angle AOB < 180^\circ$	
$\therefore m\angle AOB < 2\text{rt}$ (iii)	
$\Rightarrow m\angle ACB < 1\text{rt}$	Using (i) and (iii)
In fig (III) $m\angle AOB > 180^\circ$	
$\therefore m\angle AOB > 2\text{rt}$ (iv)	
$\Rightarrow m\angle ACB > 1\text{rt}$	Using (i) and (iv)

Corollary 1. The angles subtended by an arc at the circumference of a circle are equal.**Corollary 2.** The angles in the same segment of a circle are congruent.

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Theorem 4

12.1 (iv) The opposite angles of any quadrilateral inscribed in a circle are supplementary.



Given.

ABCD is a quadrilateral inscribed in a circle with centre O.

To prove:

$$\begin{cases} m\angle A + m\angle C = 2\text{rts} \\ m\angle B + m\angle D = 2\text{rts} \end{cases}$$

Construction:

Draw $\overline{OA}$ and $\overline{OC}$.Write $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ and $\angle 6$ as shown in the figure.

Proof:

Statements	Reasons
Standing on the same arc ADC, $\angle 2$ is a central angle	Arc ADC of the circle with centre O.

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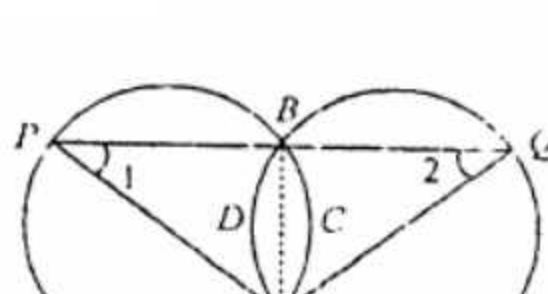
Whereas $\angle B$ is the circum angle	
$\therefore m\angle B = \frac{1}{2} (m\angle 2)$ (i)	By theorem 1
Standing on the same arc ABC, $\angle 4$ is a central angle whereas $\angle D$ is the circum angle	Arc ABC of the circle with centre O.
$m\angle D = \frac{1}{2} (m\angle 4)$ (ii)	By theorem 1
$m\angle B + m\angle D = \frac{1}{2} m\angle 2 + \frac{1}{2} m\angle 4$	Adding (i) and (ii)
$= \frac{1}{2} (m\angle 2 + m\angle 4)$	
$= \frac{1}{2} (\text{Total central angle})$	
i.e., $m\angle B + m\angle D = \frac{1}{2} (4\text{ rts}) = 2\text{rts}$	
Similarly, $m\angle A + m\angle C = 2\text{rts}$	

Corollary 1. In equal circles or in the same circle if two minor arcs are equal then angles inscribed by their corresponding major arcs also equal.**Corollary 2.** In equal circles or in the same circle, two equal arcs subtend equal angles at the circumference and vice versa.

Example 1:

Two equal circles intersect in A and B. Through B, a straight line is drawn to meet the circumferences at P and Q respectively. Prove that $m\angle P = m\angle Q$.

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Given:

Two equal circles cut each other at points A and B. A straight line PBQ drawn through B meets the circles at P and Q respectively.

To prove:

$$m\angle P = m\angle Q$$

Construction:

Join the points A and B. Also $\overline{AP}$ and $\overline{AQ}$ Write $\angle 1$ and $\angle 2$ as shown in the figure.

Proof:

Statements	Reasons
$\therefore m\angle ACB = m\angle ADB$	Arcs about the common chord AB.
$\therefore m\angle 1 = m\angle 2$	Corresponding angles made by opposite arcs
So, $m\angle APQ = m\angle AQP$	Sides opposite to equal angles in $\triangle APQ$.
Or $m\angle P = m\angle Q$	

Example 2:

ABCD is a quadrilateral circumscribed about a circle.

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Show that $m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$

Given:

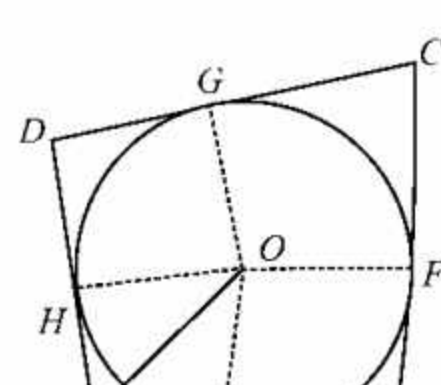
ABCD is a quadrilateral circumscribed about a circle with centre O.

So that each side becomes tangent to the circle.

To prove:

$$m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$$

Construction:

Drawn $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{BC}$, $\overline{OG} \perp \overline{CD}$ and $\overline{OH} \perp \overline{DA}$ 

Proof:

Statements	Reasons
$\therefore m\overline{AE} = m\overline{HA}$; $m\overline{EB} = m\overline{BF}$... (i)	Since tangents drawn from a point to the circle are equal in length
$m\overline{CG} = m\overline{FC}$ and $m\overline{GD} = m\overline{DH}$... (ii)	
$(m\overline{AE} + m\overline{EB}) + (m\overline{CG} + m\overline{GD})$	Adding (i) & (ii)
$= (m\overline{BF} + m\overline{FC}) + (m\overline{DH} + m\overline{HA})$	
or $m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$	

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MISCELLANEOUS EXERCISE 12

Q1. Multiple Choice Questions:

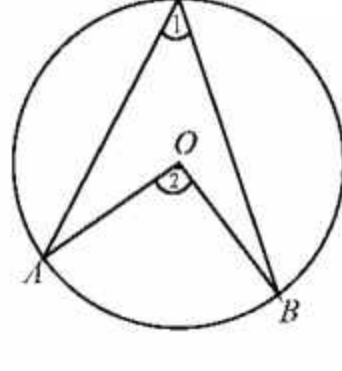
Four possible answers are given for the following question.

Tick (✓) the correct answer.

(i) A circle passes through the vertices of a right angled $\triangle ABC$ with $m\angle C = 3^\circ$ and $m\angle B = 4^\circ$, $m\angle C = 90^\circ$. Radius of the circle is:

- (a) 1.5 cm (b) 2.0 cm
(c) 2.5 cm (d) 3.5 cm

(ii) In the adjacent circular figure, central and inscribed angles stand on the same arc $\widehat{AB}$. Then



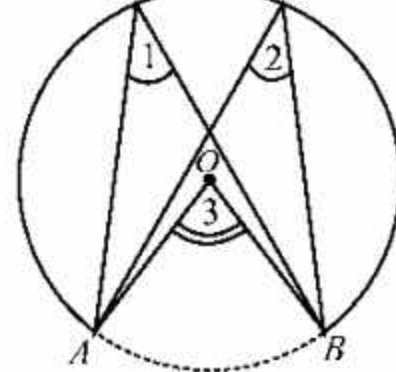
- (a) $m\angle 1 = m\angle 2$ (b) $m\angle 1 = 2m\angle 2$
(c) $m\angle 2 = 3m\angle 1$ (d) $m\angle 2 = 2m\angle 1$

(iii) In the adjacent figure if $m\angle 3 = 75^\circ$, then find $m\angle 1$ and $m\angle 2$.

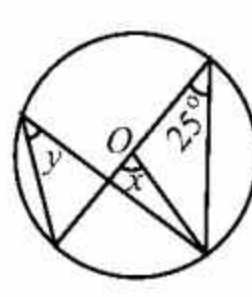
- (a) $37\frac{1}{2}, 37\frac{1}{2}$ (b) $37\frac{1}{2}, 75^\circ$

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- (c) $75^\circ, 37\frac{1}{2}$ (d) $75^\circ, 75^\circ$

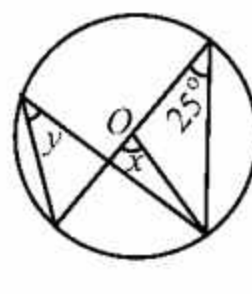


(iv) Given that O is the centre of the circle. The angle marked x will be:



- (a) $12\frac{1}{2}$ (b) 25°
(c) 50° (d) 75°

(v) Given that O is the centre of the circle. The angle marked y will be:

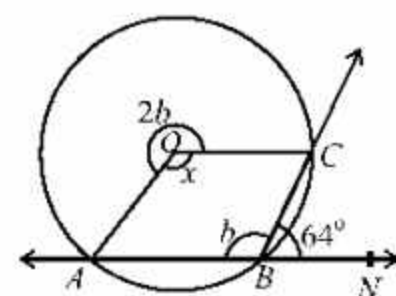


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- (a) $12\frac{1}{2}$ (b) 25°
(c) 50° (d) 75°

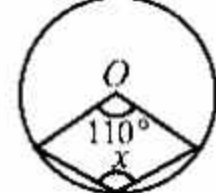
(vi) In the figure, O is the centre of the circle and $\widehat{ABN}$ is a straight line.

The obtuse angle $\angle AOC = x$ is:



- (a) 32° (b) 64°
(c) 96° (d) 128°

(vii) In the Figure, O is the centre of the circle, then the angle x is:



- (a) 55° (b) 110°
(c) 220° (d) 125°

(viii) In the figure, O is the centre of the circle then angle x is:

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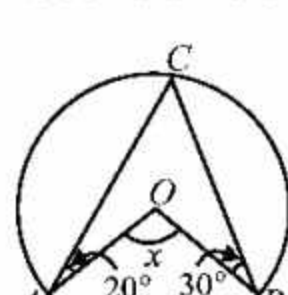
- (a) 15° (b) 30°
(c) 45° (d) 60°

(ix) In the figure, O is the centre of the circle then the angle x is:



- (a) 15° (b) 30°
(c) 45° (d) 60°

(x) In the figure, O is the centre of the circle then the angle x is:



- (a) 50° (b) 75°
(c) 100° (d) 125°

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Answers:

(i)	c	(ii)	d	(iii)	a	(iv)	c	(v)	b
(vi)	d	(vii)	d	(viii)	b	(ix)	d	(x)	c

Summary

- ✓ The angle subtended by an arc at the centre of a circle is called central angle.
- ✓ A central angle is subtended by two radii with the vertex at the centre of the circle.
- ✓ The angle subtended by an arc of a circle at its circumference is called a circumangle.
- ✓ A circumangle is subtended between any two chords of a circle, having common point on its circumference.
- ✓ A quadrilateral is called cyclic when a circle can be drawn through its four vertices.
- ✓ The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.
- ✓ Any two angles in the same segment of a circle are equal.
- ✓ The angle
 - in a semi-circle is a right angle,
 - in a segment greater than a semi-circle is less than a right angle.
 - in a segment less than a semi-circle is greater than a right angle.

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- ✓ The opposite angles of any quadrilateral inscribed in a circle are supplementary.

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