Exercise 1.1

1. Write the following quadratic equations in the standard form and point out pure quadratic equations.

i. **(x+7) (x-3) = -7**

Solution:

$(x+7)(x-3)=-7$
$x(x-3)+7(x-3)=-7$
$x^2-3x+7x-21=-7$
$x^2+4x-21+7=0$
$x^2+4x-14=0$

The above equation is a quadratic equation.

ii. $\frac{x^2+4}{3}-\frac{x}{7}=1$

Solution:

$\frac{x^2+4}{3}-\frac{x}{7}=1$

Multiply both sides by 21, we get

$21\times\frac{x^2+4}{3}-21\times\frac{x}{7}=1\times 21$
$7(x^2+4)-3x=21$
$7x^2+28-3x=21$
$7x^2-3x+28-21=0$
$7x^2-3x+7=0$

The above equation is a quadratic equation.

iii. $\frac{x}{x+1}+\frac{x+1}{x}=6$

Solution:

$\frac{x}{x+1}+\frac{x+1}{x}=6$
$\frac{x^2+(x+1)^2}{x(x+1)}=6$
$x^2+x^2+2x+1=6x(x+1)$
$2x^2+2x+1=6x^2+6x$
$2x^2-6x^2+2x-6x+1=0$
$-4x^2-4x+1=0$
$-(4x^2+4x-1)=0$
$\Rightarrow 4x^2+4x-1=0$

The above equation is a quadratic equation.

iv $\frac{x+4}{x-2}-\frac{x-2}{x}+4=0$

Solution:

$\frac{x+4}{x-2}-\frac{x-2}{x}+4=0$
$\frac{x(x+4)-(x-2)^2+4x(x-2)}{x(x-2)}=0$
$(x^2+4x)-(x^2-4x+4)+4(x^2-2x)=0$
$x^2+4x-x^2+4x-4+4x^2-8x=0$
$x^2-x^2+4x^2+4x+4x-8x-4=0$
$4x^2+8x-8x-4=0$
$4x^2-4=0$
$4(x^2-4)=0$
$x^2-1=0$

The above equation is a pure quadratic equation.

v. $\frac{x+3}{x+4}-\frac{x-5}{x}=1$

Solution:

$\frac{x+3}{x+4}-\frac{x-5}{x}=1$

$\frac{x(x+3)-(x+4)(x-5)}{x(x+4)}=1$

$(x^2+3x)-x(x-5)-4(x-5)=x(x+4)$
$x^2+3x-x^2+5x-4x+20=x^2+4x$
$x^2-x^2+3x+5x-4x+20=x^2+4x$
$4x+20=x^2+4x$
$-x^2+4x-4x+20=0$
$-x^2-20=0$
$-(x^2-20)=0$
$x^2-20=0$

The above equation is a pure quadratic equation.

vi. $\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$

Solution:

$\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$
$\frac{(x+1)(x+3)+(x+2)^2}{(x+2)(x+3)}=\frac{25}{12}$
$\frac{x(x+3)+1(x+3)+(x^2+4x+4)}{x(x+3)+2(x+3)}=\frac{25}{12}$
$\frac{x^2+3x+x+3+x^2+4x+4}{x^2+3x+2x+6}=\frac{25}{12}$

$\frac{2x^2+8x+7}{x^2+5x+6}=\frac{25}{12}$
$25(x^2+5x+6)=12(2x^2+8x+7)$
$25x^2+125x+150=24x^2+96x+84$
$x^2+29x+66=0$

The above equation is a quadratic equation.

Q2. Solve by factorization:

i. $x^2-x-20=0$

Solution:

$x^2-x-20=0$
$x^2-5x+4x-20=0$
$x(x-5)+4(x-5)=0$
$(x+4)(x-5)=0$

Either $x+4=0$ *or* $x-5=0$
$x=-4$ $\quad x=5$

Thus, solution set $=\{-4,5\}$

ii. $3y^2=y(y-5)$

Solution:

$3y^2=y(y-5)$
$3y^2=y^2-5y$
$3y^2-y^2+5y=0$
$2y^2+5y=0$
$y(2y+5)=0$
Either $y=0$ or $2y+5=0$

$2y=-5$
$y=\frac{-5}{2}$

Thus, solution set $=\left\{0,-\frac{5}{2}\right\}$

iii. $4-32x=17x^2$

Solution:

$4-32x=17x^2$
or $17x^2+32x-4=0$
$17x^2+34x-2x-4=0$
$17x(x+2)-2(x+2)=0$
$(17x-2)(x+2)=0$

Either $17x-2=0$ *or* $x+2=0$
$17x=2$ $\quad x=-2$
$x=\frac{2}{17}$

Thus, solution set $=\left\{\frac{2}{17},-2\right\}$

iv. $x^2-11x=152$

Solution:

$x^2-11x=152$
$x^2-11x-152=0$
$x^2-19x+8x-152=0$
$x(x-19)+8(x-19)=0$
$(x+8)(x-19)=0$

Either $(x+8)=0$ *or* $(x-19)=0$
$x=-8$ $\quad x=19$

Thus, solution set = {-8,19}

v. $\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$

Solution:

$\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$
$\frac{(x+1)^2+x^2}{x(x+1)}=\frac{25}{12}$
$\frac{x^2+2x+1+x^2}{x^2+x}=\frac{25}{12}$
$\frac{2x^2+2x+1}{x^2+x}=\frac{25}{12}$
$25(x^2+x)=12(2x^2+2x+1)$
$25x^2+25x=24x^2+24x+12$
$25x^2-24x^2+25x-24x-12=0$
$x^2+x-12=0$
$x^2+4x-3x-12=0$
$x(x+4)-3(x+4)=0$
$(x-3)(x+4)=0$

Either $x-3=0$ *or* $x+4=0$
$x=3$ $\quad x=-4$

Thus, solution set = {3, – 4}

vi. $\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$

Solution:

$\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$
$\frac{2}{x-9}=\frac{(x-4)-(x-3)}{(x-3)(x-4)}$
$\frac{2}{x-9}=\frac{x-4-x+3}{x^2-7x+12}$
$\frac{2}{x-9}=\frac{-1}{x^2-7x+12}$
$2(x^2-7x+12)=-1(x-9)$
$2x^2-14x+24=-x+9$
$2x^2-14x+x+24-9=0$
$2x^2-13x+15=0$
$2x^2-10x-3x+15=0$
$2x(x-5)-3(x-5)=0$

$(2x-3)(x-5)=0$

Either $2x-3=0$ *or* $x-5=0$
$2x=3$ $\quad x=5$
$x=\frac{3}{2}$ $\quad x=5$

Thus solution set $=\{5,\frac{3}{2}\}$

Q3. Solve the following equations by completing square:

i. $7x^2+2x-1=0$

Solution:

$7x^2+2x-1=0$
$7x^2+2x=1$
$\frac{7x^2}{7}+\frac{2x}{7}=\frac{1}{7}$
$x^2+\frac{2x}{7}=\frac{1}{7}$
$(x)^2+2(x)\left(\frac{1}{7}\right)+\left(\frac{1}{7}\right)^2=\frac{1}{7}+\left(\frac{1}{7}\right)^2$
$\left(x+\frac{1}{7}\right)^2=\frac{1}{7}+\frac{1}{49}$
$\left(x+\frac{1}{7}\right)^2=\frac{8}{49}$

Takin square root on both sides, we get
$x+\frac{1}{7}=\pm\sqrt{\frac{8}{49}}$
$x+\frac{1}{7}=\pm\frac{2\sqrt{2}}{7}$
$x=-\frac{1}{7}\pm\frac{2\sqrt{2}}{7}$
$x=\frac{-1\pm 2\sqrt{2}}{7}$

Thus, solution set $=\left\{\frac{-1\pm 2\sqrt{2}}{7}\right\}$

ii. $ax^2+4x-a=0$, $a\neq 0$

Solution:

$ax^2+4x=a$

$\frac{ax^2}{a}+\frac{4x}{a}=\frac{a}{a}$
$x^2+\frac{4x}{a}=1$
$(x)^2+2(x)\left(\frac{2}{a}\right)+\left(\frac{2}{a}\right)^2=1+\left(\frac{2}{a}\right)^2$
$\left(x+\frac{2}{a}\right)^2=1+\frac{4}{a^2}$
$\left(x+\frac{2}{a}\right)^2=\frac{a^2+4}{a^2}$

Taking square root on both sides, we get
$x+\frac{2}{a}=\pm\sqrt{\frac{a^2+4}{a^2}}$
$x=-\frac{2}{a}\pm\sqrt{\frac{a^2+4}{a^2}}$
$x=\frac{-2\pm\sqrt{a^2+4}}{a}$

Thus, solution set $=\left\{\frac{-2\pm\sqrt{a^2+4}}{a}\right\}$

iii. $11x^2-34x+3=0$

Solution:

$11x^2-34x+3=0$
$11x^2-34x=-3$
$\frac{11x^2}{11}-\frac{34x}{11}=-\frac{3}{11}$
$x^2-\frac{34}{11}x=-\frac{3}{11}$
$(x)^2-2(x)\left(\frac{34}{22}\right)+\left(\frac{34}{22}\right)^2=-\frac{3}{11}+\left(\frac{34}{22}\right)^2$
$\left(x-\frac{34}{22}\right)^2=\frac{3}{11}+\frac{1156}{484}$

$\left(x-\frac{34}{22}\right)^2=\frac{-132+1156}{484}$
$\left(x-\frac{34}{22}\right)^2=\frac{1024}{484}$

Taking square root on both sides we get
$\left(x-\frac{34}{22}\right)=\pm\sqrt{\frac{1024}{484}}$
$x-\frac{34}{22}=\pm\frac{32}{22}$

$x=\frac{34}{22}\pm\frac{32}{22}$
$x=\frac{34\pm 32}{22}$
$x=\frac{34+32}{22}$, $\quad x=\frac{34-32}{22}$
$=\frac{66}{22}$ $\quad =\frac{2}{22}$
$=3$ $\quad =\frac{1}{11}$

Thus solution set $=\left\{3,\frac{1}{11}\right\}$

iv. $lx^2+mx+n=0$, $l\neq 0$

Solution:

$lx^2+mx+n=0$
$lx^2+mx=-n$
$\frac{lx^2}{l}+\frac{mx}{l}=-\frac{n}{l}$
$x^2+\frac{mx}{l}=-\frac{n}{l}$
$(x)^2+2(x)\left(\frac{m}{2l}\right)+\left(\frac{m}{2l}\right)^2=-\frac{n}{l}+\left(\frac{m}{2l}\right)^2$

$\left(x+\frac{m}{2l}\right)^2=-\frac{n}{l}+\frac{m^2}{4l^2}$
$\left(x+\frac{m}{2l}\right)^2=\frac{-4ln+m^2}{4l^2}$
$\left(x+\frac{m}{2l}\right)^2=\frac{m^2-4ln}{4l^2}$

Taking square root on both sides, we get
$\sqrt{\left(x+\frac{m}{2l}\right)^2}=\pm\sqrt{\frac{m^2-4ln}{4l^2}}$
$x+\frac{m}{2l}=\pm\sqrt{\frac{m^2-4ln}{4l^2}}$
$x=-\frac{m}{2l}\pm\sqrt{\frac{m^2-4ln}{4l^2}}$
$x=\frac{-m\pm\sqrt{m^2-4ln}}{2l}$

Thus solution set $=\left\{\frac{-m\pm\sqrt{m^2-4ln}}{2l}\right\}$

v. $3x^2+7x=0$

Solution:

$3x^2+7x=0$
$\frac{3x^2}{3}+\frac{7x}{3}=\frac{0}{3}$
$x^2+\frac{7}{3}x=0$
$(x)^2+2(x)\left(\frac{7}{6}\right)+\left(\frac{7}{6}\right)^2=0+\left(\frac{7}{6}\right)^2$
$\left(x+\frac{7}{6}\right)^2=\left(\frac{7}{6}\right)^2$

Taking square root on both sides, we get
$\sqrt{\left(x+\frac{7}{6}\right)^2}=\pm\sqrt{\left(\frac{7}{6}\right)^2}$
$x+\frac{7}{6}=\pm\frac{7}{6}$

$x=-\frac{7}{6}\pm\frac{7}{6}$
$x=-\frac{7}{6}+\frac{7}{6}$ *or* $x=-\frac{7}{6}-\frac{7}{6}$
$x=0$ $\quad x=-\frac{14}{6}$
$x=-\frac{7}{3}$

Thus solution set $=\left\{0,-\frac{7}{3}\right\}$

vi. $x^2-2x-195=0$

Solution:

$x^2-2x-195=0$
$x^2-2x=195$
$(x)^2-2(x)(1)+(1)^2=195+(1)^2$
$(x-1)^2=195+1$
$(x-1)^2=196$
Taking square root on both sides
$\sqrt{(x-1)^2}=\pm\sqrt{196}$
$x-1=\pm 14$
$x=1\pm 14$
$x=1+14$ *or* $x=1-14$
$=15$ $\quad =-13$

Thus, solution set = {–13,15}

vii. $-x^2+\frac{15}{2}=\frac{7}{2}x$

Solution:

$-x^2-\frac{7}{2}x=-\frac{15}{2}$
$-\left(x^2+\frac{7}{2}x\right)=-\frac{15}{2}$
$x^2+\frac{7}{2}x=\frac{15}{2}$
$(x)^2+2(x)\left(\frac{7}{4}\right)+\left(\frac{7}{4}\right)^2=\frac{15}{2}+\left(\frac{7}{4}\right)^2$
$\left(x+\frac{7}{4}\right)^2=\frac{15}{2}+\frac{49}{16}$
$\left(x+\frac{7}{4}\right)^2=\frac{120+49}{16}$
$\left(x+\frac{7}{4}\right)^2=\frac{169}{16}$

Taking square root on both sides, we get
$\sqrt{\left(x+\frac{7}{4}\right)^2}=\pm\sqrt{\frac{169}{16}}$
$x+\frac{7}{4}=\pm\frac{13}{4}$
$x=-\frac{7}{4}\pm\frac{13}{4}$
$x=-\frac{7}{4}+\frac{13}{4}$ *or* $x=-\frac{7}{4}-\frac{13}{4}$

$x=\frac{3}{2}$ $\quad x=-5$

Thus, solution set $=\left\{\frac{3}{2},-5\right\}$

viii. $x^2+17x+\frac{33}{4}=0$

Solution:

$x^2+17x=-\frac{33}{4}$
$(x)^2+2(x)\left(\frac{17}{2}\right)+\left(\frac{17}{2}\right)^2=-\frac{33}{4}+\left(\frac{17}{2}\right)^2$
$\left(x+\frac{17}{2}\right)^2=\frac{33}{4}+\frac{289}{4}$
$\left(x+\frac{17}{2}\right)^2=\frac{256}{4}$

Taking square root on both sides,
$\sqrt{\left(x+\frac{17}{2}\right)^2}=\pm\sqrt{\frac{256}{4}}$
$x+\frac{17}{2}=\pm\frac{16}{2}$
$x=-\frac{17}{2}\pm\frac{16}{2}$
$x=-\frac{17}{2}+\frac{6}{2}$ *or* $x=-\frac{17}{2}-\frac{16}{2}$
$x=-\frac{1}{2}$ $\quad x=-\frac{33}{2}$
Thus solution set $=\left\{-\frac{1}{2},-\frac{33}{2}\right\}$

ix. $4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$

Solution:

$4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$
$\frac{4(3x+1)-8}{3x+1}=\frac{3x^2+5}{3x+1}$
$\frac{12x+4-8}{3x+1}=\frac{3x^2+5}{3x+1}$
$\frac{12x-4}{3x+1}=\frac{3x^2+5}{3x+1}$

Multiplying both sides by $(3x+1)$, we get

$12x-4=3x^2+5$
$3x^2+5-12x+4=0$
$3x^2-12x+9=0$
$3(x^2-4x+3)=0$
$\Rightarrow x^2-4x+3=0$
$x^2-4x=-3$
$(x)^2-2(x)(2)+(2)^2=-3+(2)^2$
$(x-2)^2=1$

Taking square root on both sides,
$\sqrt{(x-2)^2}=\pm\sqrt{1}$
$x-2=\pm 1$
$x=2\pm 1$
$x=2+1$ *or* $x=2-1$
$x=3$ $\quad x=1$

Thus solution set = {1,3}

x. $7(x+2a)^2+3a^2=5a(7x+23a)$

Solution:

$7(x+2a)^2+3a^2=5a(7x+23a)$
$7(x^2+4ax+4a^2)+3a^2=35ax+115a^2$
$7x^2+28ax+28a^2+3a^2=35ax+115a^2$
$7x^2+28ax-35ax+28a^2+3a^2-115a^2=0$
$7x^2-7ax-84a^2=0$
$7(x^2-ax-12a^2)=0$
$x^2-ax-12a^2=0$
$x^2-ax=12a^2$
$(x)^2-2(x)\left(\frac{a}{2}\right)+\left(\frac{a}{2}\right)^2=12a^2+\left(\frac{a}{2}\right)^2$
$\left(x-\frac{a}{2}\right)^2=12a^2+\frac{a^2}{4}$
$\left(x-\frac{a}{2}\right)^2=\frac{48a^2+a^2}{4}$
$\left(x-\frac{a}{2}\right)^2=\frac{49a^2}{4}$

Taking square root on both sides, we get
$\sqrt{\left(x-\frac{a}{2}\right)^2}=\pm\sqrt{\frac{49a^2}{4}}$
$x-2=\frac{7a}{2}$
$x=\frac{a}{2}\pm\frac{7a}{2}$
$x=\frac{a}{2}+\frac{7a}{2}$ *or* $x=\frac{a}{2}-\frac{7a}{2}$
$=\frac{8a}{2}$ $\quad =-\frac{6a}{2}$
$=4a$ $\quad =-3a$

Thus solution set = {–3a,4a}

Quadratic Formula:

Derivation of quadratic formula by using completing square method.

The quadratic equation in standard form is

$ax^2+bx+c=0,\ a\neq 0$

Dividing each term of the equation by a, we get

$x^2+\frac{b}{a}x+\frac{c}{a}=0$

Shifting constant term $\frac{c}{a}$ to the right, we have

$x^2+\frac{b}{a}x=-\frac{c}{a}$

Adding $\left(\frac{b}{2a}\right)^2$ on both sides, we obtain

$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$

$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$

or

$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$

Taking square root on both sides, we get

$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$

or $x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}\Rightarrow x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Thus, x= $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ is known as "quadratic formula".

Exercise 1.2

Q1. Solve the following equations using quadratic formula:

(i) $2-x^2=7x$

Solution:

$2-x^2=7x$

$-x^2-7x+2=0$

$-\left(x^2+7x-2\right)=0$

$\Rightarrow x^2+7x-2=0$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here a=1, b=7, c=-2

Now x= $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-7\pm\sqrt{(7)^2-4(1)(-2)}}{2(1)}$

$x=\frac{-7\pm\sqrt{49+8}}{2}$

$x=\frac{-7\pm\sqrt{57}}{2}$

Thus, solution set = $\left\{\frac{-7\pm\sqrt{57}}{2}\right\}$

(ii) $5x^2+8x+1=0$

Solution:

$5x^2+8x+1=0$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=5,b=8,c=1$

Now x= $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

x= $\frac{-8\pm\sqrt{(8)^2-4(5)(1)}}{2(5)}$

$x=\frac{-8\pm\sqrt{64-20}}{10}$

$x=\frac{-8\pm\sqrt{44}}{10}$

$x=\frac{-8\pm2\sqrt{11}}{10}$

$x=\frac{2\left(-4\pm\sqrt{11}\right)}{10}$

$x=\frac{-4\pm\sqrt{11}}{5}$

Thus solution set = $\left\{\frac{-4\pm\sqrt{11}}{5}\right\}$

(iii) $\sqrt{3}x^2+x=4\sqrt{3}$

Solution:

$\sqrt{3}x^2+x=4\sqrt{3}$

$\sqrt{3}x^2+x-4\sqrt{3}=0$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=\sqrt{3},b=1,c=-4\sqrt{3}$

Now x= $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

x= $\frac{-1\pm\sqrt{(1)^2-4\left(\sqrt{3}\right)\left(-4\sqrt{3}\right)}}{2\left(\sqrt{3}\right)}$

x= $\frac{-1\pm\sqrt{1+48}}{2\left(\sqrt{3}\right)}$

x= $\frac{-1\pm\sqrt{49}}{2\sqrt{3}}$

x= $\frac{-1\pm7}{2\sqrt{3}}$

x= $\frac{-1+7}{2\sqrt{3}}$ *or* x= $\frac{-1-7}{2\sqrt{3}}$

x= $\frac{6}{2\sqrt{3}}$ x= $\frac{-8}{2\sqrt{3}}$

x= $\frac{3}{\sqrt{3}}$ x= $-\frac{4}{\sqrt{3}}$

x= $\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

x= $\frac{3\sqrt{3}}{\left(\sqrt{3}\right)^2}$

x= $\frac{3\sqrt{3}}{3}$

$x=\sqrt{3}$

Thus solution set = $\left\{\sqrt{3},-\frac{4}{\sqrt{3}}\right\}$

(iv) $4x^2-14=3x$

Solution:

$4x^2-14=3x$

$4x^2-3x-14=0$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Now x= $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

x= $\frac{-(-3)\pm\sqrt{(-3)^2-4(4)(-14)}}{2(4)}$

x= $\frac{3\pm\sqrt{9+224}}{8}$

x= $\frac{3\pm\sqrt{233}}{8}$

x= $\frac{3\pm\sqrt{233}}{8}$

Thus solution set = $\left\{\frac{3\pm\sqrt{233}}{8}\right\}$

(v) $6x^2-3-7x=0$

Solution:

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=6,b=-7,c=-3$

Now x= $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

x= $\frac{-(-7)\pm\sqrt{(-7)^2-4(6)(-3)}}{2(6)}$

x= $\frac{7\pm\sqrt{49+72}}{12}$

x= $\frac{7\pm\sqrt{121}}{12}$

x= $\frac{7\pm11}{12}$

x= $\frac{7+11}{12}$ *or* x= $\frac{7-11}{12}$

x= $\frac{18}{12}$ x= $-\frac{4}{12}$

x= $\frac{3}{2}$ x= $-\frac{1}{3}$

Thus solution set = $\left\{-\frac{1}{3},\frac{3}{2}\right\}$

(vi) $3x^2+8x+2=0$

Solution:

$3x^2+8x+2=0$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=3,b=8,c=2$

Now x= $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

x= $\frac{-8\pm\sqrt{(8)^2-4(3)(2)}}{2(3)}$

x= $\frac{-8\pm\sqrt{64-24}}{6}$

x= $\frac{-8\pm\sqrt{40}}{6}$

x= $\frac{-8\pm2\sqrt{10}}{6}$

x= $\frac{2\left(-4\pm\sqrt{10}\right)}{6}$

x= $\frac{-4\pm\sqrt{10}}{3}$

Thus solution set = $\left\{\frac{-4\pm\sqrt{10}}{3}\right\}$

(vii) $\frac{3}{x-6}-\frac{4}{x-5}=1$

Solution:

$\frac{3}{x-6}-\frac{4}{x-5}=1$

$\frac{3(x-5)-4(x-6)}{(x-6)(x-5)}=1$

$3x-15-4x+24=(x-6)(x-5)$

$-x+9=x^2-11x+30$

$x^2-11x+x+30-9=0$

$x^2-10x+21=0$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=1,b=-10,c=21$

Now x= $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

x= $\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(21)}}{2(1)}$

x= $\frac{10\pm\sqrt{100-84}}{2}$

x= $\frac{10\pm\sqrt{16}}{2}$

x= $\frac{10\pm4}{2}$

x= $\frac{10+4}{2}$ *or* $x=\frac{10-4}{2}$

x= $\frac{14}{2}$ $x=\frac{6}{2}$

$x=7$ $x=3$

Thus, solution set = $\{3,7\}$

(viii) $\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$

Solution:

$\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$

$\frac{2x(x+2)-(x-1)(4-x)}{2x(x-1)}=\frac{7}{3}$

$\frac{\left(2x^2+4x\right)-\left(4x-x^2-4+x\right)}{2x^2-2x}=\frac{7}{3}$

$\frac{\left(2x^2+4x\right)-\left(-x^2+5x-4\right)}{2x^2-2x}=\frac{7}{3}$

$\frac{2x^2+4x+x^2-5x+4}{2x^2-2x}=\frac{7}{3}$

$7\left(2x^2-2x\right)=3\left(3x^2-x+4\right)$

$14x^2-14x=9x^2-3x+12$

$14x^2-9x^2-14x+3x-12=0$

$5x^2-11x-12=0$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=5,b=-11,c=-12$

Now x= $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

x= $\frac{-(-11)\pm\sqrt{(-11)^2-4(5)(-12)}}{2(5)}$

x= $\frac{11\pm\sqrt{121+240}}{10}$

x= $\frac{11\pm\sqrt{361}}{10}$

x= $\frac{11\pm19}{10}$

x= $\frac{11+19}{10}$ *or* $x=\frac{11-19}{10}$

x= $\frac{30}{10}$ $x=-\frac{8}{10}$

$x=3$ $x=-\frac{4}{5}$

Thus, solution set = $\left\{3,-\frac{4}{5}\right\}$

(ix) $\frac{a}{x-b}+\frac{b}{x-a}=2$

Solution:

$\frac{a}{x-b}+\frac{b}{x-a}=2$

$\frac{a(x-a)+b(x-b)}{(x-a)(x-b)}=2$

$ax-a^2+bx-b^2=2(x-a)(x-b)$

$ax+bx-a^2-b^2=2\left(x^2-ax-bx+ab\right)$

$ax+bx-a^2-b^2=2x^2-2ax-2bx+2ab$

$2x^2-2ax-ax-2bx-bx+2ab+a^2+b^2=0$

$2x^2-3ax-3bx+2ab+a^2+b^2=0$

$2x^2-3(a+b)x+\left(2ab+a^2+b^2\right)=0$

$2x^2-3(a+b)x+(a+b)^2=0$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=2,b=-3(a+b),c=(a+b)^2$

Now x= $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

x= $\frac{-[-3(a+b)]\pm\sqrt{[-3(a+b)]^2-4(2)(a+b)^2}}{2(2)}$

x= $\frac{3(a+b)\pm\sqrt{9(a+b)^2-8(a+b)^2}}{4}$

x= $\frac{3(a+b)\pm\sqrt{(a+b)^2}}{4}$

x= $\frac{3(a+b)\pm(a+b)}{4}$

x= $\frac{3(a+b)+(a+b)}{4}$ $x=\frac{3(a+b)-(a+b)}{4}$

x= $\frac{3a+3b+a+b}{4}$ $x=\frac{3a+3b-a-b}{4}$

$x=\frac{4a+4b}{4}$ $x=\frac{2a+2b}{4}$

$x=\frac{4(a+b)}{4}$ $x=\frac{2(a+b)}{4}$

$x=a+b,$ $x=\frac{1}{2}(a+b)$

Thus, solution set = $\left\{(a+b),\frac{1}{2}(a+b)\right\}$

(x) $-(l+m)-lx^2+(2l+m)x=0$, $l\neq0$

Solution:

$-(l+m)-lx^2+(2l+m)x=0$

$-lx^2+(2l+m)x-(l+m)=0$

$-\left[lx^2-(2l+m)x+(l+m)\right]=0$

$lx^2-(2l+m)x+(l+m)=0$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=l,b=-(2l+m),c=(l+m)$

Now x= $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

x= $\frac{-[-(2l+m)]\pm\sqrt{[-(2l+m)]^2-4(l)(l+m)}}{2l}$

x= $\frac{(2l+m)\pm\sqrt{(2l+m)^2-4l(l+m)}}{2l}$

x= $\frac{(2l+m)\pm\sqrt{4l^2+4lm+m^2-4l^2-4lm}}{2l}$

x= $\frac{(2l+m)\pm\sqrt{m^2}}{2l}$

x= $\frac{(2l+m)\pm m}{2l}$

x= $\frac{(2l+m)+m}{2l}$, x= $\frac{(2l+m)-m}{2l}$

x= $\frac{2l+2m}{2l}$, x= $\frac{2l+m-m}{2l}$

x= $\frac{2(l+m)}{2l}$, x= $\frac{2l}{2l}$

x= $\frac{l+m}{l}$, x= l

Thus, solution set = $\left\{\frac{l+m}{l},l\right\}$

Exercise 1.3

Q1. Solve the following equations.

(1) $2x^4 - 11x^2 + 5 = 0$

Solution:

$2x^4 - 11x^2 + 5 = 0 \quad .....(i)$

Let $x^2 = y$ *then* $x^4 = y^2$

So eq (i) becomes

$2y^2 - 11y + 5 = 0$
$2y^2 - 10y - y + 5 = 0$
$2y(y-5) - 1(y-5) = 0$
$(2y-1)(y-5) = 0$

Either $(2y-1) = 0$ *or* $(y-5) = 0$
$2y = 1 \qquad y = 5$

Put $y = \frac{1}{2}$ in $x^2 = y$, we get
$x^2 = \frac{1}{2}$
$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$
$x = \pm\frac{1}{\sqrt{2}}$

Put $y = 5$ *in* $x^2 = y$, *we get*
$x^2 = 5$
$\sqrt{x^2} = \pm\sqrt{5}$
$x = \pm\sqrt{5}$

Thus, solution set $= \left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$

(2) $2x^4 = 9x^2 - 4$

Solution:

$2x^4 = 9x^2 - 4$
$2x^4 - 9x^2 + 4 = 0 \quad .....(i)$

Let $x^2 = y$ *then* $x^4 = y^2$

So eq (i) becomes

$2y^2 - 9y + 4 = 0$
$2y^2 - 8y - y + 4 = 0$
$2y(y-4) - 1(y-4) = 0$
$(2y-1)(y-4) = 0$

Either $(2y-1) = 0$ *or* $(y-4) = 0$

$2y = 1 \qquad y = 4$

Put $y = \frac{1}{2}$ in $x^2 = y$, we get
$x^2 = \frac{1}{2}$
$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$
$x = \pm\frac{1}{\sqrt{2}}$

Put $y = 4$ *in* $x^2 = y$, *we get*
$x^2 = 4$
$\sqrt{x^2} = \pm\sqrt{4}$
$x = \pm 2$

Thus, solution set $= \left\{\pm\frac{1}{\sqrt{2}}, \pm 2\right\}$

(3) $5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

Solution:

$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$
$5x^{\frac{1}{2}} - 7x^{\frac{1}{4}} + 2 = 0 \quad .....(i)$

Let $x^{\frac{1}{4}} = y$ *then* $x^{\frac{1}{2}} = y^2$

So eq (i) becomes

$5y^2 - 7y + 2 = 0$
$5y^2 - 5y - 2y + 2 = 0$
$5y(y-1) - 2(y-1) = 0$
$(5y-2)(y-1) = 0$

Either $(5y-2) = 0$ *or* $(y-1) = 0$
$5y = 2 \qquad y = 1$
$y = \frac{2}{5} \qquad y = 1$

Put $y = \frac{2}{5}$ in $x^{\frac{1}{4}} = y$, we get
$x^{\frac{1}{4}} = y$
$x^{\frac{1}{4}} = \frac{2}{5}$

Put $y = 1$ *in* $x^{\frac{1}{4}} = y$, *we get*
$x^{\frac{1}{4}} = y$
$x^{\frac{1}{4}} = 1$

Taking power '4' on both sides we get
$\left(x^{\frac{1}{4}}\right)^4 = \left(\frac{2}{5}\right)^4$
$x = \frac{2^4}{5^4}$
$x = \frac{16}{625}$

Taking power '4' on both siedes we get
$\left(x^{\frac{1}{4}}\right)^4 = (1)^4$
$\left(x^{\frac{1}{4}}\right)^4 = 1$
$x = 1$

Thus, solution set $= \left\{\frac{16}{625}, 1\right\}$

(4) $x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$

Solution:

$x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$

$x^{\frac{2}{3}} - 15x^{\frac{1}{3}} + 54 = 0 \quad .....(i)$

Let $x^{\frac{1}{3}} = y$, *then* $x^{\frac{2}{3}} = y^2$

So eq (i) becomes

$y^2 - 15y + 54 = 0$
$y^2 - 9y - 6y + 54 = 0$
$y(y-9) - 6(y-9) = 0$
$(y-6)(y-9) = 0$

Either $(y-9) = 0$ *or* $(y-6) = 0$
$y = 9 \qquad y = 6$

Put $y = 9$ in $x^{\frac{1}{3}} = y$, we get
$x^{\frac{1}{3}} = y$
$x^{\frac{1}{3}} = 9$

Put $y = 6$ *in* $x^{\frac{1}{3}} = y$, *we get*
$x^{\frac{1}{3}} = y$
$x^{\frac{1}{3}} = 6$

Taking cube on both siedes we get
$\left(x^{\frac{1}{3}}\right)^3 = (9)^3$
$x = 729$

Taking cube on both siedes we get
$\left(x^{\frac{1}{3}}\right)^3 = (6)^3$
$x = 216$

Thus, solution set $= \{729, 216\}$

(5) $3x^{-2} + 5 = 8x^{-1}$

Solution:

$3x^{-2} + 5 = 8x^{-1}$
$3x^{-2} - 8x^{-1} + 5 = 0 \quad .....(i)$

Let $x^{-1} = y$, *then* $x^{-2} = y^2$

So eq (i) becomes

$3y^2 - 8y + 5 = 0$
$3y^2 - 5y - 3y + 5 = 0$
$y(3y-5) - 1(3y-5) = 0$
$(y-1)(3y-5) = 0$

Either $y - 1 = 0$ *or* $3y - 5 = 0$
$y = 1 \qquad 3y = 5$
$y = 1 \qquad y = \frac{5}{3}$

Put $y = 1$ in $x^{-1} = y$, we get
$x^{-1} = y$
$x^{-1} = 1$
$\frac{1}{x} = 1$
$x = 1$

Put $y = \frac{5}{3}$ *in* $x^{-1} = y$, *we get*
$x^{-1} = y$
$x^{-1} = \frac{5}{3}$
$\frac{1}{x} = \frac{5}{3}$
$x = \frac{3}{5}$

Thus, solution set $= \left\{1, \frac{3}{5}\right\}$

(6) $(2x^2+1) + \frac{3}{2x^2+1} = 4$

Solution:

$(2x^2+1) + \frac{3}{2x^2+1} = 4 \quad .....(i)$

Let $2x^2 + 1 = y$,

So eq (i) becomes

$y + \frac{3}{y} = 4$

Multiplying both sides by 'y', we get

$y^2 + 3 = 4y$
$y^2 - 4y + 3 = 0$
$y^2 - 3y - y + 3 = 0$
$y(y-3) - 1(y-3) = 0$
$(y-1)(y-3) = 0$

Either $y - 1 = 0$ *or* $y - 3 = 0$
$y = 1 \qquad y = 3$

Put $y = 1$ in $2x^2 + 1 = y$, we get
$2x^2 + 1 = 1$
$2x^2 = 1 - 1$
$2x^2 = 0$
$x^2 = 0$
$x = 0$

Put $y = 3$ *in* $2x^2 + 1 = y$, *we get*
$2x^2 + 1 = 3$
$2x^2 = 3 - 1$
$2x^2 = 2$
$x^2 = 1$
$x = \pm 1$

Thus, solution set $= \{-1, 0, 1\}$

(7) $\frac{x}{x-3} + 4\left(\frac{x-3}{x}\right) = 4$

Solution:

$\frac{x}{x-3} + 4\left(\frac{x-3}{x}\right) = 4 \quad .....(i)$

Let $\frac{x}{x-3} = y$,

So eq (i) becomes

$y + 4\left(\frac{1}{y}\right) = 4$

Multiplying both sides by 'y', we get

$y^2 + 4 = 4y$
$y^2 - 4y + 4 = 0$
$(y)^2 - 2(y)(2) + (2)^2 = 0$
$(y-2)^2 = 0$
$(y-2) = 0$

Put $y = 2$ in $\frac{x}{x-3} = y$, we get
$\frac{x}{x-3} = y$
$\frac{x}{x-3} = 2$
$2(x-3) = x$
$2x - 6 = x$
$2x - x = 6$
$x = 6$

Thus, solution set $= \{6\}$

(8) $\frac{4x+1}{4x-1} + \frac{4x-1}{4x+1} = 2\frac{1}{6}$

Solution:

$\frac{4x+1}{4x-1} + \frac{4x-1}{4x+1} = 2\frac{1}{6}$
$\frac{4x+1}{4x-1} + \frac{4x-1}{4x+1} = \frac{13}{6} \quad .....(i)$

Let $\frac{4x+1}{4x-1} = y$,

So eq (i) becomes

$y + \frac{1}{y} = \frac{13}{6}$

Multiplying both sides by '6y', we get

$6y^2 + 6 = 13y$
$6y^2 - 13y + 6 = 0$
$6y^2 - 9y - 4y + 6 = 0$
$3y(2y-3) - 2(2y-3) = 0$

$(3y-2)(2y-3) = 0$

Either $3y - 2 = 0$ *or* $2y - 3 = 0$
$3y = 2 \qquad 2y = 3$
$y = \frac{2}{3} \qquad y = \frac{3}{2}$

Put $y = \frac{2}{3}$ in $\frac{4x+1}{4x-1} = y$, we get
$\frac{4x+1}{4x-1} = y$
$\frac{4x+1}{4x-1} = \frac{2}{3}$
$3(4x+1) = 2(4x-1)$
$12x + 3 = 8x - 2$
$12x - 8x = -2 - 3$
$4x = -5$
$x = -\frac{5}{4}$

Put $y = \frac{3}{2}$ in $\frac{4x+1}{4x-1} = y$, we get
$\frac{4x+1}{4x-1} = y$
$\frac{4x+1}{4x-1} = \frac{3}{2}$
$3(4x-1) = 2(4x+1)$
$12x - 3 = 8x + 2$
$12x - 8x = 2 + 3$
$4x = 5$
$x = \frac{5}{4}$

Thus, solution set $= \left\{\pm\frac{5}{4}\right\}$

(9) $\frac{x-a}{x+a} - \frac{x+a}{x-a} = \frac{7}{12}$

Solution:

$\frac{x-a}{x+a} - \frac{x+a}{x-a} = \frac{7}{12} \quad .....(i)$

Let $\frac{x-a}{x+a} = y$,

So eq (i) becomes

$y - \frac{1}{y} = \frac{7}{12}$

Multiplying both sides by '12y', we get

$12y^2 - 12 = 7y$
$12y^2 - 7y - 12 = 0$
$12y^2 - 16y + 9y - 12 = 0$
$4y(3y-4) + 3(3y-4) = 0$

$(4y+3)(3y-4) = 0$

Either $4y + 3 = 0$ *or* $3y - 4 = 0$
$4y = -3 \qquad 3y = 4$
$y = -\frac{3}{4} \qquad y = \frac{4}{3}$

Put $y = -\frac{3}{4}$ in $\frac{x-a}{x+a} = y$, we get
$\frac{x-a}{x+a} = y$
$\frac{x-a}{x+a} = -\frac{3}{4}$
$4(x-a) = -3(x+a)$
$4x - 4a = -3x - 3a$
$4x + 3x = 4a - 3a$
$7x = a$
$x = \frac{a}{7}$

Put $y = \frac{4}{3}$ in $\frac{x-a}{x+a} = y$, we get
$\frac{x-a}{x+a} = y$
$\frac{x-a}{x+a} = \frac{4}{3}$
$4(x+a) = 3(x-a)$
$4x + 4a = 3x - 3a$
$4x - 3x = -4a - 3a$
$x = -7a$

Thus, solution set $= \left\{-7a, \frac{a}{7}\right\}$

(10) $x^4 - 2x^3 - 2x^2 + 2x + 1 = 0$

Solution:

$x^4 - 2x^3 - 2x^2 + 2x + 1 = 0$

Dividing each term by x^2, *we get*

$\frac{x^4}{x^2} - 2\frac{x^3}{x^2} - 2\frac{x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2} = 0$
$x^2 - 2x - 2 + \frac{2}{x} + \frac{1}{x^2} = 0$
$x^2 + \frac{1}{x^2} - 2x + \frac{2}{x} - 2 = 0$
$\left(x^2 + \frac{1}{x^2}\right) - 2\left(x - \frac{1}{x}\right) - 2 = 0 \quad .....(i)$

Let $x - \frac{1}{x} = y$
$\left(x - \frac{1}{x}\right)^2 = y^2$
$x^2 + \frac{1}{x^2} + 2 = y^2$
$x^2 + \frac{1}{x^2} = y^2 - 2$

So eq. (i) become

$y^2 + 2 - 2y - 2 = 0$
$y^2 - 2y = 0$
$y(y-2) = 0$

Either $y = 0$ *or* $y - 2 = 0, \Rightarrow y = 2$

Put $y = 0$ in $x - \frac{1}{x} = y$, we get
$x - \frac{1}{x} = y$
$x - \frac{1}{x} = 0$
$x^2 - 1 = 0$
$x^2 = 1$
$x = \pm 1$

Put $y = 2$ in $x - \frac{1}{x} = y$, we get
$x - \frac{1}{x} = y$
$x - \frac{1}{x} = 2$
$x^2 - 1 = 2x$
$x^2 - 2x - 1 = 0$
$x^2 - 2x - 1 = 0$
$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4+4}}{2}$
$x = \frac{2 \pm \sqrt{8}}{2}$
$x = \frac{2 \pm 2\sqrt{2}}{2}$
$x = \frac{2(1 \pm \sqrt{2})}{2}$
$x = 1 \pm \sqrt{2}$

Thus, solution set $= \{\pm 1, 1 \pm \sqrt{2}\}$

(11) $2x^4 + x^3 - 6x^2 + x + 2 = 0$

Solution:

$2x^4 + x^3 - 6x^2 + x + 2 = 0$

Dividing each term by x^2, *we get*

$\frac{2x^4}{x^2} + \frac{x^3}{x^2} - \frac{6x^2}{x^2} + \frac{x}{x^2} + \frac{2}{x^2} = 0$
$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$
$2x^2 + \frac{2}{x^2} + x + \frac{1}{x} - 6 = 0$
$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0 \quad .....(i)$

Let $x + \frac{1}{x} = y$
$\left(x + \frac{1}{x}\right)^2 = y^2$
$x^2 + \frac{1}{x^2} + 2 = y^2$
$x^2 + \frac{1}{x^2} = y^2 - 2$

So eq. (i) becomes

$2\left(y^2 - 2\right) + y - 6 = 0$

$2y^2 - 4 + y - 6 = 0$
$2y^2 + y - 10 = 0$
$2y^2 + 5y - 4y - 10 = 0$
$y(2y+5) - 2(2y+5) = 0$
$(y-2)(2y+5) = 0$

Either $y - 2 = 0$ *or* $2y + 5 = 0$
$y = 2 \qquad 2y = -5$
$y = 2 \qquad y = \frac{-5}{2}$

Put $y = 2$ in $x + \frac{1}{x} = y$, we get
$x + \frac{1}{x} = y$
$x + \frac{1}{x} = 2$
$x^2 + 1 = 2x$
$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$
$\Rightarrow x - 1 = 0$
$\Rightarrow x = 1$

Put $y = -\frac{5}{2}$ in $x + \frac{1}{x} = y$, we get
$x + \frac{1}{x} = y$
$x + \frac{1}{x} = -\frac{5}{2}$
$2x^2 + 2 = -5x$
$2x^2 + 5x + 2 = 0$

$2x^2 + 4x + x + 2 = 0$
$2x(x+2) + 1(x+2) = 0$
Either $2x + 1 = 0$ or $x + 2 = 0$
$2x = -1 \qquad x = -2$
$x = -\frac{1}{2}$

Thus, solution set $= \left\{1, -2, -\frac{1}{2}\right\}$

(12) $4.2^{2x+1} - 9.2^x + 1 = 0$

Solution:

$4.2^{2x+1} - 9.2^x + 1 = 0$
$4.2^{2x}.2 - 9.2^x + 1 = 0 \quad .....(i)$

Let $2^x = y$ Then $2^{2x} = y^2$

So eq. (i) becomes

$4.y^2.2 - 9.y + 1 = 0$
$8y^2 - 9y + 1 = 0$
$8y^2 - 8y - y + 1 = 0$
$8y(y-1) - 1(y-1) = 0$
$(8y-1)(y-1) = 0$

Either $8y - 1 = 0$ *or* $y - 1 = 0$
$8y = 1 \qquad y = 1$
$y = \frac{1}{8} \qquad y = 1$

Put $y = \frac{1}{8}$ in $2^x = y$, we get
$2^x = y$
$2^x = \frac{1}{8}$
$2^x = \frac{1}{2^3}$
$2^x = 2^{-3}$
$x = -3$

Put $y = 1$ in $2^x = y$, we get
$2^x = y$
$2^x = 1$
$2^x = 2^0$
$x = 0$

Thus, solution set $= \{-3, 0\}$

(13) $3^{2x+2} = 12.3^x - 3$

Solution:

$3^{2x+2} = 12.3^x - 3$
$3^{2x}.3^2 - 12.3^x + 3 = 0 \quad .....(i)$

Let $3^x = y$ Then $3^{2x} = y^2$

So eq. (i) becomes

$y^2.9 - 12y + 3 = 0$
$9y^2 - 12y + 3 = 0$
$9y^2 - 9y - 3y + 3 = 0$
$9y(y-1) - 3(y-1) = 0$
$(9y-3)(y-1) = 0$

Either $9y - 3 = 0$ *or* $y - 1 = 0$
$9y = 3 \qquad y = 1$
$y = \frac{3}{9} \qquad y = 1$
$y = \frac{1}{3} \qquad y = 1$

Put $y = \frac{1}{3}$ in $3^x = y$, we get
$3^x = y$
$3^x = \frac{1}{3}$
$3^x = 3^{-1}$
$x = -1$

Put $y = 1$ in $3^x = y$, we get
$3^x = y$
$3^x = 1$
$3^x = 3^0$
$x = 0$

Thus, solution set $= \{-1, 0\}$

(14) $2^x + 64.2^{-x} - 20 = 0$

Solution:

$2^x + 64.2^{-x} - 20 = 0 \quad .....(i)$

Let $2^x = y$, Then $2^{-x} = \frac{1}{y}$

So eq. (i) becomes

$y - 64.\frac{1}{y} - 20 = 0$

$y^2 - 64 - 20y = 0$

$y^2 - 20y - 64 = 0$
$y^2 - 16y - 4y - 64 = 0$
$y(y-16) - 4(y-16) = 0$
$(y-4)(y-16) = 0$

Either $y - 4 = 0$ *or* $y - 16 = 0$
$y = 4 \qquad y = 16$

Put $y = 4$ in $2^x = y$, we get
$2^x = y$
$2^x = 4$
$2^x = 2^2$
$x = 2$

Put $y = 16$ in $2^x = y$, we get
$2^x = y$
$2^x = 16$
$2^x = 2^4$
$x = 4$

Thus, solution set $= \{2, 4\}$

(15) $(x+1)(x+3)(x-5)(x-7) = 192$

Solution:

$(x+1)(x+3)(x-5)(x-7) = 192$

As $1 - 5 = 3 - 7$
$4 = 4$

So $[(x+1)(x-5)][(x+3)(x-7)] = 192$
$[x^2 - 5x + x - 5][x^2 - 7x + 3x - 21] = 192$
$(x^2 - 4x - 5)(x^2 - 4x - 21) = 192 \quad .....(i)$

Let $x^2 - 4x = y$

$(y-5)(y-21) = 192$
$y^2 - 21y - 5y + 105 = 192$
$y^2 - 26y + 105 - 192 = 0$
$y^2 - 26y - 87 = 0$
$y^2 - 29y + 3y - 87 = 0$
$y(y-29) + 3(y-29) = 0$
$(y+3)(y-29) = 0$

Either $y + 3 = 0$ *or* $y - 29 = 0$
$y = -3 \qquad y = 29$

Put $y = -3$ in $x^2 - 4x = y$, we get
$x^2 - 4x = y$
$x^2 - 4x = -3$
$x^2 - 4x + 3 = 0$
$x^2 - 3x - x + 3 = 0$

$x(x-3) - 1(x-3) = 0$
$(x-1)(x-3) = 0$

Either $x - 1 = 0$ *or* $x - 3 = 0$
$x = 1 \qquad x = 3$

Put $y = 29$ in $x^2 - 4x = y$, we get
$x^2 - 4x = y$
$x^2 - 4x = 29$
$x^2 - 4x - 29 = 0$

Here $a = 1, b = -4, c = -29$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-29)}}{2(1)}$
$x = \frac{4 \pm \sqrt{16+116}}{2}$
$x = \frac{4 \pm \sqrt{132}}{2}$
$x = \frac{4 \pm 2\sqrt{33}}{2}$
$x = \frac{2\left(2 \pm \sqrt{33}\right)}{2}$
$x = 2 \pm \sqrt{33}$

Thus, solution set $= \left\{1, 3, 2 \pm \sqrt{33}\right\}$

(16) $(x-1)(x-2)(x-8)(x+5) + 360 = 0$

Solution:

$(x-1)(x-2)(x-8)(x+5) + 360 = 0$

As $-1 - 2 = -8 + 5$
$-3 = -3$

So $[(x-1)(x-2)][(x-8)(x+5)] + 360 = 0$
$[x^2 - 2x - x + 2][x^2 + 5x - 8x - 40] + 360 = 0$
$(x^2 - 3x + 2)(x^2 - 3x - 40) + 360 = 0 \quad .....(i)$

Let $x^2 - 3x = y$

So eq (i) *becomes*

$(y+2)(y-40) + 360 = 0$
$y^2 - 40y + 2y - 80 + 360 = 0$
$y^2 - 38y + 280 = 0$
$y^2 - 28y - 10y + 280 = 0$
$y(y-28) - 10(y-28) = 0$
$(y-10)(y-28) = 0$

Either $y - 10 = 0$ *or* $y - 28 = 0$
$y = 10 \qquad y = 28$

Put $y = 10$ in $x^2 - 3x = y$, we get
$x^2 - 3x = y$
$x^2 - 3x = 10$
$x^2 - 3x - 10 = 0$
$x^2 - 5x + 2x - 10 = 0$
$x(x-5) + 2(x-5) = 0$
$(x+2)(x-5) = 0$
Either $x + 2 = 0$ *or* $x - 5 = 0$
$x = -2 \qquad x = 5$

Put $y = 28$ in $x^2 - 3x = y$, we get
$x^2 - 3x = y$
$x^2 - 3x = 28$
$x^2 - 3x - 28 = 0$
$x^2 - 7x + 4x - 28 = 0$
$x(x-7) + 4(x-7) = 0$
$(x+4)(x-7) = 0$
Either $x + 4 = 0$ *or* $x - 7 = 0$
$x = -4 \qquad x = 7$

Thus, solution set $= \{-4, -2, 5, 7\}$

Radical Equations:

An equation involving expression under the radical sign is called a radical equation.

e.g., $\sqrt{x+3}=x+1$ *and* $\sqrt{x-1}=\sqrt{x-2}+1$

Exercise 1.4

Solve the following equations.

(1) $2x+5=\sqrt{7x+16}$

Solution

$2x+5=\sqrt{7x+16}$ .....(i)

Squaring both sides, we get

$(2x+5)^2=\left(\sqrt{7x+16}\right)^2$

$4x^2+20x+25=7x+16$

$4x^2+20x+25-7x-16=0$

$4x^2+20x-7x+25-16=0$

$4x^2+13x+9=0$

$4x^2+9x+4x+9=0$

$x(4x+9)+1(4x+9)=0$

$(x+1)(4x+9)=0$

Either $x+1=0$ *or* $4x+9=0$

$x=-1$ $\quad 4x=-9$

$x=-\frac{9}{4}$

Check :

Put $x=-1$ in eq.(i), *we get*

$2(-1)+5=\sqrt{7(-1)+16}$ $\quad \Rightarrow -2+5=\sqrt{-7+16}$

$3=\sqrt{9}\Rightarrow 3=3$ (which is true)

Put $x=-\frac{9}{4}$ *in eq*(i), *we get*

$2\left(-\frac{9}{4}\right)+5=\sqrt{7\left(-\frac{9}{4}\right)+16}$

$-\frac{9}{2}+5=\sqrt{-\frac{63}{4}+16}$

$\frac{1}{2}=\sqrt{\frac{1}{4}}$

$\frac{1}{2}=\frac{1}{2}$ (which is true)

Thus, solution set $=\left\{-1,-\frac{9}{4}\right\}$

(2) $\sqrt{x+3}=3x-1$

Solution:

$\sqrt{x+3}=3x-1$ .....(i)

Squaring both sides, we get

$\left(\sqrt{x+3}\right)^2=(3x-1)^2$

$x+3=9x^2-6x+1$

$9x^2-6x+1-x-3=0$

$9x^2-7x-2=0$

$9x^2-9x+2x-2=0$

$9x(x-1)+2(x-1)=0$

$(9x+2)(x-1)=0$

Either $9x+2=0$ *or* $x-1=0$

$9x=-2$ $\quad x=1$

$x=-\frac{2}{9}$ $\quad x=1$

Check :

Put $x=-\frac{2}{9}$ in eq.(i), *we get*

$\sqrt{-\frac{2}{9}+3}=3\left(-\frac{2}{9}\right)-1$

$\sqrt{\frac{25}{9}}=-\frac{2}{3}-1$

$\sqrt{\frac{25}{9}}=-\frac{5}{3}$

$\frac{5}{3}\neq-\frac{5}{3}$ (which is not true)

Put $x=1$ *in eq*(i), *we get*

$\sqrt{1+3}=3(1)-1$

$\sqrt{4}=3-1$

$2=2$ (which is true)

Thus, solution set $=\{1\}$

(3) $4x=\sqrt{13x+14}-3$

Solution:

$4x=\sqrt{13x+14}-3$ .....(i)

$4x+3=\sqrt{13x+14}$

Squaring both sides, we get

$(4x+3)^2=\left(\sqrt{13x+14}\right)^2$

$16x^2+24x+9=13x+14$

$16x^2+24x-13x+9-14=0$

$16x^2+11x-5=0$

$16x^2+16x-5x-5=0$

$16x(x+1)-5(x+1)=0$

$(16x-5)(x+1)=0$

Either $16x-5=0$ *or* $x+1=0$

$16x=5$ $\quad x=-1$

$x=\frac{5}{16}$ $\quad x=-1$

Check :

Put $x=\frac{5}{16}$ in eq.(i), *we get*

$4\left(\frac{5}{16}\right)=\sqrt{13\left(\frac{5}{16}\right)+14}-3$

$\frac{5}{4}=\sqrt{\frac{65}{16}+14}-3$

$\frac{5}{4}=\sqrt{\frac{289}{16}}-3$

$\frac{5}{4}=\frac{17}{4}-3$

$\frac{5}{4}=\frac{5}{4}$ (which is true)

Put $x=-1$ *in eq*(i), *we get*

$4(-1)=\sqrt{13(-1)+14}-3$

$-4=\sqrt{-13+14}-3$

$-4=\sqrt{1}-3$

$-4=1-3$

$-4\neq-2$ (which is not true)

Thus, solution set $=\left\{\frac{5}{16}\right\}$

(4) $\sqrt{3x+100}-x=4$

Solution:

$\sqrt{3x+100}-x=4$ .....(i)

$\sqrt{3x+100}=4+x$

Squaring both sides, we get

$\left(\sqrt{3x+100}\right)^2=(x+4)^2$

$3x+100=x^2+8x+16$

$x^2+8x+16-3x-100=0$

$x^2+5x-84=0$

$x^2+12x-7x-84=0$

$x(x+12)-7(x+12)=0$

$(x-7)(x+12)=0$

Either $x-7=0$ *or* $x+12=0$

$x=7$ $\quad x=-12$

Check :

Put $x=7$ in eq.(i), *we get*

$\sqrt{3(7)+100}-7=4$ $\quad \Rightarrow \sqrt{21+100}-7=4$

$\sqrt{121}-7=4$ $\quad \Rightarrow 11-7=4$

$4=4$ (which is true)

Put $x=-12$ *in eq*(i), *we get*

$\sqrt{3(-12)+100}-(-12)=4$ $\quad \Rightarrow \sqrt{-36+100}+12=4$

$\sqrt{64}+12=4$ $\quad \Rightarrow 8+12=4$

$20\neq4$ (which is not true)

Thus, solution set $=\{7\}$

(5) $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

Solution:

$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$

$(x+5)+(x+21)+2\sqrt{(x+5)(x+21)}=x+60$

$x+5+x+21+2\sqrt{x^2+26x+105}=x+60$

$2x+26+2\sqrt{x^2+26x+105}=x+60$

$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$ .....(i)

Squaring both sides, we get

$2\sqrt{x^2+26x+105}=x+60-2x-26$

$2\sqrt{x^2+26x+105}=-x+34$

$2\sqrt{x^2+26x+105}=-(x-34)$

Squaring both sides, we get

$\left(2\sqrt{x^2+26x+105}\right)^2=\left[-(x-34)\right]^2$

$4\left(x^2+26x+105\right)=x^2-68x+1156$

$4x^2+104x+420=x^2-68x+1156$

$4x^2-x^2+104x+68x+420-1156=0$

$3x^2+172x-736=0$

Here a=3, b=172, c=-736

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-172\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$

$x=\frac{-172\pm\sqrt{29584+8832}}{6}$

$x=\frac{-172\pm\sqrt{38416}}{6}$

$x=\frac{-172-196}{6}$ *or* $x=\frac{-172+196}{6}$

$x=\frac{-368}{6}$ $\quad x=\frac{24}{6}$

$x=-\frac{184}{3}$ $\quad x=4$

Check :

Put $x=-\frac{184}{3}$ in eq.(i), *we get*

$\sqrt{-\frac{184}{3}+5}+\sqrt{-\frac{184}{3}+21}=\sqrt{-\frac{184}{3}+60}$

$\sqrt{-\frac{169}{3}}+\sqrt{-\frac{121}{3}}=\sqrt{-\frac{4}{3}}$

$\frac{13i}{\sqrt{3}}+\frac{11i}{\sqrt{3}}=\frac{2i}{\sqrt{3}}$

$\frac{24i}{\sqrt{3}}=\frac{2i}{\sqrt{3}}$ (which is not true)

Put $x=4$ *in eq*(i), *we get*

$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$

$\sqrt{9}+\sqrt{25}=\sqrt{64}$

$3+5=8$

$8=8$ (which is true)

Thus, solution set $=\{4\}$

(6) $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$

Solution:

$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$ .....(i)

Squaring both sides, we get

$\left(\sqrt{x+1}+\sqrt{x-2}\right)^2=\left(\sqrt{x+6}\right)^2$

$(x+1)+(x-2)+2\sqrt{(x+1)(x-2)}=x+6$

$x+1+x-2+2\sqrt{x^2-x-2}=x+6$

$2x-1+2\sqrt{x^2-x-2}=x+6$

$2\sqrt{x^2-x-2}=-x+7$

$2\sqrt{x^2-x-2}=-(x-7)$

Squaring both sides, we get

$\left(2\sqrt{x^2-x-2}\right)^2=\left[-(x-7)\right]^2$

$4(x^2-x-2)=x^2-14x+49$

$4x^2-4x-8=x^2-14x+49$

$4x^2-x^2-4x+14x-8-49=0$

$3x^2+10x-57=0$

Here a=3, b=10, c=-57

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-10\pm\sqrt{(10)^2-4(3)(-57)}}{2(3)}$

$x=\frac{-10\pm\sqrt{100+684}}{6}$

$x=\frac{-10\pm\sqrt{784}}{6}$

$x=\frac{-10\pm28}{6}$

$x=\frac{-10-28}{6}$ *or* $x=\frac{-10+28}{6}$

$x=\frac{-38}{6}$ $\quad x=\frac{18}{6}$

$x=-\frac{19}{3}$ $\quad x=3$

Check :

Put $x=-\frac{19}{3}$ in eq.(i), *we get*

$\sqrt{-\frac{19}{3}+1}+\sqrt{-\frac{19}{3}-2}=\sqrt{-\frac{19}{3}+6}$

$\sqrt{-\frac{16}{3}}+\sqrt{\frac{-25}{3}}=\sqrt{-\frac{1}{3}}$

$\frac{4i}{\sqrt{3}}+\frac{5i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$

$\frac{9i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$ (which is not true)

Put $x=3$ *in eq*(i), *we get*

$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$

$\sqrt{4}+\sqrt{1}=\sqrt{9}$

$2+1=3$

$3=3$ (which is true)

Thus, solution set $=\{3\}$

(7) $\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

Solution:

$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$ .....(i)

Squaring both sides, we get

$\left(\sqrt{11-x}-\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$

$(11-x)+(6-x)-2\sqrt{(11-x)(6-x)}=27-x$

$11-x+6-x-2\sqrt{(11-x)(6-x)}=27-x$

$17-2x-2\sqrt{x^2-17x+66}=27-x$

$-2\sqrt{x^2-17x+66}=27-x-17+2x$

$-2\sqrt{x^2-17x+66}=10+x$

Squaring both sides, we get

$\left(-2\sqrt{x^2-17x+66}\right)^2=(10+x)^2$

$4(x^2-17x+66)=100+20x+x^2$

$4x^2-68x+264=x^2+20x+100$

$4x^2-x^2-68x-20x+264-100=0$

$3x^2-88x+164=0$

Here a = 3, b = -88, c = 164

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-88)\pm\sqrt{(-88)^2-4(3)(164)}}{2(3)}$

$x=\frac{88\pm\sqrt{7744-1968}}{6}$

$x=\frac{88\pm\sqrt{5776}}{6}$

$x=\frac{88\pm76}{6}$

$x=\frac{88-76}{6}$ *or* $x=\frac{88+76}{6}$

$x=\frac{12}{6}$ $\quad x=\frac{164}{6}$

$x=2$ $\quad x=\frac{82}{3}$

Check :

Put $x=2$ in eq.(i), *we get*

$\sqrt{11-2}-\sqrt{6-2}=\sqrt{27-2}$

$\sqrt{9}-\sqrt{4}=\sqrt{25}$

$3-2=5$

$1=5$ (which is not true)

Put $x=\frac{82}{3}$ *in eq*(i), *we get*

$\sqrt{11-\frac{82}{3}}-\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$

$\sqrt{\frac{-49}{3}}-\sqrt{\frac{-64}{3}}=\sqrt{\frac{-1}{3}}$

$\frac{7i}{\sqrt{3}}-\frac{8i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$

$\frac{-i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$ (which is not true)

Thus, solution set $=\{\ \}$

(8) $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

Solution:

$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$ .....(i)

Squaring both sides, we get

$\left(\sqrt{4a+x}-\sqrt{a-x}\right)^2=\left(\sqrt{a}\right)^2$

$(4a+x)+(a-x)-2\sqrt{(4a+x)(a-x)}=a$

$4a+x+a-x-2\sqrt{4a^2-3ax-x^2}=a$

$5a-2\sqrt{4a^2-3ax-x^2}=a$

$-2\sqrt{4a^2-3ax-x^2}=a-5a$

$-2\sqrt{4a^2-3ax-x^2}=-4a$

$\Rightarrow\sqrt{4a^2-3ax-x^2}=2a$

Squaring both sides, we get

$\left(\sqrt{4a^2-3ax-x^2}\right)^2=(2a)^2$

$4a^2-3ax-x^2=4a^2$

$4a^2-4a^2-3ax-x^2=0$

$-3ax-x^2=0$

$-x(x+3a)=0$

$x(x+3a)=0$

Either $x=0$ *or* $x+3a=0$

$x=-3a$

Check :

Put $x=0$ in eq.(i), *we get*

$\sqrt{4a+0}-\sqrt{a-0}=\sqrt{a}$

$\sqrt{4a}-\sqrt{a}=\sqrt{a}$

$2\sqrt{a}-\sqrt{a}=\sqrt{a}$

$\sqrt{a}=\sqrt{a}$ (which is true)

Put $x=-3a$ in eq.(i), *we get*

$\sqrt{4a+(-3a)}-\sqrt{a-(-3a)}=\sqrt{a}$

$\sqrt{4a-3a}-\sqrt{a+3a}=\sqrt{a}$

$\sqrt{a}-\sqrt{4a}=\sqrt{a}$

$\sqrt{a}-2\sqrt{a}=\sqrt{a}$

$-\sqrt{a}=\sqrt{a}$ (which is not true)

Thus, solution set $=\{0\}$

(9) $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$

Solution:

$\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$ .....(i)

Let $x^2+x=y$

So eq.(i) becomes

$\sqrt{y+1}-\sqrt{y-1}=1$

Squaring both sides, we get

$\left(\sqrt{y+1}-\sqrt{y-1}\right)^2=1$

$(y+1)+(y-1)-2\sqrt{(y+1)(y-1)}=1$

$y+1+y-1-2\sqrt{y^2-1}=1$

$2y-2\sqrt{y^2-1}=1$

$-2\sqrt{y^2-1}=1-2y$

Squaring both sides, we get

$\left(-2\sqrt{y^2-1}\right)^2=(1-2y)^2$

$4(y^2-1)=1-4y+4y^2$

$4y^2-4=1-4y+4y^2$

$4y^2-4-1+4y-4y^2=0$

$4y-5=0$

$y=\frac{5}{4}$

Put $y=\frac{5}{4}$ *in* $x^2+x=y$, *we get*

$x^2+x=\frac{5}{4}$

$4x^2+4x=5$

$4x^2+4x-5=0$

Here a=4, b=4, c=-5

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$

$x=\frac{-4\pm\sqrt{16+80}}{8}$

$x=\frac{-4\pm\sqrt{96}}{8}$

$x=\frac{-4\pm4\sqrt{6}}{8}=\frac{4\left(-1\pm\sqrt{6}\right)}{8}=\frac{-1\pm\sqrt{6}}{2}$

Thus, solution set $=\left\{\frac{-1\pm\sqrt{6}}{2}\right\}$

(10) $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$

Solution:

$\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$ .....(i)

Let $x^2+3x=y$

So eq.(i) becomes

$\sqrt{y+8}+\sqrt{y+2}=3$

Squaring both sides, we get

$\left(\sqrt{y+8}+\sqrt{y+2}\right)^2=(3)^2$

$(y+8)+(y+2)+2\sqrt{(y+8)(y+2)}=9$

$y+8+y+2+2\sqrt{y^2+10y+16}=9$

$2y+10+2\sqrt{y^2+10y+16}=9$

$2\sqrt{y^2+10y+16}=9-2y-10$

$2\sqrt{y^2+10y+16}=-2y-1$

$2\sqrt{y^2+10y+16}=-(2y+1)$

Squaring both sides, we get

$\left(2\sqrt{y^2+10y+16}\right)^2=\left[-(2y+1)\right]^2$

$4(y^2+10y+16)=4y^2+4y+1$

$4y^2+40y+64=4y^2+4y+1$

$4y^2-4y^2+40y-4y+64-1=0$

$36y=-63$

$y=\frac{63}{36}$

Put $y=-\frac{63}{36}$ *in* $x^2+3x=y$, *we get*

$x^2+3x=-\frac{63}{36}$

$\Rightarrow 36x^2+108x=-63$

$36x^2+108x+63=0$

Here a=36, b=108, c=63

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-108\pm\sqrt{(108)^2-4(36)(63)}}{2(36)}$

$x=\frac{-108\pm\sqrt{11664+9072}}{72}$

$x=\frac{-108\pm\sqrt{2592}}{72}$

$x=\frac{-108\pm36\sqrt{2}}{72}$

$x=\frac{36\left(-3\pm\sqrt{2}\right)}{72}$

$x=\frac{-3\pm\sqrt{2}}{2}$

Thus, Solution set $=\left\{\frac{-3\pm\sqrt{2}}{2}\right\}$

(11) $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

Solution:

$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$ .....(i)

Let $x^2+3x=y$

So eq.(i) becomes

$\sqrt{y+9}+\sqrt{y+4}=5$

Squaring both sides, we get

$\left(\sqrt{y+9}+\sqrt{y+4}\right)^2=(5)^2$

$(y+9)+(y+4)+2\sqrt{(y+9)(y+4)}=25$

$y+9+y+4+2\sqrt{y^2+13y+36}=25$

$2y+13+2\sqrt{y^2+13y+36}=25$

$2\sqrt{y^2+13y+36}=25-2y-13$

$2\sqrt{y^2+13y+36}=-2y+12$

$2\sqrt{y^2+13y+36}=-2(y-6)$

$\Rightarrow\sqrt{y^2+13y+36}=-(y-6)$

Squaring both sides, we get

$\left(\sqrt{y^2+13y+36}\right)^2=\left[-(y-6)\right]^2$

$y^2+13y+36=y^2-12y+36$

$y^2-y^2+13y+12y+36-36=0$

$25y=0$

$\Rightarrow y=0$

Put $y=0$ *in* $x^2+3x=y$, *we get*

$x^2+3x=y$

$x^2+3x=0$

$x(x+3)=0$

Either $x=0$ *or* $x+3=0$

$x=-3$

Thus, Solution set $=\{-3,0\}$

Solved Miscellaneous Exercise. 1

Q1. Multiple Choice Questions:

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) Standard form of quadratic equation is:

(a) $bx+c=0, b\neq 0$ **(b)** $ax^2+bx+c=0, a\neq 0$

(c) $ax^2=bx, a\neq 0$ **(d)** $ax^2=0, a\neq 0$

(ii) The number of terms in a standard quadratic equation $ax^2+bx+c=0$ **is**

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

(iii) The number of methods to solve a quadratic equation is:

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

(iv) The quadratic formula is:

(a) $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **(b)** $\frac{b\pm\sqrt{b^2-4ac}}{2a}$

(c) $\frac{-b\pm\sqrt{b^2+4ac}}{2a}$ **(d)** $\frac{b\pm\sqrt{b^2-4ac}}{2a}$

(v) Two linear factors of x^2-15x+56 are:

(a) (x-7) and (x+8) **(b)** (x+7) and (x-8)

(c) (x-7) and (x-8) **(d)** (x+7) and (x+8)

(vi) An equation which remains unchanged when x is replaced by $\frac{1}{x}$ **is called a/an**

(a) Exponential equation **(b)** Reciprocal equation

(c) Radical Equation **(d)** None of these

(vii) An equation of the type $3^x+3^{2-x}+6=0$ is a/an:

(a) Exponential equation **(b)** Radical Equation

(c) Reciprocal equation **(d)** None of these

(viii) The solution set of equation $4x^2$-16=0 is

(a) $\{\pm 4\}$ **(b)** $\{4\}$

(c) $\{\pm 2\}$ **(d)** ± 2

(ix) An equation of the form $2x^4-3x^3+7x^2-3x+2=0$ is called a/an

(a) Reciprocal equation **(b)** Radical Equation

(c) Exponential equation **(d)** None of these

Answers:

(i)	b	**(ii)**	c	**(iii)**	c	**(iv)**	a	**(v)**	c

(vi)	b	**(vii)**	a	**(viii)**	c	**(ix)**	a		

Q2. Write short answers of the following question.

i. $x^2+2x-2=0$

Answer:

$x^2+2x-2=0$

Compare it with

$ax^2+bx+c=0$

Here $a=1, b=2, c=-2$

By using quadratic equation, we get

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-2\pm\sqrt{(2)^2-4(1)(-2)}}{2(1)}$$
$$x=\frac{-2\pm\sqrt{4+8}}{2}$$
$$x=\frac{-2\pm\sqrt{12}}{2}$$
$$x=\frac{-2\pm 2\sqrt{3}}{2}$$
$$x=\frac{2(-1\pm\sqrt{3})}{2}=-1\pm\sqrt{3}$$

Thus, the solution set $=\{-1\pm\sqrt{3}\}$

ii. Solve by factorization $5x^2=15x$

Answer:

$5x^2=15x$
$5x^2-15x=0$
$5x(x-3)=0$

Either $5x=0$ *or* $x-3=0$
$x=0$ $x=3$

Thus, the solution set $=\{0,3\}$

iii. Write in standard form $\frac{1}{x+4}+\frac{1}{x-4}=3$

Solution:

$\frac{1}{x+4}+\frac{1}{x-4}-3$ $\frac{x-4+x+4}{(x+4)(x-4)}=3$

$\frac{2x}{x^2-16}=3$ $3(x^2-16)=2x$

$3x^2-48-2x=0$

$3x^2-2x-48=0$

iv. Write the names of the method for solving a quadratic equation.

Answer: Solution of quadratic equations:

To find solution set of a quadratic equation, following methods are used:

(i) Factorization **(ii)** Completing square

(iii) Quadratic Formula

v. $Solve\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$

Answer:

$\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$

Taking square root on both sides, we get

$2x-\frac{1}{2}=\pm\frac{3}{2}$

$2x=\frac{1}{2}\pm\frac{3}{2}$

Either $2x=\frac{1}{2}+\frac{3}{2}$ *or* $2x=\frac{1}{2}-\frac{3}{2}$

$2x=2$ $2x=-1$

$x=1$ $x=-\frac{1}{2}$

Thus, the solution set= $\left\{-\frac{1}{2},1\right\}$

vi. $Solve\ \sqrt{3x+18}=x$

Answer:

$\sqrt{3x+18}=x$

Taking square root on both sides, we get

$3x+18=x^2$

$x^2-3x-18=0$

$x^2-6x+3x-18=0$

$x(x-6)+3(x-6)=0$

$(x-6)(x+3)=0$

Either $x-6=0$ *or* $x+3=0$

$x=6$ $x=-3$

Thus, the solution set= $\{-3,6\}$

vii. Define quadratic equation.

Ans: Quadratic Equation:

An equation, which contains the square of the unknown (variable) quantity, but no higher power, is called a **quadratic equation** or an equation of the second degree.

viii. Define reciprocal equation.

Ans: Reciprocal equations of the type:

$$a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)=c=0\ or\ ax^4+bx^3+cx^2+bx+a=0$$

An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$.

Replacing x by $\frac{1}{x}$ *in* $ax^4-bx^3+cx^2-bx+a=0$

$a\left(\frac{1}{x}\right)^4-b\left(\frac{1}{x}\right)^3+c\left(\frac{1}{x}\right)^2-b\left(\frac{1}{x}\right)+a=0$ which is simplified as

$a-bx+cx^2-by^3+ax^4=0$ we get the same equation.

Thus $ax^4-bx^3+cx^2-bx+a=0$ is a reciprocal equation.

ix. Define exponential equation.

Ans: Exponential equation:

In exponential equations, variable occurs in exponent.

For example, $5^{1+x}+5x^{1-x}=26$.

x. Define radical equation.

Ans: Radical equations:

An equation involving expression under the radical sign is called a **radical equation**.

e.g., $\sqrt{x+3}=x+1$ *and* $\sqrt{x-1}=\sqrt{x-2}+1$

Q3. Fill in the blanks:

(i) The standard form of quadratic equation is _____.

(ii) The number of methods to solve a quadratic equation are _____.

(iii) The name of the method to derive a quadratic formula is _____.

(iv) The solution of the equation $ax^2+bx+c=0, a\neq 0$ is _____.

(v) The solution set of $25x^2-1=0$ is _____.

(vi) An equation of the form $2^{2x}-3.2^x+5=0$ is called a/an _____ equation.

(vii) The solution set of the equation $x^2-9=0$ is _____.

(viii) An equation of the type $x^4+x^3+x^2+x+1=0$ called a/an _____.

(ix) A root of an equation, which do not satisfy the equation is called _____root.

(x) An equation involving impression of the variable under _____is called radical.

Answer

(i)	$ax^2+bx+c=0$	**(ii)**	3	**(iii)**	Completing Square
(iv)	$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	**(v)**	$\left\{\pm\frac{1}{5}\right\}$	**(vi)**	Exponential

(vii)	$\{\pm 3\}$	**(viii)**	Reciprocal	**(ix)**	Extraneous
(x)	Radical Sign				

Summary

- ✓ An equation, which contains the square of the unknown (variable) quantity, but no higher power, is called a quadratic equation or an equation of the second degree.
- ✓ A second-degree equation in one variable x, $ax^2+bx+c=0$
- ✓ Where $a\neq 0$ and a, b, care real numbers, is called the general or standard form of a quadratic equation.
- ✓ An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$.
- ✓ In exponential equations, variable occur in exponents.
- ✓ An equation involving expression under the radical sign is called a radical equation.
- ✓ Quadratic formula for $ax^2+bx+c=0$, $a\neq 0$ *is* $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.
- ✓ Any quadratic equation is solved by the following three methods.
- ✓ **(i)** Factorization **(ii)** Completing square **(iii)**Quadratic formula