Exercise 7.1

1- Write the first three terms of the following:

(i) $a_n = n + 3$ (ii) $a_n = (-1)^n n^3$ (iii) $a_n = 3n + 5$

(iv) $a_n = \frac{n+1}{2n+5}$ (v) $a_n = \frac{1}{(2n-1)^2}$ (vi) $a_n = n + 3 = 2$

(vii) $a_n = \frac{1}{3^n}$ (viii) $a_n = 3n - 5$ (ix) $a_n = (n+1)a_{n-1}, a_1 = 1$

Solution:(i)

$a_n = n + 3$

Putting values $n = 1, 2, 3$

$a_1 = 1 + 3 = 4$

$a_2 = 2 + 3 = 5$

$a_3 = 3 + 3 = 6$

First three terms 4,5,6

Solution:(ii)

$a_n = (-1)^n n^3$

Putting values $n = 1, 2, 3$

$a_1 = (-1)^1(1)^3$

$= (-1)(1) = -1$

$a_2 = (-1)^2(2)^3$

$= (+1)(8) = 8$

$= (+1)(8) = 8$

$a_3 = (-1)^3(3)^3$

$= (-1)(27) = -27$

First three terms $-1, 8-27$

Solution:(iii)

Putting $n = 1, 2, 3$

$$a_1 = 3(1) + 5$$
$$= 3 + 5 = 8$$
$$a_2 = 3(2) + 5$$
$$= 6 + 5 = 11$$
$$a_3 = 3(3) + 5$$
$$= 9 + 5 = 14$$

First three terms are *8, 11, 14.*

Solution:(iv)

$$a_n = \frac{n+1}{2n+5}$$

Putting $n = 1, 2, 3$

$$a_1 = \frac{1+1}{2(1)+5} = \frac{2}{2+5} = \frac{2}{7}$$

$$a_2 = \frac{2+1}{2(2)+5} = \frac{3}{4+5} = \frac{3}{9}$$

$$a_3 = \frac{3+1}{2(3)+5} = \frac{4}{6+5} = \frac{4}{11}$$

First three terms are $\frac{2}{7}, \frac{3}{9}, \frac{4}{11}$

Solution:(v)

$$a_n = \frac{1}{(2n-1)^2}$$

Putting $n = 1, 2, 3$

$$a_1 = \frac{1}{\{2(1)-1\}^2} = \frac{1}{(2-1)^2} = \frac{1}{(1)^2} = 1 \quad (i)$$

$$a_2 = \frac{1}{\{2(2)-1\}^2} = \frac{1}{(4-1)^2} = \frac{1}{(3)^2} = \frac{1}{9} \quad (i$$

$$a_3 = \frac{1}{\{2(3)-1\}^2} = \frac{1}{(6-1)^2} = \frac{1}{(5)^2} = \frac{1}{25} \quad (ii$$

First three terms are $1, \frac{1}{9}, \frac{1}{25}$

Solution:(vi)

$$a_n = n + 3$$

Putting $n = 1, 2, 3$

$$a_1 = 1 + 3 = 4$$
$$a_2 = 2 + 3 = 5$$
$$a_3 = 3 + 3 = 6$$

First three terms are 4, 5, 6

Solution:(vii)

$$a_n = \frac{1}{3^n}$$

Putting $n = 1, 2, 3$

$$a_1 = \frac{1}{3^1} = \frac{1}{3}$$

$$a_2 = \frac{1}{3^2} = \frac{1}{9}$$

$$a_3 = \frac{1}{3^3} = \frac{1}{27}$$

First three terms are $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$

Solution:(viii)

$$a_n = 3n - 5$$

Putting $n = 1, 2, 3$

$$a_1 = 3(1) - 5 = 3 - 5 = -2$$

$a_2 = 3(2) - 5 = 6 - 5 = 1$

$a_3 = 3(3) - 5 = 9 - 5 = 4$

First three terms are $-2, 1, 4$

Solution:(ix)

$a_n = (n + 1)$

Putting $n = 2, 3$

$a_1 = 1$

$a_2 = 2 + 1 = 3$

$a_3 = 3 + 1 = 4$

First three terms are $1, 3, 4$

2- *Find the terms indicated in the following sequences.*

(i) $2,6,11,17,\ldots a_8$ *(ii)* $1,3,12,60,\ldots a_7$ *(iii)* $1,\frac{1}{3},\frac{1}{9},\frac{1}{27},\ldots a_6$

(iv) $-1,1,3,5,\ldots,a_9$ *(v)* $\frac{1}{3},\frac{2}{5},\ldots,a_5$ *(vi)* $1,-3,5,-7,\ldots,a_9$

2(i) 2, 6, 11, 17, ... a_8

Solution:

$a_1 = 2$

$a_2 = 6$

$a_3 = 11$

$a_4 = 17$

$a_2 - a_1 = 6 - 2 = 4$

$a_3 - a_2 = 11 - 6 = 5$

$a_4 - a_3 = 17 - 11 = 6$

$a_5 - a_4 = 24 - 17 = 7$

$a_6 - a_5 = 32 - 24 = 8$

$a_7 - a_6 = 41 - 32 = 9$

$a_8 - a_7 = 51 - 41 = 10$

$$a_8 = 51$$

2(ii) **1, 3, 12, 60, ... a_7**

Solution:

$$a_1 = 1$$
$$a_2 = 3 \quad = 1 \times 3$$
$$a_3 = 12 \quad = 3 \times 4$$
$$a_4 = 60 \quad = 12 \times 5$$
$$a_5 = 360 \quad = 60 \times 6$$
$$a_6 = 2520 \quad = 360 \times 7$$
$$a_7 = \quad = 2520 \times 8 = 20160$$

2(iii) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots a_6$

Solution:-

$$a_1 = 1$$
$$a_2 = \frac{1}{3} \quad = 1 \times \frac{1}{3}$$
$$a_3 = \frac{1}{9} \quad = \frac{1}{3} \times \frac{1}{3}$$
$$a_4 = \frac{1}{27} \quad = \frac{1}{9} \times \frac{1}{3}$$
$$a_5 = \frac{1}{81} \quad = \frac{1}{27} \times \frac{1}{3}$$
$$a_6 = \frac{1}{243} \quad = \frac{1}{81} \times \frac{1}{3} = \frac{1}{243}$$

2(iv) **–1, 1, 3, 5, ... a_9**

Solution:

$a_1 = -1$

$a_2 = 1$

$a_3 = 3$

$a_4 = 5$

$a_2 - a_1 = 1 - (-1)$

$= 1 + 1 = 2$

$a_3 - a_2 = 3 - 1 = 2$

$a_4 - a_3 = 5 - 3 = 2$

$a_5 = 5 + 2 = 7$

$a_6 = 7 + 2 = 9$

$a_7 = 9 + 2 = 11$

$a_8 = 11 + 2 = 13$

$a_9 = 13 + 2 = 15$

2(v) $\frac{1}{3}, \frac{2}{5}, \ldots, a_5$

Solution:

$\frac{1}{3}, \frac{2}{5}, \ldots, a_5$

$a_1 = \frac{1}{3}$

$a_2 = \frac{2}{5}$

$a_3 = \frac{3}{7}$

$a_4 = \frac{4}{9}$

$a_5 = \frac{5}{11}$

2(vi) ***1, -3, 5, -7, ..., a_9***

Solution:

$a_1 = 1$

$a_2 = -3$

$a_3 = 5$

$a_4 = -7$

$a_5 = 9$

$a_6 = -11$

$a_7 = 13$

$a_8 = -15$

$a_9 = 17$

3- Find the next four terms of the following sequences.

(i) 12,16,21,27,... (ii) 1,3,7,15,31,... (iii) -1,2,12,40,...
(iv) 9,11,14,17,19,22... (v) 4,8,12,16,... (vi) -2,0,2,4,6,8,10,...

3(i) 12, 16, 21, 27,...

Solution:

$16 - 12 = 4$
$21 - 16 = 5$
$27 - 21 = 6$
$34 - 27 = 7$
$42 - 34 = 8$
$51 - 42 = 9$
$61 - 51 = 10$

$27 + 7 = 34$
$34 + 8 = 42$
$42 + 9 = 51$
$51 + 10 = 61$
34,42,51,61

3(ii) 1,3,7,15,31,...

Solution:

$3 - 1 = 2$
$7 - 3 = 4$
$15 - 7 = 8$
$31 - 15 = 16$
$63 - 31 = 32$
$127 - 63 = 64$

$31 + 32 = 63$
$63 + 64 = 127$
$127 + 128 = 255$

$225 - 127 = 128$

$511 - 255 = 256$ $\qquad$ $255 + 256 = 511$

1.$(1+2).(3+2^2)+(7+2^3)+(15+2^4)$
$+ (31+2^5)+(63+2^6)+(127+2^7)$....

3(iii) -2,2,12,40,.....

Solution:

$2-(-2)=2+2=4=3^1+1$
$12-2=10=3^2+1$
$40-12=28=3^3+1$

$112-40=82=3^4+1$
$356-112=244=3^5+1$
$1086-356=730=3^6+1$
$3274-1086=2188=3^7+1$

$112, 356, 1086, 3274$

3(iv) 9, $\underline{9}$ $\underline{11}$ 14 17 $\underline{19}$ 22 25 $\underline{27}$ 30 33 $\quad$ $\underline{3}$ $\underline{3}$ $\quad$ $\underline{3}$ $\underline{3}$ $\quad$ $\underline{2}$ $\quad$ $\underline{2}$

3(v) 4, 8, 12, 16,.....

Solution:

$8 - 4 = 4$
$12 - 8 = 4$
$16 - 12 = 4$
$20 - 16 = 4$
$24 - 20 = 4$
$28 - 24 = 4$
$32 - 28 = 4$

3(vi) -2, 0, 2, 4, 6, 8, 10

Solution:

$0-(-2) = 0 + 2 = 2$
$2-0 = 2$
$4-2 = 2$
$6-4 = 2$
$8-6 = 2$
$10-8 = 2$
$12-10 = 2$
$14-12 = 2$
$16-14 = 2$
$18-16 = 2$

-2, 0, 2, 4, 6, 8, 10, ...

Exercise 7.2

1- ***Find the speçified term of the following A.P.***

(i) 3, 7, 11, ...61st term. *(ii)* $-4, -7, -10, \ldots a_{19}$

(iii) 6, 4, 2, ...45th term. *(iv)* $9, 14, 19, \ldots a_{14}$

(v) $11, 6, 1, \ldots a_{18}$

Solution:-

61^{st} 3, 7, 11,.....

Here $a_1 = 3$

$d = 7 - 3 = 4$

$a_{61} = ?$

$n = 61$

We know that $a_n = a+(n-1)d$

Putting values a, d and n in the formula

$$a_{61} = 3+(61-1)(4)$$
$$= 3 + 60 \times 4$$
$$= 3+240$$
$$= 243$$

1(ii) -4, -7, -10, ... a_{19}

Solution:

Here $a = a_1 = -4$

$d = (-7) - (-4)$

$= -7 + 4 = -3$

$a_{19} = ?$

$n = 19$

We know that $a_n = a+(n-1)d$

Putting values of a, d and n in the formula

$$a_{19} = -4+(19-1)(-3)$$
$$= -4+18(-3)$$
$$= -4 - 54$$
$$= -58$$
$$a_{19} = -58$$

Solution:

Here $a = a_1 = 4$

$d = 4 - 6 = -2$

$a_{45} = ?$

$n = 45$

We know that $a_n = a+(n-1)d$

Putting values of a, d and n in the formula

$$a_{45} = 6+(45-1)(-2)$$
$$= 6+(44)(-2)$$
$$= 6-88$$
$$= -82$$

Hence 45th term is -82

1(iv) *9, 14, 19, ...a_{14}*

Solution:

Here $a = a_1 = 9$

$d = 14 - 9 = 5$

$a_{14} = ?$

$n = 14$

We know that $a_n = a+(n-1)d$

Putting values of a, d and n in the formula

$$a_{14} = 9+(14-1)(5)$$
$$= 9+(13)(5)$$
$$= 9+65$$
$$= 74$$

1(v) *11, 6, 1, ...a_{18}*

Solution:

$a = a_1 = 11$

$d = 6 - 11 = -5$

$a_{18} = ?$

$n = 18$

We know that $a_n = a+(n-1)d$

Putting values in a, d and n in the formula

$$a_{18} = 11+(18-1)(-5)$$
$$= 11+(17)(-5)$$

$= 11 - 85$

$= -74$

2- ***Find the missing element using the formula of A.P***

$a_n = a + (n - 1)\, d$

(i) $a = 2,\ a_n = 402,\ n = 26$

(ii) $a_n = 81,\ d = -3,\ n = 18$

(iii) $a = 5,\ a_n = 61,\ n = 15$

(iv) $a = 16,\ a_n = 0,\ d = -\frac{1}{4}$

(v) $a = 10,\ a_n = 400,\ d = 5$

(vi) $a_n = 261,\ d = 4,\ n = 18$

Solution: (i) $a_n = a + (n - 1)\, d$

$a = 2$

$a_n = 402$

$n = 26$

$d = ?$

Here we find *d?*

$a_n = a + (n - 1)\, d$

Putting values of $a,\ a_n,\ n$

$402 = 2 + (26 - 1)\, d$

$402 = 2 + 25\, d$

$2 + 25\, d = 402$ *OR*

$25\, d = 402 - 2$

$25\, d = 400$

Hence $d = \frac{400}{25} = 16$

Solution: (ii) $a_n = a + (n - 1)\, d$

$a_n = 81$

$d = -3$

$n = 18$

Here ***a*** is unknown

$a_n = a + (n - 1)\, d$

Putting values of $a,\ a_n,\ n$

$$81 = a + (18 - 1)(-3)$$
$$81 = a + (17)(-3)$$
$$81 = a - 51$$
$$a - 51 = 81 \quad OR$$

Therefore $a = 81 + 51$
$$= 132$$

Solution: (iii) $a_n = a + (n - 1)\,d$ *formula*

$a_n = 5$ *and*

$a_n = 61$

$n = 15$

Here d is unknown

$$a_n = a + (n - 1)\,d$$

Putting values of a, a_n, and n.

$$61 = 5 + (15 - 1)\,d$$
$$61 = 5 + 14\,d$$
$$5 + 14\,d = 61 \quad OR$$
$$14\,d = 61 - 5$$
$$14\,d = 56 \quad Therefore$$
$$d = \frac{56}{14}$$
$$d = 4$$

Solution: (iv) $a_n = a + (n - 1)\,d$ *formula*

$a = 16$ *and*

$a_n = 0$

$$d = -\frac{1}{4}$$

Here n is unknown.

$$a_n = a + (n - 1)\,d$$

Putting values of a, a_n and d.

$$0 = 16 + (n - 1)\left(-\frac{1}{4}\right)$$

Multiplying by 4.

$$0 = 64 + (n - 1)(-1)$$

$$0 = 64 - n + 1$$
$$n = 64 + 1 \qquad OR$$
$$n = 65 \qquad OR$$

Solution: ***(v)*** $a_n = a + (n - 1)\,d$ formula

$$a = 10 \qquad and$$
$$a_n = 400$$
$$d = 5$$

Here n is unknown

$$a_n = a + (n - 1)\,d$$

Putting values of a, an and d.

$$400 = 10 + (n - 1)(5)$$
$$400 - 10 = 5n - 5$$
$$5n - 5 = 390 \qquad OR$$
$$5_n = 390 + 5$$
$$5_n = 395$$
$$n = \frac{395}{5}$$
$$= 79$$

Solution: ***(vi)*** $a_n = a + (n - 1)\,d$ fromula

$$a_n = 261 \qquad and$$
$$d = 4$$
$$n = 18$$

Here a is unknown

$$a_n = a + (n - 1)\,d$$

Putting values of a_n, d and n.

$$261 = a + (18 - 1)(4)$$
$$= a + 17 \times 4$$
$$261 = a + 68$$
$$261 - 68 = a$$
$$193 = a \qquad OR$$
$$a = 193$$

3- ***Find the 15^{th} terms of an A.P where the 3^{rd} term is 8 and the common difference is $\frac{1}{3}$.***

Solution:

$$a_3 = 8$$

$$d = \frac{1}{3}$$

$$a_{15} = ?$$

First we find a

$$a_n = a + (n-1)\,d$$

$$a_3 = a + (3-1)\,d$$

$$8 = a + (2)\left(\frac{1}{3}\right)$$

$$8 = a + \left(\frac{2}{3}\right)$$

$$a = 8 - \frac{2}{3} \quad OR$$

$$a = 8 - \frac{2}{3}$$

$$a = \frac{24-2}{3} = \frac{22}{3} \quad (A)$$

$a_n = a + (n-1)\,d$ formula

$$a_{15} = \frac{22}{3} + (15-1)\left(\frac{1}{3}\right) \quad \text{from A}$$

$$= \frac{22}{3} + (14)\frac{1}{3}$$

$$= \frac{22}{3} + \frac{14}{3}$$

$$= \frac{22+14}{3}$$

$$= \frac{36}{3} = 12$$

4. Which term of an A.P 6, 2, −2..... is− 146?

Solution:

Here term ..... 6, 2, -2, ...

$a = 6$

$d = 2 - 6 = -4$

$an = -146$

$n = ?$

$a_n = a + (n-1)\,d$ *formula*

Putting values a, a_n and d in formula.

$-146 = 6 + (n-1)(-4)$ *formula*

$-146 = 6 - 4n + 4$

$4n = 6 + 4 + 146$ *OR*

$4n = 156$

$$n = \frac{156}{4}$$

$n = 39$

5. Which term of an A.P 5,2,–1 .... is −134?

Solution:

$a = 5$

$d = 2 - 5 = -3$

$n = -118$

$n = ?$

$a_n = a + (n-1)\,d$ formula

Putting values of a, d and n.

$-118 = 5 + (n-1)(-3)$

$-118 = 5 + 3n + 3$

$3n = 5 + 3 + 118$ Or

$3n = 126$

$$n = \frac{126}{3}$$

$n = 42$

6- ***How many terms are there in an A.P, in which $a_1 = a = 11$, $a_n = 68$, $d = 3$.***

Solution:- ***Here term.....***

$d = 3$

$a_n = 68$

$a = 11$

$n = ?$

$a_n = a + (n-1)\,d$ *formula*

Putting values of a, a_n and d in the formula.

$68 = 11 + (n-1)(3)$

$68 = 11 + 3n - 3$

$68 = 8 + 3n$

$8 + 3n = 68$

$3n = 68 - 8$ Or

$3n = 60$

$$n = \frac{60}{3}$$

$n = 20$

7- ***Find the 11^{th} term of a_n A.P $2 - x$, $3 - 2x$, $4 - 3x$.....***

Solution:- ***In term of a_n A.P $2 - x$, $3 - 2x$, $4 - 3x$.....***

$a = 2 - x$

$d = (3 - 2x) - (2 - x)$

$d = 3 - 2x - 2 + x = 1 - x$

$a_{11} = ?$

$\therefore n = 11$

$a_n = a + (n-1)\,d$ ***formula***

$a_{11} = (2 - x) + (11 - 1)(1 - x)$

$$= 2 - x + 10(1 - x)$$
$$= 2 - x + 10 - 10x$$

Hence $a_{11} = 12 - 11x$

8- Find the n^{th} term of an A.P, where $a_{n-5} = 3n + 9$

$a_n = ?$

Solution:-

$a_{n-5} = 3n + 9$ We put $n + 5$ in place of n.

$a_{n+5-5} = 3(n + 5) + 9$

$a_n = 3n + 15 + 9$

$a_n = 3n + 24$

9- Find the n^{th} term of an A.P: $\left(\frac{3}{4}\right)^2, \left(\frac{3}{7}\right)^2, \left(\frac{3}{10}\right)^2 \ldots\ldots$

Solution:

$a = 4$

$d = 7 - 4 = 3$

$a_n = ?$

$a_n = a + (n - 1)\,d$ ***formula***

$= 4 + (n - 1)(3)$

$= a + 3n - 3$

$= 4 + 3n - 3$

$= 3n + 1$

The nth term of the sequence $\left(\frac{3}{3n+1}\right)^2$

10- If the n^{th} term of an A.P is $3n - 5$. Find the A.P

Solution:-

$a_n = 3n - 5$

Putting $n = 1, 2, 3, 4, \ldots.$

$a_1 = 3(1) - 5 = 3 - 5 = -2$

$a_2 = 3(2) - 5 = 6 - 5 = 1$

$a_3 = 3(3) - 5 = 9 - 5 = 4$

$a_4 = 3(4) - 5 = 12 - 5 = 7$

$-2, 1, 4, 7\ldots\ldots$ *is required*

Exercise 7.3

1- Find A.M between:

(i) $-3, 7$ (ii) $x-1, x+7$

(iii) $\sqrt{7}, 3\sqrt{7}$ (iv) x^2+x+1, x^2-x+1

Solution:-(i) $-3, 7$

Here $a = -3$

$b = 7$

(A. M) $A = \dfrac{-3+7}{2}$

Here $A = \dfrac{4}{2} = 2$

Solution:-(ii) $x-1, x+7$

Here $a = x-1$

$b = x+7$

(A. M) $A = \dfrac{a+b}{2}$

$= \dfrac{(x-1)+(x+7)}{2}$

$= \dfrac{x-1+x+7}{2}$

$= \dfrac{2x+6}{2}$

$= \dfrac{2(x+3)}{2}$

$A = x+3$

Solution:-(iii) $\sqrt{7}, 3\sqrt{7}$

Here $a = \sqrt{7}$

$b = 3\sqrt{7}$

(A.M) $A = \dfrac{a+b}{2}$

$= \dfrac{\sqrt{7}+3\sqrt{7}}{2}$

$= \dfrac{\sqrt{7}(1+3)}{2}$

$= \dfrac{\sqrt{7}\ \ 4}{2}$

Hence $A = 2\sqrt{7}$

Solution:-(iv) $x^2 + x + 1, x^2 - x + 1$

Here $a = x^2 + x + 1$

$b = x^2 - x + 1$

(A.M) $A = \dfrac{a+b}{2}$

$= \dfrac{(x^2+x+1)+(x^2-x+1)}{2}$

$= \dfrac{x^2+x+1+x^2-x+1}{2}$

$= \dfrac{2x^2+2}{2}$

$= \dfrac{2(x^2+1)}{2}$

Hence $A = x^2 + 1$

2- ***If 3 and 6 are two A.Ms between a and b, find a and b.***

Solution:-

Here A. Ms between $a, 3, 6, b$

$$d = 6 - 3 = 3$$

$$b = a_4 = a_3 + d$$

$$= 6 + 3$$

$$b = 9$$

and

$$a = a_2 - d$$

$$= 3 - 3$$

$$a = 0$$

$$a = 0, b = 9$$

We can also find a and b using this method.

$$a + d = 3$$

$$a + 3 = 3$$

$$a = 3 - 3$$

$$a = 0$$

$$6 + d = b$$

and

$$6 + 3 = b$$

$$9 = b$$

$$b = 9$$

3- ***Find three A.Ms between 11 and 19.***

Solution:-

Here

$$a_1 = 11$$

$$a_5 = 19$$

$$n = 5$$

Now

$$a_n = a + (n - 1)d$$

$$a_5 = a + (5 - 1)d$$

Now

$$a_n = 11 + 4d$$

$$19 = 11 + 4d$$

$$19 - 11 = 4d$$

$$8 = 4d$$

$$d = 2$$

$$A_1 = a + d$$

$$= 11 + 2 = 13$$

$$A_2 = A_1 + d$$

$= 13 + 2 = 15$

$A_3 = A_2 + d$

$= 15 + 2 = 17$

required three A. Ms are 13, 15, 17.

4- *Find three A.Ms between* $2\sqrt{3}$ *and* $6\sqrt{3}$

Solution:-

Let A_1, A_2, A_3 be three A.Ms between $2\sqrt{3}$ and $6\sqrt{3}$. Such that $2\sqrt{3}, A_1, A_2, A_3, 6\sqrt{3}$ is an A.P.

$a_1 = 2\sqrt{3}$

$a_5 = 6\sqrt{3}$

$n = 5$

Now $a_n = a + (n - 1)d$

$6\sqrt{3} - 2\sqrt{3} = 4d$

Or $4d = 6\sqrt{3} - 2\sqrt{3}$

$= \sqrt{3}(6 - 2)$

$4d = \sqrt{3}(4)$

$d = \sqrt{3}$

$A_1 = a + d$

Now $= 2\sqrt{3} + \sqrt{3}$

$\sqrt{3} + (2 + 1)$

$A_1 = 3\sqrt{3}$

and $A_2 = a + 2d$

$= 2\sqrt{3} + 2\sqrt{3}$

$= \sqrt{3}(2 + 2) = 4\sqrt{3}\ A_2$

$A_3 = a + 3d$

$$= 2\sqrt{3} + 3\sqrt{3}$$

$$= \sqrt{3}(2+3)$$

$$A_3 = 5\sqrt{3}$$

$$3\sqrt{3}, 4\sqrt{3}, 5\sqrt{3}$$

Thus $3\sqrt{3}, 4\sqrt{3}, 5\sqrt{3}$ are the required three A.Ms between $6\sqrt{3}$ and $2\sqrt{3}$.

5- Find six A.Ms between 5 and 8.

Solution:-

Let $A_1, A_2, A_3, A_4, A_5, A_6$ be six A.Ms between 5 and 8. Such that $5, A_1, A_2, A_3, A_4, A_5, A_6, 8$ *is an A.P.*

Here $a_1 = 5$

$a_8 = 8$

$n = 8$

$a_n = a + (n-1)d$

$8 = 5 + (8-1)d$

$8 - 5 = 7d$

$3 = 7d$

$7d = 3$ Or

$d = \frac{3}{7}$ Or

Now $A_1 = a + d$

$$= 5 + \frac{3}{7} = \frac{38}{7}$$

$A_2 = a + 2d$

$$= 5 + 2 \times \frac{3}{7}$$

$$= 5 + \frac{6}{7} = \frac{41}{7}$$

$A_3 = a + 3d$

$$= 5 + 3 \times \frac{3}{7}$$

$$= 5 + \frac{9}{7} = \frac{44}{7}$$

$$A_4 = a + 4d$$

$$= 5 + 4 \times \frac{3}{7}$$

$$= 5 + \frac{12}{7} = \frac{47}{7}$$

$$A_5 = a + 5d$$

$$= 5 + 5\left(\frac{3}{7}\right)$$

$$= 5 + \frac{15}{7} = \frac{50}{7}$$

$$A_6 = a + 6d$$

$$= 5 + 6\left(\frac{3}{7}\right)$$

$$= 5 + \frac{18}{7}$$

$$= \frac{35 + 18}{7} = \frac{53}{7}$$

Thus $\frac{38}{7}, \frac{41}{7}, \frac{44}{7}, \frac{47}{7}, \frac{50}{7}, \frac{53}{7}$ are six required A.Ms.

between 5 and 8.

6. *Find seven A.Ms between 8 and 12.*

Solution:-

Let $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ be seven required A.Ms, between 8 and 12.

Such that $8, A_1, A_2, A_3, A_4, A_5, A_6, A_7, 12$ is an A. P.

$$a_1 = 8$$
$$a_9 = 12$$
$$n = 9$$
$$a_n = a + (n-1)d$$
$$12 = 8 + (a-1)d$$
$$12 - 8 = 8d$$
$$4 = 8d$$
$$8d = 4 \qquad \text{Or}$$
$$d = \frac{4}{8}$$
$$d = \frac{1}{2} \qquad \text{Or}$$
$$A_1 = a + d$$
$$= 8 + \frac{1}{2}$$
$$= \frac{16+1}{2} = \frac{17}{2}$$

Now

$$A_2 = 8 + 2d$$
$$= 8 + 2(\frac{1}{2})$$
$$= 8 + \frac{2}{2}$$
$$= \frac{16+2}{2} = \frac{18}{2}$$
$$A_3 = a + 3d$$
$$= 8 + 3(\frac{1}{2})$$
$$= 8 + \frac{3}{2}$$

$$= \frac{16+3}{2} = \frac{19}{2}$$

$A_4 = a + 4d$

$$= 8 + 4(\frac{1}{2}) \quad = \frac{8}{1} + \frac{4}{2}$$

$$= \frac{16+4}{2} = \frac{20}{2}$$

$A_5 = a + 5d$

$$= 8 + 5(\frac{1}{2})$$

$$= \frac{16+5}{2} = \frac{21}{2}$$

$A_6 = 8 + 6d$

$$= 8 + 6(\frac{1}{2})$$

$$= \frac{16+6}{2} = \frac{22}{2}$$

$A_7 = a + 7d$

$$= 8 + 7(\frac{1}{2})$$

$$= \frac{16+7}{2} = \frac{23}{2}$$

Thus $\frac{17}{2}, \frac{18}{2}, \frac{19}{2}, \frac{20}{2}, \frac{21}{2}, \frac{22}{2}, \frac{23}{2}$, are the required seven A.Ms between 8 and 12.

7- ***If the A.M between 5 and b is 10, then find the value of b.***

Solution:-

a, A, b is an A. P.

$$A = \frac{a+b}{2}$$

Here $a = 5$

$b = b$

$A = 10$

$$A = \frac{a+b}{2}$$

$$10 = \frac{5+b}{2}$$

Or $20 = 5 + b$

Or $5 + b = 20$

$b = 20 - 5$

$b = 15$

8- ***If the A.M between a and 10 is 40, then find the value of "a".***

Solution:-

a, A, b is an A. P.

$$A = \frac{a+b}{2}$$

Here $a = a$

$b = 10$

$A = 40$

$$A = \frac{a+b}{2}$$ and

$$40 = \frac{a+10}{2}$$

$$40 \times 2 = a + 10$$

$$80 = a + 10$$

$$a + 10 = 80$$

$$a = 80 - 10 \qquad \text{Or}$$

$$a = 70$$

9. *If the three A.Ms between a and b are 5,9 and 13, find a and b.*

Solution:-

5, 9 and 13 is an A.P between a and b.

$a, 5, 9, 13, b$ is A. Ms

$$d = 9 - 5 = 4$$

$$= 13 - 9 = 4$$

$$a + d = 5$$

$$a + 4 = 5$$

$$a = 5 - 4$$

$$a = 1 \qquad (i)$$

$$13 + d = b \qquad \text{and}$$

$$13 + 4 = b$$

$$17 = b$$

$$b = 17 \qquad (ii)$$

$$a = 1, b = 17$$

Exerise 7.4

1- Find the 7^{th} term of a G.P 2,8,32.....

Solution: G. P sequence is 2, 8, 32, ...

Given $a = 2$

$$r = \frac{8}{2} = 4$$

$n = 7$

$a_n = ar^{n-1}$ **formula**

$a_7 = ?$

$a_7 = 2(4)^{7-1}$

$= 2(4)^6$

$= 2 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$

$= 2 \times 4096$

$a_7 = 8192$

2- Find the 11^{th} term of a G.P 2,6,18 ....

Solution:- G. P sequence is 2, 6, 18,.....

Given $a = 2$

$$r = \frac{6}{2} = 3$$

$n = 11$

$a_{11} = ?$

$a_n = ar^{n-1}$ **formula**

$a_{11} = 2 \times 3^{11-1}$

$= 2 \times 3^{10}$

$= 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 2 \times 9 \times 9 \times 9 \times 9$

$= 2 \times 81 \times 81 \times 9$

$a_{11} = 118098$

3- ***Find the 6^{th} term of a G.P*** $-\frac{3}{2}, 3, -6.....$

Solution:-

In term of a G. P $-\frac{3}{2}, 3, -6.....$

$$a = -\frac{3}{2}$$

$$r = -\frac{6}{3} = -2$$

$$n = 6$$

$$a_6 = ?$$

$$a_n = ar^{n-1} \qquad \textbf{formula}$$

$$a_6 = \left(-\frac{3}{2}\right)(-2)^{6-1} \quad \text{Now}$$

$$= \left(-\frac{3}{2}\right)(-2)(-2)(-2)(-2)(-2)$$

$$= \left[\left(-\frac{3}{2}\right)(-2)\right][(-2)(-2)][(-2)(-2)]$$

$$= 3 \times 4 \times 4$$

$$= 48$$

4- ***Find the 5^{th} term of a G.P 4, –12, 36....***

Solution:-

In term of a G. P 4, -12, 36,....

$$a = 4$$

$$d = \frac{-12}{4} = -3$$

$$n = 5$$

$a_5 = ?$

$a_n = ar^{n-1}$ **formula**

$a_5 = 4(-3)^{5-1}$

$= 4(-3)^4$

$= 4(-3)(-3)(-3)(-3)$

$= 4 \times 9 \times 9$

$a_5 = 324$

5- Find the missing elements of the G.P

(i) $r = 10,\ a_n = 100,\ a = 1$

(ii) $a_n = 400,\ r = 2,\ a = 25$

(iii) $a = 128,\ r = \frac{1}{2},\ a_n = \frac{1}{4}$

Solution:5(i)

$r = 10$

$a_n = 100$

$a = 1$

$n = ?$

$a_n = ar^{n-1}$ **formula**

Putting values of a ' a_n ' r

$100 = (1)(10)^{n-1}$

$(10)^2 = (10)^{n-1}$

$(10)^2 = (10)^{n-1}$

Therefore $n-1 = 2$

$n = 2 + 1 = 3$

Solution:5(ii)

$a_n = 400$

$r = 2$

$a = 25$

$n = ?$

$a^n = ar^{n-1}$ *formula*

Putting values a ' r ' a_n

$400 = 25(2)^{n-1}$

$$\frac{400}{25} = 2^{n-1}$$

$$16 = 2^{n-1}$$

$$2^4 = 2^{n-1}$$

Therefore $n-1 = 4$

$$n = 4 + 1 = 5$$

Solution:5(iii)

$$a = 128$$

$$r = \frac{1}{2}$$

$$a_n = \frac{1}{4}$$

$$n = ?$$

$a^n = ar^{n-1}$ *Formula*

Putting values a, r, a_n

$$\frac{1}{4} = 128\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{4\times 128} = \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{32\times 4\times 4} = \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{2^2\times 2^2\times 2^2\times 2^3} = \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{2^9} = \left(\frac{1}{2}\right)^{n-1}$$

$$\left(\frac{1}{2}\right)^9 = \left(\frac{1}{2}\right)^{n-1}$$

$$n-1=9$$
$$n=9+1$$

Hence $n=10$

6- *Find the 11^{th} term of a G.P whose 5^{th} term is 9 and common ratio is 2.*

Solution:

$r=2$

$a_5=9$

$a_{11}=?$

$a_n=ar^{n-1}$ *formula*

$a_5=ar^{5-1}=ar^4$ *Putting values of a_5 and r.*

a_5r

$9=a(2)^4$

$9=a\times 16$

$$\frac{9}{16}=a$$

$$a=\frac{9}{16}$$

Now we want to find $a_{11}=ar^{11-1}$

putting values of a and r.

$$a_{11}=\frac{9}{16}\times 2^{10}$$

$$a_{11}=\frac{9}{16}\times 1024$$

$$a_{11}=9\times 64$$

$$=576$$

7- *Find the 13^{th} term of a G.P whose 7^{th} term is 25 and common ratio is 3.*

Solution:

$a_7=25$

$r=3$

$a_{13} = ?$

$a_n = ar^{n-1}$ ***formula***

$a_7 = ar^{7-1} = ar^6$ putting values of a_7 and r.

$25 = a(3)^6$

$25 = a \times 729$

$$\frac{25}{729} = a$$

$$a = \frac{25}{729}$$

$a_{13} = ar^{13-1}$ *Now*

Putting values of a, a_{13}

$$= \frac{25}{729}(3)^{13-1}$$

$$= \frac{25}{729} \times 3^{12}$$

$$= \frac{25}{3^6} \times 3^6 \times 3^6$$

$$= 25 \times 3^6$$

$$= 25 \times 729 = 18225$$

8- ***If a, b, c, d are in G.P, show that, $a-b, b-c, c-d$, are in G.P***

Solution:-

a, b, c, d is G.P *therefore*

$$(A)\quad \frac{b}{a}=\frac{c}{b}=\frac{d}{c}$$

$$\frac{c}{b}=\frac{d}{c} \qquad \text{and}$$

$$\frac{b}{a}=\frac{d}{c}$$

If $a-b, b-c, c-d$ is G.P.

Then $$\frac{b-c}{a-b}=\frac{c-d}{b-c}$$

$$(b-c)(b-c)=(a-b)(c-d)$$

$$b^2-bc-bc+c^2=ac-ad-bc+bd$$

$$(i)\quad b^2=ca \qquad \left(\frac{b}{a}=\frac{c}{b}\right)$$

$$\left(\frac{c}{b}=\frac{d}{c}\right) \quad c^2=bd \quad (ii)$$

$$\left(\frac{b}{a}=\frac{d}{c}\right) \quad bc=ad (iii)$$

$$b^2-bc-bc+c^2$$

$$=ca-bc-ad+bd \quad \text{From } (i),(ii),(iii)$$

$a-b,\ b-c,\ c-d$ is G.P

Second Law:

a, b, c, d is G.P

$$\frac{b}{a}=\frac{c}{b}=\frac{d}{c}$$ Therefore

$$\frac{a}{b}=\frac{b}{c}=\frac{c}{d} \quad (A) \quad \text{Or}$$

Now if $a-b,\ b-c,\ c-d$ *is G.P*

Then $\dfrac{b-c}{a-b}=\dfrac{c-d}{b-c}$ prove

Now $\dfrac{a}{b}=\dfrac{b}{c}$ from A

$$\frac{a}{b}-1=\frac{b}{c}-1$$

$$\frac{a-b}{b}=\frac{b-c}{c}$$

$$\frac{a-b}{b-c}=\frac{b}{c}$$

$$\frac{b-c}{a-b}=\frac{c}{b} \qquad (i)$$

$$\frac{b}{c}=\frac{c}{d}$$

$$\frac{b}{c}-1=\frac{c}{d}-1$$

$$\frac{b-c}{c}=\frac{c-d}{d}$$

$$\frac{b-c}{c-d}=\frac{c}{d}$$

$$\frac{c-d}{b-c}=\frac{d}{c}$$

$$\frac{c-d}{b-c}=\frac{c}{b}\ (ii)\left(\frac{c}{b}=\frac{d}{c}\right) \quad (A)$$

$$\frac{b-c}{a-b}=\frac{c-d}{b-c} \qquad (i),(ii)$$

9- ***Find the n^{th} term of a G.P, if $\frac{a_5}{a_3}=\frac{4}{9}$ and a***

Solution:

$$\frac{a_5}{a_3}=\frac{4}{9}$$

$$\frac{ar^{5-1}}{ar^{3-1}}=\frac{4}{9}$$

$$\frac{r^4}{r^2}=\frac{4}{9}$$

$$r^2=\left(\frac{2}{3}\right)^2$$

$$r=\frac{2}{3} \qquad (A)$$

$$a_2=ar^{2-1} \quad (i)$$

$$a_2=\frac{4}{9} \qquad (ii)$$

$$ar=\frac{4}{9} \qquad \text{from } (i) \text{ and } (ii)$$

$$a\times\frac{2}{3}=\frac{4}{9} \qquad \text{from } (A)$$

$$a=\frac{4}{9}\times\frac{3}{2}$$

$$a=\frac{2}{3}$$

Now $a_n = ar^{n-1}$

$$= \frac{2}{3}\left(\frac{2}{3}\right)^{n-1}$$

$$a_n = \left(\frac{2}{3}\right)^{n-1+1} = \left(\frac{2}{3}\right)^n$$

10- ***Find three consecutive numbers in G.P, whose sum is 26 and their product is 216.***

Solution:-

Three consecutive numbers according to the given condition a, ar, ar^2.

$$a + ar + ar^2 = 26$$

$$(a)(ar)(ar^2) = 216 \quad \text{and}$$

$$a^3r^3 = 216$$

$$(ar)^3 = (6)^3$$

$$ar = 6 \qquad (A)$$

$$a = \frac{6}{r} \qquad \text{Putting (i)}$$

$$a + ar + ar^2 = 26$$

$$a(1 + r + r^2) = 26$$

$$a = \frac{6}{r} \qquad (i)$$

$$(1 + r + r^2) = 26\frac{6}{r}$$

$$6(1 + r + r^2) = 26r$$

$$6 + 6r + 6r^2 - 26r = 0$$

$$6r^2 - 20r + 6 = 0 \quad \text{Dividing by 2}$$

$$3r^2 - 10r + 3 = 0$$

$$3r^2 - r - 9r + 3 = 0$$

$$r(3r - 1) - 3(3r - 1) = 0$$

$$(3r - 1)(r - 1) = 0$$

$$3r-1=0$$
$$3r=1$$
$$r=\frac{1}{3} \quad (ii)$$

and $r-3=0$

$r=3$ then (B)

$ar=6$ Now (A)

$a(3)=6$ Now (B)

$$a=\frac{6}{3}$$
$$a=2 \quad (C)$$

Take 3 or $a=2, r=\frac{1}{3}$

$$a=2$$
$$ar=2\times\frac{1}{3}=\frac{2}{3}$$
$$ar^2=2\times\left(\frac{1}{3}\right)^2=2\times\frac{1}{9}$$
$$=\frac{2}{9}$$

Three consecutive numbers are $2, \frac{2}{3}, \frac{2}{9}$

If $r=3$

Then $a=2$

$ar=2\times3=6$

$ar^2=2\times3^2=2\times9=18$

Consecutive number 2, 6, 18

11- ***Find the 30th term of a G.P*** $x, 1, \frac{1}{x}....$

Solution:

G. P is $x, 1, \frac{1}{x}....$

$$a = x \qquad (i)$$

$$r = \frac{1}{x} \qquad (ii)$$

$$a_n = ar^{n-1}$$

$$a_{30} = ar^{30-1}$$

from (i) and (ii)

$$= x \times \left(\frac{1}{x}\right)^{29}$$

$$= x \times x^{-29}$$

$$= x^{1-29}$$

$$= x^{-28}$$

$$= x^{\frac{1}{28}}$$

12- Find the p^{th} term of a G.P x, x^3, x^5,.....

Solution:-

G.P is x, x^3, x^5,.....

Therefore $a = x$

$$r = \frac{x^3}{x}$$

$$x^2 =$$

$a_n = ar^{n-1}$ **formula**

$$a_p = (x)(x^2)^{p-1}$$

$$= (x)x^{2(p-1)}$$

$$= x\, x^{2p-2}$$

$$= x^{2p-2+1}$$

$$a_p = x^{2p-1}$$

Exercise 7.5

G.M between a, b is G. $G = \pm\sqrt{ab}$

1- ***Find G.M between: (i) 9 and 5 (ii) 4 and 9 (iii) –2 and –8.***

(i) 9 and 5 (ii) 4 and 9

(iii) -2 and -8

Solution:(i)

Given 9 and 5

$$G = \pm\sqrt{ab}$$
$$= \pm\sqrt{9\times 5}$$
$$= \pm 3\sqrt{5}$$

Solution:- (ii)

4 and 9

$$G = \pm\sqrt{ab}$$
$$= \pm\sqrt{(-4)(9)}$$
$$= \pm\sqrt{36}$$
$$= \pm 6$$

Solution:- (iii)

-2 and -8

$$G = \pm\sqrt{ab}$$
$$= \pm\sqrt{(-2)(-8)}$$
$$= \pm\sqrt{16}$$
$$= \pm 4$$

2- ***Insert two G.Ms between:(i) 1 and 8 (ii) 3 and 81***

(i) 1 and 8 (ii) 3 and 81

Solution:-

Let G_1 and G_2 be the two G.Ms between *1* and *8* such that *1*, G_1, G_2 *8* is a *G.P.*

Here $a = 1$

$n = 4$

$a_4 = 8$

Since $a_n = ar^{n-1}$

$a_4 = ar^{4-1}$

$8 = ar^3$

$8 = 1 \times r^3$

$2^3 = r^3$

$r = 2$

Thus $G_1 = ar$

$= 1 \times 2 = 2$

$G_2 = ar^2$

$= 1 \times 2^2$

$= 1 \times 4 = 4$

Solution:- (ii) 3 and 81

Let G_1 and G_2 be the two GMs between 3 and 81, such that 3, G_1, G_2, 81 is a G.P.

$n = 4$

Here $a = 3$

$n = 4$

$a_4 = 81$

Since $a_n = ar^{n-1}$

$a_4 = ar^{4-1}$

$81 = 3r^3$

$r^3 = \frac{81}{3} = 27$

$r^3 = 3^3$

$r = 3$

Thus $G_1 = ar$

$= 3 \times 3$

$= 9$

$G_2 = ar^2$

$= 3(3)^2$

$= 3 \times 9 = 27$

3- ***Insert three G.Ms between:(i) 1 and 16 (ii) 2 and 32***

(i) 1 and 16

(ii) 2 and 32

Solution:-

Let G_1, G_2 and G_3 be the three G.Ms between 1 and 16, that are required. G.Ms 1, G_1, G_2, G_3 16 is a G. P.

Here $a = 1$

$n = 5$

$a_5 = 16$

Since $a_n = ar^{n-1}$

$a_5 = (1)r^{5-1}$

$16 = r^4$

$r^3 = r^4$

Thus $2^4 = r^4$

$r = 2$

$G_1 = ar$

$= 1 \times 2 = 2$

$G_2 = ar^2 = 1 \times 2^2 = 4$

$G_3 = ar^3 = 1 \times 2^3 = 8$

2, 4, 8 are required three G. Ms

Solution:- 2 and 32

Let G_1, G_2, G_3 be the three G.Ms between 2 and 32 that are required. G.Ms 2, G_1, G_2 and G_3 32 is a G. P.

Here $a = 2$

$n = 5$

$a_5 = 32$

Since $a_n = ar^{n-1}$

$a_5 = ar^{5-1}$

$32 = 2r^4$

$r^4 = 16$

Thus $r^4 = 2^4$

$r = 2$

Now $G_1 = ar$

$= 2 \times 2 = 4$

$G_2 = ar^2$

$= 2 \times 2^2 = 8$

$G_3 = ar^3$

$= 2 \times 2^3 = 16$

4, 8 and 16 are required three G.Ms.

4- *Insert four real geometric means between 3 and 96.*

Solution:- ***3 and 96***

Let G_1, G_2, G_3 and G_4 be the four G.Ms between 3 and 96 that are required. G. Ms 3, G_1, G_2, G_3 and G_4, 96 is a G. P.

Here $a = 3$

$n = 6$

$a_6 = 96$

Since $a_n = ar^{n-1}$

$a_6 = ar^{6-1}$

$96 = 3r^5$

$$\frac{96}{3} = r^5$$

$$r^5 = \frac{96}{3} = 32$$

$r^5 = 2^5$

$r = 2$

$G_1 = ar$

$= 3 \times 2 = 6$

$G_2 = 3r^2$

$= 3(2)^2 = 12$

$G_3 = 3r^3$

$= 3(2)^3 = 24$

$G_4 = 3(r)^4$

$= 3(2)^4 = 48$

6,12,24,48 are four required G.Ms.

5- ***The A.M between two numbers is 5 and their positive G.M is 4. Find the numbers.***

Solution:-

Let numbers are a and b.

(i) A is A.M $A = \frac{a+b}{2}$

(ii) G is G.M $G = \sqrt{ab}$

According to condition A = 5, G = 4

$5 = \frac{a+b}{2}$ from (i) and (ii)

$4 = \sqrt{ab}$

$16 = ab$ *from (iii)*

$10 = a + b$ *from (iv)*

$10 - b = a$ Or

$a = 10 - b$ Or

Putting values of a in *(iii)*

$16 = (10 - b)(b)$

$16 = 10b - b^2$

$b^2 - 10b + 16 = 0$ Or

$b^2 - 2b - 8b + 16 = 0$

$b(b-2) - 8(b-2) = 0$

$(b-2)(b-8) = 0$

$b - 2 = 0$

$b = 2$

$b - 8 = 0$

$b = 8$

$a + b = 10$ Now from *(iv)*

$a + 2 = 10$

$a = 10 - 2 = 8$

$a + 8 = 10$ b = 8

$a = 10 - 8 = 2$

Hence Numbers are 2,8 or 8,2

6- ***The positive G.M between two numbers is 6 and the A.M between them is 10. Find the numbers.***

Solution:-

$$\text{A is A.M} = \frac{a+b}{2} \qquad \text{(i)}$$

$$\text{G is G.M} = \sqrt{ab} \qquad \text{(ii)}$$

From (i) and (ii)

$$10 = \frac{a+b}{2} \quad \text{Or}$$

$$20 = a + b \qquad \text{(iii)}$$

$$6 = \sqrt{ab} \quad \text{Or}$$

$$36 = ab \qquad \text{(iv)}$$

$$a = 20 - b \qquad \text{from (iii)}$$

Putting values of a in (iv)

$$36 = (20 - b)(b)$$

$$36 = 20b - b^2$$

$$b^2 - 20b + 36 = 0 \quad \text{Or}$$

$$b^2 - 2b\,b - 18b + 36 = 0$$

$$b(b-2) - 18(b-2) = 0$$

$$(b-2)(b-18) = 0$$

If $b-2 = 0$

$$b-18 = 0$$

$$b-2 = 0$$

$$b = 2$$

Putting b = 2 in (iv)

$$[redacted] = 2a$$

$$a = 18$$

$b-18 = 0$ *and*

$$b = 18$$

Putting b = 18 in (iv)

$$36 = 18a$$

$$a = 2$$

Thus numbers are 2, 18 or 18, 2

7- ***Show that the A.M between the two numbers 4 and 8 is greater than their geometric mean.***

Solution:-

Here $a = 4$

$b = 8$

$$(A.M) = \frac{4+8}{2} = 6$$

$$(G.M) = \sqrt{ab}$$

$$G = \sqrt{4 \times 8}$$

$$= \sqrt{32}$$

Now $\sqrt{32}$ *is less than 6.*

Proved $6 > \sqrt{32}$

Therefore $A > G$

8- ***Insert four geometric means between 160 and 5.***

Solution:-

numbers are 5 and 160.

Let G_1, G_2, G_3 and G_4 the four G.Ms between 160 and 5.

Here $n = 6$

$a = 5$

$a_6 = 160$

Since $a_n = ar^{n-1}$

$a_6 = 5r^{6-1}$

$160 = 5r^5$

$$r^5 = \frac{160}{5}$$

$r^5 = 32 = 2^5$

$r = 2$

$G_1 = ar$

$$= 5 \times 2 = 10$$

$$G_2 = ar^2$$

$$= 5 \times 2^2 = 20$$

$$G_3 = ar^3$$

$$= 5 \times 2^3 = 40$$

$$G_4 = 5 \times 2^4 = 80$$

Hence required G.Ms are 10, 20, 40, 80.

9- ***Insert three geometric means between 486 and 6.***

Solution:- ***Let G_1, G_2 and G_3 the three G. Ms between 186 and 6.***

Here $n = 5$

$a = 486$

$a_5 = 6$

Since $a_n = ar^{n-1}$

$a_5 = ar^{5-1}$

$6 = 486r^4$

$$\frac{6}{486} = r^4$$

$$\frac{1}{81} = r^4$$

$$r^4 = \frac{1}{81} = \left(\frac{1}{3}\right)^4$$

$$r = \frac{1}{3}$$

$$G_1 = ar$$

$$= 486 \times \frac{1}{3} = 162$$

$$G_2 = 486 \times \left(\frac{1}{3}\right)^2$$

$$= 486 \times \frac{1}{9} = 54$$

$$G_3 = 486 \times \left(\frac{1}{3}\right)^3$$

$$= 486 \times \frac{1}{27} = 18$$

Hence required G.Ms are 162, 54, 18.

10- *Insert four geometric means between $\frac{1}{8}$ and 128.*

Solution:- $\frac{1}{8}$, 128

Let G_1, G_2, G_3, G_4 be the four G.Ms between $\frac{1}{8}$ and 128.

Here $n = 6$

$$a = \frac{1}{8}$$

$$a_6 = 128$$

$$a_n = ar^{n-1}$$

$$a_6 = ar^{6-1}$$

$$128 = \frac{1}{8} r^5$$

$$128 \times 8 = r^5$$

$$1024 = r^5$$

$$4^5 = r^5$$

$$r = 4$$

$$G_1 = ar$$

$$= \frac{1}{8} \times 4 = \frac{1}{2}$$

$$G_2 = ar^2$$

$$= \frac{1}{8} \times 4^2$$

$$= \frac{1}{8} \times 16 = 2$$

$$G_3 = ar^3$$

$$= \frac{1}{8} \times 4^3$$

$$= \frac{1}{8} \times 64 = 8$$

$$G_4 = \frac{1}{8} r^4$$

$$= \frac{1}{8} \times 4^4 = 32$$

$\frac{1}{2}$, 2, 8, 32 are required G.Ms.

11- Insert six geometric means between 56and $-\frac{7}{16}$

Solution:- 56, $-\frac{7}{16}$

Let $G_1, G_2, G_3, G_4, G_5, G_6$ be the six G.Ms between 56 and $-\frac{7}{16}$

Here $n = 8$

$a = 56$

$a_6 = -\frac{7}{16}$

Since $a_n = ar^{n-1}$

$a_6 = ar^{a-1}$

$$-\frac{7}{16} = 56r^7$$

$$r^7 = -\frac{7}{16 \times 56}$$

$$= -\frac{1}{16 \times 8}$$

$$= -\frac{1}{128}$$

$$r^7 = \left(-\frac{1}{2}\right)^7$$

$$r = -\frac{1}{2}$$

$$G_1 = ar$$

$$= 56\left(-\frac{1}{2}\right) = -28$$

$$G_2 = ar^2$$

$$= 56\left(-\frac{1}{2}\right)^2$$

$$= 56 \times \frac{1}{4} = 14$$

$$G_3 = ar^3$$

$$= 56 \times \left(-\frac{1}{2}\right)^3$$

$$= 56 \times \left(-\frac{1}{8}\right) = -7$$

$$G_4 = ar^4$$

$$= 56\left(-\frac{1}{2}\right)^4$$

$$= 56 \times \frac{1}{16} = \frac{7}{2}$$

$$G_5 = ar^5$$

$$= a\left(-\frac{1}{2}\right)^5$$

$$= 56 \times \left(-\frac{1}{32}\right) = -\frac{7}{4}$$

$$G_6 = ar^6$$

$$= 56 \times \left(-\frac{1}{2}\right)^6$$

$$= 56 \times \frac{1}{64} = \frac{7}{8}$$

Six required G. Ms are $-28, 14, -7, \frac{7}{2}, -\frac{7}{4}, \frac{7}{8}$.

12- Insert five geometric means between $\frac{32}{81}$ and $\frac{9}{2}$

Solution:-

Numbers are $\frac{32}{81}, \frac{9}{2}$

Let $G_1 G_2 G_3 G_4 G_5$ *be the five G. Ms between* $\frac{32}{81}$

and $\frac{9}{2}$.

Here $n = 7$

$$a = \frac{32}{81}$$

$$a_6 = +\frac{9}{2}$$

Since $a_n = ar^{n-1}$

$$a_n = ar^{6-1}$$

$$\frac{9}{2} = \frac{32}{81}r^5$$

$$r^5 = \frac{9}{2} \times \frac{32}{81}$$

$$r^5 = \frac{729}{64} = \left(\frac{3}{2}\right)^5$$

Therefore $r = \frac{3}{2}$

$$G_1 = ar$$

$$= \frac{32}{81} \times \frac{3}{2} = \frac{16}{27}$$

$$G_2 = ar^2$$

$$= \frac{32}{81} \times \left(\frac{3}{2}\right)^2$$

$$= \frac{32}{81} \times \frac{9}{4} = \frac{8}{9}$$

$$G_3 = ar^3$$

$$= \frac{32}{81} \times \left(\frac{3}{2}\right)^3$$

$$= \frac{32}{81} \times \frac{27}{8} = \frac{4}{3}$$

$$G_4 = ar^4$$

$$= \frac{32}{81} \times \left(\frac{3}{2}\right)^4$$

$$= \frac{32}{81} \times \frac{81}{16} = 2$$

$$G_5 = ar^5$$

$$= \frac{32}{81} \times \left(\frac{3}{2}\right)^5$$

$$= \frac{32}{81} \times \frac{243}{32} = 3$$

Five required G.Ms are $\frac{16}{27}, \frac{8}{9}, \frac{4}{3}, 2, 3$