Unit -1 PERCENTAGE, RATIO AND PROPORTION

PERCENTAGE

The word "percent" is a short form of the Latin word "percentum". Percent means out of hundred or per hundred. The symbol for percentage is "%".

71% of the Earth's surface is water.
Percentage of land is 29%.

Ratio

Ratio is a comparison of like quantities measured in like units. The symbol for ratio is ":"

ANTECEDENT AND CONSEQUENT

In a ratio $a : b$, 'a' is called the antecedent and 'b' is called the consequent, e.g. in ratio $2 : 5$, antecedent is 2 and consequent is 5.

THE ORDER OF RATIO

If the magnitudes of the two quantities are denoted by 'a' and 'b' then ratio from 'a' to 'b' is $a : b$.

We cannot write this ratio as $b : a$, because, $a : b \neq b : a$ since $\frac{a}{b} \neq \frac{b}{a}$.

PROPORTION

The equality of two ratios is known as proportion. The symbol for proportion is "::" or "=".

EXTREMES AND MEANS

If $a : b = c : d$, then the proportion is $a : b :: c : d$. We read it as ratio a is to b is as ratio c is to d. a, b, c and d are called the terms of the proportion.

The first and fourth terms, i.e. "a" and "d" are called the extremes, while the second and third terms "b" and "c" are called the means of the proportion.

The product of means is equal to the product of extremes, i.e.



$$ad = bc$$

DIRECT PROPORTION

The relationship between two ratios in which increase in one quantity causes a proportional increase in the other quantity or decrease in one quantity causes a decrease in the other quantity is called "direct proportion".

INVERSE PROPORTION

The relationship between two ratios in which increase in one quantity causes a proportional decrease in the other quantity or a decrease in the one quantity cause a proportional increase in the other quantity is an inverse proportion.

COMPOUND PROPORTION

The relationship between two or more proportions is known as compound proportion.

Exercise 1.1

1. Express the following percentages as fractions in their lowest form.

(i) 95% (ii) 65% (iii) 75% (iv) 25% (v) 56% (vi) 48%

(vii) 8% (viii) $33\frac{1}{2}\%$ (ix) $37\frac{1}{2}\%$ (x) $87\frac{1}{2}\%$

(xi) $5\frac{1}{4}\%$ (xii) $42\frac{1}{2}\%$

Solutions:

(i) $95\% = \frac{95}{100} = \frac{19}{20}$

(ii) $65\% = \frac{65}{100} = \frac{13}{20}$

(iii) $75\% = \frac{75}{100} = \frac{3}{4}$

(iv) $25\% = \frac{25}{100} = \frac{1}{4}$

(v) $56\% = \frac{56}{100} = \frac{14}{25}$

(vi) $48\% = \frac{48}{100} = \frac{12}{25}$

(vii) $8\% = \frac{8}{100} = \frac{2}{25}$

(viii) $33\frac{1}{2}\% = \frac{67}{2}\% = \frac{67}{2\times100} = \frac{67}{200}$

(ix) $37\frac{1}{2}\% = \frac{75}{2}\%$

$= \frac{75}{2\times100}$

$= \frac{3}{8}$

(x) $87\frac{1}{2}\% = \frac{175}{2}\%$

$= \frac{175}{2\times100}$

$= \frac{7}{8}$

(xi) $5\frac{1}{4}\% = \frac{21}{4}\%$

$= \frac{21}{4\times100}$

$= \frac{21}{400}$

(xii) $42\frac{1}{2}\% = \frac{85}{2}\%$

$= \frac{85}{2\times100}$

$= \frac{17}{40}$

2. ***Express the following fractions as percentages, giving your answer correct to 1 decimal place, where necessary.***

(i) $\frac{3}{4}$ (ii) $\frac{3}{5}$ (iii) $\frac{4}{25}$ (iv) $\frac{13}{20}$

(v) $\frac{31}{25}$ (vi) $\frac{21}{40}$ (vii) $\frac{23}{60}$ (viii) $\frac{8}{3}$

(ix) $\frac{8}{5}$ (x) $\frac{7}{8}$ (xi) $\frac{5}{8}$ (xii) $\frac{3}{8}$

Solutions:

(i) $\frac{3}{4} = \frac{3}{4}\times\frac{100}{100}$

$= \frac{3}{4}\times100\%$

$=3\times25\%$

$=75\%$

(ii) $\frac{3}{5} = \frac{3}{5} \times \frac{100}{100}$

$= \frac{3}{5} \times 100\%$

$= 3 \times 20\%$

$= 60\%$

(iii) $\frac{4}{25} = \frac{4}{25} \times \frac{100}{100}$

$= \frac{4}{25} \times 100\%$

$= 16\%$

(iv) $\frac{13}{20} = \frac{13}{20} \times \frac{100}{100}$

$= \frac{13}{20} \times 100\%$

$= 65\%$

(v) $\frac{31}{25} = \frac{31}{25} \times \frac{100}{100}$

$= \frac{31}{25} \times 100\%$

$= 124\%$

(vi) $\frac{21}{40} = \frac{21}{40} \times \frac{100}{100}$

$= \frac{21}{40} \times 100\%$

$= 52.5\%$

(vii) $\frac{23}{60} = \frac{23}{60} \times \frac{100}{100}$

$= \frac{23}{60} \times 100\%$

$= \frac{23}{3} \times 5\%$

$= \frac{115}{3}\%$

$=38.33\%$

$=38.30\%$

(viii) $\frac{8}{3} = \frac{8}{3} \times \frac{100}{100}$

$= \frac{8}{3} \times 100\%$

$= \frac{800}{3}\%$

$=266.666\%$

$=266.67\%$

(ix) $\frac{8}{5} = \frac{8}{5} \times \frac{100}{100}$

$= \frac{8}{5} \times 100\%$

$=160\%$

(x) $\frac{7}{8} = \frac{7}{8} \times \frac{100}{100}$

$= \frac{7}{8} \times 100\%$

$= \frac{700}{8}\%$

$=87.5\%$

(xi) $$\frac{5}{8} = \frac{5}{8} \times \frac{100}{100}$$

$$= \frac{5}{8} \times 100\%$$

$$= \frac{125}{2}\%$$

$$= 62.5\%$$

(xii) $$\frac{3}{8} = \frac{3}{8} \times \frac{100}{100}$$

$$= \frac{3}{8} \times 100\%$$

$$= \frac{75}{2}\%$$

$$= 37.5\%$$

3. Express the following percentages as decimals, giving your answer correct to 3 places of decimal.

(i) 47% (ii) 58% (iii) 92% (iv) 8%

(v) 12% (vi) 120% (vii) 180% (viii) 145%

(ix) $5\frac{1}{2}\%$ (x) $5\frac{1}{3}\%$ (xi) $48\frac{2}{3}\%$ (xii) $58\frac{1}{3}\%$

Solutions:

(i) $$47\% = \frac{47}{100} = .47$$

(ii) $$58\% = \frac{58}{100} = .58$$

(iii) $$92\% = \frac{92}{100} = 0.92$$

(iv) $$8\% = \frac{8}{100} = 0.08$$

(v) $12\% = \frac{12}{100} = 0.12$

(vi) $180\% = \frac{120}{100} = 1.8$

(vii) $120\% = \frac{180}{100} = 1.2$

(viii) $145\% = \frac{145}{100} = 1.45$

(ix) $5\frac{1}{2}\% = \frac{11}{2}\% = 5.5\% = \frac{5.5}{100} = 0.055$

(x) $5\frac{1}{3}\% = \frac{16}{3}\% = 5.33\% = \frac{5.33}{100} = 0.0533$

(xi) $48\frac{2}{3}\% = \frac{146}{3}\% = 48.66\% = \frac{48.7}{100} = 0.487$

(xii) $58\frac{1}{3}\% = \frac{175}{3}\% = 58.3\% = \frac{58.3}{100} = 0.583$

4. Express the following decimals as percentages.

(i)	0.5	(ii)	0.9	(iii)	1.25	(iv)	1.39
(v)	1.72	(vi)	0.22	(vii)	2.64	(viii)	3.41
(ix)	0.845	(x)	1.78	(xi)	1.58	(xii)	0.065

Solutions:

(i) $0.5 = 0.5 \times \frac{100}{100}$

$= 0.5 \times 100\%$

$= \frac{5}{10} \times 100\%$

$= 5 \times 10\%$

$= 50\%$

(ii) $0.9 = 0.9 \times \frac{100}{100}$

$= 0.9 \times 100\%$

$= \frac{9}{10} \times 100\%$

$= 9 \times 10\%$

$= 90\%$

(iii) $1.25 = 1.25 \times \frac{100}{100}$

$= 1.25 \times 100\%$

$= \frac{125}{100} \times 100\%$

$= 125\%$

(iv) $1.39 = 1.39 \times \frac{100}{100}$

$= 1.39 \times 100\%$

$= \frac{139}{100} \times 100\%$

$= 139\%$

(v) $1.72 = 1.72 \times \frac{100}{100}$

$= 1.72 \times 100\%$

$= \frac{172}{100} \times 100\%$

$= 172\%$

(vi) $0.22 = 0.22 \times \frac{100}{100}$

$= 0.22 \times 100\%$

$= \frac{22}{100} \times 100\%$

$= 22\%$

(vii) $2.64 = 2.64 \times \frac{100}{100}$

$= 2.64 \times 100\%$

$= \frac{264}{100} \times 100\%$

$= 264\%$

(viii) $3.41 = 3.41 \times \frac{100}{100}$

$= 3.41 \times 100\%$

$= \frac{341}{100} \times 100\%$

$= 341\%$

(ix) $0.845 = 0.845 \times \frac{100}{100}$

$= 0.845 \times 100\%$

$= 84.5\%$

(x) $1.78 = 1.78 \times \frac{100}{100}$

$= 1.78 \times 100\%$

$= \frac{178}{100} \times 100\%$

$= 178\%$

(xi) $1.58 = 1.58 \times \frac{100}{100}$

$= 1.58 \times 100\%$

$= 158\%$

(xii) $0.065 = 0.065 \times \frac{100}{100}$

$= 0.065 \times 100\%$

$= 6.5\%$

5. ***Complete the following table.***

Fraction	Percentage	Decimal
i. $\frac{3}{4}$	75%	0.75
ii. $\frac{4}{5}$	80%	0.80
iii. $\frac{2}{5}$	40%	0.4
iv. $\frac{31}{50}$	62%	0.62
v. $\frac{11}{25}$	44%	0.44

Exercise 1.2

1. ***If 45% of the students in a school are girls. What percentage are boys?***

Solution:

Number of Girls = 45%

Number of boys = (100 - 45)%

= 55%

2. ***If 82% of the house have a television, what percentage do not have?***

Solution:

Percentage of home which have Television = 82%

Percentage of home which have not television = (100 - 82)%

= 18%

3. ***A hockey team won 62% of their matches and 26% of them were ended in a draw. What percentage of the matches they lost?***

Solution:

won % matches of hockey team = 62%

percentage of matches which were ended = 26%

Percentage of both matches = (62+26)%

= 88%

Percentage of those matches which were lost = (100 - 88)%

= 12%

4. ***An aeroplane carries 400 passengers, 52% of the passengers were Pakistani, 17% were Chinese, 12% were from Iran and the rest were from British?***

(i) ***How many people each nationality were on the plane.***

(ii) ***What percentage were British?***

Solution:

Total number of passengers in plane = 400

Percentage of Pakistani passengers = 52

Numbers of Pakistani passengers = $\frac{52}{100} \times 400$

(i) = 208

Percentage of Chinese passengers = 17%

Thus

Number of Chinese passengers = $\frac{17}{100} \times 400$

= 68 (ii)

Percentage of Irani passengers = 12%

Number of Irani passerngers = $\frac{12}{100} \times 400$

= 48

Number of British passengers = 400 - 208 - 68 -48

= 76

Note:

Number of British passengers are subtracted from the total number of Pakistani, Chinese and Irani passengers.

5. ***Amna scored 46 out of 50 in a Maths test, 64 out of 75 in Chemistry test and 72 out of 80 in a Physics test. In which sybject did she perform best?***

Solution:

Out of 50 marks in maths = 46

Percentage scored in math = $\frac{46}{50} \times 100\%$

(i) = 92%

Out of 75 marks in Chemistry = 64

Percentage marks in Chemistry = $\frac{64}{75_3} \times 100^4$

$$= \frac{64 \times 4}{3}$$

$$= \frac{256}{3}$$

(ii) $= 85.3\%$

Out of 80 marks in Physics $= 72$

Percentage marks in Physics $= \frac{72}{80} \times 100\%$

(iii) $= 90\%$

Amna got the best marks in maths.

6. ***A table costs a carpenter Rs. 720 to make. He sells it for Rs. 920. What percentage of profit did he earn?***

Solution:

Total cost of a table = Rs. 720

Sale price of a table = Rs. 920

Percentage of Profit on Rs. 720 = (920 - 720)

= Rs. 200

$$\text{Percentage Profit} = \frac{200}{720} \times 100$$

$$= \frac{20000}{720}$$

$$= 27.78\%$$

7. ***If 8.4% of a book consists of 42 pages. Find total number of pages in the book.***

Solution:

Number of pages in 8.4% part of a book = 42

$$\text{Total number of pages in the book} = \frac{42}{84} \times 100 \times 10$$

$$= \frac{42000}{84}$$

$$= 500 \text{ Pages}$$

8. ***Out of his total income Hamza spends 20% on house rent and 70% of the rest on household expenditure. If he saves Rs. 1800, What is his total income?***

Solution:

Amount spend on house rent = 20%

Rest value = (100 - 20)%

= 80%

Amount spend on house hold expenditure = $\frac{70}{100} \times 80$

Total value = 56%

Share of % income on house rent and house hold expenditure = (20 + 56)%

= 76%

= (100 - 76)%

Saving % of income = 24%

Total income if saving will be = Rs. 100

Total income if saving will be Rs.24 = $\frac{100}{24} \times 1800$

Total income if saving will be Rs. 1800 = Rs. 7500

9. ***Raheel's income is 25% more than that of Rauf. What percent is Rauf's income less than Raheel's?***

Solution:

Let Income of Rauf = Rs. 100

Raheel's income is more than 25% extra of Rauf's = Rs. 125

If Raheel's income Rs.125 than Rauf's income = Rs. 100

If Raheel's income Rs.100 less than Rauf's income = $\frac{100}{125} \times 100$

= Rs. 80

= (100 - 80)%

Less percentage of Rauf's income than Raheel's. = 20%

Exercise 1.3

1. Find the ratio of first quantity to the second in its lowest term.

(i) Rs. 24, Rs.6 (ii) 20 Kg, 5 Kg (iii) 20cm, 80cm
(iv) 5m, 5m (v) 1500km, 1200km (vi) Rs.150,Rs.275

Solution:

Required Ratio = 24:6
= 4:1 (Dividing by 6)

(ii) 20kg, 5kg
Required Ratio = 20:5
= 4:1 (Dividing by 5)

(iii) 20cm, 80cm
Requierd Ratio = 20:80
= 1:4 (Dividing by 20)

(iv) 5m, 5m
Required Ratio = 5:5
= 1:1 (Dividing by 5)

(v) 1500km, 1200km
Required Ratio = 1500:1200
= 15:12 (Dividing by 100)
= 5:4 (Dividing by 3)

(vi) Rs. 150, Rs.275
Required Ratio = 150: 275
= 30:55 Dividing by 5
= 6:11 Dividing 5

2. Express each of the following ratios in its simplest form.

(i) $\frac{2}{3}:\frac{3}{5}$ (ii) $\frac{4}{5}.\frac{3}{4}$ (iii) $\frac{5}{6}:\frac{7}{10}$

(iv) $\frac{13}{40}:\frac{3}{20}$ (v) $\frac{2}{3}:\frac{1}{6}$ (vi) $\frac{4}{10}:20$

(vii) $\frac{15}{10}:2$ (viii) $\frac{12}{10}:\frac{28}{10}$ (ix) $\frac{2}{5}:\frac{1}{3}$

Solutions:

(i) $\frac{2}{3}:\frac{3}{5}$

$\frac{2}{3}\div\frac{3}{5}=\frac{2}{3}\times\frac{5}{3}$

$=\frac{10}{9}=10:9$

(ii) $\frac{4}{5}:\frac{3}{4}$

$\frac{4}{5}:\frac{3}{4}=\frac{4}{5}\div\frac{3}{4}$

$=\frac{4}{5}\times\frac{4}{3}$

$=\frac{16}{15}$

$=16:15$

(iii) $\frac{5}{6}:\frac{7}{10}$

$\frac{5}{6}:\frac{7}{10}=\frac{5}{6}\times\frac{10}{7}$

$=\frac{5}{3}\times\frac{5}{7}=\frac{25}{21}$

$=25:21$

(iv) $\frac{13}{40}:\frac{3}{20}$

$\frac{13}{40}:\frac{3}{20}=\frac{13}{40}\div\frac{3}{20}$

$$= \frac{13}{40} \times \frac{20}{3}$$

$$= \frac{13}{6}$$

$$= 13:6$$

(v) $\frac{2}{3} : \frac{1}{6}$

$$\frac{2}{3} : \frac{1}{6} = \frac{2}{3} \div \frac{1}{6}$$

$$= \frac{2}{3} \times \frac{6}{1}$$

$$= \frac{2}{1} \times \frac{2}{1}$$

$$= \frac{4}{1}$$

$$= 4:1$$

(vi) $\frac{4}{10} : 20$

$$\frac{4}{10} : 20 = \frac{4}{10} \div 20$$

$$= \frac{4}{10} \times \frac{1}{20}$$

$$= \frac{1}{10} \times \frac{1}{5}$$

$$= \frac{1}{50}$$

$$= 1:50$$

(vii) $\frac{15}{10} : 2$

$$\frac{15}{10} : 2 = \frac{15}{10} \div 2$$

$$= \frac{15}{10} \times \frac{1}{2}$$

$$= \frac{15}{20}$$

$$= \frac{3}{4}$$

$$= 3:4$$

(viii) $\frac{12}{10} : \frac{28}{10}$

$$\frac{12}{10} : \frac{28}{10} = \frac{12}{10} \div \frac{28}{10}$$

$$= \frac{12}{10} \times \frac{10}{28}$$

$$= \frac{12}{28}$$

$$= \frac{3}{7}$$

$$= 3:7$$

(ix) $\frac{2}{5} : \frac{1}{3}$

$\frac{2}{5} : \frac{1}{3} = \frac{2}{5} \div \frac{1}{3}$

$= \frac{2}{5} \times \frac{3}{1}$

$= \frac{6}{5}$

= 6:5

3. ***In a city 126 medical students traveled by:***

Rikshaw	Taxi	Bus	Car
14	9	75	28

Find ratio of the students who used.

(i) Rikshaw to taxi

(ii) Taxi to bus

(iii) Taxi to car.

Solution:

Number of students who travelled by car = 28

Number of students who travelled by bus = 75

Number of students who travelled by Taxi = 9

Number of students who travelled by Rikshaw = 14

(i) Rikshaw to Taxi

14 : 9

= 14 : 9

(ii) by Taxi by bus

9 : 75

$9 \div 75$

$= \frac{9}{75}$

$$= \frac{3}{25}$$

$$= 3 : 25$$

(iii) By Taxi : By car

9 : 28

4. ***In a school library, there are 75 books on Mathematics, 115 on English, 85 on Chemistry and 60 on Physics. Find the ratio of the following:***

(i) Mathematics books to English books

(ii) English books to Chemistry books

(iii) English books to Physics books

(iv) Physics books to Chemistry books

(v) Physics books to Mathematics books

(vi) Chemistry books to Mathematics books

Solution:

Number of Math's books = 75

Number of English books = 115

Number of Chemistry books = 85

Number of Physics books = 60

No. of Math books : No. of English books

75 : 115

75 : 115

$$= \frac{75}{115}$$

or **15 : 23** $$= \frac{15}{23}$$

No. of English books : No. of Chem books

$$115 : 85$$

$$115 : 85$$

$$= \frac{115}{85}$$

$$\text{or} \quad 23 : 17 = \frac{23}{17}$$

No. of English books : No. of Physics books

$$115 : 60$$

$$115 : 60$$

$$= \frac{115}{60}$$

$$\text{or} \quad 23 : 12 = \frac{23}{12}$$

No. of Physics book : No. of Chemistry book

$$60 : 85$$

$$60 : 85$$

$$= \frac{60}{85}$$

$$\text{or} \quad 12 : 17 = \frac{12}{17}$$

No. of Physics books : No. of Math

60 : books

60 : 75

$= \frac{75}{60}$

$= \frac{60}{75}$

or $4 : 5 = \frac{12}{15}$

$= \frac{4}{5}$

No. of Chemistry book : No. of Math books

85 : 75

85 : 75

$= \frac{85}{75}$

or $17 : 15 = \frac{17}{15}$

Exercise 1.4

1. Find the ratio of 6 rupees each to 72 rupees per dozen.

Solution:

$$72 \text{ rupees Perdozen mean} = \frac{72}{12}$$

$$= 6$$

Therefore there is same ratio between each Rs. 6 and Rs. 7256 thats mean Rs. 6 and every Rs. 6.

$$6 \quad : \quad 6 = \frac{6}{6}$$

$$= \frac{1}{1}$$

$$= 1 : 1$$

2. Find the ratio of Rs. 160 per meter to Rs. 150 per meter.

Solution:

Rs.150 per meter : Rs.160 per meter

$$160 \quad : \quad 150$$

$$= \frac{160}{150}$$

$$\text{or} \quad 16 : 15 = \frac{16}{15}$$

3. Find the ratio of Rs. 72 for 24 to rupees 4 each.

Solution:

$$\text{Rs.72 for 24 mean} = \frac{72}{24}$$

For each one Rs. 3 = 3

Rs. 72 for 24 : For each 4

Thus 3 : 4

4. ***A square 'A' has side 2cm and a square 'B' has side 6 cm. Find the ratio of:***

(i) The length of the side of the square A to the length of the side of the square B.

(ii) The perimeter of the square A to the perimeter of the square 'B'.

(iii) The area of the square 'A' to the area of the square 'B'.

Solution:

Length of a square A side = 2 cm

Therefore Area of A side = 2×2

$= 4 \text{ cm}^2$

$= 4 \times 2$

Perimeter of square A = 8cm

Length of the square B side = 6cm

Area of B side = 6×6

$= 36 \text{ cm}^2$

Perimeter of square B = 4×6

$= 24$ cm

Length of square A side : Length of square B side

Now $2 : 6 = \frac{2}{6}$

$2 : 6 = \frac{1}{3}$

$1 : 3$

Perimeter of square A : Perimeter of square B

(ii) $8 : 24 = \frac{8}{24}$

Now $8 : 24$

$= \frac{1}{3}$

$= 1:3$

Area of square A : Area of square B

$$4 : 36 = \frac{4}{36}$$

$$4 : 36 = \frac{1}{9}$$

$$= 1 : 9$$

5. ***If a : b = 2 : 3, find the ratio 6a : 2b.***

Solution:

$$a:b = 2:3$$

$$\text{i.e. } a = 2$$

$$b = 3$$

$$\text{then } 6a = 6\times2 = 12$$

$$\text{and } 2b = 2\times3 = 6$$

$$\text{Hence } 6a:2b = 12:6$$

$$= \frac{12}{6}$$

$$= \frac{2}{1}$$

$$= 2:1$$

6. ***A triangle has sides of lengths 3cm, 4cm, and 6cm. Find the ratio of the lengths of the sides to one another.***

Solution:

First side of length = 3 cm

Second side of length = 4 cm

Third side of length = 6 cm

(i) First side of length : Second side of length

3 : 4

(ii) Second side of length : Third side of length

$$4 : 6$$

$$\text{and} \quad 4 : 6 = \frac{4}{6}$$

$$= \frac{2}{3}$$

$$= 2 : 3$$

(iii) First side of length : Third side of length

$$3 : 6$$

$$\text{and} \quad 3 : 6 = \frac{3}{6}$$

$$= \frac{1}{2}$$

$$= 1 : 2$$

7. ***Two angles in a triangle are 54° and 72°. Find the ratio of the third angle to the sum of the first two.***

Solution:

$$\text{First angle} = 54^\circ$$

$$\text{Second angle} = 72^\circ$$

Therefore $\quad \text{Sum of two angles} = 54 + 72$

$$= 126^\circ$$

$$\text{Amount of 3rd side} = 180^\circ - 126^\circ$$

$$= 54^\circ$$

Required Ratio

Sum of remaining amount of two angles : amount of 3rd angle

$$54 : 126$$

$$\text{and} \quad 54 : 126 = \frac{54}{126}$$

$$= \frac{18\times 3}{18\times 7}$$

$$= \frac{3}{7}$$

$$= 3 : 7$$

8. ***Ali's father earns a salary of Rs. 40,000 in a month, while his father's monthly expenditures are Rs. 35,000. Find the ratio of his father's:***

(i) Income to expenditure

(ii) Expenditure to savings

(iii) Income to saving

Solution:

Salary = Rs. 40,000

Expenditure = Rs. 35,000

Therefore, Saving = 40,000−35,000

= 5000 Rupees

Income Expensis required ratios

40,000 : 35,000

Now $40{,}000 : 35{,}000 = \frac{40{,}000}{35{,}000}$

$$= \frac{8}{7}$$

$$= 8 : 7$$

Expenditure : Saving

35,000 : 5000

and $35{,}000 : 5000 = \frac{35000}{5000}$

$$= \frac{7}{1}$$

$= 7:1$

Income : Saving

40,000 : 5000

and $40{,}000 : 5000 = \frac{40{,}000}{5{,}000}$

$= \frac{8}{1}$

$= 8 : 1$

9. ***A square A has side 6cm and square B has side 8cm. Find the ratio of:***

(i) The length of the side of a square A to the length of the side of the square B.

(ii) The area of square A to the area of square B.

Solution:

Length of square A side = 6 cm

Area of A square = 6×6

$= 36\text{cm}^2$

Length of square B side = 8cm

Area of B square = 8×8

$= 64\ \text{cm}^2$

(i) Required ratios

Length of square side A : Length of square side B

6 : 8

$6 : 8 = \frac{6}{8}$

and $= \frac{3}{4}$

Area of square A side : Area of square B side

$$36 : 64$$

$$\text{and } 36 : 64 = \frac{36}{64}$$

$$= \frac{9}{16}$$

$$= 9 : 16$$

10. A family has 12 pets c which 6 are cats, 2 are dogs and rest are birds. Find the ratio of hte number of:

(i) birds to dogs

(ii) birds to pets

Solution:

$$\text{Total birds} = 12$$

$$\text{Number of Parrots} = 6$$

$$\text{Number of Sparrows} = 2$$

$$\text{Number of rest birds} = 12 - 6 - 2$$

$$= 4$$

Number of Pigeons : Number of Sparrows

$$4 : 2$$

$$\text{and } 4 : 2 = \frac{4}{2}$$

$$= \frac{2}{1}$$

$$= 2 : 1$$

Number of Pigeons : Number of total birds

$$4 : 12$$

$$\text{and } 4 : 12 = \frac{4}{12}$$

$$= \frac{1}{3}$$

$$= 1 : 3$$

Exercise 1.5

1. Find the value of x in the proportion 20 : 50 :: 8 : x?

Solution:

Product of extremes = $20x$

Product of means = 50×8

Product of extremes = Product of means

Thus $20x = 50 \times 8$

Therefore, $$x = \frac{50 \times 8}{20}$$

$$= 20$$

2. The price of 15 suits is Rs. 6750. How many such suits can be purchased by an amount of Rs. 4050?

Solution:

No. of suits purchased by = 15

Rs. 6750

No. of suits purchased by = x

Rs. 4050

Suits	Price
15 ↑	6750 ↑
x ↑	4050 ↑

Therefore,

Direct Proportion $x : 15 :: 4050 : 6750$

Product of extremes = $6750x$

Product of means = 15×4050

Product of extremes = Product of means

$6750x = 15 \times 4050$

Therefore, $$x = \frac{15 \times 4050}{6750} = 9$$

Thus 9 suits can be purchased in Rs. 4050

3. ***A motorcycle covers 90km in 2 liters of petrol. In how many liters of petrol will it cover 225km?***

Solution:

Let 1st condition

Quantity of petrol = 2 liter

Distance covered = 90 km

Second condition

Distance covered = 225km

Let Quantity of petrol = x (l)

Now

	Petrol	distance covers
Direct Proportion	2 ↓	90 ↓
	x ↓	225 ↓

Therefore $x : 2 :: 225:90$

Product of extremes = $90x$

Product of means = 2×225

Product of extremes = Product of means

Thus $90x = 2\times225$

Therefore $$x = \frac{{}^{1}\cancel{2}\times\cancel{225}^{5}}{\cancel{90}_{45_1}}$$

$$= 5$$

Thus the motorcycle will cover 225 km distance in 5 liters.

4. ***A certain journey by train takes 5 hours at the speed of 45 km/h. What will be the speed of the train to complete the same journey in 3 hours?***

Solution:

1st condition

Train speed per hour = 45km

Time = 5 hours

2nd condition = 3 hours

Time

Train speed = x hours

Now time Train speed time Time

Inverse Proportion $\uparrow \begin{matrix} 45 & 5 \downarrow \\ x & 3 \downarrow \end{matrix}$

$x : 45 :: 5 : 3$

Product of extremes = $3x$

Product of mean = 45×5

Product of extremes = Product of mean

$$3x = 45 \times 5$$

$$x = \frac{45 \times 5}{3}$$

Hence $x = 75$

5. ***Six men can paint a house in four days. How long it would take to paint the house if three men are employed?***

Solution:

1st condition

Number of men = 6 men

2nd condition

Number of days for work = 4 days

Number of mens = 3 men

Let

Number of days for work = x days

Number of days Number of men

Inverse Proportion $\uparrow \begin{matrix} 4 & \downarrow 6 \\ x & \downarrow 3 \end{matrix}$

$x : 4 :: 6 : 3$

Product of extremes = $3x$

Product of means = 4×6

Product of extremes = Product of means

$$3x = 4 \times 6$$

Thus $$x = \frac{4 \times 6}{3}$$

8 days

6. ***A manager plans to produce 100 bicycles with the help of 25 persons working 4 hours daily. How many bicycles can be made by 40 persons if they work 3 hours daily?***

Solution:

1st Condition

Number of men = 25men

daily woking hours = 4 hours

Number of ready cycles = 100cycles

2nd Condition

Number of men = 40men

Daily working hours = 3 hours

Let number of ready cycle = x

Number of cycles | Number of men | Daily time

$\uparrow \begin{matrix} 100 \\ x \end{matrix}$ $\uparrow \begin{matrix} 25 \\ 40 \end{matrix}$ $\uparrow \begin{matrix} 4 \\ 3 \end{matrix}$

There is direct proportion between number of cycles and men.

And there is direct proportion between cycles and time

$$x : 100 :: \begin{Bmatrix} 40 : 25 \\ 3 : 4 \end{Bmatrix}$$

here.

Product of extremes = $25 \times 4 \times x$

and Product of means = $100 \times 40 \times 3$

Product of extremes = Product of means

$$25 \times 4x = 100 \times 40 \times 3$$

Hence $$x = \frac{100 \times 40 \times 3}{25 \times 4}$$

$$= 120 \text{ cycles}$$

7. ***A factory makes 560 fans in 7 days with the help of 20 machines. How many fans can be made in 12 days with the help of 18 machines?***

Solution:

1st condition

Number of machines = 20

Number of days = 7

Number of fans = 560

2nd condition

Number of machines = 18

Number of days = 12

Let Number of fans = x

No. of fans	No. of machines	No. of days
↑ 560	↑ 20	↑ 7
x	18	12

There is direct proportion between numbers of fans and machines.

$$x : 560 :: \left\{ \begin{matrix} 18 : 20 \\ 12 : 7 \end{matrix} \right\}$$

Product of extremes = $20 \times 7 \times x$

Product of means = $560 \times 18 \times 12$

Thus $20 \times 7 \times x = 560 \times 18 \times 12$

$$x = \frac{560 \times 18 \times 12}{20 \times 7}$$

$= 864$ fans

8. ***A factory makes 600 soaps in 9 days with the help of 20 machines. How many soaps can be made in 12 days with the help of 18 machines?***

Solution:

1st condition

Number of machines = 20

Number of days = 9

Numbre of suits = 600

2nd condition

Number of machines = 18

Number of days = 12

Number of suits = x

No. of suits	No. of days	No. of machines
600 ↑	↑ 9	↑ 20
x	12	18

$$x : 600 :: \left\{\begin{matrix} 12 : 9 \\ 18 : 20 \end{matrix}\right\}$$

There is direct proportion between number of suits and No. days.

Product of extremes = Product of means

$9 \times 20 \times x = 600 \times 12 \times 18$.

Hence $$x = \frac{600 \times 12 \times 18}{9 \times 20}$$

= 720 Suits

9. ***If the stay of 12 men for 28 days in a hotel costs Rs. 6720. Find the cost for the stay of 8 men for 14 days in the hotel.***

Solution:

1st condition

Number of men = 12 men

Number of days = 28days

Cost = Rs. 6720

2nd condition

Number of men = 8 men

Number of days = 14days

Let cost = Rs. x

Cost	No. of men	No. of days
6720 ↑	↑ 12	↑ 28
x	8	14

There is also direct proportion between cost and men.

$$x : 6720 :: \left\{\begin{matrix} 8 : 12 \\ 14 : 28 \end{matrix}\right\}$$

There is also direct proportion between cost and days.

Here Product of extremes = $12 \times 28 \times x$

Product of means = $6720 \times 8 \times 14$

and Product of extremes = Product of means

$12 \times 28x = 6720 \times 8 \times 14$

Hence $$x = \frac{6720 \times 8 \cdot 14}{12 \times 28}$$

= Rs. 2240

2nd condition

Quantity of hay = 770

Number of days = 28

Let Number of Cows = x

No. of Cows	No. of days	No. of hay
↑ 14	↓ 18	↑ 63
x	28	770

$$x : 14 :: \begin{Bmatrix} 18 : 28 \\ 770 : 63 \end{Bmatrix}$$

Remember That:

We have some quantity of hay.

Now this quantity is enough for 14 Cows for 18 days.

Now same quantity have to finish in 20 days, then number of Cows less than before. Thus there is direct proportion between Cows and days.

Now number of days are fixed. And 63 kg hay finished 14 Cows. For these fix days for 770 kg hay, number of Cows more than before. This is direct proportion.

Now Product of extremes = Product of means

$$28 \times 63x = 14 \times 18 \times 770$$

$$x = \frac{14 \times 18 \times 770}{28 \times 63}$$

$$= 110 \quad \text{(Cows)}$$

12. ***Juice manufacturer produces 3000 bottles in a day employing 15 workers working 8 hours. Find the***

10. If the stay of 14 men for 8 days in a hotel costs Rs. 22, 400. Find the cost for the stay of 7 men for 13 days.

Solution:

1st condition

Number of men = 14

Number of days = 8

Cost = 22400

2nd condition

Number of men = 7

Number of days = 13

Let Cost = x

There is also direct proportion between cost and days.

Cost — No. of men — No. of days

22400 ↑ 14 ↑ 8

7 ↑ 13

There is direct proportion between cost and men.

Similarly $x : 22400 :: \begin{cases} 7 : 14 \\ 13 : 8 \end{cases}$

Here Prodcut of extremes = $14 \times 8 \times x$

and Product of means = $22400 \times 7 \times 13$

Product of extremes = Product of means

$$14 \times 8\, x = 13 \times 7 \times 22400$$

Hence $$x = \frac{22400 \times 7 \times 13}{14 \times 8}$$

$$= \text{Rs. } 18200$$

11. 14 cows consume 63 kg of hay in 18 days. How many cows will eat 770 kg of hay in 28 days at same rate?

Solution:

1st condition

Number of Cows = 14 Cows

Quantity of Hay = 63 kg

Number of days = 18 days

number of bottles manufactured when he employs 18 workers working 6 hours.

Solution:

1st condition

Number of employs = 15

Daily time duration = 8 hours

Number of produced bottles = 3000

2nd condition

Number of employs = 18

daily time duration = 6 hours

Let

Number of produced bottles = Rs. x

No. of bottles	Daily time duration	No. of employs
3000 ↑	↑ 8	↑ 15
x	6	18

$$x : 3000 :: \left\{ \begin{matrix} 6 : 8 \\ 18 : 15 \end{matrix} \right\}$$

Product of extremes = Product of means

$$8 \times 15x = 3000 \times 6 \times 18$$

$$x = \frac{3000 \times 6 \times 18}{8 \times 15}$$

$$= 2700 \quad \text{Bottles}$$