Exercise 6.1

Q.1 Find the H.C.F. of the following expressions.

(i) $39x^7y^3z$ **and** $91x^5y^6z^7$

Solution:

$39x^7y^3z = 3\times 13x^7y^3z$

$91x^5y^6z^7 = 13\times 7x^5y^6z^7$

H.C.F= $13x^5y^3z$

(ii) $102xy^2z, 85x^2yz$ **and** $187xyz^2$

Solution:

$102xy^2z = 2\times 3\times 17xy^2z$

$85x^2yz = 5\times 17x^2yz$

$187xyz^2 = 11\times 17xyz^2$

H.C.F. = $17xyz$

Q.2 Find the H.C.F. of the following expressions by factorization.

(i) $x^2+5x+6, \quad x^2-4x-12$

Solution:

$$x^2+5x+6 = x^2+3x+2x+6$$

$$= x(x+3)+2(x+3)$$

$$= (x+3)(x+2)$$

$$x^2-4x-12 = x^2-6x+2x-12$$

$$= x(x-6)+2(x-6)$$

$$= (x-6)(x+2)$$

H.C.F. = $x+2$

(ii) $x^3-27, x^2+6x-27, 2x^2-18$

Solution:

$$x^3-27 = (x)^3-(3)^3 = (x-3)(x^2+3x+9)$$

$$x^2+6x-27 = x^2+9x-3x-27$$

$$= x(x+9)-3(x+9)$$

$$= (x+9)(x-3)$$

$$2x^2-18 = 2(x^2-9) = 2(x^2-3^2)$$

$$= 2(x+3)(x-3)$$

H.C.F. = $x-3$

(iii) $x^3-2x^2+x,\ x^2+2x-3,\ x^2+3x-4$

Solution:

$$x^3-2x^2+x=x\left(x^2-2x+1\right)=x(x-1)^2$$

$$=x(x-1)(x-1)$$

$$x^2+2x-3=x^2+3x-x-3$$

$$=x(x+3)-1(x+3)$$

$$=(x+3)(x-1)$$

$$x^2+3x-4=x^2+4x-x-4$$

$$=x(x+4)-1(x+4)$$

$$=(x+4)(x-1)$$

H.C.F. = $x-1$

(iv) $18(x^3-9x^2+8x),\ 24\left(x^2-3x+2\right)$

Solution:

$$18(x^3-9x^2+8x)=18\left(x^2-9x+8\right)$$

$$=18x\left(x^2-8x-x+8\right)$$

$$=18x\left[x(x-8)-1(x-8)\right]$$

$$=2\times3^2\times(x-2)(x-1)$$

H.C.F. $=2\times3(x-1)=6(x-1)$

(v) $36\left(3x^4+5x^3-2x^2\right),\ 54\left(27x^4-x\right)$

Solution:

$$36\left(3x^4+5x^3-2x^2\right)=4\times9x^2\left(3x^2+5x-2\right)$$

$$=4\times9x^2\left(3x^2+6x-x-2\right)$$

$$=2^2.3^2x^2\left[3x(x+2)-1(x+2)\right]$$

$$=2^2.3^2x^2(x+2)(3x-1)$$

$$54\left(27x^4-2\right)=2\times27x\left(27x^3-1\right)$$

$$=2\times3^3x[(3x)^3-(1)]$$

$$=2\times3^3x(3x-1)(9x^2+3x+1)$$

H.C.F $=2\times 3^2x(3x-1)=18x(3x-1)$

Q.3 Find the HCF of the following by division method.

(i) $x^3+3x^2-16x+12,\quad x^3+x^2-10x+8$

Solution:

$$\begin{array}{r|c} & 1 \\ \cline{2-2} x^3+x^2-10x+8 & x^3+3x^2-16x+12 \\ & \pm x^3\pm x^2\mp 10x\pm 8 \\ \cline{2-2} & 2x^2-6x+4 \\ & 2(x^2-3x+2) \end{array}$$

By Ignoring 2

$$\begin{array}{r|c} \cline{2-2} x^2-3x+2 & x^3+x^2-10x+8 \\ & \pm x^3\mp 3x^2\pm 2x \\ \cline{2-2} & 4x^2-12x+8 \\ & \pm 4x^2\mp 12x+8 \\ \cline{2-2} & 0 \end{array}$$

H.C.F. $=x^2-3x+2$

(ii) $x^4+x^3-2x^2-x-3,\ 5x^3+3x^2-17x+6$

Solution:

$$\begin{array}{r|c} & x+2 \\ \cline{2-2} 5x^3+3x^2-17x+6 & x^4+x^3-2x^2-x-3 \\ & \times 5 \\ \cline{2-2} & 5x^4+5x^3-10x^2-5x-15 \\ & \pm 5x^4\pm 3x^3\mp 17x^2\pm 6x \\ \cline{2-2} & 2x^3+7x^2-x-15 \\ & \times 5 \\ \cline{2-2} & 10x^3+35x^2-5x-75 \\ & \pm 10x^3\pm 6x^2\mp 34x\pm 12 \\ \cline{2-2} & 29x^2+29x-87 \\ & 29(x^2+x-3) \end{array}$$

By Ignoring 29

$$\begin{array}{r|l} \multicolumn{1}{r}{} & \multicolumn{1}{c}{5x-2} \\ \cline{2-2} x^2+x-3 & 5x^3+3x^2-17x+6 \\ & \pm 5x^3 \pm 5x^2 \mp 15x \\ \cline{2-2} & -2x^2-2x+6 \\ & \mp 4x^2 \mp 2x \pm 6 \\ \cline{2-2} \end{array}$$

$$\begin{array}{r|l} \cline{2-2} & \quad 0 \end{array}$$

$H.C.F. = x^2+x-3$

(iii) $2x^5-4x^4-6x, \quad x^5+x^4-3x^3-3x^2$

Solution:

$2x^5-4x^4-6x = 2x\left(x^4-2x^3-3\right)$

$x^5+x^4-3x^3-3x^2 = x^2\left(x^3+x^2-3x-3\right)$

In this case H.C.F of $2x \text{ and } x^2 \text{ is } x$

Now we find H.C.F. of $x^4-2x^3 \text{ and } x^3+x^2-3x-3$

$$\begin{array}{r|l} \multicolumn{1}{r}{} & \multicolumn{1}{c}{x-3} \\ \cline{2-2} x^3+x^2-3x-3 & x^4-2x^3-x^2+x-3 \\ & \pm x^4 \pm x^3 \mp 3x^3 \mp 3x \\ \cline{2-2} & -3x^3+3x^2+3x-3 \\ & \mp 3x^3 \mp 3x^2 \pm 3x \pm 3 \\ \cline{2-2} & 6x^2-6x-12 \\ & 6\left(x^2-x-2\right) \end{array}$$

By Ignoring 6

$$\begin{array}{r|l} \multicolumn{1}{r}{} & \multicolumn{1}{c}{x+2} \\ \cline{2-2} x^2-x-2 & x^3+x^2-3x-3 \end{array}$$

$$\pm x^3 \mp x^2 \mp 2x$$

$$2x^2 - x - 3$$

$$\pm 2x^2 \mp 2x \mp 4$$

$$x + 1$$

Then

$$\begin{array}{r|l} & x-2 \\ \hline x+1 & x^2 - x - 2 \\ & \pm x^2 \pm x \\ \hline & -2x - 2 \\ & \mp 2x \mp 2 \\ \hline & 0 \end{array}$$

$H.C.F = x + 1$

Hence the H.C.F. of the given expression is

$$x \times (x+1) = x^2 + x$$

Q.4 Find the LCM of the following expressions.

(i) $39x^7y^3z \text{ and } 91x^5y^6z^7$

Solution:

$$39x^7y^3z = 3 \times 13x^7y^3z$$

$$91x^5y^6z^7 = 13 \times 7x^5y^6z^7$$

L.C.M. $= 3 \times 7 \times 13x^7y^6z^7$

$$= 273x^7y^6z^7$$

(ii) $102xy^2z,\ 85\,x^2yz \text{ and } 187\,xyz^2$

Solution:

$$102xy^2z = 2 \times 3 \times 17xy^2z$$

$$85x^2yz = 5 \times 17x^2yz$$

$$187xyz^2 = 11 \times 17xyz^2$$

L.C.M. $= 2 \times 3 \times 5 \times 11 \times 17x^2y^2z^2$

$$= 5610x^2y^2z^2$$

Q.5 Find the LCM of the following expressions by factorization.

(i) $x^2 + 25x + 100 \text{ and } x^2 - x - 20$

Solution:

$$x^2 + 25x + 100 = x^2 - 20x - 5x + 100$$

$$= x(x-20) - 5(x-20)$$

$$= (x-20)(x-5)$$

$$x^2-x-20 = x^2-5x+4x-20$$

$$= x(x-5)+4(x-5)$$

$$= (x-5)(x+4)$$

$$\text{L.C.M.} = (x-5)(x-20)(x+4)$$

(ii) $x^2+4x+4,\ x^2-4,\ 2x^2+x-6$

Solution:

$$x^2+4x+4=(x+2)^2$$

$$x^2-4=(x+2)(x-2)$$

$$2x^2+x-6=2x^2+4x-3x-6$$

$$=2x(x+2)-3(x+2)$$

$$=(x+2)(2x-3)$$

$$L.C.M.=(x+2)^2(x-2)(2x-3)$$

(iii) $2\left(x^4-y^4\right),3\left(x^3+2x^2y-xy^2-2y^3\right)$

Solution:

$$2\left(x^4-y^4\right)=2\left(x^2-y^2\right)\left(x^2+y^2\right)$$

$$=2(x-y)(x+y)$$

$$=2\left(x^2-y^2\right)\left(x^2+y^2\right)\left(x^2+y^2\right)$$

$$3\left(x^3+2x^2y-xy^2-2y^3\right)=3\left[x^2(x+2y)-y^2(x+2y)\right]$$

$$=3(x+2y)\left(x^2+y^2\right)$$

$$=3(x+y)(x-y)(x+2y)$$

$$L.C.M.=2.3(x-y)(x+y)\left(x^2+y^2\right)(x+2y)$$

$$=6\left(x^2-y^2\right)\left(x^2+y^2\right)(x+2y)$$

$$=6\left(x^4-y^4\right)(x+2y)$$

(iv) $4\left(x^4-1\right),\ 6\left(x^3-x^2-x+1\right)$

Solution:

$$4(x^4-1)=4(x^2-1)(x^2+1)$$

$$=2^2(x-1)(x+1)(x^2+1)$$

$$6(x^3-x^2-x+1)=6[x^2(x-1)-1(x-1)]$$

$$=2.3(x-1)(x^2+1)$$

$$=2.3(x-1)(x-1)(x+1)$$

$$=2.3(x-1)^2(x+1)$$

$$L.C.M.=2^2.3(x-1)^2(x+1)(x^2+1)$$

$$=12(x-1)^2(x+1)(x^2+1)$$

$$=12(x-1)(x-1)(x+1)(x^2+1)$$

$$=12(x-1)(x^2-1)(x^2+1)$$

$$=12(x-1)(x^4-1)$$

Q.6 **For what value of k. is $(x+4)$ the H.C.F of** $(x^2+x-(2k+2)$ *and* $2x^2+kx-12$?

Solution:

Let $p(x)=x^2+x-(2k+2)$

And $q(x)=2x^2+kx-12$

As $x+4$ *is H.C.F of* $p(x)$ *and* $q(x)$. *So* $p(x)$ *is exactly divisible by* $x+4$ *and thus* $p(-4)=0$

i.e $(-4)^2+(-4)-(2k+2)=0$

$$=16-4-2k-2=0$$

$$10-2k=0$$

$$2k=10$$

$$k=5$$

Q.7 **If $(x+3)(x-2)$ is the H.C.F of $p(x)=(x+3)(2x^2-3x+k)$ and $q(x)=(x-2)(3x^2=7x-1)$, find** *k and l.*

Solution:

$$p(x)=(x+3)(2x^2-3x+k)$$

$$q(x)=(x-2)(3x^2=7x-1)$$

H.C.F. of $p(x)$ *and* $q(x)=(x+3)(x-2)$

$(x+3)(x-2)$ is a factor of

$(x+3)(2x^2-3x+k)$

Hence $x-2$ is a factor of $2x^2-3x+k$

$\therefore$ $2(2)^2-3(2)+k=0$

$$8-6+k=0$$

$$2+k=0$$

$$k=-2$$

Similarly, $(x+3)(x-2)$ is a factor of $(x-2)(3x^2+7x-l)$

$x+3$ is a factor of $(3x^2+7x-l)$

$$3(-3)^2+7(-3)-l=0$$

$$27-21-l=0$$

$$6-l=0$$

$$l=6$$

Q.8 The L.C.M. and H.C.F. of two polynomials $p(x)$ and $q(x)$ are $2(x^4-1)$ and $(x+1)(x^2+1)$respectively. If $p(x)=x^3+x^2+x+1$, find $q(x)$.

Solution:

$$L.C.M=2(x^4-1)$$

$$H.C.F=(x+1)(x^2+1)$$

$$p(x)=x^3+x^2+x+1.$$

$$q(x)=\frac{(L.C.M)\times(H.C.F)}{P(X)}$$

$$=\frac{2(x^4-1)(x+1)(x^2+1)}{x^3+x^2+x+1}$$

$$=\frac{2(x^4-1)x^3+x+x^2+1}{x^3+x^2+x+1}$$

$$=\frac{2(x^4-1)(x^3+x^2+x+1)}{x^3+x^2+x+1}$$

$$=2(x^4-1)$$

Q.9 Let $p(x)=10(x^2-9)(x^2-3x+2)$ and $q(x)=10x(x+3)(x-1)^2$. If the H.C.F of $p(x),q(x)$ is $10(x+3)(x-1)$, find their L.C.M.

Solution:

$$p(x)=10(x^2-9)(x^2-3x+2)$$

$$q(x)=10x(x+3)(x-1)^2$$

$$H.C.F=10(x+3)(x-1)$$

$$L.C.M.=\frac{p(x)\times q(x)}{H.C.F}$$

$$=\frac{10\left(x^2-9\right)\left(x^2-3x+2\right)10x(x+3)(x-1)^2}{10(x+3)(x-1)}$$

$$=10\left(x^2-9\right)\left(x^2-3x+2\right).x(x-1)$$

$$=10\left(x^2-9\right)\left[\left(x^2-2x-x+2\right).x(x-1)\right]$$

$$=10\left(x^2-9\right)[x(x-2)-1(x-2].x.(x-1)$$

$$=10\left(x^2-9\right)(x-2)(x-1).x.(x-1)$$

$$=10\left(x^2-9\right)(x-2).x.(x-1)^2$$

$$=10x(x-2)(x-1)^2\left(x^2-9\right)$$

Q.10 Let the product of L.C.M and H.C.F. two polynomials be $(x+3)^2(x-2)(x+5)$. If one polynomial is $(x+3)(x-2)$ and the second polynomial is $x^2+kx+15$. Find the value of k.

Solution:

$$(L.C.M.)(H.C.F.)=(x+3)^2(x-2)(x+5)$$

$$p(x)=(x+3)(x-2)$$

$$q(x)=x^2+kx+15$$

$$p(x).q(x)=(L.C.M.)\times(H.C.F.)$$

$$(x+3)(x-2)\left(x^2+kx+15\right)=(x+3)^2(x-2)(x+5)$$

$$x^2+kx+15=(x+3)(x+5)$$

$$x^2+kx+15=x^2+8x+15$$

$$k=8$$

Q.11 Waqas wishes to distribute 128 bananas and also176 apples equally among a certain number of children. Find the highest number of children who can get the fruit in this way.

Solution:

Required number of children = H.C.F. of 128 and 176.

```
         1
  128 | 176
        128   2
       48 | 128
             96   1
            32 | 48
                 32   2
                16 | 32
                     32
                     --
                      0
```

H.C.F. = 16

Hence the highest number of children = 16

Exercise 6.2

Simplify each of the following as rational expression.

Q1. $\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12}$

Solution:

$$=\frac{(x^2-3x+2x-6)}{(x^2-3^2)}+\frac{(x^2+6x-4x-24)}{(x^2-4x+3x-12)}$$

$$=\frac{x(x-3)+2(x-3)}{(x-3)(x-3)}+\frac{x(x+6)-4(x-6)}{x(x-4)+3(x-4)}$$

$$=\frac{(x-3)(x+2)}{(x-3)(x-3)}+\frac{(x+6)(x-4)}{(x-4)(x+3)}$$

$$=\frac{x+2}{x+3}+\frac{x+6}{x+3}$$

$$=\frac{x+2+x+6}{x+3}$$

$$=\frac{2x+8}{x+3}$$

$$=\frac{2(x+4)}{x+3}$$

Q2. $\left[\frac{(x+1)}{(x-1)}-\frac{(x-1)}{(x+1)}-\frac{4x}{(x^4+1)}\right]+\frac{4x}{(x^4-1)}$

Solution:

$$=\frac{(x+1)^2(x^2+1)-(x-1)^2(x^2+1)-4x(x-1)(x+1)}{(x-1)(x+1)(x^2+1)}+\frac{4x}{x^4-1}$$

$$=\frac{(x^2+2x+1)(x^2+1)-(x^2-2x+1)(x^2+1)-4x(x^2-1)}{(x^2-1)(x^2+1)}+\frac{4x}{x^4-1}$$

$$=\frac{x^4+x^2+2x^3+2x+1-(x^4+x^2-2x^3-2x+x^2+1)}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{x^4+2x^3+2x^2+1-x^4+2x^3-2x^2+2x-1-4x^3+4x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{4x^3+4x-4x^3+4x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{8x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{8x+4x}{x^4-1}$$

$$=\frac{12x}{x^4-1}$$

Q3. $\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$

Solution:

$$\frac{1}{x^2-5x-3x+15}+\frac{1}{x^2-3x-x+3}-\frac{2}{x^2-5x-x+5}$$

$$=\frac{1}{x(x-5)-3(x-5)}+\frac{1}{x(x-3)-1(x-3)}-\frac{2}{x(x-5)-1(x-5)}$$

$$=\frac{1}{(x-5)(x-3)}+\frac{1}{(x-3)(x-1)}-\frac{2}{(x-5)(x-1)}$$

$$=\frac{x-1+x-5-2(x-3)}{(x-5)(x-3)(x-1)}$$

$$=\frac{2x-6-2x+6}{(x-5)(x-3)(x-1)}$$

$$=\frac{0}{(x-5)(x-3)(x-1)}$$

$$=0$$

Q4. $\frac{(x+2)(x+3)}{x^2-9}+\frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

Solution:

$$=\frac{(x+2)(x+3)}{x^2-3^2}+\frac{(x+2)2(x^2-16)}{(x-4)(x^2-3x+2x-6)}$$

$$=\frac{(x+2)(x+3)}{(x+3)(x-3)}+\frac{2(x+2)(x+4)(x-4)}{(x-4)[x(x-3)+2(x-3)]}$$

$$=\frac{x+2}{x-3}+\frac{2(x+2)(x+4)}{(x-3)(x+2)}$$

$$=\frac{x+2}{x-3}+\frac{2(x+4)}{(x-3)}$$

$$=\frac{x+2+2x+8}{x-3}$$

$$=\frac{3x+10}{x-3}$$

Q5. $\frac{x+3}{2x^2+9x+9}+\frac{1}{2(2x-3)}-\frac{4x}{4x^2-9}$

Solution:

$$=\frac{x+3}{2x^2+6x+3x+9}+\frac{1}{2(2x-3)}-\frac{4x}{(2x)^2-3^2}$$

$$=\frac{x+3}{2x(x+3)+3(x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x+3)(2x-3)}$$

$$=\frac{x+3}{(x+3)(2x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x+3)(2x-3)}$$

$$=\frac{1}{(2x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x+3)(2x-3)}$$

$$=\frac{2(2x-3)+2x+3-(4x)2}{2(2x+3)(2x-3)}$$

$$=\frac{4x-6+2x+3-8x}{2(2x+3)(2x-3)}$$

$$=\frac{-2x-3}{2(2x+3)(2x-3)}$$

$$=\frac{-(2x+3)}{2(2x+3)(2x-3)}$$

$$=\frac{-1}{2(2x-3)}$$

$$=\frac{1}{2(3-2x)}$$

Q6. $A-\frac{1}{A}$ **where** $A=\frac{a+1}{a-1}$

Solution:

$$A-\frac{1}{A}=\frac{a+1}{a-1}-\frac{a-1}{a+1}$$

$$=\frac{(a+1)^2-(a-1)^2}{(a-1)(a+1)}$$

$$=\frac{a^2+2a+1-(a^2-2a+1)}{a^2-1}$$

$$=\frac{a^2+2a+1-a^2+2a-1}{a^2-1}$$

$$=\frac{4a}{a^2-1}$$

Q7. $\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$

Solution:

$$=\left[\frac{x-1}{x-2}+\frac{2}{-(x-2)}\right]-\left[\frac{x+1}{x+2}+\frac{4}{-(x^2-4)}\right]$$

$$=\left[\frac{x-1}{x-2}+\frac{2}{x-2}\right]-\left[\frac{x+1}{x+2}+\frac{4}{x^2-4}\right]$$

$$=\frac{x-1-2}{x-2}-\frac{(x+1)(x-2)-4}{x^2-4}$$

$$=\frac{x-3}{x-2}-\frac{x^2-x-2-4}{x^2-4}$$

$$=\frac{x-3}{x-2}-\frac{x^2-x-6}{x^2-4}$$

$$=\frac{x-3}{x-2}-\frac{x^2-3x+2x-6}{x^2-2^2}$$

$$=\frac{x-3}{x-2}-\frac{x(x-3)+2(x-3)}{(x+2)(x-2)}$$

$$=\frac{x-3}{x-2}-\frac{(x-3)(x+2)}{(x+2)(x-2)}$$

$$=\frac{x-3}{x-2}-\frac{x-3}{x-2}$$

$$=0$$

Q8. What rational expression should be subtracted from $\frac{2x^2+2x-7}{x^2+x-6}$ to get $\frac{x-1}{x-2}$?

Solution:

Required expression

$$=\frac{2x^2+2x-7}{x^2+x-6}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{x^2+3x-2x-6}-\frac{x-1}{x-2}$$

$$= \frac{2x^2+2x-7}{(x+3)(x-2)} - \frac{x-1}{x-2}$$

$$= \frac{2x^2+2x-7-(x+3)(x-1)}{(x+3)(x-2)}$$

$$= \frac{2x^2+2x-7-(x^2+2x-3)}{(x+3)(x-2)}$$

$$= \frac{2x^2+2x-7-x^2-2x+3}{(x+3)(x-2)}$$

$$= \frac{x^2-4}{(x+3)(x-2)}$$

$$= \frac{(x+2)(x-2)}{(x+3)(x-2)}$$

$$= \frac{x+2}{x-3}$$

Q9. Perform the indicated operations and simplify to the lowest form
$\frac{x^2+x-6}{x^2-x-6} \times \frac{x^2-4}{x^2-9}$.

Solution:

$$= \frac{x^2+3x-2x-6}{x^2-3x+2x-6} \times \frac{x^2-2^2}{x^2-3^2}$$

$$= \frac{x(x+3)-2(x+3)}{x(x-3)+2(x-3)} \times \frac{(x+2)(x-2)}{(x+3)(x-3)}$$

$$= \frac{(x+3)(x-2)}{(x-3)(x+2)} \times \frac{(x+2)(x-2)}{(x+3)(x-3)}$$

$$= \frac{(x+3)(x-2)}{(x-3)(x+2)}$$

$$= \frac{(x-2)^2}{(x-3)^2}$$

Q10. $\frac{x^3-8}{x^2-4} \times \frac{x^2+6x+8}{x^2-2x+1}$

Solution:

$$= \frac{x^3-2^3}{x^2-2^2} \times \frac{x^2+4x+2x+8}{(x-1)^2}$$

$$= \frac{(x-2)(x^2+2x+4)}{(x+2)(x-2)} \times \frac{x(x+4)+2(x+4)}{(x-1)(x-1)}$$

$$= \frac{x^2+2x+4}{x+2} \times \frac{(x+4)(x+2)}{(x-1)(x-1)}$$

$$= \frac{(x+4)(x^2+2x+4)}{(x-1)^2}$$

Q11. $\frac{\mathbf{x^4-8x}}{\mathbf{2x^2+5x-3}} \times \frac{\mathbf{2x-1}}{\mathbf{x^2+2x+4}} \times \frac{\mathbf{x+3}}{\mathbf{x^2-2x}}$

Solution:

$$= \frac{x^4-8x}{2x^2+5x-3} \times \frac{2x-1}{x^2+2x+4} \frac{x+3}{x^2-2x}$$

$$= \frac{x(x^3-8)}{2x^2+6x-x-3} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x(x-2)}$$

$$= \frac{x(x-2)(x^2+2x+4)}{2x(x+3)-1(x+3)} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x(x-2)}$$

$$= \frac{x(x-2)(x^2+2x+4)}{(x+3)(2x-1)} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x(x-2)}$$

$$= 1$$

Q12. $\frac{\mathbf{2y^2+7y-4}}{\mathbf{3y^2-13y+4}} \div \frac{\mathbf{4y^2-1}}{\mathbf{6y^2+y-1}}$

Solution:

$$= \frac{2y^2+7y-4}{3y^2-13y+4} \times \frac{6y^2+y-1}{4y^2-1}$$

$$= \frac{2y^2+8y-y-4}{3y^2-12y-y+4} \times \frac{6y^2+3y-2y-1}{(2y)^2-1^2}$$

$$= \frac{2y(y+4)-1(y+4)}{3y(y-4)-1(y-4)} \times \frac{3y(2y+1)-1(2y+1)}{(2y+1)(2y-1)}$$

$$= \frac{(y+4)(2y-1)}{(y-4)(3y-1)} \times \frac{(2y+1)(3y-1)}{(2y+1)(2y-1)}$$

$$= \frac{y+4}{y-4}$$

Q13. $\left[\frac{\mathbf{x^2+y^2}}{\mathbf{x^2-y^2}} - \frac{\mathbf{x^2-y^2}}{\mathbf{x^2+y^2}}\right] \div \left[\frac{\mathbf{x+y}}{\mathbf{x-y}} - \frac{\mathbf{x-y}}{\mathbf{x+y}}\right]$

Solution:

$$= \frac{(x^2+y^2)^2-(x^2-y^2)^2}{(x^2-y^2)(x^2+y^2)} \div \frac{(x+y)^2-(x-y)^2}{(x+y)(x-y)}.$$

$$= \frac{x^4+2x^2y^2+y^4-(x^4-2x^2y^2+y^4)}{(x^2-y^2)(x^2+y^2)} \div \frac{x^2+2xy+y^2-(x^2-2xy+y^2)}{(x+y)(x-y)}.$$

$$= \frac{x^4+2x^2y^2+y^4-x^4+2x^2y^2-y^4}{(x^2-y^2)(x^2+y^2)} \div \frac{x^2+2xy+y^2-x^2+2xy-y^2}{(x+y)(x-y)}$$

$$= \frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)} \times \frac{4xy}{x^2-y^2}$$

$$= \frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)} \times \frac{x^2-y^2}{4xy}$$

$$= \frac{xy}{x^2+y^2}$$

Exercise 6.3

Use factorization to find the square root of following equations.

(i) $4x^2-12xy+9y^2$

Solution:

$$=(2x)^2-2(2x)(3y)+(3y)^2$$

$$=(2x-3y)^2$$

$$=\sqrt{(2x-3y)^2}$$

Required square root is $\pm(2x-3y)$

(ii) $x^2-1+\frac{1}{4x^2}$ $(x\neq 0)$

Solution

$$=(x)^2-2(x)\left(\frac{1}{2x}\right)+\left(\frac{1}{2x}\right)^2$$

$$=\left(x-\frac{1}{2x}\right)^2$$

Required square root is $\pm\left(x-\frac{1}{2x}\right)^2$

(iii) $\frac{1}{16}x^2-\frac{1}{12}xy+\frac{1}{36}y^2$

Solution:

$$=\left(\frac{1}{16}x\right)^2-2\left(\frac{1}{4}x\right)\left(\frac{1}{6}y\right)+\left(\frac{1}{6}y\right)^2$$

$$=\left(\frac{1}{4}x-\frac{1}{6}y\right)^2$$

Required square root is $\pm\left(\frac{1}{4}x-\frac{1}{6}y\right)^2$

Or $\pm\left(\frac{x}{4}-\frac{y}{6}\right)$

(iv) $4(a+b)^2-12(a^2-b^2)+9(a-b)^2$

Solution:

$$=\left[2(a+b)^2-2[2(a+b)\right]\left[3(a-b)\right]+\left[3(a-b)\right]^2$$

$$=\left[2(a+b)-3(a-b)\right]^2$$

$$=(2a+2b-3a+3b)^2$$

$$=(5b-a)^2$$

Required square root is $\pm(5b-a)^2$

(v) $\dfrac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

Solution:

$$=\frac{\left(2x^3\right)^3-2\left(2x^3\right)\left(3y^3\right)+\left(3Y^3\right)^2}{\left(3x^2\right)^2-2\left(3x^2\right)\left(4y^2\right)+\left(4y^2\right)^2}$$

$$=\frac{\left(2x^3-3y^3\right)^2}{\left(3x^2+4y^2\right)^2}$$

$$=\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2$$

Required square root is $\pm\left(\dfrac{2x^3-3y^3}{3x^2+4y^2}\right)$

(vi) $\left(x+\dfrac{1}{x}\right)^2-4\left(x-\dfrac{1}{x}\right)\qquad(x\neq 0)$

Solution:

$$=x^2+2+\frac{1}{x^2}-4\left(x-\frac{1}{x}\right)$$

$$=x^2+2+\frac{1}{x^2}-4\left(x-\frac{1}{x}\right)+4$$

$$=\left(x-\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right)+4$$

$$=\left(x-\frac{1}{x}\right)^2-2\left(x-\frac{1}{x}\right).2+(2)^2$$

$$=\left[\left(x-\frac{1}{x}\right)-2\right]^2$$

Required square root is $\pm\left[\left(x-\dfrac{1}{x}\right)-2\right]$

(vii) $\left(x^2-\dfrac{1}{x^2}\right)^2-4\left(x+\dfrac{1}{x}\right)^2+12\qquad(x\neq 0)$

Solution:

$$=x^4+2+\frac{1}{x^4}-4\left(x^2+2+\frac{1}{x^2}\right)+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)-8+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)+4$$

$= \left(x^2+\frac{1}{x^2}\right)^2-2\left(x^2+\frac{1}{x^2}\right).2+(2)^2$

$= \left[\left(x^2+\frac{1}{x^2}\right)-2\right]^2$

Required square root is $\pm\left[\left(x^2+\frac{1}{x^2}\right)-2\right]$

(viii) $(x^2+3x+2)(x^2+4x+3)(x^2+5x+6)$

Solution:

$= (x^2+2x+x+2)(x^2+3x+x+3)(x^2+3x+2x+6)$

$= [x(x+2)+1(x+2)][x(x+3)+1(x+3)]$

$[x(x+3)+2(x+3)]$

$= (x+2)(x+1)(x+3)(x+1)(x+3)(x+2)$

$= (x+1)^2(x+2)^2(x+3)^2$

$= [(x+1)(x+2)(x+3)]^2$

Required square root is $\pm[(x+1)(x+2)(x+3)]$

(ix) $(x^2+8x+7)(2x^2-x-3)(2x^2+11x-21)$

Solution:

$= (x^2+7x+x+7)(2x^2-3x+2x-3)$

$(2x^2+14x-3x-21)$

$= [x(x+7)+1(x+7)][x(2x-3)+1(2x-3)$

$= [2x(x+7)-3(x+7)]$

$= (x+7)(x+1)(2x-3)(x+1)(x+7)(2x-3)$

$= (x+7)^2(x+1)^2(2x-3)^2$

$= [(x+7)(x+1)(2x-3)]^2$

So, the required square root is $\pm[(x+7)(x+1)(2x-3)]$

Q2. Use division method to find the square root of following equations.

(i) $4x^2+12xy+9y^2+16x+24y+16$

Solution:

		$2x$	$+3y$	$+4$		
$2x$	$4x^2$	$+12xy^2$	$+9y^2$	$+6x$	$+24y$	$+16$
	$\pm4x^2$					
$4x+3y$	$12xy$	$+16x$	$+9y^2$	$+24y$	$+16$	
	$\pm12xy$		$\pm9y^2$			
$4x+6y+4$		$16x$		$+24y$	$+16$	
		$\pm16x$		$\pm24y$	±16	
			0			

The square root is $\pm(2x+3y+4)$

(ii) $x^4-10x^3+37x^2-60x+36$

Solution:

		x^2	$-5x$	$+6$	
x^2	x^4	$-10x^3$	$+37y^2$	$-60x$	$+36$
	$\pm x^4$				
$2x^2-5x$		$-10x^3$	$+37x^2$		
		$\pm10x^3$	$\pm25y^2$		
$2x^2-10x+6$			$12x^2$	$-60x$	$+36$
			$\pm12x^2$	$\pm60x$	±36
			0		

The square root is $\pm(x^2+5x+6)$

(iii) $9x^4-6x^3+7x^2-2x+1$

Solution:

$$\begin{array}{r|ccccc}
 & 3x & x & +1 & & \\ \hline
3x^2 & 9x^4 & +6x^3 & +7x^2 & -2x & +1 \\
 & \pm 9x^4 & & & & \\ \hline
6x^2-x & & -6x^3 & +7x^2 & & \\
 & & & \pm x^2 & & \\
 & & \pm 6x^3 & & & \\ \hline
6x^2-2x & & & 6x^2 & +2x & +1 \\
+1 & & & & & \\
 & & & \pm 6x^2 & \pm 2x & \pm 1 \\ \hline
 & & & 0 & &
\end{array}$$

The square root is $\pm\left(3x^2-x+1\right)$

(i) $4+25x^2-12x-24x^3+16x^4$

Solution:

$$\begin{array}{r|ccccc}
 & 4x^2 & -3x & +2 & & \\ \hline
4x^2 & 16x^4 & -24x^3 & +25x^2 & -12x & +4
\end{array}$$

$$\begin{array}{r|ccccc}
 & \pm 16x^4 & & & & \\ \hline
8x^2-3x & & -24x^3 & +25x^2 & & \\
 & & & \pm 9x^2 & & \\
 & & \pm 24x^3 & & & \\ \hline
8x^2-6z+ & & & 16x^2 & +12x & +4 \\
+2 & & & & & \\
 & & & \pm 16x^2 & \pm 12x & \pm 4 \\ \hline
 & & & 0 & &
\end{array}$$

The square root is $\pm\left(4x^2-3x+2\right)$

(ii) $\frac{x^2}{y^2}-10\frac{x}{y}+27-10\frac{x}{y}+\frac{x^2}{y^2}$ $(x\neq 0)(y\neq 0)$

Solution:

$$\begin{array}{c|ccccc}
 & & \frac{x}{y} & -5 & \frac{y}{x} & \\
\hline
\frac{x}{y} & \frac{x^2}{y^2} & -10\frac{x}{y} & +27 & -10\frac{x}{y} & +\frac{x^2}{y^2} \\
 & \pm\frac{x^2}{y^2} & & & & \\
\hline
\end{array}$$

$$\begin{array}{c|cccc}
2\frac{x}{y}-5 & -10\frac{x}{y} & +27 & & \\
 & & \pm 25 & & \\
 & \pm 10\frac{x}{y} & & & \\
\hline
 & & 2 & -10\frac{x}{y} & +\frac{x^2}{y^2} \\
2\frac{x}{y}-10 & & & & \\
+\frac{y}{x} & & & & \\
 & & \pm 2 & \pm 10\frac{x}{y} & \pm\frac{x^2}{y^2} \\
\hline
 & & 0 & & \\
\end{array}$$

The square root is $\pm\left(\frac{x}{y}-5+\frac{y}{x}\right)$

Q3. Find the value of k for which these following expressions will become perfect square.

(i) $4x^4-12x^3+37x^2-42x+k$

Solution:

$$\begin{array}{c|ccccc}
 & & 2x^2 & -3x & +7 & \\
\hline
2x^2 & 4x^4 & -12x^3 & +37x^2 & -42x & +k \\
\end{array}$$

	$\pm 4x^4$				
$84x^2-3$		$-12x^3$	$+37x^2$		
			$\pm 9x^2$		
		$\pm 12x^3$			
$4x^2-6x$			$28x^2$	$+42x$	
$+7$					$+k$
			$\pm 28x^2$		± 49
				$\pm 42x$	
		$k-49$			

The given expression will be perfect square when reminder = 0

If $k-49=0$

i.e. $k=49$

(iii) $x^4-4x^3+10x^2-kx+9$

Solution:

		x^2	$-2x$	$+3$	
x^2	x^4	$-4x^3$	$+10x^2$	$-kx$	$+9$
	$\pm x^4$				
$2x^2-2x$		$-4x^3$	$+10x^2$	$-kx$	$+9$

			$\pm 4x^2$		
		$\pm 4x^3$			
$2x^2-4x$			$6x^2$	$-kx$	$+9$
$+3$					
			$\pm 6x^2$	$\pm 12x$	
					± 9
			$-kx$	$+12x$	

The given expression will be perfect square when reminder = 0

If $-kx+12x=0$

$-k+12=0$

$-k=-12$

i.e. $k=12$

Q4. Find the value of L and m for which the following expression will become perfect square.

(i) $x^4 + 4x^3 + 16x^2 - lm + m$

Solution:

$$\begin{array}{r|lllll}
 & & x^2 & +2x & +6 & \\
\hline
x^2 & x^4 & +4x^3 & +16x^2 & +lx & +m \\
 & \pm x^4 & & & & \\
\hline
\end{array}$$

$$\begin{array}{r|llll}
\hline
2x^2+2x & 4x^3 & +16x^2 & +lx & +m \\
 & \pm 4x^3 & \pm 4x^2 & & \\
\hline
2x^2+4x & & 12x^2 & +lx & +m \\
+6 & & & & \\
 & & \pm 12x^2 & \pm 24x & \pm 36 \\
\hline
 & (l-24) & x- & (m & -36) \\
\end{array}$$

The given expression will be perfect square when reminder = 0

If $l - 24 = 0$

And $m - 36 = 0$

$\therefore$ $l = 24$ $m = 36.$

(ii) $49x^4 - 70x^3 + 109x^2 + lx - m$

Solution:

$$\begin{array}{r|lllll}
 & & 7x^2 & -5x & +6 & \\
\hline
7x^2 & 49x^4 & -70x^3 & +109x^2 & +lx & -m \\
 & \pm 49x^4 & & & & \\
\hline
14x^2+5 & & -70x^3 & +109x^2 & & \\
x & & & & & \\
\end{array}$$

$$\begin{array}{r|llll} & \pm70x^3 & \pm29x^2 & & \\ \hline 2x^2+4x+6 & & 84x^2 & +lx & -m \\ & & \pm84x^2 & \pm60x & \pm36 \\ \hline & (l+60) & x- & m & -36 \end{array}$$

The given expression will be perfect square

If $l+60=0$ And $-m-36=0$

$\therefore$ $l=-60$; $m=-36.$

Q5. To make the expression $9x^4-12x^3+22x^2-13x+12$, a perfect square

(i) What should be added to it?
(ii) What should be subtracted from it?
(iii) What should be the value of x?

Solution:

$$\begin{array}{r|llllll} & & 3x^2 & -2x & +3 & \\ \hline 3x^2 & 9x^4 & -12x^3 & +22x^2 & -13x. & +12 \\ & \pm9x^4 & & & & \\ \hline 6x^2-2x & & -12x^3 & +22x^2 & & \end{array}$$

$$\begin{array}{r|llll} & & \pm4x^2 & & \\ & \pm12x^3 & & & \\ \hline 6x^2-4x+3 & & 18x^2 & +13x & +12 \\ & & \pm18x^2 & \pm60x & \pm9 \\ \hline & & -x & +3 & \end{array}$$

(i) To make the expression a perfect square we should add $x-3$

(ii) To make the expression a perfect square we should subtract $-x+3$

(iii) To make the expression perfect square reminder = 0

$-x-3$ **Or** $x=3$.

Review Exercise 6

Q.1 Choose the correct answer.

(i) H.C.F. of $p^3q - pq^3$ and $p^5q^2 - p^2q^5$ is……

(a) $pq(p^2 - q^2)$ **(b)** $pq(p - q)$

(c) $p^2q^2(p - q)$ **(d)** $pq(p^3 - q^3)$

(ii) H.C.F of $5x^2y^2$ and $20x^3y^3$ is……..

(a) $5x^2y^2$ **(b)** $20x^3y^3$
(c) $100x^5y^5$ **(d)** $5xy$

(iii) H.C.F. of $x - 2$ and $x^2 + x - 6$ is……

(a) $x^2 + x - 6$ **(b)** $x + 3$

(c) $x - 2$ **(d)** $x + 6$

(iv) H.C.F of $a^3 + b^3$ and $a^2 - ab + b^2$ is…….

(a) $a + b$ **(b)** $a^2 - ab + b^2$

(c) $(a - b)^2$ **(d)** $a^2 + b^2$

(v) H.C.F of $x^2 - 5x + 6$ and $x^2 - x - 6$ is……..

(a) $x - 3$ **(b)** $x + 2$

(c) $x^2 - 4$ **(d)** $x - 2$

(vi) H.C.F of $a^2 - b^2$ and $a^3 - b^3$ is………

(a) $a - b$ **(b)** $a + b$

(c) $a^2 + ab + b^2$ **(d)** $a^2 - ab + b^2$

(vii) H.C.F of $x^2 + 3x + 2, x^2 + 4x + 3$ and $x^2 + 5x + 4$ is……

(a) $x + 1$ **(b)** $(x + 1)(x + 2)$

(c) $x + 3$ **(d)** $(x + 4)(x + 1)$

(viii) L.C.M. of $15x^2, 45xy$ and $30xyz$ is ……..

(a) $90xyz$ **(b)** $90x^2yz$

(c) $15xyz$ **(d)** $15x^2yz$

(ix) L.C.M. of $a^2 + b^2$ and $a^4 - b^4$ is……..

(a) $a^2 + b^2$ **(b)** $a^2 - b^2$

(c) $a^4 - b^4$ **(d)** $a - b$

(x) The product of algebraic expressions is equal to the…….. of their H.C.F. and L.C.M.

(a) Sum **(b)** Difference

(c) Product **(d)** Quotient

(xi) **Simplify** $\frac{a}{9a^2-b^2}+\frac{1}{3a-b}=\cdots$

(a) $\frac{4a}{9a^2-b^2}$ **(b)** $\frac{4a-b}{9a^2-b^2}$

(c) $\frac{4a+b}{9a^2-b^2}$ **(d)** $\frac{b}{9a^2-b^2}$

(xii) **Simplify** $\frac{a^2+5a-14}{a^2-3a-18}\times\frac{a+3}{a-2}=\cdots$

(a) $\frac{a+7}{a-6}$ **(b)** $\frac{a+7}{a-2}$

(c) $\frac{a+3}{a-6}$ **(d)** $\frac{a-2}{a+3}$

(xiii) **Simplify** $\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2-b^2}=\cdots$

(a) $\frac{1}{a+b}$ **(b)** $\frac{1}{a-b}$

(c) $\frac{a-b}{a^2+b^2}$ **(d)** $\frac{a+b}{a^2+b^2}$

(xiv) **Simplify** $\left(\frac{2x+y}{x+y}-1\right)\div\left(1-\frac{x}{x+y}\right)=\cdots$

(a) $\frac{x}{x+y}$ **(b)** $\frac{y}{x+y}$

(c) $\frac{y}{x}$ **(d)** $\frac{x}{y}$

(xv) **The square root of $a2-2a+1$ is……**

(a) $\pm(a+1)$ **(b)** $\pm(a-1)$

(c) $a-1$ **(d)** $a+1$

(xvi) **What should be added to complete the square of $x4+64$**

(a) $8x^2$ **(b)** $-8x^2$

(c) $16x^2$ **(d)** $4x^2$

(xvii) **The square root of $x4+1x4+2$ is…….**

(a) $\pm(x+\frac{1}{x})$ **(b)** $(x^2+\frac{1}{x^2})$

(c) $\pm(x-\frac{1}{x})$ **(d)** $\pm(x^2-\frac{1}{x^2})$

Answers:

(i) b	(ii) a	(iii) c	(iv) b	(v) a
(vi) a	(vii) a	(viii) b	(ix) c	(x) c
(xi) c	(xii) a	(xiii) a	(xiv) d	(xv) b
(xvi) c	(xvii) b			

Q2. Find the H.C.F of the following by factorization.

$\mathbf{8x^4 - 128}$, $\mathbf{12x^3 - 96}$

Solution:

$8x^4 - 128 = 8(x^4 - 16) = 8(x^4 - 4^2)$

$= 8(x^2 - 2^2)(x^2 + 2^2)$

H.C.F. $= 2^3(X - 2)(x + 2)(x^2 + 4)$

$12x^3 - 96 = 12(x^3 - 8) = 2^2 \times 3(x^3 - 2^3)$

$= 2^2 \times 3(x - 2)(x^2 + 2x + 4)$

H.C.F. $= 2^2(x - 2)$

$= 4(x-2)$

Q3. Find the H.C.F. of the following by division method.

$\mathbf{y^3 + 3y^2 - 3y - 9,\ y^3 + 3y^2 - 8y - 24}$

Solution:

$$\begin{array}{r|l} & 1 \\ \hline y^3 + 3y^2 - 3x - 9 & y^3 - 3y^2 - 8y + 24 \\ & \pm y^3 \pm x3y^2 \mp 3y \pm 9 \\ \hline & -5y - 15 \\ & -5(y + 3) \end{array}$$

By Ignoring -5

$$\begin{array}{r|l} & y^2 - 3 \\ \hline y + 3 & y^3 + 3y^2 - 3y - 9 \\ & \pm y^3 \mp 3y^2 \\ \hline & -3y - 9 \\ & \mp 3y \mp 9 \\ \hline & 0 \end{array}$$

So, H.C.F. is $y + 3$

Q4. Find the L.C.M. OF the following by factorization.

$$\mathbf{12x^2 - 75, 6x^2 - 13x - 5, 4x^2 - 20x + 25}$$

Solution:

$$12x^2 - 75 = 3(4x^2 - 25) = 3[(2x)^2 - 5^2)$$

$$= 3(2x - 5)(2x + 5)$$

$$6x^2 - 13x - 5 = 6x^2 - 15x + 2x - 5$$

$$= 3x(2x - 5)(3x + 1)$$

$$= (2x - 5)(3x + 1)$$

$$4x^2 - 20x + 5 = (2x)^2 - 2(2x)(5) + 25$$

$$= 3x(2x - 5)(3x + 1)$$

$$= (2x - 5)^2$$

So, L.C.M. $= 3(2x + 5)(3x + 1)(2x - 5)^2$

Q5. If H.C.F. of $x^4 + 3x^3 + 5x^2 + 26x + 56$ and $x^4 + 2x^3 - 4x^2 - x + 28$ is $x^2 + 5x + 7$. Find their L.C.M.

Solution:

Let $p(x) = x^4 + 3x^3 + 5x^2 + 26x + 56$

$Q(x) = x^4 + 2x^3 - 4x^2 - x + 28$

H.C.F. $= x^2 + 5x + 7$

L.C.M.$= \frac{p(x)\times Q(X)}{H.C.F}$

$$= \frac{(x^4+3x^3+5x^2+26x+56)(x^4+2x^3-4x^2-x+28)}{x^2+5x+7}$$

Now dividing

$$\begin{array}{r|rrrrr}
 & & x^2 & -2x & +8 & \\
\hline
x^2-5x+7 & x^4 & +3x^3 & +5x^2 & +26x & +56 \\
 & \pm x^4 & \pm 5x^3 & \pm 7^2 & & \\
\hline
 & & -2x^3 & +2x^2 & +26x & \\
 & & \pm 2x^3 & \pm 10x^2 & \pm 14x & \\
\hline
 & & & 8x^2 & +40x & +56 \\
 & & & \pm 8x^2 & \pm 40x & \pm 56 \\
\hline
\end{array}$$

0

So, L.C.M.

$$= \frac{(x^2+5x+7)(x^2+2x+8)(x^4+2x^3-4x62-x+28)}{x^2+5x+7}$$

$$= (x^2+2x+8)(x^4+2x^3-4x62-x+28)$$

Q6. Simplify

(i) $\frac{3}{x3+x^2+x+1} \quad \frac{3}{x^3-x^2+x-1}$

$$= \frac{3}{x^2(x+1)+1(x+1)} \quad \frac{3}{x^2(x-1)+1(x-1)}$$

$$= \frac{3}{(x+1)(x^2+1)} \quad \frac{3}{(x-1)(x^2+1)}$$

$$= \frac{3(x-1)-3(x+1)}{(x+1)(X-1)(x^2+1)}$$

$$= \frac{3x-3-3x-3}{(x^2-1)(x^2+1)}$$

$$= \frac{-6}{x^4-1}$$

$$= \frac{-6}{-(1-x^4)}$$

$$= \frac{6}{1-x^4}$$

(ii) $\frac{a+b}{a^2-b^2} \div \frac{a^2-ab}{a^2-2ab+b^2}$

Solution:

$$= \frac{a+b}{a^2-b^2} \times \frac{a^2-2ab+b^2}{a^2-ab}$$

$$= \frac{a+b}{a^2-b^2} \times \frac{a^2-2ab+b^2}{a^2-ab}$$

$$= \frac{1}{a}$$

Q7. Find square root by using factorization.

$\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27 \qquad (x \neq 0)$

Solution:

$$= x^2+\frac{1}{x^2}+2+10\left(x+\frac{1}{x}\right)+27-2$$

$$= \left(x+\frac{1}{x}\right)^2+10\left(x+\frac{1}{x}\right)+25$$

$$= \left(x+\frac{1}{x}\right)^2+2\left(x+\frac{1}{x}\right)+5+5^2$$

$$= \left[\left(x+\frac{1}{x}\right)+5\right]^2$$

Required square root is $\pm\left[\left(x+\frac{1}{x}\right)+5\right]$

Q8. Find square root using division method.

$\frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2} \quad (x \neq 0)(y \neq 0)$

Solution:

$$
\begin{array}{l|lllll}
 & & \frac{2x}{y} & +5 & \frac{3y}{x} & \\
\hline
\frac{2x}{y} & \frac{4x^2}{y^2} & +\frac{20x}{y} & +13 & -\frac{30x}{y} & +\frac{9y^2}{x^2} \\
 & \pm\frac{4x^2}{y^2} & & & & \\
\hline
\frac{4x}{y}+5 & & \frac{20x}{y} & +13 & & \\
\end{array}
$$

$$
\begin{array}{l|lllll}
 & & \pm\frac{20x}{y} & \pm 25 & & \\
\hline
\frac{4x}{y}+10-\frac{3y}{x} & & & -12 & -\frac{30y}{x} & +\frac{9y^2}{x^2} \\
 & & & \pm 12 & \pm\frac{30y}{x} & \pm\frac{9y^2}{x^2} \\
\hline
 & & & 0 & & \\
\end{array}
$$

So, the required root is $\pm\left(\frac{2x}{y}+5-\frac{3y}{x}\right)$