

Exercise 5.1

Q1. Factorize

(i) $2abc - 4abx + 4abd$

$$= 2ab(c - 2x + d)$$

(ii) $9xy - 12x^2y + 18y^2$
$$= 3y(3x - 4x^2 + 6y)$$

(iii) $-3x^2y - 3x + 9xy^2$
$$= -3x(xy + 1 - 3y^2)$$

(iv) $5ab^2c^3 - 10a^2b^3c - 20a^3bc^2$
$$= 5abc(bc^2 - 2b^2 - 4a^2c)$$

(v) $3x^3y(x-3y) - (7x^2y^2(x-3y))$
$$= (x-3y)(3x^3y - 7x^2y^2)$$

$$= (x-3y)x^2y(3x-7y)$$

$$= x^2y(x-3y)x^2y(3x-7y)$$

(vi) $2xy^3(x^2+5) + 8xy^2(x^2+5)$
$$= (x^2+5)(2xy^3 + 8xy^2)$$

$$= (x^2+5)(2xy^2)(y+4)$$

$$= 2xy^2(x^2+5)(y+4)$$

Q.2

(i) $5ax - 3ay - 5bx + 3by$
$$= 5ax - 5bx - 3ay + 3by$$

$$= 5x(a-b)(5x-3y)$$

(ii) $3xy + 2y - 12x - 8$
$$= 3xy - 12x + 2y - 8$$

$$= 3x(y-4) + 2(y-4)$$

$$= (y-4)(3x+2)$$

(iii) $x^3 + 3xy^2 - 2x^2 - 6y^3$
$$= x(x^2 + 3y^2) - 2y(x^2 + 3y^2)$$

$$= (x^2 + 3y^2)(x-2y)$$

(iv) $(x^2 - y^2)z + (y^2 - z^2)x$

$$\begin{aligned} &= x^2z - y^2z + y^2x - z^2x \\ &= x^2z - z^2x + y^2x - y^2z \\ &= xz(x-z) + y^2x - y^2z \\ &= (x-z)(xz + y^2) \end{aligned}$$

Q.3

(i) $144a^2 + 24a + 1$
$$= 144a^2 + 12a + 12a + 1$$

$$= 12a(12a+1) + 1(12a+1)$$

$$= (12a+1)(12a+1) = (12a+1)^2$$

(ii) $\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}$
$$= \left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2$$

$$= \left(\frac{a}{b} - \frac{b}{a}\right)^2$$

(iii) $(x+y)^2 - 14z(x+y) + 49z^2$
$$= (x+y)^2 - 2(x+y)(7z) + (7z)^2$$

$$= (x+y-7z)^2$$

(iv) $12x^2 - 36x + 27$

$$\begin{aligned} &= 3(4x^2 - 12x + 9) \\ &= 3[(2x)^2 - 2(2x)(3) + (3)^2] \\ &= 3(2x-3)^2 \end{aligned}$$

Q.4.

(i) $3x^2 - 75y^2$
$$= 3(x^2 - 25y^2)$$

$$= 3[(x^2) - (5y)^2]$$

$$= 3(x+5y)(x-5y)$$

(ii) $x(x-1) - y(y-1)$
$$= x^2 - x - y^2 + y$$

$$= x^2 - y^2 - x + y$$

$$= (x+y)(x-y) - 1(x-y)$$

$$= (x-y)(x+y-1)$$

(iii) $128am^2 - 242an^2$
$$= 2a(64m^2 - 121n^2)$$

$$= 2a[(8m)^2 - (11n)^2]$$

$$= 21(8m+11n)(8m-11n)$$

(iv) $3x - 243x^3$
$$= 3x(1 - 81x^2)$$

$$= 3x[(1)^2 - (9x^2)]$$

$$= 3x(1+9x)(1-9x)$$

Q.5

(i) $x^2 - y^2 - 6y - 9$
$$= x^2 - (y^2 + 6y + 9)$$

$$= x^2 - ((y^2) + 2(y)(3) + (3)^2)$$

$$= x^2 - (y+3)^2$$

$$= (x+(y+3))(x-(y+3))$$

$$= (x+y+3)(x-y-3)$$

(ii) $x^2 - a^2 + 2a - 1$
$$= x^2 - (a^2 - 2a + 1)$$

$$= x^2 - ((a)^2 - 2(a)(1) + (1)^2)$$

$$= x^2 - (a-1)^2$$

$$= (x)^2 - (a-1)^2$$

$$= (x+(a-1))(x-(a+1))$$

$$= (x+a-1)(x-a+1)$$

(iii) $4x^2 - y^2 - 4x - 2y + 3$
$$= 4x^2 - (y^2 + 2y + 1)$$

$$= (2x)^2 - (y+1)^2$$

$$= [2x+(y+1)][2x-(y+1)]$$

$$= (2x+y-1)(2x-y-1)$$

(iv) $x^2 - y^2 - 4x - 2y + 3$
$$= x^2 - 4x - y^2 - 2y + 3$$

$$= x^2 - 4x - y^2 - 2y + 3$$

$$= x^2 - 4x + 4 - y^2 - 2y - 1$$

$$= x^2 - 4x + 4 - (y+1)^2$$

$$= ((x-2) + (y+1))(x-2-y-1)$$

$$= (x+y-1)(x-y-3)$$

(v) $25x^2 - 10x + 1 - 36z^2$
$$= (5x)^2 - 2(5x)(1) + (1)^2 - 36z^2$$

$$= (5x-1)^2 - (6z)^2$$

$$= ((5x-1)^2 + 6z)((5x-1) - 6z)$$

$$= (5x-1+6z)(5x-1-6z)$$

(vi) $x^2 - y^2 - 4xz + 4z^2$
$$= x^2 - 4xz + 4z^2 - y^2$$

$$= (x)^2 - 2(x)(2z) + (2z)^2 - y^2$$

$$= (x-2z)^2 - (y)^2$$

$$= ((x-2z)^2 - y)((x-2z)y)$$

$$= (x-2z+y)(x-y-2z)$$

$$= (x+y-2y)(x-y-2z)$$

Exercise 5.2

Factorize

Q.1 (i) $x^4 + \frac{1}{x^4} - 3$

$$\begin{aligned} &= x^4 + \frac{1}{x^4} - 2 - 1 \\ &= x^4 - 2 + \frac{1}{x^4} - 1 \\ &= \left(x^2 - \frac{1}{x^2}\right)^2 - (1)^2 \\ &= \left(x^2 - \frac{1}{x^2} + 1\right)\left(x^2 - \frac{1}{x^2} - 1\right) \\ &= \left(x^2 - \frac{1}{x^2} + 1\right)\left(x^2 - \frac{1}{x^2} - 1\right) \end{aligned}$$

(ii) $3x^4 + 12y^4$

$$\begin{aligned} &= 3(x^4 + 4y^4) \\ &= 3(x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2) \\ &= 3(x^2 + 2y^2)^2 - 4x^2y^2 \\ &= 3\left((x^2 + 2y^2)^2 + (2xy)^2\right) \\ &= 3\left((x^2 + 2y^2)^2 + (2xy)\left((x^2 + 2y^2) - 2xy\right)\right) \\ &= 3(x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2) \end{aligned}$$

(iii) $a^4 + 3a^2b^2 + ab^4$

$$\begin{aligned} &= a^4 + 4a^2b^2 - a^2b^2 + 4b^2 \\ &= a^4 + 4a^2b^2 + 4b^2 - a^2b^2 \\ &= (a^2 + 2b^2)^2 - (ab)^2 \\ &= (a^2 + 2b^2 + ab)(a^2 + 2b^2 - ab) \\ &= (a^2 + ab + 2b^2)(a^2 - ab + 2b^2) \end{aligned}$$

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(iv) $4x^4 + 81$

$$\begin{aligned} &= (2x^2)^2 + (9)^2 + 36x^2 - 36x^2 \\ &= (2x^2 + 9)^2 - (6x)^2 \\ &= (2x^2 + 9 + 6x)(2x^2 + 9 - 6x) \\ &= (2x^2 + 6x + 9)(2x^2 - 6x + 9) \end{aligned}$$

(v) $x^4 + x^2 + 25$

$$\begin{aligned} &= x^4 + 10x^2 + 25 - 9x^2 \\ &= (x^2)^2 + 2(x^2)5 - 9x^2 \\ &= (x^2 + 5)^2 - (3x)^2 \\ &= (x^2 + 5 + 3x)(x^2 + 5 - 3x) \\ &= (x^2 + 2x + 4)(x^2 - 2x + 4) \end{aligned}$$

(vi) $x^4 + 4x^2 + 16$

$$\begin{aligned} &= x^4 + 8x^2 + 16 - 4x^2 \\ &= (x^2 + 4)^2 - (2x)^2 \\ &= (x^2 + 4 + 2x)(x^2 + 4 - 2x) \\ &= (x^2 + 2x + 4)(x^2 + 2x - 4) \end{aligned}$$

Q.2 (i) $x^2 + 14x + 48$

$$\begin{aligned} &= x^2 + 8x + 6x + 48 \\ &= x(x + 8) + 6(x + 8) \\ &= (x + 8)(x + 6) \end{aligned}$$

(ii) $x^2 - 21x + 108$

$$\begin{aligned} &= x^2 - 12x - 9x + 108 \\ &= x(x - 12) - 9(x - 12) \\ &= (x - 12)(x - 9) \end{aligned}$$

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(iii) $x^2 - 11x - 42$

$$\begin{aligned} &= x^2 - 14x + 3x - 42 \\ &= x(x - 14) + 3(x - 14) \\ &= (x - 14)(x + 3) \end{aligned}$$

(iv) $x^2 + x - 132$

$$\begin{aligned} &= x^2 + 12x - 11x - 132 \\ &= x(x + 12) - 11(x + 12) \\ &= (x + 12)(x - 11) \end{aligned}$$

Q.3 (i) $4x^2 + 12x + 5$

$$\begin{aligned} &= 4x^2 + 10x + 2x + 5 \\ &= 2x(2x + 5) + 1(2x + 5) \\ &= (2x + 5)(2x + 1) \end{aligned}$$

(ii) $30x^2 + 7x - 15$

$$\begin{aligned} &= 30x^2 + 25x - 18x - 15 \\ &= 5x(6x + 5) - 3(6x + 5) \\ &= (6x + 5)(5x - 3) \end{aligned}$$

(iii) $24x^2 - 65x + 21$

$$\begin{aligned} &= 24x^2 - 56x - 9x + 21 \\ &= 8x(3x - 7) - 3(3x - 7) \\ &= (3x - 7)(8x - 3) \end{aligned}$$

(iv) $5x^2 - 16x - 21$

$$= 5x^2 - 21x + 5x - 21$$

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$$\begin{aligned} &= x(5x - 21) + 1(5x - 21) \\ &= (5x - 21)(x + 1) \end{aligned}$$

(v) $4x^2 - 17xy + 4y^2$

$$\begin{aligned} &= 4x^2 - 16xy - xy + 4y^2 \\ &= 4x(x - 4y) - y(x - 4y) \\ &= (x - 4y)(4x - y) \end{aligned}$$

(vi) $3x^2 - 38xy - 13y^2$

$$\begin{aligned} &= 3x^2 - 39xy + xy - 13y^2 \\ &= 3x(x - 13y) + y(x - 13y) \\ &= (x - 13y)(3x + y) \end{aligned}$$

(vii) $5x^2 + 33xy - 14y^2$

$$\begin{aligned} &= 5x^2 + 35xy - 2xy - 14y^2 \\ &= 5x(x + 7y) + y(x + 7y) \\ &= (x + 7y)(5x + y) \end{aligned}$$

(viii) $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4$

$$\begin{aligned} \text{let } 5x - \frac{1}{x} &= y \\ &= y^2 + 4y + 4 \\ (y + 2)^2 &= (y + 2)(y + 2) \end{aligned}$$

by putting value of $y = 5x - \frac{1}{x}$

$$= \left(5x - \frac{1}{x} + 2\right)\left(5x - \frac{1}{x} + 2\right)$$

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Q.4 (i) $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

$$\begin{aligned} \text{let } x^2 + 5x &= y \\ (y + 4)(y + 6) - 3 \\ &= y^2 + 6y + 4y + 24 - 3 \\ &= y^2 + 10y + 21 \\ &= y^2 + 7y + 3y + 21 \\ &= y(y + 7) + 3(y + 7) \\ &= (y + 7)(y + 3) \end{aligned}$$

by putting value of $y = x^2 + 5x$

$$= (x^2 + 5x + 7)(x^2 + 5x + 3)$$

(ii) $(x^2 - 4x)(x^2 - 4x - 1) - 20$

$$\begin{aligned} \text{let } x^2 - 4x &= y \\ &= y(y - 1) - 20 \\ &= y^2 - y - 20 \\ &= y^2 - 5y + 4y - 20 \\ &= y(y - 5) + 4(y - 5) \\ &= (y - 5)(y + 5) \end{aligned}$$

by putting value of $y = x^2 - 4x$

$$\begin{aligned} &= (x^2 - 4x - 5)(x^2 - 4x + 4) \\ &= (x^2 - 5x + x - 5)(x^2 - 2x - 2x + 4) \\ &= ((x(x - 5) + 1(x - 5))((x(x - 2) - 2(x - 2)) \\ &= ((x - 5)(x + 1))((x - 2)(x - 2)) \\ &= (x - 5)(x + 1)(x - 2)^2 \end{aligned}$$

(iii) $(x + 2)(x + 3)(x + 4)(x + 5) - 15$

By using commutative property of addition

$$\begin{aligned} \text{As } 2 + 5 &= 3 + 4 \\ &= (x^2 + 7x + 10)(x^2 + 7x + 12) - 15 \end{aligned}$$

let $x^2 + 7x = y$

$$= (y + 10)(y + 12) - 15$$

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$$\begin{aligned} &= y^2 + 22y + 120 - 15 \\ &= y^2 + 22y + 105 \\ &= y^2 + 15y + 7y + 105 \\ &= y(y + 15) + 7(y + 15) \\ &= (y + 15)(y + 7) \end{aligned}$$

By putting value of $y = x^2 + 7x$

$$= (x^2 + 7x + 15)(x^2 + 7x + 7)$$

(iv) $(x + 4)(x - 5)(x + 6)(x - 7) - 504$

By using commutative property of subtraction

$$\begin{aligned} \text{As } 4 - 5 &= 6 - 7 \\ &= (x^2 - x - 20)(x^2 - x - 42) - 504 \end{aligned}$$

let $x^2 - x = y$

$$\begin{aligned} &= (y - 20)(y - 42) - 504 \\ &= y^2 - 42y - 20y + 840 - 504 \\ &= y^2 - 62y + 336 \\ &= y(y - 56) + (y - 6) \end{aligned}$$

by putting value of $y = x^2 - x$

$$\begin{aligned} &= (x^2 - x - 56)(x^2 - x - 6) \\ &= (x^2 - 8x + 7x - 56)(x^2 - 3x + 2x - 6) \\ &= x(x - 8) + 7(x - 8)(x(x - 3) + 2(x - 3)) \\ &= (x - 8)(x + 7)(x - 3)(x + 2) \end{aligned}$$

(v) $(x + 1)(x + 2)(x + 3)(x + 6) - 3x^2$

By using commutative property of multiplication

$$\begin{aligned} \text{As } 1(6) &= (2)(3) \\ &= (x^2 + 7x + 6)(x^2 + 5x + 6) - 3x^2 \\ &= (x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2 \end{aligned}$$

let $x^2 + 6 = y$

$$\begin{aligned} &= (y + 7x)(y + 5x) - 3x^2 \\ &= y^2 + 5xy + 7xy + 35x^2 - 3x^2 \\ &= y^2 + 12xy + 32x^2 \end{aligned}$$

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$$\begin{aligned} &= y^2 + 8xy + 4xy + 32x^2 \\ &= y(y + 8x) + 4x(y + 8x) \\ &= (y + 8x)(y + 4x) \end{aligned}$$

By putting value of $y = x^2 + 6$

$$\begin{aligned} &= (x^2 + 6 + 8x)(x^2 + 6 + 4x) \\ &= x\left(x + 8 + \frac{6}{x}\right)x\left(x + 4 + \frac{6}{x}\right) \\ &= x^2\left(x + \frac{6}{x} + 8\right)\left(x + \frac{6}{x} + 4\right) \end{aligned}$$

Q.5 (i) $x^3 + 48x - 12x^2 - 64$

$$\begin{aligned} &= x^3 - 12x^2 + 48x - 64 \\ &= x^3 - 3x^2 + 3x + 45x - 64 \\ &= (x - 4)^3 \end{aligned}$$

(ii) $8x^3 + 60x^2 + 150x + 125$

$$\begin{aligned} &= (2x)^3 + 3(2x)^2 \cdot 5 + 3(2x) \cdot 5^2 + 5^3 \\ &= (2x + 5)^3 \end{aligned}$$

(iii) $x^3 - 18x^2 + 108x - 216$

$$\begin{aligned} &= x^3 - 3x^2 \cdot 6 + 3x \cdot 6^2 - 6^3 \\ &= (x - 6)^3 \end{aligned}$$

(iv) $8x^3 - 125y^3 - 60x^2y + 150xy^2$

$$\begin{aligned} &= 8x^3 - 60x^2y + 150xy^2 - 125y^3 \\ &= (2x)^3 - 3(2x)^2 \cdot 5y + 3(2x) \cdot (5y)^2 - (5y)^3 \\ &= (2x - 5y)^3 \end{aligned}$$

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Q.6 (i) $27 + 8x^3$

$$\begin{aligned} &= (3)^3 + (2x)^3 \\ &= (3 + 2x)\left(3^2 - 3 \cdot 2x + (2x)^2\right) \\ &= (3 + 2x)(9 - 6x + 4x^2) \end{aligned}$$

(ii) $125x^3 - 216y^3$

$$\begin{aligned} &= (5x)^3 - (6y)^3 \\ &= (5x - 6y)\left((5x)^2 + 5x \cdot 6y + (6y)^2\right) \\ &= (5x - 6y)(25x^2 + 30xy + 36y^2) \end{aligned}$$

(iii) $64x^3 + 27y^3$

$$\begin{aligned} &= (4x)^3 + (3y)^3 \\ &= (4x + 3y)\left((4x)^2 - 4x \cdot 3y + (3y)^2\right) \\ &= (4x + 3y)(16x^2 - 12xy + 9y^2) \end{aligned}$$

(iv) $8x^3 + 125y^3$

$$\begin{aligned} &= (2x)^3 + (5y)^3 \\ &= (2x + 5y)\left((4x)^2 - 2x \cdot 5y + (5y)^2\right) \\ &= (2x + 5y)(4x^2 - 10xy + 25y^2) \end{aligned}$$

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Exercise 5.3

Q.1 Use Remainder theorem to find the remainder when

(i) $3x^3 - 10x^2 + 13x - 6$ is divided by $(x - 2)$

Solution

$$\text{Let } p(x) = 3x^3 - 10x^2 + 13x - 6$$

When $p(x)$ is divided by $x - 2$

$$\text{The remainder } R = p(2)$$

$$p(2) = 3(2)^3 - 10(2)^2 + 13(2) - 6$$

$$p(2) = 24 - 40 + 26 - 9 = 4$$

Therefore remainder = 4

(ii) $4x^3 - 4x + 3$ is divided by $(2x - 1)$

Solution

$$\text{Let } p(x) = 4x^3 - 4x + 3$$

When $p(x)$ is divided by $2x - 1$

$$\text{The remainder } R = \frac{1}{2}$$

$$p\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right) + 3$$

$$p\left(\frac{1}{2}\right) = 4\left(\frac{1}{8}\right) - 4\left(\frac{1}{2}\right) + 3$$

$$p\left(\frac{1}{2}\right) = \frac{1}{2} - 4 + 3$$

$$p\left(\frac{1}{2}\right) = \frac{1 - 4 + 6}{2} = \frac{3}{2}$$

Therefore remainder = $\frac{3}{2}$

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(iii) $6x + 2x^2 - x + 2$ divided by $(x + 2)$

Solution:

$$\text{Let } p(x) = 6x^2 + 2x^2 - x + 2$$

When $p(x)$ is divided by $x + 2$

$$\text{The remainder } R = p(-2)$$

$$p(-2) = 6(-2)^2 + 2(-2) - 2 + 2$$

$$p(-2) = 96 - 16 + 2 + 2 = 84$$

Therefore remainder = 84

(iv) $(2x - 1)^3 + 6(3 + 4x)^2 - 10$ divided by $(2x + 1)$

Solution:

$$\text{Let } p(x) = (2x - 1)^3 + 6(3 + 4x)^2 - 10$$

When $p(x)$ is divided by $2x + 1$

$$\text{The remainder } R = p\left[-\frac{1}{2}\right]$$

$$p\left(-\frac{1}{2}\right) = \left[2\left(-\frac{1}{2}\right) - 1\right]^3 + 6\left[3 + 4\left(-\frac{1}{2}\right)\right]^2 - 10$$

$$p\left(-\frac{1}{2}\right) = (-1 - 1)^3 + 6(3 - 2)^2 - 10$$

$$p\left(-\frac{1}{2}\right) = (-2)^3 + 6(1)^2 - 10$$

$$p\left(-\frac{1}{2}\right) = -8 + 6 - 10 = -12$$

Therefore remainder = -12

(v) $x^3 - 3x^2 + 4x - 14$ divided by $(x + 2)$

Solution

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$$\text{Let } p(x) = x^3 - 3x^2 + 4x - 14$$

When $p(x)$ is divided by $x + 2$

$$\text{The remainder } R = p(-2)$$

$$p(-2) = (-2)^3 - 3(-2)^2 + 4(-2) - 14$$

$$p(-2) = -8 - 12 - 8 - 14 = -42$$

Therefore remainder = -42

Q2. (i) If $(x + 2)$ is a factor of $3x^2 - 4kx - 4k^2$ then find the value(s) of k .

Solution:

$$\text{Let } p(x) = 3x^2 - 4kx - 4k^2$$

As $x + 2 = x - (-2)$ is a factor of $p(x)$

$$\text{So } p(-2) = 0$$

$$3(-2)^2 - 4k(-2) - 4k^2 = 0$$

$$12 + 8k - 4k^2 = 0$$

$$4k^2 - 8k - 12 = 0$$

$$k^2 - 2k - 3 = 0$$

$$3(1 + k) - k(1 + k) = 0$$

$$(1 + k)(3 - k) = 0$$

$$1 + k = 0; 3 - k = 0$$

$$k = -1; k = 3$$

$$\Rightarrow k = -1, 3$$

(ii) If $(x - 1)$ is a factor of $x^3 - kx^2 + 11x - 6$, then find the value(s) of k .

Solution:

$$\text{Let } p(x) = x^3 - kx^2 + 11x - 6$$

As $x - 1$ is a factor of $p(x)$ we have $p(1) = 0$

3

$$(1)^3 - k(1)^2 + 11(1) - 6 = 0$$

$$1 - k + 11 - 6 = 0$$

$$-k + 6 = 0$$

$$\Rightarrow k = 6$$

Q3. Without actual long division determine whether

(i) $(x - 2)$ and $(x - 3)$ are factors of $p(x) = x^3 - 12x^2 + 44x - 48$

Solution:

The remainder for $x - 2$ is

$$p(2) = (2)^3 - 12(2)^2 + 44(2) - 48$$

$$p(2) = 8 - 48 + 88 - 48$$

$$p(2) = 0$$

Since remainder = 0 therefore $x - 2$ is a factor of $p(x)$

The remainder for $x - 3$ is

$$p(3) = (3)^3 + 2(3)^2 + 44(3) - 48$$

$$p(3) = 3$$

Since remainder is not equal to zero therefore $x - 3$ is not a factor.

(ii) $(x - 2), (x + 3)$ and $(x - 4)$ are factors of $q(x) = x^3 + 2x^2 - 5x - 6$

Solution:

The remainder for $x - 2$ is

$$p(2) = (2)^3 + 2(2)^2 - 5(2) - 6$$

$$p(2) = 8 + 8 - 10 - 6$$

$$p(2) = 0$$

Since remainder = 0 therefore $x - 2$ is a factor of $q(x)$.

The remainder for $x + 3$ is

$$p(-3) = (-3)^3 + 2(-3)^2 - 5(-3) - 6$$

$$p(-3) = -27 + 18 + 15 - 6$$

$$p(-3) = 0$$

Since remainder = 0 therefore $x + 3$ is a factor of $q(x)$.

The remainder for $x - 4$ is

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$$p(4) = (4)^3 + 2(4)^2 - 5(4) - 6$$

$$p(4) = 64 + 32 - 20 - 6$$

$$p(4) = 70$$

Not a factor as remainder is not equal to zero.

Q4. For what value of m is the polynomial $p(x) = 4x^3 - 7x^2 + 6x - 3m$ exactly divisible by $x + 2$?

Solution:

As $p(x)$ is exactly divisible $x + 2$ therefore remainder = 0

$$4(-2)^3 - 7(-2)^2 + 6(-2) - 3m = 0$$

$$-72 - 3m = 0$$

$$-24 - m = 0$$

$$m = -24$$

Q5. Determine the value of k if $p(x) = kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + k$ leaves the same remainder when divided by $(x - 3)$.

Solution:

$$p(3) = k(3)^3 + 4(3)^2 + 3(3) - 4$$

$$p(3) = 27k + 41$$

Now

$$q(3) = (3)^3 - 4(3) + k$$

$$q(3) = 15 + k$$

According to given condition:

$$27k + 41 = 15 + k$$

$$k = -1$$

Q6. The remainder after dividing the polynomial $p(x) = x^3 + ax^2 + 7$ by $(x + 1)$ is 2b. Calculate the value of a and b if this expression leaves a remainder of (b + 5) on being divided by $(x - 2)$.

Solution:

$$p(x) = x^3 + ax^2 + 7$$

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When $p(x)$ is divided by $x + 1$, then the remainder $p(-1) = 0$

$$p(-1) = (-1)^3 + a(-1)^2 + 7$$

$$p(-1) = -1 + a + 7$$

$$p(-1) = a + 6$$

$$\text{As given remainder} = 2b$$

$$a + 6 = 2b$$

$$a - 2b = -6 \quad \dots\dots\dots(i)$$

When $p(x)$ is divided by $(x - 2)$, then the remainder $p(2) = 0$

$$p(2) = (2)^3 + a(2)^2 + 7$$

$$p(2) = 4a + 15$$

As given remainder = $b + 15$

Therefore, calculated remainder = given remainder

$$4a + 15 = b + 15$$

$$4a - b = 0 \quad \dots\dots\dots(ii)$$

Multiply eq(ii) by 2 and subtract from eq(i):

$$a - 2b = -6$$

$$-8a \mp 2b = \mp 20$$

$$-7a = 14$$

Put $a = -2$ in eq(i), we get

$$-2 - 2b = -6$$

$$-2b = -4 \text{ Or } b = 2$$

$$a = -2, b = 2$$

Q7. The polynomial $x^3 + bx^2 + mx + 24$ has factor $(x + 4)$ and it leaves a remainder of 36 when divided by $(x - 2)$. Find the value of b and m.

Solution:

$$\text{Let } p(x) = x^3 + bx^2 + mx + 24$$

As $x + 4$ is a factor of $p(x)$

$$\text{i.e. } (4)^3 + b(-4)^2 + m(-4) + 24 = 0$$

6

$$4b - m = 10 \quad \dots\dots\dots(i)$$

When $p(x)$ is divided by $x - 2$

When remainder is $p(2)$

$$\text{Then } p(2) = 36$$

$$x^3 + bx^2 + mx + 24 = 36$$

$$8 + 4b + 2m + 24 = 36$$

$$4b + 2m = 4$$

$$2b + 3m = 2 \quad \dots\dots\dots(ii)$$

By adding eq(i) and eq(ii) we get:

$$6b = 12$$

$$b = 2$$

Putting $b = 2$ in eq (i)

$$8 - m = 10$$

$$-m = 2$$

$$m = -2$$

So, $b = 2, m = -2$.

Q8. The expression $bx^3 + mx^2 - 4$ leaves remainder of -3 and 12 when divided by $(x - 1)$ and $(x + 2)$ respectively. Calculate the values of b and m.

Solution:

When $p(x)$ is divided by $x - 1$ the remainder

$$f(1)^3 + m(1)^2 - 4 = -3$$

$$f + m - 4 = -3$$

$$f + m = 1 \quad \dots\dots\dots(i)$$

When $p(x)$ is divided by $x + 2$ the remainder

$$p(-2) = 12$$

$$f(-2)^3 + m(-2)^2 - 4 = 12$$

$$-8f + 4m - 4 = 12$$

$$-8f + 4m = 16$$

7

$$10a + b = 27 \quad \dots\dots\dots(ii)$$

By multiplying eq(ii) by 2 and subtracting from eq(i):

$$11a + 2b = 36$$

$$\frac{\pm 20a \pm 2b = \pm 54}{-9a = -18}$$

$$a = 2$$

Putting in eq(ii):

$$20 + b = 27$$

$$b = 7$$

So, $a = 2$ and $b = 7$.

8

$$10a + b = 27 \quad \dots\dots\dots(ii)$$

By multiplying eq(ii) by 2 and subtracting from eq(i):

$$11a + 2b = 36$$

$$\frac{\pm 20a \pm 2b = \pm 54}{-9a = -18}$$

$$a = 2$$

Putting in eq(ii):

$$20 + b = 27$$

$$b = 7$$

So, $a = 2$ and $b = 7$.

9

Exercise 5.4

Factorize each of the following cubic polynomials by factor theorem.

1. $x^3 - 2x^2 - x + 2$

Solution:

Let $p(x) = x^3 - 2x^2 - x + 2$

Possible factors of constant zeros of $p(x)$ are $p = \pm 1, \pm 2$ and possible factors of leading coefficient 1 are $q = \pm 1$ Thus the expected zeros of $p(x)$ are

$\frac{p}{q} = \pm 1, \pm 2$

Now $p(1) = (1)^3 - 2(1)^2 - 1 + 2$

$p(1) = 1 - 2 - 1 + 2$
 $p(1) = 0$

Hence $x = 1$ is a zero of $p(x)$ therefore $x - 1$ is a factor of $p(x)$

$p(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$
 $p(-1) = -1 - 2 + 1 + 2$
 $p(-1) = 0$

Hence $x = -1$ is a zero of $p(x)$ therefore $x + 1$ is a factor of $p(x)$

$p(2) = 2^3 - 2(2)^2 - 2 + 2$
 $p(2) = 8 - 8 - 2 + 2$
 $p(2) = 0$

Hence $x = 2$ is a zero of $p(x)$ and therefore $x - 2$ is a factor of $p(x)$

Hence required factors are $(x - 1)(x + 1)(x + 2)$.

2. $x^3 - x^2 - 22x + 40$

Solution:

Let $p(x) = x^3 - x^2 - 22x + 40$

Possible factors of constant term 40 are

$p = \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

And those of leading coefficient 1 are

Thus the possible zeros of $p(x)$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

Now $p(1) = (1)^3 - (1)^2 - 22(1) + 40$

$p(1) = 1 - 1 - 22 + 40$
 $p(1) = 18 \neq 0$

So, $x - 1$ is not a factor of $p(x)$

Now $p(-1) = (-1)^3 - (-1)^2 - 22(-1) + 40$

$p(-1) = -1 - 1 + 22 + 40$
 $p(-1) = 60 \neq 0$

So, $x + 1$ is not a factor of $p(x)$

Now $p(-2) = (-2)^3 - (-2)^2 - 22(-2) + 40$

$p(-2) = -8 - 4 - 44 + 40$
 $p(-2) = -8 \neq 0$

So, $x + 2$ is not a factor of $p(x)$

Now $p(2) = (2)^3 - (2)^2 - 22(-2) + 40$

$p(2) = 8 - 4 + 44 + 40$
 $p(2) = 72 \neq 0$

So, $x - 2$ is not a factor of $p(x)$

Now $p(-4) = (-4)^3 - (-4)^2 - 22(-4) + 40$

$p(-4) = -64 - 16 + 88 + 40$
 $p(-4) = 48 \neq 0$

So, $x + 4$ is not a factor of $p(x)$

Now $p(4) = 4^3 - (4)^2 - 22(4) + 40$

$p(4) = 64 - 16 - 88 + 40$
 $p(4) = 0$

So, $x - 4$ is a factor of $p(x)$

Now $p(5) = (5)^3 - (5)^2 - 22(5) + 40$

$p(5) = 125 - 25 - 110 + 40$
 $p(5) = 30 \neq 0$

$x - 5$ is not a factor of $p(x)$

Now $p(-5) = (-5)^3 - (-5)^2 - 22(-5) + 40$

$p(-5) = -125 - 25 + 110 + 40$
 $p(-5) = 0$

So, $x + 5$ is a factor of $p(x)$

Hence required factors are $(x - 4)$ and $(x + 5)$.

3. Factorize $x^3 - 6x^2 + 3x + 10$ cubic polynomials by factor theorem.

Solution:

Let $p(x) = x^3 - 6x^2 + 3x + 10$

Possible factors of constant term 10 are

$\pm 1, \pm 2, \pm 5, \pm 10$

Now $p(-1) = (-1)^3 - 6(-1)^2 - 3(-1) + 10$

$p(-1) = -1 - 6 + 3 + 10$
 $p(-1) = 6 \neq 0$

So, $x + 1$ is not a factor

Now $p(-2) = (-2)^3 - 6(-2)^2 + 3(-2) + 10$

$p(-2) = -8 - 24 - 6 + 10$
 $p(-2) = -28 \neq 0$

So, $x + 2$ is not a factor

Now $p(2) = (2)^3 - 6(2)^2 + 3(2) + 10$

$p(2) = 8 - 24 + 6 + 10$
 $p(2) = 0$

So, $x - 2$ is a factor

Now $p(-4) = (-4)^3 - 6(-4)^2 + 3(-4) + 10$

$p(-4) = -64 - 96 - 12 + 10$
 $p(-4) = -162 \neq 0$

So, $x + 4$ is not a factor

Now $p(-5) = (-5)^3 - 6(-5)^2 + 3(-5) + 10$

$p(-5) = -125 - 150 - 15 + 10$
 $p(-5) = 280 \neq 0$

So, $x + 5$ is not a factor

Now $p(5) = (5)^3 - 6(5)^2 + 3(5) + 10$

$p(5) = 125 - 150 + 15 + 10$
 $p(5) = 0$

So, $x - 5$ is a factor

Hence required factors are $(x - 2)$ and $(x - 5)$.

4. $x^3 + x^2 - 10x + 8$

Solution:

Let $p(x) = x^3 + x^2 - 10x + 8$

Possible factors of constant term 8 are

$p = \pm 1, \pm 2, \pm 4, \pm 8$

Thus the expected zeros of $p(x)$ are

$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8$
 $\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8$

$p(1) = (1)^3 + (1)^2 - 10(1) + 8$
 $= 1 + 1 - 10 + 8 = 0$

So, $x - 1$ is a factor of $p(x)$

$p(-1) = (-1)^3 + (-1)^2 - 10(-1) + 8$
 $= -1 + 1 + 10 + 8 = 18 \neq 0$

$x + 1$ is not a factor of $p(x)$

$p(2) = (2)^3 + (2)^2 - 10(2) + 8$
 $= 8 + 4 - 20 + 8 = 0$

$x + 2$ is not a factor of $p(x)$

$p(-4) = (-4)^3 + (4)^2 - 10(4) + 8$
 $= -64 + 16 - 40 + 8 = 0$

$x - 4$ is not a factor of $p(x)$

$p(-4) = (-4)^3 + (4)^2 - 10(4) + 8$
 $= -64 + 16 - 40 + 8 = 0$

So, $x + 4$ is a factor of $p(x)$

Hence required factors are $(x - 1)(x - 2)(x + 4)$.

Q5. $x^3 - 2x^2 + 5x + 6$

Solution:

Let $p(x) = x^3 - 2x^2 + 5x + 6$

Possible factors of constant term 6 are

$p = \pm 1, \pm 2, \pm 3, \pm 6$

Thus the possible zeros of $p(x)$ are

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6$
 $\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6$

$p(1) = 1^3 - 2(1)^2 - 5(1) + 6$
 $= -1 - 2 + 5 + 6 = 8 \neq 0$

So, $x - 1$ is a factor of $p(x)$

$p(-1) = 1^3 - 2(1)^2 - 5(1) + 6$
 $= -1 - 2 + 5 + 6 = 8 \neq 0$

$x + 1$ is not a factor of $p(x)$

$p(-2) = -2^3 - 2(2)^2 - 5(-2) + 6$
 $= -8 - 8 + 10 + 6 = 0$

$x + 2$ is a factor of $p(x)$

$p(3) = 3^3 - 2(3)^2 - 5(3) + 6$
 $= 27 - 18 + 15 + 6 = 0$

So, $x - 3$ is a factor of $p(x)$

Hence required factors are $(x - 1)(x - 2)(x + 2)$.

Q6. $x^3 + 5x^2 - 2x - 24$

Solution:

Let $p(x) = x^3 + 5x^2 - 22x + 40$

Possible factors of constant term 40 are

$p = \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

And those of leading coefficient 1 are Thus the possible zeros of $p(x)$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

Now $p(1) = 1 - 1 - 22 + 44 = 18 \neq 0$

So, $x - 1$ is not a factor of $p(x)$

$p(x) = 1^3 + 1^2 - 22(-1) + 40$
 $= 8 - 4 - 44 + 40 = 0$
 $= -1 - 1 + 22 + 40 = 60 \neq 0$

$x + 1$ is not a factor of $p(x)$

$p(2) = -2^3 - (-2)^2 - 22(2) + 40$
 $= 8 - 4 - 44 + 40 = 0$

$x - 2$ is a factor of $p(x)$

$(-2)^2 - 22(-2) + 40$
 $= 8 - 4 + 44 + 40 = 72 \neq 0$

$x + 2$ is not a factor of $p(x)$

$p(4) = 4^3 - (4)^2 - 22(4) + 40$
 $= 64 - 16 - 88 + 40 = 48 \neq 0$

$x - 4$ is not a factor of $p(x)$

$p(-4) = -4^3 - (4)^2 - 22(4) + 40$

$x + 4$ is not a factor of $p(x)$

$p(5) = 5^3 - (5)^2 - 110 + 40$
 $= 125 - 25 - 110 + 40 = 30 \neq 0$

$x - 5$ is not a factor of $p(x)$

$p(-5) = -5^3 - (-5)^2 - 22(-5) + 40$
 $= 125 - 25 + 110 + 40 = 0$

So, $x + 5$ is a factor of $p(x)$

Hence required factors are $(x - 2)(x - 4)(x + 5)$.

Q7. $3x^3 - x^2 - 12x + 4$

So, $x - 1$ is not a factor of $p(x)$

$p(x) = 1^3 + 1^2 - 22(-1) + 40$
 $= 8 - 4 - 44 + 40 = 0$
 $= -1 - 1 + 22 + 40 = 60 \neq 0$

$x + 1$ is not a factor of $p(x)$

$p(-2) = 3(-2)^3 - (-2)^2 - 12(-2) - 24$
 $= -24 - 4 + 4 + 4 = 0$

So, $x + 2$ is a zero of $p(x)$

$p(\frac{1}{3}) = 3(\frac{1}{3})^3 - (\frac{1}{3})^2 - 12(\frac{1}{3}) + 4$
 $= \frac{1}{9} - \frac{1}{9} - 4 + 4 = 0$

So, $3x - 1$ is zero of $p(x)$

Hence $(x - 2), (x + 2)$ and $(3x - 1)$ are factors of $p(x)$

Hence required functions are $(x - 2)(x + 2)(3x - 1)$.

Q8. $2x^3 + x^2 - 2x - 1$

Solution:

Let $p(x) = 2x^3 + x^2 - 2x - 1$

Possible factors of the constant term -1 are $p = \pm 1$ and those of leading coefficient 2 are $q = \pm 1, \pm 2$

Thus the possible zeros $p(x)$ are $\frac{p}{q} = \pm 1, \pm \frac{1}{2}$

$p(1) = 2(1)^3 + (1)^2 - 2(1) - 1$
 $= 2 + 1 - 2 - 1 = 0$

So, $x - 1$ is a zero of $p(x)$

$p(\frac{1}{2}) = 2(\frac{1}{2})^3 + (\frac{1}{2})^2 - 2(\frac{1}{2}) - 1$
 $= \frac{1}{4} + \frac{1}{4} - 1 - 1 = 0$

So, $x = \frac{1}{2}$ is a zero of $p(x)$

Hence, $x + 1$ and $2x + 1$ are factors of $p(x)$

Hence, required factors are $(x + 1)(x - 1)(2x + 1)$.

Review Exercise 5

Q1. Multiple choice questions. Choose the correct answers.

1. The factors of $x^2 - 5x + 6$ are
- (a) $x + 1, x - 6$

(b) $x - 2, x - 3$

(c) $x + 6, x - 1$

(d) $x + 2, x + 3$
2. Factors of $8x^3 - 27y^3$ are
- (a) $(2x + 3y), (4x^2 + 9y^2)$

(b) $(2x + 3y), (4x^2 - 9y^2)$

(c) $(2x + 3y), (4x^2 - 6xy + 9y^2)$

(d) $(2x - 3y), (4x^2 + 6xy + 9y^2)$
3. Factors of $3x^3 - x - 2$ are
- (a) $(x + 1), (3x - 2)$

(b) $(x + 1), (3x + 2)$

(c) $(x - 1), (3x + 2)$

(d) $(x - 1), (3x - 2)$
4. Factors of $a^4 - x - 2$ are
- (a) $(a - b), (a + b), (a^2 + 4b^2)$

(b) $(a^2 - 2b^2), (a^2 + 2b^2)$

(c) $(a - b), (a + b), (a^2 - 4b^2)$

(d) $(a - 2b), (a^2 - 2b^2)$
5. What will be added to complete the square of $9a^2 - 12ab$?
- (a) $-16b^2$

(b) $16b^2$

(c) $4b^2$

(d) $-4b^2$
6. Find m so that $x^2 + 4x + m$ is a complete square...
- (a) 8

(b) -8

(c) 4

(d) 16
7. Factors of $5x^2 - 17xy - 12y^2$ are....
- (a) $(x + 4y), (5x + 3y)$

(b) $(x - 4y), (5x - 3y)$

- (c) $(x - 4y), (5x + 4y)$
- (d) $(x + 4y), (5x - 4y)$

8. Factors of $27x^3 - \frac{1}{x^3}$
- (a) $\left(3x - \frac{1}{x}\right), \left(9x^2 + 3 + \frac{1}{x^2}\right)$

(b) $\left(3x + \frac{1}{x}\right), \left(9x^2 + 3 + \frac{1}{x^2}\right)$

(c) $\left(3x - \frac{1}{x}\right), \left(9x^2 - 3 + \frac{1}{x^2}\right)$

(d) $\left(3x + \frac{1}{x}\right), \left(9x^2 - 3 + \frac{1}{x^2}\right)$
- ANSWERS:
- (1) b (2) c (3) d (4) b (5) c (6) c

(7) c (8) a
- Q2. Complete the following items. Fill in the blanks
- (1) $x^2 + 5x + 6 =$ _____

(2) $4a^2 - 16 =$ _____

(3) $4a^2 + 4ab +$ _____ is a complete square.

(4) $\frac{x^2}{y^2} - 2 + \frac{x^2}{y^2} =$ _____

(5) $(x + y)(x^2 - xy + y^2) =$ _____

(6) Factored form of $x^4 - 16$ is _____

(7) If $x + 2$ is factor of $p(x) = x^2 + 2kx + 8$, then $k =$ _____.
- ANSWERS:
- (1) $(x + 2), (x + 3)$ (2) $4(a - 2)(a + 2)$ (3) b^2 (4) $\left(\frac{x}{y} - \frac{y}{x}\right)^2$ (5) $x^3 + y^3$

(6) $(x - 2)(x + 2)(x^2 + 4)$ (7) -3
- 2
- Mathematics
- Q3. Factorize the following:
- (1) $x^2 + 8x + 16 - 4y^2$

Solution:

$$= (x + 4)^2 - (2y)^2$$
$$= (x + 4 + 2y)(x + 4 - 2y)$$
$$= (x + 2y + 4)(x - 2y + 4)$$

(2) $4x^2 - 16y^2$

Solution:

$$= 4(x^2 - 4y^2)$$
$$= 4[(x)^2 - (2y)^2]$$
$$= 4(x + 2y)(x - 2y)$$

(3) $9x^2 + 27x + 8$

Solution:

$$= 9x^2 + 24x + 3x + 8$$
$$= 3x(3x + 8) + 1(3x + 8)$$
$$= (3x + 8)(3x + 1)$$

(4) $1 - 64z^3$

Solution;

$$= (1)^3 - (4z)^3$$
$$= (1 - 4z)[(1)^2 + 1(4z) + (4z)^2]$$
$$= (1 - 4z)(1 + 4z + 16z^2)$$

(5) $8x^3 - \frac{1}{27y^3}$

Solution:

$$= (2x)^3 - \left(\frac{1}{3y}\right)^3$$
$$= \left(2x - \frac{1}{3y}\right)\left[(2x) + 2x\frac{1}{3y} + \left(\frac{1}{3y}\right)^2\right]$$
$$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$$
- 3
- Mathematics
- (6) $2y^3 + 5y - 3$

Solution:

$$= 2y^3 + 6y - y - 3$$
$$= 2y(y + 3) - 1(y + 3)$$
$$= (y + 3)(2y - 1)$$

(7) $x^3 + x^2 - 4x - 4$

Solution:

$$= x^2(x + 1) - 4(x + 1)$$
$$= (x + 1)(x^2 - 4)$$
$$= (x + 1)(x^2 - 2^2)$$
$$= (x + 1)(x + 2)(x - 2)$$

(8) $25m^2n^2 + 10mn + 1$

Solution:

$$= (5mn)^2 + 2(5mn)(1) + (1)^2$$
$$= (5mn + 1)^2$$

(9) $1 - 12pq + 36p^2q^2$

Solution:

$$= (1)^2 - 2(1)(6pq) + (6pq)^2$$
$$= (1 - 6pq)^2$$
- 4
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