

Exercise 4.1

Polynomials

A polynomial in the variable x is an algebraic expression of the form,

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0, a_n \neq 0$$

Where n , the highest power of x , is a non-negative integer called the degree of the polynomial and each coefficient a_i is a real number.

Q1. Identify whether the following algebraic expressions are polynomials (yes or no)

(i) $3x^2 + \frac{1}{x} - 5$

(ii) $3x^3 - 4x^2 - x\sqrt{x} + 3$

(iii) $x^2 - 3x + \sqrt{2}$

(iv) $\frac{3x}{2x-1} + 8$

Solution:

(i) No

(ii) No

(iii) Yes

(iv) No

Q2. State whether each of the following expression is a rational expression or not.

(i) $\frac{3\sqrt{x}}{3\sqrt{x}+5}$

(ii) $\frac{x^3 - 2x^2 + \sqrt{3}}{2x^2 - 1}$

(iii) $\frac{x^2 + 6x + 9}{x^2 - 9}$

(iv) $\frac{2\sqrt{x} + 3}{2\sqrt{x} - 3}$

Solution:

(i) No

(ii) Yes

(iii) Yes

(iv) No

Q3. Reduce the following rational expressions to the lowest forms.

1

(i) $\frac{120x^2 y^2 z^3}{30x^3 y z^2}$

(ii) $\frac{8a(x+1)}{2(x^2-1)}$

(iii) $\frac{(x+y)^2 - 4xy}{(x-y)^2}$

(iv) $\frac{(x^3 - y^3)(x^2 - 2xy + y^2)}{(x-y)(x^2 + xy + y^2)}$

(v) $\frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

(vi) $\frac{x^3 - 4x + 4}{2x^2 - 8}$

(vii) $\frac{64x^3 - 64x}{(8x^2 + 8)(2x+2)}$

(viii) $\frac{9x^2 - (x^2 - 4)^2}{4 + 3x - x^2}$

Solution:

1. $\frac{120x^2 y^2 z^3}{30x^3 y z^2} = \frac{30 \times 4y^{3-1}xz^{3-2}}{30x^{3-2}} = \frac{4y^2 z^2}{x}$

2. $\frac{8a(x+1)}{2(x^2-1)} = \frac{2(4a)(x+1)}{2(x+1)(x-1)} = \frac{4a}{x-1}$

3. $\frac{(x+y)^2 - 4xy}{(x-y)^2}$

$= \frac{x^2 + y^2 + 2xy - 4xy}{(x-y)^2}$

$= \frac{x^2 + y^2 - 2xy}{(x-y)^2}$

$= \frac{(x-y)^2}{(x-y)^2}$

$= \boxed{1}$

4. $\frac{(x^3 - y^3)(x^2 - 2xy + y^2)}{(x-y)(x^2 + xy + y^2)}$

$= \frac{(x^3 - y^3)(x-y)^2}{(x-y)(x^2 + xy + y^2)}$

2

$= \frac{(x^3 - y^3)(x-y)^2}{(x-y)(x^2 + xy + y^2)}$

$= \frac{(x-y)(x^2 + xy + y^2)(x-y)}{(x^2 + xy + y^2)}$

$= \boxed{(x-y)^2}$

5. $\frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

$= \frac{(x+2)(x+1)(x-1)}{(x+1)(x+2)(x-2)}$

$= \frac{x-1}{x-2}$

6. $\frac{x^3 - 4x + 4}{2x^2 - 8}$

$= \frac{x^3 - 4x + 2^2}{2(x^2 - 4)}$

$= \frac{(x-2)^2}{2(x+2)(x-2)}$

$= \boxed{\frac{x-2}{2(x+2)}}$

7. $\frac{64x^3 - 64x}{(8x^2 + 8)(2x+2)}$

$= \frac{64x(x^2 - 1)}{8(x^2 + 1)2(x+1)}$

$= \frac{64x(x^2 + 1)(x^2 - 1)}{16(x^2 + 1)(x+1)}$

$= \frac{4x(x^2 - 1)}{x+1}$

$= \frac{4x(x+1)(x-1)}{x+1}$

3

$= \boxed{4x(x-1)}$

8. $\frac{9x^2 - (x^2 - 4)^2}{4 + 3x - x^2}$

$= \frac{(3x)^2 - (x^2 - 4)^2}{4 + 3x - x^2}$

$= \frac{[3x + (x^2 - 4)][3x - (x^2 - 4)]}{4 + 3x - x^2}$

$= \frac{(x^2 + 3x - 4)(4 + 3x - x^2)}{4 + 3x - x^2}$

$= \boxed{x^2 + 3x - 4}$

Q4. Evaluate

(a) $\frac{x^3 y - 2z}{xz}$ for

i. $x = 3, y = -1, z = -2$

ii. $x = -1, y = -9, z = 4$

(b) $\frac{x^2 y^3 - 5z^4}{xyz}$ for $x = 4, y = -2, z = -1$

Solution:

(a) $\frac{x^3 y - 2z}{xz}$

(i) Putting $x = 3, y = -1, z = -2$ in the above equation, we get

$\frac{x^3 y - 2z}{xz}$

$= \frac{(3)^3(-1) - 2(-2)}{3(-2)}$

$= \frac{-27 + 4}{-6}$

4

$= \frac{-23}{-6}$

$= \frac{23}{6}$

$= \boxed{\frac{23}{6}}$

(ii) Putting $x = -1, y = -9, z = 4$ in the above equation, we get

$\frac{x^3 y - 2z}{xz}$

$= \frac{(-1)^3(-9) - 2(4)}{(-1)(4)}$

$= \frac{9 - 8}{-4}$

$= \boxed{-\frac{1}{4}}$

(b) $\frac{x^2 y^3 - 5z^4}{xyz}$

Putting $x = 4, y = -2, z = -1$ in the above equation, we get

$= \frac{(4)^2(-2)^3 - 5(-1)^4}{4(-2)(-1)}$

$= \frac{4(-2)(-1)}{4(-2)(-1)}$

$= \frac{16(-8) - 5(1)}{8} = \frac{-128 - 5}{8} = -\frac{133}{8}$

$= \boxed{-16\frac{7}{8}}$

Q5. Perform the indicated operation and simplify.

i. $\frac{15}{2x-3y} - \frac{4}{3y-2x}$

ii. $\frac{1+2x}{1-2x} - \frac{1-2x}{1+2x}$

5

iii. $\frac{x^2 - 25}{x^2 - 36} \cdot \frac{x+5}{x+6}$

iv. $\frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{x^2 - y^2}$

v. $\frac{x-2}{x^2 + 6x + 9} - \frac{x}{2x^2 - 18}$

vi. $\frac{1}{x-1} - \frac{1}{x+1} + \frac{2}{x^2 + 1} - \frac{4}{x^4 - 1}$

Solution:

i. $\frac{15}{2x-3y} - \frac{4}{3y-2x}$

$= \frac{15}{2x-3y} - \frac{4}{-2x+3y}$

$= \frac{15}{2x-3y} + \frac{4}{-(2x-3y)}$

$= \frac{15}{2x-3y} - \frac{4}{2x-3y}$

$= \frac{15+4}{2x-3y}$

$= \frac{19}{2x-3y}$

ii. $\frac{1+2x}{1-2x} - \frac{1-2x}{1+2x}$

$= \frac{(1+2x)^2 - (1-2x)^2}{(1-2x)(1+2x)}$

$= \frac{1+4x+4x^2 - (1-4x+4x^2)}{(1-2x)(1+2x)}$

$= \frac{1+4x+4x^2 - 1+4x-4x^2}{(1-2x)(1+2x)}$

$= \frac{8x}{1-4x^2}$

6

iii. $\frac{x^2 - 25}{x^2 - 36} \cdot \frac{x+5}{x+6}$

$= \left(\frac{(x^2 - 5)}{(x+6)(x-6)} \right) \left(\frac{(x-5)(x-6)}{(x-6)(x+6)} \right)$

$= \left(\frac{(x^2 - 5) - (x+5)(x-6)}{(x-6)(x+6)} \right)$

$= \left(\frac{(x^2 - 25) - x^2 + 6x - 5x + 30}{(x+6)(x-6)} \right)$

$= \frac{x+5}{(x+6)(x-6)}$

$= \frac{x+5}{x^2 - 36}$

iv. $\frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{x^2 - y^2}$

$= \frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{(x+y)(x-y)}$

$= \frac{x(x+y) - y(x-y) - 2xy}{(x+y)(x-y)}$

$= \frac{x^2 + xy - xy + y^2 - 2xy}{(x+y)(x-y)}$

$= \frac{x^2 + y^2 - 2xy}{(x+y)(x-y)}$

$= \frac{(x-y)^2}{(x+y)(x-y)}$

$= \frac{x-y}{x+y}$

v. $\frac{x-2}{x^2 + 6x + 9} - \frac{x+2}{2x^2 - 18}$

7

$= \frac{x-2}{(x+3)^2} - \frac{x+2}{2(x^2 - 9)}$

$= \frac{x-2}{(x+3)^2} - \frac{x+2}{2(x+3)(x-3)}$

$= \frac{(x-2)2(x-3) - (x+2)(x+3)}{2(x+3)^2(x-3)}$

$= \frac{2(x^2 - 5x + 6) - (x^2 + 5x + 6)}{2(x+3)^2(x-3)}$

$= \frac{2x^2 - 10x + 12 - x^2 - 5x - 6}{2(x+3)^2(x-3)}$

$= \frac{x^2 - 15x + 6}{2(x+3)^2(x-3)}$

vi. $\frac{1}{x-1} - \frac{1}{x+1} + \frac{2}{x^2 + 1} - \frac{4}{x^4 - 1}$

$= \frac{1}{x-1} - \frac{1}{x+1} + \frac{2}{x^2 + 1} - \frac{4}{(x^2 + 1)(x+1)(x-1)}$

$= \frac{(x^2 + 1)(x+1) - (x^2 + 1)(x-1) - 2(x+1)(x-1) - 4}{(x^2 + 1)(x+1)(x-1)}$

$= \frac{x^3 + x^2 + x + 1 - (x^3 - x^2 + x - 1) - 2(x^2 - 1) - 4}{(x^2 + 1)(x+1)(x-1)}$

$= \frac{x^3 + x^2 + x + 1 - x^3 + x^2 - x + 1 - 2x^2 + 2 - 4}{(x^2 + 1)(x+1)(x-1)}$

$= \frac{2}{x^2 - 1}$

Q6 Perform the indicated operation and simplify.

(i) $\left(x^2 - 49 \right) \frac{5x+2}{x+7}$

(ii) $\frac{4x-12}{x^2-9} + \frac{18-2x^2}{x^2+6x+9}$

(iii) $\frac{x^4 y^6}{x^2 - y^2} \div (x^4 + x^2 y^2 + y^4)$

(iv) $\frac{x^2 - 1}{x^2 + 2x + 1} \cdot \frac{x+5}{1-x}$

(v) $\frac{x^2 + xy}{y(x+y)} \cdot \frac{x^2 + xy}{y(x+y)} \cdot \frac{x^2 - x}{xy - 2y}$

8

Solution:

(i) $\left(x^2 - 49 \right) \frac{5x+2}{x+7}$

$= \frac{(x+7)(x-7)}{1} \cdot \frac{(5x+2)}{x+7}$

$= \frac{(x+7)(x-7)(5x+2)}{x+7}$

$= (x-7)(5x+2)$

$= 5x^2 + 2x - 35x - 14$

$= 5x^2 - 33x - 14$

(ii) $\frac{4x-12}{x^2-9} + \frac{18-2x^2}{x^2+6x+9}$

$= \frac{4x-12}{x^2-9} + \frac{x^2+6x+9}{18-2x^2}$

$= \frac{4(x-3)}{(x+3)(x-3)} + \frac{(x+3)^2}{2(9-x^2)}$

$= \frac{2(x-3) \times (x+3) \times (x+3)}{(x+3)(x-3)(3-x)(3+x)}$

$= \frac{2}{3-x}$

(iii) $\frac{x^4 - y^6}{x^2 - y^2} \div (x^4 + x^2 y^2 + y^4)$

$= \frac{x^4 - y^6}{x^2 - y^2} \times \frac{1}{(x^4 + x^2 y^2 + y^4)}$

$= \frac{(x^2 + y^2)(x^2 - y^2)}{(x+y)(x-y)} \times \frac{1}{x^2 + 2x^2 y^2 + y^4 - x^2 y^2}$

9

$= \frac{(x+y)(x^2 - xy + y^2)(x-y)(x^2 + xy + y^2)}{(x+y)(x-y)((x^2 + y^2)^2 - (xy)^2)}$

$= \frac{(x^2 - xy + y^2)(x^2 + xy + y^2)}{[(x^2 + y^2)^2 + (xy)^2][(x^2 + y^2)^2 - (xy)^2]}$

$= \frac{(x^2 - xy + y^2)(x^2 + xy + y^2)}{(x^2 - xy + y^2)(x^2 + xy + y^2)}$

$= \boxed{1}$

(iv) $\frac{x^2 - 1}{x^2 + 2x + 1} \times \frac{x+5}{1-x}$

$= \frac{(x+1)(x-1)}{(x+1)^2} \times \frac{x+5}{1-x}$

$= \frac{-(x+1)(1-x)(x+5)}{(x+1)((x+1)(1-x))}$

$= \boxed{\frac{-(x+5)}{x+1}}$

(v) $\frac{x^2 + xy}{y(x+y)} \cdot \frac{x^2 + xy}{y(x+y)} \cdot \frac{x^2 - x}{xy - 2y}$

$= \frac{x^2 + xy}{y(x+y)} \cdot \frac{x^2 + xy}{y(x+y)} \cdot \frac{xy - 2y}{x^2 - x}$

$= \frac{x(x+y)}{y(x+y)} \cdot \frac{x(x+y)}{y(x+y)} \cdot \frac{y(x-2)}{x(x-1)}$

$= \boxed{\frac{x(x-2)}{y(x-1)}}$

10

Exercise 4.2

Q1 (I) If $a + b = 10$ and $a - b = 6$ then find the value of $(a^2 + b^2)$.

(II) If $a + b = 5$ and $a - b = \sqrt{17}$, then find the value of ab .

Solution:

(I) $a+b=10$, $a-b=6$

$$(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$(10)^2 + (6)^2 = 2(a^2 + b^2)$$

$$100 + 36 = 2(a^2 + b^2)$$

$$2(a^2 + b^2) = 136$$

$$a^2 + b^2 = 68$$

(II) $a+b=5$, $a-b=\sqrt{17}$

$$(a+b)^2 - (a-b)^2 = 4ab$$

$$(5)^2 - (\sqrt{17})^2 = 4ab$$

$$25 - 17 = 4ab$$

or

$$4ab = 8$$

$$ab = 2$$

Q2. If $a^2 + b^2 + c^2 = 45$ and $a + b + c = -1$, find the value of $ab + bc + ca$.

Solution:

$$a^2 + b^2 + c^2 = 45, \quad a + b + c = -1$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$(-1)^2 = 45 + 2(ab + bc + ca)$$

$$1 = 45 + 2(ab + bc + ca)$$

$$1 - 45 = 2(ab + bc + ca)$$

$$2(ab + bc + ca) = -44$$

$$\Rightarrow ab + bc + ca = -22$$

Q3. If $m+n+p=10$ and $mn+np+mp=27$, find the value of $m^2 + n^2 + p^2$.

Solution:

$$(m + n + p)^2 = m^2 + n^2 + p^2 + 2(mn + np + mp)$$

$$(10)^2 = m^2 + n^2 + p^2 + 2(27)$$

$$100 - m^2 + n^2 + p^2 = 54$$

So

$$m^2 + n^2 + p^2 = 100 - 54 = 46$$

Q.4 If $x+y+z=78$, and $xy+yz+zx=59$, find the value of $x+y+z$.

Solution:

$$x^2 + y^2 + z^2 = 78, \quad xy + yz + zx = 59$$

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$$

$$= 78 + 2(59)$$

$$(x + y + z)^2 = 78 + 118 = 196$$

$$x + y + z = \pm\sqrt{196} = \pm 14$$

Q.5 If $x+y+z=12$ and $x^2 + y^2 + z^2 = 64$ find the value of $xy+yz+zx$.

Solution:

$$x + y + z = 12$$

$$x^2 + y^2 + z^2 = 64$$

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(x + y + z)$$

$$(12)^2 = 64 + 2(xy + yz + zx)$$

$$xy + yz + zx = 40$$

Q6. If $x+y+z=7$ and $xy=12$, then the value of $x^3 + y^3 + z^3$

Solution:

$$x + y + z = 7$$

$$xy = 12$$

$$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$$

$$(7)^3 = x^3 + y^3 + 3(12)(7)$$

$$343 = x^3 + y^3 + 252$$

or

$$x^3 + y^3 = 343 - 252 = 91$$

Q7. If $3x+4y=11$ and $xy=12$, then find the value of $27x^3 + 64y^3$

Solution:

$$3x + 4y = 11, \quad xy = 12 \quad (3x + 4y)^3 = (3x)^3 + (4y)^3 + 3(3x)(4y)(3x + 4y)$$

$$(11)^3 = 27x^3 + 64y^3 + 36xy(11)$$

$$1331 = 27x^3 + 64y^3 + 4752$$

or

$$27x^3 + 64y^3 = 1331 - 4752$$

$$27x^3 + 64y^3 = -3421$$

Q8. If $x-y=4$ and $xy=21$, the find the value of $x^3 - y^3$.

Solution:

$$x - y = 4,$$

$$xy = 21$$

$$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$$

$$(4)^3 = x^3 - y^3 - 3(21)(4)$$

$$64 = x^3 - y^3 - 252$$

$$x^3 - y^3 = 64 + 252 = 316$$

Q9. If $5x-6=13$ and $xy=6$, then find the value if $125x^3 - 216y^3$

Solution:

$$5x - 6y = 13$$

$$xy = 6$$

$$(5x - 6y)^3 = (5x)^3 - (6y)^3 - 3(5x)(6y)(5x - 6y)$$

$$(13)^3 = 125x^3 - 216y^3 - 1170(6)$$

$$2197 = 125x^3 - 216y^3 - 7020$$

$$125x^3 - 216y^3 = 2197 + 7020 = 9217$$

Q10. If $x + \frac{1}{x} = 3$, then find the value of $x^3 + \frac{1}{x^3}$

Solution:

$$x + \frac{1}{x} = 3 \quad \left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right)(3) = x^3 + \frac{1}{x^3} + 9$$

$$27 = x^3 + \frac{1}{x^3} + 9$$

$$x^3 + \frac{1}{x^3} = 27 - 9 = 18$$

Q11. If $x - \frac{1}{x} = 7$ then find the value of $x^3 - \frac{1}{x^3}$

Solution:

$$x - \frac{1}{x} = 7$$

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x\right)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$(7)^3 = x^3 - \frac{1}{x^3} - 21$$

$$343 = x^3 - \frac{1}{x^3} - 21$$

$$x^3 - \frac{1}{x^3} = 343 + 21 = 364$$

Q12. If $\left(3x + \frac{1}{3x}\right)$ then find the value of $\left(27x^3 - \frac{1}{27x^3}\right)$

Solution:

$$\left(3x + \frac{1}{3x}\right)^3 = (3x)^3 + \frac{1}{(3x)^3} + 3\left(3x\right)\left(\frac{1}{3x}\right)\left(3x + \frac{1}{3x}\right)$$

$$(5)^3 = 27^3 + \frac{1}{27x^3} + 3(5)$$

$$125 = 27x^3 + \frac{1}{27x^3} + 15$$

So

$$27x^3 + \frac{1}{27x^3} = 125 - 15 = 110$$

Q13. If $\left(5x - \frac{1}{5x}\right) = 6$, then find the value of $\left(125x^3 - \frac{1}{125x^3}\right)$

Solution:

Q14. Factorize

(i) $x^3 - y^3 - x + y$

$$= (x - y)(x^2 + xy + y^2) - 1(x - y)$$

$$= (x - y)(x^2 + y^2 - 1)$$

(ii) $8x^3 - \frac{1}{27y^3}$

$$= (2x)^3 - \left(\frac{1}{3y}\right)^3$$

$$= \left(2x - \frac{1}{3y}\right)\left((2x)^2 + 2x \cdot \frac{1}{3y} + \left(\frac{1}{3y}\right)^2\right)$$

$$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \left(\frac{1}{3y}\right)^2\right)$$

$$= \left(2x - \frac{1}{3y}\right)\left(4x^2 \cdot \frac{2x}{3y} + \frac{1}{9y^2}\right)$$

Q15. Find the product, using formulas.

(i) $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

$$= (x^2 + y^2)\left((x^2)^2 - x^2 \cdot y^2 + (y^2)^2\right)$$

$$= (x^2)^3 + (y^2)^3 = x^6 + y^6$$

(ii) $(x^4 - y^4)(x^8 + x^4y^4 + y^8)$

$$= (x^2 + y^2)\left((x^2)^2 - x^2 \cdot y^2 + (y^2)^2\right)$$

$$= (x^2)^3 - (y^2)^3 = x^6 - y^6$$

(iii) $(x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$

$$= ((x - y)(x^2 + xy + y^2))((x + y)(x^2 - xy + y^2))((x^2 + y^2)(x^4 - x^2y^2 + y^2))$$

$$= (x^3 - y^3)(x^3 + y^3)((x^2)^3 + (y^2)^3)$$

$$= ((x^2)^3 - (y^2)^3)(x^6 + y^6)$$

$$= (x^6 - y^6)(x^6 + y^6)$$

$$= (x^6)^2 - (y^6)^2$$

$$= x^{12} - y^{12}$$

(iv) $(2x^2 - 1)(2x^2 + 1)(4x^4 + 2x^2 + 1)$

$$= ((2x^2 - 1)(4x^4 + 2x^2 + 1))((2x^2 + 1)(4x^4 - 2x^2 + 1))$$

$$= (2x^2 - 1)((2x^2)^2 + 2x^2 \cdot 1 + (1)^2)(2x^2 + 1)(4x^4 - 2x^2 + 1)$$

$$= (2x^2 - 1)((2x^2)^2 + 2x^2 \cdot 1 + (1)^2)(2x^2 + 1)(2x^2 + 1)^2((2x^2)^2 - (2x^2)(1) + (1)^2)$$

$$= (((2x^2)^3 - (1)^3))((2x^2)^3 + (1)^3)$$

$$= (8x^6 - 1)(8x^6 + 1)$$

$$= (8x^6)^2 - (1)^2$$

$$= 64x^{12} - 1$$

Exercise 4.3

Q1. Express each of the following surds in the simplest form.

Solution:

(i) $\sqrt{180}$
 $=\sqrt{9 \times 4 \times 5}$
 $=\sqrt{9} \times \sqrt{4} \times \sqrt{5}$
 $=3.2.\sqrt{5}$
 $=6\sqrt{5}$

(ii) $3\sqrt{162}$
 $=3\sqrt{81 \times 2}$
 $=3.\sqrt{81}.\sqrt{2}$
 $=3.9.\sqrt{2}$
 $=27\sqrt{2}$

(iii) $\frac{3}{4}\sqrt[3]{128}$
 $=\frac{3}{4}\sqrt[3]{64 \times 2}$
 $=\frac{3}{4}\sqrt[3]{64}.\sqrt[3]{2}$
 $=\frac{3}{4}.4.\sqrt[3]{2}$
 $=3.\sqrt[3]{2}$

(iv) $\sqrt[3]{96x^6y^7z^8}$
 $=\sqrt[3]{32.3.x^5.x.y^5.y^2.z^5.z^3}$
 $=\sqrt[3]{(2xyz)^5.3xy^2z^3}$

$=2xyz\sqrt[3]{3xy^2z^3}$

Q2. Simplify

Solution:

(i) $\frac{\sqrt{18}}{\sqrt{3}.\sqrt{2}}$
 $=\frac{\sqrt{18}}{\sqrt{6}}$
 $=\sqrt{\frac{18}{6}}$
 $=\sqrt{3}$

(ii) $\frac{\sqrt{21}.\sqrt{9}}{\sqrt{63}}$
 $=\frac{\sqrt{21 \times 9}}{\sqrt{63}}$
 $=\frac{\sqrt{189}}{\sqrt{63}}$
 $=\sqrt{\frac{189}{63}}$
 $=\sqrt{3}$

(iii) $\sqrt[3]{243x^5y^{10}z^{15}}$
 $=\sqrt[3]{3^5}.\sqrt[3]{x^5}.\sqrt[3]{y^{10}}.\sqrt[3]{z^{15}}$
 $=3xy^2z^3$

(iv) $\frac{4}{5}\sqrt[3]{125}$

$=\frac{4}{5}\sqrt{5^2}$
 $=\frac{4}{5}.5$
 $=4$

(v) $\sqrt{21} \times \sqrt{7} \times \sqrt{3}$
 $=\sqrt{21} \times \sqrt{21}$
 $=\left(\sqrt{21}\right)^2$
 $=21$

Q3. Simplify by combining similar terms.

Solution:

(i) $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$
 $=\sqrt{9 \times 5}-3\sqrt{4 \times 5}+4\sqrt{5}$
 $=\sqrt{9}.\sqrt{5}-3.\sqrt{4}.\sqrt{5}+4.\sqrt{5}$
 $=3\sqrt{5}-3 \times 2\sqrt{5}+4\sqrt{5}$
 $=3\sqrt{5}-6\sqrt{5}+4\sqrt{5}$
 $=(3-6+4)\sqrt{5}$
 $=\sqrt{5}$

(ii) $4\sqrt{12}+5\sqrt{27}-3\sqrt{75}+\sqrt{300}$
 $=4\sqrt{4 \times 3}+5\sqrt{9 \times 3}-3\sqrt{25 \times 3}+\sqrt{100 \times 3}$
 $=4 \times 2 \times \sqrt{3}+5 \times 3 \times \sqrt{3}-3 \times 5 \times \sqrt{3}+10 \times \sqrt{3}$
 $=(8+15-15+10)\sqrt{3}$
 $=18\sqrt{3}$

(iii) $\sqrt{3}(2\sqrt{3}+3\sqrt{3})$

$=\sqrt{3}[(2+3)\sqrt{3}]$
 $=\left(\sqrt{3}\right)^2(5)$
 $=3 \times 5$
 $=15$

(iv) $2(6\sqrt{5}-3\sqrt{5})$
 $=2[(6-3)\sqrt{5}]$
 $=2.3\sqrt{5}$
 $=6\sqrt{5}$

Q4. Simplify

Solution:

(i) $(3+\sqrt{3})(3-\sqrt{3})$
 $=(3)^2-\left(\sqrt{3}\right)^2$
 $=9-3$
 $=6$

(ii) $(\sqrt{5}+\sqrt{3})^2$
 $=\left(\sqrt{5}\right)^2+\left(\sqrt{3}\right)^2+2\sqrt{5}.\sqrt{3}$
 $=5+3+2\sqrt{15}$
 $=8+2\sqrt{15}$

(iii) $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$
 $=\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2$
 $=5-3$
 $=2$

(iv) $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$
 $=\left(\sqrt{2}\right)^2-\left(\frac{1}{\sqrt{3}}\right)^2$
 $=2-\frac{1}{3}$
 $=\frac{6-1}{3}$
 $=\frac{5}{3}$

(v) $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})(x+y)(x^2+y^2)$
 $=\left(\left(\sqrt{x}\right)^2-\left(\sqrt{y}\right)^2\right)(x+y)(x^2+y^2)$
 $=(x-y)(x+y)(x^2+y^2)$
 $=(x^2-y^2)(x^2+y^2)$
 $=(x^2)^2-(y^2)^2$
 $=x^4-y^4$

Exercise 4.4

Q1. Rationalize the denominator of the following.

i)
$$\frac{3}{4\sqrt{3}}$$
$$= \frac{3}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{3\sqrt{3}}{4 \times 3}$$
$$= \frac{\sqrt{3}}{4}$$

ii)
$$\frac{14}{\sqrt{98}}$$
$$= \frac{14}{\sqrt{98}} \times \frac{\sqrt{98}}{\sqrt{98}}$$
$$= \frac{14\sqrt{98}}{98}$$
$$= \frac{1}{7} \sqrt{49 \times 2}$$
$$= \frac{1}{7} \times 7 \times \sqrt{2}$$
$$= \sqrt{2}$$

iii)
$$\frac{6}{\sqrt{8}\sqrt{27}}$$
$$= \frac{6}{\sqrt{8}\sqrt{27}} \times \frac{\sqrt{6}\sqrt{27}}{\sqrt{6}\sqrt{27}}$$

$$= \frac{6}{8 \times 27} \times \sqrt{6 \times 27}$$
$$= \frac{1}{36} \sqrt{4 \times 2} \times \sqrt{9 \times 3}$$
$$= \frac{1}{4 \times 9} \times 2 \times 3 \times \sqrt{2} \times \sqrt{3}$$
$$= \frac{1}{6} \sqrt{6}$$
$$= \frac{\sqrt{6}}{6}$$

iv)
$$\frac{1}{3+2\sqrt{5}}$$
$$= \frac{1}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$
$$= \frac{3-2\sqrt{5}}{(3)^2-(2\sqrt{5})^2}$$
$$= \frac{3-2\sqrt{5}}{9-(4 \times 5)}$$
$$= \frac{3-2\sqrt{5}}{-11}$$
$$= -\frac{1}{11}(3-2\sqrt{5})$$

v)
$$\frac{15}{\sqrt{31}-4}$$
$$= \frac{15}{\sqrt{31}-4} \times \frac{\sqrt{31}+4}{\sqrt{31}+4}$$
$$= \frac{15(\sqrt{31}+4)}{(\sqrt{31})^2-(4)^2}$$

$$= \frac{15(\sqrt{31}+4)}{31-16}$$
$$= \frac{15(\sqrt{31}+4)}{15}$$
$$= \sqrt{31}+4$$

vi)
$$\frac{2}{\sqrt{5}-\sqrt{3}}$$
$$= \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$
$$= 2 \frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}$$
$$= 2 \frac{\sqrt{5}+\sqrt{3}}{5-3}$$
$$= \frac{2(\sqrt{5}+\sqrt{3})}{2}$$
$$= \sqrt{5}+\sqrt{3}$$

vii)
$$\frac{\sqrt{5}-1}{\sqrt{3}+1}$$
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$
$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$$
$$= \frac{(\sqrt{3})^2-2\sqrt{3}+(1)^2}{3-1}$$

$$= \frac{3-2\sqrt{3}+1}{2}$$
$$= \frac{4-2\sqrt{3}}{2}$$
$$= \frac{2(2-\sqrt{3})}{2}$$
$$= 2-\sqrt{3}$$

viii)
$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$
$$= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$
$$= \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$
$$= \frac{(\sqrt{5})^2+2\sqrt{15}+(\sqrt{3})^2}{5-3}$$
$$= \frac{5+3+2\sqrt{15}}{2}$$
$$= \frac{8+2\sqrt{15}}{2}$$
$$= \frac{2(4+\sqrt{15})}{2}$$
$$= 4+\sqrt{15}$$

Q2. Find the conjugate of $x+\sqrt{y}$

i) $3+\sqrt{7}$ iv) $2+\sqrt{5}$
ii) $4-\sqrt{5}$ v) $5+\sqrt{7}$
iii) $2+\sqrt{3}$ vi) $4-\sqrt{15}$

vii) $7-\sqrt{6}$ viii) $9-\sqrt{2}$

Solution:
i) **Conjugate of $3+\sqrt{7}$ is $3-\sqrt{7}$**
ii) **Conjugate of $4-\sqrt{5}$ is $4+\sqrt{5}$**
iii) **Conjugate of $2+\sqrt{5}$ is $2-\sqrt{5}$**
iv) **Conjugate of $2+\sqrt{5}$ is $2-\sqrt{5}$**
v) **Conjugate of $5+\sqrt{7}$ is $5-\sqrt{7}$**
vi) **Conjugate of $4-\sqrt{15}$ is $4+\sqrt{15}$**
vii) **Conjugate of $7+\sqrt{6}$ is $7+\sqrt{6}$**
viii) **Conjugate of $9-\sqrt{2}$ is $9+\sqrt{2}$**

Q3.

(i) If $x=2-\sqrt{3}$, find $\frac{1}{x}$

(ii) If $x=4-\sqrt{17}$, find $\frac{1}{x}$

(iii) If $x=\sqrt{3}+2$, find $x+\frac{1}{x}$

Solution:

i) $x=2-\sqrt{3}$
$$\frac{1}{x} = \frac{1}{2-\sqrt{3}}$$
$$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$
$$= \frac{2+\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \frac{2+\sqrt{3}}{4-3}$$
$$= 2+\sqrt{3}$$

ii) $x=4-\sqrt{17}$

$$\frac{1}{x} = \frac{1}{4-\sqrt{17}}$$
$$= \frac{1}{4-\sqrt{17}} \times \frac{4+\sqrt{17}}{4+\sqrt{17}}$$
$$= \frac{4+\sqrt{17}}{(4)^2-(\sqrt{17})^2}$$
$$= \frac{4+\sqrt{17}}{16-17}$$
$$= -(4+\sqrt{17})$$
$$= -4-\sqrt{17}$$

iii) $x=\sqrt{3}+2$
$$\frac{1}{x} = \frac{1}{\sqrt{3}+2}$$
$$= \frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$$
$$= \frac{\sqrt{3}-2}{(\sqrt{3})^2-(2)^2}$$
$$= \frac{\sqrt{3}-2}{3-4} = \frac{\sqrt{3}-2}{-1}$$
$$= -(\sqrt{3}-2)$$
$$= -\sqrt{3}+2$$
$$= 2-\sqrt{3}$$

$$\therefore x+\frac{1}{x} = \sqrt{3}+2+2-\sqrt{3} = [4]$$

Q4. Simplify:

(i) $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

(ii) $\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2+\sqrt{5}}$

(iii) $\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$

Solution:

(i)
$$\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$$
$$= \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})+(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$$
$$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}-\sqrt{6}+\sqrt{3}+\sqrt{5}-\sqrt{10}-\sqrt{6}+\sqrt{6}}{(\sqrt{5})^2-(\sqrt{3})^2}$$
$$= \frac{2\sqrt{5}-2\sqrt{6}}{5-3}$$
$$= \frac{2(\sqrt{5}-\sqrt{6})}{2}$$
$$= \sqrt{5}-\sqrt{6}$$

(ii)
$$\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2+\sqrt{5}}$$
$$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$
$$= \frac{2-\sqrt{3}}{4-3} + \frac{2(\sqrt{5}+\sqrt{3})}{5-3} + \frac{2-\sqrt{5}}{4-5}$$
$$= \frac{2-\sqrt{3}}{1} + \frac{2(\sqrt{5}+\sqrt{3})}{2} + \frac{2-\sqrt{5}}{-1}$$
$$= 2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2+\sqrt{5}$$
$$= 2\sqrt{5}$$

(iii)
$$\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$$
$$= \frac{2}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$
$$= \frac{2(\sqrt{3}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2} - \frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$$
$$= \frac{2(\sqrt{3}-\sqrt{2})}{5-3} + \frac{(\sqrt{3}-\sqrt{2})}{3-2} - \frac{3(\sqrt{5}-\sqrt{2})}{5-2}$$
$$= \sqrt{3}-\sqrt{2}+\sqrt{3}-\sqrt{2}-(\sqrt{5}-\sqrt{2})$$
$$= \sqrt{3}-\sqrt{2}-\sqrt{5}+\sqrt{2}=[0]$$

Q5. i)

If $x=2+\sqrt{3}$, find the value of $x-\frac{1}{x}$ and $\left(x-\frac{1}{x}\right)^2$

ii) $x=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ find the value of $x+\frac{1}{x}$, $x^2+\frac{1}{x^2}$ and $x^3+\frac{1}{x^3}$

Hint: $a^2+b^2=(a+b)^2-2ab$
and
 $a^3+b^3=(a+b)^3-3ab(a+b)$

Solution:

i) $x=2+\sqrt{3}$
$$\frac{1}{x} = \frac{1}{2+\sqrt{3}}$$
$$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$
$$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$
$$= \frac{2-\sqrt{3}}{4-3} = [2-\sqrt{3}]$$

ii) $x=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$
$$\frac{1}{x} = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$
$$x+\frac{1}{x} = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} + \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$
$$= \frac{(\sqrt{5}-\sqrt{2})^2+(\sqrt{5}+\sqrt{2})^2}{(\sqrt{5})^2-(\sqrt{2})^2}$$
$$= \frac{(\sqrt{5})^2+(\sqrt{2})^2+(\sqrt{5})^2+(\sqrt{2})^2}{5-2}$$
$$= \frac{5+2+5+2}{3} = \frac{14}{3}$$
$$\left(x+\frac{1}{x}\right)^2 = \left(\frac{14}{3}\right)^2$$
$$x^2+\frac{1}{x^2}+2 = \frac{196}{9}$$
$$= \frac{196}{9} - 2$$
$$= \frac{196-8}{9} = \frac{178}{9}$$
$$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$$
$$= \left(\frac{14}{3}\right)^3 - 3\left(\frac{14}{3}\right)$$

$$= \frac{2744-378}{27} = \frac{2366}{27}$$

Q6. Determine the rational number a and b if

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} = a+b\sqrt{3}$$

Solution:

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} = a+b\sqrt{3}$$
$$= \frac{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = a+b\sqrt{3}$$
$$= \frac{3-2\sqrt{3}+1+3+2\sqrt{3}+1}{3-1} = a+b\sqrt{3}$$
$$\frac{8}{2} = a+b\sqrt{3}$$
$$4 = a+b\sqrt{3}$$
$$4+0 = a+b\sqrt{3}$$

By comparing both sides of the equation
$$\Rightarrow a=4, \quad b=0$$

REVIEW EXERCISE 4

Q1. Multiple Choice Questions. Choose the correct answers.

(i) $4x+3y-2$ is an algebraic.....

- (a) expression (b) sentence
(c) equation (d) in equation

(ii) The degree of polynomial $4x^4+2x^{2y}$ is

- (a) 1 (b) 2 (c) 3 (d) 4

(iii) a^2+b^3 is equal to.....

- a) $....(a-b)(a^2+ab+b^2)$
b) $....(a+b)(a^2-ab+b^2)$
c) $....(a-b)(a^2-ab+b^2)$
d) $....(a-b)(a^2+ab-b^2)$

(iv) Conjugate of surd $a+\sqrt{b}$ is.....

- (a) $-a+\sqrt{b}$ (b) $a-\sqrt{b}$
(c) $a+\sqrt{b}$ (d) $-a-\sqrt{b}$

(v) $(3+\sqrt{2})(3-\sqrt{2})$ is equal to.....

- (a) 7 (b) -7
(c) 1 (d) -1

(vi) $\frac{1}{a-b}-\frac{1}{a+b}$ is equal to.....

1

(a) $\frac{2a}{a^2+b^2}$ (b) $\frac{2b}{a^2-b^2}$

(c) $\frac{-2a}{a^2-b^2}$ (d) $\frac{-2b}{a^2+b^2}$

(vii) $\frac{a^2-b^2}{a+b}$ is equal to.....

- (a) $(a-b)^2$ (b) $(a+b)^2$
(c) $a+b$ (d) $a-b$

(viii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$ is equal to.....

- (a) a^2+b^2 (b) a^2-b^2
(c) $a-b$ (d) $a+b$

Answer:

(i)	c	(ii)	d	(iii)	b	(iv)	b
(v)	a	(vi)	b	(vii)	d	(viii)	c

Q2. Fill in the blanks:

i. the degree of polynomial $x^3y^2+3xy+y^3$ is

ii. $x^2-4=$

iii. $x^3+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)(\dots\dots\dots)$

iv. $2(a^2+b^2)=(a+b)^2+(\dots\dots\dots)^2$.

v. $\left(x-\frac{1}{x}\right)^2=$

vi. Order of surd $\sqrt[3]{x}$ is

vii. $\frac{1}{2-\sqrt{3}}=$

2

Answers:

i. 3

ii. $(x+2)(x-2)$

iii. $x^3+2+\frac{1}{x^2}$

iv. $(a+b)^2(a-b)^2$

v. $x^2-2+\frac{1}{x^2}$

vi. 3

vii. $2+\sqrt{3}$

Q3. If $x+\frac{1}{x}=3$ find these.

i. $x^2+\frac{1}{x^2}$

ii. $x^4+\frac{1}{x^4}$

Solution:

i. $\left(x+\frac{1}{x}\right)^2=(3)^2$

$$x^2+\frac{1}{x^2}+2=9$$

$$x^2+\frac{1}{x^2}=7$$

ii. $\left(x^2+\frac{1}{x^2}\right)^2=(7)^2$

$$x^4+\frac{1}{x^4}=47$$

Q4. If $x-\frac{1}{x}=2$ find these:

i. $x^2+\frac{1}{x^2}$

ii. $x^4+\frac{1}{x^4}$

Solution:

3

i. $\left(x-\frac{1}{x}\right)^2=(2)^2$

$$x^2+\frac{1}{x^2}-2=4$$

$$x^2+\frac{1}{x^2}=6$$

ii. $\left(x^2+\frac{1}{x^2}\right)^2=(6)^2$

$$x^4+\frac{1}{x^4}+2=36$$

$$x^4+\frac{1}{x^4}=34$$

Q5. Find the value of x^3+y^3 and xy if $x+y=5$ and $x-y=3$

Solution:

$$x+y=5, x-y=3$$

$$4xy=(x+y)^2-(x-y)^2$$

$$=(5)^2-(3)^2$$

$$xy=4$$

Now,

$$x^3+y^3=(x+y)^3-3xy(x+y)$$

$$=(5)^3-3(4)(5)$$

$$=125-60$$

$$=65$$

6. If $p=2+(3)^{1/2}$, find:

Solution:

4

$$\frac{1}{p}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$=2-\sqrt{3}$$

1. $P+\frac{1}{p}=2+\sqrt{3}+2-\sqrt{3}=4$

2. $p-\frac{1}{p}=2+\sqrt{3}-(2-\sqrt{3})=2\sqrt{3}$

3. $=p^2+\frac{1}{p^2}=(p+\frac{1}{p})^2-2=(4)^2-2=16-2=14$

4. $=p^2-\frac{1}{p^2}=(p+\frac{1}{p})(p-\frac{1}{p})=4(2\sqrt{3})=8(\sqrt{3})$

Q7. Simplify

Solution:

$$\frac{1}{q}=\frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$=\sqrt{5}-2$$

1. $q+\frac{1}{q}=\sqrt{5}+2+\sqrt{5}-2=2\sqrt{5}$

2. $q-\frac{1}{q}=\sqrt{5}+2-(\sqrt{5}-2)=4$

3. $q^2+\frac{1}{q^2}=(q+\frac{1}{q})^2-2=(2\sqrt{5})^2-2=20-2=18$

4. $q^2-\frac{1}{q^2}=(q+\frac{1}{q})(q-\frac{1}{q})=(2\sqrt{5})(4)=8\sqrt{5}$

Q8. Simplify:

Solution:

5

$$\frac{\sqrt{a^2+2}+\sqrt{a^2+2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}=\frac{\sqrt{a^2+2}+\sqrt{a^2+2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}\times\frac{\sqrt{a^2+2}+\sqrt{a^2+2}}{\sqrt{a^2+2}+\sqrt{a^2+2}}$$

$$=\frac{a^2+2+2\sqrt{a^2-4}+a^2-2}{a^2+2-a^2+2}$$

1. $=\frac{2a^2-2\sqrt{a^2-4}}{4}$

$$=\frac{2(a^2-\sqrt{a^2-4})}{4}$$

$$=\frac{a^2-\sqrt{a^2-4}}{2}$$

$$=\frac{1}{a-\sqrt{a^2-x^2}}-\frac{1}{a+\sqrt{a^2-x^2}}=\frac{a+\sqrt{a^2-x^2}-(a-\sqrt{a^2-x^2})}{(a-\sqrt{a^2-x^2})(a+\sqrt{a^2-x^2})}$$

$$=\frac{2\sqrt{a^2-x^2}}{(a^2-(\sqrt{a^2-x^2})^2)}$$

2. $=\frac{2\sqrt{a^2-x^2}}{a^2-(a^2-x^2)}$

$$=\frac{2\sqrt{a^2-x^2}}{a^2-x^2+x^2}$$

$$=\frac{2\sqrt{a^2-x^2}}{x^2}$$

6