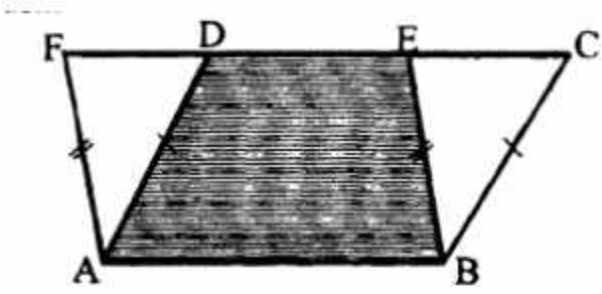


THEOREM 16.1.1

Parallelograms on the same base and between the same parallel lines (or of the same altitude) are equal in area.

Solution:



Given:

Two parallelograms ABCD and ABEF having the same base  $\overline{AB}$  and between the same parallel lines AB and DE.

To Prove:

Area of parallelogram ABCD = Area of parallelogram ABEF

Proof

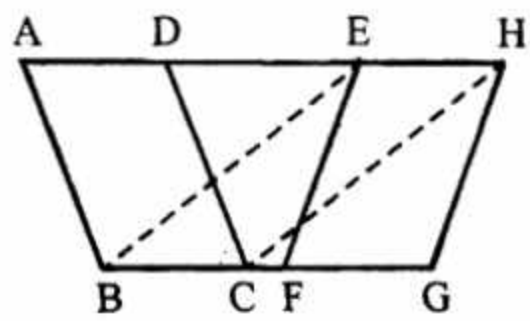
| Statements                                                                                       | Reasons                           |
|--------------------------------------------------------------------------------------------------|-----------------------------------|
| area of (parallelogram ABCD)<br>= area of (quadrilateral ABED) + area of $\triangle CBE$ ... (1) | Area addition axiom               |
| area of (parallelogram ABEF)<br>= area of (quadrilateral ABED) + area of $\triangle DAF$ ... (2) |                                   |
| $m\overline{CB} = m\overline{DA}$                                                                | opposite sides of a parallelogram |
| $m\overline{BE} = m\overline{AF}$                                                                | opposite sides of a parallelogram |

|                                                                                           |                                   |
|-------------------------------------------------------------------------------------------|-----------------------------------|
| $m\angle CBE = m\angle DAF$                                                               | opposite sides of a parallelogram |
| $\therefore \triangle CBE \cong \triangle DAF$                                            | S.A.S. congruent axiom            |
| $\therefore \text{area of } (\triangle CBE) = \text{area of } (\triangle DAF)$<br>... (3) | congruent area axiom              |
| Hence area of (parallelogram ABCD)<br>= area of (parallelogram ABEF)                      | from (1), (2) and (3)             |

THEOREM 16.1.2

Parallelograms on equal bases and having the same (or equal) altitude are equal in area.

Solution



Given

Parallelograms ABCD, EFGH are on the equal bases  $\overline{BC}$  and  $\overline{FG}$ , having equal altitudes.

To Prove

area of (parallelogram ABCD) = area of (parallelogram EFGH)

Construction

Place the parallelograms ABCD and EFGH so that their equal bases  $\overline{BC}, \overline{FG}$  are in the straight line BCFG. Join  $\overline{BE}$  and  $\overline{CH}$ .

Proof

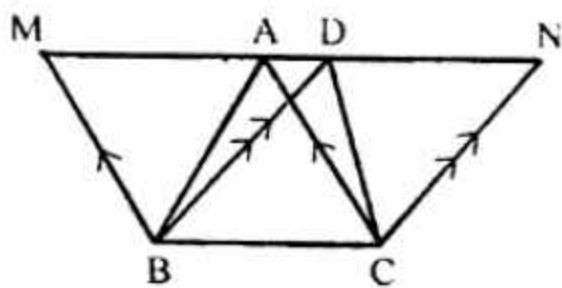
| Statements                                                                                                                                                                        | Reasons                                                                   |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------|
| The given $\parallel^{\text{gm}}$ ABCD and EFGH are between the same parallels<br>Hence ADEH is a straight line $\parallel$ to BC<br>$\therefore m\overline{BC} = m\overline{FG}$ | Their altitudes are equal (given)<br><br>Given<br>EFGH is a parallelogram |

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                    |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $= m\overline{EH}$<br>Now $m\overline{BC} = m\overline{EH}$ and they are $\parallel$<br>$\therefore \overline{BE}$ and $\overline{CH}$ are both equal and $\parallel$<br>Hence EBCH is a parallelogram<br>Now $\parallel^{\text{gm}}$ ABCD = $\parallel^{\text{gm}}$ EBCH ....(i)<br><br>But $\parallel^{\text{gm}}$ EBCH = $\parallel^{\text{gm}}$ EFGH .....(ii)<br>Hence,<br>area ( $\parallel^{\text{gm}}$ ABCD) = area ( $\parallel^{\text{gm}}$ EFGH)<br>same parallels | A quadrilateral with two opposite<br>Being on the same base BC and<br>between the same parallels<br>Being on the same base same parallels<br><br>From (i) and (ii) |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|

THEOREM 16.1.3

Triangles on the same base and of the same (i.e., equal) altitudes are equal in area.

Solution



Given

$\Delta$ s ABC, DBC on the same base  $\overline{BC}$ , and having equal altitudes.

To Prove

Area of ( $\Delta ABC$ ) = Area of ( $\Delta DBC$ )

Construction

Draw  $\overline{BM} \parallel$  to  $\overline{CA}$ ,  $\overline{CN} \parallel$  to  $\overline{BD}$  meeting  $\overline{AD}$  produced in M, N.

Proof

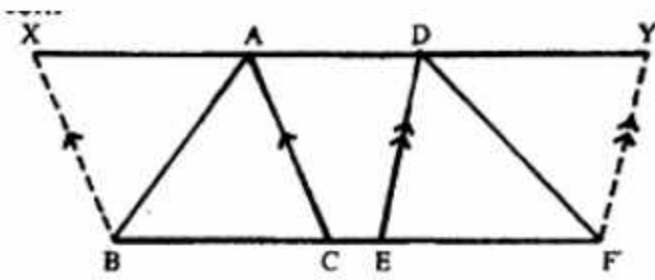
| Statements                                                                                                                                                                                                                      | Reasons                                                                                                                                         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| $\Delta ABC$ and $\Delta DEF$ are between the same $\parallel^{\text{gs}}$<br>Hence MADN is parallel to $\overline{BC}$<br>$\therefore$ Area ( $\parallel^{\text{gm}}$ BCAM) = Area ( $\parallel^{\text{gm}}$ BCND)<br>.....(i) | Their altitudes are equal<br><br>These $\parallel^{\text{gm}}$ are on the same base $\overline{BC}$ and between the same $\parallel^{\text{s}}$ |
| But $\Delta ABC = \frac{1}{2} \parallel^{\text{gm}}$ BCAM) .....(ii)                                                                                                                                                            | Each diagonal of a $\parallel^{\text{gm}}$ bisects it into two                                                                                  |

|                                                                                                                                   |                                                     |
|-----------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------|
| and $\Delta DEF =$ (Area of ( $\parallel^{\text{gm}}$ EFYD) .....(iii)<br>Hence,<br>Area ( $\Delta ABC$ ) = Area ( $\Delta DBC$ ) | congruent triangles<br><br>From (i), (ii) and (iii) |
|-----------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------|

THEOREM 16.1.4

Triangles on equal bases and of equal altitudes are equal in area.

Solution:



Given:

$\Delta$ s ABC, DEF on equal bases  $\overline{BC}$  and  $\overline{EF}$  having altitudes equal.

To prove:

$\text{Area } (\Delta ABC) = \text{Area } (\Delta DEF)$

Construction:

Place the  $\Delta$ s ABC and DEF so that their equal bases  $\overline{BC}$  and  $\overline{EF}$  are in the same straight line BCEF and their vertices on the same side of it. Draw  $\overline{BX} \parallel$  to  $\overline{CA}$  and  $\overline{FY} \parallel$  to  $\overline{ED}$ , meeting  $\overline{AD}$  produced in X, Y respectively.

Proof

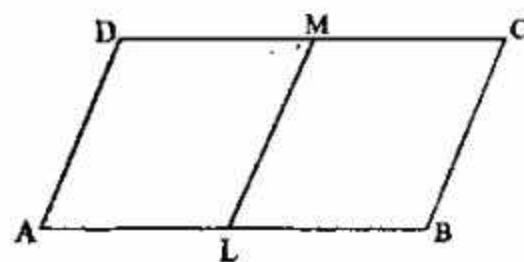
| Statements                                                                                                                | Reasons                                                                                                                             |
|---------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------|
| $\Delta ABC, \Delta DEF$ are between the same parallels<br>$\therefore$ XADY is $\parallel$ to BCEF                       | Their altitudes are equal (given)                                                                                                   |
| $\therefore \text{Area } (\parallel^{\text{gm}} \text{ BCAX}) = \text{Area } (\parallel^{\text{gm}} \text{ EFYD})$<br>(i) | These $\parallel^{\text{gm}}$ are on equal bases and between the same parallels<br>Diagonal of a $\parallel^{\text{gm}}$ bisects it |

|                                                                                                                                                                                                                  |                                           |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------|
| But $\Delta ABC = \frac{1}{2} (\parallel^{\text{gm}} \text{ BCAX})$<br>and $\Delta DEF = \frac{1}{2} (\parallel^{\text{gm}} \text{ EFYD})$<br>$\therefore \text{Area } (\Delta ABC) = \text{Area } (\Delta DEF)$ | (ii)<br>(iii)<br>From (i), (ii) and (iii) |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------|

## EXERCISE 16.1

**Q1.** Show that the line segment joining the mid points of opposite sides of a parallelogram divides it into two equal parallelograms.

**Solution:**



**To prove:**

Area of parallelogram  $ALMD$  = Area of parallelogram  $LBCM$ .

**Proof:**

$\overline{AB} \parallel \overline{CD}$  opposite sides of parallelogram  $ABCD$ .

As  $L$  is midpoint of  $\overline{AB}$

$$\overline{AL} \cong \overline{LB}$$

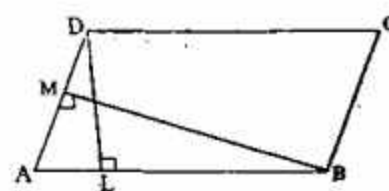
The parallelogram  $ALMD$  and  $LBCM$  are on equal bases ( $\overline{AL} \cong \overline{LB}$ ) and between the same parallel lines  $AB$  and  $DC$ .

$\therefore$  They are equal areas

Hence Area of parallelogram  $ALMD$  = Area of parallelogram  $LBCM$ .

**Q2.** In a parallelogram  $ABCD$ ,  $m\overline{AB} = 10$  cm. The altitudes corresponding to sides  $AB$  and  $AD$  are respectively 7 cm and 8 cm. Find  $\overline{AD}$ .

**Solution:**



1

**Given:**

$ABCD$  is a parallelogram.

$m\overline{AB} = 10$  cm,  $\overline{DL}$  and  $\overline{BM}$  are altitudes

$m\overline{DL} = 7$  cm,  $m\overline{BM} = 8$  cm

**To prove:**

$$m\overline{AD} = ?$$

**Proof:**

Area of a parallelogram = base  $\times$  altitude

Area of a parallelogram  $ABCD$

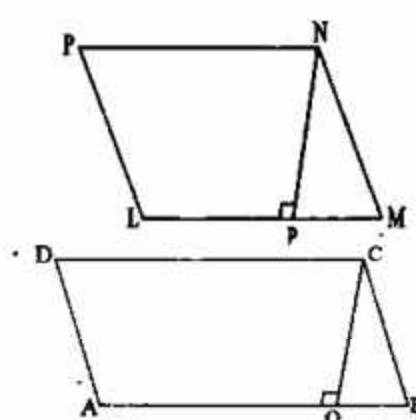
$$m\overline{AB} \times m\overline{DL} = m\overline{AD} \times m\overline{BM}$$

$$10 \times 7 = m\overline{AD} \times 8$$

$$m\overline{AD} = \frac{10 \times 7}{8} = \frac{35}{8} = 8.75 \text{ cm}$$

**Q3.** If two parallelograms of equal areas have the same or equal bases, their altitudes are equal.

**Solution**



**Given:**

In a parallelogram  $ABCD$ ,  $\overline{CQ}$  is altitude and in parallelogram  $LMNP$ ,  $\overline{NP}$  is altitude. Areas of parallelogram  $ABCD$  = Area of parallelogram  $LMNP$  and  $m\overline{AB} = m\overline{LM}$

2

**To prove:**

$$m\overline{CQ} = m\overline{NP}.$$

**Proof:**

Area of a parallelogram  $ABCD$  = Area of parallelogram  $LMNP$  (Given)

We know that area of a parallelogram = base  $\times$  altitude

$$m\overline{AB} \times m\overline{CQ} = m\overline{LM} \times m\overline{NP}$$

but  $m\overline{AB} = m\overline{LM}$  (Given)

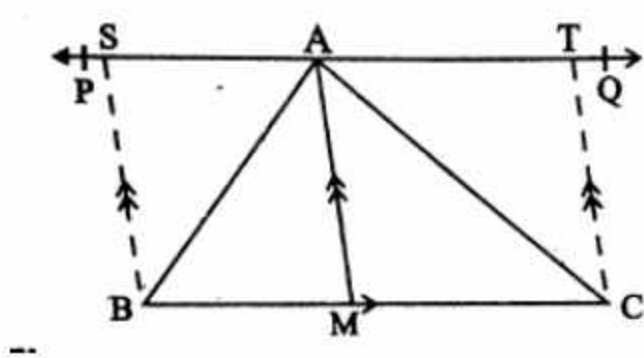
$$m\overline{CQ} = m\overline{NP}$$

3

EXERCISE 16.2

Q1. Show that a median of a triangle divides it into two triangles of equal area.

Solution:



Given

In  $\triangle ABC$ ,  $\overline{AM}$  is median

i.e.  $m\overline{BM} = m\overline{MC}$

To prove:

Area  $\triangle ABM$  = Area  $\triangle ACM$

Construction:

Draw  $\overline{PQ} \parallel \overline{BC}$ , Draw  $\overline{BS} \parallel \overline{AM}$  and  $\overline{CT} \parallel \overline{AM}$

Proof:

$\overline{BS} \parallel \overline{MA}$  (Construction)

$\overline{BM} \parallel \overline{SA}$  (Construction)

$\therefore$  BMAS is a parallelogram.

Similarly, AMCT is also a parallelogram.

Parallelograms BMAS and they are between the same parallel lines  $\overline{BC}$  and  $\overline{PQ}$ .

$\therefore$  They have equal areas.

So, Area parallelogram BMAS = Area parallelogram AMCT

$\Rightarrow \frac{1}{2}$  (area parallelogram BMAS)

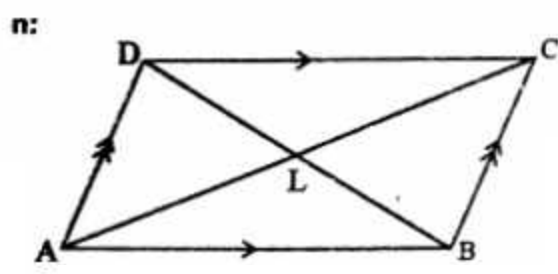
$\Rightarrow \frac{1}{2}$  (area parallelogram AMCT)

Area  $\triangle ABM$  = Area  $\triangle AMC$

So, a median of a triangle divides it into two triangles of equal area.

Q2. Prove that a parallelogram is divided by its diagonals into four triangles of equal area.

Solution:



Given:

In parallelogram ABCD,  $\overline{AC}$  and  $\overline{BD}$  are its diagonal, which meet at L.

To prove:

Triangles ABL, BCL, CDL and ADL have equal area.

Proof:

Triangles ABC and ABD have the same base  $\overline{AB}$  and are between the same parallel lines  $\overline{AB}$  and  $\overline{DC}$ .

They have equal area,

or Area  $\triangle ABC$  = Area  $\triangle ABD$

or Area  $\triangle ABL$  + Area  $\triangle BCL$  ~ Area  $\triangle ABL$  + Area  $\triangle ADL$

$\Rightarrow$  Area  $\triangle BCL$  = Area  $\triangle ADL$  (i)

Similarly, Area  $\triangle ABC$  = Area  $\triangle BCD$

Area  $\triangle BCL$  = Area  $\triangle ABL$

Area  $\triangle BCL$  = Area  $\triangle CDL$

$\Rightarrow$  Area  $\triangle ABL$  ~ Area  $\triangle CDL$  (ii)

As diagonals of a parallelogram bisect each other,

L is midpoint of  $\overline{AC}$ .

So,  $\overline{BL}$  is a median of  $\triangle ABC$

Area  $\triangle BCL$  = Area  $\triangle ABL$  (iii)

From (i), (ii) and (iii) we get

Area  $\triangle ABL$  = Area  $\triangle BCL$  = Area  $\triangle CDL$  = Area  $\triangle ADL$

Q3. Divide a triangle into six equal triangular parts.

Solution:

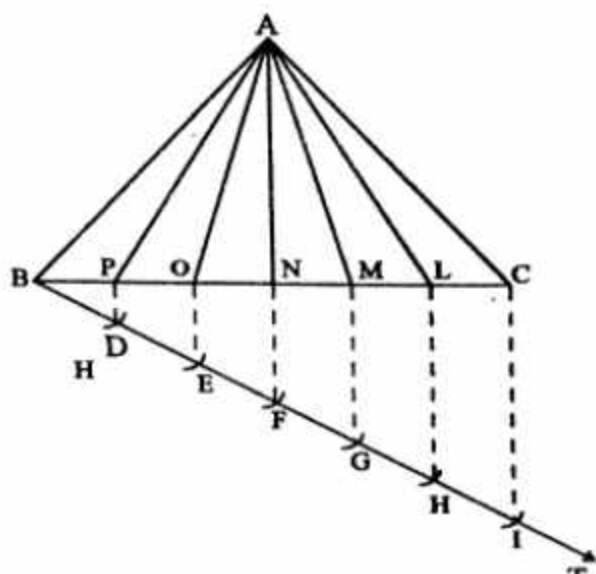
Given:

$\triangle ABC$

Required:

To divide  $\triangle ABC$  into six equal triangular parts.

Construction:



(i) Draw the ray  $\overline{BT}$  making an acute angle CBT.

(ii) On  $\overline{BT}$  mark six points D; E; F; G; H and I such that

$m\overline{BD} = m\overline{DE} = m\overline{EF} = m\overline{FG} = m\overline{GH} = m\overline{HI}$

(iii) Join IC.

(iv) Draw  $\overline{HL}$ ,  $\overline{GM}$ ,  $\overline{FN}$ ,  $\overline{EO}$ ,  $\overline{DP}$  each parallel to  $\overline{IC}$ .

(v) Join A to L, M, N, O and P. So, BAP, PAO, OAN, NAM, MAL and LAC are required six equal parts.

REVIEW EXERCISE 16

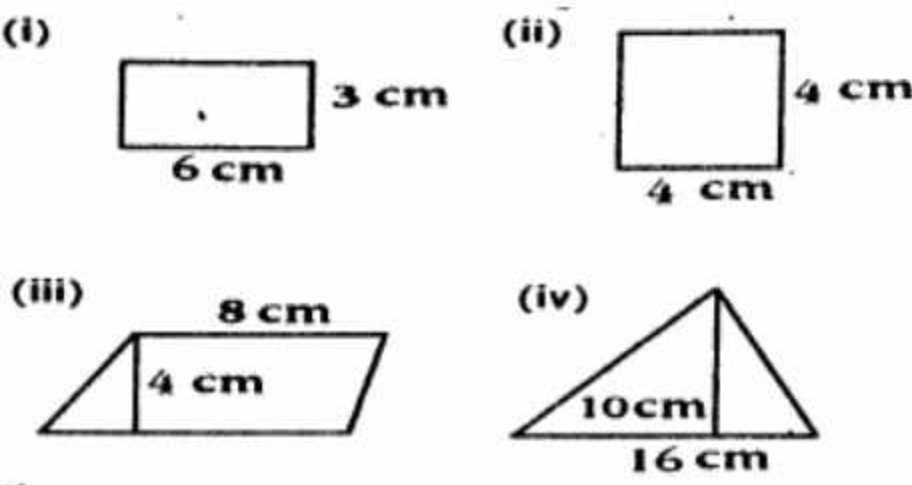
Q1. Which of the following are true and which are false?

- (i) Area of a figure means region enclosed by bounding lines of closed figure.
- (ii) Similar figures have same area.
- (iii) Congruent figures have same area.
- (iv) A diagonal of a parallelogram divides it into two non-congruent triangles.
- (v) Altitude of a triangle means perpendicular from vertex to the opposite side (base).
- (vi) Area of a parallelogram is equal to the product of base and height.

Answers:

|       |        |         |        |       |        |
|-------|--------|---------|--------|-------|--------|
| (i) T | (ii) F | (iii) T | (iv) F | (v) T | (vi) T |
|-------|--------|---------|--------|-------|--------|

Q2. find the area of the following.



Solution:

(i)  $\text{Area} = 6 \times 3 = 18\text{ cm}^2$

(ii)  $\text{Area} = 4 \times 4 = 16\text{ cm}^2$

(iii)  $\text{Area} = 8 \times 4 = 32\text{ cm}^2$

(iv)  $\text{Area} = \frac{1}{2} \times 10 \times 16 = 80\text{ cm}^2$

Q3. Define the following.

Solution:

(i) **Area of a figure:**

The region enclosed by the bounding lines of a closed figure is called the area of the figure.

The area of a closed region is expressed in square units (say Sq. m or m<sup>2</sup>).

(ii) **Triangular Region:**

The interior of a triangle is the part of the plane, enclosed by the triangle.

A triangle region is the union of a triangle and its interior i.e., the three-line segment forming the triangle and its interior.

By area of a triangle, we mean the area of its triangular region.

(iii) **Rectangular Region:**

The interior of a rectangle is the part of the plane enclosed by the rectangle.

A rectangular region is the union of a rectangle and its interior.

A rectangular region can be divided into two or more than two triangular regions in many ways.

(iv) **Altitude or Height of a triangle**

If one side of a triangle is taken as its base the perpendicular to that side, from the opposite vertex is called altitude or height of the triangle.