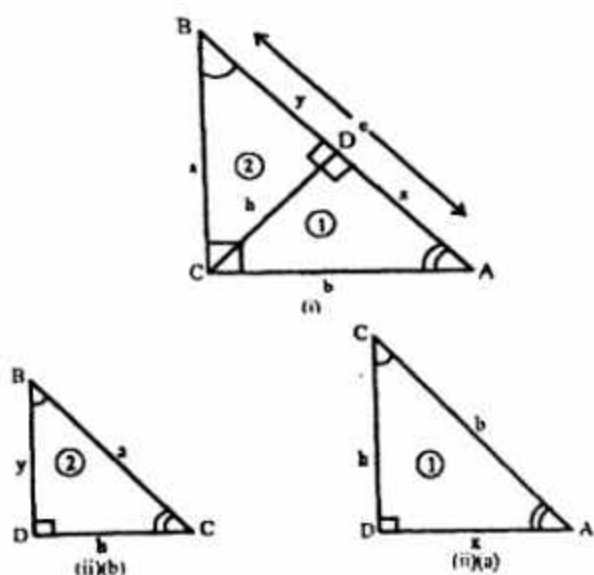


THEORM 15.1.1

PYTHAGORAS’ THEOREM

In a right-angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



Given

$\triangle ACB$ is a right-angled triangle in which $m\angle C = 90^\circ$ and $m\overline{BC} = a$, $m\overline{AC} = b$ and $m\overline{AB} = c$.

To prove

$c^2 = a^2 + b^2$

Construction

Draw CD perpendicular from C on AB.

Let $m\overline{CD} = h$, $m\overline{AD} = x$ and $m\overline{BD} = y$. Line segment CD splits $\triangle ABC$ into two $\triangle$ s ADC and BDC which are separately shown in the figures (ii) -a and (ii) b respectively.

Proof

Statements	Reasons
In $\triangle ADC \leftrightarrow \triangle ACB$	Refer to figure ii (a) and (i) common-self congruent
$\angle A \cong \angle A$	
$\angle ADC \cong \angle ACB$	Construction given both measure 90°
$\angle C \cong \angle B$	$\angle C$ and $\angle B$, complements of $\angle A$
$\therefore \triangle ADC \cong \triangle ACB$	Congruency of three angles
$\therefore \frac{x}{b} = \frac{b}{c}$	(Measure of corresponding sides of similar triangles is similar)
or $x = \frac{b^2}{c}$(i)	
Again, in correspondence $\triangle BDC \leftrightarrow \triangle BCA$	Refer to figure ii(b) and (i)
$\angle B \cong \angle B$	Common self-congruent
$\angle BDC \cong \angle BCA$	Construction given, both measure 90°
$\angle C \cong \angle A$	$\angle C$ and $\angle A$ at complements of $\angle M$
$\therefore \triangle BDC \sim \triangle BCA$	Congruency of three angles.
$\therefore \frac{y}{a} = \frac{a}{c}$	Sides of similar triangles are proportional. (Theorem 6)
$\therefore y = \frac{a^2}{c}$(ii)	
	Supposition

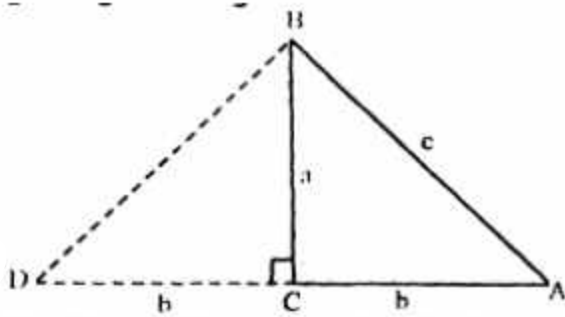
But $y + x = c$	
$\therefore \frac{a^2}{c} + \frac{b^2}{c} = c$	By (i) and (ii)
or $a^2 + b^2 = c^2$	
i.e. $c^2 = a^2 + b^2$	Multiplying both sides with c.

THEOREM 15.1.2

(CONVERSE OF PYTHAGORAS' THEOREM

15.1.1)

In a triangle if the sum of the squares of the measure of two sides is equal to the square of the measure of the third side, the triangle is a right-angled triangle.



Given

In a $\triangle ABC$, $m\overline{AB} = c$, $m\overline{BC} = a$ and $m\overline{AC} = b$ such that $a^2 + b^2 = c^2$

To prove

$m\angle ACB = 90^\circ$, $\triangle ACB$ is a right-angled triangle.

Construction

Draw $\overline{CD}$ Perpendicular to $\overline{BC}$ such that $\overline{CD} = \overline{CA}$. Join B and D.

Proof

Statements	Reasons
$\triangle DCB$ is a right-angled triangle	Construction
$\therefore (m\overline{BD})^2 = a^2 + b^2$	Pythagoras theorem
But $a^2 + b^2 = c^2$	Given

$\therefore (m\overline{BD})^2 = c^2$	
or $m\overline{BD} = c$	
Now in	
$\triangle DCB \leftrightarrow \triangle ACB$	
$\overline{CD} \cong \overline{CA}$	Construction
$\overline{BC} \cong \overline{BC}$	Common
$\overline{DB} \cong \overline{AB}$	Each is equal to c
$\therefore \triangle DCB \cong \triangle ACB$	S.S.S $\cong$ S.S.S
$\therefore \angle DCB \cong \angle ACB$	Corresponding angles of congruent triangles.
But $m\angle DCB = 90^\circ$	Construction
$\therefore m\angle ACB = 90^\circ$	
and the $\triangle ACB$ is a right-angled triangle.	

EXERCISE 15.1

Q1. Verify that the Δ s having the following measures of sides are right-angled.

(i) $a = 5\text{cm}$, $b = 12\text{cm}$, $c = 13\text{cm}$

Solution:

By Pythagoras theorem

$$\begin{aligned}a^2 + b^2 &= (5)^2 + (12)^2 \\&= 25 + 144 = 169 \\c^2 &= (13)^2 = 169 \\\therefore a^2 + b^2 &= c^2\end{aligned}$$

Thus, the triangle is right angled triangle.

(ii) $a = 1.5\text{ cm}$, $b = 2\text{cm}$, $c = 2.5\text{cm}$

Solution:

By Pythagoras theorem

$$\begin{aligned}a^2 + b^2 &= (1.5)^2 + (2)^2 \\&= 2.25 + 4 = 6.25 \\c^2 &= (2.5)^2 = 6.25 \\\therefore a^2 + b^2 &= c^2\end{aligned}$$

Thus, the triangle is right angled triangle.

(iii) $a = 9\text{cm}$, $b = 12\text{cm}$, $c = 15\text{ cm}$

Solution:

By Pythagoras theorem

$$\begin{aligned}a^2 + b^2 &= (9)^2 + (12)^2 \\&= 81 + 144 = 225 \\c^2 &= (15)^2 = 225 \\\therefore a^2 + b^2 &= c^2\end{aligned}$$

Hence the triangle is right angled triangle.

(iv) $a = 16\text{cm}$, $b = 30 = \text{cm}$, $c = 34\text{cm}$

Solution:

By Pythagoras theorem

$$\begin{aligned}a^2 + b^2 &= (16)^2 + (2)^2 \\&= 256 + 900 = 1156 \\c^2 &= (34)^2 = 1156 \\\therefore a^2 + b^2 &= c^2\end{aligned}$$

Hence the triangle is right angled triangle.

Q2. Verify that $a^2 + b^2$, $a^2 - b^2$ and $2ab$ are the measures of the sides of a right-angled triangle where a and b are any two real numbers ($a > b$)

Solution:

Let ABC be triangle such that

$$\overline{AB} = a^2 + b^2,$$

$$\therefore \overline{BC} = a^2 - b^2,$$

$$\overline{AC} = 2ab$$

By Pythagoras theorem

$$\left[\overline{AB}\right]^2 = (a^2 + b^2)^2 = a^4 + b^4 + 2a^2b^2$$

and

$$\begin{aligned}\left[\overline{AC}\right]^2 + \left[\overline{BC}\right]^2 &= (2ab)^2 + (a^2 - b^2)^2 \\&= 4a^4 + a^4 + b^4 - 2a^2b^2 \\&= a^4 + b^4 + 2a^2b^2\end{aligned}$$

$$\text{So } \left[\overline{AB}\right]^2 = \left[\overline{AC}\right]^2 + \left[\overline{BC}\right]^2$$

Hence ABC is a right-angled triangle.

Q3. The three sides of a triangle are of measure 8, x and 17 respectively. For what value of x will it become base of a right-angled triangle?

Solution:

If x is the base of right-angled triangle then -17 is the measure of hypotenuse.

By Pythagoras Theorem

$$\begin{aligned}(\text{hypotenuse})^2 &= (\text{base})^2 + (\text{perpendicular})^2 \\(17)^2 &= (x)^2 + (8)^2 \\289 &= x^2 + 64 \\x^2 &= 289 - 64 \\x^2 &= 225 \\x &= 15\end{aligned}$$

Q4. In an isosceles Δ , the base $m\overline{BC} = 28\text{ cm}$, and $m\overline{AB} = m\overline{AC} = 50\text{ cm}$. If $m\overline{AD} \perp m\overline{BC}$, then find

(i) length of AD (ii) Area of ΔABC

Solution:



(i) $\overline{AD} \perp \overline{BC}$

$\therefore D$ is mid-point for BC

$$\text{So } m\overline{BD} = \frac{1}{2}(28) = 14\text{ cm}$$

From right angled ΔABD

$$\left[m\overline{AB}\right]^2 = \left[m\overline{BD}\right]^2 + \left[m\overline{AD}\right]^2$$

$$(50)^2 = (14)^2 + \left[m\overline{AD}\right]^2$$

$$\left[m\overline{AD}\right]^2 = (50)^2 - (14)^2 = 2500 - 196 = 2304$$

$$m\overline{AD} = 48\text{cm}$$

(ii) Area of ΔABC

$$\begin{aligned}&= (m\overline{BC})(m\overline{AD}) \\&= (28)(48) \\&= 672\text{cm}^2\end{aligned}$$

Q5. In a quadrilateral ABCD, the diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other. Prove that $m\overline{AB}^2 + m\overline{CD}^2 = m\overline{AD}^2 + m\overline{BC}^2$

Solution:

The diagram AC and BD of the quadrilateral ABCD meet at O perpendicular in the right triangles ΔAOB

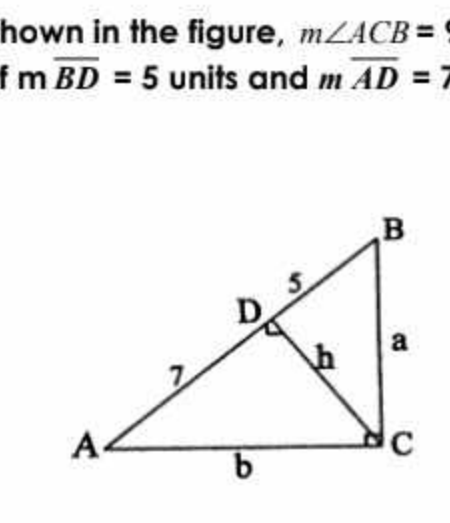
$$m\overline{AB}^2 = m\overline{AO}^2 + m\overline{OB}^2 \quad \text{(i)}$$

In the right triangle ΔBOC

$$m\overline{BC}^2 = m\overline{OB}^2 + m\overline{OC}^2 \quad \text{(ii)}$$

In the right triangle ΔDOC

$$m\overline{DC}^2 = m\overline{OC}^2 + m\overline{OD}^2 \quad \text{(iii)}$$



Adding (i) and (ii)

$$m\overline{AB}^2 + m\overline{BC}^2 = m\overline{AO}^2 + m\overline{OB}^2 + m\overline{OC}^2 + m\overline{OD}^2$$

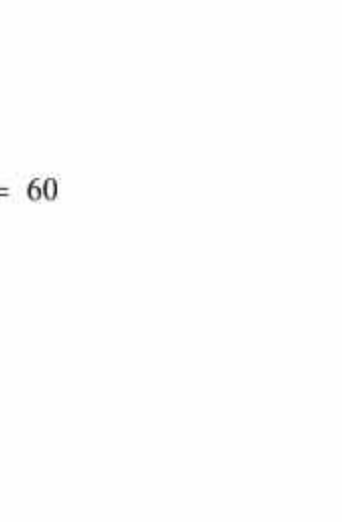
Adding (i) and (iv)

$$m\overline{AD}^2 + m\overline{BC}^2 = m\overline{AO}^2 + m\overline{OB}^2 + m\overline{OC}^2 + m\overline{OD}^2$$

$$\text{Hence } m\overline{AB}^2 + m\overline{CD}^2 = m\overline{AD}^2 + m\overline{BC}^2$$

Q6. (i) In the ΔABC as shown in the figure, $m\angle ACB = 90^\circ$ and $\overline{CD} \perp \overline{AE}$. Find the lengths a , h and b if $m\overline{BD} = 5$ units and $m\overline{AD} = 7$ units.

Solution:



$$m\overline{AB} = 5 + 7 = 12$$

In right angled ΔBDC

$$a^2 = 25 + h^2 \quad (1)$$

In right angled ΔADC

$$b^2 = 49 + h^2 \quad (2)$$

In right angled ΔABC

$$a^2 + b^2 = 144 \quad (3)$$

Adding (1) and (2)

$$a^2 + b^2 = 74 + 2h^2$$

from (3) and (4)

$$74 + 2h^2 = 144$$

$$2h^2 = 144 - 74 = 70$$

$$h^2 = 35$$

$$h = \sqrt{35}\text{units}$$

Put $h^2 = 35$ in (1)

$$a^2 = 25 + 35 = 60$$

$$a = \sqrt{60}$$

$$a = 2\sqrt{15}\text{ units}$$

Put $h^2 = 35$ in (2)

$$b^2 = 49 + 35$$

$$b^2 = 84$$

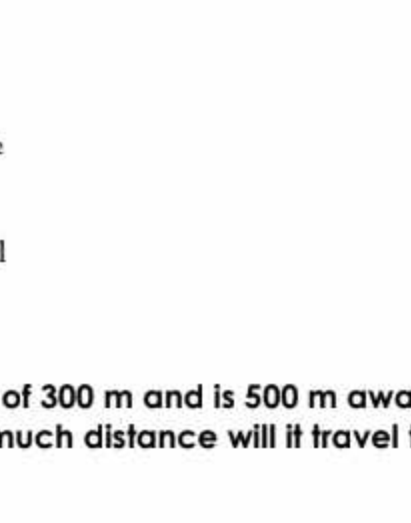
$$b = \sqrt{84} = 2\sqrt{21}\text{units}$$

$$\text{So, } a = 2\sqrt{15}\text{units}$$

$$h = \sqrt{35}\text{units}$$

$$b = 2\sqrt{21}\text{units}$$

(ii) Find the value of x in the shown figure.



From ΔADC

$$\left(m\overline{AC}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{DC}\right)^2$$

$$(13)^2 = \left(m\overline{AD}\right)^2 + (5)^2$$

$$169 = \left(m\overline{AD}\right)^2 + 25$$

$$\left(m\overline{AD}\right)^2 = 169 - 25 = 144$$

$$\therefore m\overline{AD} = 12\text{cm}$$

From ΔABD

$$\left(m\overline{AB}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{BD}\right)^2$$

$$(15)^2 = (12)^2 + (x)^2$$

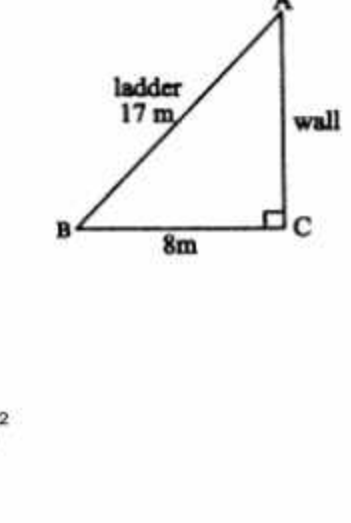
$$225 = 144 + x^2$$

$$x^2 = 225 - 144 = 81$$

$$\therefore x = 9\text{ cm}$$

Q7. A plane is at a height of 300 m and is 500 m away from the airport as shown in the figure. How much distance will it travel to land the airport?

Solution:



$$m\overline{BC} = 500\text{ m, } m\overline{AC} = 300\text{ m}$$

By Pythagoras theorem

$$m\overline{AB}^2 = m\overline{BC}^2 + m\overline{AC}^2$$

$$m\overline{AB}^2 = (500)^2 + (300)^2$$

$$m\overline{AB}^2 = 250000 + 90000$$

$$m\overline{AB}^2 = 340000$$

$$m\overline{AB}^2 = \sqrt{34 \times 10000}$$

$$m\overline{AB} = 100\sqrt{34}\text{ m}$$

Q8. A ladder 17 m long rests against a vertical wall. The foot of the ladder is 8 m away from the base of the wall. How high up the wall will the ladder reach?

Solution:



By Pythagoras Theorem

$$\left[m\overline{AB}\right]^2 = \left[m\overline{AC}\right]^2 + \left[m\overline{BC}\right]^2$$

$$(17)^2 = \left[m\overline{AC}\right]^2 + (8)^2$$

$$\left[m\overline{AC}\right]^2 = (17)^2 - (8)^2$$

$$\left[m\overline{AC}\right]^2 = 289 - 64 = 225$$

$$m\overline{AC} = \sqrt{225} = 15\text{ cm}$$

Q9. A student travels to his school by the route as shown in the figure. Find $m\overline{AD}$, the direct distance from his house to school.

Solution:

A is house, B is bus stop and D is school. Produce $\overline{AB}$ and draw $\overline{DL} \parallel \overline{DC}$ to meet $\overline{AB}$ produced at L.

We have to find AD

$$m\overline{LD} = m\overline{BC} = 6\text{km}$$

$$m\overline{BL} = m\overline{CD} = 3\text{km}$$

$$m\overline{AL} = m\overline{AB} + m\overline{BL} = 2 + 3 = 5\text{ km}$$

ALD is a right angled Δ

By Pythagoras theorem

$$m\overline{AD}^2 = m\overline{AL}^2 + m\overline{LD}^2$$

$$= (5)^2 + (6)^2 = 25 + 36 = 61$$

$$m\overline{AD} = \sqrt{61}\text{ km}$$

REVIEW EXERCISE 15

Q1. Which of the following is true and which are false?

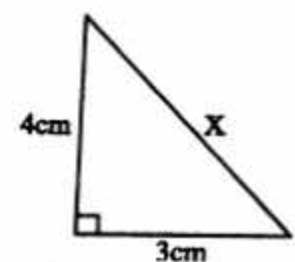
- (i) In a right-angled triangle greater angle is of 90°.
- (ii) In a right-angled triangle right angle is of 60°.
- (iii) In a right triangle hypotenuse is a side opposite to right angle.
- (iv) If a, b, c are sides of right-angled triangle with c as longer side then $c^2 = a^2 + b^2$
- (v) If 3 cm and 4 cm are two sides of a right-angled triangle, then hypotenuse is 5 cm.
- (vi) If hypotenuse of an isosceles right triangle is $\sqrt{2}$ cm then each of other side is of length 2 cm.

Answers:

(i) T	(ii) F	(iii) T	(iv) T	(v) T	(vi) F
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Q2. Find the unknown value in each of the following figures.

(i)

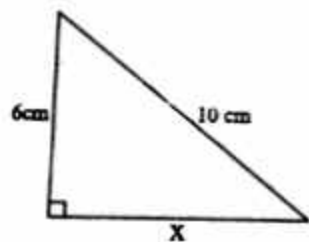


By Pythagoras Theorem

$$x^2 = 4^2 + 3^2$$
$$x^2 = 16 + 9$$
$$x^2 = 25$$

$$x^2 = \sqrt{25}$$
$$x = 5cm$$

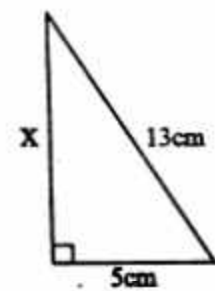
(ii)



By Pythagoras Theorem

$$(10)^2 = (6)^2 + (x)^2$$
$$100 = 36 + x^2$$
$$x^2 = 100 - 36 = 64$$
$$x^2 = \sqrt{64}$$
$$x^2 = 8cm$$

(iii)

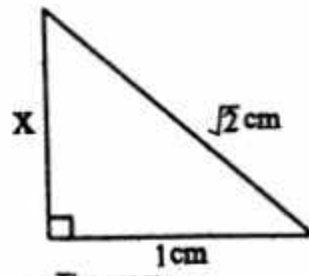


By Pythagoras Theorem

$$(13)^2 = (x)^2 + (5)^2$$
$$169 = x^2 + 25$$
$$x^2 = 169 - 25$$
$$x^2 = \sqrt{144}$$

$$x = 12cm$$

(iv)



By Pythagoras Theorem

$$(\sqrt{2})^2 = (x)^2 + (1)^2$$
$$2 = x^2 + 1$$
$$x^2 = \sqrt{1} = 1cm$$