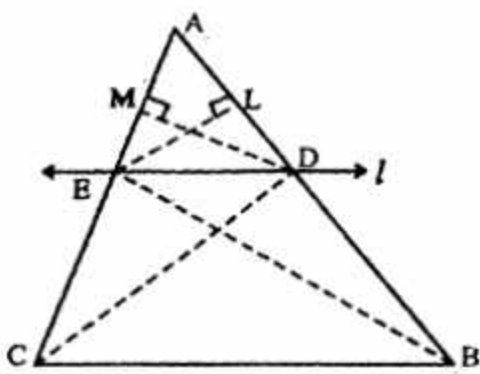


THEOREM 14.1.1

A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.



Given

In $\triangle ABC$, the line l is intersecting the sides $\overline{AC}$ and $\overline{AB}$ at points E and D respectively such that $\overline{ED} \parallel \overline{CB}$.

To prove

$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}.$

Construction

Join B to E and C to D and draw $\overline{DM} \perp \overline{EL}$ perpendiculars from D and E on $\overline{AC} \perp \overline{AB}$ to meet at the points M and L respectively.

Proof

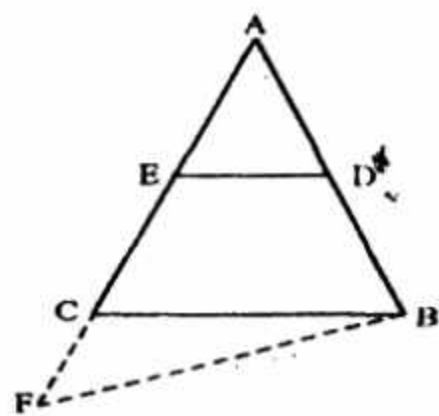
Statements	Reasons
In triangles BED and AED $m\overline{EL}$ is the common perpendicular. $\therefore \triangle BED = \frac{1}{2} \times m\overline{BD} \times m\overline{EL} \dots (i)$	 Area of a triangle = $\frac{1}{2}$ (base) (height)

And $\triangle AED = \frac{1}{2} \times m\overline{AD} \times m\overline{EL} \dots (ii)$ $\frac{\triangle BED}{\triangle AED} = \frac{m\overline{BD}}{m\overline{AD}} \dots (a)$ Similarly $\frac{\triangle CDE}{\triangle ADE} = \frac{m\overline{EC}}{m\overline{AE}} \dots (b)$ But $\triangle BED = \triangle CDE$ $\therefore$ From (a) and (b), we have $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$ $\therefore$ Hence $m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$	Dividing (i) by (ii) Areas of triangles with common base and same altitudes are equal. $\overline{ED} \parallel \overline{CB}$ Given, that so altitudes are equal. Taking reciprocal of both sides
--	--

Theorem 14.1.2

(Converse of Theorem 14.1.1)

If a line segment intersects the two sides of a triangle in the same ratio, then it is parallel to the third side.



Solution

Given

In $\triangle ABC$, $\overline{ED}$ intersects $\overline{AB}$ and $\overline{AC}$ such that $m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$

To prove

$\overline{ED} \parallel \overline{CB}$

Construction

If $\overline{ED} \nparallel \overline{CB}$ then draw $\overline{BF} \parallel \overline{DE}$ to meet $\overline{AC}$ produced at F.

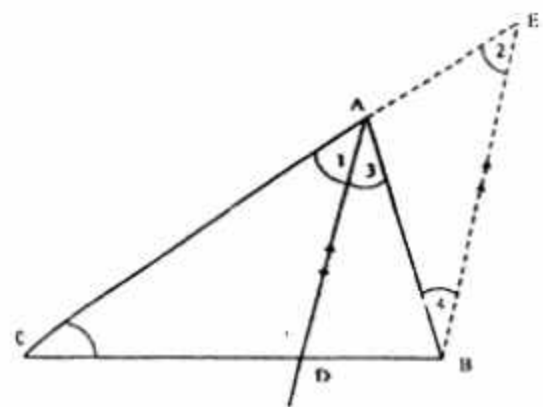
Proof

Statements	Reasons
In $\triangle ABF$ $\overline{DE} \parallel \overline{BF}$	Construction

$\therefore \frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EF}}$ (i)	A line parallel to one side of a triangle divides the other two sides proportionally (Theorem 4.1)
But $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$ (ii)	Given
$\therefore \frac{m\overline{AE}}{m\overline{EF}} = \frac{m\overline{AE}}{m\overline{EC}}$	From (i) and (ii)
or $m\overline{EF} = m\overline{EC}$ Which is possible only if point F is coincident with C. $\therefore$ Our supposition is wrong Hence $\overline{ED} \parallel \overline{CB}$	Property of real numbers

THEOREM 14.1.3

The internal bisector of an angle of a triangle divides the side opposite to it in the ratio of the lengths of the sides containing the angle.



Given

In $\triangle ABC$ internal angle bisector of $\angle A$ meets $\overline{CB}$ at the point D.

To prove

$$m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$$

Construction

Draw a line segment $\overline{BE} \parallel \overline{DA}$ to meet $\overline{CA}$ produced at E.

Proof

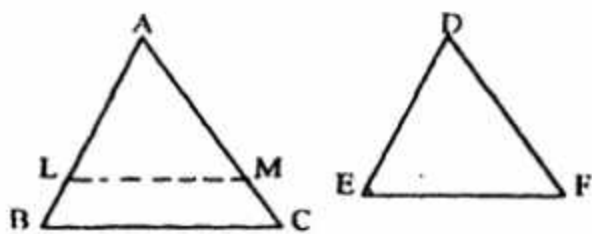
Statements	Reasons
$\therefore \overline{AD} \parallel \overline{EB}$ and $\overline{EC}$ intersects there at A and E,	Construction
So $m\angle 1 = m\angle 2$(i)	Corresponding angles
Again $\overline{AD} \parallel \overline{EB}$ and $\overline{AB}$ intersects them.	
So, $m\angle 3 = m\angle 4$(ii)	Alternate angles

But $m\angle 1 = m\angle 3$	Construction (Given)
$\therefore m\angle 2 = m\angle 4$	
and $\overline{AE} \cong \overline{AB}$	
Now $\overline{AD} \parallel \overline{EB}$	Construction
$\therefore \frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{EA}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally,
Or $\frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{EA}}{m\overline{AC}}$	$\therefore m\overline{EA} = m\overline{AB}$ (proved)
$m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$	

THEOREM 14.1.4

If two triangles are similar, then the measures of their corresponding sides are proportional.

Solution



Given

$\triangle ABC \leftrightarrow \triangle DEF$

i.e., $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$.

To prove

$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$

Construction

(a) Suppose that $m\overline{AB} > m\overline{DE}$

(b) $m\overline{AB} < m\overline{DE}$

On $\overline{AB}$ take a point L such that $m\overline{AL} = m\overline{DE}$

On $\overline{AC}$ take a point M such that $m\overline{AM} = m\overline{DF}$. Join L and M by the line segment $\overline{LM}$.

Proof

Statements	Reasons
In $\triangle ALM \leftrightarrow \triangle DEF$ $\angle A \cong \angle D$	Given

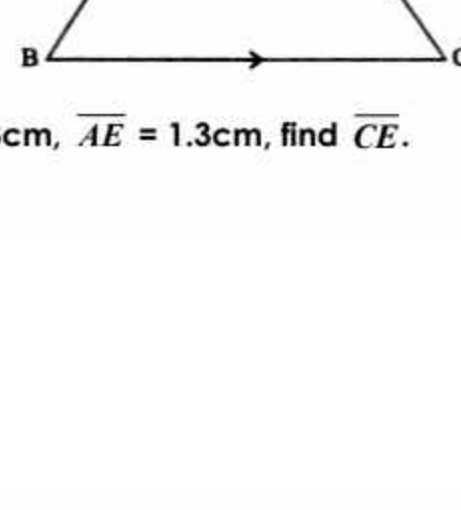
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\triangle ALM \cong \triangle DEF$	S.A.S Postulate
and $\angle L \cong \angle E$, $\angle M \cong \angle F$	Corresponding angles of congruent triangles
	Given
Now $\angle E \cong \angle B$ and $\angle F \cong \angle C$	Transitivity of congruence
$\therefore \angle L \cong \angle B$, $\angle M \cong \angle C$	Corresponding angles are equal.
Thus $\overline{LM} \parallel \overline{BC}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
Hence $\frac{m\overline{AL}}{m\overline{AB}} = \frac{m\overline{AM}}{m\overline{AC}}$	
Or $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}}$	$m\overline{AL} = m\overline{DE}$ and $m\overline{AM} = m\overline{DF}$ (construction)
Similarly, by intercepting segments on $\overline{BA}$ and $\overline{BC}$, we can prove that	
$\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{EF}}{m\overline{BC}}$ (ii)	
Thus $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}} = \frac{m\overline{EF}}{m\overline{BC}}$	By (i) and (ii)
Or $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$	By taking reciprocals.

(b) If $m\overline{AB} < m\overline{DE}$, it can Similarly, be proved by taking intercepts on the sides of $\triangle DEF$. If $m\overline{AB} = m\overline{DE}$ Then in the correspondence of $\triangle ABC \leftrightarrow \triangle DEF$ $\angle A \cong \angle D$ $\angle B \cong \angle E$ and $\overline{AB} \cong \overline{DE}$ so $\triangle ABC \cong \triangle DEF$ Thus $= \frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{EC}}{m\overline{EF}} = 1$ Hence the result is true for all cases.	Given Given Construction A.S.A $\cong$ A.S.A $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$
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EXERCISE 14.1

Q1. In $\triangle ABC$, $DE \parallel BC$.

Solution:

(i) $\overline{AD} = 1.5\text{cm}$, $\overline{BD} = 3\text{cm}$, $\overline{AE} = 1.3\text{cm}$, find $\overline{CE}$.In $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$

$$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$$

$$\frac{1.5}{3} = \frac{1.3}{m\overline{EC}}$$

$$\therefore 1.5(m\overline{EC}) = (1.3)3$$

$$\begin{aligned} m\overline{EC} &= \frac{1.3 \times 3}{1.5} = \frac{13 \times 3 \times 10}{10 \times 15} \\ &= \frac{13 \times 3}{15} = \frac{13}{5} = \mathbf{2.6\text{ cm}} \end{aligned}$$

(ii) $\overline{AD} = 2.4\text{cm}$, $\overline{AE} = 3.2\text{cm}$, $\overline{EC} = 4.8\text{ cm}$, find $\overline{AB}$.In $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$

$$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$$

1

Mathematics

$$\frac{1.5}{m\overline{DB}} = \frac{1.3}{4.8}$$

$$\therefore 3.2(m\overline{DB}) = (2.4)(4.8)$$

$$\begin{aligned} m\overline{DB} &= \frac{2.4 \times 4.8}{3.2} = \frac{24 \times 10 \times 48}{10 \times 32 \times 10} \\ &= \frac{36}{10} = 3.6\text{ cm} \end{aligned}$$

$$m = m + m = 2.4 + 3.6$$

$$= 6.0\text{ cm}$$

(iii) $\frac{\overline{AD}}{\overline{DB}} = \frac{3}{5}$, $\overline{AC} = 4.8\text{cm}$, find $\overline{AE}$.In $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$

$$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$$

$$\frac{3}{5} = \frac{\overline{AE}}{\overline{EC}}$$

$$\frac{3}{5} + 1 = \frac{\overline{AE}}{\overline{EC}} + 1$$

$$\frac{8}{5} = \frac{\overline{AE} + \overline{EC}}{\overline{EC}} = \frac{\overline{AC}}{\overline{EC}}$$

$$\frac{8}{5} = \frac{4.8}{\overline{EC}}$$

$$8\overline{EC} = 4.85 = 24$$

$$\overline{EC} = \frac{24}{8} = 3$$

$$m\overline{AE} = m\overline{AC} - m\overline{EC} = 4.8 - 3 = \mathbf{1.8\text{cm}}$$

2

Mathematics

(iv) $\overline{AD} = 2.4\text{cm}$, $\overline{AE} = 3.2\text{cm}$, $\overline{DE} = 2\text{cm}$, $\overline{BC} = 5\text{cm}$, find $\overline{AB}$, $\overline{DB}$, $\overline{AC}$, $\overline{CE}$.In $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$

$$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}} = \frac{\overline{DE}}{\overline{BC}}$$

$$\frac{2.4}{\overline{AB}} = \frac{3.2}{\overline{AC}} = \frac{2}{5}$$

$$(\overline{AB}) = 5(2.4) = 12$$

$$\overline{AB} = \frac{12}{2} = \mathbf{6\text{cm}}$$

$$2(\overline{AC}) = 5(3.2) = 16$$

$$= \frac{16}{2} = \mathbf{8\text{cm}}$$

$$\overline{DE} = \overline{AB} - \overline{AD}$$

$$= 6 - 2.4 = \mathbf{3.6\text{cm}}$$

$$\overline{CE} = \overline{AC} - \overline{AE}$$

$$= 8 - 3.2 = \mathbf{4.8\text{cm}}$$

(v) $\overline{AD} = 4x-3$, $\overline{AE} = 8x-7$, $\overline{BD} = 3x-1$, $\overline{CE} = 5x-3$, find x .In $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

$$\Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$$(4x-3)(5x-3) = (8x-7)(3x-1)$$

$$\Rightarrow 20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$$

$$\text{or } 20x^2 - 24x - 27x + 29x + 9 - 7 = 0$$

3

Mathematics

$$-4x^2 + 2x + 2 = 0$$

$$\text{or } 2x^2 - x - 1 = 0$$

$$2x^2 - 2x + x - 1 = 0$$

$$2x(x-1) + (x-1) = 0$$

$$(x-1)(2x+1) = 0$$

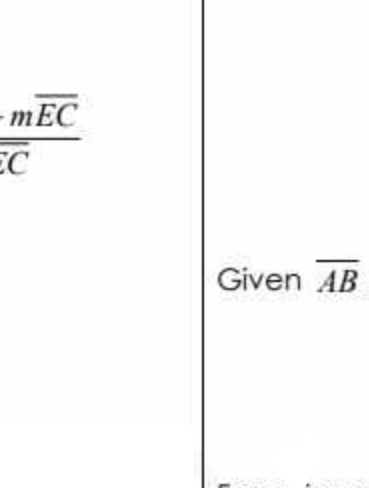
$$x = 1, -\frac{1}{2}$$

For $x = -\frac{1}{2}$ sides become negative.So, $x = 1$.Q2. If $\triangle ABC$ is an isosceles triangle, $\angle A$ is vertex angle and $\overline{DE}$ intersects the sides $\overline{AB}$ and $\overline{AC}$ as shown in the figure so that

$$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$$

Prove that $\triangle ADE$ is also an isosceles triangle

Solution:



Given

In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$

$$\text{and } \frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$$

To prove

 $\triangle ADE$ is isosceles.

Proof

4

Mathematics

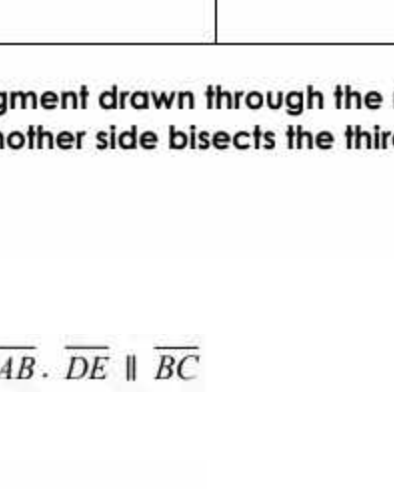
Statements	Reasons
$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$	Given
$\frac{m\overline{AD}}{m\overline{DB}} + 1 = \frac{m\overline{AE}}{m\overline{EC}} + 1$	
Or $\frac{m\overline{AD} + m\overline{DB}}{m\overline{DB}} = \frac{m\overline{AE} + m\overline{EC}}{m\overline{EC}}$	
i.e. $\frac{m\overline{AB}}{m\overline{DB}} = \frac{m\overline{AC}}{m\overline{EC}}$	Given $\overline{AB} \cong \overline{AC}$
$\Rightarrow m\overline{DB} = m\overline{EC}$	From figure
$m\overline{AB} - m\overline{DB} = m\overline{AC} - m\overline{EC}$	
$m\overline{AD} = m\overline{AE}$	
Or $\overline{AD} = \overline{AE}$	
$\therefore \triangle ADE$ is isosceles.	

Q3. In an equilateral triangle ABC shown in the figure, $m\overline{AE} : m\overline{AC}$ $= m\overline{AD} : \overline{AB}$ m. Find all the three angles of $\triangle ADE$ and name it also.

Solution:

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Mathematics



Given

ABC is equilateral triangle

$$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$$

To prove

All angles of $\triangle ADE$.

Proof

Statements	Reasons
$\frac{m\overline{AC}}{m\overline{AE}} = \frac{m\overline{AB}}{m\overline{AD}}$	Given
$\frac{m\overline{AC}}{m\overline{AE}} - 1 = \frac{m\overline{AB}}{m\overline{AD}} - 1$	
$\frac{m\overline{AC} - m\overline{AE}}{m\overline{AE}} = \frac{m\overline{AB} - m\overline{AD}}{m\overline{AD}}$	
$\frac{m\overline{EC}}{m\overline{AE}} = \frac{m\overline{DB}}{m\overline{AD}}$	Given

Q4. Prove that the line segment drawn through the mid-point of one side of a triangle and parallel to another side bisects the third side.

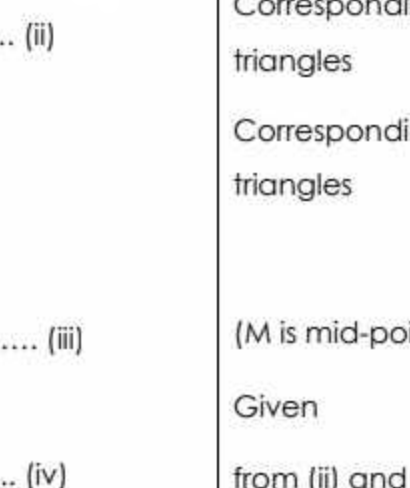
Solution:

Given

In $\triangle ABC$, D is mid-point of $\overline{AB}$, $\overline{DE} \parallel \overline{BC}$

To prove

$$\overline{EA} \cong \overline{EC}$$



Construction

Take $\overline{EF} \parallel \overline{AB}$

Proof

Statements	Reasons
In $\triangle BLN \leftrightarrow \triangle ALM$	
$\overline{BL} \cong \overline{AL}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{NL} \cong \overline{ML}$	Construction
$\therefore \triangle BLN \cong \triangle ALM$	S.A.S. postulate
And $\angle A \cong \angle 3$	 (i)
$\overline{NB} \cong \overline{AM}$	 (ii)
$\overline{NB} \parallel \overline{AM}$	Corresponding angles of congruent triangles
Thus $\overline{NB} \parallel \overline{MC}$	 (iii)
$\overline{MC} \cong \overline{AM}$	 (iv)
$\overline{NB} \cong \overline{AM}$	 (v)
$\therefore \triangle BCMN$ is a parallelogram	Opposite sides of a parallelogram (BCMN)
$\overline{BC} \parallel \overline{LM}$	

9

Mathematics

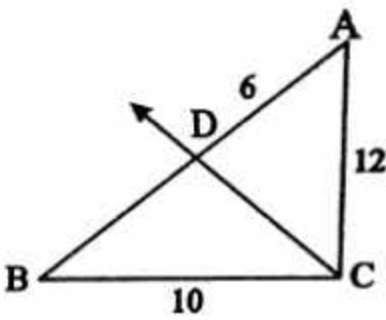
Or $\overline{BC} \parallel \overline{NL}$	(vi)
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EXERCISE 14.2

Q1. In $\triangle ABC$ as shown in the figure, CD bisects $\angle C$ and meets AB at D . $\overline{mBD}$ is equal to

- (a) 5 (b) 16 (c) 10 (d) 18

Solution:



In $\triangle ABC$, CD bisect $\angle C$ meets AB at D .
As CD is the internal bisector of $\angle C$

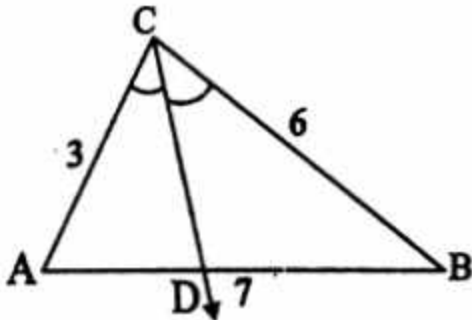
So $\frac{\overline{mBD}}{\overline{mDA}} = \frac{\overline{mBC}}{\overline{mCA}}$

$$\frac{\overline{mBD}}{6} = \frac{10}{12}$$
$$\overline{mBD} = 6 \times \frac{10}{12}$$
$$= \boxed{5}$$

The correct answer is **(a)**.

Q2. In $\triangle ABC$ shown in the figure, $\overline{CD}$ bisects $\angle C$. If $\overline{mAC} = 3$, $\overline{mCB} = 6$ and $\overline{mAB} = 7$, then find $\overline{mAD}$ and $\overline{mDB}$.

Solution:



In $\triangle ABC$, $\overline{mAC} = 3$, $\overline{mCB} = 6$ and $\overline{mAB} = 7$

Let $\overline{mAD} = x$

then $\overline{mDB} = 7 - x$

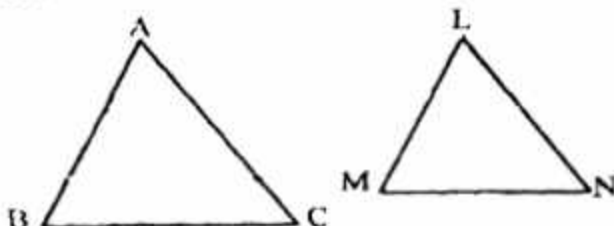
As $\overline{CD}$ is internal bisector of C

So $\frac{\overline{mAD}}{\overline{mDB}} = \frac{\overline{mAC}}{\overline{mCB}}$

$$\frac{x}{7-x} = \frac{3}{6}$$
$$6x = 21 - 3x$$
$$9x = 21$$
$$x = \frac{21}{9}$$
$$\overline{mAD} = \frac{21}{9} = \frac{7}{3}$$
$$\overline{mDB} = \overline{mAB} - \overline{mAD}$$
$$\overline{mDB} = 7 - \frac{21}{9} = \frac{63-21}{9} = \frac{42}{9} = \frac{14}{3}$$

Q3. Show that in any correspondence of two triangles, if two angles of one triangle is congruent to the corresponding angles of the other, then the triangles are similar.

Solution:



Let the two triangles be $\triangle ABC$ and $\triangle LMN$

It is given that,

$$m\angle A = m\angle L$$

$$m\angle B = m\angle M$$

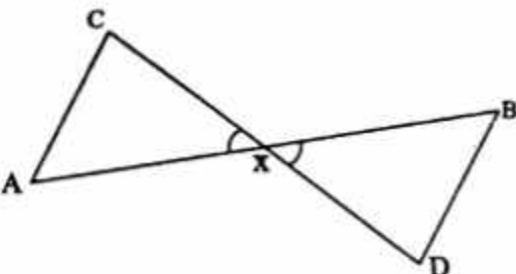
As sum of the angles of a triangle is 180°

$$m\angle A + m\angle B + m\angle C = 180^\circ$$
$$m\angle L + m\angle M + m\angle N = m\angle L + m\angle M + m\angle N$$
$$m\angle L + m\angle M + m\angle C = m\angle L + m\angle M + m\angle N$$
$$m\angle C = m\angle N$$

$\therefore$ The two triangles $\triangle ABC$ and $\triangle LMN$ are similar.

Q4. If line segments $\overline{AB}$ and $\overline{CD}$ are intersecting at point X and $\frac{\overline{mAX}}{\overline{mXB}} = \frac{\overline{mCX}}{\overline{mXD}}$ then show that $\triangle AXC$ and $\triangle BXD$ are similar.

Solution:



Given

Line segments $\overline{AB}$ and $\overline{CD}$ are intersecting at point X and $\frac{\overline{mAX}}{\overline{mXB}} = \frac{\overline{mCX}}{\overline{mXD}}$

To prove:

$\triangle AXC$ and $\triangle BXD$ are similar.

Proof

Statements	Reasons
$\frac{\overline{mAX}}{\overline{mXB}} = \frac{\overline{mCX}}{\overline{mXD}}$	Given
SO, $\overline{AC} \parallel \overline{BD}$	
In $\triangle AXC$ and $\triangle BXD$	
$m\angle AXC = m\angle BXD$	Vertical angles
$m\angle A = m\angle B$	Alternate angles
$m\angle C = m\angle D$	Alternate angles
So, the triangles are similar.	

REVIEW EXERCISE 14

Q1. Which of the following are true and which are false?

- (i) Congruent triangles are of same size and shape.
 (ii) Similar triangles are of same shape but different sizes.
 (iii) Symbol used for congruent is ' $\cong$ '.
 (iv) Symbol used for similarity is ' $\sim$ '.
 (v) Congruent triangles are similar.
 (vi) Similar triangles are congruent.
 (vii) A line segment has only one mid-point.
 (viii) One and only one line can be drawn through two points.
 (ix) Proportion is non-equality of two ratios.
 (x) Ratio has no unit.

Answers:

(i) T	(ii) T	(iii) F	(iv) F	(v) T	(vi) F
(vii) T	(viii) T	(ix) F	(x) T		

Q2. Define the following:

- (i) Ratio (ii) Proportion (iii) Congruent Triangles (iv) Similar Triangle

Solution:

(i) Ratio

The ratio of two quantities a and b of same kind is denoted as a:b and is defined as:

1

The ratio $a:b = \frac{a}{b}$ is the Comparison of two like a and b quantities 'a' and 'b' are called terms of 'a' ratio 'b'. Terms must be expressed in the same units.

(ii) Proportion

The statement of equality of two ratios is called proportion.

i.e. if $a:b = c:d$ then a, b, c and d are said to be in proportion.

(iii) Congruent Triangle



Two triangles said to be congruent written symbolically as $\cong$, if there exists a correspondence between them such that all the corresponding sides and angles are congruent.

$$\text{if } \begin{cases} AB \cong DE \\ BC \cong EF \\ CA \cong FD \end{cases} \text{ and } \begin{cases} \angle A \cong \angle D \\ \angle B \cong \angle E \\ \angle C \cong \angle F \end{cases}$$

Then $\triangle ABC \cong \triangle DEF$

(iv) Similar Triangle

if in $\triangle ABC \leftrightarrow \triangle DEF$

$$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$$

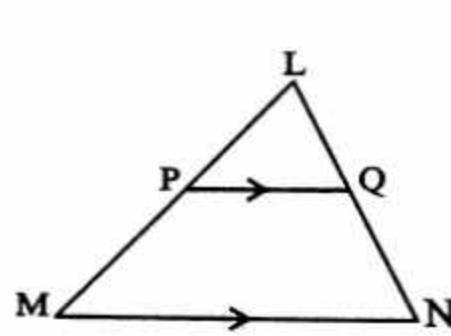
$$\text{and } \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

Then $\triangle ABC$ and $\triangle DEF$ are called a similar triangle which is symbolically written as $\triangle ABC \sim \triangle DEF$.

Q3. In $\triangle LMN$ shown in the figure, $\overline{MN} \parallel \overline{PQ}$

2

Solution:



(i) If $m\overline{LM} = 5\text{cm}$, $m\overline{LP} = 2.5\text{cm}$, $m\overline{LQ} = 2.3\text{cm}$, $m\overline{LN} = ?$

As $\overline{PQ} \parallel \overline{MN}$

$$\frac{m\overline{LP}}{m\overline{LM}} = \frac{m\overline{LQ}}{m\overline{LN}}$$

$$\frac{2.5}{5} = \frac{2.3}{m\overline{LN}}$$

$$2.5(m\overline{LN}) = 2.3 \times 5 = 11.5$$

$$m\overline{LN} = \frac{11.5}{2.5} = \frac{115}{25} = \frac{23}{5} = 4.6\text{cm}$$

(ii) If $m\overline{LM} = 6\text{cm}$, $m\overline{LQ} = 2.5\text{cm}$, $m\overline{QN} = 5\text{cm}$, $m\overline{LP} = ?$

As $\overline{PQ} \parallel \overline{MN}$

$$\frac{m\overline{LP}}{m\overline{LM}} = \frac{m\overline{LQ}}{m\overline{LN}}$$

$$\frac{m\overline{LP}}{6} = \frac{2.5}{7.5}$$

$$m\overline{LP} = \frac{2.5}{7.5} \times 6 = \frac{25}{75} \times 6 = 2\text{cm}$$

3

Q4. In the shown figure, let $m\overline{PA} = 8x - 7$, $m\overline{PB} = 4x - 3$, $m\overline{AQ} = 5x - 3$, $m\overline{BR} = 3x - 1$. Find the value of x if $\overline{AB} \parallel \overline{QR}$.

Solution:

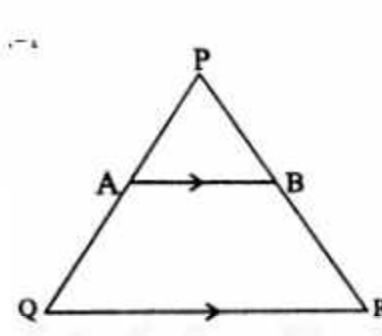
$$m\overline{PA} = 8x - 7, m\overline{PB} = 4x - 3$$

$$m\overline{AQ} = 5x - 3, m\overline{BR} = 3x - 1$$

As $\overline{AB} \parallel \overline{QR}$

$$\frac{m\overline{PA}}{m\overline{AQ}} = \frac{m\overline{PB}}{m\overline{BR}}$$

$$\frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$$



$$(8x-7)(3x-1) = (4x-3)(5x-3)$$

$$24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9$$

$$24x^2 - 20x^2 - 29x + 27x + 7 = 9$$

$$4x^2 - 2x + 7 = 9$$

$$4x^2 - 2x - 2 = 0$$

$$2x^2 - x - 1 = 0$$

$$2x^2 - 2x + x - 1 = 0$$

$$2x(x-1) + (x-1) = 0$$

$$(x-1)(2x+1) = 0$$

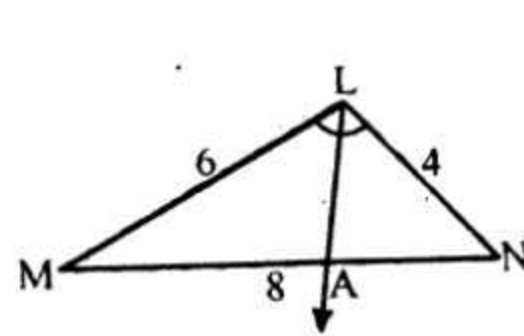
$$\left(x = 1, -\frac{1}{2}\right)$$

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$x=1$ is the required value.

Q5. In $\triangle LMN$ shown in the figure, $\overline{LA}$ bisects $\angle L$. If $m\overline{LN} = 4$, $m\overline{LM} = 6$, $m\overline{MN} = 8$, then find $m\overline{MA}$ and $m\overline{AN}$.

Solution:



$$m\overline{LN} = 4, m\overline{LM} = 6, m\overline{MN} = 8$$

$\overline{LA}$ bisects $\angle L$

$$\frac{m\overline{MA}}{m\overline{NA}} = \frac{m\overline{LM}}{m\overline{LN}} = \frac{6}{4}$$

$$\text{i.e. } m\overline{MA} : m\overline{NA} = 6:4$$

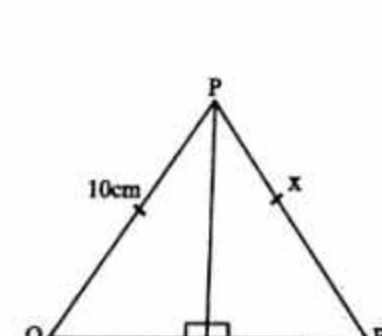
$$\text{but } m\overline{MN} : m\overline{MA} + m\overline{AN} = 8$$

$$m\overline{MA} = \frac{6}{10} \times 8 = \frac{48}{10} = 4.8$$

$$\text{and } m\overline{AN} = \frac{4}{10} \times 8 = \frac{32}{10} = 3.2$$

Q6. In isosceles $\triangle PQR$ shown in the figure, find the value of x and y.

Solution:



$$\overline{PQ} \cong \overline{PR}$$

$$\Rightarrow x = 10\text{cm}$$

$$\overline{PM} \perp \overline{QR} \text{ where } PQR \text{ is an isosceles triangle}$$

$$\therefore m\overline{MQ} = m\overline{MR}$$

$$\Rightarrow y = 6\text{cm}$$

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$$\overline{PQ} \cong \overline{PR}$$

$$\Rightarrow x = 10\text{cm}$$

$$\overline{PM} \perp \overline{QR} \text{ where } PQR \text{ is an isosceles triangle}$$

$$\therefore m\overline{MQ} = m\overline{MR}$$

$$\Rightarrow y = 6\text{cm}$$

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