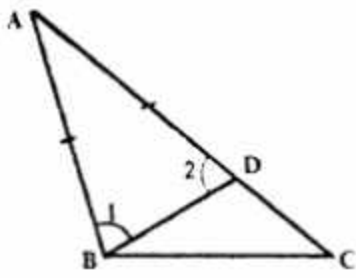


THEOREM 13.1.1

If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it.



Given

In $\triangle ABC$, $m\overline{AC} > m\overline{AB}$

To Prove

$m\angle ABC > m\angle ACB$

Construction

On AC take a point D such that $\overline{AD} \cong \overline{AB}$. Join B to D so that $\triangle ADB$ is an isosceles triangle.

Proof

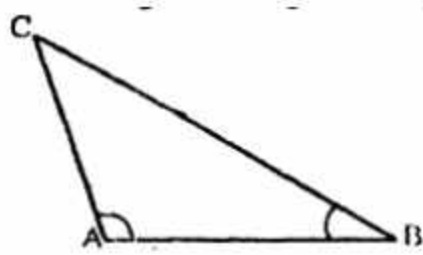
Statements	Reasons
In $\triangle ABD$ $m\angle 1 = m\angle 2 \dots\dots (i)$	Angles opposite to congruent sides,
In $\triangle BCD$ $m\angle 2 > m\angle ACB \dots\dots (ii)$	An exterior angle of a triangle is greater than a non-adjacent interior angle

$\therefore m\angle 1 > m\angle ACB \dots\dots (iii)$	By (i) and (ii)
$m\angle ABC = m\angle 1 + m\angle DBC$	Postulate of addition of angles.
$\therefore m\angle ABC > m\angle 1 \dots\dots (iv)$	By (iii) and (iv)
Hence $m\angle ABC > m\angle ACB$	Transitive property of inequality of real numbers

THEORM 13.1.2

Converse of THEORM 13.1.1

If two angles of a triangle are unequal in measure, the side opposite to the greater angle is longer than the side opposite to the smaller angle.



Given

In $\triangle ABC$, $m\angle A > m\angle B$

To Prove

$m\overline{BC} > m\overline{AC}$

Proof

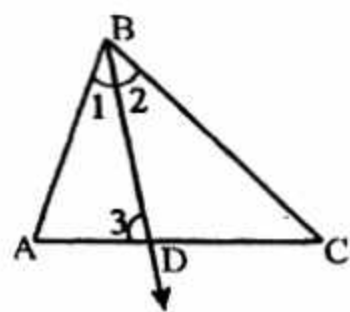
Statements	Reasons
If, $m\overline{BC} \not> m\overline{AC}$, then either (i) $m\overline{BC} = m\overline{AC}$ or (ii) $m\overline{BC} < m\overline{AC}$ From (i) if $m\overline{BC} = m\overline{AC}$, then $m\angle A = m\angle B$ which is impossible.	Trichotomy property of real numbers Angles opposite to congruent sides are congruent Contrary to the given.

From (ii) if $m\overline{BC} < m\overline{AC}$, then $m\angle A < m\angle B$ This is also impossible to contrary to what is given $\therefore m\overline{BC} \neq m\overline{AC}$ and $m\overline{BC} \not< m\overline{AC}$ Hence $m\overline{BC} > m\overline{AC}$	The angle opposite to longer side is greater than angle opposite to smaller side Trichotomy property of real numbers.
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THEOREM 13.1.3

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Solution



Given

ABC is triangle

To Prove

- (i) $m\overline{AB} + m\overline{BC} > m\overline{AC}$
- (ii) $m\overline{AC} + m\overline{AB} > m\overline{BC}$
- (iii) $m\overline{AC} + m\overline{BC} > m\overline{AB}$

Construction

Take a point D on CA such that $\overline{AD} \cong \overline{AB}$. Join B to D and name the angles. $\angle 1$, $\angle 2$ as shown in the given figure.

Proof

Statements	Reasons
In ΔCBD	
$m\angle 3 > m\angle 2$ (i)	Exterior angle is greater than non-adjacent interior angle
$m\angle 2 = m\angle 1$ (ii)	Construction

$\therefore m\angle 3 > m\angle 1$	By (i) and (ii)
and $m\overline{AB} > m\overline{AD}$ (iii)	In ΔABD the side opposite to the larger angle is greater than that of the side opposite to the smaller angle.
similarly	
$m\overline{BC} > m\overline{DC}$	Adding (iii) and (iv)
$m\overline{AB} + m\overline{BC} > m\overline{AD} + m\overline{DC}$	$\therefore m\overline{AD} + m\overline{DC} = m\overline{AC}$
or $m\overline{AB} + m\overline{BC} > m\overline{AC}$	
Similarly, by drawing angle bisector of $\angle A$ and $\angle C$ it can be proved that	
$m\overline{AC} + m\overline{AB} > m\overline{BC}$	
and $m\overline{AC} + m\overline{BC} > m\overline{AB}$	

Theorem 13.1.4

From a point, outside a line, the perpendicular is the shortest distance from the point to the line.

Given

A line $\vec{AB}$ and a point C (not lying on $\vec{AB}$) and a point D on $\vec{AB}$ such that $\overline{CD}$ is perpendicular to $\vec{AB}$.

To Prove

$m\overline{CD}$ is the shortest distance from the point C to $\vec{AB}$

Construction

Take a point E on $\vec{AB}$. Join C and E to form a $\triangle CDE$.

Proof

Statements	Reasons
In $\triangle CDE$	
$m\angle CDB > m\angle CED$	An exterior angle of a triangle is greater than non-adjacent interior angle.
But $m\angle CDB = m\angle CDE$	Supplement of right angle.
$\therefore m\angle CDE > m\angle CED$	
or $m\angle CED < m\angle CDE$	Reflexive property of inequality
or $m\overline{CD} < m\overline{CE}$	Side opposite to greater angle is greater.
But E is any point on AB	
Hence $m\overline{CD}$ is the shortest distance from C to $\vec{AB}$.	

EXERCISE 13.1

Q1. Two sides of a triangle measure 10 cm and 15 cm. Which of the following measure is possible for the third side?

(a) 5 cm (b) 20 cm (c) 25 cm (d) 30 cm

Solution:

(a) Measure of sides are 10 cm, 15 cm and 5 cm

$$\text{As } 10 + 5 = 15$$

Since the 'sum of two sides is equal to the third side therefore:

So, **5 cm is not possible.**

(b) Sides are 10 cm, 15 cm and 20 cm

$$10 + 15 > 20$$

$$10 + 20 > 15$$

$$15 + 20 > 10$$

Since the sum of two sides is greater than third side therefore:

20 cm is possible for third side.

(c) Sides are 10 cm, 15 cm and 25 cm

Since the sum of two sides is equal to the third side therefore:

So, **25 cm is not possible.**

(d) Sides are 10 cm, 15 cm and 30 cm

$$10 + 15 < 30$$

Since the sum of two sides is equal to the third side therefore:

So, **30 cm is not possible.**

Q2. O is interior point of the ΔABC . Show that

$$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2} (m\overline{AB} + m\overline{BC} + m\overline{CA})$$

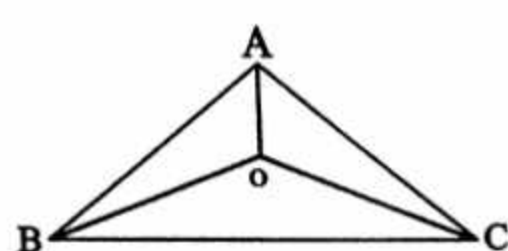
Solution:

Given:

O is a point in side a triangle ABC. O is joined with A, B and C.

To prove:

$$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2} (m\overline{AB} + m\overline{BC} + m\overline{CA})$$



Proof

Statements	Reasons
In ΔOAB $m\overline{OA} + m\overline{OB} > m\overline{AB}$	In a triangle the sum of measure of two sides is greater than measure of third side
In ΔOBC $m\overline{OB} + m\overline{OC} > m\overline{AC}$	
In ΔCOA	

$m\overline{OC} + m\overline{OA} > m\overline{AC}$ Adding (i), (ii), (iii) $2(m\overline{OA} + m\overline{OB} + m\overline{OC})$ $> m\overline{AB} + m\overline{BC} + m\overline{AC}$ $m\overline{OA} + m\overline{OB} + m\overline{OC}$ $> \frac{1}{2} (m\overline{AB} + m\overline{BC} + m\overline{CA})$	Dividing both sides by 2
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3. In the ΔABC , $m\angle B = 70^\circ$ and $m\angle C = 45^\circ$. Which of the sides of the triangle is longest and which is the shortest?

Solution:

$$m\angle B = 70^\circ$$

$$m\angle C = 45^\circ$$

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$m\angle A + 70^\circ + 45^\circ = 180^\circ$$

$$m\angle A + 115^\circ = 180^\circ$$

$$m\angle A = 180^\circ - 115^\circ = 65^\circ$$

Since the largest angle is B. So, the longest side is opposite to B is $\overline{AC}$
(Longest)

Since the smallest angle is C. So, the shortest side is opposite to C is $\overline{AB}$
(Shortest)

Q4. Prove that in a right-angled triangle, the hypotenuse is longer than each of the other two sides.

Solution:



In the right-angled triangle ABC, $m\angle B = 90^\circ$ and $\overline{AC}$ is hypotenuse.

$\angle A$ and $\angle C$ both are acute angles

$$\therefore m\angle B > m\angle A$$

$$\text{And } m\angle B > m\angle C$$

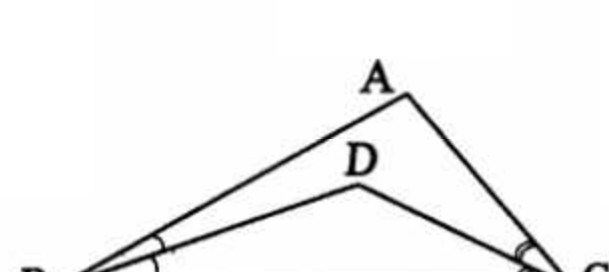
$$\therefore \angle B \text{ is the largest angle}$$

$$\therefore \text{Side opposite to } \angle B$$

Which is hypotenuse is longer than each of the other two sides.

Q5. In the triangle figure, $\overline{AB} > \overline{AC}$. $\overline{BD}$ and $\overline{CD}$ are the bisectors of $\angle B$ and $\angle C$ respectively. Prove that $\overline{BD} > \overline{DC}$.

Solution:



$\overline{BD}$ is bisector of $\angle B$

$$\therefore m\angle ABD = m\angle CBD$$

$$\text{So, } m\angle CBD = m\angle ABC$$

Similarly, as $\overline{CD}$ is bisector of $\angle C$.

$$\text{So, } m\angle BCD = m\angle ACB$$

It is given that $\overline{AB} > \overline{AC}$

$$\therefore \angle ACB < \angle ABC \quad \text{Opposite angles}$$

$$\Rightarrow \frac{1}{2} m\angle ACB > \frac{1}{2} m\angle ABC$$

$$\angle ACB < \angle ABC \quad \text{Opposite angles}$$

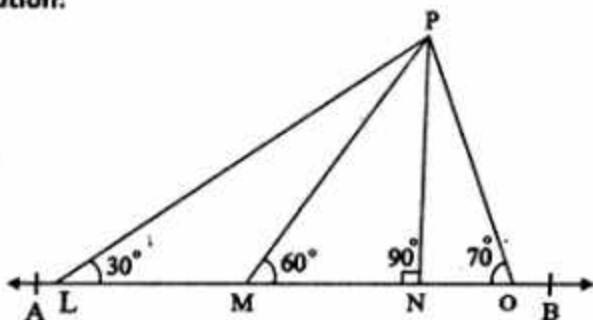
$$m\angle BCD > \frac{1}{2} m\angle ABC$$

EXERCISE 13.2

Q1. In the figure, P is any point and $\overline{AB}$ is a line. Which of the following is the shortest distance between the point P and the line $\overline{AB}$?

- (a) $m\overline{PL}$ (b) $m\overline{PM}$ (c) $m\overline{NP}$ (d) $m\overline{PO}$

Figure:



We all know that from a point outside the line, the perpendicular is the shortest distance from the point to the line.

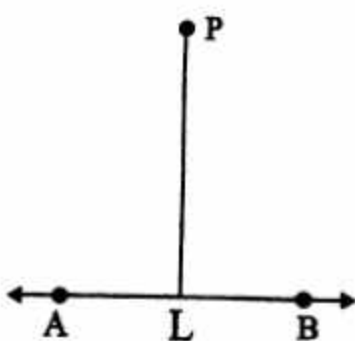
As $\overline{PN}$ is perpendicular to $\overline{AB}$.

So $\overline{PN}$ is the shortest distance.

Q2. In the figure, P is any point lying away from the line AB. Then $m\overline{PL}$ will be the shortest distance if

- (a) $m\angle PLA = 80^\circ$ (b) $m\angle PLB = 100^\circ$ (c) $m\angle PLA = 90^\circ$

Solution



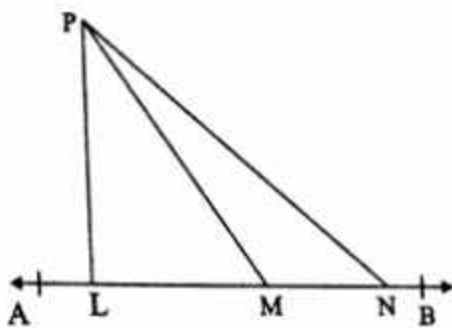
We know that for a point outside a line, the shortest distance from the point to the line is perpendicular to the line.

As $m\overline{PL}$ is shortest,

So, $\overline{PL}$ is perpendicular to $\overline{AB}$.

So, $m\angle PLA = 90^\circ$

Q3. In the figure, $\overline{PL}$ is perpendicular to the line AB and $m\overline{LN} > m\overline{LM}$. Prove that $m\overline{PN} > m\overline{PM}$.



Given

$\overline{PL}$ is perpendicular to $\overline{AB}$ and $m\overline{LN} > m\overline{LM}$

To prove

$m\overline{PN} > m\overline{PM}$

Proof

Statements	Reasons
In $\triangle LPN$ $m\angle PLN = 90^\circ$ $\therefore m\angle PLN < 90^\circ$ (i)	Given Angle of a triangle
In $\triangle PLM$ $m\angle PMN > m\angle PLM$ $\therefore m\angle PMN < 90^\circ$ (ii)	Exterior angle $\angle PLM = 90^\circ$ from (i) and (ii)
In $\triangle PMN$	opposite sides

$m\angle PMN > m\angle PNL$ $m\overline{PN} > m\overline{PM}$	
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REVIEW EXERCISE 13

Q1. Which of the following are true and which are false?

- (i) The angle opposite to the longer side is greater.
- (ii) In a right-angled triangle greater angle is of 60°.
- (iii) In an isosceles right-angled triangle, angles other than right angles are each of 45°.
- (iv) A triangle having two congruent sides is called equilateral triangle.
- (v) A perpendicular from a point to line is shortest distance.
- (vi) Perpendicular to line form an angle of 90°.
- (vii) A point outside the line is collinear
- (viii) Sum of two sides of triangle is greater than the third.
- (ix) The distance between a line and a point on it is zero.
- (x) Triangle can be formed of lengths 2 cm, 3 cm and 5 cm.

Answers:

(i) T	(ii) F	(iii) T	(iv) F	(v) T	(vi) T
(vii) F	(viii) T	(ix) T	(x) F		

Q2. What will be angle for shortest distance from an outside point to the line?

Solution:

The shortest distance between a point and a line is perpendicular from the point to the line. So, the angle for shortest distance from an outside point is

90°.

Q3. If 13 cm, 12 cm, and 5 cm are the lengths of a triangle, then verify that difference of measures of any two sides of a triangle is less than the measure of the third side.

Solution

Let the three sides be

$a = 13\text{ cm}, b = 12\text{ cm}, c = 5\text{ cm}$
 $a - b = 13 - 12 = 1 < 5 = c$

i.e. $a - b < c$ (i)

$12 - 5 = 7 < 13$

$b - c < a$ (ii)

$13 - 5 = 8 < 12$

i.e. $a - c < b$ (iii)

From (i), (ii) and (iii) we find that the difference of measures of any two sides of a triangle is less than the measure of the third side.

Q4. If 10 cm, 6 cm and 8 cm are the lengths of a triangle, then verify that sum of measures of two sides of a triangle is greater than the third side.

Solution

Let the measure of the sides of the triangle be

$a = 10\text{ cm}, b = 6\text{ cm}, c = 8\text{ cm}$

$10 + 6 = 16 > 8$

$a + b < c$ (i)

$6 + 8 = 14 > 10$

$b + c < a$ (ii)

$8 + 10 = 18 > 6$

$c + a > b$

From (i), (ii) and (iii) we conclude that the sum of measures of two sides of a triangle is greater than the third side.

Q5. 3 cm, 4 cm and 7 cm are not the lengths of the triangle. Give the reason.

Solution

Let $a = 3\text{ cm}, b = 4\text{ cm}, c = 7\text{ cm}$

$a + b = 3 + 4 = 7 = c$

i.e. $a + b = c$

Since we know that for a triangle sum of measures of two sides should be greater than measure of the third side.

$3 + 4 \nless 7$

So, 3 cm, 4 cm and 7 cm are not the, lengths of the triangle

Q6. If 3 cm and 4 cm are the lengths two sides of a right-angle triangle then what should be the third length of the triangle.

Solution

Let the three sides be

$3^2 + 4^2 = a^2$ (Pythagoras theorem)

$9 + 16 = a^2$

$a^2 = 25$

$\Rightarrow a = 5$

The third Length is 5 cm.