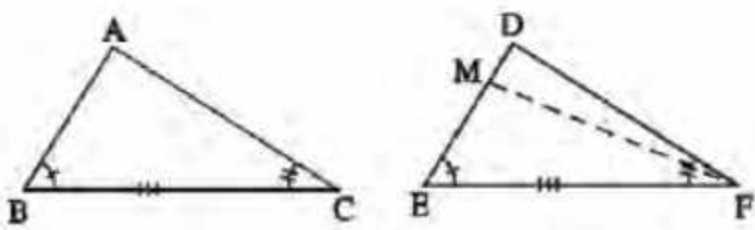


THEOREM 10.1.1

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding side and angles of the other, then the triangles are congruent.
(A.S.A. $\cong$ A.S.A.)



Given:

In $\triangle ABC \leftrightarrow \triangle DEF$
 $\angle B \cong \angle E, \overline{BC} \cong \overline{EF}, \angle C \cong \angle F.$

To prove:

$\triangle ABC \cong \triangle DEF$

Construction

Suppose $\overline{AB} \neq \overline{DE}$, take a point M on DE such that $\overline{AB} \cong \overline{ME}$. Join M to F

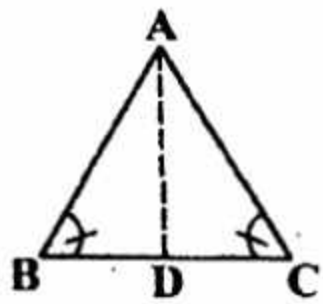
Proof:

Statements	Reasons
In $\triangle ABC \leftrightarrow \triangle D'EF$ $\overline{AB} \cong \overline{D'E}$ (i) $\overline{BC} \cong \overline{EF}$ (ii)	Construction Given Given

$\angle B \cong \angle E$ (iii) $\therefore \triangle ABC \cong \triangle D'EF$ So, $\angle C \cong \angle D'FE$ But, $\angle C \cong \angle DFE$ $\therefore \angle DFE \cong \angle D'FE$ This is possible only if D and D' are the same points, and $\overline{D'E} \cong \overline{DE}$ So, $\overline{AB} \cong \overline{DE}$ (iv) Thus from (ii), (iii) and (iv), we have $\triangle ABC \cong \triangle DEF$	S.A.S. postulate (Corresponding angles of congruent triangles) Given Both congruent to $\angle C$ Proved that D and D' are the same points S.A.S. postulate
---	--

THEOREM 10.1.2

If two angles of a triangle are congruent, then the sides opposite to them are also congruent.



Given:

In $\triangle ABC$,

$\angle B \cong \angle C$

To prove:

$\overline{AB} \cong \overline{AC}$

Construction:

Draw the bisector of $\angle A$, to meet BC at point D.

Proof:

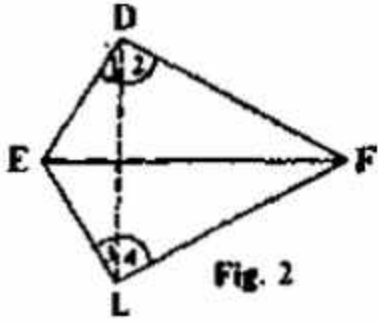
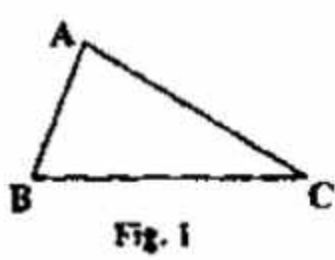
Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$\angle B \cong \angle C$	Given
$\angle BAD \cong \angle CAD$	Construction
	S.A.A. $\cong$ S.A.A.

$\therefore \triangle ABD \cong \triangle ACD$	(Corresponding angles of congruent
Hence $\overline{AB} \cong \overline{AC}$	triangles)

THEOREM 10.1.3

If in a given correspondence of two triangles, the three sides of one triangle are congruent to the corresponding three sides of the other triangle then the triangles are congruent.

Solution:



Given

In $\triangle ABC \leftrightarrow \triangle DEF$

$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$

To Prove

$\triangle ABC \cong \triangle DEF$

Construction

Suppose that in $\triangle DEF$ the side $\overline{EF}$ is not smaller than any of the remaining two sides. On $\overline{EF}$ construct a $\triangle LEF$ in which, $\angle FEL \cong \angle B$ and $\overline{LE} \cong \overline{AB}$. Join D and M. As shown in the above figures we label some of the angles as 1, 2, 3 and 4.

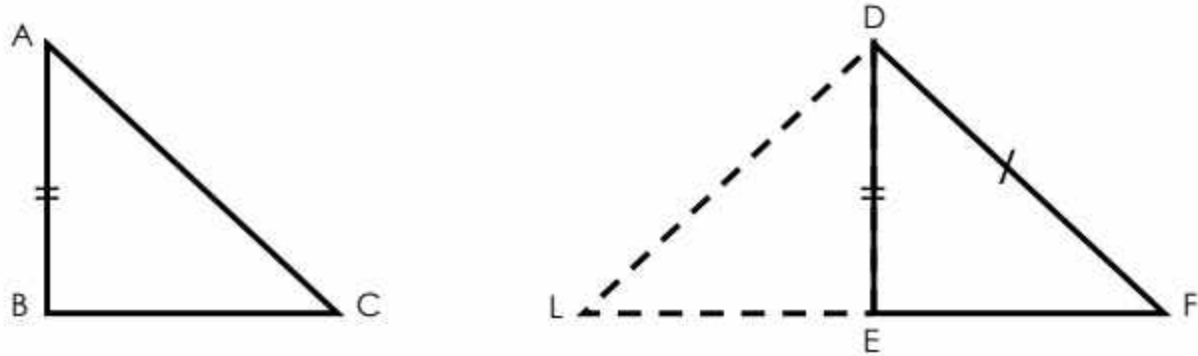
Proof

Statements	Reasons
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In $\triangle ABC \leftrightarrow \triangle LEF$ $\overline{BC} \cong \overline{EF}$ $\angle B = \angle FEL$ $\overline{AB} \cong \overline{LE}$ $\therefore \triangle ABC \cong \triangle LEF$ And $\overline{CA} \cong \overline{FL}$ (i) Also $\overline{CA} \cong \overline{FD}$ (ii) $\therefore \overline{FL} \cong \overline{FD}$ In $\triangle FDL$ $\angle 2 \cong \angle 4$ (iii) Similarly, $\angle 1 \cong \angle 3$ (iv) $\therefore m\angle 2 + m\angle 1 = m\angle 4 + m\angle 3$ Now, in $\triangle DEF \leftrightarrow \triangle LEF$ $\overline{FD} \cong \overline{FL}$ And $m\angle EDF \cong m\angle ELF$ $\overline{DE} \cong \overline{LE}$ $\therefore \triangle DEF \cong \triangle LEF$ Also $\triangle ABC \cong \triangle LEF$ Hence $\triangle ABC \cong \triangle DEF$	Given Construction Construction S.A.S postulate corresponding sides of congruent triangles) Given From (i) and (ii) $\overline{FM} \cong \overline{FD}$ (proved) {from (iii) and (iv) Proved Proved Each one $\cong \overline{AB}$ S.A.S. postulate Proved Each $\triangle \cong \triangle LEF$ (Proved)
--	--

THEOREM 10.1.4

If in the correspondence of the two right-angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other, then the triangles are congruent.



Given:

In $\triangle ABC \leftrightarrow \triangle DEF$

$\angle B \cong \angle E$ (Right angles)

$\overline{CA} \cong \overline{FD}$, $\overline{AB} \cong \overline{DE}$

To Prove:

$\triangle ABC \cong \triangle DEF$

Construction:

Produce $\overline{EF}$ to point L such that $EL \cong BC$ and join points D and L.

Proof:

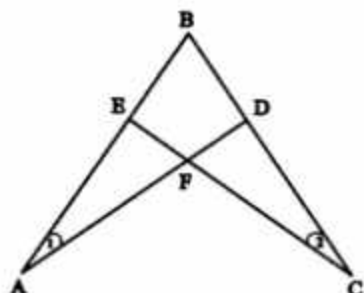
Statements		Reasons
$m\angle DEF + m\angle DEL = 180^\circ$	(i)	Supplementary angles
Now $m\angle DEF = 90^\circ$	(ii)	Given

$\therefore m\angle DEL = 90^\circ$	From (i) and (ii)
In $\triangle ABC \leftrightarrow \triangle DEL$	
$\overline{BC} \cong \overline{EL}$	Construction
$\angle ABC \cong \angle DEL$	Each equal to 90°
$\overline{AB} \cong \overline{DE}$	Given
$\therefore \triangle ABC \cong \triangle DEL$	S.A.S. postulate
And $\angle C \cong \angle L$	Corresponding angles of congruent triangles
$\overline{CA} \cong \overline{LD}$	Corresponding sides of congruent triangles
But $\overline{CA} \cong \overline{FD}$	Given
$\therefore \overline{LD} \cong \overline{FD}$	Each is congruent to $\overline{CA}$
In $\triangle DLF$	
$\angle F \cong \angle L$	$\overline{FD} \cong \overline{LD}$ (proved)
But $\angle C \cong \angle L$	Proved
$\angle C \cong \angle F$	Each is congruent to $\angle L$.
In $\triangle ABC \leftrightarrow \triangle ADEF$	Given
$\overline{AB} \cong \overline{DE}$	Given
$\angle ABC \cong \angle DEF$	Proved
$\angle C \cong \angle F$	S.A.A. S.A.A
$\therefore \triangle ABC \cong \triangle DEF$	

Exercise 10.1

Q1. In the given figure, $AB \cong CB$, $\angle 1 \cong \angle 2$. Prove that $\triangle ABD \cong \triangle CBE$.

Solution:



Given:

In the given figure $\angle 1 \cong \angle 2$ and $\overline{AB} \cong \overline{CB}$

To prove:

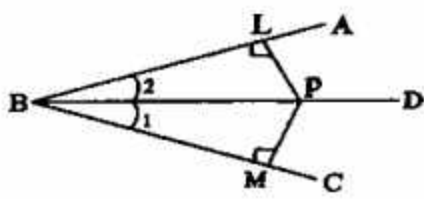
$\triangle ABD \cong \triangle CBE$

Proof:

Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle CBE$	
$\overline{AB} \cong \overline{CB}$	Given
$\angle BAD \cong \angle BCE$	Given $\angle 1 \cong \angle 2$
$\angle ABD \cong \angle CBE$	Common
$\therefore \triangle ABD \cong \triangle CBE$	S.A.A $\cong$ S.A.A

Q2. From a point on the bisector of an angle, perpendiculars are drawn to the arms of the angle. Prove that these perpendiculars are equal in measure.

Solution:



Given:

$\overline{BD}$ is bisector of $\angle ABC$, P is point on $\overline{BD}$ and $\overline{PL}$ and $\overline{PM}$ are perpendicular to $\overline{AB}$ and $\overline{CB}$ respectively

To prove:

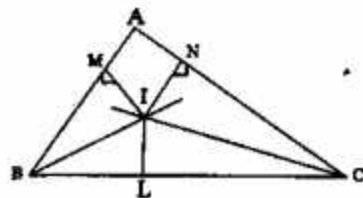
$\overline{PL} \cong \overline{PM}$

Proof:

Statements	Reasons
In $\triangle BLP \leftrightarrow \triangle BMP$	
$\overline{BP} \cong \overline{BP}$	Common
$\angle BLP \cong \angle BMP$	Each right angle (given)
$\angle LBP \cong \angle MBP$	Given BD is bisector of angle B
$\triangle BLP \cong \triangle BMP$	S.A.A $\cong$ S. A. A.
So $\overline{PL} \cong \overline{PM}$	Corresponding sides of $\cong$ Δ 's.

Q3. In a triangle ABC, the bisectors of $\angle B$ and $\angle C$ meet in a point I. Prove that I is equidistant from the three sides of $\triangle ABC$.

Solution:



Given:

In $\triangle ABC$ the bisector of $\angle B$ and $\angle C$ meet at I, IL, IM and IN are perpendiculars to the three sides of $\triangle ABC$.

To prove:

$\overline{IL} \cong \overline{IM} \cong \overline{IN}$

Proof:

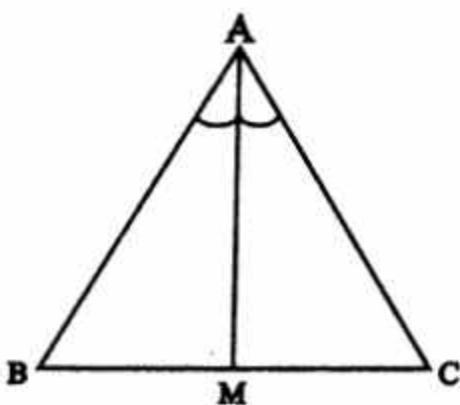
Statements	Reasons
In $\triangle ILB \leftrightarrow \triangle IMB$	
$\overline{BI} \cong \overline{BI}$	Common
$\angle IBL \cong \angle IBM$	Given BI is bisector of $\angle B$
$\angle ILB \cong \angle IMB$	Given each $\angle$ is right angles.
$\triangle ILB \cong \triangle IMB$	S.A.A $\cong$ S.A.A.
$\overline{IL} \cong \overline{IM}$ (i)	Corresponding sides of $\cong$ Δ 's.
Similarly	
$\triangle IAC \cong \triangle INC$	
So $\overline{IL} \cong \overline{IN}$ (ii)	Corresponding sides of $\cong$ Δ 's.

From (i) and (ii)	
$\overline{IL} \cong \overline{IM} \cong \overline{IN}$	
I is equidistant from the three sides of $\triangle ABC$.	

Exercise 10.2

Q1. Prove that any two medians of an equilateral triangle are equal in measure.

Solution:



Given:

In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$ and M is mid point of BC.

To prove:

$\overline{AM}$ bisects $\angle A$ and $\overline{AM}$ is perpendicular to $\overline{BC}$.

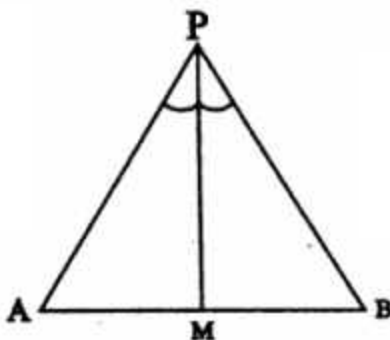
Proof:

Statements	Reasons
In $\triangle ABM \leftrightarrow \triangle ACM$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BM} \cong \overline{CM}$	Given M is mid point of $\overline{BC}$.
$\overline{BM} \cong \overline{AM}$	Common
$\therefore \triangle ABM \cong \triangle ACM$	S. S. S. $\cong$ S. S. S.
	Corresponding sides of $\cong$ $\triangle$'s.

So $\angle BAM \cong \angle CAM$	
$\therefore \overline{AM}$ bisects $\angle A$	Corresponding sides of $\cong$ $\triangle$'s.
Also $\angle AMB \cong \angle AMC$	
but $m\angle AMB \cong \angle AMC = 180^\circ$	$\overline{BC}$ is a straight line.
$\therefore m\angle AMB = \angle AMC = 90^\circ$	
i. e. $\overline{AM}$ is perpendicular to $\overline{BC}$.	

Q2. Prove that a point, which is equidistant from the end points of a line segment, is on the right bisector of the line segment.

Solution:



Given:

$\overline{AB}$ is a line segment and P is a point such that

$\overline{PA} \cong \overline{PB}$

To prove:

P is on right bisector of $\overline{AB}$

Construction:

Draw $\overline{PM}$ bisector of $\angle P$ meeting $\overline{AB}$ at M.

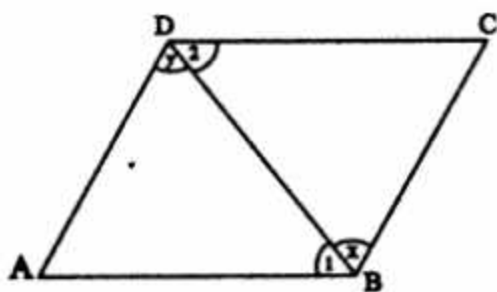
Proof:

Statements	Reasons
In $\triangle APM \leftrightarrow \triangle BPM$	
$\overline{PA} \cong \overline{PB}$	Given
$\angle APM \cong \angle BPM$	Construction.
$\overline{PM} \cong \overline{PM}$	Common
$\therefore \triangle APM \cong \triangle BPM$	S. S. S. $\cong$ S. S. S.
So $\overline{AM} \cong \overline{BM}$	Corresponding sides of $\cong$ $\triangle$'s.
$\angle PMA \cong \angle PMB$	
but $m\angle PMA \cong \angle PMB = 180^\circ$	$\overline{BC}$ is a straight line.
$\therefore m\angle PMA = \angle PMB = 90^\circ$	
So $\overline{PM}$ is right bisector of $\overline{AB}$	
or P is on right bisector of $\overline{AB}$.	

Exercise 10.3

Q1. In the given figure, $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$. Prove that $\angle A \cong \angle C$, $\angle ABC \cong \angle ADC$.

Solution:



Given:

In the figure, $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$

To prove:

$\angle A \cong \angle C$
 $\angle ABC \cong \angle ADC$

Construction:

Join B to D

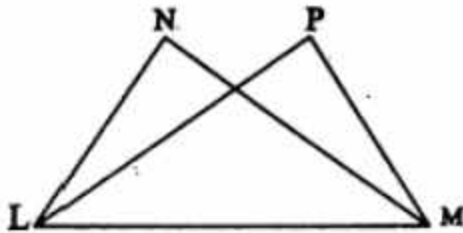
Proof:

Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{CD}$	Given

$\overline{BD} \cong \overline{DB}$	Common
$\therefore \triangle ABC \cong \triangle CDB$	S. S. S. $\cong$ S. S. S.
$\therefore \angle A \cong \angle C$	Corresponding sides of $\cong$ Δ 's.
$\hat{1} \cong \hat{2}$ and $\angle x \cong \angle y$	Corresponding sides of $\cong$ Δ 's.
$\therefore$ by adding above equations $\hat{1} + \hat{x} = \hat{2} + \hat{y}$ or $\angle ABC \cong \angle ADC$	Addition of angles

Q2. In the figure, $\overline{LN} \cong \overline{MP}$, $\overline{MN} \cong \overline{LP}$. Prove that $\angle N \cong \angle P$, $\angle NML \cong \angle PLM$.

Solution:



Given:

In the figure

$\overline{LN} \cong \overline{MP}$ and $\overline{LP} \cong \overline{MN}$

To prove:

$\angle N \cong \angle P$
 $\angle NML \cong \angle PLM$

Proof

Statements	Reasons
In $\triangle LMN \leftrightarrow \triangle MLP$	
$\overline{LN} \cong \overline{MP}$	Given
$\overline{LP} \cong \overline{MN}$	Given
$\overline{LM} \cong \overline{ML}$	Common
$\therefore \triangle LMN \cong \triangle MLP$	S. S. S. $\cong$ S. S. S.
$\therefore \angle N \cong \angle P$	Corresponding sides of $\cong$ Δ 's.
$\angle NML \cong \angle PLM$	Corresponding sides of $\cong$ Δ 's.

Q3. Prove that the median bisecting the base of an isosceles triangle bisects the vertex angle and it is perpendicular to the base.

Solution:

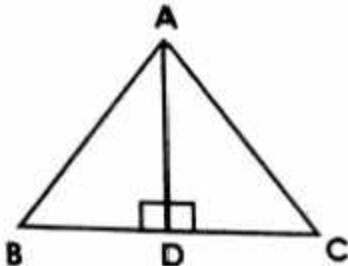
Given:

An isosceles triangle ABC with the base $\overline{BC}$ and $\overline{AD}$ bisects at point D.

i.e. $\overline{BD} \cong \overline{DC}$ and $\overline{AB} \cong \overline{AC}$

To prove:

$\angle BAD \cong \angle CAD$
and $\overline{AD} \perp \overline{BC}$

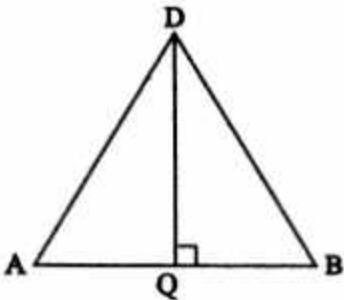


Proof

Statements	Reasons
In the correspondence of $\triangle ABD \leftrightarrow \triangle ACD$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{AD} \cong \overline{AD}$	Common
$\overline{BD} \cong \overline{DC}$	Given
$\therefore \triangle ABD \cong \triangle ACD$	S. S. S. postulate
Thus $\angle BAD \cong \angle CAD$	Corresponding angles of congruent triangle
$m\angle ADB = m\angle ADC = 180^\circ$ $m\angle ADC = m\angle ADC = 180^\circ$ $\Rightarrow 2 m\angle ADC = 180^\circ$ $\Rightarrow m\angle ADC = 90^\circ$	Supplementary angles As $m\angle ADC = m\angle ADB$
Hence $\overline{AD} \perp \overline{BC}$	

Exercise 10.4

Q1. In $\triangle PAB$ of figure, $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$ proved that $\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$.



Solution:
Given:

In $\triangle PAB$,
 $\overline{PQ} \perp \overline{AB}$, and $\overline{PA} \cong \overline{PB}$

To prove:
 $\overline{AQ} \cong \overline{BQ}$

$\angle APQ \cong \angle BPQ$

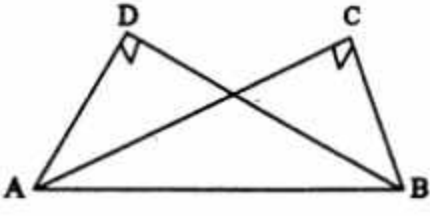
Proof:

Statements	Reasons
In $\triangle APQ \leftrightarrow \triangle BPQ$	
$\overline{PA} \cong \overline{PB}$	Given
$\angle AQP \cong \angle BQP$ $\overline{PQ} \cong \overline{PQ}$	Given $\overline{PQ} \perp \overline{AB}$
	Common

1

$\therefore \triangle APQ \cong \triangle BPQ$	H.S $\cong$ H.S
so $\overline{AQ} \cong \overline{BQ}$	Corresponding sides of $\cong$ $\triangle$'s.
$\angle APQ \cong \angle BPQ$	Corresponding sides of $\cong$ $\triangle$'s.

Q2. In the figure, $m\angle C = m\angle D = 90^\circ$ and $\overline{BC} \cong \overline{AD}$. prove that $\overline{AC} \cong \overline{BD}$ and $\angle BAC \cong \angle ABD$.



Solution:
Given:

In the figure
 $m\angle C = m\angle D = 90^\circ$ and $\overline{BC} \cong \overline{AD}$

To prove:
 $\angle ABC \cong \angle ABD$

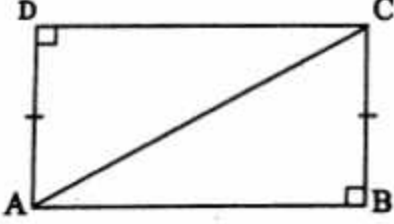
Proof:

Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle BAC$	
$\overline{D} \cong \overline{C}$	Given
$\overline{AD} \cong \overline{BC}$	Given
	Common

2

$\overline{AB} \cong \overline{BA}$	H.S $\cong$ H.S
$\therefore \triangle ABD \cong \triangle BAC$	Corresponding sides of $\cong$ $\triangle$'s.
so $\overline{AC} \cong \overline{BD}$	Corresponding sides of $\cong$ $\triangle$'s.
$\angle BAC \cong \angle ABD$	

Q3. In the figure, $m\angle B = m\angle D = 90^\circ$ and $\overline{AD} \cong \overline{BC}$. Prove that ABCD is rectangle.



Solution:
Given:

In the figure,
 $m\angle B = m\angle D = 90^\circ$ and $\overline{AD} \cong \overline{BC}$

To prove:
ABCD is rectangle.

Construction:
Join A to C.

Proof:

Statements	Reasons
In $\triangle ABC \leftrightarrow \triangle CDA$	

3

$\overline{B} \cong \overline{D}$	Given each angle = 90° .
$\overline{AC} \cong \overline{CA}$	Given
$\overline{BC} \cong \overline{DA}$	Common
$\therefore \triangle ABC \cong \triangle CDA$	H.S $\cong$ H.S
$\therefore$ So $\overline{AB} \cong \overline{CD}$	Corresponding sides of $\cong$ $\triangle$'s.
$\angle ACB \cong \angle CAD$ Hence ABCD is a rectangle.	Corresponding sides of $\cong$ $\triangle$'s.

4

Review Exercise 10

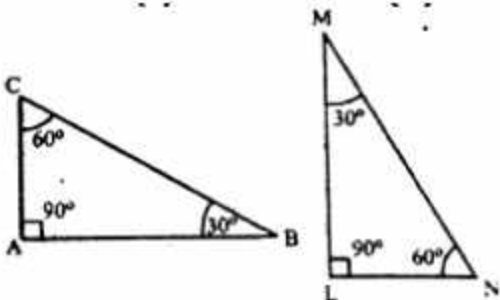
1. Which of the following are true and which are false?
- (i) A ray has two end points
 - (ii) In a triangle, there can be only one right angle
 - (iii) Three points are said to be collinear, if they lie on same line
 - (iv) Two parallel lines intersect at a point.
 - (v) Two lines can intersect only at one point.
 - (vi) A triangle of congruent sides has non-congruent angles.

Answers:

(i) F	(ii) T	(iii) T	(iv) F	(v) T	(vi) F
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2. If $\triangle ABC \cong \triangle LMN$, then

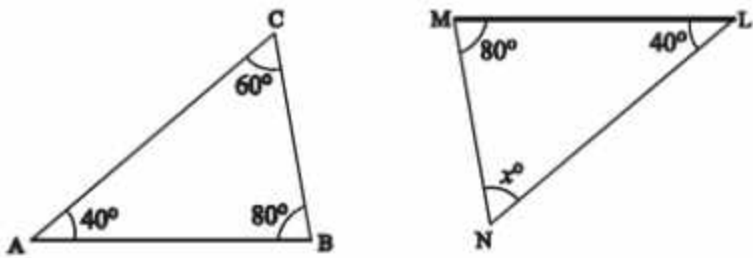
- (i) $m\angle M \cong$
- (ii) $m\angle N \cong$
- (iii) $m\angle A \cong$



Solution:

(i) F	(ii) T	(iii) T
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3. If $\triangle ABC \cong \triangle LMN$, then find the unknown x.



Solution:

Given that: $\triangle ABC \cong \triangle LMN$

$\angle C \cong \angle M$

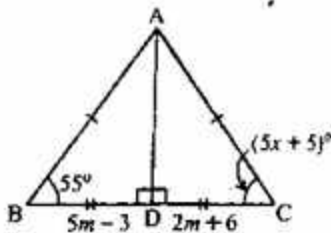
or $m\angle C \cong m\angle M$

$\Rightarrow 60^\circ \cong x$

$\Rightarrow x \cong 60^\circ$

Q4. Find the value unknowns for the given congruent triangles.

Solution:



$\triangle ADB \cong \triangle ADC$

$BD \cong CD$

Corresponding sides of $\triangle$'s.

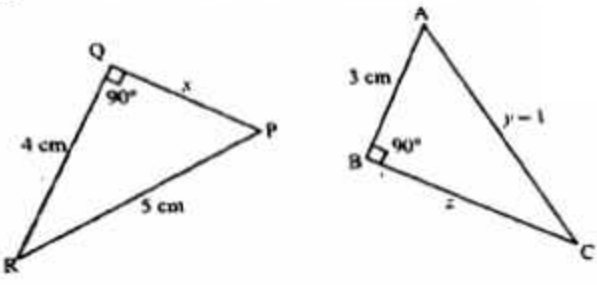
$\Rightarrow BD \cong CD$
 $\Rightarrow 5m - 3 = 2m + 6$
or $5m - 2m = 6 + 3$
 $3m = 9$
 $\angle B \cong \angle C$

Corresponding sides of $\triangle$'s.

$\Rightarrow m\angle B \cong m\angle C$
 $55 = (5x + 5)$
 $55 = 5x + 5$
 $5x = 55 - 5 = 50$
 $x = 10^\circ$

Q5. If $\triangle PQR \cong \triangle ABC$, then find the unknowns

Solution:



$\triangle PQR \cong \triangle ABC$

$\therefore \overline{BD} \cong \overline{CD}$

Corresponding sides of $\triangle$ s.

$\Rightarrow x = 3 \text{ cm}$

$\overline{PR} \cong \overline{AC}$

Corresponding sides of $\triangle$'s.

$5 = y - 1$
 $y = 5 + 1$
 $\Rightarrow y = 6$

Also $\overline{QR} \cong \overline{BC}$

Corresponding sides of $\triangle$'s.

$m\overline{QR} \cong m\overline{BC}$

$4 \text{ cm} = z$

or $z = 4 \text{ cm}$