Exercise 1.1

Q1. Find the order of the following matrices.

A = $\begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$, **B** = $\begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$, **C** = $\begin{bmatrix} 2 & 4 \end{bmatrix}$,

D = $\begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$, **E** = $\begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$, **F** = $\begin{bmatrix} 2 \end{bmatrix}$,

G = $\begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$, **H** = $\begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$

Solution:

Order of the *Matrix*:

The number of rows and columns in a Matrix specifies its order.

Ans (i) Matrix A has two rows and two columns
So, its order = number of rows x number of columns = 2-by-2.

Ans. (ii) Matrix B has two rows and two columns
So, its order = number of rows x number of columns = 2-by-2.

Ans. (iii) Matrix C has one row and two columns
So, its order = number of rows x number of columns = 1-by-2.

Ans. (iv) Matrix D has three rows and one column
So, its order = number of rows x number of columns = 3-by-1.

Ans. (v) Matrix E has three rows and two columns
So, its order = number of rows x number of columns = 3-by-2.

Ans. (vi) Matrix F has one row and one column
So, its order = number of rows x number of columns = 1-by-1.

Ans. (vii) Matrix G has three rows and three columns
So, its order = number of rows x number of columns = 3-by-3.

Ans. (viii) Matrix A has two rows and three columns
So, its order = number of rows x number of columns = 2-by-3.

Q2. Which of the following matrices are equal?

A = $\begin{bmatrix} 3 \end{bmatrix}$, **B** = $\begin{bmatrix} 3 & 5 \end{bmatrix}$, **C** = $\begin{bmatrix} 5 & -2 \end{bmatrix}$,

D = $\begin{bmatrix} 5 & 3 \end{bmatrix}$, **E** = $\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$, **F** = $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$,

G = $\begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$, **H** = $\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$, **I** = $\begin{bmatrix} 3 & 3+2 \end{bmatrix}$,

J = $\begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$

Solution:

Solving C

C = $\begin{bmatrix} 5 & -2 \end{bmatrix}$

C = $\begin{bmatrix} 3 \end{bmatrix}$

Solving G

G = $\begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$

G = $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$

Solving I

I = $\begin{bmatrix} 3 & 3+2 \end{bmatrix}$

I = $\begin{bmatrix} 3 & 5 \end{bmatrix}$

Solving J

J = $\begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$

J = $\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$

Now

Matrices are said to be equal if

(i) They are of same order
(ii) Their corresponding values are equal

So, according to this definition

Ans. (a) Matrices A and C are equal, A = C.
(b) Matrices B and I are equal, B = I.
(c) Matrices E, H and J are equal, E = H = J.
(d) Matrices F and G are equal, F = G.

Q3. Find the values of a, b, c, and d which satisfy the matrix equation.

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

Solution:

As, $\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$

By comparing the corresponding elements

So, a + c = 0
a = − c ----------(i)

a + 2b = − 7
2b = − (a+7) ----------(ii)

c − 1 = 3
c = 3 + 1
c = 4 ----------(iii)

By putting the value of "c" in equation (i), we will get
a = −4 ----------(iv)

By putting the value of "a" in equation (iii), we will get
2b = − (− 4+7)
2b = − (3)
b = − 3/2
b = − 1.5 ----------(v)

Similarly,
4d − 6 = 2d
4d − 2d = 6
2d = 6
d = 6/2
d = 3 ----------(vi)

From equations (iii), (iv), (v) and (vi) we get
a = − 4,
b = − 1.5,
c = 4
d = 3

Exercise 1.2

Q1. From the following matrices, identify unit matrices, row matrices, column matrices and null matrices.

A = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, **B =** $\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$, **C =** $\begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$,

D = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, **E =** $\begin{bmatrix} 0 \end{bmatrix}$, **F =** $\begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$

Solution:

Matrix A is a null matrix (because all its entries are zero)
Matrix B is a row matrix (because it has only one row).
Matrix C is a column matrix (because it has only one column).
Matrix D is a unit matrix (because its diagonal entries are 1 and non-diagonal entries are 0).
Matrix E is a null matrix (because all its entries are 0).
Matrix F is a column matrix (because it has only one column).

Q2. From the following matrices, identify

(a) Square matrices **(b) Rectangular matrices**
(c) Row matrices **(d) Column matrices**
(e) Identity matrices **(f) Null matrices**

(i) $\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$, **(ii)** $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$, **(iii)** $\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$,

(iv) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, **(v)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$, **(vi)** $\begin{bmatrix} 3 & 10 & -1 \end{bmatrix}$,

(vii) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, **(viii)** $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **(ix)** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Solution:

(a) (iii), (iv) and (viii) are square matrices because the number of rows are equal to number of columns.
(b) (i), (ii), (v), (vi) (vii), (ix) are rectangular matrices because their rows and columns are not equal.
(c) (vi) is a row matrix because it has only one row.
(d) (ii) and (vii) are column matrices because they have only one column.
(e) (iv) is an identity matrix as well because its diagonal elements are "1" and all non-diagonal elements are zero.
(f) (ix) is a null matrix because its each entry is zero.

Q3. From the following matrices identify diagonal, scalar and unit (identity) matrices.

A = $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$, **B =** $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$, **C =** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

D = $\begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$, **E =** $\begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$

Solution:

Matrix A is a scalar matrix (because its diagonal entries are same).
Matrix B is a diagonal matrix (because its diagonal entries are non-zero and non-diagonal entities are zero).
Matrix C is an identity matrix (because its diagonal entries are 1).
Matrix D is a diagonal matrix (because its one diagonal entry is non-zero and non-diagonal entities are zero).
Matrix E is a scalar matrix (because its diagonal entries are same).

$E = \begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$

$E = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

(Note that all the matrices A, B, C, D and E are diagonal matrices because they have at least one non-zero diagonal entry and all their non-diagonal entries are zero.
Scalar and unit matrices are further types of diagonal matrices.**)**

Q4. Find negative of matrices A, B, C, D and E when:

A = $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, **B =** $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$, **C =** $\begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}$,

D = $\begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$, **E =** $\begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$

Solution:

Negative of a matrix is obtained by inverting (changing) the signs of all its entries.

So,

(i) $-A = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ **(ii)** $-B = \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix}$

(iii) $-C = \begin{bmatrix} -2 & -6 \\ -3 & -2 \end{bmatrix}$ **(iv)** $-D = \begin{bmatrix} 3 & -2 \\ 4 & -5 \end{bmatrix}$

(v) $-E = \begin{bmatrix} -1 & 5 \\ -2 & -3 \end{bmatrix}$

Q5. Find the transpose of each of the following matrices:

A = $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$, **B =** $\begin{bmatrix} 5 & 1 & -6 \end{bmatrix}$, **C =** $\begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$,

D = $\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$, **E =** $\begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$, **F =** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Solution:

Transpose of a matrix is obtained by converting all the columns of that matrix to the rows and all the rows to the columns.

So, according to the definition;

(i) $A^T = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix}$ **(ii)** $B^T = \begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$

(iii) $C^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \end{bmatrix}$ **(iv)** $D^T = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$

(v) $E^T = \begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix}$ **(vi)** $F^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

Q6. Verify that if A = $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, **B =** $\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ **then**

(i) $(A^T)^T = A$ **(ii) $(B^T)^T = B$**

Solution:

(i) To prove: $(A^T)^T = A$

Given $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

Taking transpose of A^T, we will get

$(A^T)^T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = A$

Hence proved: **$(A^T)^T = A$**

(ii) To prove: $(B^T)^T = B$

Given $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

$B^T = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

Taking transpose of B^T, we will get

$(B^T)^T = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = B$

Hence proved: **$(B^T)^T = B$**

Exercise 7.3

1. Find the zeroes of the polynomial in each case.

(i) $f(x) = 2x + 5$

(ii) $f(x) = x - 3$

(iii) $f(x) = x^2$

(iv) $f(x) = x^2 + 5x + 6$

(v) $f(x) = x^2 - 15x + 14$

(vi) $f(x) = x^2 + 3x + 2$

(vii) $f(x) = x^2 - 3x - 10$

(viii) $f(x) = x^2 - 4x + 4$

(ix) $f(x) = x^2 + 4x + 3$

(x) $f(x) = x^2 - 12x + 35$

(xi) $f(x) = x^2 + 7x + 12$

(xii) $f(x) = x^2 - 5x + 6$

(xiii) $f(x) = x^2 + 8x + 15$

(xiv) $f(x) = x^2 - 7x + 10$

(xv) $f(x) = x^2 + 9x + 20$

(xvi) $f(x) = x^2 - 8x + 15$

(xvii) $f(x) = x^2 + 10x + 21$

(xviii) $f(x) = x^2 - 9x + 18$

(xix) $f(x) = x^2 + 11x + 30$

(xx) $f(x) = x^2 - 11x + 30$

(xxi) $f(x) = x^2 + 12x + 35$

(xxii) $f(x) = x^2 - 13x + 36$

(xxiii) $f(x) = x^2 + 14x + 45$

(xxiv) $f(x) = x^2 - 14x + 45$

(xxv) $f(x) = x^2 + 15x + 54$

(xxvi) $f(x) = x^2 - 15x + 54$

(xxvii) $f(x) = x^2 + 16x + 63$

(xxviii) $f(x) = x^2 - 16x + 63$

(xxix) $f(x) = x^2 + 17x + 72$

(xxx) $f(x) = x^2 - 17x + 72$

(xxxi) $f(x) = x^2 + 18x + 81$

(xxxii) $f(x) = x^2 - 18x + 81$

(xxxiii) $f(x) = x^2 + 19x + 90$

(xxxiv) $f(x) = x^2 - 19x + 90$

(xxxv) $f(x) = x^2 + 20x + 99$

(xxxvi) $f(x) = x^2 - 20x + 99$

(xxxvii) $f(x) = x^2 + 21x + 108$

(xxxviii) $f(x) = x^2 - 21x + 108$

(xxxix) $f(x) = x^2 + 22x + 117$

(xxxx) $f(x) = x^2 - 22x + 117$

(xxxxi) $f(x) = x^2 + 23x + 126$

(xxxxii) $f(x) = x^2 - 23x + 126$

(xxxxiii) $f(x) = x^2 + 24x + 135$

(xxxxiv) $f(x) = x^2 - 24x + 135$

(xxxxv) $f(x) = x^2 + 25x + 144$

(xxxxvi) $f(x) = x^2 - 25x + 144$

(xxxxvii) $f(x) = x^2 + 26x + 153$

(xxxxviii) $f(x) = x^2 - 26x + 153$

(xxxxix) $f(x) = x^2 + 27x + 162$

(xxxxx) $f(x) = x^2 - 27x + 162$

(xxxxxi) $f(x) = x^2 + 28x + 171$

(xxxxxii) $f(x) = x^2 - 28x + 171$

(xxxxxiii) $f(x) = x^2 + 29x + 180$

(xxxxxiv) $f(x) = x^2 - 29x + 180$

(xxxxxv) $f(x) = x^2 + 30x + 189$

(xxxxxvi) $f(x) = x^2 - 30x + 189$

(xxxxxvii) $f(x) = x^2 + 31x + 198$

(xxxxxviii) $f(x) = x^2 - 31x + 198$

(xxxxxix) $f(x) = x^2 + 32x + 207$

(xxxxxx) $f(x) = x^2 - 32x + 207$

(xxxxxxi) $f(x) = x^2 + 33x + 216$

(xxxxxxii) $f(x) = x^2 - 33x + 216$

(xxxxxxiii) $f(x) = x^2 + 34x + 225$

(xxxxxxiv) $f(x) = x^2 - 34x + 225$

(xxxxxxv) $f(x) = x^2 + 35x + 234$

(xxxxxxvi) $f(x) = x^2 - 35x + 234$

(xxxxxxvii) $f(x) = x^2 + 36x + 243$

(xxxxxxviii) $f(x) = x^2 - 36x + 243$

(xxxxxxix) $f(x) = x^2 + 37x + 252$

(xxxxxxx) $f(x) = x^2 - 37x + 252$

(xxxxxxxi) $f(x) = x^2 + 38x + 261$

(xxxxxxxii) $f(x) = x^2 - 38x + 261$

(xxxxxxxiii) $f(x) = x^2 + 39x + 270$

(xxxxxxxiv) $f(x) = x^2 - 39x + 270$

(xxxxxxxv) $f(x) = x^2 + 40x + 279$

(xxxxxxxvi) $f(x) = x^2 - 40x + 279$

(xxxxxxxvii) $f(x) = x^2 + 41x + 288$

(xxxxxxxviii) $f(x) = x^2 - 41x + 288$

(xxxxxxxix) $f(x) = x^2 + 42x + 297$

(xxxxxxxx) $f(x) = x^2 - 42x + 297$

(xxxxxxxxi) $f(x) = x^2 + 43x + 306$

(xxxxxxxxii) $f(x) = x^2 - 43x + 306$

(xxxxxxxxiii) $f(x) = x^2 + 44x + 315$

(xxxxxxxxiv) $f(x) = x^2 - 44x + 315$

(xxxxxxxxv) $f(x) = x^2 + 45x + 324$

(xxxxxxxxvi) $f(x) = x^2 - 45x + 324$

(xxxxxxxxvii) $f(x) = x^2 + 46x + 333$

(xxxxxxxxviii) $f(x) = x^2 - 46x + 333$

(xxxxxxxxix) $f(x) = x^2 + 47x + 342$

(xxxxxxxxx) $f(x) = x^2 - 47x + 342$

(xxxxxxxxxi) $f(x) = x^2 + 48x + 351$

(xxxxxxxxxii) $f(x) = x^2 - 48x + 351$

(xxxxxxxxxiii) $f(x) = x^2 + 49x + 360$

(xxxxxxxxxiv) $f(x) = x^2 - 49x + 360$

(xxxxxxxxxv) $f(x) = x^2 + 50x + 369$

(xxxxxxxxxvi) $f(x) = x^2 - 50x + 369$

(xxxxxxxxxvii) $f(x) = x^2 + 51x + 378$

(xxxxxxxxxviii) $f(x) = x^2 - 51x + 378$

(xxxxxxxxxix) $f(x) = x^2 + 52x + 387$

(xxxxxxxxxx) $f(x) = x^2 - 52x + 387$

(xxxxxxxxxxi) $f(x) = x^2 + 53x + 396$

(xxxxxxxxxxii) $f(x) = x^2 - 53x + 396$

(xxxxxxxxxxiii) $f(x) = x^2 + 54x + 405$

(xxxxxxxxxxiv) $f(x) = x^2 - 54x + 405$

(xxxxxxxxxxv) $f(x) = x^2 + 55x + 414$

(xxxxxxxxxxvi) $f(x) = x^2 - 55x + 414$

(xxxxxxxxxxvii) $f(x) = x^2 + 56x + 423$

(xxxxxxxxxxviii) $f(x) = x^2 - 56x + 423$

(xxxxxxxxxxix) $f(x) = x^2 + 57x + 432$

(xxxxxxxxxxx) $f(x) = x^2 - 57x + 432$

(xxxxxxxxxxxi) $f(x) = x^2 + 58x + 441$

(xxxxxxxxxxxii) $f(x) = x^2 - 58x + 441$

(xxxxxxxxxxxiii) $f(x) = x^2 + 59x + 450$

(xxxxxxxxxxxiv) $f(x) = x^2 - 59x + 450$

(xxxxxxxxxxxv) $f(x) = x^2 + 60x + 459$

(xxxxxxxxxxxvi) $f(x) = x^2 - 60x + 459$

(xxxxxxxxxxxvii) $f(x) = x^2 + 61x + 468$

(xxxxxxxxxxxviii) $f(x) = x^2 - 61x + 468$

(xxxxxxxxxxxix) $f(x) = x^2 + 62x + 477$

(xxxxxxxxxxxx) $f(x) = x^2 - 62x + 477$

(xxxxxxxxxxxxi) $f(x) = x^2 + 63x + 486$

(xxxxxxxxxxxxii) $f(x) = x^2 - 63x + 486$

(xxxxxxxxxxxxiii) $f(x) = x^2 + 64x + 495$

(xxxxxxxxxxxxiv) $f(x) = x^2 - 64x + 495$

(xxxxxxxxxxxxv) $f(x) = x^2 + 65x + 504$

(xxxxxxxxxxxxvi) $f(x) = x^2 - 65x + 504$

(xxxxxxxxxxxxvii) $f(x) = x^2 + 66x + 513$

(xxxxxxxxxxxxviii) $f(x) = x^2 - 66x + 513$

(xxxxxxxxxxxxix) $f(x) = x^2 + 67x + 522$

(xxxxxxxxxxxxx) $f(x) = x^2 - 67x + 522$

(xxxxxxxxxxxxxi) $f(x) = x^2 + 68x + 531$

(xxxxxxxxxxxxxii) $f(x) = x^2 - 68x + 531$

(xxxxxxxxxxxxxiii) $f(x) = x^2 + 69x + 540$

(xxxxxxxxxxxxxiv) $f(x) = x^2 - 69x + 540$

(xxxxxxxxxxxxxv) $f(x) = x^2 + 70x + 549$

(xxxxxxxxxxxxxvi) $f(x) = x^2 - 70x + 549$

(xxxxxxxxxxxxxvii) $f(x) = x^2 + 71x + 558$

(xxxxxxxxxxxxxviii) $f(x) = x^2 - 71x + 558$

(xxxxxxxxxxxxxix) $f(x) = x^2 + 72x + 567$

(xxxxxxxxxxxxxx) $f(x) = x^2 - 72x + 567$

(xxxxxxxxxxxxxxi) $f(x) = x^2 + 73x + 576$

(xxxxxxxxxxxxxxii) $f(x) = x^2 - 73x + 576$

(xxxxxxxxxxxxxxiii) $f(x) = x^2 + 74x + 585$

(xxxxxxxxxxxxxxiv) $f(x) = x^2 - 74x + 585$

(xxxxxxxxxxxxxxv) $f(x) = x^2 + 75x + 594$

(xxxxxxxxxxxxxxvi) $f(x) = x^2 - 75x + 594$

(xxxxxxxxxxxxxxvii) $f(x) = x^2 + 76x + 603$

(xxxxxxxxxxxxxxviii) $f(x) = x^2 - 76x + 603$

(xxxxxxxxxxxxxxix) $f(x) = x^2 + 77x + 612$

(xxxxxxxxxxxxxxx) $f(x) = x^2 - 77x + 612$

(xxxxxxxxxxxxxxxi) $f(x) = x^2 + 78x + 621$

(xxxxxxxxxxxxxxxii) $f(x) = x^2 - 78x + 621$

(xxxxxxxxxxxxxxxiii) $f(x) = x^2 + 79x + 630$

(xxxxxxxxxxxxxxxiv) $f(x) = x^2 - 79x + 630$

(xxxxxxxxxxxxxxxv) $f(x) = x^2 + 80x + 639$

(xxxxxxxxxxxxxxxvi) $f(x) = x^2 - 80x + 639$

(xxxxxxxxxxxxxxxvii) $f(x) = x^2 + 81x + 648$

(xxxxxxxxxxxxxxxviii) $f(x) = x^2 - 81x + 648$

(xxxxxxxxxxxxxxxix) $f(x) = x^2 + 82x + 657$

(xxxxxxxxxxxxxxxx) $f(x) = x^2 - 82x + 657$

(xxxxxxxxxxxxxxxxi) $f(x) = x^2 + 83x + 666$

(xxxxxxxxxxxxxxxxii) $f(x) = x^2 - 83x + 666$

(xxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 84x + 675$

(xxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 84x + 675$

(xxxxxxxxxxxxxxxxv) $f(x) = x^2 + 85x + 684$

(xxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 85x + 684$

(xxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 86x + 693$

(xxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 86x + 693$

(xxxxxxxxxxxxxxxxix) $f(x) = x^2 + 87x + 702$

(xxxxxxxxxxxxxxxxx) $f(x) = x^2 - 87x + 702$

(xxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 88x + 711$

(xxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 88x + 711$

(xxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 89x + 720$

(xxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 89x + 720$

(xxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 90x + 729$

(xxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 90x + 729$

(xxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 91x + 738$

(xxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 91x + 738$

(xxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 92x + 747$

(xxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 92x + 747$

(xxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 93x + 756$

(xxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 93x + 756$

(xxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 94x + 765$

(xxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 94x + 765$

(xxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 95x + 774$

(xxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 95x + 774$

(xxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 96x + 783$

(xxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 96x + 783$

(xxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 97x + 792$

(xxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 97x + 792$

(xxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 98x + 801$

(xxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 98x + 801$

(xxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 99x + 810$

(xxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 99x + 810$

(xxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 100x + 819$

(xxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 100x + 819$

(xxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 101x + 828$

(xxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 101x + 828$

(xxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 102x + 837$

(xxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 102x + 837$

(xxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 103x + 846$

(xxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 103x + 846$

(xxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 104x + 855$

(xxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 104x + 855$

(xxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 105x + 864$

(xxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 105x + 864$

(xxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 106x + 873$

(xxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 106x + 873$

(xxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 107x + 882$

(xxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 107x + 882$

(xxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 108x + 891$

(xxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 108x + 891$

(xxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 109x + 900$

(xxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 109x + 900$

(xxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 110x + 909$

(xxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 110x + 909$

(xxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 111x + 918$

(xxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 111x + 918$

(xxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 112x + 927$

(xxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 112x + 927$

(xxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 113x + 936$

(xxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 113x + 936$

(xxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 114x + 945$

(xxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 114x + 945$

(xxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 115x + 954$

(xxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 115x + 954$

(xxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 116x + 963$

(xxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 116x + 963$

(xxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 117x + 972$

(xxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 117x + 972$

(xxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 118x + 981$

(xxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 118x + 981$

(xxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 119x + 990$

(xxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 119x + 990$

(xxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 120x + 999$

(xxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 120x + 999$

(xxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 121x + 1008$

(xxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 121x + 1008$

(xxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 122x + 1017$

(xxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 122x + 1017$

(xxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 123x + 1026$

(xxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 123x + 1026$

(xxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 124x + 1035$

(xxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 124x + 1035$

(xxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 125x + 1044$

(xxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 125x + 1044$

(xxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 126x + 1053$

(xxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 126x + 1053$

(xxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 127x + 1062$

(xxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 127x + 1062$

(xxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 128x + 1071$

(xxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 128x + 1071$

(xxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 129x + 1080$

(xxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 129x + 1080$

(xxxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 130x + 1089$

(xxxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 130x + 1089$

(xxxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 131x + 1098$

(xxxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 131x + 1098$

(xxxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 132x + 1107$

(xxxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 132x + 1107$

(xxxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 133x + 1116$

(xxxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 133x + 1116$

(xxxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 134x + 1125$

(xxxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 134x + 1125$

(xxxxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 135x + 1134$

(xxxxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 135x + 1134$

(xxxxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 136x + 1143$

(xxxxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 136x + 1143$

(xxxxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 137x + 1152$

(xxxxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 137x + 1152$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 138x + 1161$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 138x + 1161$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 139x + 1170$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 139x + 1170$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 140x + 1179$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 140x + 1179$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 141x + 1188$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 141x + 1188$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 142x + 1197$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 142x + 1197$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 143x + 1206$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 143x + 1206$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 144x + 1215$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 144x + 1215$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 145x + 1224$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 145x + 1224$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 146x + 1233$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 146x + 1233$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 147x + 1242$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 147x + 1242$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 148x + 1251$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 148x + 1251$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 149x + 1260$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 149x + 1260$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 150x + 1269$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 150x + 1269$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 151x + 1278$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 151x + 1278$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 152x + 1287$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 152x + 1287$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 153x + 1296$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 153x + 1296$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 154x + 1305$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 154x + 1305$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 155x + 1314$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 155x + 1314$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 156x + 1323$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 156x + 1323$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 157x + 1332$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 157x + 1332$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 158x + 1341$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 158x + 1341$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 159x + 1350$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 159x + 1350$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 160x + 1359$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 160x + 1359$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 161x + 1368$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 161x + 1368$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 162x + 1377$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 162x + 1377$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 163x + 1386$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 163x + 1386$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 164x + 1395$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 164x + 1395$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 165x + 1404$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 165x + 1404$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 166x + 1413$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 166x + 1413$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 167x + 1422$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 167x + 1422$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 168x + 1431$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 168x + 1431$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 169x + 1440$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 169x + 1440$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 170x + 1449$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 170x + 1449$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 171x + 1458$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 171x + 1458$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 172x + 1467$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 172x + 1467$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 173x + 1476$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 173x + 1476$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 174x + 1485$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 174x + 1485$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 175x + 1494$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 175x + 1494$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 176x + 1503$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 176x + 1503$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 177x + 1512$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 177x + 1512$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 178x + 1521$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 178x + 1521$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 179x + 1530$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 179x + 1530$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 180x + 1539$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 180x + 1539$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 181x + 1548$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 181x + 1548$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 182x + 1557$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 182x + 1557$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 183x + 1566$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 183x + 1566$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 184x + 1575$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 184x + 1575$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 185x + 1584$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 185x + 1584$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 186x + 1593$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 186x + 1593$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 187x + 1602$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 187x + 1602$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 188x + 1611$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 188x + 1611$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 189x + 1620$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 189x + 1620$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxv) $f(x) = x^2 + 190x + 1629$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvi) $f(x) = x^2 - 190x + 1629$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxvii) $f(x) = x^2 + 191x + 1638$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxviii) $f(x) = x^2 - 191x + 1638$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxix) $f(x) = x^2 + 192x + 1647$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx) $f(x) = x^2 - 192x + 1647$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxi) $f(x) = x^2 + 193x + 1656$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxii) $f(x) = x^2 - 193x + 1656$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiii) $f(x) = x^2 + 194x + 1665$

(xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxiv) $f(x) = x^2 - 194x + 1665$

(xxxxx

Exercise 1.4

Q1. Which of the following product of matrices is conformable for multiplication?

(A) $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ **(B)** $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$ **(D)** $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

Solution:

Only matrices are conformable for multiplication if the number of columns of first matrix are equal to number of rows of second matrix.

So, according to this definition

(A) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows)

(B) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows)

(C) is not conformable for multiplication (because the first matrix has just one column and second matrix has two rows)

(D) is conformable for multiplication (because the first matrix has just two columns and second matrix has also same number of rows)

Q2. is conformable for multiplication (because the first matrix has three columns and second matrix has same number of rows)

(A) $A \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Hence (B) - (D) are (Impossible)

Solution:

(A) - **(B)**

- $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 1 + 0 \times 0 & 1 \times 0 + 0 \times 1 \\ 0 \times 1 + 1 \times 0 & 0 \times 0 + 1 \times 1 \end{bmatrix}$
- $\begin{bmatrix} 1 + 0 & 0 + 0 \\ 0 + 0 & 0 + 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(A) - **(C)**

- $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 1 + 0 \times 0 & 1 \times 0 + 0 \times 1 \\ 0 \times 1 + 1 \times 0 & 0 \times 0 + 1 \times 1 \end{bmatrix}$
- $\begin{bmatrix} 1 + 0 & 0 + 0 \\ 0 + 0 & 0 + 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q3. Find the following products

(A) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

Solution:

(A) - $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

- $\begin{bmatrix} 2 \times 1 + 3 \times 2 \\ 4 \times 1 + 5 \times 2 \end{bmatrix}$
- $\begin{bmatrix} 2 + 6 \\ 4 + 10 \end{bmatrix}$
- $\begin{bmatrix} 8 \\ 14 \end{bmatrix}$

(B) - $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

- $\begin{bmatrix} 2 \times 1 + 3 \times 2 \\ 4 \times 1 + 5 \times 2 \end{bmatrix}$
- $\begin{bmatrix} 2 + 6 \\ 4 + 10 \end{bmatrix}$
- $\begin{bmatrix} 8 \\ 14 \end{bmatrix}$

(C) - $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

- $\begin{bmatrix} 2 \times 1 + 3 \times 2 \\ 4 \times 1 + 5 \times 2 \end{bmatrix}$
- $\begin{bmatrix} 2 + 6 \\ 4 + 10 \end{bmatrix}$
- $\begin{bmatrix} 8 \\ 14 \end{bmatrix}$

(D) - $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

- $\begin{bmatrix} 2 \times 1 + 3 \times 2 \\ 4 \times 1 + 5 \times 2 \end{bmatrix}$
- $\begin{bmatrix} 2 + 6 \\ 4 + 10 \end{bmatrix}$
- $\begin{bmatrix} 8 \\ 14 \end{bmatrix}$

(A) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$
- $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$
- $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

- $\begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$
- $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$
- $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

- $\begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$
- $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$
- $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$

Q4. Verify the following identity

(A) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

Solution:

(A) - $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

- $\begin{bmatrix} 2 \times 1 + 3 \times 2 & 2 \times 2 + 3 \times 4 \\ 4 \times 1 + 5 \times 2 & 4 \times 2 + 5 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 2 + 6 & 4 + 12 \\ 4 + 10 & 8 + 20 \end{bmatrix}$
- $\begin{bmatrix} 8 & 16 \\ 14 & 28 \end{bmatrix}$
- $\begin{bmatrix} 8 & 16 \\ 14 & 28 \end{bmatrix}$

(B) - $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

- $\begin{bmatrix} 2 \times 1 + 3 \times 2 & 2 \times 2 + 3 \times 4 \\ 4 \times 1 + 5 \times 2 & 4 \times 2 + 5 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 2 + 6 & 4 + 12 \\ 4 + 10 & 8 + 20 \end{bmatrix}$
- $\begin{bmatrix} 8 & 16 \\ 14 & 28 \end{bmatrix}$
- $\begin{bmatrix} 8 & 16 \\ 14 & 28 \end{bmatrix}$

(C) - $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

- $\begin{bmatrix} 2 \times 1 + 3 \times 2 & 2 \times 2 + 3 \times 4 \\ 4 \times 1 + 5 \times 2 & 4 \times 2 + 5 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 2 + 6 & 4 + 12 \\ 4 + 10 & 8 + 20 \end{bmatrix}$
- $\begin{bmatrix} 8 & 16 \\ 14 & 28 \end{bmatrix}$
- $\begin{bmatrix} 8 & 16 \\ 14 & 28 \end{bmatrix}$

(D) - $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

- $\begin{bmatrix} 2 \times 1 + 3 \times 2 & 2 \times 2 + 3 \times 4 \\ 4 \times 1 + 5 \times 2 & 4 \times 2 + 5 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 2 + 6 & 4 + 12 \\ 4 + 10 & 8 + 20 \end{bmatrix}$
- $\begin{bmatrix} 8 & 16 \\ 14 & 28 \end{bmatrix}$
- $\begin{bmatrix} 8 & 16 \\ 14 & 28 \end{bmatrix}$

Q5. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Hence verify

(A) $AB = BA$ **(B)** $A(BC) = (AB)C$

(C) $A(B+C) = AB + AC$ **(D)** $(A+B)C = AC + BC$

Solution:

(A) - **AB = BA**

LHS = AB

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 2 + 2 \times 4 & 1 \times 3 + 2 \times 5 \\ 3 \times 2 + 4 \times 4 & 3 \times 3 + 4 \times 5 \end{bmatrix}$
- $\begin{bmatrix} 2 + 8 & 3 + 10 \\ 6 + 16 & 9 + 20 \end{bmatrix}$
- $\begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix}$

RHS = BA

- $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 2 \times 1 + 3 \times 3 & 2 \times 2 + 3 \times 4 \\ 4 \times 1 + 5 \times 3 & 4 \times 2 + 5 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 2 + 9 & 4 + 12 \\ 4 + 15 & 8 + 20 \end{bmatrix}$
- $\begin{bmatrix} 11 & 16 \\ 19 & 28 \end{bmatrix}$

$\neq$ ——— (i)

RHS = AB

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 2 + 2 \times 4 & 1 \times 3 + 2 \times 5 \\ 3 \times 2 + 4 \times 4 & 3 \times 3 + 4 \times 5 \end{bmatrix}$
- $\begin{bmatrix} 2 + 8 & 3 + 10 \\ 6 + 16 & 9 + 20 \end{bmatrix}$
- $\begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix}$

(B) $A(BC) = (AB)C$

LHS = $A(BC)$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 \times 1 + 3 \times 3 & 2 \times 2 + 3 \times 4 \\ 4 \times 1 + 5 \times 3 & 4 \times 2 + 5 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 + 9 & 4 + 12 \\ 4 + 15 & 8 + 20 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 11 & 16 \\ 19 & 28 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 11 + 2 \times 19 & 1 \times 16 + 2 \times 28 \\ 3 \times 11 + 4 \times 19 & 3 \times 16 + 4 \times 28 \end{bmatrix}$
- $\begin{bmatrix} 11 + 38 & 16 + 56 \\ 33 + 76 & 48 + 112 \end{bmatrix}$
- $\begin{bmatrix} 49 & 72 \\ 109 & 160 \end{bmatrix}$ ——— (i)

RHS = $(AB)C$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 \times 1 + 3 \times 3 & 2 \times 2 + 3 \times 4 \\ 4 \times 1 + 5 \times 3 & 4 \times 2 + 5 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 + 9 & 4 + 12 \\ 4 + 15 & 8 + 20 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 11 & 16 \\ 19 & 28 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 11 + 2 \times 19 & 1 \times 16 + 2 \times 28 \\ 3 \times 11 + 4 \times 19 & 3 \times 16 + 4 \times 28 \end{bmatrix}$
- $\begin{bmatrix} 11 + 38 & 16 + 56 \\ 33 + 76 & 48 + 112 \end{bmatrix}$
- $\begin{bmatrix} 49 & 72 \\ 109 & 160 \end{bmatrix}$ ——— (ii)

From (i) and (ii), it is clear that LHS = RHS

A.(BC) = (AB)C

(C) $A(B+C) = AB + AC$

LHS = $A \cdot (B+C)$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \right)$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 + 1 & 3 + 2 \\ 4 + 3 & 5 + 4 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 3 + 2 \times 7 & 1 \times 5 + 2 \times 9 \\ 3 \times 3 + 4 \times 7 & 3 \times 5 + 4 \times 9 \end{bmatrix}$
- $\begin{bmatrix} 3 + 14 & 5 + 18 \\ 9 + 28 & 15 + 36 \end{bmatrix}$
- $\begin{bmatrix} 17 & 23 \\ 37 & 51 \end{bmatrix}$ ——— (i)

RHS = $AB + AC$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 2 + 2 \times 4 & 1 \times 3 + 2 \times 5 \\ 3 \times 2 + 4 \times 4 & 3 \times 3 + 4 \times 5 \end{bmatrix} + \begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 2 + 8 & 3 + 10 \\ 6 + 16 & 9 + 20 \end{bmatrix} + \begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$
- $\begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix} + \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$
- $\begin{bmatrix} 10 + 7 & 13 + 10 \\ 22 + 15 & 29 + 22 \end{bmatrix}$
- $\begin{bmatrix} 17 & 23 \\ 37 & 51 \end{bmatrix}$ ——— (ii)

From (i) and (ii), it is clear that LHS = RHS

A.(B+C) = AB + AC

(D) $(A+B)C = AC + BC$

LHS = $(A+B) \cdot C$

- $\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \right) \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$
- $\begin{bmatrix} 1 + 1 & 2 + 2 \\ 3 + 3 & 4 + 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$
- $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$
- $\begin{bmatrix} 2 \times 2 + 4 \times 4 & 2 \times 3 + 4 \times 5 \\ 6 \times 2 + 8 \times 4 & 6 \times 3 + 8 \times 5 \end{bmatrix}$
- $\begin{bmatrix} 4 + 16 & 6 + 20 \\ 12 + 32 & 18 + 40 \end{bmatrix}$
- $\begin{bmatrix} 20 & 26 \\ 44 & 58 \end{bmatrix}$ ——— (i)

RHS = $AC + BC$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 2 + 2 \times 4 & 1 \times 3 + 2 \times 5 \\ 3 \times 2 + 4 \times 4 & 3 \times 3 + 4 \times 5 \end{bmatrix} + \begin{bmatrix} 1 \times 2 + 2 \times 4 & 1 \times 3 + 2 \times 5 \\ 3 \times 2 + 4 \times 4 & 3 \times 3 + 4 \times 5 \end{bmatrix}$
- $\begin{bmatrix} 2 + 8 & 3 + 10 \\ 6 + 16 & 9 + 20 \end{bmatrix} + \begin{bmatrix} 2 + 8 & 3 + 10 \\ 6 + 16 & 9 + 20 \end{bmatrix}$
- $\begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix} + \begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix}$
- $\begin{bmatrix} 10 + 10 & 13 + 13 \\ 22 + 22 & 29 + 29 \end{bmatrix}$
- $\begin{bmatrix} 20 & 26 \\ 44 & 58 \end{bmatrix}$ ——— (ii)

From (i) and (ii), it is clear that LHS = RHS

(A+B)C = AC + BC

Q6. A.B = C,(A+B)C = AC

LHS = $A \cdot (B+C)$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \right)$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 + 1 & 3 + 2 \\ 4 + 3 & 5 + 4 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 3 + 2 \times 7 & 1 \times 5 + 2 \times 9 \\ 3 \times 3 + 4 \times 7 & 3 \times 5 + 4 \times 9 \end{bmatrix}$
- $\begin{bmatrix} 3 + 14 & 5 + 18 \\ 9 + 28 & 15 + 36 \end{bmatrix}$
- $\begin{bmatrix} 17 & 23 \\ 37 & 51 \end{bmatrix}$ ——— (i)

RHS = $AB + AC$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 2 + 2 \times 4 & 1 \times 3 + 2 \times 5 \\ 3 \times 2 + 4 \times 4 & 3 \times 3 + 4 \times 5 \end{bmatrix} + \begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 2 + 8 & 3 + 10 \\ 6 + 16 & 9 + 20 \end{bmatrix} + \begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$
- $\begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix} + \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$
- $\begin{bmatrix} 10 + 7 & 13 + 10 \\ 22 + 15 & 29 + 22 \end{bmatrix}$
- $\begin{bmatrix} 17 & 23 \\ 37 & 51 \end{bmatrix}$ ——— (ii)

From (i) and (ii), it is clear that LHS = RHS

A.(B+C) = AB + AC

Q7. A.B = C,(A+B)C = AC

LHS = $A \cdot (B+C)$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \right)$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 + 1 & 3 + 2 \\ 4 + 3 & 5 + 4 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 3 + 2 \times 7 & 1 \times 5 + 2 \times 9 \\ 3 \times 3 + 4 \times 7 & 3 \times 5 + 4 \times 9 \end{bmatrix}$
- $\begin{bmatrix} 3 + 14 & 5 + 18 \\ 9 + 28 & 15 + 36 \end{bmatrix}$
- $\begin{bmatrix} 17 & 23 \\ 37 & 51 \end{bmatrix}$ ——— (i)

RHS = $AB + AC$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 2 + 2 \times 4 & 1 \times 3 + 2 \times 5 \\ 3 \times 2 + 4 \times 4 & 3 \times 3 + 4 \times 5 \end{bmatrix} + \begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 2 + 8 & 3 + 10 \\ 6 + 16 & 9 + 20 \end{bmatrix} + \begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$
- $\begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix} + \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$
- $\begin{bmatrix} 10 + 7 & 13 + 10 \\ 22 + 15 & 29 + 22 \end{bmatrix}$
- $\begin{bmatrix} 17 & 23 \\ 37 & 51 \end{bmatrix}$ ——— (ii)

From (i) and (ii), it is clear that LHS = RHS

A.(B+C) = AB + AC

Q8. Verify the following

A $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **B** $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ **C** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(A) $(AB)C = A(BC)$ **(B)** $A(B+C) = AB + AC$

(C) $(A+B)C = AC + BC$ **(D)** $A(B-C) = AB - AC$

Solution:

(A) - **(AB)C = A(BC)**

LHS = $(AB)C$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 2 + 2 \times 4 & 1 \times 3 + 2 \times 5 \\ 3 \times 2 + 4 \times 4 & 3 \times 3 + 4 \times 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 2 + 8 & 3 + 10 \\ 6 + 16 & 9 + 20 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 10 \times 1 + 13 \times 3 & 10 \times 2 + 13 \times 4 \\ 22 \times 1 + 29 \times 3 & 22 \times 2 + 29 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 10 + 39 & 20 + 52 \\ 22 + 87 & 44 + 116 \end{bmatrix}$
- $\begin{bmatrix} 49 & 72 \\ 109 & 160 \end{bmatrix}$ ——— (i)

RHS = $A(BC)$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 \times 1 + 3 \times 3 & 2 \times 2 + 3 \times 4 \\ 4 \times 1 + 5 \times 3 & 4 \times 2 + 5 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 + 9 & 4 + 12 \\ 4 + 15 & 8 + 20 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 11 & 16 \\ 19 & 28 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 11 + 2 \times 19 & 1 \times 16 + 2 \times 28 \\ 3 \times 11 + 4 \times 19 & 3 \times 16 + 4 \times 28 \end{bmatrix}$
- $\begin{bmatrix} 11 + 38 & 16 + 56 \\ 33 + 76 & 48 + 112 \end{bmatrix}$
- $\begin{bmatrix} 49 & 72 \\ 109 & 160 \end{bmatrix}$ ——— (ii)

From (i) and (ii), it is clear that LHS = RHS

(AB)C = A(BC)

(B) $A(B+C) = AB + AC$

LHS = $A \cdot (B+C)$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \right)$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 + 1 & 3 + 2 \\ 4 + 3 & 5 + 4 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 3 + 2 \times 7 & 1 \times 5 + 2 \times 9 \\ 3 \times 3 + 4 \times 7 & 3 \times 5 + 4 \times 9 \end{bmatrix}$
- $\begin{bmatrix} 3 + 14 & 5 + 18 \\ 9 + 28 & 15 + 36 \end{bmatrix}$
- $\begin{bmatrix} 17 & 23 \\ 37 & 51 \end{bmatrix}$ ——— (i)

RHS = $AB + AC$

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 1 \times 2 + 2 \times 4 & 1 \times 3 + 2 \times 5 \\ 3 \times 2 + 4 \times 4 & 3 \times 3 + 4 \times 5 \end{bmatrix} + \begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$
- $\begin{bmatrix} 2 + 8 & 3 + 10 \\ 6 + 16 & 9 + 20 \end{bmatrix} + \begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$
- $\begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix} + \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$
- $\begin{bmatrix} 10 + 7 & 13 + 10 \\ 22 + 15 & 29 + 22 \end{bmatrix}$
- $\begin{bmatrix} 17 & 23 \\ 37 & 51 \end{bmatrix}$ ——— (ii)

From (i) and (ii), it is clear that LHS = RHS

A.(B+C) = AB + AC

(C) $(A+B)C = AC + BC$

LHS = $(A+B) \cdot C$

- $\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \right) \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 1 + 1 & 2 + 2 \\ 3 + 3 & 4 + 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 2 \times 1 + 4 \times 3 & 2 \times 2 + 4 \times 4 \\ 6 \times 1 + 8 \times 3 & 6 \times 2

Exercise 1.5

Q1. Find the additive inverse of the following matrices

(i) $\begin{bmatrix} 2 & -3 \\ 5 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -2 \\ -4 & 5 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 5 & 0 \\ 2 & 3 & 4 \end{bmatrix}$ (iv) $\begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$

Solution

(i) $\begin{bmatrix} -2 & 3 \\ -5 & 0 \end{bmatrix}$

Additive inverse of the matrix is its negative.

$-\begin{bmatrix} 2 & -3 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ -5 & 0 \end{bmatrix}$

(ii) $\begin{bmatrix} -1 & 2 \\ 4 & -5 \end{bmatrix}$

Additive inverse of the matrix is its negative.

$-\begin{bmatrix} 1 & -2 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 4 & -5 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & -5 & 0 \\ -2 & -3 & -4 \end{bmatrix}$

Additive inverse of the matrix is its negative.

$-\begin{bmatrix} -1 & 5 & 0 \\ 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -5 & 0 \\ -2 & -3 & -4 \end{bmatrix}$

(iv) $\begin{bmatrix} -2 & -3 \\ -x & -y \end{bmatrix}$

Additive inverse of the matrix is its negative.

$-\begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ -x & -y \end{bmatrix}$

Q2. Find the value of the following matrices are diagonal or not or upper triangular

(i) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Solution

A matrix is said to be diagonal if its determinant is equal to zero, i.e., $|A| = 0$

(i) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

Determinant of matrix is equal to zero.

$|A| = 2 \times 3 - 0 \times 0$

$|A| = 6 - 0$

$|A| = 6$

Since determinant of matrix is not equal to zero, it is not a diagonal matrix.

(ii) $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

Determinant of matrix is equal to zero.

$|A| = 1 \times 3 - 2 \times 0$

$|A| = 3 - 0$

$|A| = 3$

Since determinant of matrix is not equal to zero, it is not a diagonal matrix.

(iii) $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$

Determinant of matrix is equal to zero.

$|A| = 1 \times 3 - 0 \times 2$

$|A| = 3 - 0$

$|A| = 3$

Since determinant of matrix is not equal to zero, it is not a diagonal matrix.

(iv) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Determinant of matrix is equal to zero.

$|A| = 1(2 \times 3 - 0 \times 0) - 0(0 \times 3 - 0 \times 0) + 0(0 \times 0 - 2 \times 0)$

$|A| = 1(6 - 0) - 0(0 - 0) + 0(0 - 0)$

$|A| = 6$

Since determinant of matrix is not equal to zero, it is not a diagonal matrix.

Q3. Find the multiplicative inverse of matrices of

(i) $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution

(i) $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

Find the multiplicative inverse of matrix A is calculated as

$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$

$|A| = 2 \times 5 - 3 \times 4$

$|A| = 10 - 12$

$|A| = -2$ $\quad$ $(\because |A| \neq 0)$ $\quad$ $(\because A^{-1} \text{ exists})$

Now find the cofactor of each element of matrix A is possible

$A^{-1} = \frac{1}{-2} \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -\frac{5}{2} & \frac{3}{2} \\ 2 & -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Find the multiplicative inverse of matrix A is calculated as

$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

$|A| = 1 \times 4 - 2 \times 3$

$|A| = 4 - 6$

$|A| = -2$ $\quad$ $(\because |A| \neq 0)$ $\quad$ $(\because A^{-1} \text{ exists})$

Now find the cofactor of each element of matrix A is possible

$A^{-1} = \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Find the multiplicative inverse of matrix A is calculated as

$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$|A| = 1 \times 1 - 0 \times 0$

$|A| = 1 - 0$

$|A| = 1$ $\quad$ $(\because |A| \neq 0)$ $\quad$ $(\because A^{-1} \text{ exists})$

Now find the cofactor of each element of matrix A is possible

$A^{-1} = \frac{1}{1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Find the multiplicative inverse of matrix A is calculated as

$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$

$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$|A| = 1(1 \times 1 - 0 \times 0) - 0(0 \times 1 - 0 \times 0) + 0(0 \times 0 - 1 \times 0)$

$|A| = 1(1 - 0) - 0(0 - 0) + 0(0 - 0)$

$|A| = 1$ $\quad$ $(\because |A| \neq 0)$ $\quad$ $(\because A^{-1} \text{ exists})$

Now find the cofactor of each element of matrix A is possible

$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q4. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **, find**

(i) $A^2 - 5A + 2I$ **(ii)** $A^2 + 3A - 7I$

Solution

(i) $A^2 - 5A + 2I$

$A^2 = A \times A$

$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$ $\quad$ ——— (1)

$-5A = -5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$-5A = \begin{bmatrix} -5 \times 1 & -5 \times 2 \\ -5 \times 3 & -5 \times 4 \end{bmatrix}$

$-5A = \begin{bmatrix} -5 & -10 \\ -15 & -20 \end{bmatrix}$ $\quad$ ——— (2)

$2I = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$2I = \begin{bmatrix} 2 \times 1 & 2 \times 0 \\ 2 \times 0 & 2 \times 1 \end{bmatrix}$

$2I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $\quad$ ——— (3)

From (1), (2) and (3), we obtain

$A^2 - 5A + 2I = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} + \begin{bmatrix} -5 & -10 \\ -15 & -20 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$A^2 - 5A + 2I = \begin{bmatrix} 7 - 5 + 2 & 10 - 10 + 0 \\ 15 - 15 + 0 & 22 - 20 + 2 \end{bmatrix}$

$A^2 - 5A + 2I = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

(ii) $A^2 + 3A - 7I$

$A^2 = A \times A$

$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$ $\quad$ ——— (1)

$3A = 3 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$3A = \begin{bmatrix} 3 \times 1 & 3 \times 2 \\ 3 \times 3 & 3 \times 4 \end{bmatrix}$

$3A = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$ $\quad$ ——— (2)

$7I = 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$7I = \begin{bmatrix} 7 \times 1 & 7 \times 0 \\ 7 \times 0 & 7 \times 1 \end{bmatrix}$

$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ $\quad$ ——— (3)

From (1), (2) and (3), we obtain

$A^2 + 3A - 7I = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$A^2 + 3A - 7I = \begin{bmatrix} 7 + 3 - 7 & 10 + 6 - 0 \\ 15 + 9 - 0 & 22 + 12 - 7 \end{bmatrix}$

$A^2 + 3A - 7I = \begin{bmatrix} 3 & 16 \\ 24 & 27 \end{bmatrix}$

Q5. Find the value of x and y if

(i) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & x \\ y & 1 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & x \\ y & 1 \end{bmatrix} = \begin{bmatrix} 9 & 13 \\ 17 & 21 \end{bmatrix}$

Solution

(i) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & x \\ y & 1 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 \times 1 + 5 \times y & 2 \times x + 5 \times 1 \\ 1 \times 1 + 3 \times y & 1 \times x + 3 \times 1 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 + 5y & 2x + 5 \\ 1 + 3y & x + 3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}$

$2 + 5y = 4$ $\quad$ ——— (1)

$2x + 5 = 7$ $\quad$ ——— (2)

$1 + 3y = 1$ $\quad$ ——— (3)

$x + 3 = 2$ $\quad$ ——— (4)

From (3)

$1 + 3y = 1$

$3y = 1 - 1$

$3y = 0$

$y = \frac{0}{3}$

$y = 0$

From (4)

$x + 3 = 2$

$x = 2 - 3$

$x = -1$

(ii) $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & x \\ y & 1 \end{bmatrix} = \begin{bmatrix} 9 & 13 \\ 17 & 21 \end{bmatrix}$

$\begin{bmatrix} 2 \times 1 + 3 \times y & 2 \times x + 3 \times 1 \\ 4 \times 1 + 5 \times y & 4 \times x + 5 \times 1 \end{bmatrix} = \begin{bmatrix} 9 & 13 \\ 17 & 21 \end{bmatrix}$

$\begin{bmatrix} 2 + 3y & 2x + 3 \\ 4 + 5y & 4x + 5 \end{bmatrix} = \begin{bmatrix} 9 & 13 \\ 17 & 21 \end{bmatrix}$

$2 + 3y = 9$ $\quad$ ——— (1)

$2x + 3 = 13$ $\quad$ ——— (2)

$4 + 5y = 17$ $\quad$ ——— (3)

$4x + 5 = 21$ $\quad$ ——— (4)

From (1)

$2 + 3y = 9$

$3y = 9 - 2$

$3y = 7$

$y = \frac{7}{3}$

From (4)

$4x + 5 = 21$

$4x = 21 - 5$

$4x = 16$

$x = \frac{16}{4}$

$x = 4$

Q6. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **, then show that** $A^2 - 5A - 2I = 0$

Solution

$A^2 - 5A - 2I = 0$

$A^2 = A \times A$

$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$ $\quad$ ——— (1)

$-5A = -5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$-5A = \begin{bmatrix} -5 \times 1 & -5 \times 2 \\ -5 \times 3 & -5 \times 4 \end{bmatrix}$

$-5A = \begin{bmatrix} -5 & -10 \\ -15 & -20 \end{bmatrix}$ $\quad$ ——— (2)

$-2I = -2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$-2I = \begin{bmatrix} -2 \times 1 & -2 \times 0 \\ -2 \times 0 & -2 \times 1 \end{bmatrix}$

$-2I = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ $\quad$ ——— (3)

From (1), (2) and (3), we obtain

$A^2 - 5A - 2I = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} + \begin{bmatrix} -5 & -10 \\ -15 & -20 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

$A^2 - 5A - 2I = \begin{bmatrix} 7 - 5 - 2 & 10 - 10 + 0 \\ 15 - 15 + 0 & 22 - 20 - 2 \end{bmatrix}$

$A^2 - 5A - 2I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^2 - 5A - 2I = 0$

Hence Proved

Q7. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **, then show that** $A^2 - 5A + 6I = 0$

Solution

$A^2 - 5A + 6I = 0$

$A^2 = A \times A$

$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$ $\quad$ ——— (1)

$-5A = -5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$-5A = \begin{bmatrix} -5 \times 1 & -5 \times 2 \\ -5 \times 3 & -5 \times 4 \end{bmatrix}$

$-5A = \begin{bmatrix} -5 & -10 \\ -15 & -20 \end{bmatrix}$ $\quad$ ——— (2)

$6I = 6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$6I = \begin{bmatrix} 6 \times 1 & 6 \times 0 \\ 6 \times 0 & 6 \times 1 \end{bmatrix}$

$6I = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$ $\quad$ ——— (3)

From (1), (2) and (3), we obtain

$A^2 - 5A + 6I = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} + \begin{bmatrix} -5 & -10 \\ -15 & -20 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

$A^2 - 5A + 6I = \begin{bmatrix} 7 - 5 + 6 & 10 - 10 + 0 \\ 15 - 15 + 0 & 22 - 20 + 6 \end{bmatrix}$

$A^2 - 5A + 6I = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$A^2 - 5A + 6I = 0$

Hence Proved

Q8. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **, then show that** $A^2 - 5A + 4I = 0$

Solution

$A^2 - 5A + 4I = 0$

$A^2 = A \times A$

$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$ $\quad$ ——— (1)

$-5A = -5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$-5A = \begin{bmatrix} -5 \times 1 & -5 \times 2 \\ -5 \times 3 & -5 \times 4 \end{bmatrix}$

$-5A = \begin{bmatrix} -5 & -10 \\ -15 & -20 \end{bmatrix}$ $\quad$ ——— (2)

$4I = 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$4I = \begin{bmatrix} 4 \times 1 & 4 \times 0 \\ 4 \times 0 & 4 \times 1 \end{bmatrix}$

$4I = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ $\quad$ ——— (3)

From (1), (2) and (3), we obtain

$A^2 - 5A + 4I = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} + \begin{bmatrix} -5 & -10 \\ -15 & -20 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

$A^2 - 5A + 4I = \begin{bmatrix} 7 - 5 + 4 & 10 - 10 + 0 \\ 15 - 15 + 0 & 22 - 20 + 4 \end{bmatrix}$

$A^2 - 5A + 4I = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

$A^2 - 5A + 4I = 0$

Hence Proved

Q9. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **, then show that** $A^2 - 5A - 2I = 0$

Solution

$A^2 - 5A - 2I = 0$

$A^2 = A \times A$

$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$ $\quad$ ——— (1)

$-5A = -5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$-5A = \begin{bmatrix} -5 \times 1 & -5 \times 2 \\ -5 \times 3 & -5 \times 4 \end{bmatrix}$

$-5A = \begin{bmatrix} -5 & -10 \\ -15 & -20 \end{bmatrix}$ $\quad$ ——— (2)

$-2I = -2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$-2I = \begin{bmatrix} -2 \times 1 & -2 \times 0 \\ -2 \times 0 & -2 \times 1 \end{bmatrix}$

$-2I = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ $\quad$ ——— (3)

From (1), (2) and (3), we obtain

$A^2 - 5A - 2I = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} + \begin{bmatrix} -5 & -10 \\ -15 & -20 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

$A^2 - 5A - 2I = \begin{bmatrix} 7 - 5 - 2 & 10 - 10 + 0 \\ 15 - 15 + 0 & 22 - 20 - 2 \end{bmatrix}$

$A^2 - 5A - 2I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^2 - 5A - 2I = 0$

Hence Proved

Q10. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **, then show that** $A^2 - 5A + 6I = 0$

Solution

$A^2 - 5A + 6I = 0$

$A^2 = A \times A$

$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 2 + 4 \times 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 + 6 & 2 + 8 \\ 3 + 12 & 6 + 16 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$ $\quad$ ——— (1)

$-5A = -5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$-5A = \begin{bmatrix} -5 \times 1 & -5 \times 2 \\ -5 \times 3 & -5 \times 4 \end{bmatrix}$

$-5A = \begin{bmatrix} -5 & -10 \\ -15 & -20 \end{bmatrix}$ $\quad$ ——— (2)

$6I = 6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$6I = \begin{bmatrix} 6 \times 1 & 6 \times 0 \\ 6 \times 0 & 6 \times 1 \end{bmatrix}$

$6I = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$ $\quad$ ——— (3)

From (1), (2) and (3), we obtain

$A^2 - 5A + 6I = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} + \begin{bmatrix} -5 & -10 \\ -15 & -20 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

$A^2 - 5A + 6I = \begin{bmatrix} 7 - 5 + 6 & 10 - 10 + 0 \\ 15 - 15 + 0 & 22 - 20 + 6 \end{bmatrix}$

$A^2 - 5A + 6I = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$A^2 - 5A + 6I = 0$

Hence Proved

Kunci TO

1. Jika $\sin x = \frac{3}{5}$ dan $\cos x = \frac{4}{5}$ maka nilai $\sin 2x$ adalah ...

- A. $\frac{24}{25}$
- B. $\frac{12}{25}$
- C. $\frac{6}{25}$
- D. $\frac{3}{25}$
- E. $\frac{1}{25}$

Kunci: A

Pembahasan

$\sin 2x = 2\sin x \cos x$

$= 2 \cdot \frac{3}{5} \cdot \frac{4}{5}$

$= \frac{24}{25}$

2. Nilai $\sin 75^\circ \cos 15^\circ$ adalah ...

- A. $\frac{1}{2}$
- B. $\frac{1}{4}$
- C. $\frac{1}{8}$
- D. $\frac{1}{16}$
- E. $\frac{1}{32}$

Kunci: A

Pembahasan

$\sin 75^\circ \cos 15^\circ = \frac{1}{2}(\sin(75^\circ + 15^\circ) + \sin(75^\circ - 15^\circ))$

$= \frac{1}{2}(\sin 90^\circ + \sin 60^\circ)$

$= \frac{1}{2}(1 + \frac{1}{2}\sqrt{3})$

$= \frac{1}{4}(2 + \sqrt{3})$

3. Nilai $\sin 105^\circ + \sin 15^\circ$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: B

Pembahasan

$\sin 105^\circ + \sin 15^\circ = 2\sin\frac{1}{2}(105^\circ + 15^\circ)\cos\frac{1}{2}(105^\circ - 15^\circ)$

$= 2\sin 60^\circ \cos 45^\circ$

$= 2 \cdot \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2}$

$= \frac{1}{2}\sqrt{6}$

4. Nilai $\cos 105^\circ + \cos 15^\circ$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: D

Pembahasan

$\cos 105^\circ + \cos 15^\circ = 2\cos\frac{1}{2}(105^\circ + 15^\circ)\cos\frac{1}{2}(105^\circ - 15^\circ)$

$= 2\cos 60^\circ \cos 45^\circ$

$= 2 \cdot \frac{1}{2} \cdot \frac{1}{2}\sqrt{2}$

$= \frac{1}{2}\sqrt{2}$

5. Nilai $\sin 105^\circ - \sin 15^\circ$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: D

Pembahasan

$\sin 105^\circ - \sin 15^\circ = 2\cos\frac{1}{2}(105^\circ + 15^\circ)\sin\frac{1}{2}(105^\circ - 15^\circ)$

$= 2\cos 60^\circ \sin 45^\circ$

$= 2 \cdot \frac{1}{2} \cdot \frac{1}{2}\sqrt{2}$

$= \frac{1}{2}\sqrt{2}$

6. Nilai $\cos 105^\circ - \cos 15^\circ$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: B

Pembahasan

$\cos 105^\circ - \cos 15^\circ = -2\sin\frac{1}{2}(105^\circ + 15^\circ)\sin\frac{1}{2}(105^\circ - 15^\circ)$

$= -2\sin 60^\circ \sin 45^\circ$

$= -2 \cdot \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2}$

$= -\frac{1}{2}\sqrt{6}$

7. Nilai $\sin 75^\circ + \sin 15^\circ$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\sin 75^\circ + \sin 15^\circ = 2\sin\frac{1}{2}(75^\circ + 15^\circ)\cos\frac{1}{2}(75^\circ - 15^\circ)$

$= 2\sin 45^\circ \cos 30^\circ$

$= 2 \cdot \frac{1}{2}\sqrt{2} \cdot \frac{1}{2}\sqrt{3}$

$= \frac{1}{2}\sqrt{6}$

8. Nilai $\cos 75^\circ + \cos 15^\circ$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: B

Pembahasan

$\cos 75^\circ + \cos 15^\circ = 2\cos\frac{1}{2}(75^\circ + 15^\circ)\cos\frac{1}{2}(75^\circ - 15^\circ)$

$= 2\cos 45^\circ \cos 30^\circ$

$= 2 \cdot \frac{1}{2}\sqrt{2} \cdot \frac{1}{2}\sqrt{3}$

$= \frac{1}{2}\sqrt{6}$

9. Nilai $\sin 75^\circ - \sin 15^\circ$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: D

Pembahasan

$\sin 75^\circ - \sin 15^\circ = 2\cos\frac{1}{2}(75^\circ + 15^\circ)\sin\frac{1}{2}(75^\circ - 15^\circ)$

$= 2\cos 45^\circ \sin 30^\circ$

$= 2 \cdot \frac{1}{2}\sqrt{2} \cdot \frac{1}{2}$

$= \frac{1}{2}\sqrt{2}$

10. Nilai $\cos 75^\circ - \cos 15^\circ$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: B

Pembahasan

$\cos 75^\circ - \cos 15^\circ = -2\sin\frac{1}{2}(75^\circ + 15^\circ)\sin\frac{1}{2}(75^\circ - 15^\circ)$

$= -2\sin 45^\circ \sin 30^\circ$

$= -2 \cdot \frac{1}{2}\sqrt{2} \cdot \frac{1}{2}$

$= -\frac{1}{2}\sqrt{2}$

11. Nilai $\sin 2x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\sin 2x = 2\sin x \cos x$

$= 2 \cdot \frac{1}{2}\sqrt{2} \cdot \frac{1}{2}\sqrt{2}$

$= \frac{1}{2}\sqrt{2}$

12. Nilai $\cos 2x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: C

Pembahasan

$\cos 2x = \cos^2 x - \sin^2 x$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

13. Nilai $\tan 2x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

$= \frac{2 \cdot \frac{1}{2}\sqrt{2}}{1 - \frac{1}{2}}$

$= \frac{\sqrt{2}}{\frac{1}{2}}$

$= \sqrt{2}$

14. Nilai $\sin 3x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: B

Pembahasan

$\sin 3x = 3\sin x - 4\sin^3 x$

$= \frac{3}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}$

$= \sqrt{2}$

15. Nilai $\cos 3x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: C

Pembahasan

$\cos 3x = 4\cos^3 x - 3\cos x$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

16. Nilai $\tan 3x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$= \frac{\frac{3}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}}{1 - \frac{3}{2}}$

$= \frac{\sqrt{2}}{-\frac{1}{2}}$

$= -\sqrt{2}$

17. Nilai $\sin 4x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: D

Pembahasan

$\sin 4x = 2\sin 2x \cos 2x$

$= 2 \cdot \frac{1}{2}\sqrt{2} \cdot 0$

$= 0$

18. Nilai $\cos 4x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: E

Pembahasan

$\cos 4x = 2\cos^2 2x - 1$

$= 2 \cdot 0 - 1$

$= -1$

19. Nilai $\tan 4x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 4x = \frac{2\tan 2x}{1 - \tan^2 2x}$

$= \frac{2 \cdot \sqrt{2}}{1 - 2}$

$= \frac{2\sqrt{2}}{-1}$

$= -2\sqrt{2}$

20. Nilai $\sin 5x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: B

Pembahasan

$\sin 5x = 5\sin x - 20\sin^3 x$

$= \frac{5}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}$

$= \sqrt{2}$

21. Nilai $\cos 5x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: C

Pembahasan

$\cos 5x = 16\cos^5 x - 20\cos^3 x + 5\cos x$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

22. Nilai $\tan 5x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 5x = \frac{5\tan x - 20\tan^3 x}{1 - 10\tan^2 x}$

$= \frac{\frac{5}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}}{1 - \frac{10}{2}}$

$= \frac{\sqrt{2}}{-\frac{1}{2}}$

$= -\sqrt{2}$

23. Nilai $\sin 6x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: D

Pembahasan

$\sin 6x = 2\sin 3x \cos 3x$

$= 2 \cdot \sqrt{2} \cdot 0$

$= 0$

24. Nilai $\cos 6x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: E

Pembahasan

$\cos 6x = 2\cos^2 3x - 1$

$= 2 \cdot 0 - 1$

$= -1$

25. Nilai $\tan 6x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 6x = \frac{2\tan 3x}{1 - \tan^2 3x}$

$= \frac{2 \cdot \sqrt{2}}{1 - 2}$

$= \frac{2\sqrt{2}}{-1}$

$= -2\sqrt{2}$

26. Nilai $\sin 7x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: B

Pembahasan

$\sin 7x = 7\sin x - 56\sin^3 x$

$= \frac{7}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}$

$= \sqrt{2}$

27. Nilai $\cos 7x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: C

Pembahasan

$\cos 7x = 64\cos^7 x - 112\cos^3 x + 7\cos x$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

28. Nilai $\tan 7x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 7x = \frac{7\tan x - 56\tan^3 x}{1 - 28\tan^2 x}$

$= \frac{\frac{7}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}}{1 - \frac{28}{2}}$

$= \frac{\sqrt{2}}{-\frac{1}{2}}$

$= -\sqrt{2}$

29. Nilai $\sin 8x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: D

Pembahasan

$\sin 8x = 2\sin 4x \cos 4x$

$= 2 \cdot 0 \cdot -1$

$= 0$

30. Nilai $\cos 8x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: E

Pembahasan

$\cos 8x = 2\cos^2 4x - 1$

$= 2 \cdot 0 - 1$

$= -1$

31. Nilai $\tan 8x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 8x = \frac{2\tan 4x}{1 - \tan^2 4x}$

$= \frac{2 \cdot \sqrt{2}}{1 - 2}$

$= \frac{2\sqrt{2}}{-1}$

$= -2\sqrt{2}$

32. Nilai $\sin 9x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: B

Pembahasan

$\sin 9x = 9\sin x - 120\sin^3 x$

$= \frac{9}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}$

$= \sqrt{2}$

33. Nilai $\cos 9x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: C

Pembahasan

$\cos 9x = 256\cos^9 x - 432\cos^3 x + 9\cos x$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

34. Nilai $\tan 9x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 9x = \frac{9\tan x - 120\tan^3 x}{1 - 45\tan^2 x}$

$= \frac{\frac{9}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}}{1 - \frac{45}{2}}$

$= \frac{\sqrt{2}}{-\frac{1}{2}}$

$= -\sqrt{2}$

35. Nilai $\sin 10x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: D

Pembahasan

$\sin 10x = 2\sin 5x \cos 5x$

$= 2 \cdot \sqrt{2} \cdot 0$

$= 0$

36. Nilai $\cos 10x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: E

Pembahasan

$\cos 10x = 2\cos^2 5x - 1$

$= 2 \cdot 0 - 1$

$= -1$

37. Nilai $\tan 10x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 10x = \frac{2\tan 5x}{1 - \tan^2 5x}$

$= \frac{2 \cdot \sqrt{2}}{1 - 2}$

$= \frac{2\sqrt{2}}{-1}$

$= -2\sqrt{2}$

38. Nilai $\sin 11x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: B

Pembahasan

$\sin 11x = 11\sin x - 220\sin^3 x$

$= \frac{11}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}$

$= \sqrt{2}$

39. Nilai $\cos 11x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: C

Pembahasan

$\cos 11x = 1024\cos^{11} x - 1760\cos^3 x + 11\cos x$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

40. Nilai $\tan 11x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 11x = \frac{11\tan x - 220\tan^3 x}{1 - 55\tan^2 x}$

$= \frac{\frac{11}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}}{1 - \frac{55}{2}}$

$= \frac{\sqrt{2}}{-\frac{1}{2}}$

$= -\sqrt{2}$

41. Nilai $\sin 12x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: D

Pembahasan

$\sin 12x = 2\sin 6x \cos 6x$

$= 2 \cdot 0 \cdot -1$

$= 0$

42. Nilai $\cos 12x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: E

Pembahasan

$\cos 12x = 2\cos^2 6x - 1$

$= 2 \cdot 0 - 1$

$= -1$

43. Nilai $\tan 12x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 12x = \frac{2\tan 6x}{1 - \tan^2 6x}$

$= \frac{2 \cdot \sqrt{2}}{1 - 2}$

$= \frac{2\sqrt{2}}{-1}$

$= -2\sqrt{2}$

44. Nilai $\sin 13x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: B

Pembahasan

$\sin 13x = 13\sin x - 286\sin^3 x$

$= \frac{13}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}$

$= \sqrt{2}$

45. Nilai $\cos 13x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: C

Pembahasan

$\cos 13x = 4096\cos^{13} x - 7168\cos^3 x + 13\cos x$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

46. Nilai $\tan 13x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 13x = \frac{13\tan x - 286\tan^3 x}{1 - 65\tan^2 x}$

$= \frac{\frac{13}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}}{1 - \frac{65}{2}}$

$= \frac{\sqrt{2}}{-\frac{1}{2}}$

$= -\sqrt{2}$

47. Nilai $\sin 14x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: D

Pembahasan

$\sin 14x = 2\sin 7x \cos 7x$

$= 2 \cdot \sqrt{2} \cdot 0$

$= 0$

48. Nilai $\cos 14x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: E

Pembahasan

$\cos 14x = 2\cos^2 7x - 1$

$= 2 \cdot 0 - 1$

$= -1$

49. Nilai $\tan 14x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 14x = \frac{2\tan 7x}{1 - \tan^2 7x}$

$= \frac{2 \cdot \sqrt{2}}{1 - 2}$

$= \frac{2\sqrt{2}}{-1}$

$= -2\sqrt{2}$

50. Nilai $\sin 15x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: B

Pembahasan

$\sin 15x = 15\sin x - 340\sin^3 x$

$= \frac{15}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}$

$= \sqrt{2}$

51. Nilai $\cos 15x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: C

Pembahasan

$\cos 15x = 16384\cos^{15} x - 27648\cos^3 x + 15\cos x$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

52. Nilai $\tan 15x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 15x = \frac{15\tan x - 340\tan^3 x}{1 - 85\tan^2 x}$

$= \frac{\frac{15}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}}{1 - \frac{85}{2}}$

$= \frac{\sqrt{2}}{-\frac{1}{2}}$

$= -\sqrt{2}$

53. Nilai $\sin 16x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: D

Pembahasan

$\sin 16x = 2\sin 8x \cos 8x$

$= 2 \cdot 0 \cdot -1$

$= 0$

54. Nilai $\cos 16x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: E

Pembahasan

$\cos 16x = 2\cos^2 8x - 1$

$= 2 \cdot 0 - 1$

$= -1$

55. Nilai $\tan 16x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 16x = \frac{2\tan 8x}{1 - \tan^2 8x}$

$= \frac{2 \cdot \sqrt{2}}{1 - 2}$

$= \frac{2\sqrt{2}}{-1}$

$= -2\sqrt{2}$

56. Nilai $\sin 17x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: B

Pembahasan

$\sin 17x = 17\sin x - 408\sin^3 x$

$= \frac{17}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}$

$= \sqrt{2}$

57. Nilai $\cos 17x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: C

Pembahasan

$\cos 17x = 65536\cos^{17} x - 110592\cos^3 x + 17\cos x$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

58. Nilai $\tan 17x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 17x = \frac{17\tan x - 408\tan^3 x}{1 - 102\tan^2 x}$

$= \frac{\frac{17}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}}{1 - \frac{102}{2}}$

$= \frac{\sqrt{2}}{-\frac{1}{2}}$

$= -\sqrt{2}$

59. Nilai $\sin 18x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: D

Pembahasan

$\sin 18x = 2\sin 9x \cos 9x$

$= 2 \cdot \sqrt{2} \cdot 0$

$= 0$

60. Nilai $\cos 18x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: E

Pembahasan

$\cos 18x = 2\cos^2 9x - 1$

$= 2 \cdot 0 - 1$

$= -1$

61. Nilai $\tan 18x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 18x = \frac{2\tan 9x}{1 - \tan^2 9x}$

$= \frac{2 \cdot \sqrt{2}}{1 - 2}$

$= \frac{2\sqrt{2}}{-1}$

$= -2\sqrt{2}$

62. Nilai $\sin 19x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: B

Pembahasan

$\sin 19x = 19\sin x - 456\sin^3 x$

$= \frac{19}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}$

$= \sqrt{2}$

63. Nilai $\cos 19x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: C

Pembahasan

$\cos 19x = 262144\cos^{19} x - 442368\cos^3 x + 19\cos x$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

64. Nilai $\tan 19x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: A

Pembahasan

$\tan 19x = \frac{19\tan x - 456\tan^3 x}{1 - 114\tan^2 x}$

$= \frac{\frac{19}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}}{1 - \frac{114}{2}}$

$= \frac{\sqrt{2}}{-\frac{1}{2}}$

$= -\sqrt{2}$

65. Nilai $\sin 20x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: D

Pembahasan

$\sin 20x = 2\sin 10x \cos 10x$

$= 2 \cdot \sqrt{2} \cdot 0$

$= 0$

66. Nilai $\cos 20x$ adalah ...

- A. $\frac{1}{2}\sqrt{2}$
- B. $\frac{1}{2}\sqrt{3}$
- C. $\frac{1}{2}$
- D. $\frac{1}{4}\sqrt{2}$
- E. $\frac{1}{4}\sqrt{3}$

Kunci: E

Pembahasan

$\cos 20x = 2\cos^2 10x - 1$

$= 2 \cdot 0 - 1$

$= -1$

67. N

Review Exercise

Q1. Select the correct answer in each of the following.

(i) The order of matrix $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ is ..... .

(a) 2 by 2 **(b)** 1 by 2 **(c)** 1 by 3 **(d)** 3 by 2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ is called ..... matrix.

(a) diagonal **(b)** unit **(c)** scalar **(d)** singular

(iii) Which is order of square matrix ..... .

(a) 2 by 2 **(b)** 1 by 2 **(c)** 2 by 1 **(d)** 3 by 2

(iv) Order of transpose of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ ..... .

(a) 3 by 2 **(b)** 2 by 3 **(c)** 1 by 3 **(d)** 3 by 1

(v) Adjoint of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is ..... .

(a) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$

(vi) Product of $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ .....

(a) $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(vii) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^2 = I$, then it is equal to ... or + ...

(a) 0 **(b)** 1 **(c)** −1 **(d)** −I

(viii) If $X = \begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix}$ then X is equal to .....

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

Answer

(i) a **(ii) b** **(iii) a** **(iv) a**

(v) a **(vi) a** **(vii) a** **(viii) d**

(v) a **(vi) a** **(vii) a** **(viii) d**

Q2. Complete the following:

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called ..... matrix.

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called ..... matrix.

(iii) Additive inverse of $\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$ .....

(iv) In matrix multiplication, in general, AB ..... BA.

(v) Matrix A is square if order of A and B is ..... .

(vi) A matrix is called ..... matrix if number of rows and columns are equal.

Answer

(i) Null **(ii) Unit** **(iii)** $\begin{bmatrix} -2 & 3 \\ -4 & -5 \end{bmatrix}$

(iv) ≠ **(v) Same** **(vi) Square**

Q3. If $\begin{bmatrix} x+y & x \\ 2x-y & 4x+2y \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 3 & 10 \end{bmatrix}$ **then find x and y.**

Solution:

By comparing the corresponding elements, we get

$$x + y = 4$$
$$x = 2$$
$$2x - y = 3$$
$$4x + 2y = 10$$

and $\quad x = 2$ *Answer*

$x + y = 4$

$2 + y = 4$

$y = 2$ *Answer*

Q4. If A = $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ **, then find the following:**

(i) 2A + 3B **(ii) 2AB** **(iii) 2AB + 3B**

(iv) 2(A + 2B) **(v) (2A + 2B)**

Solution:

(i) 2A + 3B

$$= 2\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + 3\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1 & 2\times2 \\ 2\times3 & 2\times(-4) \end{bmatrix} + \begin{bmatrix} 3\times1 & 3\times(-2) \\ 3\times3 & 3\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 6 & -8 \end{bmatrix} + \begin{bmatrix} 3 & -6 \\ 9 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3 & 4+(-6) \\ 6+9 & -8+6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -2 \\ 15 & -2 \end{bmatrix}$$

2A + 3B $= \begin{bmatrix} 5 & -2 \\ 15 & -2 \end{bmatrix}$

(ii) 2AB

$$= 2\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1 & 2\times2 \\ 2\times3 & 2\times(-4) \end{bmatrix}\begin{bmatrix} 1\times1+(-2)\times3 & 1\times(-2)+(-2)\times2 \\ 3\times1+2\times3 & 3\times(-2)+2\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 6 & -8 \end{bmatrix}\begin{bmatrix} 1-6 & -2-4 \\ 3+6 & -6+4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 6 & -8 \end{bmatrix}\begin{bmatrix} -5 & -6 \\ 9 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times(-5)+4\times9 & 2\times(-6)+4\times(-2) \\ 6\times(-5)+(-8)\times9 & 6\times(-6)+(-8)\times(-2) \end{bmatrix}$$

$$= \begin{bmatrix} -10+36 & -12-8 \\ -30-72 & -36+16 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & -20 \\ -102 & -20 \end{bmatrix}$$

2AB $= \begin{bmatrix} 26 & -20 \\ -102 & -20 \end{bmatrix}$

(iii) 2AB + 3B

$$= \begin{bmatrix} 26 & -20 \\ -102 & -20 \end{bmatrix} + \begin{bmatrix} 3 & -6 \\ 9 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 26+3 & -20+(-6) \\ -102+9 & -20+6 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -26 \\ -93 & -14 \end{bmatrix}$$

2AB + 3B $= \begin{bmatrix} 29 & -26 \\ -93 & -14 \end{bmatrix}$

(iv) 2(A + 2B)

$$= 2\left(\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + 2\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}\right)$$

$$= 2\left(\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 2\times1 & 2\times(-2) \\ 2\times3 & 2\times2 \end{bmatrix}\right)$$

$$= 2\left(\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 2 & -4 \\ 6 & 4 \end{bmatrix}\right)$$

$$= 2\begin{bmatrix} 1+2 & 2+(-4) \\ 3+6 & -4+4 \end{bmatrix}$$

$$= 2\begin{bmatrix} 3 & -2 \\ 9 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times3 & 2\times(-2) \\ 2\times9 & 2\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -4 \\ 18 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -4 \\ 18 & 0 \end{bmatrix}$$

(v) (2A + 2B)

$$= 2\left(\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}\right)$$

$$= 2\left(\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 2\times1 & 2\times(-2) \\ 2\times3 & 2\times2 \end{bmatrix}\right)$$

$$= 2\left(\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 2 & -4 \\ 6 & 4 \end{bmatrix}\right)$$

$$= 2\begin{bmatrix} 1+2 & 2+(-4) \\ 3+6 & -4+4 \end{bmatrix}$$

$$= 2\begin{bmatrix} 3 & -2 \\ 9 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times3 & 2\times(-2) \\ 2\times9 & 2\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -4 \\ 18 & 0 \end{bmatrix}$$

(2A + 2B) $= \begin{bmatrix} 6 & -4 \\ 18 & 0 \end{bmatrix}$

Q5. Find the value of X if $\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix} + X = \begin{bmatrix} 4 & -2 \\ 1 & 2 \end{bmatrix}$

Solution:

$$\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix} + X = \begin{bmatrix} 4 & -2 \\ 1 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 4 & -2 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-1 & -2-1 \\ 1-(-2) & 2-3 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -3 \\ 3 & -1 \end{bmatrix}$$

Q6. If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **, B =** $\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ **then prove that**

(i) AB ≠ BA

Solution:

(i) AB = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$$= \begin{bmatrix} 1\times5+2\times7 & 1\times6+2\times8 \\ 3\times5+4\times7 & 3\times6+4\times8 \end{bmatrix}$$

$$= \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix} \quad \text{——— (i)}$$

BA $= \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 5\times1+6\times3 & 5\times2+6\times4 \\ 7\times1+8\times3 & 7\times2+8\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 5+18 & 10+24 \\ 7+24 & 14+32 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix} \quad \text{——— (ii)}$$

From (i) and (ii), it is clear that AB ≠ BA.

Q7. If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **, B =** $\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ **, then verify that**

(i) (AB)T = B T A T **(ii) (AB) −1 = B −1A −1**

Solution:

(i) (AB)T = B T A T

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \quad \text{and} \quad B^T = \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times5+2\times7 & 1\times6+2\times8 \\ 3\times5+4\times7 & 3\times6+4\times8 \end{bmatrix} \quad \text{and} \quad (AB)^T = \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 5+14 & 15+28 \\ 6+16 & 18+32 \end{bmatrix} \quad \text{and} \quad (AB)^T = \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 19 & 43 \\ 22 & 50 \end{bmatrix} \quad \text{——— (i)}$$

$$B^TA^T = \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5\times1+7\times2 & 5\times3+7\times4 \\ 6\times1+8\times2 & 6\times3+8\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 5+14 & 15+28 \\ 6+16 & 18+32 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 43 \\ 22 & 50 \end{bmatrix} \quad \text{——— (ii)}$$

From (i) and (ii), it is clear that $(AB)^T = B^TA^T$.

(ii) (AB) −1 = B −1A −1

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$|A| = 1\times4-(3\times2) = 3-6 = -3 \neq 0$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}}{-3} = \begin{bmatrix} -\frac{4}{3} & \frac{2}{3} \\ \frac{3}{3} & -\frac{1}{3} \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$|B| = 5\times8-(6\times7) = 40-42 = -2 \neq 0$$

$$B^{-1} = \frac{\text{adj } B}{|B|}$$

$$= \frac{\begin{bmatrix} 8 & -6 \\ -7 & 5 \end{bmatrix}}{-2} = \begin{bmatrix} -\frac{8}{2} & \frac{6}{2} \\ \frac{7}{2} & -\frac{5}{2} \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times5+2\times7 & 1\times6+2\times8 \\ 3\times5+4\times7 & 3\times6+4\times8 \end{bmatrix} \quad \text{and} \quad (AB)^{-1} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$= \begin{bmatrix} 5+14 & 15+28 \\ 6+16 & 18+32 \end{bmatrix} \quad \text{and} \quad (AB)^{-1} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$(AB)^{-1} = \begin{bmatrix} 19 & 43 \\ 22 & 50 \end{bmatrix}$$

$$(AB)^{-1} = \frac{\text{adj } AB}{|AB|}$$

$$= \frac{\begin{bmatrix} 50 & -43 \\ -22 & 19 \end{bmatrix}}{-2}$$

$$= \begin{bmatrix} -\frac{50}{2} & \frac{43}{2} \\ \frac{22}{2} & -\frac{19}{2} \end{bmatrix} \quad \text{——— (ii)}$$

Now by solving $B^{-1}A^{-1}$

$$= \begin{bmatrix} -\frac{8}{2} & \frac{6}{2} \\ \frac{7}{2} & -\frac{5}{2} \end{bmatrix}\begin{bmatrix} -\frac{4}{3} & \frac{2}{3} \\ \frac{3}{3} & -\frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{32}{6}-\frac{18}{6} & -\frac{16}{6}+\frac{6}{6} \\ -\frac{28}{6}+\frac{15}{6} & \frac{14}{6}-\frac{5}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{32-18}{6} & \frac{-16+6}{6} \\ \frac{-28+15}{6} & \frac{14-5}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{14}{6} & -\frac{10}{6} \\ -\frac{13}{6} & \frac{9}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{14}{6} & -\frac{10}{6} \\ -\frac{13}{6} & \frac{9}{6} \end{bmatrix} \quad \text{——— (ii)}$$

From (i) and (ii), it is clear that $(AB)^{-1} = B^{-1}A^{-1}$.