



## 10.1 FREQUENCY DISTRIBUTION

### Frequency

The number of times each value appears is called the frequency of that value. For example if 3 students get marks from 10% to 20%, the frequency is 3 if 5 students get marks from 20% to 30%, frequency is 5.

### Frequency Table

The table which gives the frequency of each score is called a frequency table.

#### 10.1.1 Grouped Frequency Table

The local health authority wanted to collect some information about the heights of five years old children in their area. At their first visit to school for medical examination, the height of each child of five years was recorded, e.g., the following figures are the recorded heights (*in cm to the nearest cm*) of ninety children.

|     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 99  | 107 | 102 | 98  | 115 | 95  | 106 | 110 | 108 | 105 |
| 118 | 102 | 114 | 108 | 94  | 104 | 113 | 102 | 105 | 95  |
| 105 | 110 | 109 | 101 | 106 | 108 | 107 | 107 | 101 | 109 |
| 108 | 105 | 116 | 109 | 114 | 110 | 97  | 110 | 113 | 116 |
| 112 | 101 | 92  | 105 | 104 | 115 | 111 | 103 | 110 | 99  |
| 93  | 104 | 103 | 113 | 107 | 94  | 102 | 117 | 116 | 104 |
| 99  | 114 | 106 | 114 | 98  | 109 | 107 | 104 | 106 | 107 |
| 109 | 113 | 112 | 100 | 109 | 113 | 118 | 104 | 94  | 114 |
| 107 | 96  | 108 | 103 | 112 | 106 | 115 | 111 | 115 | 101 |

In the table the heights are listed in a random order (*the same order the children came for the medical examination*). We are to arrange and organize this data in some manner for clear understanding to get any result. We group the heights as:

90 – 94cm ; 95 – 99cm ; 100 – 104cm ;  
105 – 109cm ; 110 – 114cm ; 115 – 119cm

The heights are arranged in order of magnitude. Counting the number of heights in each group gives the following frequency table.

| Heights (cm) | Tally | Frequency |
|--------------|-------|-----------|
| 90 – 94      |       | 5         |
| 95 – 99      |       | 9         |
| 100 – 104    |       | 17        |
| 105 – 109    |       | 28        |
| 110 – 114    |       | 21        |
| 115 – 119    |       | 10        |
|              |       | Total: 90 |

### 10.1.2 Histogram

When a bar chart is constructed so that the area of each bar is proportional to the number of items in each group, it is called a histogram.

## Histogram with equal class intervals

Consider the following examples:

### EXAMPLE

Construct a histogram with equal intervals with the help of following frequency.

| Weights (kg) | 50 – 59 | 60 – 69 | 70 – 79 | 80 – 89 | 90 – 99 | 100 – 109  |
|--------------|---------|---------|---------|---------|---------|------------|
| Frequency    | 15      | 30      | 35      | 15      | 3       | 2          |
|              |         |         |         |         |         | Total: 100 |

### SOLUTION:

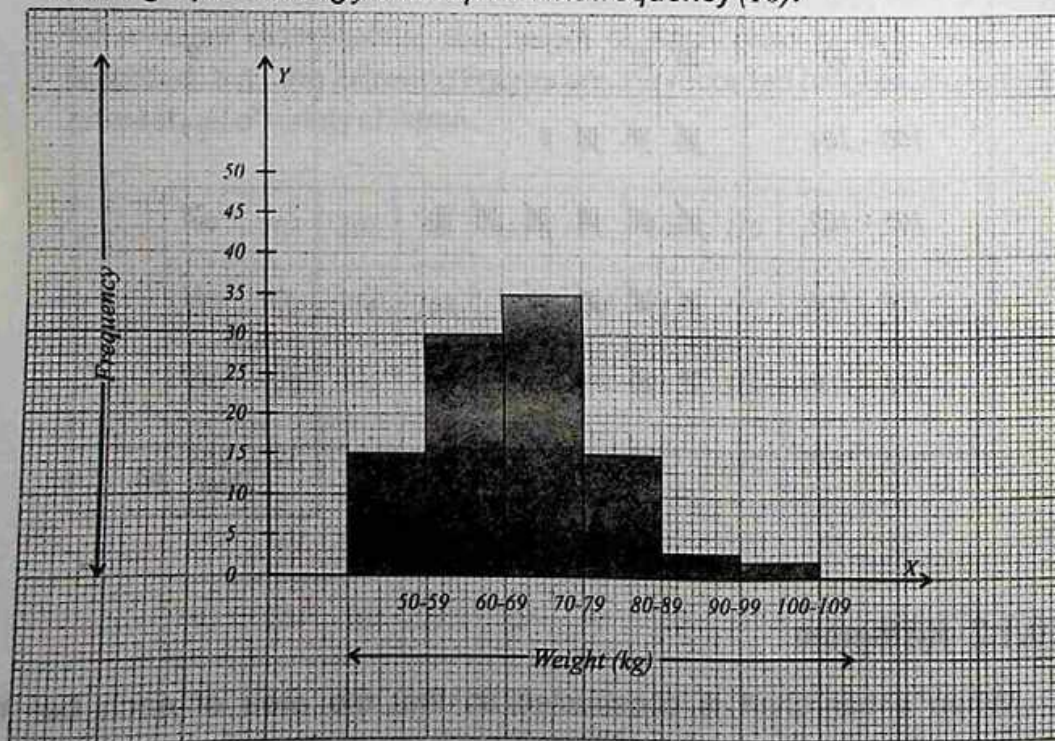
The following bar chart illustrates the frequency table.

Weight (kg) has been taken along  $x$ -axis towards right of 'O'.

Frequency has been taken above  $x$ -axis.

One big square along  $x$ -axis represents 10kg weight.

One big square along  $y$ -axis represents frequency (10).



It is the area of a bar that gives the impression of the number of items in a group.

The diagram is a histogram with equal intervals. Each group covers the same span, so each bar has the same width. Hence the area of each bar is proportional to the number of items in the group.

## Histogram with unequal class intervals

### EXAMPLE

Consider the following frequency table which shows the result of a survey on the per month pay of 100 men.

| Pay (rupees) | 0 - 1999 | 2000 - 3999 | 4000 - 5999 | 6000 - 9999 | 10000 - 19999 |
|--------------|----------|-------------|-------------|-------------|---------------|
| Frequency    | 20       | 36          | 25          | 14          | 5             |

### SOLUTION:

The intervals of the group are not equal.

The first three groups each cover a span of Rs.2000.

The fourth group intervals Rs.4000 twice the interval of the first three.

The fifth group intervals Rs.10,000 five times the interval of the first three.

To illustrate this distribution on histogram we must make:

The width of the first three bars equal.

The width of the fourth bar twice that of the first three.

The width of the fifth bar five times that of the first three.

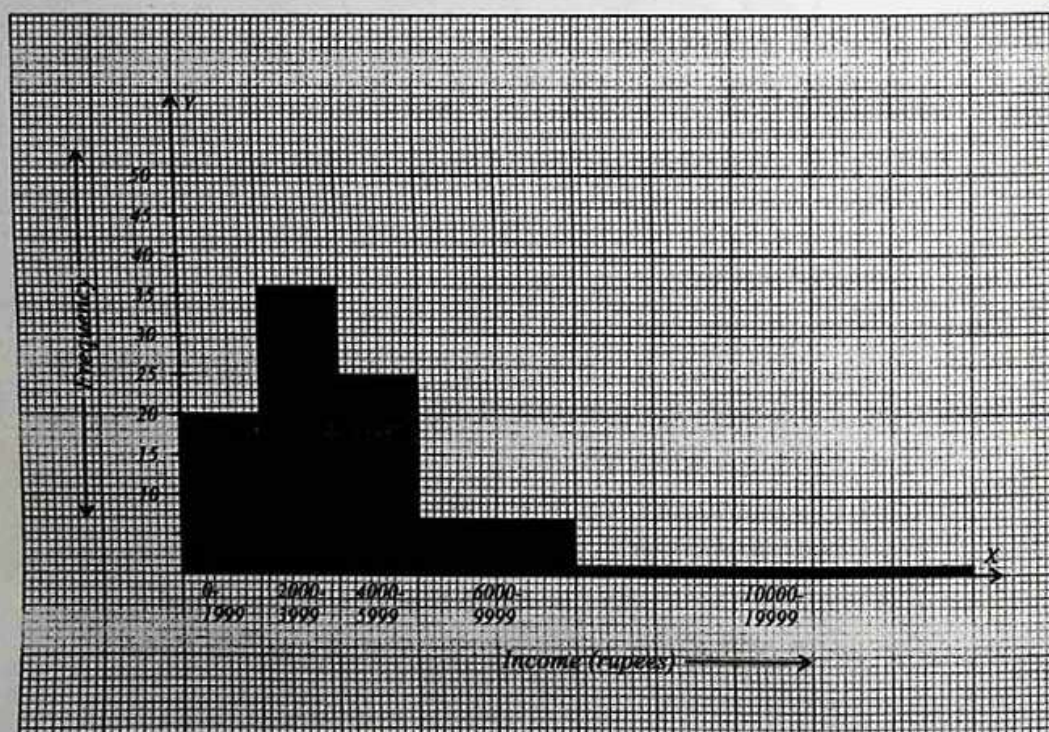
Now consider the 14 incomes in the group Rs.6000 - Rs.9999. If we suppose that these incomes are evenly spread throughout this group, then these are:

Seven men with incomes in the group Rs.6000 - Rs.7999

and seven men with incomes in the group Rs.8000 - Rs.9999.

The fourth bar is therefore 7 units high for the whole group, because the width of this bar is twice that of the others used. Its area is proportional to the frequency of the group.

If we also assume that the income in the fifth group are evenly spread throughout the group, then there is one man with an income in each of the subgroups with interval Rs.2000. The fifth bar is therefore one unit high for the whole group. The area of the fifth bar is proportional to the frequency of the group.



*Note:* We do not use frequency to label vertical axis, because it is the area of the bar which represents the frequency and not the height.

### 10.1.3 Frequency Polygon

A frequency polygon is a many sided closed figure. It is constructed by plotting frequencies against their class marks (midpoints) and then joining the resulting points by means of straight lines. A frequency polygon can also be obtained by joining the mid points of the tops of the rectangles in the histogram.

#### EXAMPLE

The data in the frequency table shows the mass (in kg) of 40 people upon joining a weight loss program.

Represent the given data using a:

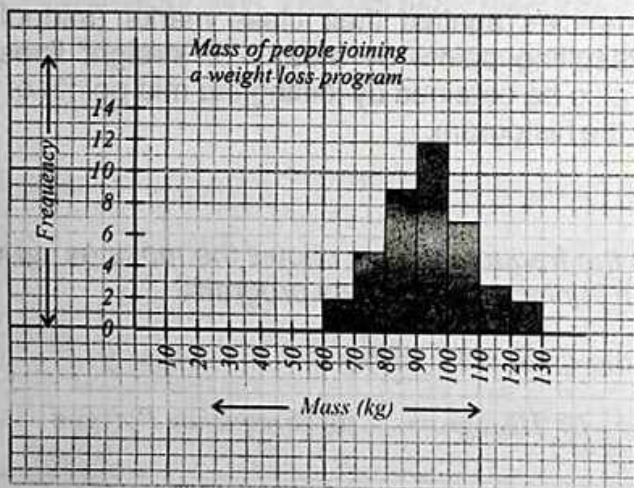
(i) Histogram      (ii) Frequency Polygon

| Class intervals | Frequency |
|-----------------|-----------|
| 60 — 70         | 2         |
| 70 — 80         | 5         |
| 80 — 90         | 9         |
| 90 — 100        | 12        |
| 100 — 110       | 7         |
| 110 — 120       | 3         |
| 120 — 130       | 2         |
| Total: 40       |           |

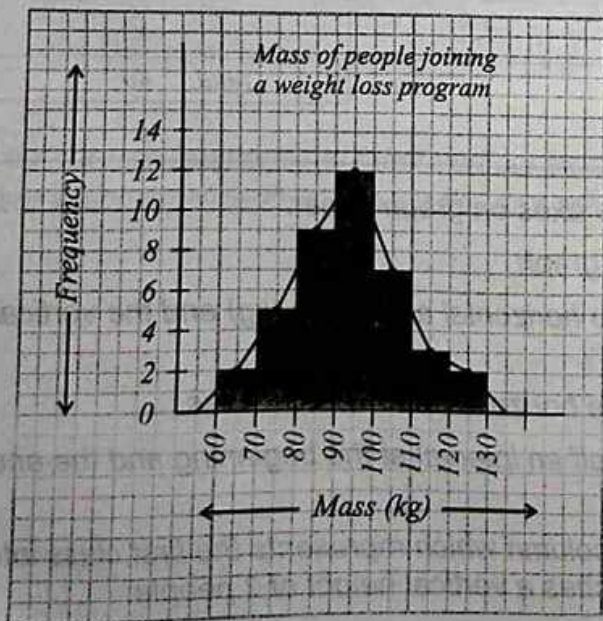
#### SOLUTION:

- Draw the axes on graph paper.
- Title the graph.
- Label the horizontal axis mass (kg) and the vertical axis frequency.
- Scale the horizontal and vertical axes.
- Leave half an interval at the beginning and the end of the graph.
- Draw a column which represents the first class interval and reaches a vertical height of 2 people.

Repeat step 3 for each of the other class intervals.



- (ii) Mark the mid points of the tops of the rectangles obtained in the histogram from part A. Join the mid points by straight line intervals. Close the polygon by drawing lines which meet the horizontal axis a half column width before the first column and a half column width after the last column.



**EXERCISE - 10.1**

- 1- Fifty Junior school children joined the school's computer club. Their ages were recorded.

|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 10 | 8  | 9  | 10 | 7  | 8  | 8  | 11 | 10 | 9  |
| 7  | 8  | 9  | 9  | 10 | 11 | 11 | 10 | 9  | 8  |
| 8  | 7  | 9  | 7  | 10 | 7  | 10 | 8  | 9  | 11 |
| 10 | 11 | 8  | 10 | 9  | 8  | 9  | 7  | 11 | 10 |
| 9  | 10 | 10 | 11 | 10 | 11 | 7  | 11 | 10 | 9  |

Make a frequency table showing the number of each age and illustrate this information with a bar chart.

- 2- The local fish and chip shop had 56 customers on Saturday evening. They spent the following amount in rupees.

|     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 270 | 110 | 45  | 96  | 250 | 490 | 325 | 45  |
| 382 | 136 | 125 | 450 | 420 | 380 | 150 | 250 |
| 85  | 250 | 320 | 525 | 218 | 210 | 216 | 120 |
| 155 | 430 | 250 | 40  | 510 | 150 | 510 | 245 |
| 320 | 120 | 316 | 150 | 260 | 45  | 180 | 310 |
| 273 | 280 | 85  | 280 | 318 | 45  | 210 | 282 |
| 462 | 316 | 218 | 316 | 325 | 45  | 560 | 315 |

use groups Rs.0-99, Rs.100-199, Rs.200-299, Rs.300-399, Rs.400-499, Rs.500-599 to make a frequency tables illustrate the data with a bar chart.

- 3- The weights to the nearest gram of 30 bags of popcorn sold at a festival are given as:

|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 69 | 83 | 75 | 65 | 68 | 68 | 73 | 70 | 80 | 79 |
| 70 | 76 | 63 | 86 | 69 | 65 | 66 | 74 | 86 | 68 |
| 70 | 60 | 67 | 74 | 65 | 65 | 67 | 88 | 81 | 63 |

Make a frequency table, illustrate the data with a bar chart.

## 10.2 CUMULATIVE FREQUENCY

The cumulative frequency shows the total number of observations (scores), which are below a certain value. We explain this with the help of following example:

### EXAMPLE-1

*All the students of class 9 took a maths test. Here are their marks as percentage. They have been grouped in 10. Find cumulative frequency.*

| Marks in %           | Frequency |
|----------------------|-----------|
| 1 - 10               | 3         |
| 11 - 20              | 6         |
| 21 - 30              | 11        |
| 31 - 40              | 13        |
| 41 - 50              | 18        |
| 51 - 60              | 24        |
| 61 - 70              | 14        |
| 71 - 80              | 6         |
| 81 - 90              | 3         |
| 91 - 100             | 2         |
| Total Frequency: 100 |           |

- (i) How many students scored 10 % or less ?
- (ii) How many students scored 20 % or less ?
- (iii) How many students scored 30 % or less ?
- (iv) How many students scored 40 % or less ?
- (v) How many students scored 50 % or less?
- (vi) How many students scored 60 % or less?
- (vii) How many students scored 70 % or less?
- (viii) How many students scored 80 % or less?
- (ix) How many students scored 90 % or less?
- (x) How many students scored 100 % or less?

**SOLUTION:**

The following table shows cumulative frequencies.

| Marks %       | Cumulative Frequency |                |
|---------------|----------------------|----------------|
| 10 % or less  | 3                    | 3              |
| 20 % or less  | 9                    | $3 + 6 = 9$    |
| 30 % or less  | 20                   | $9 + 11 = 20$  |
| 40 % or less  | 33                   | $20 + 13 = 33$ |
| 50 % or less  | 51                   | $33 + 18 = 51$ |
| 60 % or less  | 75                   | $51 + 24 = 75$ |
| 70 % or less  | 89                   | $75 + 14 = 89$ |
| 80 % or less  | 95                   | $89 + 6 = 95$  |
| 90 % or less  | 98                   | $95 + 3 = 98$  |
| 100 % or less | 100                  | $98 + 2 = 100$ |

**EXAMPLE-2**

The marks in a science test are given in the table.  
Complete the cumulative frequency.

| Marks    | Frequency | Marks       | Cumulative frequency |
|----------|-----------|-------------|----------------------|
| 1 - 10   | 5         | 10 or less  |                      |
| 11 - 20  | 6         | 20 or less  |                      |
| 21 - 30  | 8         | 30 or less  |                      |
| 31 - 40  | 16        | 40 or less  |                      |
| 41 - 50  | 23        | 50 or less  |                      |
| 51 - 60  | 18        | 60 or less  |                      |
| 61 - 70  | 12        | 70 or less  |                      |
| 71 - 80  | 10        | 80 or less  |                      |
| 81 - 90  | 2         | 90 or less  |                      |
| 91 - 100 | 0         | 100 or less |                      |

**SOLUTION:**

| Marks    | Frequency | Marks       | Cumulative Frequency |
|----------|-----------|-------------|----------------------|
| 1 – 10   | 5         | 10 or less  | 5                    |
| 11 – 20  | 6         | 20 or less  | 11                   |
| 21 – 30  | 8         | 30 or less  | 19                   |
| 31 – 40  | 16        | 40 or less  | 35                   |
| 41 – 50  | 23        | 50 or less  | 58                   |
| 51 – 60  | 18        | 60 or less  | 76                   |
| 61 – 70  | 12        | 70 or less  | 88                   |
| 71 – 80  | 10        | 80 or less  | 98                   |
| 81 – 90  | 2         | 90 or less  | 100                  |
| 91 – 100 | 0         | 100 or less | 100                  |

**10.2.2 Cumulative Frequency Polygon**

When the cumulative frequencies are plotted against the end points of their respective class intervals and joined together, the resultant graph is called a cumulative frequency polygon, or an **ogive**.

Therefore, an ogive can be considered as a line graph of the cumulative frequency results.

**EXAMPLE**

*The data in the frequency table shows the number of fish caught by 28 competitors in fishing competition.*

*Construct a cumulative frequency polygon (that is, an ogive) for the given data.*

| Number of Fish Caught | Frequency |
|-----------------------|-----------|
| 0 ——— 5               | 3         |
| 5 ——— 10              | 5         |
| 10 ——— 15             | 6         |
| 15 ——— 20             | 8         |
| 20 ——— 25             | 4         |
| 25 ——— 30             | 2         |
| Total: 28             |           |

**SOLUTION:**

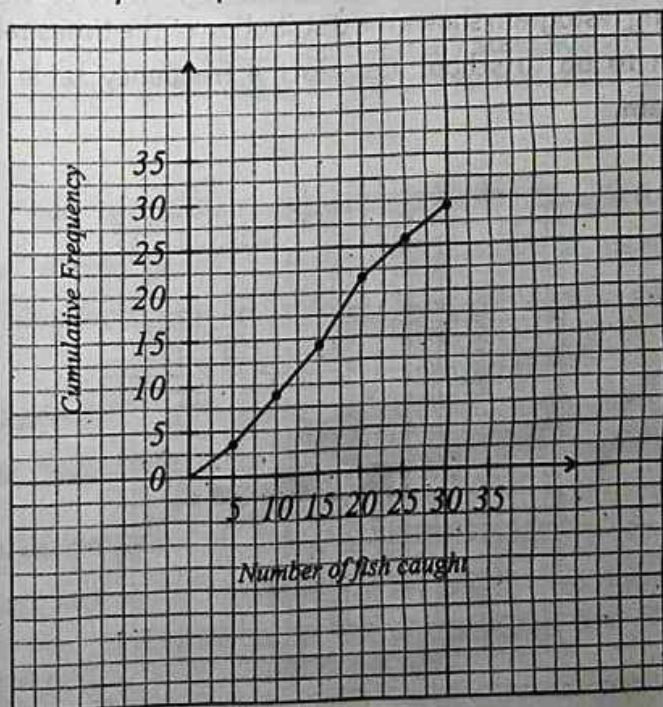
Find cumulative frequencies, draw the axes on graph paper, title the graph, label the horizontal axis "Number of fish caught" and the vertical axis "Cumulative frequency".

| Number of fish caught  | Frequency ( $f$ ) | Cumulative Frequency ( $cf$ ) |
|------------------------|-------------------|-------------------------------|
| 0 ——— 5                | 3                 | 3                             |
| 5 ——— 10               | 5                 | $3 + 5 = 8$                   |
| 10 ——— 15              | 6                 | $8 + 6 = 14$                  |
| 15 ——— 20              | 8                 | $14 + 8 = 22$                 |
| 20 ——— 25              | 4                 | $22 + 4 = 26$                 |
| 25 ——— 30              | 2                 | $26 + 2 = 28$                 |
| Total: $\Sigma f = 28$ |                   |                               |

Plot each of the cumulative frequencies against the end point of its respective class intervals.

The first point of an ogive curve has a cumulative frequency value of zero corresponding to the lowest possible value of the initial class interval. Therefore the first point of the ogive will be  $(0,0)$ .

The next point will be  $(5,3)$  followed by  $(10,8)$  and so on. Connect each of the plotted points with straight line segments.



**EXERCISE - 10.2**

- 1- Draw a histogram to represent the frequency table in each of the following tables.

- (i) The table shows the distribution of ages of 100 people attending a school function.

|                    |      |       |       |       |       |
|--------------------|------|-------|-------|-------|-------|
| <i>Age (years)</i> | 0-19 | 20-39 | 40-59 | 60-79 | 80-89 |
| <i>Frequency</i>   | 43   | 24    | 17    | 10    | 6     |

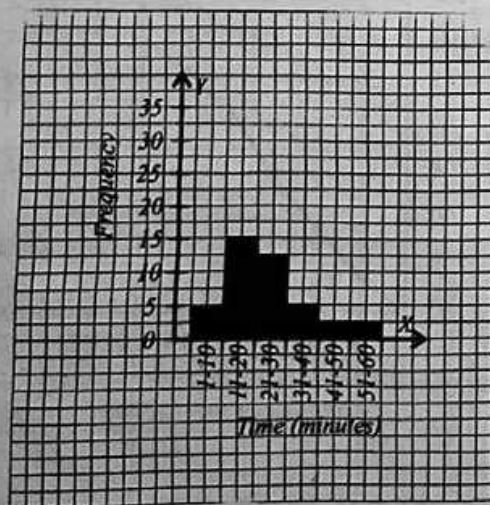
- (ii) The table shows the results of a survey on the weekly earnings of 100 sixteen-year old boys.

|                        |     |       |       |       |       |       |
|------------------------|-----|-------|-------|-------|-------|-------|
| <i>Weekly earnings</i> | 0-9 | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 |
| <i>Frequency</i>       | 45  | 10    | 11    | 21    | 10    | 3     |

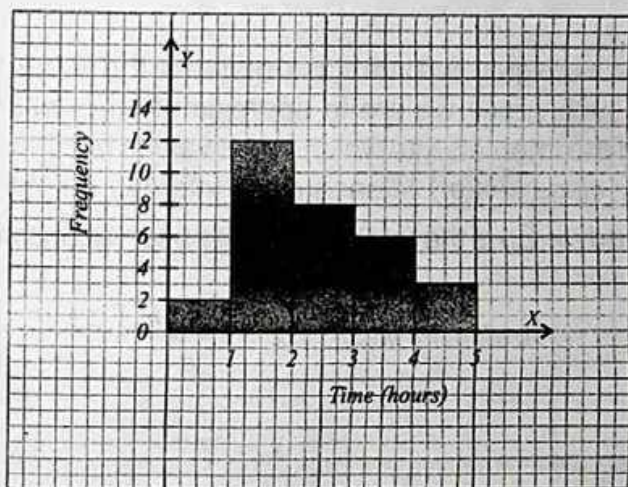
- (iii) The table shows the distribution of the average marks of 40 children in the end-of-year examinations.

|                     |      |       |       |       |        |
|---------------------|------|-------|-------|-------|--------|
| <i>Average mark</i> | 1-20 | 21-40 | 41-60 | 61-80 | 81-100 |
| <i>Frequency</i>    | 2    | 4     | 19    | 12    | 3      |

- 2- Following histogram shows the distribution of the times taken by 50 children to go to school. Construct a frequency table from the histogram.



- 3- Following histogram is based on the number of hours that 30 children spent watching television on a particular Saturday. Construct a frequency table from the histogram.



## 10.3 MEASURES OF CENTRAL TENDENCY

Measures of central tendency are summary statistics which measure the middle (or center) of the data. These are known as mean, median and mode.

- (i) The mean is the average of all observations in a set of data.
- (ii) The median is the middle observation in an ordered set of data.
- (iii) The mode is the most frequent observation in a data set.

### 10.3.1 Calculate Arithmetic Mean or Mean, Median, Mode, Geometric Mean, Harmonic Mean from Ungrouped/Grouped Data

#### Arithmetic Mean by definition

By definition the Arithmetic mean of an ungrouped data is obtained by adding all numbers (scores) in the set together and then the total is divided by the number of scores in that set.

$$\text{Arithmetic Mean} = \frac{\text{Sum of all scores}}{\text{Number of scores}}$$

Symbolically this is written as:  $\bar{x} = \frac{\sum x}{n}$ , Where the symbol " $\Sigma$ " indicates the "sum".

When the data are grouped into class intervals, the actual values (or data) are lost. In such cases we are to approximate the real values with the midpoints of the intervals into which these values fall. If ' $x$ ' represents the midpoint (or class center) of each class interval,  $f$  is the corresponding frequency and  $(n = \sum f)$  is the total number of observations in a set. Then

$$\text{Arithmetic mean} = \bar{x} = \frac{\sum (fx)}{n}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

### EXAMPLE-1

The total monthly income of 8 persons are Rs.3000, Rs.4000, Rs.3500, Rs.4500, Rs.3800, Rs.4200, Rs.3600 and Rs.5400. Find their arithmetic mean.

**SOLUTION:**  $x_1 = 3000$ ,  $x_2 = 4000$ ,  $x_3 = 3500$ ,  $x_4 = 4500$ ,  
 $x_5 = 3800$ ,  $x_6 = 4200$ ,  $x_7 = 3600$ ,  $x_8 = 5400$ ,

$$\text{Arithmetic mean} = \bar{x} = \frac{\sum x}{n}$$

$$= \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8}{8}$$

$$= \frac{3000 + 4000 + 3500 + 4500 + 3800 + 4200 + 3600 + 5400}{8}$$

$$= \frac{32000}{8}$$

$$\bar{x} = \text{Rs.}4000$$

**EXAMPLE-2**

Find the arithmetic mean for the following distribution showing marks obtained by 50 students in mathematics in the annual exam.

| Marks     | 20 - 24 | 25 - 29 | 30 - 34 | 35 - 39 | 40 - 44 | 45 - 49 | 50 - 54 |
|-----------|---------|---------|---------|---------|---------|---------|---------|
| Frequency | 1       | 4       | 8       | 11      | 15      | 9       | 2       |

**SOLUTION:**

| Marks   | Frequency ( $f$ ) | Class Mark ( $x$ ) | $fx$ |
|---------|-------------------|--------------------|------|
| 20 - 24 | 1                 | 22                 | 22   |
| 25 - 29 | 4                 | 27                 | 108  |
| 30 - 34 | 8                 | 32                 | 256  |
| 35 - 39 | 11                | 37                 | 407  |
| 40 - 44 | 15                | 42                 | 630  |
| 45 - 49 | 9                 | 47                 | 423  |
| 50 - 54 | 2                 | 52                 | 104  |

$$\begin{aligned}\bar{x} &= \frac{\sum (fx)}{n} = \frac{22 + 108 + 256 + 407 + 630 + 423 + 104}{50} \\ &= \frac{1950}{50} \\ &= 39 \text{ marks}\end{aligned}$$

**Arithmetic Mean using Deviation from Assumed Means**

The computation of mean using the formula  $\bar{x} = \frac{\sum fx}{\sum f}$  is easy, provided the values of  $x_i$  and  $f_i$  are not large. If  $x_i$  and  $f_i$  are large we can save a considerable time taking deviations from assumed means. If 'A' is an assumed mean (which may be any number) and  $D_i$  denotes the deviations of  $X_i$  from 'A' i.e.

$$D_i = X_i - A, \text{ then } X_i = D_i + A, \text{ we have}$$

$$\begin{aligned}\bar{X} &= A + \frac{\sum_{i=1}^k D_i}{n} \\ &= A + \frac{\sum_{i=1}^n D}{n} \quad (\text{ungrouped data})\end{aligned}$$

$$\begin{aligned}\text{and } \bar{X} &= A + \frac{\sum_{i=1}^k f_i D_i}{\sum f_i} \\ &= A + \frac{\sum f D}{\sum f} \quad (\text{grouped data})\end{aligned}$$

**EXAMPLE-1**

Find the arithmetic mean from the following values using formula:

$$\bar{X} = A + \frac{\sum D}{n}$$

184, 191, 172, 168, 187, 189, 196, 186, 193, 195.

**SOLUTION:**

Taking deviation from the assumed mean  $A=180$ ,  $D=X-A$ , we get

$$\begin{aligned}D &= 184 - 180, 191 - 180, 172 - 180, 168 - 180, 187 - 180, 189 - 180, \\ &\quad 196 - 180, 189 - 180, 193 - 180, 195 - 180\end{aligned}$$

$$= 4, 11, -8, -12, 7, 9, 16, 9, 13, 15$$

$$\bar{X} = A + \frac{\sum D}{n} = 180 + \frac{4+11-8-12+7+9+16+9+13+15}{10}$$

$$= 180 + \frac{64}{10}$$

$$= 180 + 6.4$$

$$= 186.4$$

**EXAMPLE-2**

Find the mean weight of 120 students for the distribution of weights in the following table. Using formula:

$$\bar{X} = A + \frac{\Sigma f D}{\Sigma f}$$

| Weight (Pounds)      | Class Mark (X) | Frequency (f) | $D = X - 144.5$                  | $fD$  |
|----------------------|----------------|---------------|----------------------------------|-------|
| 110 - 119            | 114.5          | 1             | - 30                             | - 30  |
| 120 - 129            | 124.5          | 4             | - 20                             | - 80  |
| 130 - 139            | 134.5          | 17            | - 10                             | - 170 |
| 140 - 149            | 144.5 ← A      | 28            | 0                                | 0     |
| 150 - 159            | 154.5          | 25            | 10                               | 250   |
| 160 - 169            | 164.5          | 18            | 20                               | 360   |
| 170 - 179            | 174.5          | 13            | 30                               | 390   |
| 180 - 189            | 184.5          | 6             | 40                               | 240   |
| 190 - 199            | 194.5          | 5             | 50                               | 250   |
| 200 - 209            | 204.5          | 2             | 60                               | 120   |
| 210 - 219            | 214.5          | 1             | 70                               | 70    |
| $n = \Sigma f = 120$ |                |               | $\Sigma f D = 1680 - 280 = 1400$ |       |

**SOLUTION:**  $\bar{X} = A + \frac{\Sigma f D}{\Sigma f}$

$$= 144.5 + \frac{1400}{120}$$

$$= 144.5 + 11.67$$

$$= 156.17 \text{ pounds}$$

## Median (ungrouped data)

The median is middle value of any set of data arranged in numerical order. In the set of  $n$  numbers, the median is located at the  $\frac{n+1}{2}$ th score.

The median is:

- the middle score for an odd number of scores arranged in numerical order.
- the average of the two middle scores for an even number of scores arranged in numerical order.

### EXAMPLE

For the data set 6, 2, 4, 3, 4, 5, 4, 5. Find the median.

#### SOLUTION:

2 3 4 4 4 5 5 6 (Check the scores are arranged in numerical order)

2 3 4 4 4 5 5 6 {Locate the position of the median using the rule  $\frac{n+1}{2}$  where  $n=8$ . This places the median as the 4.5th score = 4th score + 0.5 (5th score - 4th score)}

$$\text{Median} = \frac{4+4}{2}$$

$$= \frac{8}{2}$$

$$= 4$$

(obtain the average of two middle scores)

The median is 4.

## Median (grouped data)

The median is obtained by drawing an ogive of the data and estimating the median from the 50th percentage.

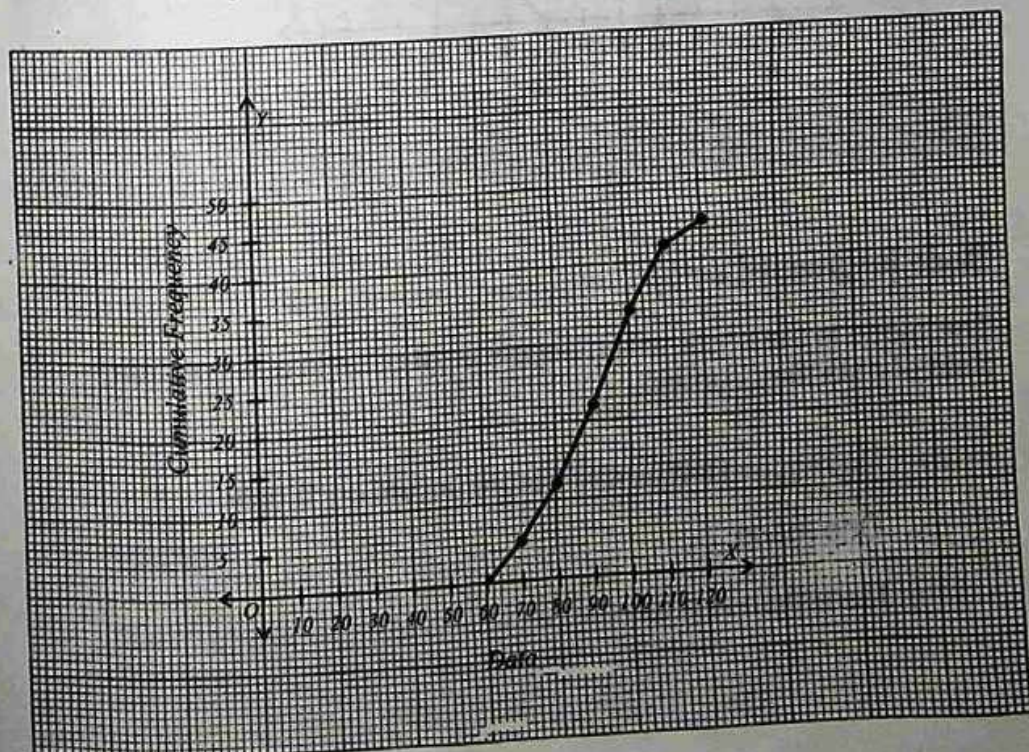
**EXAMPLE**

For the given data estimate the median.

| Class interval | Frequency<br>(f) | Cumulative Frequency<br>(Cf) |
|----------------|------------------|------------------------------|
| 60 - 70        | 5                | 5                            |
| 70 - 80        | 7                | 12                           |
| 80 - 90        | 10               | 22                           |
| 90 - 100       | 12               | 34                           |
| 100 - 110      | 8                | 42                           |
| 110 - 120      | 3                | 45                           |
| Total          | 45               |                              |

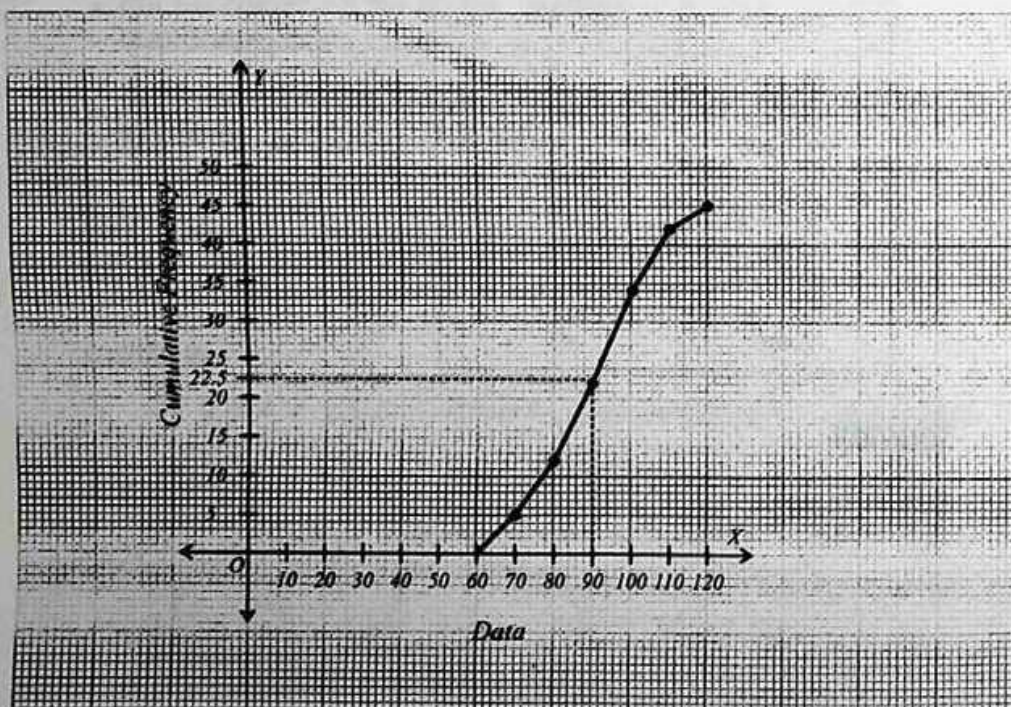
**SOLUTION:**

Draw the ogive.



Locate the middle of the cumulative frequency axis, that is 22.5 and label it.

Draw a horizontal line from the point to the ogive and then vertically to the horizontal axis. Read the value of the median from the  $x$ -axis. The median for the given data is approximately 90.



### Mode (ungrouped data)

The mode is the score which occurs most often in a set of data. Sets of data may contain:

- (i) No mode: That is, each score occurs once only.
- (ii) One mode
- (iii) More than one mode

**EXAMPLE**

For the data set 6, 2, 4, 3, 4, 5, 4, 5, find the mode.

**SOLUTION:**

We work through the set and see that value in the data which occurs the greatest number of times. The mod is 4.

$$\begin{array}{ccccccc}
 & & & & \downarrow & \downarrow & \\
 2 & 3 & 4 & 4 & 4 & 5 & 5 & 6 \\
 & & \uparrow & \uparrow & \uparrow & & & 
 \end{array}$$

**Mode (grouped data)**

We do not find a mode, because exact scores are lost. We can however, find a model class. This is the class interval that has the highest frequency.

**EXAMPLE**

Find the model class from the following table.

| Class interval | Frequency |
|----------------|-----------|
| 60 - 70        | 5         |
| 70 - 80        | 7         |
| 80 - 90        | 10        |
| 90 - 100       | 12        |
| 100 - 110      | 8         |
| 110 - 120      | 3         |
| Total          | 45        |

**SOLUTION:**

The model class is the class interval with the highest frequency.  
The model class is the 90 - 100 class interval.

### Geometric Mean (ungrouped data)

The geometric mean "G" of a set of  $n$  positive values  $x_1, x_2, \dots, x_n$  is the  $n$ th root of the product of the values. Thus

$$G = \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdots x_n}$$

$$= (x_1 \cdot x_2 \cdot x_3 \cdots x_n)^{\frac{1}{n}}$$

#### EXAMPLE

Geometric mean of 2, 4 and 8 is:

**SOLUTION:**

$$G = \sqrt[3]{2 \times 4 \times 8} = \sqrt[3]{64}$$

$$= [(4)^3]^{\frac{1}{3}}$$

$$= 4$$

### Geometric Mean (grouped data)

Let  $x_1, x_2, \dots, x_k$  represent the class marks in a frequency distribution with  $f_1, f_2, \dots, f_k$  as the corresponding class frequencies (where  $f_1 + f_2 + \dots + f_k = \Sigma f = n$ ). Since  $x_1$  occurs  $f_1$  times and so on.  $x_k$  occurs  $f_k$  times, then the product of original values may be written as :

$$\underbrace{x_1 \cdot x_1 \cdots x_1}_{f_1 \text{ times}} \quad \underbrace{x_2 \cdot x_2 \cdots x_2}_{f_2 \text{ times}} \quad \cdots \quad \underbrace{x_k \cdot x_k \cdots x_k}_{f_k \text{ times}}$$

or  $x_1^{f_1} \cdot x_2^{f_2} \cdots x_k^{f_k}$  and the geometric mean is:

$$G = \sqrt[n]{x_1^{f_1} \cdot x_2^{f_2} \cdots x_k^{f_k}}$$

$$= (x_1^{f_1} \cdot x_2^{f_2} \cdots x_k^{f_k})^{1/n}$$

This is sometimes called the weighted geometric mean with weights  $f_1, f_2, \dots, f_k$ .

### EXAMPLE

Find the geometric mean for the following frequency distribution.

|     |   |   |   |   |
|-----|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 |
| $f$ | 2 | 3 | 4 | 1 |

### SOLUTION:

$$\text{Here } \Sigma f = 2 + 3 + 4 + 1 = 10$$

$$\begin{aligned}
 G &= (x_1^{f_1} \cdot x_2^{f_2} \dots x_k^{f_k})^{1/n} \\
 &= ((1)^2 \cdot (2)^3 \cdot (3)^4 \cdot (4)^1)^{1/10} \\
 &= (1 \times 8 \times 81 \times 4)^{1/10} \\
 &= (2592)^{1/10} = 2.1946
 \end{aligned}$$

### Harmonic Mean (ungrouped data)

The harmonic mean  $H$  of a set of  $n$  values  $x_1, x_2, \dots, x_n$  is the reciprocal of the arithmetic mean of the reciprocal of the values.

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^n \left( \frac{1}{x_i} \right)} = \frac{n}{\sum \left( \frac{1}{x} \right)}$$

**EXAMPLE**

Find the harmonic mean of the values 2, 4 and 8.

$$\text{SOLUTION: } H = \frac{3}{\frac{1}{2} + \frac{1}{4} + \frac{1}{8}} = \frac{3}{\frac{4+2+1}{8}} = \frac{3}{\frac{7}{8}} = \frac{3 \times 8}{7} = \frac{24}{7} = 3.43$$

**The Harmonic Mean (grouped data)**

Let  $x_1, x_2, \dots, x_k$  represent the class marks and  $f_1, f_2, \dots, f_k$  as the corresponding class frequencies (where  $f_1 + f_2 + \dots + f_k = \sum f_i = n$ ).

Then the reciprocal of the class marks will be  $\frac{1}{x_1}, \frac{1}{x_2}, \dots, \frac{1}{x_k}$ . Since the

reciprocals occur with frequencies  $f_1, f_2, \dots, f_k$ . The total value of the

reciprocals in the first class is  $\frac{f_1}{x_1}$  in the second class is  $\frac{f_2}{x_2}$ , ... in the

$k^{\text{th}}$  class is  $\frac{f_k}{x_k}$ . Then  $\frac{f_1}{x_1} + \frac{f_2}{x_2} + \dots + \frac{f_k}{x_k} = \sum_{i=1}^k \frac{f_i}{x_i} = \sum f\left(\frac{1}{x}\right)$ .

The harmonic mean is defined as:

$$H = \frac{\sum_{i=1}^k f_i}{\sum_{i=1}^k f_i \left(\frac{1}{x_i}\right)} = \frac{\sum f}{\sum f\left(\frac{1}{x}\right)} = \frac{n}{\sum f\left(\frac{1}{x}\right)}$$

Sometimes this is also called weighted harmonic mean with weights  $f_1, f_2, \dots, f_k$ .

### 10.3.2 Properties of Arithmetic Mean

Following are the properties of Arithmetic mean.

(i) The sum of deviations of values from their mean is zero, symbolically:

$$\sum (x_i - \bar{X}) = 0 \text{ or } \sum f_i (x_i - \bar{X}) = 0$$

(ii) If  $n_1$  values have mean  $\bar{x}_1$ ,  $n_2$  values have mean  $\bar{x}_2, \dots, n_k$  values have mean  $\bar{x}_k$ , then mean of all the values is:

$$\bar{X} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2 + \dots + n_k \bar{x}_k}{n_1 + n_2 + \dots + n_k}$$

$$= \frac{\sum_{i=1}^k n_i \bar{x}_i}{\sum_{i=1}^k n_i}$$

(iii) The sum of squares of the deviations of the values  $x_i$  from any value "a" is minimum if and only if  $a = \bar{X}$ , symbolically  $\sum (x_i - a)^2$  is a minimum if and only if  $a = \bar{X}$ .

### 10.3.3 Weighted Mean and Moving Average

If  $x_1, x_2, \dots, x_k$  have weights  $w_1, w_2, \dots, w_k$ , then the weighted arithmetic mean or the weighted mean (denoted by  $\bar{x}_w$ ) is defined as:

$$\bar{x}_w = \frac{w_1 x_1 + w_2 x_2 + \dots + w_k x_k}{w_1 + w_2 + \dots + w_k}$$

$$= \frac{\sum_{i=1}^k w_i x_i}{\sum_{i=1}^k w_i}$$

## Moving Average

The average calculated by using “ $n$ ” consecutive values of the observed series, for example we have to calculate 3 year moving average, then we take first three values from the series, add them and place against the middle of its time period. Then repeat the operation by dropping 1st value from the beginning and including first value after the preceding total. Mathematical form:

$$a_1 = \frac{1}{3}(y_1 + y_2 + y_3)$$

$$a_2 = \frac{1}{3}(y_2 + y_3 + y_4) \text{ and soon.}$$

## Quartiles

We know that the median of an array is the middle value (or the mean of the two middle values). It divides a set of data into two equal parts. There are certain other values which divided a set of data into four equal parts called as first, second and third quartiles. These are denoted by  $Q_1$ ,  $Q_2$  and  $Q_3$  respectively.

The first and third quartiles are also called as lower and upper quartiles respectively. The second quartile is the median.

$$Q_1 = \text{value of } \left(\frac{n+1}{4}\right) \text{th item.}$$

$$Q_2 = \text{value of } 2\left(\frac{n+1}{4}\right) \text{th item or } \left(\frac{n+1}{2}\right) \text{th item.}$$

$$Q_3 = \text{value of } \frac{3(n+1)}{4} \text{th item.}$$

**EXAMPLE**

Find all the quartiles from the following marks obtained by 20 students in statistic test.

53 74 82 42 39 20 81 68 58 28

67 54 93 70 30 55 36 37 29 61

**SOLUTION:**

The marks of  $n=20$  students are arranged in ascending order as follows:

20 28 29 30 36 37 39 42 53 54

55 58 61 67 68 70 74 81 82 93

$$Q_1 = \text{value of } \left(\frac{n+1}{4}\right)\text{th or } \left(\frac{20+1}{4}\right)\text{th or } 5.25\text{th item from below.}$$

The value of the 5th item is 36 and that of the 6th item is 37. Thus the first quartile is a value of 0.25th of the way between 36 and 37, which is 36.25.

$$\text{Thus } Q_1 = 36.25$$

$$Q_2 = \text{value of } 2\left(\frac{n+1}{4}\right)\text{th or } \frac{2(20+1)}{4}\text{th or } \frac{21}{2}\text{th or } 10.5\text{th item from below.}$$

The value of the 10th item is 54 and that of the 11th item is 55. Thus the second quartile is a value of 0.5th of the way between 54 and 55, which is 54.5.

$$\text{Thus } Q_2 = 54.5$$

$$Q_3 = \text{value of } \frac{3(n+1)}{4}\text{th or } \frac{3(20+1)}{4}\text{th or } \frac{3 \times 21}{4}\text{th or } \frac{63}{4}\text{th or } 15.75\text{th item from below.}$$

The value of the 15th item is 68 and that of the 16th item is 70. Thus the first quartile is a value of 0.75th of the way between 68 and 70, which is  $68 + 2(0.75) = 69.5$ .

$$\text{Thus } Q_3 = 69.5$$

**EXAMPLE**

The marks of 100 students in an examination are given in the following table.

| Marks   | No. of Students | Cumulative Frequency |
|---------|-----------------|----------------------|
| 1 — 10  | 2               | 2                    |
| 11 — 20 | 12              | 14                   |
| 21 — 30 | 25              | 39                   |
| 31 — 40 | 29              | 68                   |
| 41 — 50 | 15              | 83                   |
| 51 — 60 | 10              | 93                   |
| 61 — 70 | 4               | 97                   |
| 71 — 80 | 3               | 100                  |

Use ogive to estimate the quartile, that is

- (i) The upper quartile.  
(ii) The lower quartile.

$$\frac{3}{4} \text{ of the total frequency} \\ = \frac{3}{4} \times 100 = 3 \times 25 = 75$$

From the curve the upper quartile = 44

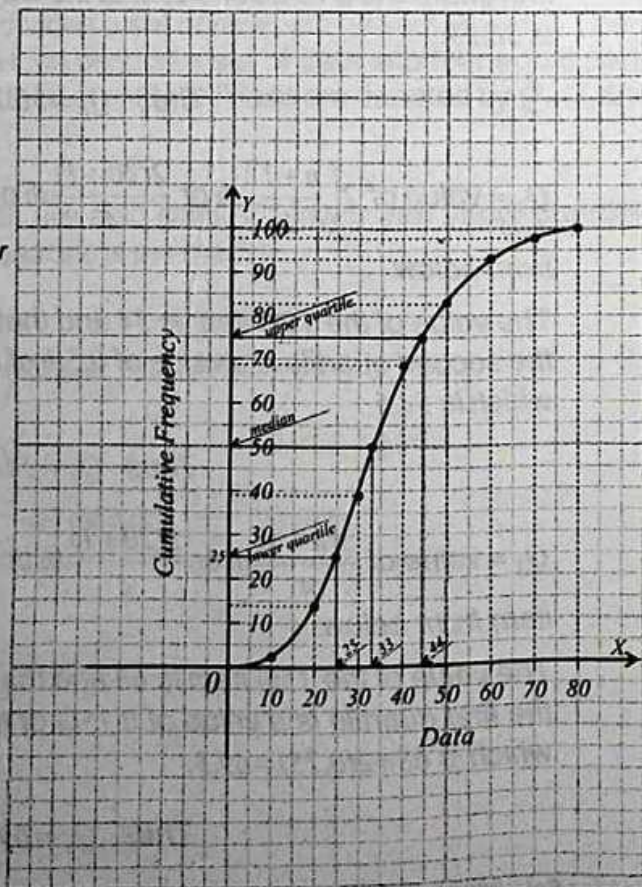
$$\frac{1}{4} \text{ of the total frequency} \\ = \frac{1}{4} \times 100 = 25$$

From the curve the lower quartile = 25

$$\frac{1}{2} \text{ of the total frequency} \\ = \frac{1}{2} \times 100 = 50$$

From the curve the median mark = 33

Two small squares along x-axis represents 10 units.  
Two small squares along y-axis represents 10 units.



### 10.3.4 Estimate Median, Quartile and Mode Graphically, Graphic Location of Median

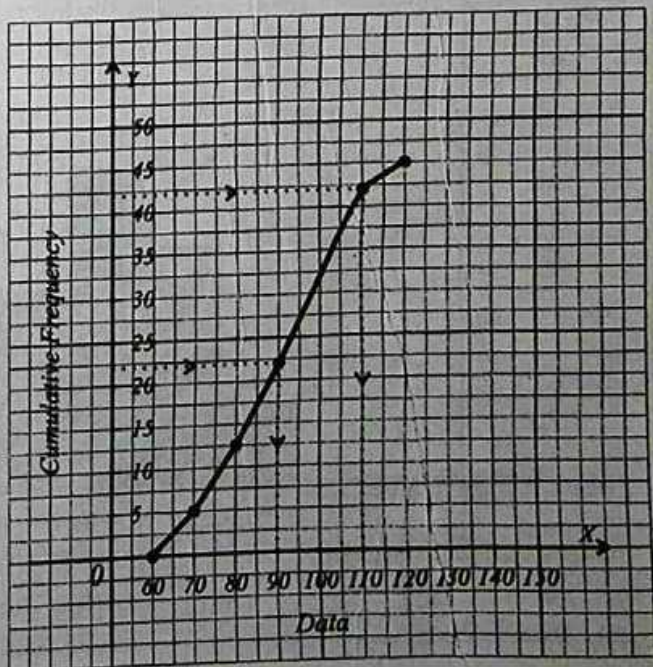
The approximate value of the median can be located from an ogive (a cumulative frequency polygon). In an ogive, the median is the value of  $x$  corresponding to  $\frac{n}{2}$ . Thus to locate median, we mark  $\frac{n}{2}$  along the

$y$ -axis and draw a perpendicular from this point of  $y$ -axis and extend it so as to intersect the ogive. Then we drop a perpendicular on the  $x$ -axis from the point of intersection. The point at which the perpendicular intersects the  $x$ -axis is the value of the median.

For the given data estimate the median, Mode

| Class-intervals | Frequency | Cumulative Frequency |
|-----------------|-----------|----------------------|
| 60 — 70         | 5         | 5                    |
| 70 — 80         | 7         | 12                   |
| 80 — 90         | 10        | 22                   |
| 90 — 100        | 12        | 34                   |
| 100 — 110       | 8         | 42                   |
| 110 — 120       | 3         | 45                   |
|                 | 45        | —                    |

- Draw the ogive
- Locate the middle of the cumulative frequency axis, that is 22.5 and label it.
- Draw a horizontal line from this point to the ogive and then vertically to the horizontal axis.
- Read the value of the median from the  $x$ -axis. The median for the given data is approximately 90.



**EXAMPLE**

Calculate mode by graphic method from the given data.

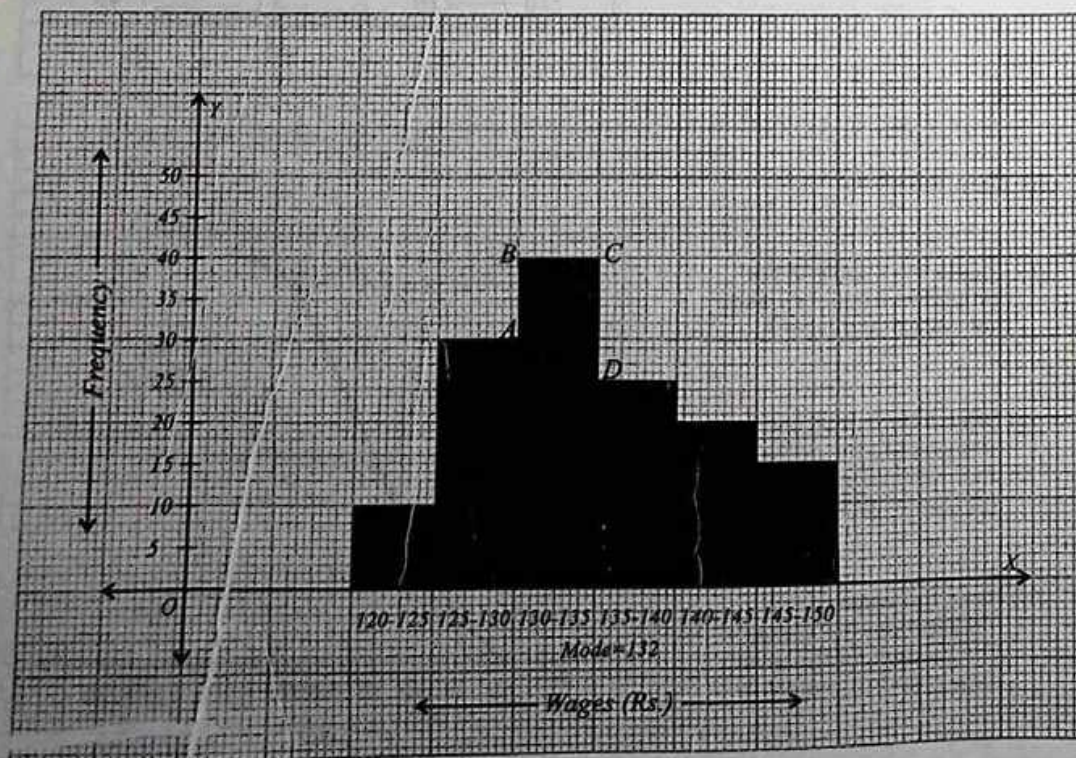
| Wages<br>Rs. | 120 - 125 | 125 - 130 | 130 - 135 | 135 - 140 | 140 - 145 | 145 - 150 |
|--------------|-----------|-----------|-----------|-----------|-----------|-----------|
| $f$          | 10        | 30        | 40        | 25        | 20        | 15        |

**SOLUTION:**

Wages in rupees are taken along  $x$ -axis while the frequency along the  $y$ -axis. One big square along  $x$ -axis represents 5 units while one big square along  $y$ -axis represents 10 units. We join the extreme ends of the bar with maximum frequency 30 and 25 as shown in the figure.

The two line-segments  $AC$  and  $BD$  cut each other at point  $P$ . We drop a perpendicular from  $P$  which meets  $x$ -axis at  $M$ . The value of  $M$  is 132.

Thus Mode = 132.



## Graphic Location of Quartiles

### EXAMPLE

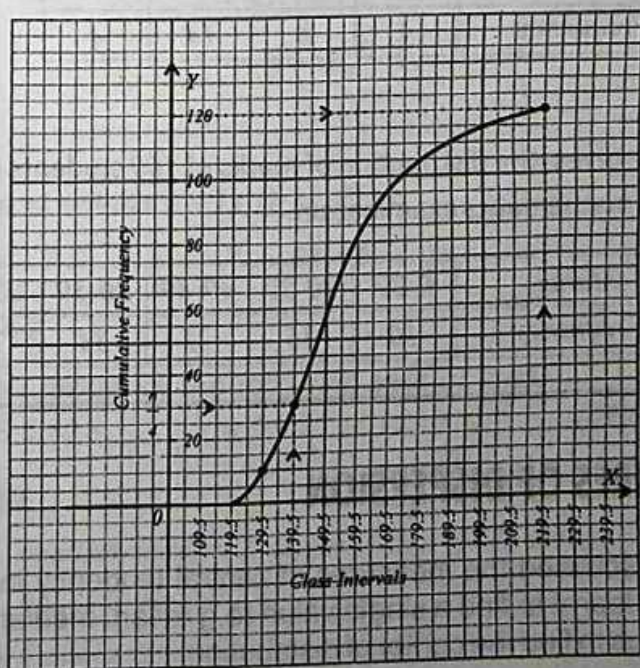
The following table shows a frequency distribution of grades on a final examination in mathematics. Locate quartiles graphically.

| Grade          | 90-99 | 80-89 | 70-79 | 60-69 | 50-59 | 40-49 | 30-39 |
|----------------|-------|-------|-------|-------|-------|-------|-------|
| No of Students | 9     | 32    | 43    | 21    | 11    | 3     | 1     |

### SOLUTION:

For the graphic location of quartile, students are shown in the following table. We write the given data as follow:

| Grade | Number of Students ( <i>f</i> ) | Class Intervals | Cumulative Frequency |
|-------|---------------------------------|-----------------|----------------------|
| 30-39 | 1                               | 29.5 - 39.5     | 1                    |
| 40-49 | 3                               | 39.5 - 49.5     | 4                    |
| 50-59 | 11                              | 49.5 - 59.5     | 15                   |
| 60-69 | 21                              | 59.5 - 69.5     | 36                   |
| 70-79 | 43                              | 69.5 - 79.5     | 79                   |
| 80-89 | 32                              | 79.5 - 89.5     | 111                  |
| 90-99 | 9                               | 89.5 - 99.5     | 120                  |



**EXERCISE - 10.3**

1- Represent the given data using Frequency polygon.

(i) The table shows the distribution of marks of 30 children in a test.

|                  |             |              |              |              |
|------------------|-------------|--------------|--------------|--------------|
| <i>Marks</i>     | <i>0-39</i> | <i>40-49</i> | <i>50-79</i> | <i>80-99</i> |
| <i>Frequency</i> | 8           | 8            | 10           | 4            |

(ii) The table shows the distribution of time (in seconds) taken for 40 children to complete the obstacle race.

|                      |             |              |              |              |
|----------------------|-------------|--------------|--------------|--------------|
| <i>Time (second)</i> | <i>1-40</i> | <i>41-50</i> | <i>51-60</i> | <i>61-70</i> |
| <i>Frequency</i>     | 8           | 15           | 7            | 10           |

(iii) The table shows the distribution of weights of 30 bags of chips from a fish and chip shop.

|                        |             |              |              |              |
|------------------------|-------------|--------------|--------------|--------------|
| <i>Weights (grams)</i> | <i>1-50</i> | <i>51-60</i> | <i>61-70</i> | <i>71-80</i> |
| <i>Frequency</i>       | 4           | 8            | 14           | 4            |

(iv) The table gives the distribution of marks of 100 students in an end-of-terms mathematics examination.

|                  |             |              |              |              |              |
|------------------|-------------|--------------|--------------|--------------|--------------|
| <i>Marks</i>     | <i>0-29</i> | <i>30-39</i> | <i>40-49</i> | <i>50-59</i> | <i>60-99</i> |
| <i>Frequency</i> | 10          | 15           | 25           | 34           | 16           |

**10.4 MEASURES OF DISPERSION**

We have discussed the measures of central tendency in the preceding section, we learnt to find a single value (e.g mean, median, mode) which would help us to find the center of distribution.

For this let us consider the following distributions.

|    |                                |                |
|----|--------------------------------|----------------|
| 1. | 63, 63, 63, 63, 63, 63, 63, 63 | $\bar{X} = 63$ |
| 2. | 62, 62, 62, 63, 63, 64, 64, 64 | $\bar{X} = 63$ |
| 3. | 48, 49, 57, 63, 69, 68, 74, 78 | $\bar{X} = 63$ |
| 4. | 40, 41, 47, 52, 62, 87, 88, 94 | $\bar{X} = 63$ |

The above mentioned distributions have the same mean i.e. 63, but these distributions differ greatly in their dispersion, i.e. the extent to which the values are spread out from the average.

Though there is a great difference in the dispersion of the values of the distribution, yet each of these distributions is described by the same mean, i.e. 63. We therefore need a measure to see how dispersed the data is.

The measures used for this purpose are called "Measures of Dispersion".

## Range

The simplest measure of dispersion is range. Range is the difference between the largest value and the smallest value in the data. If the smallest value is denoted by  $X_o$  and the largest value is denoted by  $X_m$ , then the range denoted by  $R$  is given by:

$$R = X_m - X_o$$

For Example

For the set of values 6, 8, 13, 11, 18, 27, 23

$$R = 27 - 6 = 21$$

## EXAMPLE

Find the range for the following sets of data.

(i) 2, 9, 3, 6, 12, 8

(ii) 6, 8, 10, 14, 13, 12

(iii) 10, 18, 9, 6, 22

## SOLUTION:

(i) smallest value = 2, largest value = 12, Range =  $12 - 2 = 10$

(ii) Range =  $14 - 6 = 8$

(iii) Range =  $22 - 6 = 16$

## Variance

Variance is defined as the square of the standard deviation, i.e. the mean of the squared deviation from the mean. It is given by:

$$Var = \frac{\sum (x - \bar{X})^2}{n} \text{ (for ungrouped data)}$$

$$Var = \frac{\sum f(x - \bar{X})^2}{\sum f} \text{ (for grouped data)}$$

## Standard Deviation

It is defined as the positive square root of the mean of the squared deviations of the values from their mean. The standard deviation of a set of  $n$  values,  $x_1, x_2, \dots, x_n$  is denoted by  $S$ , where:

$$S = \sqrt{\frac{\sum (x - \bar{X})^2}{n}} \text{ (for ungrouped data)}$$

$$S = \sqrt{\frac{\sum f(x - \bar{X})^2}{\sum f}} \text{ (for grouped data)}$$

$$\text{where } n = f_1 + f_2 + \dots + f_k = \sum f$$

### EXAMPLE-1

Find the standard deviation for the values, 1, 2, 3, 4, 6, 8, 11.

Also find the variance in this case.

**SOLUTION:** Here  $\bar{X} = \frac{1+2+3+4+6+8+11}{7} = \frac{35}{7} = 5$

$$S = \sqrt{\frac{\sum (x - \bar{X})^2}{n}}$$

$$= \sqrt{\frac{(1-5)^2 + (2-5)^2 + (3-5)^2 + (4-5)^2 + (6-5)^2 + (8-5)^2 + (11-5)^2}{7}}$$

$$= \sqrt{\frac{16+9+4+1+1+9+36}{7}} = \sqrt{\frac{76}{7}}$$

$$= \sqrt{10.86} = 3.295$$

Thus standard deviation =  $S = 3.63318$

$$\text{Variance} = \frac{\Sigma(x - \bar{X})^2}{n} = \frac{76}{7} = 10.86$$

### EXAMPLE-2

Find the standard deviation for the frequency distribution of marks obtained by 50 students in English at a certain examination. Also find the variance in this case.

| Marks     | 20-24 | 25-29 | 30-34 | 35-39 | 40-44 | 45-49 | 50-54 |
|-----------|-------|-------|-------|-------|-------|-------|-------|
| Frequency | 1     | 4     | 8     | 11    | 15    | 9     | 2     |

**SOLUTION:** Here

| Marks               | Frequency (f) | Class Mark (X)   | fx                 | $x - \bar{X}$ | $(x - \bar{X})^2$ | $f(x - \bar{X})^2$ |
|---------------------|---------------|------------------|--------------------|---------------|-------------------|--------------------|
| 20-24               | 1             | 22               | 22                 | -17           | 289               | 289                |
| 25-29               | 4             | 27               | 108                | -12           | 144               | 576                |
| 30-34               | 8             | 32               | 256                | -7            | 49                | 392                |
| 35-39               | 11            | 37               | 407                | -2            | 4                 | 44                 |
| 40-44               | 15            | 42               | 630                | 3             | 9                 | 135                |
| 45-49               | 9             | 47               | 423                | 8             | 64                | 576                |
| 50-54               | 2             | 52               | 104                | 13            | 169               | 338                |
| $n = \Sigma f = 50$ |               | $\Sigma f = 259$ | $\Sigma fx = 1950$ |               |                   | 2350               |

$$\bar{X} = \frac{\Sigma f x}{\Sigma f} = \frac{1950}{50} = 39 \text{ marks}$$

$$S = \sqrt{\frac{\Sigma f(x - \bar{X})^2}{n}} = \sqrt{\frac{2350}{50}} = \sqrt{47}$$

Thus standard deviation =  $S.D = 6.85$

$$\text{Variance} = \frac{\Sigma f(x - \bar{X})^2}{n} = \frac{2350}{50} = 47$$

**EXERCISE - 10.4**

- 1- Construct a cumulative frequency polygon (that is, an ogive) for the given data.

- (i) The table shows the distribution of weights (in kilograms) of 60 boys of ten years of age.

|                    |       |       |       |       |       |
|--------------------|-------|-------|-------|-------|-------|
| <i>Weight (kg)</i> | 31-36 | 37-39 | 40-42 | 43-45 | 46-54 |
| <i>Frequency</i>   | 8     | 10    | 18    | 12    | 12    |

- (ii) The table shows the distribution of times taken (in minutes) for 50 children of five years age to eat their school dinners.

|                       |     |     |     |       |       |       |       |
|-----------------------|-----|-----|-----|-------|-------|-------|-------|
| <i>Time (minutes)</i> | 4-5 | 6-7 | 8-9 | 10-11 | 12-15 | 16-19 | 20-29 |
| <i>Frequency</i>      | 5   | 4   | 10  | 9     | 6     | 6     | 10    |

- (iii) The table shows the distribution of the ages of people boarding buses at the bus station between 08.30 to 09.00 in the morning.

|                    |     |       |       |       |       |
|--------------------|-----|-------|-------|-------|-------|
| <i>Age (years)</i> | 0-9 | 10-19 | 20-29 | 30-39 | 40-69 |
| <i>Frequency</i>   | 10  | 20    | 30    | 20    | 15    |

- (iv)

|                  |      |       |       |       |       |
|------------------|------|-------|-------|-------|-------|
| <i>Classes</i>   | 5-10 | 10-15 | 15-20 | 20-25 | 25-30 |
| <i>Frequency</i> | 10   | 15    | 20    | 30    | 15    |

- (v) The table gives the distribution of weights (kilograms) of 100 people.

|                           |       |       |       |       |       |         |
|---------------------------|-------|-------|-------|-------|-------|---------|
| <i>Weight (kilograms)</i> | 50-59 | 60-69 | 70-79 | 80-89 | 90-99 | 100-109 |
| <i>Frequency</i>          | 15    | 30    | 35    | 15    | 3     | 2       |

## Review Exercise - 10

## 1- Encircle the correct answer.

- (i) When a bar graph is constructed, so that the area of each bar is proportional to the number of items in each group is called.  
 (a) curve (b) ogive (c) histogram (d) bar diagram
- (ii) The summary statistics which measure the middle (or center) of the data is called:  
 (a) mean (b) mode  
 (c) median (d) all of these
- (iii) If all numbers in a set are added together and then the total is divided by the number of scores in the set is called  
 (a) mean (b) mode (c) median (d) weighted mean
- (iv) The middle values of data arranged in numerical order is called  
 (a) mode (b) median (c) mean (d) geometric mean
- (v) The score which occurs most often in a set of data is called  
 (a) mode (b) mean (c) median (d) geometric mean
- (vi)  $\bar{X} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$  is called  
 (a) mean value of  $x_1, x_2, \dots, x_n$  (b) arithmetic mean  
 (c) geometric mean (d) weighted mean
- (vii)  $H = \frac{n}{\sum \left( \frac{1}{x} \right)}$  is called  
 (a) harmonic mean (b) mode (c) mean (d) arithmetic mean
- (viii)  $\bar{X}_w = \frac{\sum wx}{\sum w}$  is called  
 (a) arithmetic mean (b) weighted mean  
 (c) geometric mean (d) mean
- (ix)  $\sum (x_i - \bar{X}) = 0$  is one of the properties of  
 (a) arithmetic mean (b) geometric mean  
 (c) harmonic mean (d) mode

**2- Fill in the blanks.**

- (i) When a bar graph is constructed, so that the area of each bar is proportion to the number of items in each group is called a \_\_\_\_\_.
- (ii) The summary statistic which measure the middle (or center) for the data is called \_\_\_\_\_.
- (iii) If all numbers in a set are added together and then the total is divided by the number of scores in the set is called \_\_\_\_\_.
- (iv) The middle value of data arranged in numerical order is called \_\_\_\_\_.
- (v) The score which occurs most often in a set of data is called \_\_\_\_\_.
- (vi)  $\bar{X} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$  is called the \_\_\_\_\_.
- (vii) The  $n$ th root of the product of the values of a set of  $n$  positive values is called \_\_\_\_\_.
- (viii)  $H = \frac{n}{\sum \left( \frac{1}{x} \right)}$  is called the \_\_\_\_\_.
- (ix)  $\bar{X}_w = \frac{\sum wx}{\sum w}$  is called the \_\_\_\_\_.
- (x)  $\sum (x_i - \bar{X}) = 0$  is one of the properties of \_\_\_\_\_.

**3-** Find the standard deviation of the values 2, 3, 6, 8, 11.

**4-** Find the standard deviation and variance for a set of ungrouped values, when  $n = 15$ ,  $\sum (x - \bar{x})^2 = 1444$

**5-** For the data 3, 5, 6, 8, 8, 9, 10, find

(i) Mean (ii) Median (iii) Mode

**6-** Find the mean, median and mode for the set of the value 4, 6, 7, 4, 8, 9, 7, 10.

**SUMMARY**

- ✦ When a bar chart is construed, so that the area of each bar is proportional to the number of items in each group is called a histogram.
- ✦ Cumulative frequency is a running total of class frequencies.
- ✦ When the cumulative frequencies are plotted against the end point of their respective class intervals and joined together, the resultant graph is called a cumulative frequency polygon or ogive.
- ✦ These are summary statistics which measure the middle (or center) of the data.
- ✦ To obtain the mean of a data, all numbers in the set are added together and then the total is divided by the number of scores in that set.
- ✦ The middle value of data arranged in numerical order is called median.
- ✦ The mode is the score which occurs most often in a set of data.
- ✦ Arithmetic mean of an ungrouped data is obtained by adding all numbers (scores) in the set together and then the sum is divided by the number of scores in that set.
- ✦ The geometric mean " $G$ " of a set of  $n$  positive values  $x_1, x_2, \dots, x_n$  is the  $n$ th root of the product of the values.
- ✦ The harmonic mean  $H$  of a set of  $n$  values  $x_1, x_2, \dots, x_n$  is the reciprocal of the arithmetic mean of the reciprocal of the values.

## SUMMARY

- ✦ Variance is defined as the square of the standard deviation, i.e. the mean of the squared deviation from the mean.
- ✦ The simplest measure of dispersion is range. Range is the difference between the largest value and the smallest value in the data.
- ✦ The average calculated by using "n" consecutive values of the observed series, for example we have to calculate 3 years moving average, then we take first three values from the series, add them and place against the middle of its time period. Then repeat the operation by dropping 1st value from the beginning and including first value after the preceding total.
- ✦ Standard deviation is defined as the positive square root of the mean of the squared deviations of the values from their mean, The standard deviation of a set of  $n$  values,  $x_1, x_2, \dots, x_n$  is denoted by  $S$ .
- ✦ If  $x_1, x_2, \dots, x_k$  have weights  $w_1, w_2, \dots, w_k$ , then the weighted arithmetic mean or the weighted mean (denoted by  $\bar{x}_w$ ) is defined as:

$$\bar{x}_w = \frac{\sum_{i=1}^k w_i x_i}{\sum_{i=1}^k w_i}$$

# Answers

## Exercise 1.1

- 1- (i)  $\frac{19}{20}$  (ii)  $\frac{13}{20}$  (iii)  $\frac{3}{4}$  (iv)  $\frac{1}{4}$  (v)  $\frac{14}{25}$  (vi)  $\frac{12}{25}$  (vii)  $\frac{2}{25}$  (viii)  $\frac{67}{200}$   
 (ix)  $\frac{3}{8}$  (x)  $\frac{7}{8}$  (xi)  $\frac{21}{400}$  (xii)  $\frac{17}{40}$
- 2- (i) 75% (ii) 60% (iii) 16% (iv) 65% (v) 124% (vi) 52.5% (vii) 38.3%  
 (viii) 266.66% (ix) 160% (x) 87.5% (xi) 62.5% (xii) 37.5%
- 3- (i) 0.47 (ii) 0.58 (iii) 0.92 (iv) 0.08 (v) 0.12 (vi) 1.20  
 (vii) 1.80 (viii) 1.45 (ix) 0.055 (x) 0.0533 (xi) 0.486 (xii) 0.583
- 4- (i) 50% (ii) 90% (iii) 125% (iv) 139% (v) 172% (vi) 22%  
 (vii) 264% (viii) 341% (ix) 84.5% (x) 178% (xi) 158% (xii) 6.5%
- 5- (i) 80% (ii)  $\frac{2}{5}$ , 0.4 (iv)  $\frac{31}{50}$ , 62% (v)  $\frac{11}{25}$ , 0.44

## Exercise 1.2

- 1- 55% 2- 18% 3- 12% 4- (i)  $P=208, C=68, I=48, B=76$  (ii) 19%  
 5- Math 6- 27.78% 7- 500 pages 8- Rs.7500 9- 20%

## Exercise 1.3

- 1- (i) 4:1 (ii) 4:1 (iii) 1:4 (iv) 1:1 (v) 5:4 (vi) 6:11
- 2- (i) 10:9 (ii) 16:15 (iii) 25:21 (iv) 13:6 (v) 4:1  
 (vi) 1:50 (vii) 3:4 (viii) 3:7 (ix) 6:5
- 3- (i) 14:9 (ii) 3:25 (iii) 9:28
- 4- (i) 15:23 (ii) 23:17 (iii) 23:12 (iv) 12:17 (v) 4:5 (vi) 17:15

## Exercise 1.4

- 1- 1:1 2- 16:15 3- 3:4 4- (i) 1:3 (ii) 1:3 (iii) 1:9 5- 2:1
- 6- (i) 3:4 (ii) 2:3 (iii) 1:2 7- 3:7 8- (i) 8:7 (ii) 7:1 (iii) 8:1
- 9- (i) 3:4 (ii) 9:16 10- (i) 2:1 (ii) 1:3

## Exercise 1.5

- 1- 20                      2- 9 suits                      3- 5 liters                      4- 75 km / hr  
 5- 8 days                      6- 120 bicycles                      7- 864 fans                      8- 720 soaps  
 9- Rs. 2240                      10- Rs. 18200                      11- 110 cows                      12- 2700 bottles

## Review Exercise -1

### 1- Encircle the Correct Answer.

- (i) b    (ii) b    (iii) a    (iv) d    (v) c    (vi) b    (vii) c    (viii) a    (ix) a    (x) b

### 2- Fill in the blanks.

- (i) 450    (ii)  $\frac{3}{20}$     (iii) 28 %    (iv) 67%    (v) 29 %    (vi) antecedent  
 (vii) consequent    (viii) extremes    (ix) means    (x) 10 : 9  
 3- 30%    4- Rs. 7000    5- (i) 2:1    (ii) 22 : 5    (iii) 11:3  
 6- 2 : 3    7- 16 days    8- Rs. 9720

## Exercise 2.1

- 1- Rs. 27750    2- Rs. 7500    3- Rs. 15,000    4- Rs. 36,250    5- Rs. 60,000  
 6- Rs. 17500    7- widow Rs. 93750; son Rs. 262500; daughter Rs. 131250  
 8- widow Rs. 50,000; daughter Rs. 87500    9- Rs. 3,75,000  
 10- widow Rs. 2,50,000; son Rs. 8,75,000  
 11- widow Rs. 60,000; son Rs. 84,000, daughter Rs. 42000    12- 100,000

## Review Exercise 2

### 1- Encircle the Correct Answer.

- (i) a    (ii) c    (iii) a    (iv) a    (v) c    (vi) b    (vii) a    (viii) c    (ix) a    (x) a

### 2- Fill in the blanks.

- (i) 2.5 %    (ii) 10 %    (iii) 5 %    (iv) Rs. 5000    (v) Rs. 10,000  
 (vi)  $\frac{1}{8}$     (vii)  $\frac{1}{4}$     (viii)  $\frac{1}{2}$     (ix) 2:1    (x)  $\frac{2}{3}$   
 3- Rs. 37500    4- Rs. 24500    5- widow : Rs. 5,62,500, Sons : Rs. 19,68,750  
 6- widow : Rs. 6,00,000, Son : Rs. 8,40,000 each, daughter Rs. 4,20,000 each.

### Exercise 3.1

- 1- (i) Rs.1045 (ii) Rs.1463 (iii) Rs.10560 (iv) Rs.119700 (v) Rs.494.40 (vi) Rs.729.60  
 2- Rs.2,73,000 3- (i) Rs.640 (ii) Rs.925 (iii) Rs.1560 (iv) Rs.3000 (v) Rs.75  
 4-  $12\frac{1}{2}\%$  5- 25% 6- Rs.1260 7- Rs.213 8- Rs.1215

### Exercise 3.2

- 1- (i) Rs.684.32 (ii) Rs.2622 (iii) Rs.364.08 2- (i) Rs.560 (ii) Rs.975 (iii) Rs.2500  
 3- 5% 4- 7.5% 5- Rs.400 6- Rs.920 7- Rs.11730.60 8- 20%

### Exercise 3.3

- 1- 60,000; 40,000; 1,00,000  
 2- Ali's share Rs.22,500; Daniyal's share Rs.33,750; Abdullah's share Rs.56,250  
 3- 180; 240; 300  
 4- Profit 1st partner Rs.1,00,000; 2nd partner Rs.1,50,000; 3rd partner Rs.3,50,000  
 Amount 1st partner Rs.600,000. 2nd partner Rs.900,000. 3rd partner Rs.2100,000  
 5- 39200 6- (a) Rs.70 Total amount Rs.154

### Review Exercise 3

#### 1- Encircle the Correct Answer.

- (i) c (ii) b (iii) b (iv) b (v) a

#### 2- Fill in the blanks.

- (i) cost price (ii) sale price (iii) profit (iv)  $\frac{\text{loss}}{\text{CP}} \times 100$  (v)  $100 - \text{discount}\%$   
 3- 5.5% 4- 16.6% 5- 9000; 15000; 9000 6- (i) 60000 (ii) 36000

### Exercise 4.1

- 1- 161.406 2- Rs.111724.5 3- Loss Rs.15  
 4- (i) Rs.12615 (ii) Rs.12517.42 (iii) Rs.97.58 5- (i) Rs.4.63 (ii) 1.89375 Riyals  
 6- 15772 7- Profit = Rs.1099.58 8- Profit = Rs.4759.5  
 9- T.T buying is applicable for Rs.10055.20 (ii) Rs.1857.37

### Exercise 4.2

- 1- Rs.1100 2- Rs.132 3- 10 years 4- 4.25% per year 5- Rs.1019.50 6-  $7\frac{1}{2}\%$  P.A.  
 7- (a) Rs.6720, Rs.18720 (b) 4 years, Rs.720 (c) Rs.300, Rs.408 (d) 4%, Rs.4200 (e) Rs.3600, 5%  
 (f) 7%, Rs.1989 (g) 6%, Rs.540 (h) Rs.1200,  $1\frac{1}{2}$  years 8- Rs.16 9- Rs.6180 10- Rs.20,000  
 11- (i) Rs.94.50 (ii) Rs.257.34 (iii) Rs.1244.03 (iv) Rs.149.4 (v) Rs.2422.94  
 12- Rs.5829.57 13- Rs.903.13 14- 9550.87

### Exercise 4.3

- 1- Rs.1520.50      2- Rs.88.50      3- Rs.65000      4- Rs.7084      5- Rs.3600  
 6- Rs.35000      7- Rs.157.49      8- Rs.3040      9- Rs.6480

### Exercise 4.4

1-

| Amount of policy | Yearly premium | Half yearly premium | Quarterly premium | Monthly premium |
|------------------|----------------|---------------------|-------------------|-----------------|
| (i) 50,000       | 2250 + 125     | 1235                | 641.50            | 213.75          |
| (ii) 100,000     | 4500 + 200     | 2444                | 1269.00           | 423.00          |
| (iii) 150,000    | 6750 + 200     | 3614                | 1876.50           | 625.30          |
| (iv) 200,000     | 9000 + 200     | 4784                | 2484.00           | 828.00          |

- 2- Rs.(50,000 + 1,20,000 + 4500) = Rs.1,74,500      3- Rs.(150000 + 135000 + 121500 + 0) = Rs.406500  
 4- Rs.700,000, Rs.159050      5- Rs.4,32,250      6- Rs.(36,125 + 32512.50) = Rs.68,637.50  
 7- Rs.(26,250 + 23,625 + 21,262.50 + 19136.25) = 90273.75 benefit Rs.9726.25  
 8- Rs.800,000 (approx.), Rs.23400, Zero.

### Exercise 4.5

- 1- (a) i. Rs.90 ii. 25% (b) i. Rs.150 ii.  $16\frac{2}{3}\%$  (c) i. Rs.3000 ii. 12%      2- (i) Rs.21 (ii) Rs.288  
 3- (a) i. Rs.236 ii. 18% (b) i. Rs.517.50 ii. 15% (c) i. Rs.1960 ii. 22.5%  
 4- (a) i. Rs.63 ii. 7% (b) i. Rs.200 ii. 15% (c) i. Rs.75 ii.  $16\frac{2}{3}\%$   
 5- (i) Rs.3716.80 (ii) Rs.516.80 (iii) 16.15%

### Review Exercise 4

#### 1- Encircle the Correct Answer.

- (i) a      (ii) b      (iii) a      (iv) c      (v) a      (vi) a      (vii) a      (viii) c      (ix) a

#### 2- Fill in the blanks.

- (i) cheque      (ii) pay order      (iii) ATM      (iv) profit      (v) rate  
 (vi) time      (vii) mark-up      (viii) premium      (ix) insurer      (x) insured

- 3- Rs.515,850      4- Rs.700,000, Rs.159,050      5- Markup Rs.2,770      Principal Rs.27,230

### Exercise 5.1

- 1- Rs.28000      2- Rs.6400, Rs.46400      3- Rs.18,00,000, Rs.28,00,000  
 4- Rs.2,40,000, Rs.3,20,000      5- Rs.11250, Rs.13950      6- Rs.16350      7- Rs.21375  
 8- Rs.6875

## Exercise 5.2

- 1- (i) Rs.5124.29 (ii) Rs.9168.056 (iii) Rs.3622.84 (iv) Rs.950.12  
 (v) Rs.14771.5 (vi) Rs.9848.75
- 2- (i) Rs.1155.39 (ii) Rs.718.09 (iii) Rs.1540 (iv) Rs.1021.41
- 3- (i) Rs.3153.50 (ii) Rs.3808.00 (iii) Rs.4462.50 (iv) Rs.1606.50  
 (v) Rs.2856 (vi) Rs.1874.25

## Exercise 5.3

- 1- 48 hrs, Rs.1920    2- Rs.4800    3- Rs.17640    4- Rs.26500    5- Rs.41,500
- 6- (i) Rs.1975 (ii) Rs.2725 (iii) Rs.2875    7- Rs.10,800

## Review Exercise 5

### 1- Encircle the Correct Answer.

- (i) a    (ii) b    (iii) a    (iv) b    (v) a    (vi) a    (vii) c

### 2- Fill in the blanks.

- (i) tax    (ii) direct tax    (iii) indirect tax    (iv) sales tax    (v) exercise duty  
 (vi) property tax    (vii) income tax    (viii) Rs.90,000    (ix) Rs.1200    (x) 4,50,000
- 3- Rs.19200    4- Rs.24750    5- Rs.15867.37
- 6- (i) Rs.1017.48    (ii) Rs.736.02    (iii) Rs.989.23    (iv) Rs.1055    7- Rs.70500

## Exercise 6.1

- 1- (i) Radical  $\sqrt{3}$ ; Radicand 3    (ii) Radical  $\sqrt{a}$ ; Radicanda    (iii) Radical  $\sqrt{11}$ ; Radicand 11  
 (iv) Radical  $\sqrt{6}$ ; Radicand 6    (v) Radical  $\sqrt{5}$ ; Radicand 5    (vi) Radical  $\sqrt{13}$ ; Radicand 13
- 2- (i)  $a^{3/2}$     (ii)  $a^{3/5}$     (iii)  $a^{-k/p}$     (iv)  $a^{-k/h}$
- 3- (i)  $\sqrt{25}, 5$     (ii)  $\sqrt[3]{64}, 4$     (iii)  $\sqrt[4]{81}, 3$     (iv)  $\sqrt[5]{27}, 3$     (v)  $\sqrt[3]{(27)^2}, 9$     (vi)  $\sqrt[3]{\frac{1}{8}}, \frac{1}{2}$   
 (vii)  $\sqrt{(1000)^2}, 100$     (viii)  $\sqrt{64}, 8$
- 4- (i)  $a^8$     (ii)  $a^5$     (iii)  $3a^3$     (iv)  $2a^3$     (v)  $x^8$     (vi)  $3x^3$     (vii)  $5x^3, y^5$   
 (viii)  $(8+y)^{7/2}$     (ix)  $2x^{1/2}y^{3/2}$     (x)  $\frac{x^{5/4}y^{3/2}}{z^{1/2}}$     (xi)  $\frac{2x^{1/3}}{(x+y)^{1/3}}$     (xii)  $\frac{y^{n/p}}{a^{m/p}}$
- 5- (i)  $\sqrt{21}$     (ii)  $\sqrt[3]{512}$     (iii)  $\sqrt[4]{2187}$     (iv)  $\frac{1}{\sqrt[4]{2}}$     (v)  $\sqrt[3]{59}$     (vi)  $\sqrt{\frac{1}{3}}$     (vii)  $\sqrt[12]{a^{11}}$   
 (viii)  $\sqrt{x^{247}y}$     (ix)  $y\sqrt{x^9}$     (x)  $\sqrt[4]{x}$     (xi)  $\sqrt[4]{x^4y^3}$     (xii)  $\sqrt[24]{a^{11}b^{11}}$     (xiii)  $x^{29/60}y^{11/10}$

## Exercise 6.2

| Q.No.    | (i) | (ii) | (iii) | (iv)  | (v) | (vi) |
|----------|-----|------|-------|-------|-----|------|
| Base     | $x$ | $x$  | $4y$  | $x-2$ | $x$ | $x$  |
| Exponent | 3   | 9    | 3     | 3     | 5   | 2    |

- 2-  $a^6 b^9$     3-  $\frac{x^{4/3}}{y}$     4-  $\frac{1}{y^{2a+2b}}$     5-  $\frac{1}{2x^{1/2} y^{1/2}}$     6-  $\frac{4}{9a^2 x^2}$     7-  $16ac^4$   
 8-  $\frac{3}{ba^{1/2}}$     9-  $\frac{a^{11/2} c^{3/2}}{b^3}$     10-  $b^3$     11-  $\frac{x^{11} y^6}{16}$     12-  $ab^{13}$     13-  $a^4 b^3 c$   
 14-  $\frac{4b}{9ac^9}$     15-  $2^{16} \times 3^8$     16-  $2^{14} \times 3^7$     17-  $\frac{1}{2a^4 b^6}$     18-  $2187$   
 19-  $\frac{3^7}{2^6}$     20-  $\frac{2^3}{3^6}$     21-  $\frac{125}{2187}$     22-  $a^{7/6} b^{11/12}$     23-  $a^{5/6} b^{3/2}$     24-  $a^3 b^2 c^{3/2}$   
 25-  $a^{1/2} b^{23/2}$     26-  $a^{11/12}$

- 27- (i)  $4^{4/3}$     (ii)  $2^{1/2}$     (iii)  $10x^{n/15}$     (iv)  $x^{11/20}$     (v)  $2y^{5/7}$     (vi)  $5x^2$   
 28- (i)  $ab^{3/2}$     (ii)  $x^{4/5} y^{5/9}$     (iii)  $6a^{8/5} b^{17/15}$     (iv)  $2x^{19/28} y^{2/5}$     (v)  $x^{19/6} y^{5/6} z^{5/6}$   
 29- (i)  $3^{1/6}$     (ii)  $x^{4/5}$     (iii)  $\frac{1}{2} x^{3/20}$     (iv)  $\frac{5}{4} y^{7/20}$     (v)  $x^{5/3} y^{7/5}$     (vi)  $a^{7/45} b^{4/15}$   
 (vii)  $2x^{2/15} y^{3/4}$     (viii)  $\frac{1}{4} a^{11/20} b^{7/20}$

## Exercise 6.3

- 1-  $5.1 \times 10^{-2}$     2-  $8.899 \times 10^1$     3-  $4.24 \times 10^{-1}$     4-  $2.566324 \times 10^6$   
 5-  $7.5 \times 10^{-7}$     6-  $8600$     7-  $0.00001345$     8-  $0.0000000051$   
 9-  $0.0000000525$     10-  $0.0006365$     11-  $4.8 \times 10^{-2}$     12-  $1.044 \times 10^6$   
 13-  $3.126 \times 10^{-4}$     14-  $4.5 \times 10^3 \text{ m}$     15-  $6.4 \times 10^6$

### Exercise 6.4

- 1- (i) 3 (ii) 3 (iii) 0 (iv) 1 (v) 2 (vi) 4  
 2- (i) 1.7170 (ii) 0.7996 (iii) 1.7873 (iv) 3.7559 (v) 5.4771  
 3- (i) 0.8044 (ii) 1.8044 (iii) 3.8044 4- (i) 0.01090 (ii) 1.444 (iii) 26530

### Exercise 6.5

- 1- (i) 2 (ii) 2 (iii)  $\frac{5}{2}$  2- (i) 1 (ii) 3 (iii) 4 (iv) 2 (v) 3  
 3- (i)  $\log \frac{1.3472 \times 22.79}{5}$  (ii)  $\log \frac{22.13 \times 0.354 \times 7}{3}$  (iii)  $\log \frac{57.86 \times 4.385}{2.391 \times 3.072}$   
 4- (i) 1.923 (ii) 19.19 (iii) 0.9945  
 8- (i) 0.3291 (ii) 19.19 (iii) 14.139 (iv) 0.3466 (v) 160.4 (vi) 48.01  
 (vii) 1964 (viii) 2.082 9- 27.71 10- 67.39 11- 0.1224

### Review Exercise 6

#### 1- Encircle the Correct Answer.

- (i) b (ii) a (iii) b (iv) c (v) a (vi) b (vii) a (viii) b (ix) d (x) a

#### 2- Fill in the blanks.

- (i) radical (ii) radicand (iii) exponent (iv) base (v) common logarithm  
 (vi) characteristic (vii) mantissa

- 3- (i)  $x^{3/2} y^{-5/6}$  (ii)  $a^{7/8} b^{7/12}$  4- (i)  $x^{1/3} y^{19/24}$  (ii)  $\frac{1}{40}$  5- (i) 0.2983 (ii) 5.158 (iii) 0.2465

### Exercise 7.1

- 1- (i) 4, 5, 6 (ii) -1, 8, -27 (iii) 8, 11, 14 (iv)  $\frac{2}{7}, \frac{1}{3}, \frac{4}{11}$  (v)  $1, \frac{1}{9}, \frac{1}{25}$  (vi) 4, 5, 6  
 (vii)  $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$  (viii) -2, 1, 4 (ix) 1, 3, 12  
 2- (i) 51 (ii) 20160 (iii)  $\frac{1}{243}$  (iv) 15 (v)  $\frac{5}{11}$  (vi) 17  
 3- (i) 34, 42, 51, 61 (ii) 63, 127, 255, 511 (iii) 112, 288, 704, 1664  
 (iv) 25, 27, 30, 33 (v) 20, 24, 28, 32 (vi) 12, 14, 16, 18

### Exercise 7.2

- 1- (i) 243 (ii) -58 (iii) -82 (iv) 74 (v) -74  
 2- (i) 16 (ii) 132 (iii) 4 (iv) 65 (v) 79 (vi) 193 3- 12 4- 39  
 5- 42 6- 20 7-  $12-11x$  8-  $3n+24$  9-  $\left(\frac{3}{3n+1}\right)^2$  10-  $-2, 1, 4, 7, 10, 13, \dots$

### Exercise 7.3

- 1- (i) 2 (ii)  $x+3$  (iii)  $2\sqrt{7}$  (iv)  $x^2+1$  2- 0, 9 3- 13, 15, 17  
 4-  $\frac{9\sqrt{2}}{4}, \frac{7\sqrt{2}}{2}, \frac{19\sqrt{2}}{4}$  5-  $\frac{38}{7}, \frac{41}{7}, \frac{44}{7}, \frac{47}{7}, \frac{50}{7}, \frac{53}{7}$  6-  $\frac{17}{2}, 9, \frac{19}{2}, 10, \frac{21}{2}, 11, \frac{23}{2}$   
 7- 15 8- 70 9- 1, 17

### Exercise 7.4

- 1- 8192 2- 118098 3- 48 4- 324 5- (i) 3 (ii) 5 (iii) 10  
 6- 576 7- 18225 9-  $\left(\pm\frac{2}{3}\right)^n$  10- 2, 6, 18 or 18, 6, 2  
 11-  $\frac{1}{x^{28}}$  12-  $x^{2p-1}$

### Exercise 7.5

- 1- (i)  $\pm 3\sqrt{5}$  (ii)  $\pm 6$  (iii)  $\pm 2\sqrt{2}$  2- (i) 2, 4 (ii) 9, 27 3- (i) 2, 4, 8 (ii) 2, 8, 16  
 4- 6, 12, 24, 48 5- 2, 8; 8, 2 6- 2, 18 8- 80, 40, 20, 10  
 9- 162, 54, 18 10-  $\frac{1}{2}, 2, 8, 32$  11-  $-28, 14, -7, \frac{7}{2}, -\frac{7}{4}, \frac{7}{8}$  12-  $\frac{16}{27}, \frac{8}{9}, \frac{4}{3}, 2, 3$

## Review Exercise 7

### 1- Encircle the Correct Answer.

(i) *b* (ii) *c* (iii) *a* (iv) *a* (v) *c* (vi) *b* (vii) *a* (viii) *c* (ix) *a* (x) *b*

### 2- Fill in the blanks.

(i)  $a_n$  (ii) 15 (iii)  $n$ th term (iv) 10 (v)  $\frac{a+b}{2}$   
 (vi) common ratio (vii)  $ar^{n-1}$  (viii)  $\pm\sqrt{ab}$  (ix) 4 (x)  $3n+24$

3-  $a_n = 2n+1$ , 37 4-  $a_n = \left(\frac{3}{2n+3}\right)^3$  5- 32 6-  $(3)^{11}$  7- 1, 2, 4, 8  
 8- 2, 6, 18, or 18, 6, 2

## Exercise 8.1

1. (i) {1, 4, 6, 7, 8, 9} (ii) {3, 4, 5, 6, 7, 8, 9} (iii) {4, 7} (iv) {4}  
 (v) {1, 3, 4, 5, 6, 7, 8, 9} (vi) {4}
8. (i) *A* (ii) *A* (iii) *A* (iv)  $\Phi$  (v)  $\Phi$  (vi)  $A' \cup B'$   
 (vii)  $A' \cap B'$  (viii) *A* (ix)  $\Phi$  (x)  $\Phi$

## Exercise 8.2

1.  $\{(3,1), (3,3), (5,1), (5,3), (6,1), (6,3)\}, \{(1,3), (1,5), (1,6), (3,3), (3,5), (3,6)\}$   
 $\text{Dom } A \times B = \{3, 5, 6\}$   $\text{Range } A \times B = \{1, 3\}$   
 $\text{Dom } B \times A = \{1, 3\}$   $\text{Range } B \times A = \{3, 5, 6\}$
2.  $\{(-2, -2), (-2, 1)\}, \{(-2, 4), (1, -2)\}$   $\text{Dom} = \{-2\}$   $\text{Dom} = \{-2, 1\}$   
 $\text{Range} = \{-2, 1\}$   $\text{Range} = \{4, -2\}$
3. (i)  $2^9$  (ii)  $2^{12}$  4.  $\{(2, 2), (3, 3), (3, 2)\}$

## Review Exercise 8

### 1- Encircle the Correct Answer.

(i) *b* (ii) *b* (iii) *a* (iv) *b* (v) *a* (vi) *a* (vii) *b* (viii) *b* (ix) *c* (x) *d*

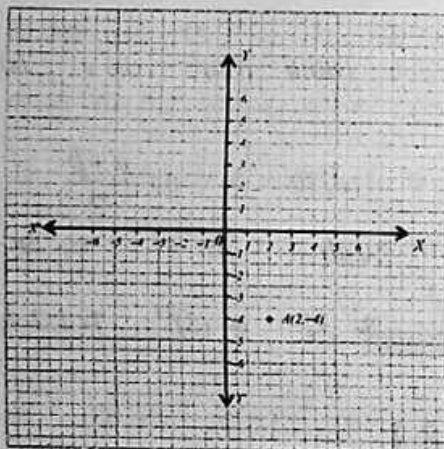
### 2- Fill in the blanks.

- (i)  $A' \cap B'$  (ii)  $A' \cup B'$  (iii)  $(A \cup B) \cup C$  (iv)  $(A \cap B) \cap C$   
 (v) commutative law as under union (vi) commutative law under intersection  
 (vii) binary relation (viii) {1, 3, 5} (ix) {2, 4, 6} (x) set *B*  
 6- (i)  $2^{16}$  (ii)  $2^6$  7-  $\text{Dom}(R) = \{2, 4\}$   $\text{Range}(R) = \{2, 5\}$

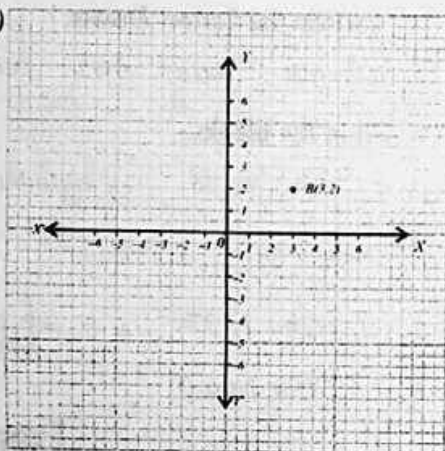
## Exercise 9.1

1-

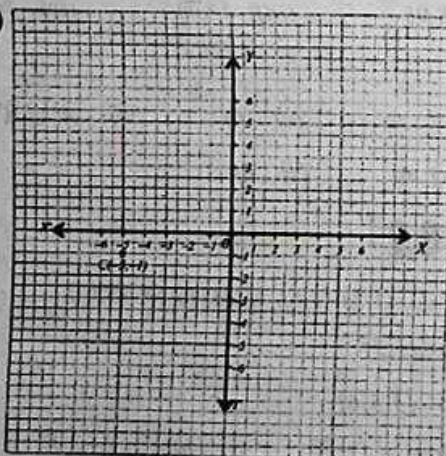
(i)



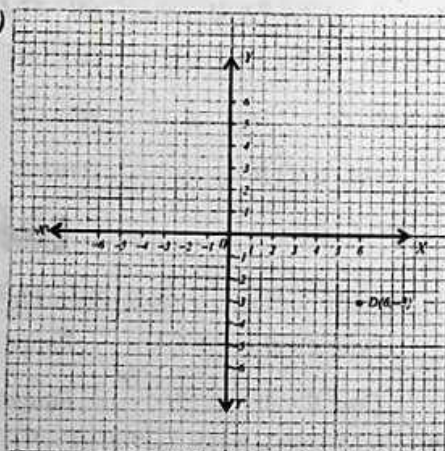
(ii)



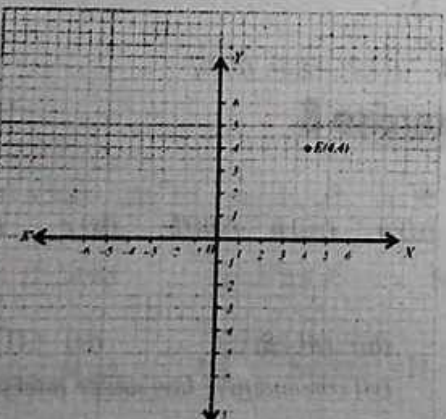
(iii)



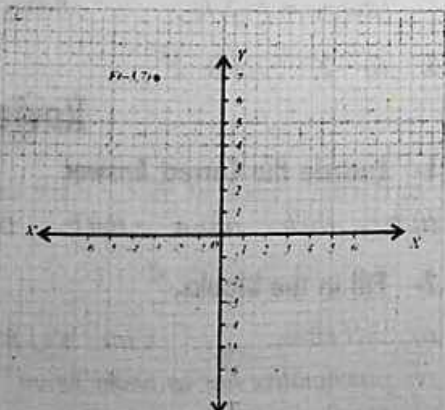
(iv)



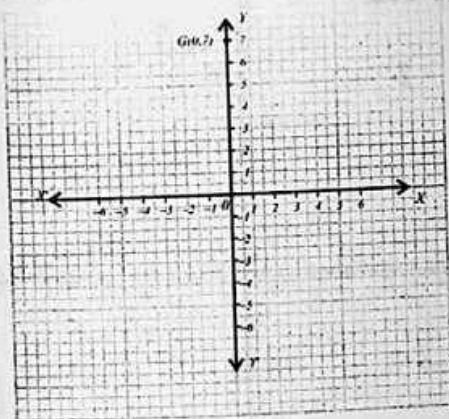
(v)



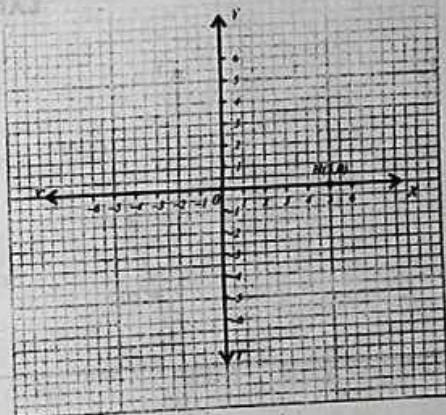
(vi)



(vii)

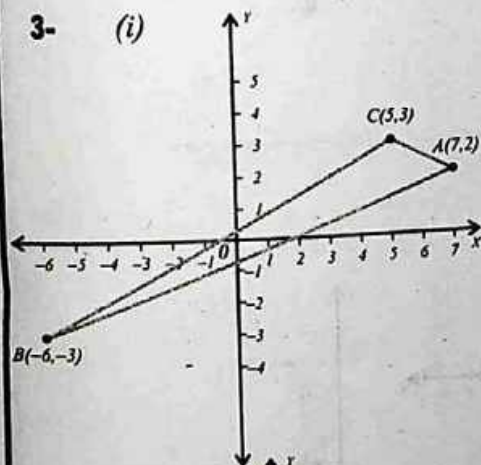


(viii)

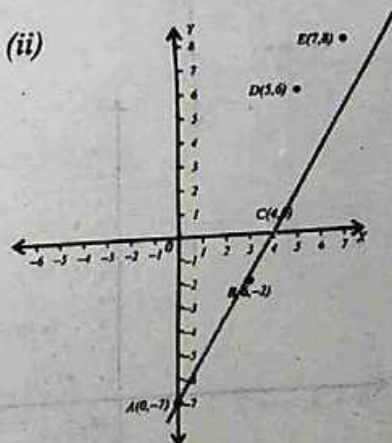


- 2- (i)  $(0, 0)$  (ii)  $(-5, 0)$  (iii)  $(3, 0)$  (iv)  $(0, 4)$  (v)  $(0, -6)$

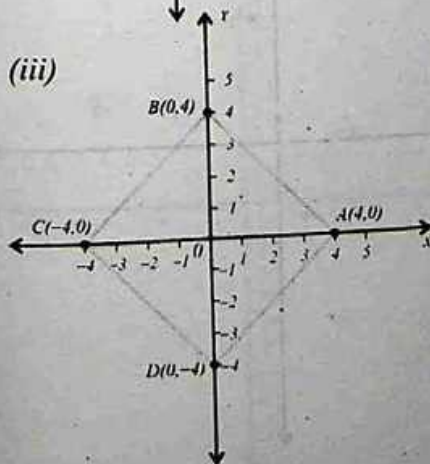
3- (i)



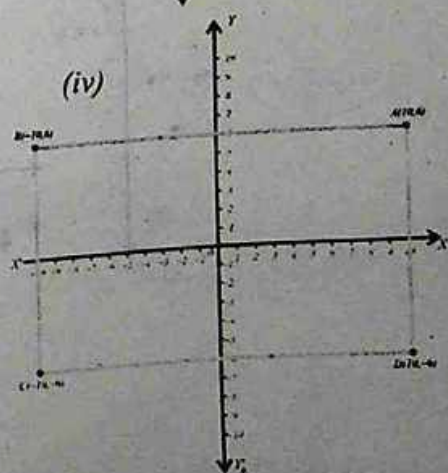
(ii)



(iii)

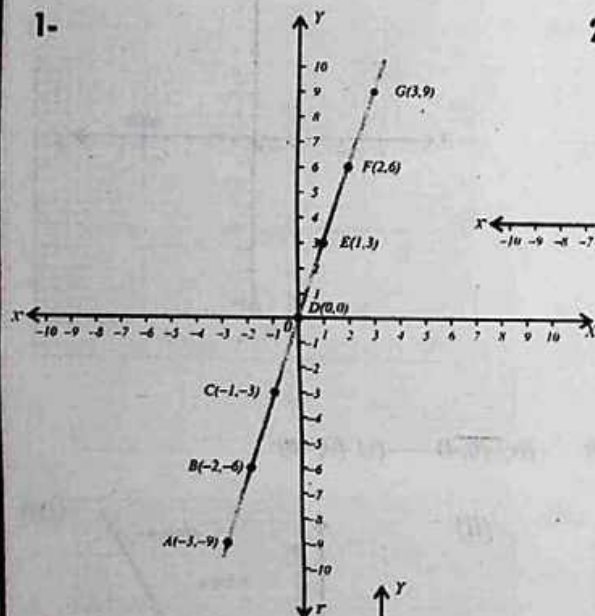


(iv)

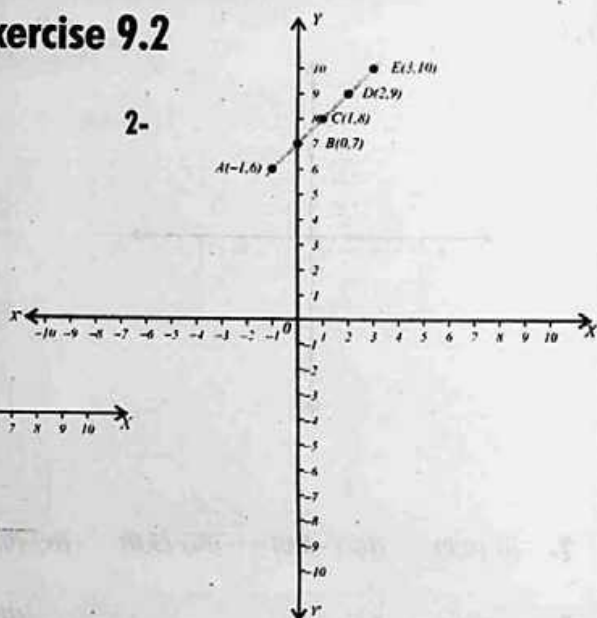


## Exercise 9.2

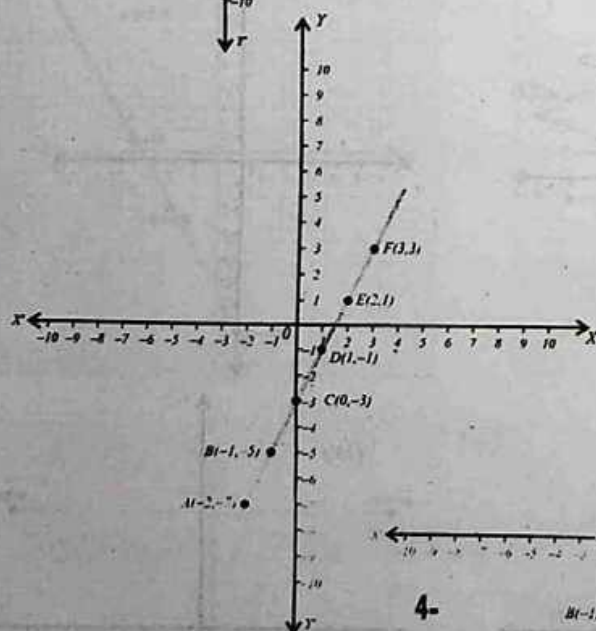
1-



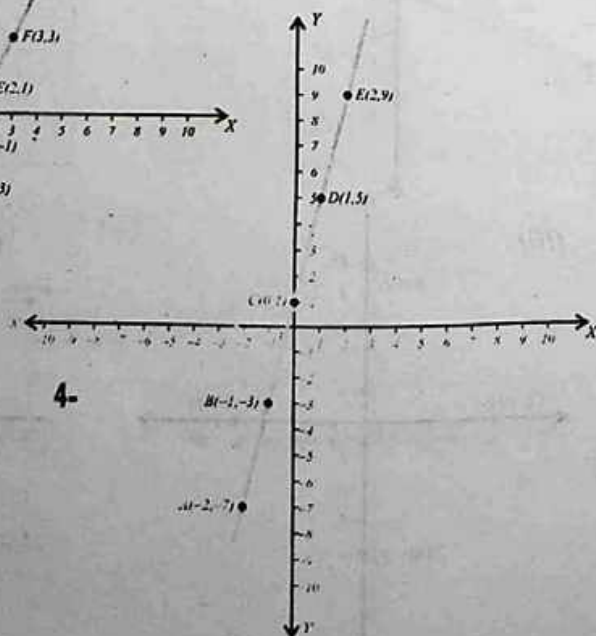
2-



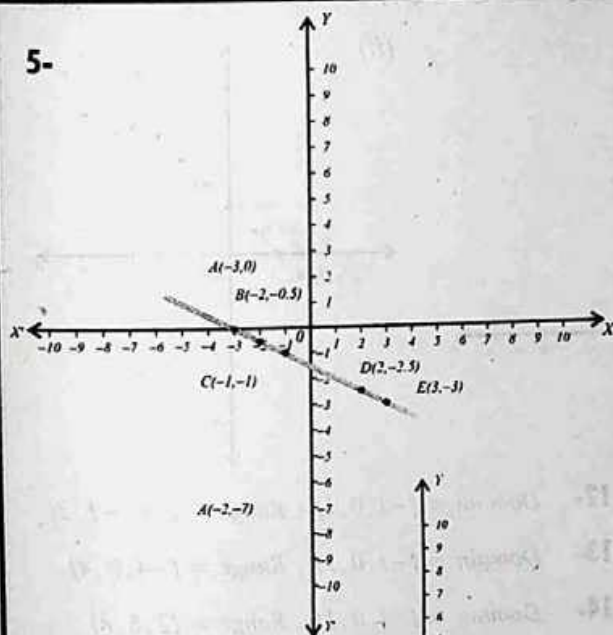
3-



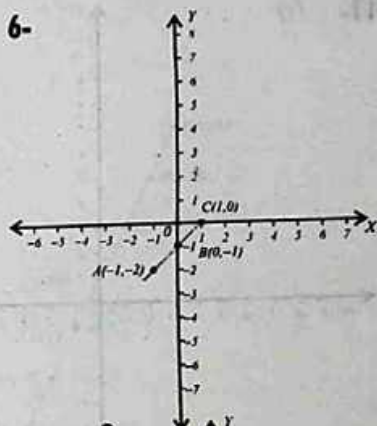
4-



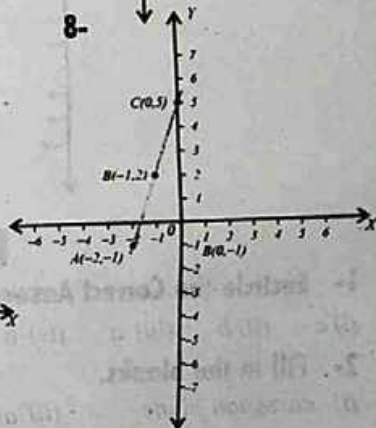
5-



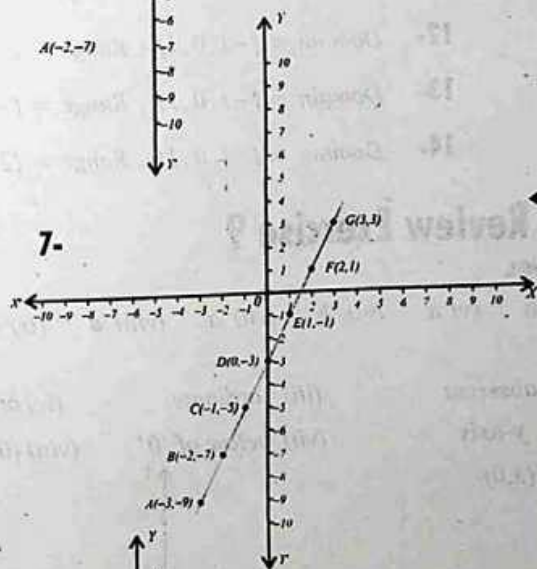
6-



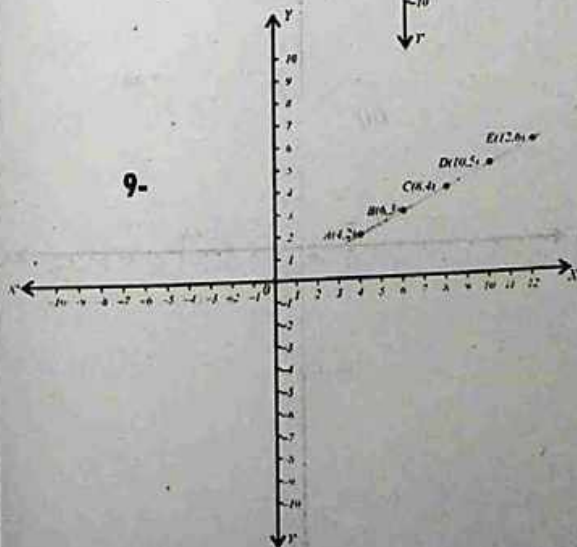
8-



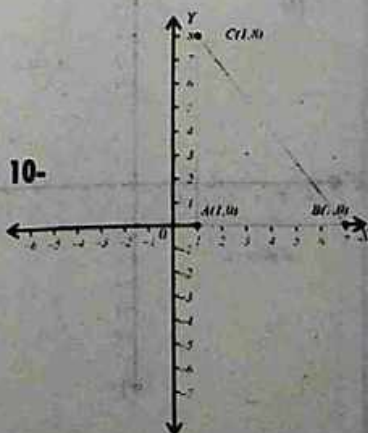
7-



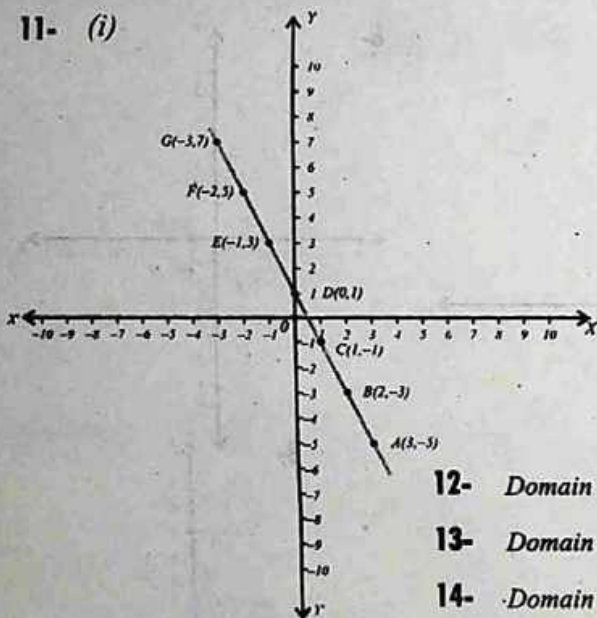
9-



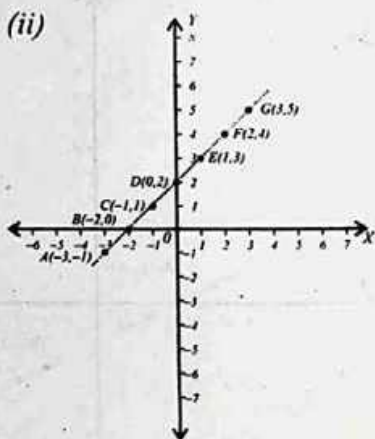
10-



11- (i)



(ii)



12- Domain =  $\{-1, 0, 1\}$ , Range =  $\{-4, -1, 2\}$

13- Domain =  $\{-1, 0, 1\}$ , Range =  $\{-4, 0, 4\}$

14- Domain =  $\{-1, 0, 1\}$ , Range =  $\{2, 5, 8\}$

## Review Exercise 9

1- Encircle the Correct Answer.

(i) c (ii) b (iii) a (iv) b (v) a (vi) b (vii) a (viii) a (ix) b

2- Fill in the blanks.

(i) cartesian plane

(ii) abscissa

(iii) ordinate

(iv) ordered pair

(v) x-axis

(vi) y-axis

(vii) below of '0'

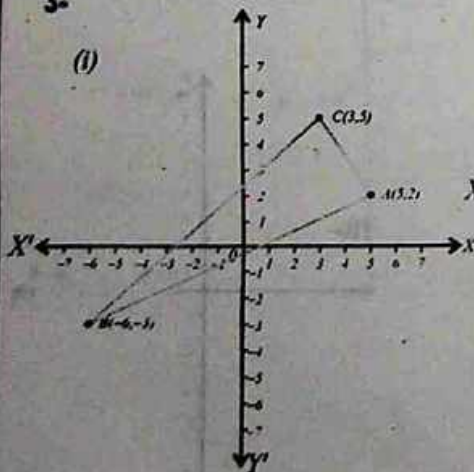
(viii) (0, 0)

(ix)  $y = c$ 

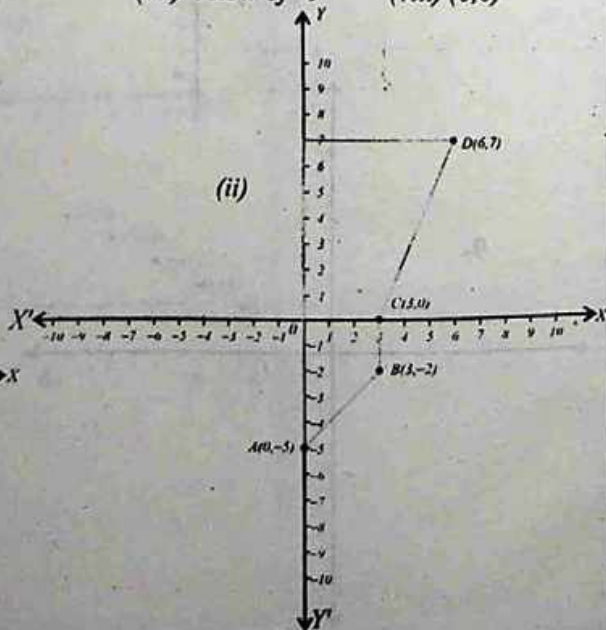
(x) (3, 0)

3-

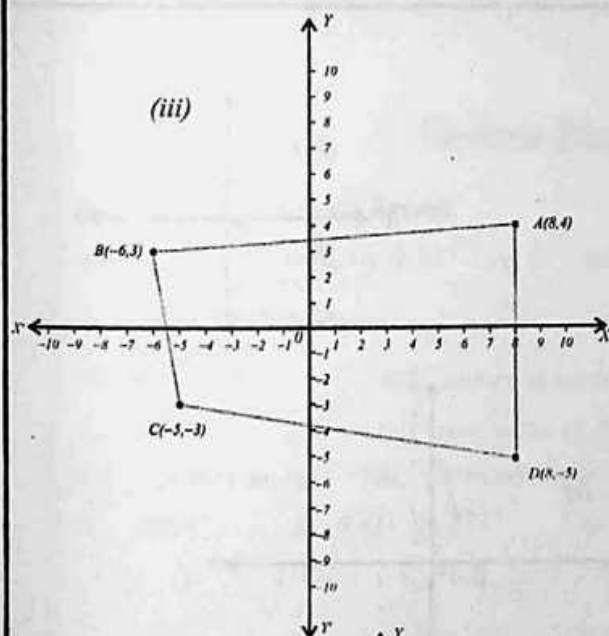
(i)



(ii)

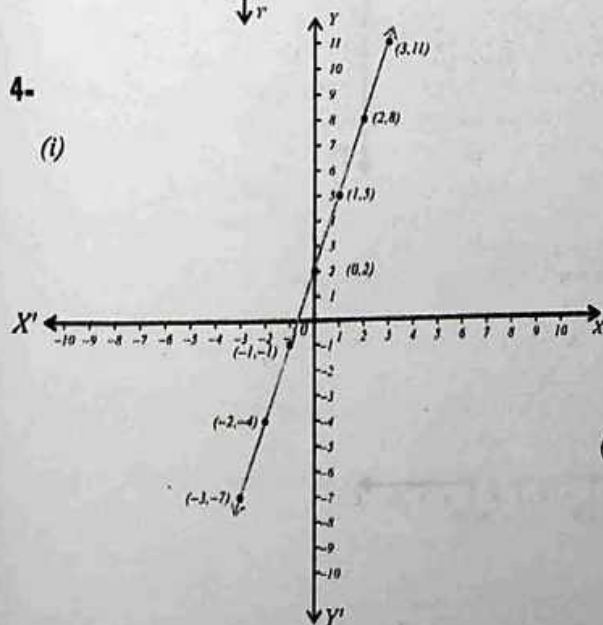


(iii)

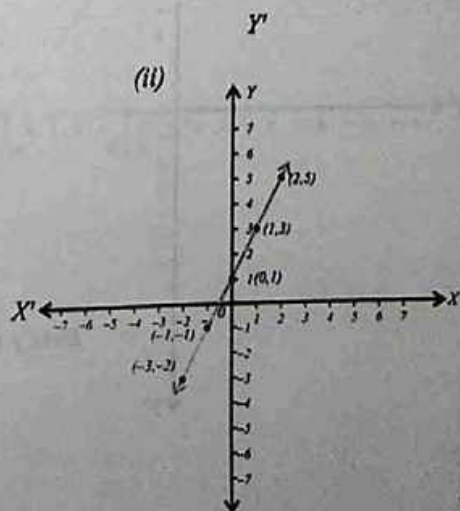


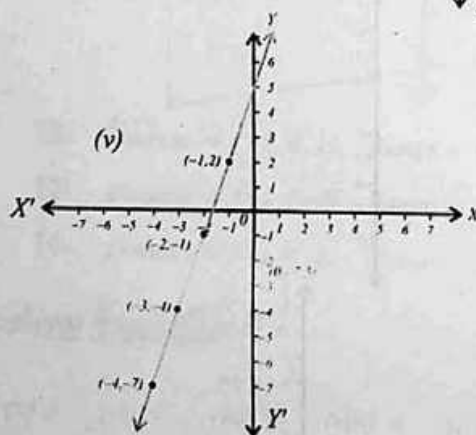
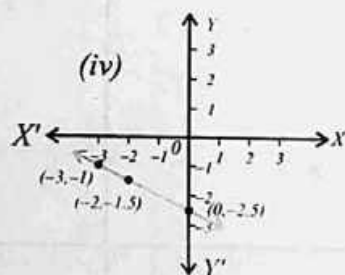
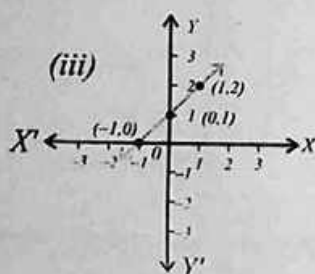
4-

(i)

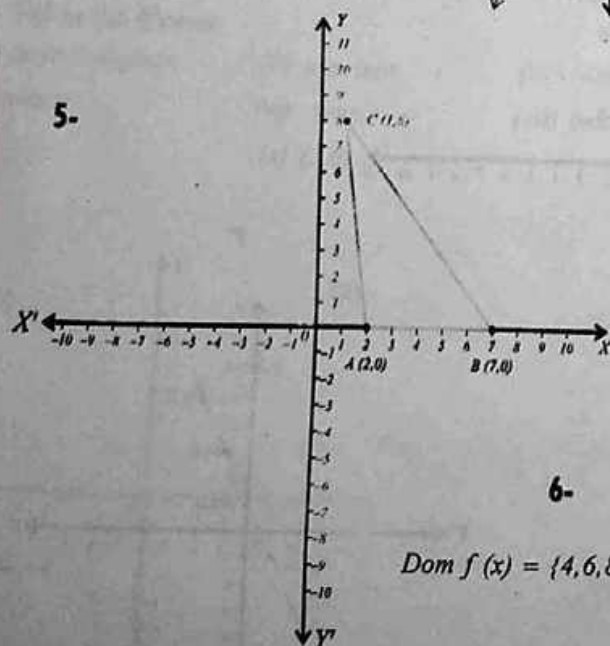


(ii)





5-



6-

$$\text{Dom } f(x) = \{4, 6, 8, 10, 12\}, \text{ Range } f(x) = \{2, 3, 4, 5, 6\}$$

## Review Exercise 10

### 1- Encircle the Correct Answer.

(i) c    (ii) d    (iii) a    (iv) b    (v) a    (vi) a    (vii) a    (viii) b    (ix) a    (x) a

### 2- Fill in the blanks.

(i) histogram                      (ii) measure of central tendency    (iii) mean    (iv) median

(v) mode                              (vi) mean value of  $x_1, x_2, \dots, x_n$

(vii) geometric mean    (viii) harmonic mean    (ix) weighted mean    (x) arithmetic mean

3- 3.286                      4- 9.811; 96.255                      5- 7; 8; 8                      6- 6.875                      7- 4.7

## GLOSSARY

### Unit-1 PERCENTAGE, RATIO AND PROPORTION

**Percentage:** Percentage means out of hundred.

**Ratio:** A comparison between the two like quantities is called a ratio.

**Antecedent:** In a ratio  $a:b$ , " $a$ " is called the antecedent.

**Consequent:** In a ratio  $a:b$ , " $b$ " is called the consequent.

**Proportion:** The equality of two ratios is called proportion.

**Extremes and Means:** If  $a:b::c:d$ , then " $a$ " and " $d$ " the first and fourth terms are called extremes and " $b$ " and " $c$ " the 2nd and 3rd terms are called means.

**Direct Proportion:** The relation between two ratios in which an increase in one quantity causes a proportional increase in the other quantity or decreases in one quantity causes a decrease in another quantity is called direct proportion.

**Inverse Proportion:** The relationship between two ratios in which increase in one quantity causes a proportional decreases in the other quantity or vice versa is called inverse proportion.

**Compound Proportion:** The relationship between two or more proportions is called a compound proportion.

### Unit-2 ZAKAT, USHR AND INHERITANCE

**Zakat:** Zakat is a transfer payment which Sahib-e-Nisab Muslims pay at given rates by them according to sharia or through the Islamic rate to the poor and needy in or after month of Rajab.

**Sahib-e-Nisab Muslim:** A Muslim who owns or keeps in his / her possession at least 7.5 tola gold or 52.5 tola silver or cash money equivalent to value for one year is considered a sahib-e-nisab Muslim.

**Exposed wealth:** This includes agriculture goods camels, sheep, goats, minerals, business inventories etc.

**Unexposed wealth:** This includes gold, silver, cash money, liquid assets etc.

**Rate of Zakat:** Rate of zakat is 2.5 % of the total value of the goods or money.

**Ushr:** It is a tax paid at the rate of 10 % from agriculture produce of land which is irrigated by natural resources and 5 % by artificial.

**Inheritance:** Distribution of remaining inheret amongst the heirs according to sharia after the burial of a deceased.

### Unit-3 BUSINESS MATHEMATICS

**Cost Price:** The price at which a particular item is purchased is called cost price. It is denoted by " $CP$ ".

**Sale Price:** The price at which an article is sold out is called the sale price. It is denoted by " $SP$ ".

**Profit:** If the selling price of an article is greater than its cost price, then the difference of these two is the profit earned. It is denoted by " $P$ ".

**Loss:** If the selling price of an article is less than its cost price, then the difference of these two is the loss. It is denoted by " $L$ ".

**Discount:** Some times a rebate is declared on the selling price of an article, this rebate is called the discount.

**Marked Price (MP):** The price tagged on a card of each and every article in a shop is known as the marked price, It is denoted by "MP".

#### Unit-4 FINANCIAL MATHEMATICS

**Current Account:** A running account which continuously remains in operation due to its liquidity.

**Saving Account:** It is meant to encourage thrift and promote saving among the persons of small means. The bank pays nominal interest half yearly on the basis of monthly balance to the depositors.

**PLS Saving Account:** Profit and loss sharing account opened with small amount with profit earned on loss sustained at the end of each half year / full year depending upon the mode of payment.

**Fixed / Time deposit Account:** The deposits kept with the bank in a account for a certain period of time ranging from 3 months to 5 years.

**Foreign Currency Accounts:** Account maintained with the bank in foreign currency like dollars, pounds and Euro etc.

**Negotiable Instruments:** It means a promissory note, a bill exchange or cheque payable whether to be order or bearer of the instruments.

**Insured:** The person or entity whose insurance is being done is called "the insured".

**Insurer:** The company under taker the act of insurance is called the insurer.

**Insured or Insurant:** A person to whom an insurance policy issued, the beneficiary in a contract issuance is called insured or insurant.

**Insurance Policy:** The contract which is executed between two parties is called insurance policy.

**Premium:** The periodic installment to be paid by the insured is called premium.

**Maturity:** The time-period agreed upon by both the parties (insured and insurer is called maturity.)

**Bonus:** The agreed amount to be paid back on maturity or expiry of the agreed period, includes the actual amount paid in installments plus profit is termed as bonus.

**Cheque:** A bill of exchange drawn on a specific banker and not expressed to be payable other wise on demand.

**Pay Order:** A cheque like instruerent issued by bank on the request of its customers.

**Bank Draft:** An order to pay money, drawn by one office of a bank upon another office of the same bank for a sum of money payable to order on demand.

**On-Line Banking:** This system indicates that a direct connection is made to centralize computer system for authorization or validation before a transaction is executed.

**ATM Card:** It is a payment card issued to a person for activating automated teller machine computer based terminal which allow consumers to make deposits and with draws.

**Credit Card:** A card indicating that the holder has been granted a line of credit enabling the holder to make purchases and or with draw cash.

**ATM:** A machine installed by the bank to dispense cash to its account holders.

**Profit:** Profit is the amount which is paid by the bank on the deposits maintained by the client with the bank.

**Principal:** The amount / capital borrowed or lent is called principal.

**Rate:** The percentage of interest charged is called rate.

**Time:** The period of the loan or deposit is called the time.

**Amount:** When the interest is added to the principal, the sum is called the amount.

**Mark Up:** The interest earned by the bank is named as mark up.

**Leasing:** Lease is a contract where by the owner of an asset, the lessor, gives the hirer, the lessee, the right to use the asset for a specific period in exchange of rental payment.

**Down Payment:** The customer is required to deposit the payment with the bank along with the application form.

## Unit-5 CONSUMER MATHEMATICS

**Tax:** Money that must be paid to the state, charged as a proportion of income and profits or added to the cost of same goods and services.

**Direct Tax:** These are the taxes which are charged on income, property and profits in the form of income tax, property tax.

**Indirect Tax:** Indirect taxes include duties, motor vehicle taxes, goods and services taxes (GST) general sale tax and value added taxes etc.

**Sales Tax:** When we buy article we have to pay a certain amount of tax as the value added tax in addition to the price of the article. This tax usually given as a certain percentage of the selling price. In Pakistan sales tax of 17% is imposed on goods bought and services rendered.

**Excise Duty:** It is the form of a tax which the buyer pay on a manufactured item at the time of purchase.

**Property Tax:** A property tax is charged on the owner of land, house, flats or building at a standard rate of 16% on annual value of the property.

**Income Tax:** It is the tax charged on all taxable incomes during the year from 1st July to next 30th June.

## Unit-6 EXPONENTS AND LOGARITHMS

**Rational Number:** A number of the form  $\frac{p}{q}$ ,  $q \neq 0$ ,  $p, q \in I$  is called a rational number.

**Irrational Number:** A number of form  $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$  are called irrational numbers.

**Radicals:** If  $\sqrt[n]{a}$  is irrational, where "a" is rational number, then  $\sqrt[n]{a}$  is called a radical.

**Radicands:** The symbol  $\sqrt[n]{\phantom{x}}$  is called radicands.

**Base and Exponent:** For any real number "a" and a positive integer "n" we define  $a^n = a \times a \times a \times \dots \times a$  (n times). Here "a" is called the base and "n" the exponent.

**Pure Radical:** A radical which has unity (one) only as rational factor, the other factor being irrational is called a pure radical.

**Mixed Radical:** A radical which has a rational factor other than unity, the other factor being irrational is called a mixed radical.

**Common Logarithm:** The logarithm calculated to the base 10 is called a common logarithm.

**Characteristic and Mantissa:** The logarithm of a number consists of two parts, the integral part is called the characteristic and the decimal part is called the mantissa.

## Unit-7 ARITHMETIC AND GEOMETRIC SEQUENCES

**Sequence:** A sequence is a function whose domain  $D$  is a set of positive integers.

**Arithmetic Sequence:** A sequence in which each term is obtained from the previous term by adding a fixed number is called an arithmetic sequence.

**Arithmetic Mean:** A number " $A$ " is said to be an arithmetic mean between the two numbers  $a$  and  $b$  if  $a, A, b$  is arithmetic sequence.

**Geometric Sequence:** A sequence in which each term is obtained from the previous term by multiplying with a common ratio is called a geometric sequence.

**Geometric Mean:** A number " $G$ " is said to be a geometric mean between the two numbers  $a$  and  $b$  if  $a, G, b$  is a geometric sequence.

## Unit-8 SETS AND FUNCTIONS

**Set:** A collection of well defined distinct objects is called a set.

$N = \{1, 2, 3, \dots\}$  is called set of natural numbers.

$M = \{0, 1, 2, 3, \dots\}$  is called set of whole numbers.

$I = \{\dots, -1, 0, 1, 2, 3, \dots\}$  is called set of integers.

$Q = \{p/q, q \neq 0, p, q \in I\}$  is called set of rational numbers.

$Q' = A$  set of irrational numbers.

$R = QUQ' = A$  set of real numbers.

**Universal Set:** If there are some sets under consideration there happens to be a set, which is a super set of each one of the given sets, such a set is called the universal set denoted by  $U$ .

**Complement of a Set:** Let  $A$  be a sub-set of a universal set  $U$ . Then complement of  $A$  is denoted by  $A'$  or  $U-A$  is the set of all those element of  $U$  which are not in  $A$ .

**Binary Relation:** Let  $A$  and  $B$  any two sets, then any sub-set the cartesian product  $A \times B$  is called a binary relation from  $A$  to  $B$ .

**Function:** Any binary relation ' $f$ ' between two non-empty sets  $A$  and  $B$  such that:

(i)  $Dom f = A$

(ii) First element in any two of the ordered pairs of  $f$  are not repeated, then  $f$  is called a function from  $A$  to  $B$ .

## Unit-9 LINEAR GRAPHS

**Plane:** The walls of a class room, the black board, the top of the desk and the top of the table are all examples of a plane.

**Cartesian Plane:** A Cartesian plane consists of two number lines  $OX$  and  $OY$  intersecting at right angle at " $O$ ".

**Ordinate:** The perpendicular distance of a point from  $X$ -axis is called an ordinate.

## Unit-10 BASIC STATISTICS

**Histogram:** When a bar chart is construed, so that the area of each bar is proportional to the number of items in each group is called a histogram.

**Cumulative Frequency:** Cumulative frequency is running total of class frequency.

**Cumulative Frequency Polygon or (ogive):** When the cumulative frequencies are plotted against the end point of their respective class intervals are joined together, the resultant graph is called a cumulative frequency polygon or ogive.

**Measures of Central Tendency:** These are summary statistics which measure the middle (or center) of the data.

**Mean:** To obtain the mean of a ungrouped data, all numbers in the set are added together and then the total is divided by the number of scores in that set.

**Median:** The middle value of data arranged in numerical order is called median.

**Mode:** The mode is the score which occurs most often in a set of data.

**Arithmetic Mean:** It is defined as the value obtained by dividing the sum of the values by their numbers. Thus the mean value of  $x_1, x_2, \dots, x_n$  denoted by  $\bar{x}$  is :

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

**Properties of arithmetic mean or mean:** (i) The sum of deviations of values from their mean is zero. Symbolically:  $\sum(x_i - \bar{x}) = 0$  or  $\sum f_i(x_i - \bar{x}) = 0$

$$(ii) \quad \bar{x} = \frac{\sum n\bar{x}}{\sum n}$$

(iii)  $\sum(x_i - a)^2$  is a minimum if and only if  $a = \bar{x}$ .

## SYMBOLS

| Symbol            | Stands for                  | Symbol                    | Stands for                |
|-------------------|-----------------------------|---------------------------|---------------------------|
| <                 | is less than                | $\because$                | because / as              |
| >                 | is greater than             | $\therefore$              | therefore / so            |
| $\leq$            | is less than or equal to    | :                         | ratio                     |
| $\geq$            | is greater than or equal to | $\propto$                 | is proportional to        |
| =                 | is equal to                 | $\infty$                  | varies                    |
| $\neq$            | is not equal to             |                           | tally mark                |
| $\nless$          | is not less than            | $\Sigma$                  | summation                 |
| $\ngtr$           | is not greater than         | $\overline{AB}$           | line segment AB           |
| $\in$             | belongs to / element        | $\overrightarrow{AB}$     | ray AB                    |
| $\forall$         | for all                     | $\overleftrightarrow{AB}$ | line                      |
| $\sqrt{\quad}$    | square root                 | $\angle$                  | angle                     |
| $ x $             | absolute value of x         | $\triangle$               | triangle                  |
| $\Rightarrow$     | implies that                | $\sim$                    | is similar to             |
| $\Leftrightarrow$ | if and only if              | $\cong$                   | is congruent to           |
| $\wedge$          | and                         | $\approx$                 | is approximately equal to |
| $\cup$            | union                       | $\parallel$               | is parallel to            |
| $\vee$            | or                          | $\widehat{AB}$            | arc AB                    |
| $\cap$            | intersection                | $\leftrightarrow$         | correspondence            |
| $\notin$          | not element / not           | U                         | universal set             |

# INDEX

## A

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## LOGARITHMS

|    | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|----|------|------|------|------|------|------|------|------|------|------|---|---|----|----|----|----|----|----|----|
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 |      |      |      |      |      | 4 | 9 | 13 | 17 | 21 | 26 | 30 | 34 | 38 |
|    |      |      |      |      |      | 0212 | 0253 | 0294 | 0334 | 0374 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 |      |      |      |      |      | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
|    |      |      |      |      |      | 0607 | 0645 | 0682 | 0719 | 0755 | 4 | 7 | 11 | 15 | 19 | 22 | 26 | 30 | 33 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 |      |      |      |      |      | 3 | 7 | 11 | 14 | 18 | 21 | 25 | 28 | 32 |
|    |      |      |      |      |      | 0969 | 1004 | 1038 | 1072 | 1106 | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 |      |      |      |      |      | 3 | 7 | 10 | 13 | 16 | 20 | 23 | 26 | 30 |
|    |      |      |      |      |      | 1303 | 1335 | 1367 | 1399 | 1430 | 3 | 7 | 10 | 13 | 16 | 19 | 22 | 25 | 29 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 |      |      |      |      |      | 3 | 6 | 9  | 12 | 15 | 19 | 22 | 25 | 28 |
|    |      |      |      |      |      | 1614 | 1644 | 1673 | 1703 | 1732 | 3 | 6 | 9  | 12 | 15 | 17 | 20 | 23 | 26 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 |      |      |      |      |      | 3 | 6 | 9  | 11 | 14 | 16 | 20 | 23 | 26 |
|    |      |      |      |      |      | 1903 | 1931 | 1959 | 1987 | 2014 | 3 | 6 | 8  | 11 | 14 | 17 | 19 | 22 | 25 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 |      |      |      |      |      | 3 | 5 | 8  | 11 | 14 | 17 | 19 | 22 | 24 |
|    |      |      |      |      |      | 2175 | 2201 | 2227 | 2253 | 2279 | 3 | 5 | 8  | 10 | 13 | 16 | 18 | 21 | 23 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 |      |      |      |      |      | 3 | 5 | 8  | 10 | 13 | 15 | 18 | 20 | 23 |
|    |      |      |      |      |      | 2430 | 2455 | 2480 | 2504 | 2529 | 2 | 5 | 7  | 10 | 12 | 15 | 17 | 20 | 22 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 |      |      |      |      |      | 2 | 5 | 7  | 9  | 12 | 14 | 16 | 19 | 21 |
|    |      |      |      |      |      | 2672 | 2695 | 2718 | 2742 | 2765 | 2 | 5 | 7  | 9  | 11 | 14 | 16 | 18 | 21 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 |      |      |      |      |      | 2 | 4 | 7  | 9  | 11 | 13 | 16 | 18 | 20 |
|    |      |      |      |      |      | 2900 | 2923 | 2945 | 2967 | 2989 | 2 | 4 | 6  | 8  | 11 | 13 | 15 | 17 | 19 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 2 | 4 | 6  | 8  | 11 | 13 | 15 | 17 | 19 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 | 2 | 4 | 6  | 8  | 10 | 12 | 14 | 16 | 18 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 | 2 | 4 | 6  | 8  | 10 | 12 | 14 | 15 | 17 |
| 23 | 3617 | 3636 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3785 | 2 | 4 | 6  | 7  | 9  | 11 | 13 | 15 | 17 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 | 2 | 4 | 5  | 7  | 9  | 11 | 12 | 14 | 16 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 | 2 | 3 | 5  | 7  | 9  | 10 | 12 | 14 | 15 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 | 2 | 3 | 5  | 7  | 8  | 10 | 11 | 13 | 15 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 | 2 | 3 | 5  | 6  | 8  | 9  | 11 | 13 | 14 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 | 2 | 3 | 5  | 6  | 8  | 9  | 11 | 12 | 14 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 | 1 | 3 | 4  | 6  | 7  | 9  | 10 | 12 | 13 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 | 1 | 3 | 4  | 6  | 7  | 9  | 10 | 11 | 13 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 | 1 | 3 | 4  | 6  | 7  | 8  | 10 | 11 | 12 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 | 1 | 3 | 4  | 5  | 7  | 8  | 9  | 11 | 12 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 | 1 | 3 | 4  | 5  | 6  | 8  | 9  | 10 | 12 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 | 1 | 3 | 4  | 5  | 6  | 8  | 9  | 10 | 11 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 | 1 | 2 | 4  | 5  | 6  | 7  | 9  | 10 | 11 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 | 1 | 2 | 4  | 5  | 6  | 7  | 8  | 10 | 11 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 | 1 | 2 | 3  | 5  | 6  | 7  | 8  | 9  | 10 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 | 1 | 2 | 3  | 5  | 6  | 7  | 8  | 9  | 10 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 | 1 | 2 | 3  | 4  | 5  | 7  | 8  | 9  | 10 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 | 1 | 2 | 3  | 4  | 5  | 6  | 8  | 9  | 10 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 7  | 8  |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 | 1 | 2 | 3  | 4  | 5  | 6  | 6  | 7  | 8  |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 | 1 | 2 | 3  | 4  | 4  | 5  | 6  | 7  | 8  |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 | 1 | 2 | 3  | 4  | 4  | 5  | 6  | 7  | 8  |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 | 1 | 2 | 3  | 4  | 4  | 5  | 6  | 7  | 8  |

## LOGARITHMS

|    | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|------|------|------|------|------|------|------|------|------|------|---|---|---|---|---|---|---|---|---|
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 7 |
| 53 | 7243 | 7251 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| 55 | 7404 | 7412 | 7419 | 7427 | 7435 | 7443 | 7451 | 7459 | 7466 | 7474 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 56 | 7482 | 7490 | 7497 | 7505 | 7513 | 7520 | 7528 | 7536 | 7543 | 7551 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 57 | 7559 | 7566 | 7574 | 7582 | 7589 | 7597 | 7604 | 7612 | 7619 | 7627 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 58 | 7634 | 7642 | 7649 | 7657 | 7664 | 7672 | 7679 | 7686 | 7694 | 7701 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 59 | 7709 | 7716 | 7723 | 7731 | 7738 | 7745 | 7752 | 7760 | 7767 | 7774 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 60 | 7782 | 7789 | 7796 | 7803 | 7810 | 7818 | 7825 | 7832 | 7839 | 7846 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 61 | 7853 | 7860 | 7868 | 7875 | 7882 | 7889 | 7896 | 7903 | 7910 | 7917 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 62 | 7924 | 7931 | 7938 | 7945 | 7952 | 7959 | 7966 | 7973 | 7980 | 7987 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 63 | 7993 | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 64 | 8062 | 8069 | 8075 | 8082 | 8089 | 8096 | 8102 | 8109 | 8116 | 8122 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 65 | 8129 | 8136 | 8142 | 8149 | 8156 | 8162 | 8169 | 8176 | 8182 | 8189 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 66 | 8195 | 8202 | 8209 | 8215 | 8222 | 8228 | 8235 | 8241 | 8248 | 8254 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 67 | 8261 | 8267 | 8274 | 8280 | 8287 | 8293 | 8299 | 8306 | 8312 | 8319 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 68 | 8325 | 8331 | 8338 | 8344 | 8351 | 8357 | 8363 | 8370 | 8376 | 8382 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| 69 | 8388 | 8395 | 8401 | 8407 | 8414 | 8420 | 8426 | 8432 | 8439 | 8445 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 70 | 8451 | 8457 | 8463 | 8470 | 8476 | 8482 | 8488 | 8494 | 8500 | 8506 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 71 | 8513 | 8519 | 8525 | 8531 | 8537 | 8543 | 8549 | 8555 | 8561 | 8567 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 72 | 8573 | 8579 | 8585 | 8591 | 8597 | 8603 | 8609 | 8615 | 8621 | 8627 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 73 | 8633 | 8639 | 8645 | 8651 | 8657 | 8663 | 8669 | 8675 | 8681 | 8686 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 74 | 8692 | 8698 | 8704 | 8710 | 8716 | 8722 | 8727 | 8733 | 8738 | 8745 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 75 | 8751 | 8756 | 8762 | 8768 | 8774 | 8779 | 8785 | 8791 | 8797 | 8802 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 76 | 8808 | 8814 | 8820 | 8825 | 8831 | 8837 | 8842 | 8848 | 8854 | 8859 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 77 | 8865 | 8871 | 8876 | 8882 | 8887 | 8893 | 8899 | 8904 | 8910 | 8915 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 78 | 8921 | 8927 | 8932 | 8938 | 8943 | 8949 | 8954 | 8960 | 8965 | 8971 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 79 | 8976 | 8982 | 8987 | 8993 | 8998 | 9004 | 9009 | 9015 | 9020 | 9025 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 80 | 9031 | 9036 | 9042 | 9047 | 9053 | 9058 | 9063 | 9069 | 9074 | 9079 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 81 | 9085 | 9090 | 9096 | 9101 | 9106 | 9112 | 9117 | 9122 | 9128 | 9133 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 82 | 9138 | 9143 | 9149 | 9154 | 9159 | 9165 | 9170 | 9175 | 9180 | 9186 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 83 | 9191 | 9196 | 9201 | 9206 | 9212 | 9217 | 9222 | 9227 | 9232 | 9238 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 84 | 9243 | 9248 | 9253 | 9258 | 9263 | 9269 | 9274 | 9279 | 9284 | 9289 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 85 | 9294 | 9299 | 9304 | 9309 | 9315 | 9320 | 9325 | 9330 | 9335 | 9340 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 86 | 9345 | 9350 | 9355 | 9360 | 9365 | 9370 | 9375 | 9380 | 9385 | 9390 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 87 | 9395 | 9400 | 9405 | 9410 | 9415 | 9420 | 9425 | 9430 | 9435 | 9440 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 88 | 9445 | 9450 | 9455 | 9460 | 9465 | 9469 | 9474 | 9479 | 9484 | 9489 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 89 | 9494 | 9499 | 9504 | 9509 | 9513 | 9518 | 9523 | 9528 | 9533 | 9538 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 90 | 9542 | 9547 | 9552 | 9557 | 9562 | 9566 | 9571 | 9576 | 9581 | 9586 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 91 | 9590 | 9595 | 9600 | 9605 | 9609 | 9614 | 9619 | 9624 | 9628 | 9633 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 92 | 9638 | 9643 | 9647 | 9652 | 9657 | 9661 | 9666 | 9671 | 9675 | 9680 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 93 | 9685 | 9689 | 9694 | 9699 | 9703 | 9708 | 9713 | 9717 | 9722 | 9727 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 94 | 9731 | 9736 | 9741 | 9745 | 9750 | 9754 | 9759 | 9763 | 9768 | 9773 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 95 | 9777 | 9782 | 9786 | 9791 | 9795 | 9800 | 9805 | 9809 | 9814 | 9818 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 96 | 9823 | 9827 | 9832 | 9836 | 9841 | 9845 | 9850 | 9854 | 9859 | 9863 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 97 | 9868 | 9872 | 9877 | 9881 | 9886 | 9890 | 9894 | 9899 | 9903 | 9908 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 98 | 9912 | 9917 | 9921 | 9926 | 9930 | 9934 | 9939 | 9943 | 9948 | 9952 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 99 | 9956 | 9961 | 9965 | 9969 | 9974 | 9978 | 9983 | 9987 | 9991 | 9996 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 |

## ANTI LOGARITHMS

|     | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 1 2 3 | 4 5 6 | 7 8 9 |
|-----|------|------|------|------|------|------|------|------|------|------|-------|-------|-------|
| .00 | 1000 | 1002 | 1005 | 1007 | 1009 | 1012 | 1014 | 1016 | 1019 | 1021 | 0 0 1 | 1 1 1 | 2 2 2 |
| .01 | 1023 | 1026 | 1027 | 1030 | 1033 | 1035 | 1038 | 1040 | 1042 | 1045 | 0 0 1 | 1 1 1 | 2 2 2 |
| .02 | 1047 | 1050 | 1052 | 1054 | 1057 | 1059 | 1062 | 1064 | 1067 | 1069 | 0 0 1 | 1 1 1 | 2 2 2 |
| .03 | 1072 | 1074 | 1076 | 1079 | 1081 | 1084 | 1086 | 1089 | 1091 | 1094 | 0 0 1 | 1 1 1 | 2 2 2 |
| .04 | 1096 | 1099 | 1102 | 1104 | 1107 | 1109 | 1112 | 1114 | 1117 | 1119 | 0 0 1 | 1 1 1 | 2 2 2 |
| .05 | 1122 | 1125 | 1127 | 1130 | 1132 | 1135 | 1138 | 1140 | 1143 | 1146 | 0 1 1 | 1 1 2 | 2 2 2 |
| .06 | 1148 | 1151 | 1153 | 1156 | 1159 | 1161 | 1164 | 1167 | 1169 | 1172 | 0 1 1 | 1 1 2 | 2 2 2 |
| .07 | 1175 | 1178 | 1180 | 1183 | 1186 | 1189 | 1191 | 1194 | 1197 | 1199 | 0 1 1 | 1 1 2 | 2 2 2 |
| .08 | 1202 | 1205 | 1208 | 1211 | 1213 | 1216 | 1219 | 1222 | 1225 | 1227 | 0 1 1 | 1 1 2 | 2 2 3 |
| .09 | 1230 | 1235 | 1236 | 1239 | 1242 | 1245 | 1247 | 1250 | 1253 | 1256 | 0 1 1 | 1 1 2 | 2 2 3 |
| .10 | 1259 | 1262 | 1265 | 1268 | 1271 | 1274 | 1276 | 1279 | 1282 | 1285 | 0 1 1 | 1 1 2 | 2 2 3 |
| .11 | 1288 | 1291 | 1294 | 1297 | 1300 | 1303 | 1306 | 1309 | 1312 | 1315 | 0 1 1 | 1 2 2 | 2 2 3 |
| .12 | 1318 | 1321 | 1324 | 1327 | 1330 | 1334 | 1337 | 1340 | 1343 | 1346 | 0 1 1 | 1 2 2 | 2 2 3 |
| .13 | 1349 | 1352 | 1355 | 1358 | 1361 | 1365 | 1368 | 1371 | 1374 | 1377 | 0 1 1 | 1 2 2 | 2 3 3 |
| .14 | 1380 | 1384 | 1387 | 1390 | 1393 | 1396 | 1400 | 1403 | 1406 | 1409 | 0 1 1 | 1 2 2 | 2 3 3 |
| .15 | 1413 | 1416 | 1419 | 1422 | 1426 | 1429 | 1432 | 1435 | 1439 | 1442 | 0 1 1 | 1 2 2 | 2 3 3 |
| .16 | 1445 | 1449 | 1452 | 1455 | 1459 | 1462 | 1466 | 1469 | 1472 | 1476 | 0 1 1 | 1 2 2 | 2 3 3 |
| .17 | 1479 | 1483 | 1486 | 1489 | 1493 | 1496 | 1500 | 1503 | 1507 | 1510 | 0 1 1 | 1 2 2 | 2 3 3 |
| .18 | 1514 | 1517 | 1521 | 1524 | 1528 | 1531 | 1535 | 1538 | 1542 | 1545 | 0 1 1 | 1 2 2 | 2 3 3 |
| .19 | 1549 | 1552 | 1556 | 1560 | 1563 | 1567 | 1570 | 1574 | 1578 | 1581 | 0 1 1 | 1 2 2 | 2 3 3 |
| .20 | 1585 | 1589 | 1592 | 1596 | 1600 | 1603 | 1607 | 1611 | 1614 | 1618 | 0 1 1 | 1 2 2 | 3 3 3 |
| .21 | 1622 | 1626 | 1629 | 1633 | 1637 | 1641 | 1644 | 1648 | 1652 | 1656 | 0 1 1 | 2 2 2 | 3 3 3 |
| .22 | 1660 | 1663 | 1667 | 1671 | 1675 | 1679 | 1683 | 1687 | 1690 | 1694 | 0 1 1 | 2 2 2 | 3 3 3 |
| .23 | 1698 | 1702 | 1706 | 1710 | 1714 | 1718 | 1722 | 1726 | 1730 | 1734 | 0 1 1 | 2 2 2 | 3 3 4 |
| .24 | 1738 | 1742 | 1746 | 1750 | 1754 | 1758 | 1762 | 1766 | 1770 | 1774 | 0 1 1 | 2 2 2 | 3 3 4 |
| .25 | 1778 | 1782 | 1786 | 1791 | 1795 | 1799 | 1803 | 1807 | 1811 | 1816 | 0 1 1 | 2 2 2 | 3 3 4 |
| .26 | 1820 | 1824 | 1828 | 1832 | 1837 | 1841 | 1845 | 1849 | 1854 | 1858 | 0 1 1 | 2 2 3 | 3 3 4 |
| .27 | 1862 | 1866 | 1871 | 1875 | 1879 | 1884 | 1888 | 1892 | 1897 | 1901 | 0 1 1 | 2 2 3 | 3 3 4 |
| .28 | 1905 | 1910 | 1914 | 1919 | 1923 | 1928 | 1932 | 1936 | 1941 | 1945 | 0 1 1 | 2 2 3 | 3 4 4 |
| .29 | 1950 | 1954 | 1959 | 1963 | 1968 | 1972 | 1977 | 1982 | 1986 | 1991 | 0 1 1 | 2 2 3 | 3 4 4 |
| .30 | 1995 | 2000 | 2004 | 2009 | 2014 | 2018 | 2023 | 2028 | 2032 | 2037 | 0 1 1 | 2 2 3 | 3 4 4 |
| .31 | 2042 | 2046 | 2051 | 2056 | 2061 | 2065 | 2070 | 2075 | 2080 | 2084 | 0 1 1 | 2 2 3 | 3 4 4 |
| .32 | 2089 | 2094 | 2099 | 2104 | 2109 | 2113 | 2118 | 2123 | 2128 | 2133 | 0 1 1 | 2 2 3 | 3 4 4 |
| .33 | 2138 | 2143 | 2148 | 2153 | 2158 | 2163 | 2168 | 2173 | 2178 | 2183 | 0 1 1 | 2 2 3 | 3 4 4 |
| .34 | 2188 | 2193 | 2198 | 2203 | 2208 | 2213 | 2218 | 2223 | 2228 | 2234 | 1 1 2 | 2 3 3 | 4 4 5 |
| .35 | 2239 | 2244 | 2249 | 2254 | 2259 | 2265 | 2270 | 2275 | 2280 | 2286 | 1 1 2 | 2 3 3 | 4 4 5 |
| .36 | 2291 | 2296 | 2301 | 2307 | 2312 | 2317 | 2323 | 2328 | 2333 | 2339 | 1 1 2 | 2 3 3 | 4 4 5 |
| .37 | 2344 | 2350 | 2355 | 2360 | 2366 | 2371 | 2377 | 2382 | 2388 | 2393 | 1 1 2 | 2 3 3 | 4 4 5 |
| .38 | 2399 | 2404 | 2410 | 2415 | 2421 | 2427 | 2432 | 2438 | 2443 | 2449 | 1 1 2 | 2 3 3 | 4 4 5 |
| .39 | 2455 | 2460 | 2466 | 2472 | 2477 | 2483 | 2489 | 2495 | 2500 | 2506 | 1 1 2 | 2 3 3 | 4 5 5 |
| .40 | 2512 | 2518 | 2523 | 2529 | 2535 | 2541 | 2547 | 2553 | 2559 | 2564 | 1 1 2 | 2 3 4 | 4 5 5 |
| .41 | 2570 | 2576 | 2582 | 2588 | 2594 | 2600 | 2606 | 2612 | 2618 | 2624 | 1 1 2 | 2 3 4 | 4 5 5 |
| .42 | 2630 | 2636 | 2642 | 2649 | 2655 | 2661 | 2667 | 2673 | 2679 | 2685 | 1 1 2 | 2 3 4 | 4 5 6 |
| .43 | 2692 | 2698 | 2704 | 2710 | 2716 | 2723 | 2729 | 2735 | 2742 | 2748 | 1 1 2 | 3 3 4 | 4 5 6 |
| .44 | 2754 | 2761 | 2767 | 2773 | 2780 | 2786 | 2793 | 2799 | 2805 | 2812 | 1 1 2 | 3 3 4 | 4 5 6 |
| .45 | 2818 | 2825 | 2831 | 2838 | 2844 | 2851 | 2858 | 2864 | 2871 | 2877 | 1 1 2 | 3 3 4 | 5 5 6 |
| .46 | 2884 | 2891 | 2897 | 2904 | 2911 | 2917 | 2924 | 2931 | 2938 | 2944 | 1 1 2 | 3 3 4 | 5 5 6 |
| .47 | 2951 | 2958 | 2965 | 2972 | 2979 | 2985 | 2992 | 2999 | 3006 | 3013 | 1 1 2 | 3 3 4 | 5 5 6 |
| .48 | 3020 | 3027 | 3034 | 3041 | 3048 | 3055 | 3062 | 3069 | 3076 | 3083 | 1 1 2 | 3 4 4 | 5 6 6 |
| .49 | 3090 | 3097 | 3105 | 3112 | 3119 | 3126 | 3133 | 3141 | 3148 | 3155 | 1 1 2 | 3 4 4 | 5 6 6 |

## ANTI LOGARITHMS

|     | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 1 2 3 | 4 5 6   | 7 8 9    |
|-----|------|------|------|------|------|------|------|------|------|------|-------|---------|----------|
| .50 | 3162 | 3170 | 3177 | 3184 | 3192 | 3199 | 3206 | 3214 | 3221 | 3228 | 1 1 2 | 3 4 4   | 5 6 7    |
| .51 | 3236 | 3243 | 3251 | 3258 | 3266 | 3273 | 3281 | 3289 | 3296 | 3304 | 1 2 2 | 3 4 5   | 5 6 7    |
| .52 | 3311 | 3319 | 3327 | 3334 | 3342 | 3350 | 3357 | 3365 | 3373 | 3381 | 1 2 2 | 3 4 5   | 5 6 7    |
| .53 | 3388 | 3396 | 3404 | 3412 | 3420 | 3428 | 3436 | 3443 | 3451 | 3459 | 1 2 2 | 3 4 5   | 6 6 7    |
| .54 | 3467 | 3475 | 3483 | 3491 | 3499 | 3508 | 3516 | 3524 | 3532 | 3540 | 1 2 2 | 3 4 5   | 6 6 7    |
| .55 | 3548 | 3556 | 3565 | 3573 | 3581 | 3589 | 3597 | 3606 | 3614 | 3622 | 1 2 2 | 3 4 5   | 6 7 7    |
| .56 | 3631 | 3639 | 3648 | 3656 | 3664 | 3673 | 3681 | 3690 | 3698 | 3707 | 1 2 3 | 3 4 5   | 6 7 8    |
| .57 | 3715 | 3724 | 3733 | 3741 | 3750 | 3758 | 3767 | 3776 | 3784 | 3793 | 1 2 3 | 3 4 5   | 6 7 8    |
| .58 | 3802 | 3811 | 3819 | 3828 | 3837 | 3846 | 3855 | 3864 | 3873 | 3882 | 1 2 3 | 4 4 5   | 6 7 8    |
| .59 | 3890 | 3899 | 3908 | 3917 | 3926 | 3936 | 3945 | 3954 | 3963 | 3972 | 1 2 3 | 4 5 6   | 6 7 8    |
| .60 | 3981 | 3990 | 3999 | 4009 | 4018 | 4027 | 4036 | 4046 | 4055 | 4064 | 1 2 3 | 4 5 6   | 6 7 8    |
| .61 | 4074 | 4083 | 4093 | 4102 | 4111 | 4121 | 4130 | 4140 | 4150 | 4159 | 1 2 3 | 4 5 6   | 7 8 9    |
| .62 | 4169 | 4178 | 4188 | 4198 | 4207 | 4217 | 4227 | 4236 | 4246 | 4256 | 1 2 3 | 4 5 6   | 7 8 9    |
| .63 | 4266 | 4276 | 4285 | 4295 | 4305 | 4315 | 4325 | 4335 | 4345 | 4355 | 1 2 3 | 4 5 6   | 7 8 9    |
| .64 | 4365 | 4375 | 4385 | 4395 | 4406 | 4416 | 4426 | 4436 | 4446 | 4457 | 1 2 3 | 4 5 6   | 7 8 9    |
| .65 | 4467 | 4477 | 4487 | 4498 | 4508 | 4519 | 4529 | 4539 | 4550 | 4560 | 1 2 3 | 4 5 6   | 7 8 9    |
| .66 | 4571 | 4581 | 4592 | 4603 | 4613 | 4624 | 4634 | 4645 | 4656 | 4667 | 1 2 3 | 4 5 6   | 7 9 10   |
| .67 | 4677 | 4688 | 4699 | 4710 | 4721 | 4732 | 4742 | 4753 | 4764 | 4775 | 1 2 3 | 4 5 7   | 8 9 10   |
| .68 | 4786 | 4797 | 4808 | 4819 | 4831 | 4842 | 4853 | 4864 | 4875 | 4887 | 1 2 3 | 4 6 7   | 8 9 10   |
| .69 | 4898 | 4909 | 4920 | 4932 | 4943 | 4955 | 4966 | 4977 | 4989 | 5000 | 1 2 3 | 5 6 7   | 8 9 10   |
| .70 | 5012 | 5023 | 5035 | 5047 | 5058 | 5070 | 5082 | 5093 | 5105 | 5117 | 1 2 4 | 5 6 7   | 8 9 11   |
| .71 | 5129 | 5140 | 5152 | 5164 | 5176 | 5188 | 5200 | 5212 | 5224 | 5236 | 1 2 4 | 5 6 7   | 8 10 11  |
| .72 | 5248 | 5260 | 5272 | 5284 | 5297 | 5309 | 5321 | 5333 | 5346 | 5358 | 1 2 4 | 5 6 7   | 9 10 11  |
| .73 | 5370 | 5383 | 5395 | 5408 | 5420 | 5433 | 5445 | 5458 | 5470 | 5483 | 1 3 4 | 5 6 8   | 9 10 11  |
| .74 | 5495 | 5508 | 5521 | 5534 | 5546 | 5559 | 5572 | 5585 | 5598 | 5610 | 1 3 4 | 5 6 8   | 9 10 12  |
| .75 | 5623 | 5636 | 5649 | 5662 | 5675 | 5689 | 5702 | 5715 | 5728 | 5741 | 1 3 4 | 5 7 8   | 9 10 12  |
| .76 | 5754 | 5768 | 5781 | 5794 | 5808 | 5821 | 5834 | 5848 | 5861 | 5875 | 1 3 4 | 5 7 8   | 9 11 12  |
| .77 | 5888 | 5902 | 5916 | 5929 | 5943 | 5957 | 5970 | 5984 | 5998 | 6012 | 1 3 4 | 5 7 8   | 10 11 12 |
| .78 | 6026 | 6039 | 6053 | 6067 | 6081 | 6095 | 6109 | 6124 | 6138 | 6152 | 1 3 4 | 6 7 8   | 10 11 13 |
| .79 | 6166 | 6180 | 6194 | 6209 | 6223 | 6237 | 6252 | 6266 | 6281 | 6295 | 1 3 4 | 6 7 9   | 10 11 13 |
| .80 | 6310 | 6324 | 6339 | 6353 | 6368 | 6383 | 6397 | 6412 | 6427 | 6442 | 1 3 4 | 6 7 9   | 10 12 13 |
| .81 | 6457 | 6471 | 6486 | 6501 | 6516 | 6531 | 6546 | 6561 | 6577 | 6592 | 2 3 5 | 6 8 9   | 11 12 14 |
| .82 | 6607 | 6622 | 6637 | 6653 | 6668 | 6683 | 6699 | 6714 | 6730 | 6745 | 2 3 5 | 6 8 9   | 11 12 14 |
| .83 | 6761 | 6776 | 6792 | 6808 | 6823 | 6839 | 6855 | 6871 | 6887 | 6902 | 2 3 5 | 6 8 9   | 11 13 14 |
| .84 | 6918 | 6934 | 6950 | 6966 | 6982 | 6998 | 7015 | 7031 | 7047 | 7063 | 2 3 5 | 6 8 10  | 11 13 15 |
| .85 | 7079 | 7096 | 7112 | 7129 | 7145 | 7161 | 7178 | 7194 | 7211 | 7228 | 2 3 5 | 7 8 10  | 12 13 15 |
| .86 | 7244 | 7261 | 7278 | 7295 | 7311 | 7328 | 7345 | 7362 | 7379 | 7396 | 2 3 5 | 7 8 10  | 12 13 15 |
| .87 | 7413 | 7430 | 7447 | 7464 | 7482 | 7499 | 7516 | 7534 | 7551 | 7568 | 2 3 5 | 7 9 10  | 12 14 16 |
| .88 | 7586 | 7603 | 7621 | 7638 | 7656 | 7674 | 7691 | 7709 | 7727 | 7745 | 2 4 5 | 7 9 11  | 12 14 16 |
| .89 | 7762 | 7780 | 7798 | 7816 | 7834 | 7852 | 7870 | 7889 | 7907 | 7925 | 2 4 5 | 7 9 11  | 13 14 16 |
| .90 | 7943 | 7962 | 7980 | 7998 | 8017 | 8035 | 8054 | 8072 | 8091 | 8110 | 2 4 6 | 7 9 11  | 13 15 17 |
| .91 | 8128 | 8147 | 8166 | 8185 | 8204 | 8222 | 8241 | 8260 | 8279 | 8299 | 2 4 6 | 8 9 11  | 13 15 17 |
| .92 | 8318 | 8337 | 8356 | 8375 | 8395 | 8414 | 8433 | 8453 | 8472 | 8492 | 2 4 6 | 8 10 12 | 14 15 17 |
| .93 | 8511 | 8531 | 8551 | 8570 | 8590 | 8610 | 8630 | 8650 | 8670 | 8690 | 2 4 6 | 8 10 12 | 14 16 18 |
| .94 | 8710 | 8730 | 8750 | 8770 | 8790 | 8810 | 8831 | 8851 | 8872 | 8892 | 2 4 6 | 8 10 12 | 14 16 18 |
| .95 | 8913 | 8933 | 8954 | 8974 | 8995 | 9016 | 9036 | 9057 | 9078 | 9099 | 2 4 6 | 8 10 12 | 15 17 19 |
| .96 | 9120 | 9141 | 9162 | 9183 | 9204 | 9226 | 9247 | 9268 | 9290 | 9311 | 2 4 7 | 9 11 13 | 15 17 20 |
| .97 | 9333 | 9354 | 9376 | 9397 | 9419 | 9441 | 9462 | 9484 | 9506 | 9528 | 2 4 7 | 9 11 13 | 16 18 20 |
| .98 | 9550 | 9572 | 9594 | 9616 | 9638 | 9661 | 9683 | 9705 | 9727 | 9750 | 2 4 7 | 9 11 14 | 16 18 20 |
| .99 | 9772 | 9795 | 9817 | 9840 | 9863 | 9886 | 9908 | 9931 | 9954 | 9977 | 2 5 7 | 9 11 14 | 16 18 20 |