

Simple Harmonic Motion

Students Learning Outcomes

After studying this chapter, the students will be able to:

- ◆ describe simple examples of free oscillations.
- ◆ use the terms displacement, amplitude, period, frequency, angular frequency and phase difference in the context of oscillations.
- ◆ express the period of simple harmonic motion in terms of both frequency and angular frequency.
- ◆ express that simple harmonic motion occurs when acceleration is proportional to the displacement from a fixed point and in opposite direction.
- ◆ use $a = -\omega^2 x$ to solve problem.
- ◆ use the equation $v = v_0 \cos$ and $v = \pm \omega \sqrt{x_0^2 - x^2}$.
- ◆ analyze graphical representation of the variation of displacement, velocity and acceleration for simple harmonic motion.
- ◆ analyze the interconversion of kinetic energy and potential energy during simple harmonic motion.
- ◆ apply $\frac{1}{2}m\omega^2 x_0^2$ for total energy of a system undergoing simple harmonic motion.
- ◆ describe that resistive force acting on an oscillating system causes damping.
- ◆ use the terms light, critical and heavy damping.
- ◆ sketch displacement time graph to illustrate light, critical and heavy damping.
- ◆ state that resonance involves a maximum amplitude of oscillation and this occurs when an oscillating system is forced to oscillate at its natural frequency.
- ◆ describe practical examples of free and forced oscillations.
- ◆ describe practical examples of damped oscillations. (with particular reference to the efforts of the degree of damping and the importance of critical damping in cases such as a car suspension system).
- ◆ justify qualitatively the factors which determine the frequency response and sharpness of the resonance.
- ◆ identify the use of standing waves and resonance in applications. [(such as rubens tubes, Chladni plates and acoustic levitation knowledge of wave harmonic modes is not required)].
- ◆ justify the importance of critical damping in car suspension system.
- ◆ justify that there are some circumstances in which resonance is useful (such as tuning a radio, microwave oven and other circumstances in which resonance should be avoided such as airplanes, swing or a suspension bridge).

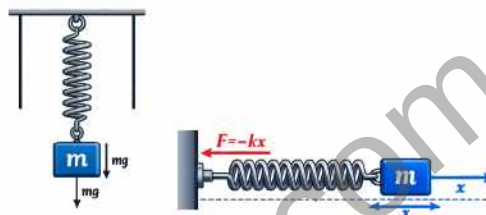
In this chapter, we will discuss concept of simple harmonic motion (S.H.M), a special type of oscillatory motion where a restoring force always pulls the object back towards its mean position. We will also explore the mathematics and physics behind S.H.M, understanding its characteristics, equations and real world applications.

Interesting Information

Many natural phenomena show periodic motion. Like motion of Earth around the Sun, the motion of moon around the Earth, water waves, sound waves, a tuning fork, a rocking chair and many others.

14.1: OSCILLATORY MOTION

Oscillatory motion is a to and fro motion about its mean position and it is a periodic motion which repeats itself after an equal interval of time.



We come across many examples of oscillatory motion in our daily life. Some examples of oscillatory motion are given below:

- (i) A simple pendulum vibrating about its mean position when displaced from its rest position.
- (ii) A mass oscillating at the end of a spring in a horizontal or vertical plane when pulled and released.
- (iii) A vibrating tuning fork.
- (iv) The atoms or molecules in a solid substance oscillate about their mean position.
- (v) Motion of a swing.
- (vi) Air molecules oscillate when sound waves travel through air.
- (vii) Most of musical instruments like guitar and violin have strings attached to them to produce music by vibratory motion.
- (viii) The wings of birds during flying.



All the bodies that undergo vibrational or oscillation motion have an equilibrium position or mean position. When the body is displaced from this mean position then there is a restoring force which brings it back to its equilibrium position and it causes vibration or oscillation motion of the body.

Terms Related to Oscillatory Motion

We will discuss some important terms related to oscillatory motion.

Vibration: "One complete round trip (cycle) of vibrating body about its mean position is

called vibration". Alternatively, vibration can also be defined motion of the body from one extreme position to other extreme position and back to first extreme position crossing mean position. For example, motion of bob of pendulum from A to B and back from B to A is one vibration as shown in Fig. 14.1.

Instantaneous Displacement (x): "The distance of a vibrating body from its mean position at any instant, is called instantaneous displacement". The distance of bob of simple pendulum from O to C is called displacement and denoted by " x ". i.e., $\overline{OC} = x$.

Amplitude (x_0): "The magnitude of the maximum displacement of the vibrating body on either side of its mean position is called amplitude denoted by x_0 , the amplitude of bob of simple pendulum is shown in Fig. 14.1 is $\overline{OA}$ or $\overline{OB}$. i.e., $\overline{OA} = \overline{OB} = x_0$.

Amplitude of a wave measures the energy carried by a wave. The greater the amplitude of the wave, greater will be the energy carried by the wave and vice versa.

Time Period: Time period is defined as the time taken to complete one vibration or one cycle. It is represented by T and its SI unit is second (s).

Time period in terms of frequency f and angular frequency ω is expressed as:

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Frequency: Frequency is defined as the number of vibrations or oscillations (n) completed by the vibrating body in one second. It is denoted by f .

Mathematically, $f = \frac{n}{t}$

It is expressed in terms of the reciprocal of time period i.e.; $f = \frac{1}{T}$

The unit of frequency is hertz (Hz). One hertz is defined as one oscillation per second. The dimension of frequency is $[T^{-1}]$.

Angular Frequency: Angular displacement per unit time is called angular frequency. It is represented by ω and it can be expressed as:

$$\omega = \frac{\theta}{t}$$

Now for one revolution $\theta = 2\pi$ radian and $t = T$ (time period).

So $\omega = \frac{2\pi}{T}$

As $f = \frac{1}{T}$ Therefore, $\omega = 2\pi f$

The SI unit of angular frequency is rad s^{-1} and its dimension is $[T^{-1}]$.

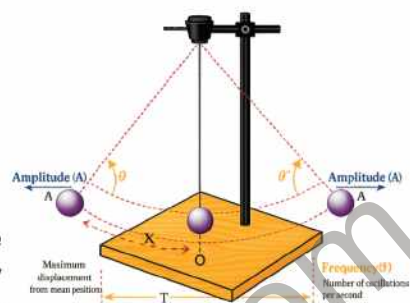


Fig. 14.1: Illustration of different terms of oscillatory motion with the help of vibrating simple pendulum

Brain teaser

If x_0 is the amplitude of simple pendulum, then

- (i) what will be distance covered in one cycle (vibration) by a simple pendulum?

Brain teaser

A fish connected to a spring makes 10 vibrations in 20 seconds. What is its period and frequency?

14.2 SIMPLE HARMONIC MOTION

Simple harmonic motion is a type of oscillation or vibratory motion produced under the action of restoring force.

The necessary conditions for the execution of simple harmonic motion are:

- The restoring force shall be directly proportional to the displacement from the mean position.
- The restoring force shall be proportional to the inertia of body.
- The force and displacement should follow Hooke's law ($F = -kx$), where k is spring constant depends upon the nature of material of spring.
- Acceleration of oscillating object should be proportional to the displacement ($a \propto -x$), the negative sign indicates that acceleration is directed towards mean position.

Interesting Information

It must be noted that not all periodic vibrations are examples of simple harmonic motion since all restoring forces are not proportional to the displacement. Any restoring force can cause oscillatory motion. An electrocardiogram traces the periodic pattern of a beating heart, but the motion of the recorder needle is not a simple harmonic motion. As the restoring force in this case is not always proportional to the displacement from the mean position.

14.3 PRACTICAL S.H.M SYSTEMS

Mass Attached to An Elastic Spring

Consider a body of mass ' m ' attached with a spring of spring constant ' k ' lying on smooth horizontal surface (Fig. 14.2). Initially the body is at rest position O called mean position. Now we apply some force F on the body and we displace the body from O to A through displacement x towards right. According to Hooke's law, the applied force is proportional to displacement, i.e.;

$$F \propto x \text{ or } F = kx \dots\dots\dots (14.1)$$

The spring will exert the force on the body due to elastic restoring force at the same time which is equal in magnitude and opposite in direction to the applied force.

$$F = -kx \dots\dots\dots (14.2)$$

When the body is released by removing the applied force, it will move towards ' O ' and will cross the mean position O due to inertia and reaches the point B , compresses the spring it returns and starts oscillating between A and B .

Also, according to Newton's second law of motion:

$$F = ma \dots\dots\dots (14.3)$$

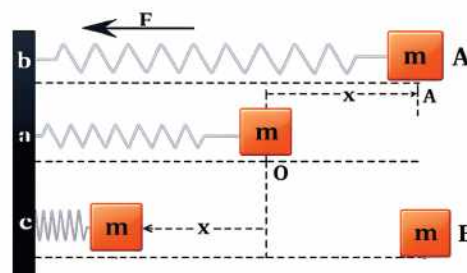


Fig. 14.2: Mass attached with spring performing S.H.M

Comparing Eqs. (14.2) and (14.3), we have

$$ma = -kx$$

or
$$a = -\frac{k}{m}x \dots\dots\dots (14.4)$$

where 'k' is spring constant depending upon the nature (physical shape and structure of a spring). The negative sign shows that acceleration and displacement are always in opposite direction or acceleration is always towards mean position.

As, k/m is a constant, so, Eq. (14.4), can be written as

$$a = -(\text{constant})x$$

or
$$a \propto -x$$

This is mathematical form of simple harmonic motion.

It shows that acceleration of the body executing S.H.M is always directly proportional to the displacement and is always directed towards the mean position.

Time period and frequency

Here
$$\omega^2 = \frac{k}{m}$$

or
$$\omega = \sqrt{\frac{k}{m}}$$

As
$$\omega = 2\pi f$$

or
$$f = \frac{\omega}{2\pi}$$

or
$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

or
$$T = \frac{1}{f} = 2\pi \sqrt{\frac{m}{k}}$$

Point to ponder!

Real world applications that utilize S.H.M principle include:

- Pendulum clocks, musical instruments, and vehicle suspension system etc.

Example 14.1 A particle executing simple harmonic motion has an angular frequency of 2 rad s^{-1} . Determine its frequency and time period.

Solution

Angular frequency $\omega = 2 \text{ rad s}^{-1}$, Frequency $f = ?$

Time period $t = ?$

As
$$\omega = 2\pi f,$$

therefore,
$$2 \text{ rad s}^{-1} = 2\pi f$$

$$f = \frac{1}{\pi}$$

$$f \approx 0.318 \text{ Hz}$$

$$T = ?$$

$$T = \frac{1}{f}$$

$$T = \frac{1}{0.318}$$

$$T \approx 3.14 \text{ s}$$

As

or

The frequency is approximately 0.318 Hz, and the time period is approximately 3 seconds.

Simple Pendulum

A simple pendulum consists of a small heavy solid bob of mass m suspended from a rigid support through a light inextensible string of certain length.

When the pendulum is displaced from its mean position O through an angle to the extreme position P and released, then a restoring force acts on the pendulum towards the mean position. Due to this restoring force, the pendulum starts oscillation to and fro under the action of gravity along a curved path about the mean position O (Fig. 14.3-b).

At extreme position P, the weight of the body makes an angle θ with the direction of the string. We can resolve weight into its rectangular components. As the pendulum has no motion along the direction of the string, therefore, the component $mg \cos \theta$ and tension T are along the same line but in opposite direction, so they cancel the effects of each other. The component $mg \sin \theta$ provides the necessary restoring force towards mean position and is responsible for the motion of simple pendulum. Thus;

$$F = -mg \sin \theta$$

Negative sign shows that the acceleration of the pendulum is always directed towards mean position. According to Newton's 2nd law:

$$F = ma$$

Comparing above equations

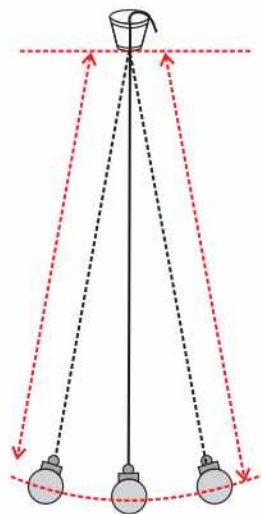
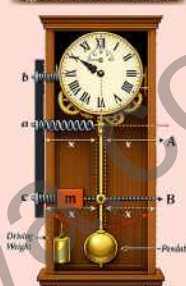
$$ma = -mg \sin \theta$$

$$a = -g \sin \theta \quad \dots\dots\dots (14.5)$$

Interesting Information

Dutch Mathematician astronomer and physicist Christiaan Huygens patented the first pendulum clock in 1656. He finalized the mathematical formula that relates pendulum length to time, using simple harmonic motion.

Huygens' First Pendulum Clock (1656)



Brain teaser

All vibrating bodies produce sound. Why a vibrating simple pendulum does not produce sound?

If the angle of the simple pendulum is small, then the $\sin\theta$ can be replaced by the angle θ itself, expressed in radians. That is, for small angles:

$$\sin\theta \approx \theta$$

So, Eq.(14.5) is written as:

$$a = -g\theta \quad \text{..... (14.6)}$$

By the definition of an angular displacement,

$$\theta = \frac{S}{\ell}$$

where S is the actual path length followed by the pendulum. Thus,

$$a = -\left(\frac{g}{\ell}\right)S$$

In Fig. (14.3 (b)) x is very nearly equal to the arc of length S of the circular path when the angle is small (about 10 or less). Hence,

$$a = -\left(\frac{g}{\ell}\right)x \quad \text{..... (14.7)}$$

If length ℓ of the pendulum is fixed and 'g' remains constant for a given place and (g/ℓ) is constant, then Eq. (14.7) can be rewritten as;

$$a = -(\text{constant})x$$

$$a \propto -x$$

This is the mathematical form of S.H.M and it is concluded that the motion of a simple pendulum is S.H.M.

$$\text{As } a = -\omega^2 x \quad \text{..... (14.8)}$$

Comparing Eqs. (14.7) and (14.8), we have

$$\omega^2 = \frac{g}{\ell}$$

$$\text{or } \omega = \sqrt{\frac{g}{\ell}} \quad \text{..... (14.9)}$$

As $\omega = 2\pi f$, therefore,

$$2\pi f = \sqrt{\frac{g}{\ell}} \quad \text{or} \quad f = \frac{1}{2\pi} \sqrt{\frac{g}{\ell}}$$

$$T = \frac{1}{f} = 2\pi \sqrt{\frac{\ell}{g}} \quad \text{..... (14.10)}$$

Table 14.1: Values of θ and $\sin\theta$ for small angle

$\theta(\text{degree})$	$\theta(\text{radian})$	$\sin \theta$
0.00	0.0000	0.0000
1.00	0.0175	0.0175
2.00	0.0349	0.0349
3.00	0.0524	0.0524
4.00	0.0698	0.0698
5.00	0.0873	0.0872
6.00	0.1048	0.1046
7.00	0.1222	0.1219
8.00	0.1397	0.1392
9.00	0.1571	0.1565

Brain teaser

A pendulum is shifted from Lahore to Karachi will its time period change?

Do you know?

Pendulum is derived from Latin word *Pendulus* means hanging.

The above expression shows that the time period of simple pendulum is directly proportional to the square root of the length of the string and inversely proportional to the square root of acceleration due to gravity. The time period of motion of the pendulum is independent of the mass m of the bob and amplitude.

A pendulum that completes one vibration in two seconds, i.e., its time period is two seconds is known as a second pendulum.

Using simple pendulum to measure g

The simple pendulum can be used to determine the gravitational acceleration at a particular location. We measure the length ' ℓ ' of the pendulum and then set the pendulum into motion. The time period T of the simple pendulum is measured using a stopwatch and the acceleration due to gravity is calculated by using Eq. (14.10) in the following form:

$$g = \frac{4\pi^2 \ell}{T^2}$$

Example 14.2 What is the length of a second pendulum? Also find its frequency at a place where the value of g is taken to be 9.8 m s^{-2} .

Solution $T = 2 \text{ s}$

$$\ell = ?$$

As $T = 2\pi \sqrt{\frac{\ell}{g}}$

or $T^2 = \frac{4\pi^2 \ell}{g}$

or $\ell = \frac{gT^2}{4\pi^2} = \frac{(9.8 \text{ m s}^{-2})(2 \text{ s})^2}{4(3.14)^2}$

$$= 0.994 \text{ m}$$

or $\ell = 99.4 \text{ cm}$

Brain teaser

Would the time period of a simple pendulum be same on Earth and moon, if its length is kept constant?

Brain teaser

While measuring the time period of a simple pendulum it is recommended to take small amplitude, why?

14.4 SIMPLE HARMONIC MOTION AND UNIFORM CIRCULAR MOTION

In order to get an insight into simple S.H.M, we will co-relate it with uniform circular motion which we have already studied. The different parameters such as displacement, velocity, acceleration and time period of simple harmonic motion (S.H.M) can be understood by relating it with uniform circular motion. It has been observed that when a particle moves in a circular path, its projection on diameter of the circle executes S.H.M.

To study the simple harmonic motion, consider a turntable of radius r with a ball attached to its rim. A beam of light casts a shadow of the ball on the screen (Fig 14.4).

When the turntable rotates with constant angular speed ω , then the ball also moves along it with uniform circular motion. Its shadow on the screen oscillates executing to and fro motion across the screen in the form of simple harmonic motion, like a body attached to a spring.

Quantitative Analysis

Consider a motion of particle P along the circumference of circle of radius x_0 with uniform angular velocity ω . Its linear velocity at point P is along the tangent ($v_p = r\omega$). Let Q be the projection which oscillates along the diameter of circle about the mean position O. When the body is at point P its projection is at distance x from the mean position.

Instantaneous Displacement

If A is the initial position of particle P, then the angle which the radius OP sweeps out in time t is $\theta = \omega t$ with OQ as shown in Fig. 14.5.

Considering triangle POQ:

$$\frac{\overline{OQ}}{\overline{OP}} = \cos \theta$$

or
$$\frac{x}{x_0} = \cos \omega t$$

$$x = x_0 \cos \theta \dots\dots\dots (14.11)$$

Equation (14.11) gives the instantaneous displacement of point Q which is executing simple harmonic motion.

Instantaneous Velocity

Considering Fig. 14.6, the line PR is the horizontal component of velocity v_p of the particle and it is parallel to the diameter AB of the circle. Therefore, as Q vibrates horizontally, so instantaneous velocity of the point Q is horizontal component of velocity v_p .

$$v = (v_p)_x$$

$$v = v_p \cos (90 - \theta)$$

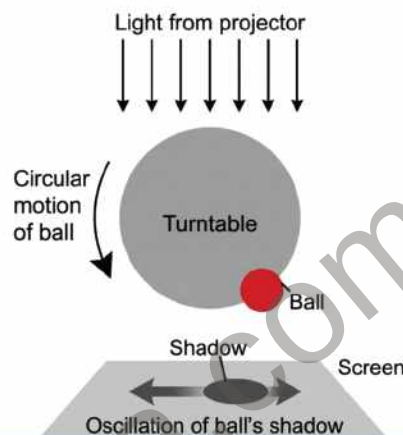


Fig. 14.4: The oscillation of the shadow of ball on screen. The ball is attached with uniformly rotating turn table.

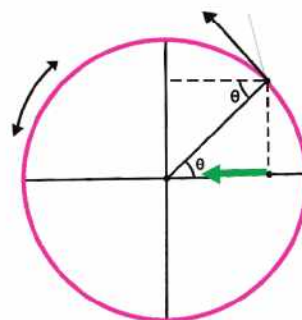


Fig. 14.5: A point P moving on circular path at constant angular velocity

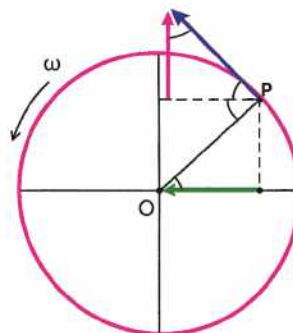


Fig. 14.6: Velocity of the point P and its horizontal component

Since $v_p = x_o \omega$ and $\cos(90 - \theta) = \sin \theta$

$$v = x_o \omega \sin \theta \quad \text{or} \quad v = x_o \omega \sin \omega t \quad \dots (14.12)$$

But $\sin^2 \theta + \cos^2 \theta = 1$

Therefore, equation 14.12 becomes:

$$\sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$v = x_o \omega \sqrt{1 - \cos^2 \theta}$$

$$v = x_o \omega \sqrt{1 - \frac{x^2}{x_o^2}} \quad \left(\because \cos \theta = \frac{x}{x_o} \right)$$

$$v = \omega \sqrt{x_o^2 - x^2} = \sqrt{\frac{k}{m}} (x_o^2 - x^2) \dots (14.13)$$

It may be noted that at mean position $x = 0$ and velocity is maximum. Thus,

$$v_{\max} = v_o = x_o \omega = \sqrt{\frac{k}{m}} x_o \dots (14.14)$$

Instantaneous Acceleration

Acceleration a_p of the particle at point P is directed towards the centre of the circle as shown in Fig. 14.7. The horizontal component of a_p is along the diameter. Thus, the acceleration of projection Q is equal to the horizontal component of a_p .

$$a = (a_p) \cos \theta$$

$$a = a_p \cos \theta$$

As $a = -x_o \omega^2$ (centripetal acceleration) and $\cos \theta = x/x_o$.

The negative sign indicates that the direction of acceleration is always directed towards the mean position.

$$a = -x_o \omega^2 \left(\frac{x}{x_o} \right) = -\omega^2 x$$

$$a = -x \omega^2 \dots (14.15)$$

As particle is moving in the circle with uniform angular frequency ω , therefore, Eq. (14.15) can be rewritten as;

$$a \propto -x$$

This expression is the mathematical condition of S.H.M i.e., acceleration is directly proportional to the displacement and negative sign shows that its direction is towards mean position, therefore, it is concluded that when a particle is moving along a

Interesting Information

The expressions such as time period, displacement, velocity and acceleration which are true for projection of a particle are also applicable for particle moving in a circle.

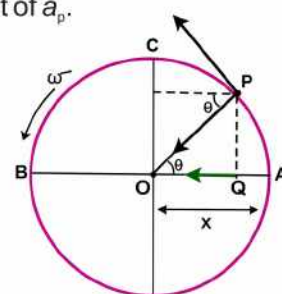


Fig. 14.7: Acceleration of the point P and its horizontal component

circumference of a circle, then its projection executes S.H.M.

Example 14.3 A 0.2 kg mass is attached to a spring with a spring constant of 10 N m^{-1} . The mass is displaced by 0.1 m from its equilibrium position and released from rest. Find:

Solution (a) acceleration at $x = 0.05 \text{ m}$ (b) velocity at $x = 0.05 \text{ m}$

$$m = 0.2 \text{ kg}$$

$$k = 10 \text{ N m}^{-1}$$

$$x_0 = 0.1 \text{ m}$$

Angular frequency $\omega = ?$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{10 \text{ N m}^{-1}}{0.2 \text{ kg}}} = \sqrt{50} = 5\sqrt{2} \text{ rad s}^{-1} \approx 7.07 \text{ rad s}^{-1}$$

(a) Acceleration at $x = 0.05 \text{ m}$

$$\begin{aligned} a &= -\omega^2 x \\ &= -(7.07 \text{ rad s}^{-1})^2 \times 0.05 \text{ m} \\ &= -50 \times 0.05 \\ &= -2.5 \text{ m s}^{-2} \end{aligned}$$

(b) Velocity at $x = 0.05 \text{ m}$

$$\begin{aligned} v &= \omega \sqrt{(x_0^2 - x^2)} \\ v &= 7.07 \text{ rad s}^{-1} \sqrt{(0.1 \text{ m})^2 - (0.05 \text{ m})^2} \\ &= 7.07 \sqrt{(0.01) - (0.0025)} \\ &= \sqrt{0.0075} \\ &\approx 7.07 \times 0.0866 \\ v &\approx 0.612 \text{ m s}^{-1} \end{aligned}$$

The results are: $a \approx -2.5 \text{ m s}^{-2}$
 $v \approx 0.612 \text{ m s}^{-1}$

14.5 PHASE

The angle $\theta = \omega t$ which specifies the displacements x as well as the direction of motion of the point oscillating S.H.M. is called phase.

Phase is the quantity which shows the state of motion of an oscillator. In circular motion, the displacement of projection of the body moving in a circle, executing S.H.M. on the diameter of the circle, is given by: $x = x_0 \cos \theta$

or $x = x_0 \cos(\omega t + 180^\circ) \dots \dots \dots (14.17)$

Where x is instantaneous displacement, x_0 is maximum displacement; ω is the angular velocity its graphical representation is shown in Fig. 14.8 (a). The general way of

showing this equation is:

$$x = x_0 \cos(\omega t + \phi) \dots\dots\dots (14.18)$$

The time varying quantity $(\omega t + \phi)$, is called the phase of the motion. It describes the state of motion at a given time. The constant ϕ is called the phase constant (or phase angle). The value of ϕ depends on ϕ displacement and velocity of the particle at $t = 0$.

The quantity ϕ represents the phase difference between the states of motion of two oscillators. Let us explain, with the help of graph plotted between x and t .

If $\phi = 0^\circ$, then Eq. (14.18) becomes:

$$x = x_0 \cos \omega t$$

Putting $t = 0, T/4, T/2, 3T/4, \dots$, we obtain a graph, as shown in Fig. 14.8 (a). This graph shows that:

at $t = 0, T/2$ and T (corresponding to $\theta = 0, \pi$ and 2π), the point is at the extreme positions. At $t = T/4$ and $3T/4$ (corresponding to $\theta = \pi/2$ and $3\pi/2$), the point is at mean position.

If $\phi = 90^\circ$, then Eq. (14.18) becomes:

$$x = x_0 \cos(\omega t + 90^\circ)$$

Putting $t = 0, T/4, T/2, 3T/4, T \dots$, we obtain graph, as shown in Fig. 14.8 (b). This graph shows that:

at $t = 0, T/2$ and T (corresponding to $\theta = 0, \pi$ and 2π), the point is at mean positions.

At $t = T/4$ and $3T/4$ (corresponding to $\theta = \pi/2$ and $3\pi/2$), the point is at extreme position.

If $\phi = 180^\circ$, then Eq. (14.18) becomes:

$$x = x_0 \cos(\omega t + 180^\circ)$$

Putting $t = 0, T/4, T/2, 3T/4, T \dots$, we obtain a graph, as shown in Fig. 14.8 (c).

It can be noted that:

When the phase difference between two oscillating systems is 180° , they are said to be oscillating out of phase.

When phase difference between oscillating systems is 0° or 360° , they are said to be oscillating in phase.

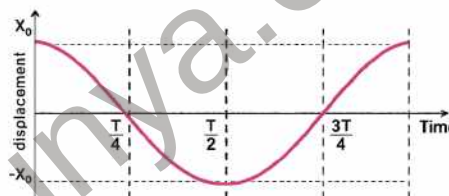


Fig. 14.8 (a): Graph of $x = x_0 \cos(\omega t + \phi)$ for $\phi = 0^\circ$

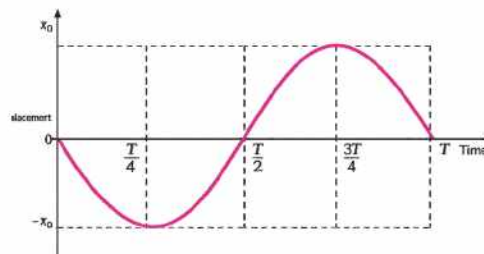


Fig. 14.8 (b): Graph of $x = x_0 \cos(\omega t + \phi)$ for $\phi = 90^\circ$

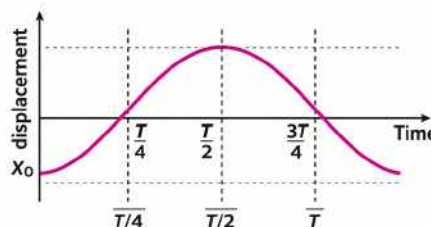


Fig. 14.8 (c): Graph of $x = x_0 \cos(\omega t + \phi)$ for $\phi = 180^\circ$

Example 14.4 A 0.5 kg mass is attached to a spring with a spring constant of 200 N m^{-1} . The mass is displaced by 0.2 m from its equilibrium position and released from rest. Assuming simple harmonic motion, find:

- (a) Displacement at $t = 0.5 \text{ s}$
- (b) Velocity at $t = 0.5 \text{ s}$
- (c) Acceleration at $t = 0.5 \text{ s}$

Solution Mass $m = 0.5 \text{ kg}$
 Spring constant $k = 200 \text{ N m}^{-1}$
 Amplitude $x_0 = 0.2 \text{ m}$
 Angular frequency $\omega = ?$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200 \text{ N m}^{-1}}{0.5 \text{ kg}}} = 20 \text{ rad s}^{-1}$$

- (a) Displacement at $t = 0.5 \text{ s}$

$$\begin{aligned} x(t) &= x_0 \cos(\omega t) \\ &= 0.2 \cos(20 \times 0.5) \\ &= 0.2 \cos(10) \\ &= 0.2 \times 0.839 \\ &= 0.168 \text{ m} \end{aligned}$$

- (b) Velocity at $t = 0.5 \text{ s}$

$$\begin{aligned} v(t) &= x_0 \omega \sin(\omega t) \\ &= 0.2 \times 20 \sin(20 \times 0.5) \\ &= 4 \sin(10) \\ v(t) &= 4 \times 0.544 = 2.176 \text{ m s}^{-1} \end{aligned}$$

- (c) Acceleration at $t = 0.05 \text{ s}$

$$\begin{aligned} a(t) &= -x_0 \omega^2 \cos(\omega t) \\ &= 0.2 \times (20)^2 \cos(20 \times 0.5) \\ &= -80 \cos(10) = -80(0.8391) \\ a(t) &= -67.12 \text{ m s}^{-2} \end{aligned}$$

14.6 GRAPHICAL REPRESENTATION OF S.H.M

The graphical representations of displacement, velocity and acceleration of a body executing SHM is given in Fig. 14.9 (a, b and c).

(i) The displacement of the particle executing SHM is given by the expression:

$$x = x_0 \cos \omega t \quad \text{..... (14.19)}$$

This shows that the particle will have maximum displacement (x_0) at extreme position.

(ii) The velocity of the particle executing SHM is given by the expression:

$$v = x_0 \omega \sin (\omega t) \quad \text{..... (14.20)}$$

The velocity of the particle is maximum (i.e., $v = x_0 \omega$) at the mean position and zero at the extreme positions.

(iii) The acceleration of the particle executing SHM is given by the expression:

$$a = -\omega^2 x$$

Putting $x = x_0 \cos \omega t$, we have:

$$a = -x_0 \omega^2 \cos (\omega t) \quad \text{..... (14.21)}$$

The acceleration will be maximum (i.e., $x_0 \omega^2$) at the extreme positions, and zero at the mean position.

From the graph of Eqs. (14.19), (14.20) and (14.21) shown in Figs. 14.9 (a, b and c) can be seen that:

- The phase difference between velocity and displacement is $\pi/2$.
- The phase difference between acceleration and displacement is $\pi/2$.

However, it must be noted that the phase difference between displacement, velocity and acceleration is $\pi/2$.

14.7 CONSERVATION OF ENERGY IN S.H.M

When a body is executing simple harmonic motion, it possesses both potential energy as well as kinetic energy. Its potential energy is on account of its displacement from mean position and the kinetic energy is due to its velocity. These energies vary during the oscillation, but the total energy at any instant remains constant in the absence of

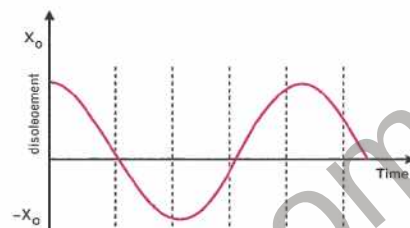


Fig. 14.9 (a): Graphical representation of displacement of oscillation performing S.H.M

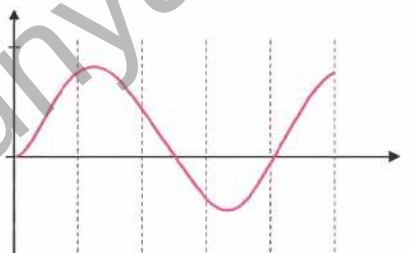


Fig. 14.9 (b): Graphical representation of velocity of oscillation performing S.H.M

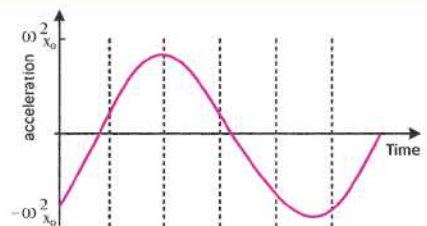


Fig. 14.9 (c): Graphical representation of an acceleration of oscillation performing S.H.M

unbalanced frictional forces. In case of mass-spring system shown in Fig (14.10), when the mass is displaced from the mean position O then there is a restoring force (F) whose value is zero at mean position when $x = 0$ and its value is maximum at either extreme position where $x = x_0$. Thus, average value of force from the mean position to the extreme position is:

$$F_{av} = \frac{0 + F}{2} = \frac{F}{2}$$

When displacement = 0 ; Force = 0

When displacement = x_0 ; Force = $F = kx_0$

$$F_{av} = \frac{0 + kx_0}{2}$$

$$F_{av} = \frac{1}{2} kx_0$$

When the spring is stretched to its maximum displacement x_0 , work is done on the spring which is given by

$$W = \frac{1}{2} kx_0(x_0) = \frac{1}{2} kx_0^2$$

This work done on the mass attached to a spring is stored in terms of potential energy, called elastic potential energy. So, we have the expression for P.E. at any instant x given by

$$(P.E.)_{inst} = \frac{1}{2} kx^2 \dots\dots\dots (14.22)$$

It is clear from Eq. (14.22) that the potential energy of oscillator is zero at $x = 0$ and maximum at $x = \pm x_0$ i.e., the extreme position on either side and total energy is entirely elastic potential energy.

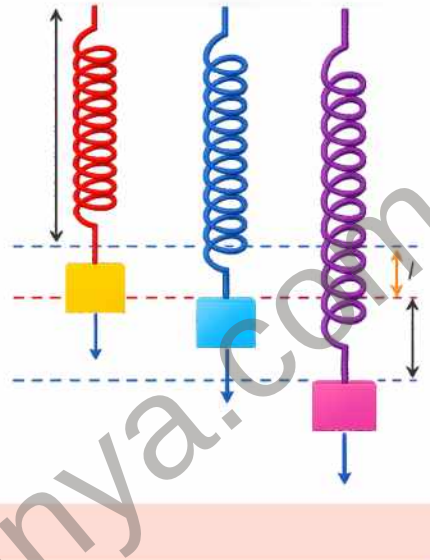
$$(P.E.)_{max} = \frac{1}{2} kx_0^2 \dots\dots\dots (14.23)$$

After the removal of force, the mass attached to a spring starts its motion with velocity v . As the instantaneous velocity v of the mass executing simple harmonic motion is given by;

$$v = \omega \sqrt{x_0^2 - x^2}$$

As $\omega = \sqrt{\frac{k}{m}}$, so above Eq. becomes:

$$v = \sqrt{\frac{k}{m} (x_0^2 - x^2)} \dots\dots\dots (14.24)$$



At mean position $x = 0$, the mass gets maximum velocity i.e.,

$$v_{\max} = v_o = \sqrt{\frac{k}{m}} x_o \dots\dots\dots (14.25)$$

Hence, at any instant where the displacement is x , the kinetic energy of mass attached to spring is given by

$$K.E. = \frac{1}{2} mv^2 = \frac{1}{2} k(x_o^2 - x^2) \dots\dots\dots (14.26)$$

At mean position ($x = 0$), the elastic P.E. is zero but the velocity is maximum, hence, total energy is entirely kinetic energy.

$$T.E = K.E._{\max} = \frac{1}{2} mv_{\max}^2 = \frac{1}{2} kx_o^2 \dots\dots\dots (14.27)$$

The total energy is the sum of kinetic and potential energy of the system at displacement ' x ', so we may write; $T.E = K.E + P.E$

Substituting the values of $K.E.$ and $P.E.$ from above Eqs., we have

$$T.E = \frac{1}{2} k(x_o^2 - x^2) + \frac{1}{2} kx^2 = \frac{1}{2} kx_o^2 \dots\dots\dots (14.28)$$

As, $\omega = \sqrt{\frac{k}{m}}$ or $k = m\omega^2$, so Eq. 14.28 can be written as;

$$T.E = \frac{1}{2} km\omega^2 x_o^2 \dots\dots\dots (14.29)$$

From Eqs. (14.23), (14.27) and (14.28), it is proved that total energy of mass spring system is constant and is directly proportional to the square of the amplitude of oscillation. This statement of conservation of energy is equally valid for all bodies

executing S.H.M. When $K.E.$ of mass is maximum, mass passes through the centre of oscillation, the $P.E.$ of mass spring is zero ($x = 0$), conversely when the $P.E.$ of spring is maximum, the mass is at its extreme position on either side, the $K.E.$ of mass is zero ($x = x_o$).

The energy oscillates between $K.E.$ and $P.E.$, but their sum remains constant. This can be illustrated by plotting graph of $K.E.$, $P.E.$ and $T.E.$ verses displacement as shown in Fig. (14.11).

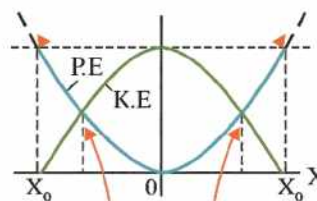


Fig. 14.11: Graphical representation of energy ($K.E.$, $P.E.$ and $T.E.$) against displacement, for S.H.M

Example 14.5 A body of 0.010 kg executes simple harmonic motion of period 6 s. one second after it has passed its mean position, its velocity is 0.06 meter per second. Find its kinetic energy, potential energy and Total energy.

Solution

$$m = 0.010 \text{ kg}$$

$$T = 6 \text{ s}$$

$$v = 0.06 \text{ m s}^{-1} \text{ (at } t = 1 \text{ s)}$$

Angular frequency (ω)

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6} = \pi/3 \text{ rad s}^{-1}$$

$$K.E. = \left(\frac{1}{2}\right)mv^2$$

$$= \left(\frac{1}{2}\right) \times 0.010 \text{ kg} \times (0.06 \text{ m s}^{-1})^2$$

$$= 1.8 \times 10^{-5} \text{ J}$$

To find the total energy (E_T), first, find the amplitude x_0 , using the formula of velocity;

$$v = x_0 \omega \sin(\omega t)$$

$$0.06 \text{ m s}^{-1} = x_0 \left(\frac{\pi}{3}\right) \sin\left(\frac{\pi}{3} \times 1\right)$$

$$0.06 \text{ m s}^{-1} = x_0 \frac{\pi}{3} (0.866)$$

$$0.06 \text{ m s}^{-1} = x_0 (1.047)(0.866)$$

$$0.06 \text{ m s}^{-1} = 0.90x_0$$

$$x_0 = 0.066 \text{ m}$$

Total energy E_T of system is given by

$$E_T = \frac{1}{2}m\omega^2 x_0^2$$

Substituting the values, we have:

$$E_T = \frac{1}{2} \times 0.01 \left(\frac{\pi}{3}\right)^2 (0.066)^2$$

$$E_T = 2.4 \times 10^{-5} \text{ J}$$

$$P.E. = E_T - K.E.$$

$$= 2.4 \times 10^{-5} - 1.8 \times 10^{-5}$$

$$P.E. = 0.6 \times 10^{-5} \text{ J}$$

$$P.E. = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$$

- Note:** (i) $P.E.$ can also be calculated by expression $P.E. = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$ where x being displacement of a particle which can be found out by $x = x_0 \cos \omega t$.
- (ii) The total energy of a system can also be found out by $E_T = K.E. + P.E.$

14.8 FREE AND FORCED OSCILLATIONS

A body is said to be executing free vibrations if it oscillates with its natural frequency without the interference of an external force.

For example, a simple pendulum shown in Fig. 14.12 vibrates freely with its natural frequency that depends only upon its length when it is slightly displaced from its mean position.

In free oscillations, the total energy of the body remains constant i.e., energy is conserved. As we are assuming in the absence of resistance force the amplitude of the oscillation remains constant.

If a freely oscillating system is subjected to an external force, then forced vibrations will take place. Such as when swing is struck repeatedly, then forced vibrations are produced as shown in Fig. 14.13.

The vibrations of a factory floor caused by the running of heavy machinery are an example of forced vibrations. Another example of forced vibration is loud music produced by sounding wooden boards of string instruments.

14.9 DAMPED OSCILLATION

The oscillations with decreasing amplitude in the presence of various resistive forces are called damped oscillations and the resistive forces are called damping forces.

In an ideal free oscillation system, we have studied that the total mechanical energy of the oscillating body remains constant, as we discussed in mass spring system, motion of a body in a circle and also in case of a simple pendulum. But in practical life, as we observe daily, some dissipating forces such as air resistance, friction, etc. The amplitude of the oscillation gradually decreases with time due to these forces and finally it comes to rest.

For example, a girl is swinging on a swing as shown in Fig. 14.14. Damping occurs and

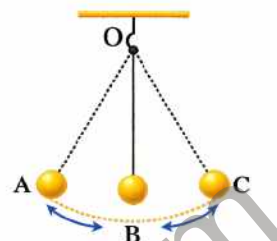


Fig 14.12: Illustration of Free oscillation of simple pendulum



Fig 14.13: Illustration of forced oscillations of a swing

Interesting Information

- Guitar string
- Bell ringing
- Atomic oscillation
- Bridge vibration
- Building sway are the examples of free oscillations

Interesting Information

Light damping: It reduces gradually the energy and amplitude of vibrating body. A swing in playground is the best example of light damping.

Heavy damping: It is a type of damping which takes longer time before the body comes to a rest. For example, a pendulum submerged in thick oil experiences a heavy resistance from oil.

Critical damping: It is a type of damping in which object returns to equilibrium position in the shortest possible time, e.g., shock absorbers in a car.

the swing will oscillate with smaller and smaller amplitude and eventually stops. Graphically the damped oscillation of the oscillating body is shown in Fig. 14.15. Similarly, the amplitude of oscillating simple pendulum decreases gradually with time till it become zero and touching an oscillating tuning fork with our finger are examples of damped oscillation.



Fig 14.14: Damped oscillation of swing

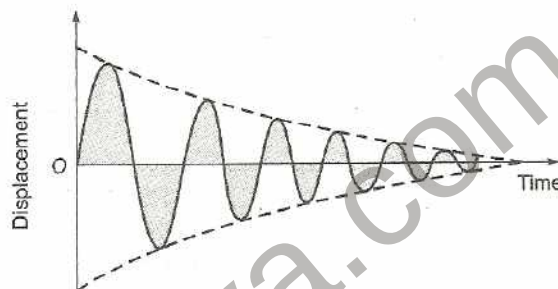


Fig 14.15: Graphical representation of damped oscillation

Application of Damped Oscillation

Shock absorbers used in the suspension system of a car shown in Fig. 14.16 are one practical application of damped oscillation. Damping system is required to ensure a comfortable ride for the passengers when the car is moving on a bumpy, rough road by producing excessive oscillations by damping using shock absorbers in such vehicles.



Fig. 14.16: Suspension system of a vehicle

14.10 RESONANCE

In damped oscillation, the oscillator cannot maintain its natural frequency for long duration due to the resistive forces and the amplitude of the oscillation decreases gradually with time. But we can maintain constant amplitude by applying a periodic external force which is called a driving force. Thus, when the oscillating body is subjected to a periodic driving force, then such oscillation is called forced oscillation and its frequency is called driving frequency. The vibration of a vehicle caused by the running of engine is an example of forced vibration. In forced oscillation, the amplitude of the oscillation depends upon the relation between the driving frequency and the natural frequency of the body.

If the frequency of the driving force is same as the natural frequency of the oscillating body, the amplitude of vibration is very much increased. This phenomenon is known as resonance. It also occurs when the applied force has frequency an integral multiple of the natural frequency of the body. If f is the natural frequency of a body, then resonance will take place at:

$$f_2 = 2f_1, f_3 = 3f_1, f_4 = 4f_1, \dots, f_n = nf_1$$

To demonstrate the resonance phenomenon, we perform a simple experiment. The experimental setup consists of two pair of pendulums A & B and C and D such that the length of A and B is ℓ_1 , and length of C and D is ℓ_2 . All the pendulums are suspended by a horizontal rod (Fig. 14.17).

Now we introduce another pendulum P whose length can be varied i.e. either ℓ_1 or ℓ_2 . Consider the case when the length of pendulum P is equal to ℓ_1 .

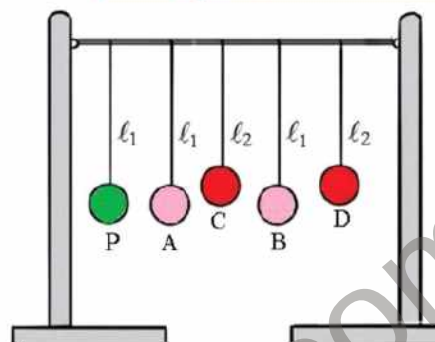


Fig 14.17: A set of five different pendulums of different lengths

If the pendulum 'P' is set into vibration, this vibration reaches the other pendulums through the rod. Then the pendulums A and B receive a driving force through the rod and they also start vibration and their amplitude increases due to the resonance phenomenon because their lengths, natural frequency and natural periods are same. At the same time, the pendulums C and D whose natural frequencies are different from natural frequency of P do not oscillate i.e. they continue to remain at rest. If the length of the pendulum P is made equal to ℓ_2 , and allowed to vibrate, then the pendulums C and D start vibration due to resonance while pendulum 'A and B' remain at rest.

Interesting Information

The resonance is the oscillation up and down back and forth motion caused by seismic waves. During an earth quake, buildings oscillate. An earthquake occurred in Pakistan magnitude 7.6 on reactor scale on 8 October, 2005 in Azad Jammu and Kashmir, a territory under Pakistan. It was centered near the city of Muzaffarabad. Over 86000 people died during this earthquake.

Some Circumstances in which Resonance is Useful

1. Heating / Cooking Food in Micro Wave Oven

Microwave ovens generate super high frequency electromagnetic waves (3GHz-30GHz and wavelength of about 12 cm and scatter them throughout the oven. The frequency of microwave excites water molecules in food causing them to oscillate at the same frequency as the microwaves and cause them to collide with one another. Friction generated by the collisions changes the kinetic energy of the water into heat that warms the food.



Fig. 14.18: Microwave oven

Food containing water molecules can only be heated by the microwave oven as shown in Fig. 14.18. The plastic or glass container do not heat up since they do not contain water molecules.

2. Radio Tuning:

The tuning of an analogue radio set for a certain station is also based on resonance in which we need to turn the knob of a radio to tune a station. Here, actually, we change the

natural frequency of the electrical circuit of receiver to make it equal to the transmission frequency of radio station. When two frequencies match with each other, then resonance occurs and energy absorbed by the station is maximum and this is the only station we hear. A simple radio is shown in Fig. 14.19.

Following are some circumstances in which resonance should be avoided:

1. Bridge Vibration

The soldiers are advised to break their steps while crossing a bridge. If the soldiers march in steps, then it is possible that the frequency of their footsteps becomes equal to the natural frequency of the bridge and the bridge may be set into vibrations with large amplitude due to the resonance and bridge may collapse.

2. Aeroplane Wing

For equilibrium, the structure of an aeroplane has been designed with its two wings. These wings experience the forces such as; aerodynamics force, turbulence, engine vibration, etc. When the frequencies of these forces are matching with the natural frequency of the wings, the resonance will occur and it may be proved dangerous.



Fig. 14.19: Radio

Interesting Information

- The frequency of an object with which it oscillates is called resonance frequency.
- The sound board of a piano, violin and guitar resonates with the vibration of string. When a musician strikes the string of a musical instrument such as a guitar or violin, the string produces sound waves in the form of vibrations. These vibrations transfer their energy to the hollow wooden box of instrument. The sound box resonates, amplifying the sound waves. The amplified sound waves are then emitted, producing a louder and richer sound.
- Magnetic resonance imaging (MRI) is an improved medical diagnostic technique in which strong radio frequency radiations are used to cause nuclei to oscillate, energy is absorbed by the nuclei. The patterns of the energy absorbed can be used to produce a computer enhanced photograph.
- The amplitude of a swing can be increased by applying a suitable periodic force on it regularly. It must be noted that natural frequency of swing matches the frequency of pushes, amplifying the motion.

14.11 SHARPNESS OF RESONANCE

We have seen that at resonance, the amplitude of the oscillator becomes very large. The amplitude as well as its sharpness, both depend upon the damping. Smaller the damping, greater will be the amplitude and more sharp will be the resonance.

A heavily damped system has a fairly flat resonance curve as is shown in an amplitude frequency graph in Fig. 14.20.

The effect of damping can be observed by

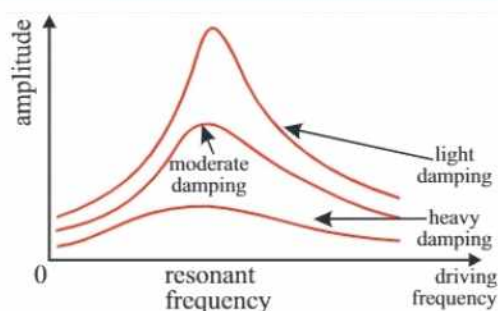


Fig. 14.20: Damping system illustration

attaching a pendulum having light mass such as a pith ball as its bob and another of the same length carrying a heavy mass such as a lead bob of equal size, to a rod (Fig. 14.17). They are set into vibrations by a third pendulum of equal length, attached to the same rod. It is observed that amplitude of the lead bob is much greater than that of the pith-ball. The damping effect for the pith-ball due to air resistance is much greater than for the lead bob.

Application of Resonance and Standing Waves

There are many applications of resonance in different devices that generate and use standing waves.

There is relationship between standing wave and resonance phenomenon which can be studied by some experimental examples such as Rubens tube, chaaladni plates and acoustic levitations.

A Rubens tube is a long tube which is an experimental apparatus used to explain the acoustic standing wave. It consists of a metal pipe with holes drilled a long top and sealed at both ends. One sealed end is attached to a small speaker or frequency generator while the other end is connected to supply of a flammable gas as shown in Fig (14.21). The pipe is filled with gas and gas leaks from holes and produce a resonant flame.

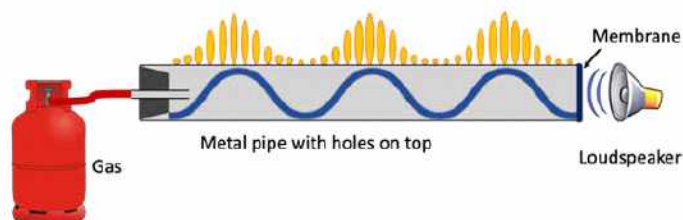
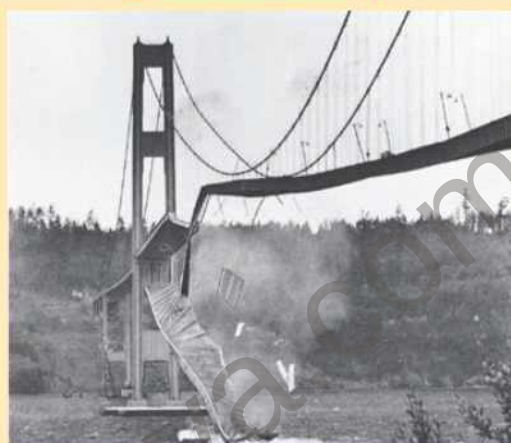


Fig. 14.21: Rubens tube apparatus

When sound waves of resonance frequency are produced by frequency generator, a standing wave is formed inside the tube. The standing wave creates points with

Point to ponder!



The resonance effect is very important in the design of bridges and other civil engineering projects. On July, 1940 the newly constructed Tacoma Narrows Bridge (Washington) was opened for traffic. Only four months after this, a mild wind set up the bridge in resonant vibrations. In a few hours the amplitude became so large that the bridge could not stand the stress and a part broke off and went into the water below.

Interesting Information



A Rubens tube also known as standing wave flame tube or simply flame tube is a physics apparatus for demonstrating acoustic standing waves in a tube invented by a German physics **Henrich Rubens in 1905.**

oscillating (higher and lower) pressure within the tube. Less gas will escape from the holes in the tube where pressure is low, hence, the flame will be lower at these points. Large quantity of gas will escape from holes in the tube where pressure is high, hence, flame will be high at these points.

These low and high flame reveal the nodes and antinodes of standing waves inside the tube as shown in Fig. (14.21).

Chladni Plate

A Chladni plate is a good example of resonance and it demonstrates a standing waves pattern. It is usually a circular or square shaped metal plate which is fixed horizontally to a vibrator and is driven transversally at its centre as shown in Fig. (14.20). When the plate is vibrated at the frequency of a resonant normal mode, the nodes and antinodes are formed over the surface of the plate.

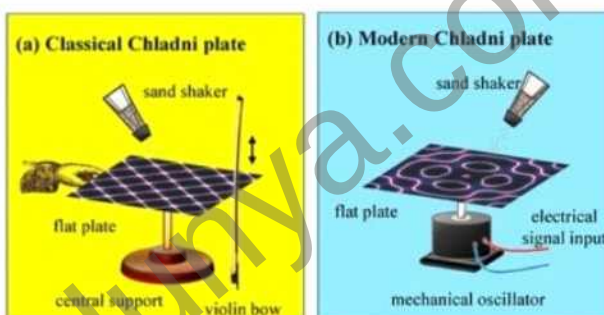


Fig. 14.22: Chladni plates

The metal plate does not vibrate at points where nodes are formed while the rest of the plate is vibrating with its maximum amplitude. In order to observe such pattern, sand is sprinkled evenly in the form of a thin layer over the plate. Hence, the sand will vibrate and shift from antinode to nodes because the nodes are stationary.

Acoustic Levitation

Acoustic levitation is a method for suspending matter in air against gravity using radiation pressure from high intensity sound waves. This method explains the relations between resonance and standing waves. In which the high frequency sound waves (ultrasonic) are used to suspend the small particles such as bits of styrofoam or droplets of water against gravity in air without any physical contact.

Working: Basically, an acoustic levitator consists of a source of sound (transducer) and a reflector. The high frequency sound waves from the transducer is bounced off by the reflector. The interaction between the incident and reflected waves forms a standing wave, where the pressure of sound in the area of node is minimum and pressure is maximum in the area of antinode. Such difference between higher and lower pressure exerts a force on small particles of

Interesting Information



Chladni plates, invented by the physicist, musician and musical instrument maker **Ernst Chladni (1756-1827)** in the late 18th century are used to demonstrate the complex pattern of standing waves vibrations.

Styrofoam against the gravity. As a result, the particles are trapped at node as shown in Fig. (14.23). In this case, the nodes of the standing waves act as acoustic traps.

Activity

The teacher may perform activity in class room to show relationship between resonance and standing waves using resonance tube or Chladni plates.

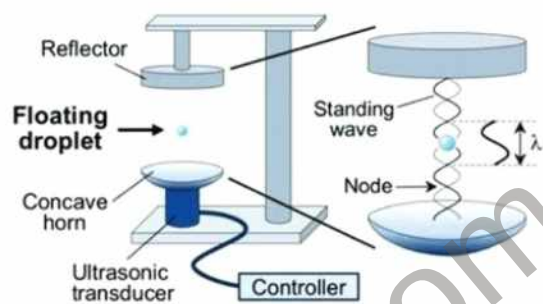


Fig. 14.23: Acoustic levitation setup

QUESTIONS

Multiple Choice Questions

Choose the correct answer:

14.1 While determining time period of simple pendulum, we keep the amplitude:

- (a) large
- (b) small
- (c) maximum
- (d) zero

14.2 In simple harmonic motion, the acceleration of a body is zero, when:

- (a) velocity is zero
- (b) displacement is zero
- (c) both velocity and displacement are zero
- (d) both velocity and displacement are maximum

14.3 If a tunnel is bored through the centre of Earth and a stone is dropped into it, then:

- (a) stone will stop at the centre of Earth
- (b) stone will move out from other side of the tunnel
- (c) stone will perform S.H.M.
- (d) none of these

14.4 Which of the following variables has zero value at the extreme position in S.H.M?

- (a) Acceleration
- (b) Speed
- (c) Displacement
- (d) Angular frequency

14.5 Which of the following force is responsible for S.H.M?

- (a) Applied force
- (b) Frictional force
- (c) Restoring force
- (d) Elastic force

14.6 The velocity of a particle moving with S.H.M at the mean position is:

- (a) zero
- (b) minimum
- (c) maximum
- (d) none of these

14.7 A child swinging on a swing in sitting position, stands up, then time period of the swing will:

- (a) increase
- (b) decrease
- (c) remain same
- (d) become zero

14.8 The total energy of a particle executing S.H.M is proportional to:

- (a) frequency of oscillation
- (b) velocity of motion
- (c) amplitude of motion
- (d) square of amplitude of motion

14.9 The period of simple pendulum increases how many times if its length is doubled?

- (a) $\sqrt{2}$ times
- (b) 2 times
- (c) 4 times
- (d) 8 times

Short Answer Questions

- 14.1 Define simple harmonic motion and write its equation.
- 14.2 State the basic conditions for frictionless system to execute simple harmonic motion.
- 14.3 Differentiate between free and forced oscillations.
- 14.4 Why we cannot construct an ideal simple pendulum? Explain briefly.
- 14.5 How does sharpness of resonance occur?
- 14.6 What would happen to the time period if the Earth suddenly stop rotating?
- 14.7 What is restoring force? What provides restoring force for simple harmonic oscillator in the following cases? (i) Simple pendulum (ii) Spring
- 14.8 What role do shock absorbers play in oscillations of vehicles in a bumpy road? Explain briefly.

Constructed Response Questions

- 14.1 Why marching troops are asked to break their steps while crossing a bridge? Explain.
- 14.2 A girl is swinging in the sitting position. Describe, how will the period of swing be changed if she stands up?
- 14.3 Will a pendulum that keeps correct time at Karachi, be accurate at Murree or at the top of mount Everest? Explain.
- 14.4 A wire hangs from a dark high tower so that its upper end is not visible. How can we determine the length of a tower?
- 14.5 Will the period of a vibrating spring increase, decrease or remain constant by addition of more mass? Explain.
- 14.6 Can a simple pendulum vibrate at centre of the Earth? Explain.

Comprehensive Questions

- 14.1 Show that motion of mass attached with spring executes S.H.M. Also find an expression for its time period and frequency.
- 14.2 Show that motion of the projection of a particle moving around a circle with constant speed is S.H.M and derive a relation for its instantaneous velocity.
- 14.3 What is a simple pendulum? Show that motion of a simple pendulum is S.H.M. Also derive an expression for time period of a simple pendulum, on what factors does it depend?
- 14.4 Derive expressions for $K.E.$ and $P.E.$ of the mass-spring system executing S.H.M. Also prove that its total energy remains same.
- 14.5 Explain resonance. Give some of its advantages and disadvantages.

- 14.6 Discuss physical significance of phase in simple harmonic motion, and explain how does it relate to timing and synchronization of oscillation?

Numerical Problems

- 14.1 A body is executing S.H.M of amplitude 0.05 m and frequency 10 Hz. Find the value of (a) acceleration (b) velocity. (Ans: 197 m s^{-2} , 3.14 m s^{-1})
- 14.2 A mass of 0.5 kg is suspended from a spring. The spring is stretched 0.098 m. Find:
(i) the period of oscillation of mass when it is given a small displacement.
(ii) frequency of oscillation of spring. (Ans: 0.628 s, 1.59 Hz)
- 14.3 A ball of mass 0.5 kg is attached to an elastic spring. The spring is compressed by 10 cm by a force of 45 N. Calculate the energy stored in the spring. What will be the maximum velocity of the ball if the compressing force is suddenly removed? (Ans: 2.25 J, 3 m s^{-1})
- 14.4 A 80 kg body executes S.H.M with amplitude 30 cm. The restoring force is 60 N, when the displacement is 30 cm. Find:
(i) time period (ii) acceleration, Speed, K.E. and P.E, when displacement is 12 cm. (Ans: 1.3 s, 3.0 m s^{-2} , 1.4 m s^{-1} , 7.6 J, 1.44 J)
- 14.5 What should be the length of a simple pendulum whose time period is one second? What is its frequency? (Ans: 0.25 m, 1 Hz)
- 14.6 A block of mass 5 Kg is dropped from a height of 0.8 m onto a spring of spring constant 1960 N m^{-1} . Find the displacement through which the spring will be compressed. (Ans: 0.2 m)