Chapter 2

Set, Functions and Groups

Exercise 2.1

Q1. Write the following set in set builder notation.

i. $\{1,2,3,\ldots\ldots,1000\}$

ii. $\{0,1,\ldots\ldots\ldots,100\}$

iii. $\{0,\pm 1, \pm 2,\ldots.\pm 1000\}$

iv. $\{0,-1,-2,\ldots.\ -500\}$

v. $\{100,101,102,\ldots\ldots 400\}$

vi. $\{-100,-101,-102,\ldots\ldots,-500\}$

vii. **{Pechawar, Lahore, Karachi, Quetta}**

viii. **{January, June, July}**

x. **The set of all old natural number**

xi. **The set of all real numbers beteen 1 and 2**

xii. **The set of all integers between -100 and 1000**

Solution.

i. $\{1,2,3,\ldots\ldots,1000\}$

$= \{x \mid x \in N \wedge x \leq 1000\}$

ii. $\{0,1,\ldots\ldots\ldots,100\}$

$= \{x \mid x \in W \wedge x \leq 100\}$

iii. $\{0,\pm 1, \pm 2,\ldots.\pm 1000\}$

$= \{x \mid x \in Z \wedge -1000 \leq x \leq 1000\}$

iv. $\{0,-1,-2,\ldots.\ -500\}$

$= \{x \mid x \in Z \wedge -500 \leq x \leq 0\}$

v. $\{100,101,102,\ldots\ldots 400\}$

$= \{x \mid x \in N \wedge 100 \leq x \leq 400\}$

vi. $\{-100,-101,-102,\ldots\ldots -500\}$

$= \{x \mid x \in z \wedge -500 \leq x \leq -100\}$

vii. {Pechawar, Lahore, Karachi, Quetta}

={x | x is the capital of province of pakistan}

viii. {January, June, July}

={x | x is the name month which starts with J}

ix. The set of all odd natural numbers

$=\{x \mid x \in N \wedge x \text{ is odd}\}$

$=\{x \mid x \in \wedge\ x > 0\}$

x. The set of rational numbers.

$= \{x \mid x = \frac{p}{q}, p,q \in z \wedge q \neq 0 \}$

$= \{x \mid x = \frac{p}{q}, \text{ where } p,q \in z \wedge q \neq 0 \}$

xi. The set of all real numbers beteen 1 and 2

$= \{x \mid x \in R \wedge 1 < x < 2\}$

xii. The set of all integers between -100 and 1000

$=\{x \mid x \in R \wedge -100 < x < 1000\}$

Q2. Write each of the following sets in the descriptive and tabular forms:

i. $\{x \mid x \in N \wedge x \leq 10\}$

ii. $\{x \mid x \in N \wedge 4 < x < 12\}$

iii. $\{x \mid x \in Z \wedge -5 < x < 5\}$

iv. $\{x \mid x \in E \wedge 2 < x \leq 4\}$

v. $\{x \mid x \in P \wedge x < 12\}$

vi. $\{x \mid x \in O \wedge 3 < x < 12\}$

vii. $\{x \mid x \in E \wedge 4 \leq x \leq 10\}$

viii. $\{x \mid x \in E \wedge 4 < x < 6\}$

ix. $\{x \mid x \in O \wedge 5 \leq x \leq 5\}$

x $\{x \mid x \in N \wedge 5 \leq x < 7\}$

xi. $x \mid x \in N \wedge x + 4 = 0\}$

xii. $\{x \mid x \in Q \wedge x^2 = 2\}$

xiii. $x \mid x \in N\ R \wedge x = x\}$

xiv. $\{x \mid x \in Q \wedge x = -x\}$

xv. $\{x \mid x \in R \wedge x \neq 2\}$

xvi. $\{x \mid x \in R \wedge x \neq Q\}$

Solution:

i. $\{x \mid x \in N \wedge x \leq 10\}$

Descriptive form

The set of first ten natural numbers.

Tabular form

$\{1,2,3,\ldots\ldots,10\}$

ii. $\{x \mid x \in N \wedge 4 < x < 12\}$

Descriptive form

The set of first ten natural numbers between 4 and 12.

Tabular form

$\{5,6,7,8,9,10,11\}$

iii. $\{x \mid x \in Z \wedge -5 < x < 5\}$

Descriptive form

The set of integers between -5 and 5.

Tabular form

$\{0,\pm 1,\pm 2,\pm 3,\pm 4\}$

iv. $\{x \mid x \in E \wedge 2 < x \leq 4\}$

Descriptive form

The set of even integers between 2 and 5.

Tabular form

$\{4\}$

v. $\{x \mid x \in P \wedge x < 12\}$

Descriptive form

The set of prime numbers less than 12.

Tabular form

$\{2,3,5,77,11\}$

vi. $\{x \mid x \in O \wedge 3 < x < 12\}$

Descriptive form

The set of even integers between 3 and 12.

Tabular form

$\{5,7,9,11\}$

vii. $\{x \mid x \in E \wedge 4 \le x \le 10\}$

Descriptive form

The set of even integers between 4 and 10.

Tabular form

$\{4,6,8,10\}$

viii. $\{x \mid x \in E \wedge 4 < x < 6\}$

Descriptive form

The set of even integers between 4 and 6.

Tabular form

$\{\}$

ix. $\{x \mid x \in O \wedge 5 \le x \le 5\}$

Descriptive form

The set of odd integers between 3 and 9.

Tabular form

$\{5,7\}$

x. $\{x \mid x \in N \wedge 5 \le x < 7\}$

Descriptive form

The set of odd integers greater or equal to 5 and less than 7.

Tabular form

$\{5\}$

xi. $\{x \mid x \in N \wedge x + 4 = 0\}$

Descriptive form

The set of natural numbers x satisfying $x+4 = 0$

Or

The set of those numbers which added to 4 give zero

Tabular form

$= \emptyset or \{\}$

xii. $\{x \mid x \in Q \wedge x^2 = 2\}$

Descriptive form

The set of rational numbers x satisfying $x^2 = 2$

Or

The set of rational numbers x for which $x^2 = 2$

Tabular form

$= \emptyset or \{\}$

Since $x^2 = 2$ => $x = \sqrt{2} \notin Q$

xiii. $\{x \mid x \in N\ R \wedge x = x\}$

Descriptive form

The set of real numbers x satisfying $x = x$

Or

The set of real numbers x for which $x = x$

Tabular form

Not possible

xiv. $\{x \mid x \in Q \wedge x = -x\}$

Descriptive form

The set of real numbers x satisfying $x = -x$

Or

The set of those rational numbers x for which $x = -x$

Tabular form

$\{0\}$

There is no rational number which is equal to its additive inverse.

xv. $\{x \mid x \in R \wedge x \neq 2\}$

Descriptive form

The set of all real numbers except 2.

Tabular form

$R - \{2\}$

xvi. $\{x \mid x \in R \wedge x \neq Q\}$

Descriptive form

The set of all rational numbers.

Or

The set of all real numbers which are not rational numbers.

Tabular form

$Q' = R - Q$

Q3. Which of the following set are finite and which of these are infinite?

i. **The set of students of our class**

ii. **The set of all schools in Pakistan.**

iii. **The set of natural numbers between 3 and 10**

iv. **The set of rational numbers between 3 and 10**

v. **The set of real numbers between 0 and 1**

vi. **The set of rational numbers between 0 and 1**

vii. **The set of whole numbers between 0 and 1**

viii. **The set of all leaves of trees in Pakistan.**

ix. $\{a,b,c\}$

x. $\{1,2,3,4,\ldots.\}$

xi. $\{1,2,3,\ldots\ldots\ldots,10000000\}$

xii. $\{x \mid x \in R \wedge x = -x\}$

xiii. $\{x \mid x \in R \wedge x^2 = -16\}$

xiv. $\{x \mid x \in R \wedge x^2 = 5\}$

xv. $\{x \mid x \in Q \wedge 0 \le x \le 1\}$

Solution:

i.	The set of students of our class	**Finite**
ii.	The set of all schools in Pakistan.	**Finite**
iii.	The set of natural numbers between 3 and 10	**Finite**
iv.	The set of rational numbers between 3 and 10	**Infinite**
v.	The set of real numbers between 0 and 1	**Infinite**
vi.	The set of rational numbers between 0 and 1	**Infinite**
vii.	The set of whole numbers between 0 and 1	**Finite**
viii.	The set of all leaves of trees in Pakistan.	**Infinite**
ix.	P(N)	**Infinite**
x.	$\{1,2,3,4,\ldots.\}$	**Infinite**
xi.	$\{1,2,3,\ldots\ldots\ldots,10000000\}$	**Finite**
xii.	$\{x \mid x \in R \wedge x = -x\}$	**Finite**
xiii.	$\{x \mid x \in R \wedge x^2 = -16\}$	**Finite**
xiv.	$\{x \mid x \in R \wedge x^2 = 5\}$	**Finite**
xv.	$\{x \mid x \in Q \wedge 0 \le x \le 1\}$	**Infinite**

Q4. Write two proper subset of each of the following sets:-

i. $\{a,b,c\}$ **ii.** $\{0,1\}$ **iii.** **N** **iv.** **Z**

v. **Q** **vi.** **R** **vii.** **W** **viii.** $\{x \mid x \in Q \wedge 0 < x \le 2\}$

Solution:

i. $\{a,b,c\}$

Proper subsets are

$S_1 = \{a\}$; $S_2 = \{b\}$

ii. $\{0,1\}$

Proper subsets are

$S_1 = \{0\}$; $S_2 = \{1\}$

iii. **N**

Proper subsets are

$S_1 = P$; $S_2 = O$

iv. **Z**

Proper subsets are

$S_1 = N$; $S_2 = W$

v. **Q**

Proper subsets are

$S_1 = N$; $S_2 = Z$

vi. **R**

Proper subsets are

$S_1 = Q$; $S_2 = Z$

vii. **W**

Proper subsets are

$S_1 = N$; $S_2 = P$

viii. $\{x \mid x \in Q \wedge O < x \leq 2\}$

Proper subsets are

$S_1 = \{x \mid x \in Q \wedge O < x > 2\}$; $S_2 = \{2\}$

Q5. Is there any set which has no proper subset? If so name that set

Solution:

Yes, empty set { } has no proper set.

Q6. What is the difference between $\{a,b\}$ and $\{\{a,b\}\}$?

Solution:

$\{a,b\}$ means that set has two elements anddd b and $\{\{a,b\}\}$ means that set has one element $\{a,b\}$

Q7. Which of the following sentences are true and which of them are false?

i. $\{1,2\} = \{2,1\}$ **ii.** $\varnothing \subseteq \{\{a\}\}$ **iii.** $\{a\} \supseteq \{\{a\}\}$ **iv.** $\{a\} \in \{\{a\}\}$

v. $a \in \{\{a\}\}$ **vi.** $\varnothing \in \{\{a\}\}$

Solution:

i. $\{1,2\} = \{2,1\}$ **True**

ii. $\varnothing \subseteq \{\{a\}\}$ **True**

iii. $\{a\} \supseteq \{\{a\}\}$ **False**

iv. $\{a\} \in \{\{a\}\}$ **True**

v. $a \in \{\{a\}\}$ **False**

vi. $\varnothing \in \{\{a\}\}$ **False**

Q8. What is the number of elements of the power set of each of the following sets?

i. { } **ii.** $\{0,1\}$ **iii.** $\{1,2,3,4,5,6,7\}$

iv. $\{1,2,3,4,5,6,7\}$ **v.** $\{a,\{b,c\}\}$ **vi.** $\{\{a,b\},\{b,c\},\{d,e\}\}$

Solution:

i. { }

$n = 0$

$n\{P(s)\} = 2^n = 2^0 = 1$

ii. $\{0,1\}$

$n = 2$

$n\{P(s)\} = 2^n = 2^2 = 4$

iii. $\{1,2,3,4,5,6,7\}$

$n = 7$

$n\{P(s)\} = 2^n = 2^7 = 128$

iv. $\{1,2,3,4,5,6,7\}$

$n = 8$

$n\{P(s)\} = 2^n = 2^8 = 256$

v. $\{a,\{b,c\}\}$

$n = 2$

$n\{P(s)\} = 2^n = 2^2 = 4$

vi. $\{\{a,b\},\{b,c\},\{d,e\}\}$

$n = 3$

$n\{P(s)\} = 2^n = 2^3 = 8$

Q9. Write down the power set of each of the following sets:-

i. $\{9,11\}$ **ii.** $\{+,-,\times,\div\}$ **iii.** $\{\varnothing\}$ **iv.** $\{a,\{b,c\}\}$

Solution:

i. $\{9,11\}$

$P(S) = \{\varnothing,\{9\},\{11\},\{9,11\}\}$

ii. $\{+,-,\times,\div\}$

$P(S) = \{\varnothing, \{+\},\{-\},\{\times\},\{\div\},\{+,\div\},\{+,-\}\{+,\times\},\{+,\div\},\{-,\times\},\{-,\div\},\{\times,\div\},\{+,-,\times\},\{+,-,\div\},\{+,\times,\div\},\{-,\times,\div\},\{+,-,\times,\div\}\}$

iii. $\{\varnothing\}$

$P(S) = \{\varnothing,\{\varnothing\}\}$

iv. $\{a,\{b,c\}\}$

$P(S) = \{\varnothing, \{a\},\{b,c\},\{a,\{b,c\}\}$

Q10. Which pairs of sets are equivalent? Which of them are also equal?

i. $\{a,b,c\}$, $\{1,2,3\}$

ii. **The set of the first 10 whole numbers,** $\{0,1,2,3,\ldots\ldots,9\}$

iii. **Set of angles of a quadrilateral ABCD.**

Set of the sides of the same quadrilateral.

iv. **Set of the sides of the hexagon ABCDEF.**

v. $\{1,2,3,\ldots\ldots\}$ $\{2,4,,6,8,\ldots\ldots\}$

vi. $\{1,2,3,\ldots\ldots\}$, $\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\ldots\ldots\}$

vii. $\{5,10,15,20,\ldots\ldots,55555\}$, $\{5,10,15,20,\ldots\ldots\}$

Solution:

i. $\{a,b,c\}$, $\{1,2,3\}$

Equivalent sets because each set has three elements.

ii. **The set of the first 10 whole numbers,** $\{0,1,2,3,\ldots\ldots,9\}$

Equal sets because each set have same 10 elements.

iii. **Set of angles of a quadrilateral ABCD.**

Set of the sides of the same quadrilateral.

Equivalent sets because each set has four elements.

iv. **Set of the sides of the hexagon ABCDEF.**

The set of the angles of the same hexagon.

Equivalent sets because each set has six elements.

v. $\{1,2,3,\ldots\ldots\}$ $\{2,4,,6,8,\ldots\ldots\}$

Equivalent sets because they have one-to-one (1-1) correspondence between them.

vi. $\{1,2,3,\ldots\ldots\}$, $\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\ldots\ldots\}$

Equivalent sets because they have one-to-one (1-1) correspondence between them.

vii. $\{5,10,15,20,\ldots\ldots,55555\}$, $\{5,10,15,20,\ldots\ldots\}$

1st set is finite but 2nd set is infinite, so they are not equivalent set.

Exercise 2.2

Q1. **Exhibit $A \cup B$ and $A \cap B$ by Venn Diagram in the following cases:**

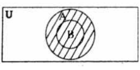
i. $A \subseteq B$ **ii.** $B \subseteq A$ **iii.** $A \cup A'$

iv. **A and B are disjoint sets. V.** **A and B are overlapping sets.**

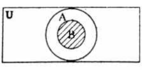
Solution:

i. $A \subseteq B$

a. $A \cup B$

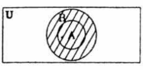


b. $A \cap B$

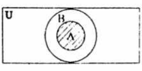


ii. $B \subseteq A$

a. $A \cup B$



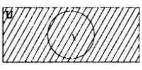
b. $A \cap B$



iii. $A \cup A'$

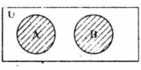
$= A \cup (U - A)$

$= U$

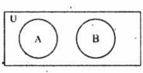


iv. **A and B are disjoint sets.**

a. $A \cup B$

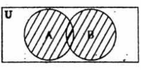


b. $A \cap B$

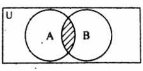


v. **A and B are overlapping sets.**

a. $A \cup B$



b. $A \cap B$

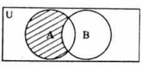


Q2 **Exhibit $A - B$ and $B - A$ by Venn Diagram when:**

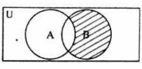
i. A and B are overlapping sets **ii. $A \subseteq B$** **iii. $B \subseteq A$**

i. **A and B are overlapping sets.**

a. $A - B$

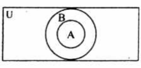


b. $A - B$

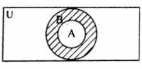


ii. $A \subseteq B$

a. $A - B$

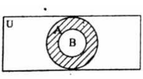


b. $A - B$

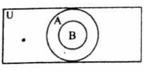


iii. $B \subseteq A$

a. $A - B$



b. $A - B$



Q3. **Under what conditions on a A and B are the following statement true?**

i. $A \cup B = A$ **ii.** $A \cup B = B$

iii. $A - \varnothing = \varnothing$ **iv.** $A \cap B = B$

v. $n(A \cup B) = n(A)+n(B)$ **vi.** $n(\cap B) = n(A)$

vii. $A - B = a$ **viii.** $N(A \cap B) = 0$

ix. $A \cup B = U$ **x.** $A \cup B = B \cup A$

xi. $n(A \cap B) = n(B)$ **xii.** $U - A = \varnothing$

Solution:

i. $A \cup B = A$

It is true iff $B \subseteq A$

ii. $A \cup B = B$

It is true iff $A \subseteq B$

iii. $A - \varnothing = \varnothing$

It is true iff $A = \varnothing$

iv. $A \cap B = B$

It is true iff $B \subseteq A$

v. $n(A \cup B) = n(A)+n(B)$

It is true iff $A \cap B = \varnothing$

vi. $\mathbf{n(\cap B) = n(A)}$

It is true iff $A \subseteq B$

vii. $\mathbf{A - B = a}$

It is true iff $A \cap B = \emptyset$

viii. $\mathbf{n(A \cap B) = 0}$

It is true iff $A \cap B = \emptyset$

ix. $\mathbf{A \cup B = U}$

It is true iff $B = A'$ or $A = B'$

x $\mathbf{A \cup B = B \cup A}$

It is true because it is commutative property w.r.t union.

xi. $\mathbf{n(A \cap B) = n(B)}$

It is true iff $B \subseteq A$

xii. $\mathbf{U - A = \emptyset}$

It is true iff $A = U$

Q4. Let $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{2,4,6,8,10\}$, $B=\{1,2,3,4,5\}$ $C=\{1,3,5,7,9\}$

List the members of each of the following sets:

---

i. A^c **ii.** B^c **iii.** $A \cup B$ **iv.** $A - B$

v. $A \cap C$ **vi.** $Ac \cup Cc$ **vii.** $Ac \cup C$ **viii.** $U=U\text{-}U$

Solution:

i. $\mathbf{A^c}$

$= \{1,2,3,\ldots..,10\} - \{2,4,6,8,10\}$

$=\{1,3,5,7,9\}$

Hence $\mathbf{A^c} = \{1,3,5,7,9\}$

ii. $\mathbf{B^c}$

$= \{1,2,3,\ldots..,10\} - \{1,3,5,7,9\}$

$=\{2,4,6,8,10\}$

Hence $\mathbf{B^c} = \{2,4,6,8,10\}$

iii. $\mathbf{A \cup B}$

$= \{2,4,6,8,10\} \cup \{1,3,5,7,9\}$

$= \{1,2,3,4,5,6,7,8,9,10\}$

Hence $\mathbf{A \cup B} = \{1,2,3,4,5,6,7,8,9,10\}$

iv. $\mathbf{A - B}$

$= \{2,4,6,8,10\} - \{1,3,5,7,9\}$

$= \emptyset$

Hence $A\text{-}B = \emptyset$

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v. $\mathbf{A \cap C}$

$= \{2,4,6,8,10\} \cap \{1,3,5,7,9\}$

$= \emptyset$

Hence $A \cap B = \emptyset$

vi. $\mathbf{A^c \cup C^c}$

$A^c = U\text{-}A$

$=\{1,2,3,4,5,6,7,8,9,10\}\text{-}\{2,4,6,8,10\}$

$=\{1,3,5,7,9\}$

$C^c = U\text{-}C$

$=\{1,2,3,4,5,6,7,8,9,10\}\text{-}\{1,3,5,7,9\}$

$=\{2,4,6,8,10\}$

$A^c \cup C^c = \{1,3,5,7,9\} \cup \{2,4,6,8,10\}$

$= \{1,2,3,4,5,6,7,8,9,10\}$

Hence $A^c \cup C^c = \{1,2,3,4,5,6,7,8,9,10\}$

vii. $\mathbf{A^c \cup C}$

$A^c = U\text{-}A$

$=\{1,2,3,4,5,6,7,8,9,10\}\text{-}\{2,4,6,8,10\}$

$=\{1,3,5,7,9\}$

$A^c \cup C = \{1,3,5,7,9\} \cup \{1,3,5,7,9\}$

Hence $A^c \cup C = \{1,3,5,7,9\}$

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viii. $\mathbf{U^c=U\text{-}U}$

$=\{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5,6,7,8,9,10\}$

$=\emptyset$

Hence $U^c=U\text{-}U = \emptyset$

Q5. Using the Venn diagram if necessary, find the single sets equal to the following.

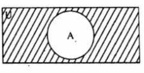
i. A^c **ii.** $A \cap U$ **iii.** $A \cup U$ **iv.** $A \cup \emptyset$

v. $\emptyset \cap \emptyset$

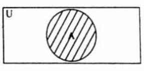
Solution:

i. $\mathbf{A^c}$

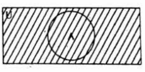
$= U\text{-}A$



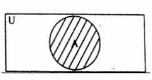
ii. $\mathbf{A \cap U}$



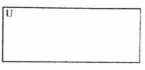
iii. $\mathbf{A \cup U}$



iv. $\mathbf{A \cup \emptyset}$



v. $\mathbf{\emptyset \cap \emptyset}$



Q6. Use Venn diagram to verify the following: -

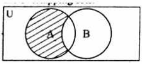
i. $\mathbf{A\text{-}B = A \cup B^c}$ **ii.** $\mathbf{(A\text{-}B)^c \cap B = B}$

Solution:

i. $\mathbf{A\text{-}B = A \cup B^c}$

L.H.S

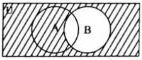
A-B, if A and B are overlapping sets:



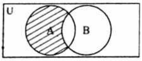
R.H.S

$A \cup B^c$

$B^c = U - B$



⇨ **$A \cup B^c$**



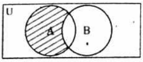
Hence Proved

ii. **$(A-B)^c \cap B = B$**

L.H.S

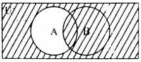
Where A and B are overlapping sets:

A-B



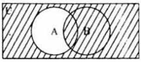
$(A-B)^c$

$= U - (A-B)$



$(A-B)^c \cap B$

R.H.S



Hence Proved

Exercise 2.3

1. Verify the commutative properties of union and intersection for the following pairs of sets: -

i. $A = \{1,2,3,4,5\}$, $B = \{4,6,8,10\}$ **ii.** N,Z

iii. $A = \{x \mid x \in \sum R \wedge x \geq 0\}$, $B = R$ **iv.** $A = \{1,2,3,4,5\}$; $B = \{4,6,8,10\}$

Solution

i. $A = (1,2,3,4,5)$, $B = \{4,6,8,10\}$

Commutative property of Union.

$A \cup B = B \cup A$

$L.H.S = A \cup B = \{1,2,3,4,5\} \cup \{4,6,8,10\}$

$= \{1,2,3,4,5,6,8,10\}$

$R.H.S = B \cup A = \{4,6,8,10\} \cup \{1,2,3,4,5\}$

$= \{1,2,3,4,5,6,8,10\}$

Hence, $A \cup B = B \cup A$

Commutative property of intersection.

$A \cap B = B \cap A$

$L.H.S = A \cap B = \{1,2,3,4,5\} \cap \{4,6,8,10\}$

$= \{4\}$

$R.H.S = (B \cap A) = \{1,2,3,4,5\} \cap \{4,6,8,10\}$

$= \{4\}$

Hence, $A \cap B = B \cap A$

ii. $A = N$; $B = Z$

Commutative property of union.

$A \cup B = B \cup A$

$L.H.S = A \cup B = N \cup Z = Z$

$R.H.S = B \cup A = Z \cup N = Z$

Hence, $A \cup B = B \cup A$

Commutative property of intersection.

$A \cap B = B \cap A$

$L.H.S = A \cap B = N \cap Z = N$

$R.H.S = B \cap A = Z \cap N = N$

Hence, $A \cap B = B \cap A$

iii. $A = \{x \mid x \in \sum R \wedge x \geq 0\}$; $B = R$

Commutative property of union

$A \cup B = B \cup A$

$L.H.S = A \cup B = \{x \mid x \in \sum R \wedge x \geq 0\} = R$

$R.H.S = B \cup A = \{x \mid x \in \sum R \wedge x \geq 0\} = R$

Hence, $A \cup B = B \cup A$

Commutative property of intersection

$A \cap B = B \cap A$

$L.H.S = A \cap B = \{x \mid x \in \sum R \wedge x \geq 0\} \cap R$

$= \{x \mid x \in \sum R \wedge x \geq 0\}$

$R.H.S = B \cap A = R \cap \{x \mid x \in \sum R \wedge x \geq 0\}$

$= \{x \mid x \in \sum R \wedge x \geq 0\}$

Hence, $A \cap B = B \cap A$

Q2. Verify the properties for the sets, A, B and C given below:

i. Associativity of Union

ii. Associativity of intersection.

iii. Distributivity of Union over intersection.

iv. Distributivity of intersection

a. $A = \{1,2,3,4\}$ $B = \{3,4,5,6,7,8\}$, $C = \{5,6,7,9.10\}$

b. $A = \{\ \}$, $B = \{0\}$, $C = \{0,1,2\}$

c. N,Z,Q

Solution:

a. $A = (1,2,3,4)$; $B = (3,4,5,6,7,8)$ $C = (5,6,7,9,10)$

i. Associativity of union

$(A \cup B) \cup C = A \cup (B \cup C)$

$L.H.S = (A \cup B) \cup C$

$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6,7,8\}$

$= \{1,2,3,4,5,6,7,8\}$

$(A \cup B) \cup C = (1,2,3,4,5,6,7,8) \cup (5,6,7,9,10)$

$= \{1,2,3,4,5,6,7,8,9,10\}$

$R.H.S = A \cup (B \cup C)$

$B \cup C = \{3,4,5,6,7,8,9,10\}$

$= \{3,4,5,6,7,8,9,10\}$

$A \cup (B \cup C) = \{1,2,3,4\} \cup \{3,4,5,6,7,8.9,10\}$

$= \{1,2,3,4,5,6,7,8.9,10\}$

Hence Proved $(A \cup B) \cup C = A \cup (B \cup C)$

ii. Associativity of intersection

$(A \cap B) \cap C = A \cap (B \cap C)$

$L.H.S = (A \cap B) \cap C$

$A \cap B = \{1,2,3,4\} \cap \{3,4,5,6,7,8\}$

$= \{3,4\}$

$(A \cap B) \cap C = \{3,4\} \cap \{5,6,7,8,9,10\}$

$= \{\ \}$

$R.H.S = A \cap (B \cap C)$

$B \cap C = \{3,4,5,6,7,8\} \cap \{5,6,7,9,10\}$

$= \{5,6,7\}$

$A \cap (B \cap C) = \{1,2,3,4\} \cap \{1,2,3,4\} \cap (5,6,7) = \{\ \}$

Hence Proved $(A \cap B) \cap C = A \cap (B \cap C)$

iii. Distributivity of union over intersection.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$L.H.S = A \cup (B \cap C)$

$B \cap C = \{3,4,5,6,7,8\} \cap \{5,6,7,9,10\}$

$= \{5,6,7\}$

$A \cup (B \cap C) = \{1,2,3,4\} \cup \{5,6,7\}$

$= \{1,2,3,4,5,6,7\}$

$R.H.S\ A \cup B = \{1,2,3,4\} \cup \{3,4,5,6,7,8\}$

$= \{1,2,3,4,5,6,7,8\}$

$R.H.S = (A \cup B) \cap (A \cup C)$

$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6,7,8\}$

$= \{1,2,3,4,5,6,7,8\}$

$A \cup C = \{1,2,3,4\} \cup \{5,6,7,9,10\}$

$= \{1,2,3,4,5,6,7,9,10\}$

$(A \cup B) \cap (A \cup C) = \{1,2,3,4,5,6,7,8\} \cap \{1,2,3,4,5,6,7,8,9,10\}$

Hence Proved, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b. $A = \{\ \}$; $B = \{0\}$; $C = \{0,1,2\}$

i. Associativity of union

$(A \cup B) \cup C = A \cup (B \cup C)$

$R.H.S = (A \cup B) \cup C$

$A \cup B = \{\ \} \cup \{0\} = \{0\}$

$(A \cup B) \cup C = \{0\} \cup \{0,1,2\} = \{0,1,2\}$

$R.H.S = A \cup (B \cup C)$

$B \cup C = \{0\} \cup \{0,1,2\} = \{0,1,2\}$

AU(BUC) = { } U {0,1,2} = {0,1,2}

Hence Proved, (AUB)UC=AU (BUC)

ii. Associativity of intersection.

(A∩B)∩C = A∩ (B∩C)

L.H.S = (A∩B)∩C

A∩B = { } ∩ {0} = { }

(A∩B) ∩C = { }∩{0,1,2} = { }

R.H.S = A∩(B∩C)

B∩C = {0)∩{0,1,2} = {0}

A∩(B∩C) = { } U {0} = { }

Hence Proved, (A∩B)∩C=A∩ (B∩C)

iii. Distributivity of union over intersection.

AU(B∩C) = (AUB) ∩ (AUC)

L.H.S = A U (B∩C)

B∩C = {0} ∩ {0,1,2}

= {0}

AU(B∩C) = { }U{0}

= {0}

R.H.S AUB = { }U{0}

= {0}

AU = { } U {0,1,2}

= {0}

Hence Proved, A U (B∩C) = (AUB) ∩ (AUC)

iii. Distributivity of intersection over union.

A∩(BUC) = (A∩B) U (A∩C)

L.H.S = A ∩ (BUC)

BUC = {0} U {0,1,2}

= {0,1,2}

A∩(BUC) = { }∩{0,1,2}

= { }

R.H.S = (A∩B) U (A∩C)

A∩C = { } ∩ {0,1,2}

= { }

A∩B = { }∩{0}

= {1,2,3,4,5,6,7,9,10}

(*A* ∩B) U (A∩C) = { } U{ }

Hence Proved, A ∩ (BUC) = (A∩B)U (A∩C)

c. A=N; B = Z, C = Q

(AUB) UC = AU (BUC)

R.H.S = (AUB)UC

AUB = N UZ = Z

(AUB) UC = ZUQ= Q

R.H.S = AU(BUC)

BUC = ZUQ = Q

Hence Proved, (AUB)UC = AU(BUC)

ii. Associativity of intersection

(A∩B)∩C = A∩(B∩C)

L.H.S = (A∩B)∩C

(A∩B∩C = N∩Q = N

R.H.S = A∩(B∩C)

(B∩C) = Z∩Q =Z

A∩(B∩C) = N∩Z = Z

Hence Proved, (A∩B)∩C = A∩ (B∩C)

iii. Distributivity of union over intersection.

AU(B∩C) = (AUB) ∩ (AUC)

L.H.S = A U (B∩C)

B∩C = Z ∩ Q = Z

AU(B∩C) = NUZ = Z

R.H.S = (AUB) ∩ (AUC)

AUB = NUZ=Z

AUC = N U Q = Q

(AUB) ∩ (AUC) = Z ∩ Q = Z

Hence Proved, A U (B∩C) = (AUB) ∩ (AUC)

iii. Distributivity of intersection over union.

A∩(BUC) = (A∩B) U (A∩C)

L.H.S = A ∩ (BUC)

BUC = Z U Q = Q

A∩(BUC) = N∩Q = N

R.H.S = (A∩B) U (A∩C)

A∩C = N ∩ Q = N

A∩B = N∩Z = N

(*A* ∩B) U (A∩C) = N U N = N

Hence Proved, A ∩ (BUC) = (A∩B)U (A∩C)

Q3. Verify De Morgan's Laws for the following sets:

U = {1,2,3,....,20}, A = {2,4,6,....20} and B = {1,3,5,19}

Solution:

i. (A∪B)' = A'∩B'

L.H.S = AUB = {2,4,6,.....,20}U{1,3,5,.....,19}

= {1,2,3,....,20}

(AUB)' = U - (AUA) = {1,2,3,.....,20}- {1,2,3,.....,20}

=ø

R.H.S A'∩B'

A' = U - A = {1,2,3,.....,20} – {1,3,5,....,19}

= {2,4,6,.....,20}

B' = U - B = {1,2,3,.....,20} – {2,4,6,....,20}

= {1,3,5,.....,19}

A'∩B' = {1,3,5,.....,19} ∩ {2,4,6,.....,20}

= ø

Hence Proved

(A∩B)' =

L.H.S = (A∩B)'

A∩B ={2,4,6,….20} ∩ {1,3,5,…..19} = ø

(A∩B)' = U - (A∩B) = {1,2,3,...,20} = { }

={1,2,3....,20)

A' = U - A = {1,2,3,4,5,….20} – {2,4,6,,.....,20}

= {1,2,3,…...,19}

B' = U-B = {1,2,3,....20} – {1,2,3,……,19}

= {2,4,6,,.....,20}

R.H.S =(A∩B)' = {1,2,3,……,19} - {2,4,6,....,20}

={1,2,3,4,5,...,20}

Hence Proved.

Q4. Let U = The set of the English alphabet.

A = {x | x is vowel}, B = {y | y is a consonant},

Verify De Morgan's Law for these sets.

Solution:

i. (A∪B)' = A'∩B'

L.H.S = (AUB)'

$A \cup B = \{x|x \text{ is a vowel}\} \cup \{y\,|\,y \text{ is a consonant}\}$

= The set of the English alphabet

$(A \cup B)' = U - (A \cup B)$ = The set of the English alphabet - The set of the English alphabet

$= \varnothing$

R.H.S $= A' \cap B'$

$A' = U - A$ = The set of the English alphabet

$= \{x|x \text{ is a vowel}\}$

$B' = U - B$ = The set of the English alphabet - $\{y\,|\,y \text{ is a consonant}\}$

$= \{x\,|\,x \text{ is a vowel}\}$

$A' \cap B' = (y|y \text{ is consonant}) \cap \{x|x \text{ is a vowel})$

$= \varnothing$

Hence proved, $(A \cup B)' = A' \cap B'$

ii. $\mathbf{(A \cap B)' = A' \cup B'}$

L.H.S $= (A \cap B)'$

$ArB = \{x|x \text{ is a vowel}) \cap \{Ay \text{ is a consonant}\}$

$= \varnothing$

$(A \cap B)' = U - (A \cap B)$ = The set of the English alphabet – $\varnothing$

$(A \cap B)'$ = The set of the English alphbet

R.H.S $= A' \cup B'$

$A' = U - A$ = The set of the English alphabet - $\{x\,|\,x \text{ is a vowel})$

$= \{y\,|\,y \text{ is a consonant}\}$

$B' = U - B$ = The set of the English alphabet - $\{x|x \text{ is a consonant}\}$

$= \{x\,|\,x \text{ is a vowel}\}$

$A' \cup B' = \{y|y \text{ is a consonant}\} \cup \{x\,|\,x \text{ is a vowel}\}$

= The set of the English alphabet

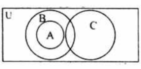
Hence proved: $(A \cap B)' = A' \cup B'$

Q5. With the help of Venn diagrams, verify the two distributive properties in the following cases w.r.t union and intersection.

i. $A \subseteq B$, $A \cap C = \varnothing$ and B and C are overlapping.

ii. A and B are overlapping; B and C are overlapping but A and C are disjoint

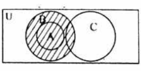
Solution:



Prove, Distributive of union over intersection

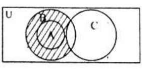
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $= A \cup (B \cap C)$

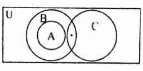


R.H.S $= (A \cup B) \cap (A \cup C)$

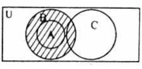
$A \cup B$



$A \cup C$



$(A \cup B) \cap (A \cup C)$



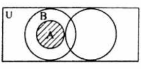
Hence Proved, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Prove, Distributive of intersection over union

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

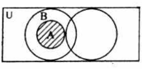
L.H.S $= A \cap (B \cup C)$

$A \cap (B \cup C)$

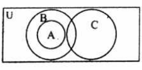


R.H.S $= (A \cap B) \cup (A \cap C)$

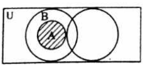
$A \cap B$



$A \cap C$

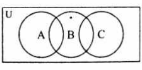


$(A \cap B) \cup (A \cap C)$



Hence Proved, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ii. A and B are overlapping; B and C are overlapping but A and C are disjoint

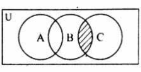


Prove, Distributive of union over intersection

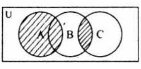
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $= A \cup (B \cap C)$

$B \cap C$

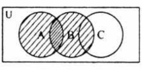


$A \cup (B \cap C)$

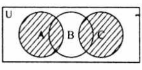


R.H.S $= (A \cup B) \cap (A \cup C)$

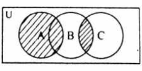
$A \cup B$



$A \cup C$



$(A \cup B) \cap (A \cup C)$



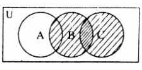
Hence Proved, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Prove, Distributive of intersection over union

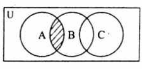
A∩(BUC) = (A∩B) U (A∩C)

L.H.S = A ∩ (BUC)

BUC

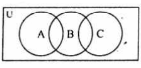


A∩(BUC)

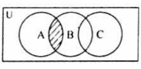


R.H.S = (A∩B) U (A∩C)

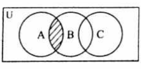
A∩C



A∩B



(*A* ∩B) U (A∩C)



Hence Proved, A ∩ (BUC) = (A∩B)U (A∩C)

Q6. Taking any set say A = (1,2,3,4,5) verify the following:-

i. A ∪ ø = A **ii. A∩A = A**

Solution:

i. A∪ø=A

R.H.S = A U ø = A = R.H.S

Hence, Proved

ii. A∪A = A

R.H.S AUA = AUA=A = R.H.S

Hence, Proved

iii. A∩A = A

R.H.S A∩A = A = R.H.S

Hence, Proved

Q7. If U = (1,2,3,4,...20) and A = (1,3,5,.....,19), verify the following.

i. A∪A' = U **ii. A∩U = A** **iii. A∩A' = ø**

i. A∪A' = U

L.H.S = AUA'

A' = U-A = {1,2,3,.....,20} – {1,3,5,.....,19}

= {2,4,6,.....,20}

AUA' = {1,3,5,.....,19) U {2,4,6,....,20}

= (1,2,3,4,.....,20)

Hence, Proved

ii. A∩U=A

L.H.S = A ∩ U

A ∩ U = {1,2,3,5,.....,19} ∩ {1,2,3,.....,20}

={1,3,5,.....,19} = A = R.H.S

Hence, Proved

iii. A∩A' =ø

L.H.S = A∩A'

A' = U - A = (1,2,3,.....,20) - (1,3,5,.....,19) = {2, 4, 6,..... 20)

A∩A' = (1,3,5,.....,19)∩{2,4,6,....,20} ={ } = R.H.S

Hence, Proved

Q8. From suitable properties of union intersection deduce the following results:

i. A∩ (A∪B) = A∪(A∩B)

ii. A∪(A∩B) = A ∩ (A∪B)

Solution:

i. A∩(A∪B) = A∪(A∩B)

L.H.S. = A∩(AUB)

= (A∩A) U (A∩B)

(Distributive property of intersection over union)

= AU(A∩B)' **{∴ A∩A = A}**

= R.H.S

Hence, Proved A∩(AUB) = AU(A∩B)

ii. A∪(A∩B) = A∩(A∪B)

L.H.S = AU(AUB)

AU(A∩B) = (A∩A)U{A∩B}

(Distributive property of intersection over union

= A∩(A∩B) {.'. AUA = A)

= R.H.S

Hence Proved: AU(A∩B) = (A∩A)U{A∩B}

Q9. Using Venn diagrams, verify the following results.

i. A∩B' = A iff A∩B= ø

ii. (A-B) ∪ B = A∪B

iii. (A-B) ∩ B = ø

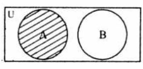
iv. A∪B' = A ∪(A'∩B')

Solution:

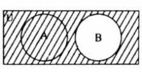
i. A∩B' = A iff A∩B= ø

if A∩B= ø

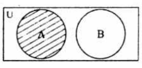
Then L.H.S = A∩B'



B' = U-B



A∩B'



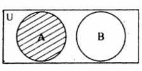
Hence, Proved

A∩B' = A iff A∩B= ø

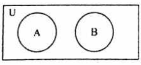
Conversely

A∩B= ø iff A∩B' = A

A∩B' = A



Then A∩B

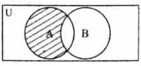


Hence Proved, $A\cap B=\varnothing$ iff $A\cap B' = A$

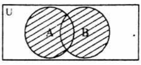
ii. **$(A-B)\cup B = A\cup B$**

L.H.S = $(A-B)\cup B$

$A-B$



$(A-B)\cup B$



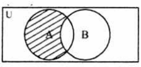
R.H.S $=A\cup B$

Hence Proved $(A-B)\cup B = A\cup B$

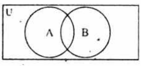
iii. **$(A-B)\cap B=\varnothing$**

L.H.S = $(A-B)\cap B$

$A-B$



$(A-B)\cap B$

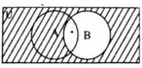


R.H.S = $\varnothing$

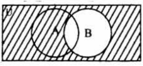
Hence Proved $(A-B)\cap B=\varnothing$

iv. **$A\cup B' = A\cup(A'\cap B')$**

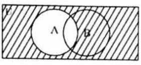
R.H.S $= A\cup(A'\cap B')$



$B' = U-B$

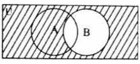


$A' = U-A$



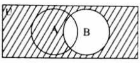
$A'\cap B'$

$A\cup(A'\cap B')$



L.H.S

$A\cup B'$



Hence Proved $A\cup B' = A\cup(A'\cap B')$

Exercise 2.4

Q1. Write the converse, inverse and contra positive of the following conditional:

i) $\sim p \to q$ **ii)** $q \to p$ **iii)** $\sim p \to \sim q$ **iv)** $\sim q \to \sim p$

Solution:

i) $\sim p \to q$

		Converse	Inverse	Contra Positive
i)	$\sim p \to q$	$q \to \sim p$	$p \to \sim q$	$\sim p \to p$

ii) $q \to p$

		Converse	Inverse	Contra Positive
ii)	$q \to p$	$p \to q$	$\sim q \to \sim p$	$\sim p \to \sim q$

iii) $\sim p \to \sim q$

		Converse	Inverse	Contra Positive
iii)	$\sim p \to \sim q$	$\sim p \to \sim q$	$p \to q$	$q \to p$

iv) $\sim q \to \sim p$

		Converse	Inverse	Contra Positive
iv)	$\sim q \to \sim p$	$\sim p \to \sim q$	$q \to p$	$p \to q$

Q2. Construct truth tables for the following statements.

i. $(p \to \sim p) \vee (p \to q)$

ii. $(p \wedge \sim p) \to q$

iii. $(\sim(p \to q) \leftrightarrow (p \wedge \sim q)$

Solution:

i. $(p \to \sim p) \vee (p \to q)$

p	q	$\sim p$	$\sim q$	$p \to \sim p$	$p \to q$	$(p \to \sim p) \vee (p \to q)$
T	T	F	F	F	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

ii. $(p \wedge \sim p) \to q$

p	q	$\sim p$	$p \wedge \sim p$	$(p \wedge \sim p) \to q$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	T

iii. $(\sim(p \to q) \leftrightarrow (p \wedge \sim q)$

p	q	$\sim p$	$(p \to q$	$\sim(p \to q)$	$p \wedge \sim q$	$(\sim(p \to q) \to \sim(p \wedge q)$	$(\sim(p \to q) \to \sim(p \wedge q)$	$(\sim(p \to q) \leftrightarrow \sim(p \wedge q)$
T	T	F	T	F	F	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	F	F	T	T	T
F	F	T	T	F	F	T	T	T

Q3. Show that each of the following statements is a tautology.

- **i.** $(p \wedge q) \to p$
- **ii.** $p \to (p \vee q)$
- **iii.** $\sim(p \to q) \to p)$
- **iv.** $\sim q \wedge (p \to q) \to \sim p$

olution:

i. $(p \wedge q) \to p$

P	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

ii. $p \to (p \vee q)$

P	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

iii. $\sim(p \to q) \to p$

P	q	$\sim(p \to q)$	$\sim(p \to q) \to p$
T	T	F	T
F	F	F	T

iv. $\sim q \wedge (p \to q) \to \sim p$

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \wedge (p \to q)$	$\sim q \wedge (p \to q) \to \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Q4. Determine whether each of the following is a tautology, a contingency or an absurdity.

i. $p \wedge \sim p$

ii. $p \to (p \to q)$

iii. $q \vee (\sim q \vee p))$

Solution:

i. $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

ii. $p \to (q \to p)$

P	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

iii. $q \vee (\sim q \vee p))$

P	Q	$\sim q$	$\sim q \vee p$	$q \vee (\sim q \vee p)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

Q5. Prove that $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

Solution:

L.H.S = $p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$

T	F	F	T	F	F	T	T
F	T	T	F	F	F	F	F
F	F	T	T	T	F	T	T

R.H.S = $p \vee (\sim p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \vee (\sim p \wedge \sim q)$
T	T	F	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	F	T	T	T	T

So, L.H.S = R.H.S

Hence Prove.

Exercise 2.5

Q1. Convert the following theorems to logical and prove them by constructing truth tables.

i. (A∩B)' = A'∪B' **ii. (A∪B)∩C = (A∪B)∪C**

iii. (A∩B)∩C = A∩(B∩C) **iv. A∪(B∩C) = (A∪B) ∩ (A∪C)**

Solution:

i. (A∩B)' = A'∪B'

L.H.S (A∩B)' → $\sim(p \wedge q)$

P	**Q**	$p \wedge q$	$\sim(p \wedge q)$
F	F	F	T

R.H.S A'∪B' → $(\sim p \vee \sim q)$

P	**Q**	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

ii. (A∪B)∩C = (A∪B)∪C

L.H.S (A∪B)∩C → $(p \vee q) \vee r$

p	**Q**	**r**	$p \vee q$	$(p \vee q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
F	F	T	T	T
F	F	F	F	F

R.H.S (A∪B)∪C → $p \vee (q \vee r)$

p	**Q**	**r**	$q \vee r$	$p \vee (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	F

iii. (A∩B)∩C = A∩(B∩C)

L.H.S (A∩B)∩C → $(p \wedge q) \wedge r$

T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

R.H.S A∩(B∩C) → $p \wedge (q \wedge r)$

P	**q**	**r**	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

iv. A∪(B∩C) = (A∪B) ∩ (A∪C)

L.H.S A∪(B∩C) → $p \vee (q \wedge r)$

p	**q**	**r**	$q \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

R.H.S (A∪B) ∩ (A∪C) → $(p \vee q) \wedge (p \vee r)$

p	**Q**	**r**	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	F	T	F	F	F
F	F	F	F	F	F

L.H.S = R.H.S

Hence Proved

Exercise 2.6

Q1. For A = {1,2,3,4}, find the following relations in A. State the domain and range of each relation. Also draw the graph of each.

i. {(x,y) I y = x} **ii. {(x,y) I y+x = 5}** **iii. {(x,y) I x+y<5)**

iv. {(x,y) I x+y>5}

Solution:

i. {(x,y) I y = x}

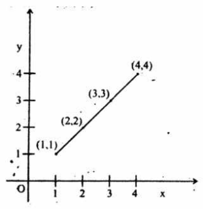
Where. A = {1,2,3,4}

Sc r = {(1,1), (2,2), (3,3),(4,4)

and

Dom (r) = {1,2,3,4}

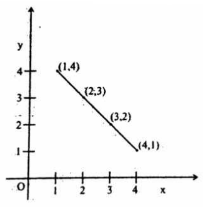
And range (r) = {1,2,3,4}



ii. {(x,y) I y+x = 5}; Where A = {1,2,3,4}

r = {(1,4), (2,3), (3,2), (2,1)}

So, Dom (r) = {1,2,3,4} = {1,2,3,4}

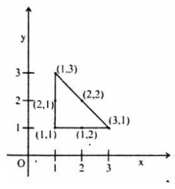


iii. {(x,y) I x+y<5), where A = {1,2,3,4}

r ={(1.1),(1,2),(1,3),(2,1), (2,2). (3.1)}

So. Dom(r)= {1.2,3}

& range(r)= {1,2,3}

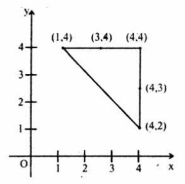


iv. {(x,y) | x+y>5}

r ={(2,4), (3,3), (3,4), (4,2), (4,3), (4,4)}

So Dom (r)= {2,3,4}

& Range (r)= {4,3,2} or {2,3,4}



Q2. Repeat Q-1 when A = R the set of real number. Which of the real lines are functions?

Solution:

i. A ϵ R

r ={(x,y) | y = x}

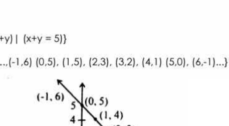
r ={(1,1), (2,2), (3,3)....} U {-1, -1}

(-2,-2),..... }

or

= {... (- 2,- 2), (-1, -1), (0,0), (1,1),

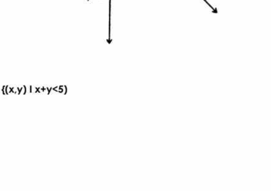
(2,2), (3,3).... }



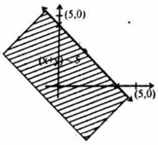
ii. A ϵ R

r = {(x+y) | (x+y = 5)}

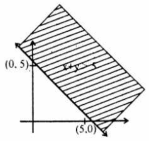
r = { .....(-1,6) (0,5), (1,5), (2,3), (3,2), (4,1) (5,0), (6,-1)...}



iii. {(x,y) I x+y<5)



iv. $\{(x,y) \mid x+y>5\}$



Q3. Which of the following diagrams represent functions and of which type?

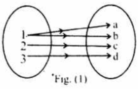


Fig. (1)

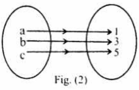


Fig. (2)

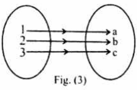


Fig. (3)

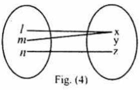


Fig. (4)

Solution:

i. Fig (1), does not represent a function because element '1' has not unique image.

ii. Fig (2), represent a (1-1) one—to—one function. So , it is on to function.

iii. Fig. (3), represent a (1-1) one-to-one function, so, it is onto (injunction) function.

iv. Fig (4), represents a function which in onto function

Q4. Find the inverse of each of the following relations. Tell whether each relation and its inverse is a function or not.

i. $\{(2,1), (3,2), (4,3), (5,4), (6,5)\}$

ii. $\{(1,3), (2,5), (3,7), (4,9), (5,11)\}$

iii. $\{(x,y) \mid y = 2x+3 \in R\}$

iv $\{(x,y) \mid y^2 = 4ax, x \geq 0\}$

v. $\{(x,y)\ x^2+y^2 = 9, |x, y \mid x,y| \leq 3\}$

Solution:

i. $r_1= \{(2,1), (3,2), (4,3), (5,4), (6,5)\}$

$r_1^{-1}=\{(2,1), (3,2), (4,3), (5,4), (6,5\}$

dom $(r_1^{-1})= \{1,2,3,4,5\}$

So, it is a function

ii. $r_2 = \{(1,3), (2,5), (3,7), (4,9), (5,11)\}$

$r_2^{-1} =\{(1,3), (2,5), (3,7), (4,9), (5,11)\}$

dom $(r_2^{-1}) = \{3,5,7,9,11)\}$

So, it is a function

iii. $r_3 = ((x,y) \mid y = 2x+3, \in R)$

$r_3^{-1}=\{(x,y) \mid y = 2x+3, \in R\}$

It is a function

iv. $x_4=\{(x,y) \mid y^2 = 4ax, x \geq 0\}$

$r_4^{-1} =\{(x,y) \mid x = \frac{y^2}{49}, y \in R\}$

It is not a function

v. $r_5 =\{(x,y)\ x^2+y^2 = 9, | x,y | \leq |3|\}$

$r_5^{-1} = \{(x,y)\ x^2+y^2 = 9, |x, y | \leq |3|\}$

It is not a function

Exercise 2.7

Q1. Complete the table, indicating by a tick mark those properties which are satisfied by the specified set of numbers.

property	Set of numbers	Natural	Whole	Integers	Rational	Reals
Closure	⊕					
Closure	⊗					
Associative	⊕					
Associative	⊗					
Identity	⊕					
Identity	⊗					
Inverse	⊕					
inverse	⊗					
Commutative	⊕					
Commutative	⊗					

Solution:

Property	Set of numbers	Natural	Whole	Integers	Rational	Reals
Closure	⊕	✓	✓	✓	✓	✓
Closure	⊗	✓	✓	✓	✓	✓
Associative	⊕	✓	✓	✓	✓	✓
Associative	⊗	✓	✓	✓	✓	✓
Identity	⊕	X	✓	✓	✓	✓
Identity	⊗	✓	✓	✓	✓	✓
Inverse	⊕	x	x	✓	✓	✓
inverse	⊗	x	x	x	x	x
Commutative	⊕	✓	✓	✓	✓	✓
Commutative	⊗	✓	✓	✓	✓	✓

Q2. What are the field axioms? In what respect does the field of real numbers differ from that of complex numbers?

Solution:

Field

A non-empty set 'F' is said to be field if for all x,y, Z, $\widehat{G}$ ϵ F, the following axiom are satisfied.

i. **A field (F, +) is in ablain group i.e.**

a. $x+y \in F$; $\forall x, y \in F$
b. $(x + y) + z = z + (x + y)$ $\forall x,y \in F$
c. $x +0 = 0+x = x$; $\forall x \in F$
d. $x-x = -x+x = 0$, $\forall x \in F$

e. $x+y = y + x$; $\forall x, y \in F$

ii. **A field (F,.) is an abehain group i.e**

a. $x.y \in F$; $\forall x, y \in F$
b. $x.(y.z) = (x.y).z$; $\forall x,y,z, \in F$
c. $x.1 = 1.x = x$; $\forall x \in F$
d. $x.\frac{1}{x} = \frac{1}{x}.x = 1$, $\forall x \in F$
e. $x.y = y . x$; $\forall x, y \in F$

iii. **∀ x, y, z ϵ F, the right distributive law holds i.e.**

$x (y+Z) = xy+ xZ$.

The field of real numbers differs. From the field of complex numbers in the since that an order relation can be defixed in the field of real numbers, that is we can say that one real number is smaller than the others real number. So, they form two different fields.

Q3. Show that the adjoining table is that of multiplication of the elements of the set of residue classes modulo.5.

∗	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Solution:

As $0 \ast 0 = 0$
$1 \ast 0 = 0$
$2 \ast 0 = 0$
$3 \ast 0 = 0$
$4 \ast 0 = 0$

'This show it is a product table

$2 \ast 3 = 6$ gives 1
$2 \ast 4 = 8$ gives 2

and

$3 \ast 2 = 6$ gives 1
$3 \ast 3 = 9$ gives 4
$3 \ast 4 = 12$ gives 3
$4 \ast 2 = 3$ gives 3
$4 \ast 3 = 12$ gives 2
$4 \ast 4 = 16$ gives 1

This table shows that all elements of table is smaller than 5 So, the table is a multiplication able of the set of residue Classes modulo 5.

Q4. Prepare a table of addition of the elements of the set of residue classes module 4.

Solution:

The table of addition of the element of-the set of residue classes 'n' is given below dements 0,1.2,3.

⊕	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

5. Which of the following binary operations shown in tables (a) or (b) is commutative?

∗	a	b	c	d
a	a	c	b	d
b	b	c	b	a
c	c	d	b	c
d	a	a	b	B

∗	a	b	c	d
a	a	c	b	d
b	c	d	b	a
c	b	b	a	c
d	d	a	c	d

Solution:

As the multiplication table is not symmetric that is when row and columns interchange the

table change.

In table (a)

$a \ast c = b$, $c \ast a = c$

Shows, $a \ast c \neq c \ast a$

In table (b)

$a \ast c = b$; $c \ast a$

Shows, $a \ast c \neq c \ast a$

Both, a and tables have no commutative property.

Q6. Supply the missing elements of the third row of the given table so that the operation $\ast$ may be associative.

$\ast$	a	b	c	d
A	a	b	c	d
B	b	a	c	d
C	-	-	-	-
D	d	c	c	d

ClassNotes

Solution:

Let x,y z, t be the required clement. If they are associative with binary operation $\ast$.

i.e

$(d \ast b) \ast a = d \ast (b \ast a)$...(1)

using the table, we have $d \ast b = c$, $b \ast a = b$, put these values in eq (i)

$c \ast a = d \ast b$

→ $x = c$ [.'. $c \ast a = x$, $d \ast b = c$].

ii. $(d \ast b) \ast b = d \ast (b \ast b)$

$c \ast b = d \ast a$ [.'. $d \ast b = c$, $b \ast b = a$]

→ $y = d$ [.'. $c \ast b = y$, $d \ast a = d$]

iii. $(d \ast b) \ast c = d \ast (b \ast c)$

$c \ast c = d \ast c = c$ [.'. $d \ast b = c$, $d \ast c = c$]

→ $Z = c$ [.'. $c \ast b = c$, $d \ast c = c$]

iv. $(d \ast b) \ast d = d \ast (b \ast d)$

$c \ast d = d \ast d$ [.'. $d \ast b = c$, $d \ast d = d$]

→ $t = d$ [.'. $c \ast d = t$, $d \ast d$]

ClassNotes

Q7. What operation is represented by the adjoining table? Name the identity element of the relevant set, if it exists. Is the operation associative? Find the inverse of 0 1 2 3 if they exist.

$\ast$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Solution:

$0 \ast 0 = 0$

$0 \ast 1 = 1$

$0 \ast 2 = 2$

$0 \ast 3 = 3$

This shows that the operation is addition.

Identity element = 0

Because it does not change any number when it is added to it

$0 \ast 2 = 2 \ast 0$

Associative property

$1 \ast (2 \ast 3) = (1 \ast 2) \ast 3$.

$1 \ast 1 = 3 \ast 3$

$2 = 2$

Hence proved they have associative property

Inverse of

0 is 0

1 is 3

2 is 2

3 is 1

Exercise 2.8

Q1. Operation performed on the two-member set G = {0,1) is shown in the adjoining table. Answer the questions: -

i. Name the identity element if it exists?

ii. What is the inverse of 1?

iii. Is the set G, under the given operation a group? Abelian or nob-Abelian?

$\oplus$	0	1
0	0	1
1	1	0

Solution:

i. Identity element = 0

i.e $0 \oplus 0 = 0$

$1 \oplus 0 = 0 \oplus 1 = 1$

ii. Inverse of 1 = 1

i.e $1 \oplus 1 = 1 \oplus 1 = 0$

iii. The set G = {0,1}

1. Closure property holds

$0 + 1 = 0$

2. Associative law holds

$(0+0) + 1 = 0 + (0+1)$

$0+1 = 0 + (1)$

$1 = 1$

3. Additive inverse exists

4. Inverse law holds.

$1+1 = 0 = 1+1,$

So, set G={0,1} is a group.

5. Commutative law

$0+1 = 1+0$

$1 = 1$

commutative law holds

Hence, set G is abelian

Q2. The operation ⊕ as performed on the set {0,1,2,3} is shown in the adjoining table, show that the set is an Abelian group?

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Solution:

1. Associative property

$1 \oplus (2 \oplus 3) = (1 \oplus 2) \oplus 3$

$1 \oplus 1 = 3 \oplus 3.$

$2 = 2$

Associative law hold

2. Closure law.

$2 \oplus 3 = 1$ (1 belongs to given table)

3. Inverse law

$2 \oplus 2 = 2 \oplus 2 = 0$

$3 \oplus 1 = 1 \oplus 3 = 0$

4. Identity exists

$0 \oplus 1 = 1 \oplus 0 = 1$

$0 \oplus 2 = 2 \oplus 2 = 2$

5. Commutative law

Or

$1+2 = 2+1$

$3 = 3$

So, commutative law holds

Hence, the set is an Abelian group

Q3. For each of the following sets, determine whether or not the set forms a group with respect to the indicated operation.

<u>**Set**</u>	<u>**Operation**</u>
i. The set of rational numbers	×
ii. The set of rational numbers	+
iii. The set of positive rational numbers	×
iv. The set of integers	+
v. The set of integers.	×

Solution:

i. The set of rational number under multiplication.

Set of rational number = Q.

Let $a \in Q$.

$a \times 0 = 0 \times a \neq 1$

Therefore, the set of rational numbers is not a group w.r.t the binary operation multiplication.

ii. The set of rational numbers under addition.

a. Closure property

$a + b = c \quad [\because a, b, c \in Q]$

b. Associative property.

$(a+b) + c = a + (b+c) \quad [\because a, b, c \in Q]$

c. Identity element

$a+0 = 0+a = a \quad [\because a, b, c \in Q]$

d. Inverse element

$a+(-a) = (-a) + a = 0 \quad (\because -a, \in Q]$

e. Commutative property

$a+b = b+a \quad [\because a,b, \in Q]$

Therefore, the set of rational numbers under.

iii. The set of positive rational numbers under multiplication.

Let 'Q' denotes the set of positive rational numbers, the "$0 \notin Q$".

i. Closure law

$x.y \in Q \quad [\forall x,y \in Q]$

product of two rational numbers is also positive rational number.

ii. Associative law

$(x.y) . Z = x. (y.Z) \quad [\forall x, y, Z \in Q]$

Product of three rational numbers is also positive rational number so, Associative law hold.

iii. Identity element

$1 \in Q$ $[\forall x \in Q]$

as $1 . x = 1 = x.1$

Hence, '1' is the identity element of Q'.

iv. Inverse element.

$x.\frac{1}{x} = 1 = \frac{1}{x}x,$ $[\because x, \frac{1}{x} \in Q']$

Hence, Q' (positive rational numbers) is group under multiplication.

v. Commutative law

$x.y = y.x$ $[x,y \in Q']$

It is that the group (Q') Abelian group.

iv. The set of integers under addition.

Let 'z' be set of integers $Z = \{0, \pm1, \pm2, \pm3, ...\}$

Then

i. Closure law

$a+b \in Z$ $[\forall a,b \in Z]$

ii. Associative law

$a+(b+c) = (a+b)+c$ $[\forall a, b \in Z]$

iii. Identity element.

$a+(-a) = o = (-a)+a$ $[\forall a, b,c \in Z]$

iv. Inverse element

$a+(-a) = 0 = (-a)+a$ $[\forall a,-a, \in Z]$

Hence integers under addition is a croon.

v. Commutative property

$a+b = b+a$ $[\forall a, b, \in Z]$

Hence, this group is Abelian group.

v. The set of integers under multiplication.

Let 'Z' be set of integers then

$Z = \{0, \pm1, \pm2, \pm3, ...\}$

i. Closure law

$a.b \in Z$ $[\forall a,b \in Z]$

ii. Associative law

$(a.b). c = a.(b.c)$ $[\forall a, b \in Z]$

iii. Identity law

$a.1 = 1 = 1.a$ $[\forall a, \in Z \text{ and } 1 \in Z]$

iv. Inverse element

$a.\frac{1}{a} = 1 = \frac{1}{a}.a$ $[\forall a, \in \text{and } \frac{1}{a} \in Z]$

because $a , \frac{1}{a} \in Z$

by this it show, the set of integers under multiplication.

Q4. Show that the adjoining table represents the sums of the elements of the set {E,0).

What is the identity element of this set? Show that this set is an abelian group.

⊕	E	0
E	E	0
0	0	E

Solution:

a. Sum of the elements of the set {E, 0}

Even ⊕ Even = Even

Even ⊕ odd = odd

Odd ⊕ even = odd

Odd ⊕ odd = even

b. Identity element

$E+E = E = E+E$ $[\because 0 \in E)$

Or $E+0 = E = 0+E$

c.

i. closure property

$E+0 = 0$ $[\because E, 0 \in \text{set}]$

ii. Associative property

$E+(E+0) = (E+E)+0$

$E+0 = 0+0$

$E = E$

iii. Inverse law

$E + E = E$

$0 + 0 = E$

iv. Identity element

E

v. commutative law

$E+0 = 0+E = 0$

ClassNotes

So, set {E, 0} is a abelian element.

Q5. Show that the set $\{1, \omega, \omega^2\}$, when $\omega^3 = 1$ is an Abelian group w.r.t. or multiplication.

⊕	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

i. Closure property

$1 \oplus \omega = \omega$

or $\omega^2 \oplus 1 = \omega^2$

Closer property holds

ii. Associative property

$\omega \oplus (\omega^2 \oplus 1) = 1 \oplus 1$

$\omega^3 = 1$ $[\because \omega^3 = 1]$

$1 = 1$

Associative property holds

iii. Inverse law

$\omega \oplus \omega^2 = 1$

Or $\omega^2 \oplus \omega = 1$

Inverse hold

iv. Identity element

$\omega^2 \oplus 1 = \omega^2$

$\omega \oplus 1 = \omega$

Inverse element

v. Commutative property

$\omega \oplus \omega^2 = \omega^2$

Commutative property holds

Hence, set $(1, \omega, \omega^2)$ is an Abelian group w.r.t. multiplication.

Q6. If G is group under the operation $\ast$ and a, b $\in$ G, find the solutions of the equations.

$a \ast x = b, \qquad x \ast a = b$

Solution:

i. **$a \ast x = b$**

multiply a^{-1} by the left side of equation.

$a^{-1} \ast (a \ast x) = a^{-1} \ast b$

$(a^{-1} \ast a) \ast x = a^{-1} \ast b$

$[\because a^{-1} \ast (a \ast x) = (a^{-1} \ast a) \ast x]$

$e \ast x = a^{-1} \ast b$

$[a^{-1} \ast a = e]$

➔ $x = (a^{-1} \ast b)$

$[e \ast x = x]$

Hence, solution of equation is $x = a^{-1} \ast b$

ii. **$x \ast a = b$**

Multiply a^{-1} by the right side of equation.

$(x \ast a) \ast a^{-1} = b \ast a^{-1}$

$x \ast (a \ast a^{-1}) = b \ast a^{-1}$

$[\because x \ast a)] = a^{-1} = x \ast (a \ast a^{-1})$

$x \ast e = b \ast a^{-1} \qquad [\because a \ast a^{-1} = e]$

$x = b \ast (\because x \ast e = x]$

$x = b \ast a^{-1}$

Hence, solution of equation is $x = b \ast a^{-1}$

Q7. Show that set consisting of elements of the form $2+\sqrt{3b}$ (a, b being rational), is an abelian group w.r.t. addition.

Solution:

Let

$G= (2+ \sqrt{3b};\ |\ , a, b \in Q)$, let x, y, z $\in$ G any three elements of G, such that

$x= a+ \sqrt{3b}; \quad y = c+ \sqrt{3d}$

$z= f+ \sqrt{3}f$

$\{a, b, c, d, e, f \in Q\}$

i. closure properties

$x+ y = u\ [\because U \in Q]$

$\because$ closure property hold

ii. Associative property

$(x+y)+z = z+(y+z)$

$\because$ Associative property hold

iii. Since, '0' is a rational number, so, additive identity present in set G.

iv. Since, any $x = a+ \sqrt{3b} \in G$, then

$-x = -a- \sqrt{3b} \in G$

So, inverse of each element of \`G' is also is G.

v. $x + y = y + x$; show that commutative property w.r.t. addition holds is G.

Hence G is an abelian group w.r.t. addition.

Q8. Determine whether, (P(S) $\ast$), where $\ast$ stands for intersection is a semi-group, a monoid or neither. If it is a monoid, specify its identity.

Solution:

Let '(P(s))' be the power set of any set 's' then 'ø' and 's' will belong to that set.

$\because$Let $\ast$ represents intersection then

i. Closure Law

Let A, B $\in$ P(s)

Then $A \ast B$ will contain elements of 's' because $\ast$ is multiplication which will be some subset of 's' belong to P(s)

ii. Associative Law

Let A, B. C $\in$ P(s)

$A \ast (B \ast C) = (A \ast B) \ast C$

As $A \cap (B \cap C) = (A \cap B) \cap C$

iii. As s $\in$ P(s)

Then y A $\in$ P(s)

$S \ast A = A \ast S= A \qquad [\because s \cap A = A = A \cap A]$

→ 's' is the identity element

Hence, P(s) is a monoid, Therefore, a semi group as well.

Its identity is 's'

Q9. Complete the following table to obtain a semi-group under $\ast$.

$\ast$	a	b	c
A	c	a	b
b	a	b	c
c	--	--	a

Solution:

Let x , y be required element

➔ In semi-group 'Associative law' hold

$(a \ast a) \ast a = a \ast (a \ast a) \quad [\because a \ast a = c]$

➔ $c \ast a = a \ast c$

➔ $c \ast = b$

so, $x = b.$

➔ In semi-aroup 'Associative law' hold

$(a \ast a) \ast b = a \ast (a \ast a)$

$c \ast b = a \ast a \quad [\because a \ast a = c]$

$[a \ast b = a]$

➔ $y = a \qquad [a \ast a = a]$

So, y =

Hence x = b and y = a

Q10. Prove that all 2 x 2 non-singular matrices over the real field from a non-abelian group under multiplication.

Solution:

Let 'S' denotes the set of all non-singular 2 x matrices over the real field.

Then:

i. Closure law

$A_{2\times2} \times B_{2\times2} = C_{2\times2} \quad [\because A,B,C \in S\ ]$

ii. Associative law

$A. (B.C)=(A.B).C \qquad [\because A,B,C \in S\ ]$

iii. Identity Element

$I_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, is a non singular matrix.

iv. Inverse law

$A_{2\times2} \,.\, A_{2\times2}^{-1} = A_{(2\times2)}^{-1} \,.\, A = I_{2\times2} \quad [\ \because A \in S]$

v. Commutative law

$A_{(2\times2)} \,.\, B_{(2\times2)} \neq B_{(2\times2)} \,.\, A_{(2\times2)} \qquad [\because A.B \in S]$

So, by this 'S' the set of 2 x 2 non-singular matrixes over the real field is a non-abelian group under multiplication.