Chapter 1

NUMBER SYSTEMS

The set of real numbers can be written as $R = Q \cup Q'$ where Q is the set of rational numbers and Q' is the set of irrational numbers.

RATIONAL NUMBER

Rational number is a number which can be put in the form $\frac{p}{q}$ where $p, q \in z \wedge q \neq 0$.

Thus $\sqrt{16}$, $\frac{3}{4}$, 2.7 are rational numbers.

Decimal Representation of Rational Numbers

(1) Terminating Decimals

A decimal which has only a finite number of digits in its decimal part, is called a terminating decimal. Terminating decimal represents a rational number. Thus 3.7, 0.0005, 207.9 are rational numbers.

(2) Recurring Decimals

Recurring decimal is a decimal in which one or more digits repeat indefinitely. Every recurring decimal represents a rational number. Thus $0.333\ldots..$, $1.57575757\ldots..$ are rational numbers.

Irrational Number

Irrational number is a number which can not be put into the form $\frac{p}{q}$ where $p, q, \in z \wedge q \neq 0$. Thus $\sqrt{2}$, $\sqrt{3}$, $\sqrt{\frac{5}{6}}$ are irrational numbers.

Note: A non terminating, non recurring decimal represents an irrational number.

Binary Operation

A binary operation in a set A is a rule usually denoted by $*$ that assigns to any pair of elements of A, taken in a definite order, another element of A.

Properties of Real Numbers

1. Addition Laws:

(i) Closure Law of Addition

$\forall\ a, b \in R \quad a + b \in R$

(ii) Associative Law of Addition

$\forall\ a, b, c \in R,\ a + (b + c) = (a + b) + c$

(iii) Additive Identity

$\forall\ a \in R\ ,\ \exists\ 0 \in R$ such that $a + 0 = 0 + a = a$

(v) Commutative Law for Addition

$\forall\ a, b \in R\ , a + b = b + a$

2. Multiplication Laws:

(vi) Closure Law of Multiplication

$\forall\ a, b \in R\ ,\ a \cdot b \in R$

(vii) Associative Law for Multiplication

$\forall\ a, b, c \in R\ ,\ a\,(bc) = (ab)\,c$

(viii) Multiplicative Identity

$\forall\ a \in R\ ,\ \exists\ 1 \in R$ such that $a \cdot 1 = 1 \cdot a = a$

(ix) Multiplicative Inverse

$\forall\ a\ (\neq 0) \in R\ ,\ \exists\ a^{-1} \in R$ such that $a \cdot a^{-1} = a^{-1} \cdot a = 1$

(x) Commutative Law of Multiplication

$\forall\ a, b \in R\ ,\ ab = ba$

3. Multiplication – Addition Law

(xi) Distributive Property

$\forall\ a, b, c \in R$

$a\,(b + c) = ab + ac$ (left distributive property)

$(a + b)\,c = ac + bc$ (Right distributive property)

Any set possessing all the above 11 properties is called a field.

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4. Properties of Equality

(i) Reflexive property: $\forall\ a \in R\ ,\ a = a$

(ii) Symmetric property: $\forall\ a, b \in R\ ,\ a = b \Rightarrow b = a$

(iii) Transitive property: $\forall\ a, b, c \in R\ ,\ a = b \wedge b = c \Rightarrow a = c$

(iv) Additive property: $\forall\ a, b, c, \in R\ ,\ a = b \Rightarrow a + c = b + c \wedge c+a = b+c$

(v) Multiplicative property: $\forall\ a, b, c, \in R\ ,\ a = b \Rightarrow ac = bc \wedge ca = cb$

(vi) Cancellation property w.r.t. addition: $\forall\ a, b, c \in R\ ,\ a + c = b + c \Rightarrow a = b$

(vii) Cancellation property w.r.t. Multiplication: $\forall\ a, b, c \in R\ ,\ ac = bc \Rightarrow a = b\ ,\ c \neq 0$

5. Properties of Inequalities

Trichotomy Property

$\forall\ a, b \in R$ either $a = b$ or $a > b$ or $a < b$

Transitive Property

$\forall\ a, b, c \in R$

(i) $a > b \wedge b > c \Rightarrow a > c$

(ii) $a < b \wedge b < c \Rightarrow a < c$

Additive Property

$\forall\ a, b, c \in R$

(a) (i) $a > b \Rightarrow a + c > b + c$ (ii) $a < b \Rightarrow a + c < b + c$

(b) (i) $a > b \wedge c > d \Rightarrow a + c > b + d$ (ii) $a < b \wedge c < d \Rightarrow a + c < b + d$

Multiplicative Properties

(a) $\forall\ a, b, c \in R$ and $c > 0$

(i) $a > b \Rightarrow ac > bc$ (ii) $a < b \Rightarrow ac < bc$

(b) $\forall\ a, b, c \in R$ and $c < 0$

(i) $a > b \Rightarrow ac < bc$ (ii) $a < b \Rightarrow ac > bc$

(c) $\forall\ a, b, c, d \in R$ and a, b, c, d are all positive,

(i) $a > b \wedge c > d \Rightarrow ac > bd$ (ii) $a < b \wedge c < d \Rightarrow ac < bd$

EXERCISE 1.1

Q.1 Which of the following sets have closure property w.r.t. addition and multiplication

(i) {0}

The set is closed w.r.t. addition because $0 + 0 = 0 \in \{0\}$

The set is closed w.r.t. multiplication because $0.0 = 0 \in \{0\}$

(ii) {1}

The set is not closed w.r.t. addition because $1 + 1 = 2 \notin \{1\}$

The set is closed w.r.t. multiplication because $1.1 = 1 \in \{1\}$

(iii) {0, – 1}

+	0	– 1
0	0	– 1
– 1	– 1	– 2

The set is not closed w.r.t. addition because $-2 \notin \{0, -1\}$

•	0	– 1
0	0	0
– 1	0	1

The set is not closed w.r.t. multiplication because $1 \notin \{0, -1\}$

(iv) {1, – 1}

+	1	– 1
1	2	0
– 1	0	– 2

The set is not closed w.r.t. addition because $-2, 0, 2 \notin \{-1, 1\}$

•	1	– 1
1	1	– 1
– 1	– 1	1

The set is closed w.r.t. multiplication.

Q.2 Name the properties used in the following equations (letters, where used, represents real numbers)

Solution:

(i) $4 + 9 = 9 + 4$ Commutative property w.r.t. '+'

$=$ Commutative property w.r.t. '+'

(ii) $(a+1)+\frac{3}{4} = a+\left(1+\frac{3}{4}\right)$ Associative property w.r.t. '+'

(iii) $(\sqrt{3}+\sqrt{5})+\sqrt{7} = \sqrt{3}+(\sqrt{5}+\sqrt{7})$ Associative property w.r.t. '+'

(iv) $100+0 = 100$ Additive Identity

(v) $100 \times 1 = 100$ Multiplicative Identity

(vi) $4.1+(-4.1) = 0$ Additive Inverse

(vii) $a-a = 0$ Additive Inverse.

(viii) $\sqrt{2} \times \sqrt{5} = \sqrt{5} \times \sqrt{2}$ Commutative property w.r.t. '.'

(ix) $a(b-c) = ab-ac$ Left distributive property.

(x) $(x-y)z = xz-yz$ Right distributive property.

(xi) $4 \times (5 \times 8) = (4 \times 5) \times 8$ Associative property w.r.t. '.'

(xii) $a(b+c-d) = ab+ac-ad$ Left distributive property

Q.3 Name the properties used in the following inequalities.

Solution:

(i) $-3 < -2 \Rightarrow 0 < 1$ Additive property.

(ii) $-5 < -4 \Rightarrow 20 > 16$ Multiplication property.

(iii) $1 > -1 \Rightarrow -3 > -5$ Additive property.

(iv) $a < 0 \Rightarrow -a > 0$ Multiplicative property.

(v) $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$ Multiplicative property.

(vi) $a > b \Rightarrow -a < -b$ Multiplicative property.

Q.4 Prove the following Rules of Addition

(i) $\frac{a}{c}+\frac{b}{c} = \frac{a+b}{c}$

Solution:

L.H.S

$= \frac{a}{c}+\frac{b}{c}$

$= a \cdot \frac{1}{c}+b \cdot \frac{1}{c}$ ¤ $\frac{a}{b} = a \cdot \frac{1}{b}$

$= (a+b) \cdot \frac{1}{c}$ Distributive property

$= \frac{a+b}{c}$ ¤ $a \cdot \frac{1}{c} = \frac{a}{c}$

(ii) $\dfrac{a}{b}+\dfrac{c}{d}=\dfrac{ad+bc}{bd}$

Solution:

$$\text{L.H.S.} = \frac{a}{b}+\frac{c}{d}$$

$$= \frac{a}{b}\cdot 1+\frac{c}{d}\cdot 1 \qquad \text{Multiplicative Identity}$$

$$= \frac{a}{b}\cdot\left(d\cdot\frac{1}{d}\right)+\frac{c}{d}\left(b\cdot\frac{1}{b}\right) \qquad \text{Multiplicative Inverse}$$

$$= \frac{a}{b}\cdot\frac{d}{d}+\frac{c}{d}\cdot\frac{b}{b} \qquad \because\ d\cdot\frac{1}{d}=\frac{d}{d},\ b\cdot\frac{1}{b}=\frac{b}{b}$$

$$= \frac{ad}{bd}+\frac{cb}{db}$$

$$= \frac{ad}{bd}+\frac{bc}{bd} \qquad \text{Commutative Property w.r.t. '.'}$$

$$= ad\cdot\frac{1}{bd}+bc\cdot\frac{1}{bd} \qquad \because\ \frac{a}{b}=a\cdot\frac{1}{b}$$

$$= (ad+bc)\cdot\frac{1}{bd} \qquad \text{Distributive Property}$$

$$= \frac{ad+bc}{bd} \qquad \because\ a\cdot\frac{1}{b}=\frac{a}{b}$$

$$= \text{R.H.S.}$$

Q.5 Prove that $-\dfrac{7}{12}-\dfrac{5}{18}=\dfrac{-21-10}{36}$

Solution:

$$\text{L.H.S.} = -\frac{7}{12}-\frac{5}{18}$$

$$= -\frac{7}{12}\cdot 1-\frac{5}{18}\cdot 1 \qquad \text{Multiplicative Identity}$$

$$= -\frac{7}{12}\left(3\cdot\frac{1}{3}\right)-\frac{5}{18}\cdot\left(2\cdot\frac{1}{2}\right) \qquad \text{Multiplicative Inverse}$$

$$= -\frac{7}{12}\cdot\frac{3}{3}-\frac{5}{18}\cdot\frac{2}{2} \qquad \because\ a\cdot\frac{1}{b}=\frac{a}{b}$$

$$= -\frac{21}{36}-\frac{10}{36}$$

$= -21 \cdot \frac{1}{36} - 10 \cdot \frac{1}{36}$ $\because \frac{a}{b} = a \cdot \frac{1}{b}$

$= (-21 - 10) \cdot \frac{1}{36}$ Distributive Property

$= \frac{-21 - 10}{36}$

$=$ R.H.S.

Q.6 Simplify by justifying each step:

(i) $\frac{\mathbf{4 + 16x}}{\mathbf{4}}$

Solution:

$\frac{4 + 16x}{4}$

$= \frac{1}{4} \cdot (4 + 16x)$ $\because \frac{a}{b} = \frac{1}{b} \cdot a$

$= \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot 16\,x$ Distributive Property

$= \frac{1}{4} \cdot 4 \cdot 4x$ Multiplicative Inverse

$= 1 + 1 \cdot 4x$ Multiplicative Inverse

$= 1 + 4x$ Multiplicative Identity

(ii) $\frac{\frac{\mathbf{1}}{\mathbf{4}} + \frac{\mathbf{1}}{\mathbf{5}}}{\frac{\mathbf{1}}{\mathbf{4}} - \frac{\mathbf{1}}{\mathbf{5}}}$

Solution:

$\frac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}}$

$= \left(\frac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}} \right) \cdot 1$ Multiplicative Identity

$$= \left(\frac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}\right) . 20 . \frac{1}{20}$$ Multiplicative Inverse

$$= \frac{\left(\frac{1}{4}+\frac{1}{5}\right) . 20}{\left(\frac{1}{4}-\frac{1}{5}\right) . 20} \quad \because \quad 20 . \frac{1}{20} = \frac{20}{20}$$

$$= \frac{\frac{1}{4} . 20 + \frac{1}{5} . 20}{\frac{1}{4} . 20 + \frac{1}{5} . 20}$$ Distributive Property

$$= \frac{\left(\frac{1}{4} . 4\right)5 + \left(\frac{1}{5} . 5\right)4}{\frac{1}{4} . 4.5 + \frac{1}{5} . 5.4} = \frac{1.5 + 1.4}{1.5 - 1.4}$$ Multiplicative Inverse

$$= \frac{5+4}{5-4}$$ Multiplicative Identity

$$= \frac{9}{1} = 9$$

(iii) $\dfrac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}$

$$= \left(\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}\right) . 1$$ Multiplicative Identity

$$= \left(\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}\right) . bd . \frac{1}{bd}$$ Multiplicative Inverse

$$= \frac{\left(\frac{a}{b}+\frac{c}{d}\right) . bd}{\left(\frac{a}{b}-\frac{c}{d}\right) . bd} \quad \because \quad bd . \frac{1}{bd} = \frac{bd}{bd}$$

$$= \frac{\frac{a}{b} . bd + \frac{c}{d} . bd}{\frac{a}{b} . bd - \frac{c}{d} . bd}$$ Distributive Property

$$= \frac{a \cdot \frac{1}{b} \cdot b \cdot d + c \cdot \frac{1}{d} \cdot b \cdot d}{a\frac{1}{b} \cdot b \cdot d - c\frac{1}{d} \cdot b \cdot d} \qquad \because \quad \frac{1}{b} = a \cdot \frac{1}{b} \;,\; \frac{c}{d} = c \cdot \frac{1}{d}$$

$$= \frac{a\left(\frac{1}{b} \cdot b\right)d + c \cdot \left(\frac{1}{d} \cdot d\right)b}{a\left(\frac{1}{b} \cdot b\right)d - c\left(\frac{1}{d} \cdot d\right)b} \qquad \text{Commutative Property}$$

$$= \frac{a \cdot 1 \cdot d + c \cdot 1 \cdot b}{a \cdot 1 \cdot d - c \cdot 1 \cdot b} \qquad \text{Multiplicative Inverse}$$

$$= \frac{ad + cb}{ad - cb} \qquad \text{Multiplicative Identity}$$

(iv) $\dfrac{\dfrac{1}{a} - \dfrac{1}{b}}{1 - \dfrac{1}{a} \cdot \dfrac{1}{b}}$

$$= \left(\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}}\right) \cdot 1 \qquad \text{Multiplicative Identity}$$

$$= \left(\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}}\right) \cdot ab \cdot \frac{1}{ab} \qquad \text{Multiplicative Inverse}$$

$$= \frac{\left(\frac{1}{a} - \frac{1}{b}\right) \cdot ab}{\left(1 - \frac{1}{a} \cdot \frac{1}{b}\right) \cdot ab} \qquad \because \quad ab \cdot \frac{1}{ab} = \frac{ab}{ab} \cdot$$

$$= \frac{\frac{1}{a} \cdot ab - \frac{1}{b} \cdot ab}{ab - \frac{1}{a} \cdot \frac{1}{b} \cdot ab} \qquad \text{Distributive Property}$$

$$= \frac{\left(\frac{1}{a} \cdot a\right)b - \left(\frac{1}{b}\ b\right)a}{ab - \left(\frac{1}{a} \cdot a\right)\left(\frac{1}{b} \cdot b\right)} \qquad \text{Commutative Property}$$

$$= \frac{1 \cdot b - 1 \cdot a}{ab - 1 \cdot 1} \qquad \text{Multiplicative Inverse}$$

$$= \frac{b - a}{ab - 1} \quad \text{Multiplicative Identity}$$

$$= \qquad \text{M}^{\text{cative ative}}$$

EXERCISE 1.2

Q.1 Verify the addition properties of complex numbers.

Solution:

Addition properties of complex numbers are:

(i) Closure property

$\forall$ (a, b) , (c, d) $\in$ C

(a, b) + (c, d) = a + ib + c + id = a + c + i (b + d) = (a + c, b + d) $\in$ C

(ii) Associative Property

$\forall$ (a, b) , (c, d) , (e, f) $\in$ C

[(a, b) + (c, d)] + (e, f) = (a, b) + [(c, d) + (e, f)]

L.H.S.

$$[(a, b) + (c, d)] + (e, f) = [a + ib + c + id] + (e + if)$$
$$= a + ib + c + id + e + if$$
$$= a + c + e + i\,(b + d + f)$$
$$= (a + c + e\ ,\ b + d + f)$$

R.H.S.

$$(a, b) + [(c, d) + (e, f)]$$
$$= (a + ib) + [c + id + e + if]$$
$$= a + ib + c + id + e + if$$
$$= a + c + e + i\,(b + d + f)$$
$$= (a + c + e\ ,\ b + d + f)$$

$\Rightarrow$ L.H.S. = R.H.S.

(iii) Additive Identity

$\forall$ (a, b) $\in$C $\quad \exists$ (0, 0) $\in$ C

such that (a, b) + (0, 0) = (0, 0) + (a, b) = (a, b)

(iv) Additive Inverse

$\forall$ $(a, b) \in C$ $\quad \exists$ $(-a, -b) \in C$

such that $(a, b) + (-a, -b) = (0, 0) = (-a, -b) + (a, b)$

(v) Commutative Property

$\forall$ $(a, b), (c, d) \in C$

$$(a, b) + (c, d) = a + ib + c + id$$
$$= a + c + i(b + d)$$
$$= c + a + i(d + b) = (c + id) + (a + ib) = (c, d) + (a, b)$$

Q.2 Verify the multiplication properties of complex numbers.

Solution:

Multiplication Properties of complex numbers are:

Closure property

$\forall$ $(a, b), (c, d) \in C$

$$(a, b) . (c, d) = (a + ib)(c + id) = ac + aid + ibc + i^2bd$$
$$= ac - bd + i(ad + bc) = (ac - bd, ad + bc) \in C$$

Associative Property

$\forall$ $(a, b), (c, d) (e, f) \in C$

$$[(a, b) . (c, d)] . (e, f) = (a, b) [(c, d) . (e, f)]$$

L.H.S.

$$[(a, b) . (c, d)] . (e, f) = [(a + ib)(c + id)] . (e + if)$$
$$= [ac + iad + ibc + i^2bd] . (e + if)$$
$$= [ac - bd + iad + ibc] . (e + if)$$
$$= ace + iacf - bde - ibdf + iade + i^2adf + ibce + i^2bcf$$
$$= ace - adf - bcf - bde + i(acf - bdf + ade + bce)$$

Now R.H.S.

$$(a, b) [(c, d) . (e, f)] = (a + ib) [(c + id)(e + if)]$$
$$= (a + ib) [ce + icf + ide + i^2df]$$
$$= (a + ib) [ce - df + icf + ide]$$
$$= ace - adf + iacf + iade + ibce - ibdf + i^2bcf + i^2bde$$
$$= ace - adf - bcf - bde + i(acf - bdf + ade + bce)$$

$\Rightarrow$ L.H.S. = R.H. S

Multiplicative Identity

$\forall$ (a, b) $\in$ C $\exists$ (1, 0) $\in$ C such that

(a, b) . (1, 0) = (a + ib) . (1 + 0 i) = a + 0i + ib + 0 = a + ib = (a, b)

and

(1, 0) (a, b) = (1 + 0i) . (a + ib) = a + ib + 0i + 0 = a + ib = (a, b)

$\Rightarrow$ (1,0) is multiplicative identity in C.

Multiplicative Inverse

$\forall$ (a + ib) $\in$ C $\exists$ $(a + ib)^{-1} \in C$

where

$$(a+ib)^{-1} = \frac{1}{a+ib} = \frac{1}{a+ib} \cdot \frac{a-ib}{a-ib}$$

$$= \frac{a-ib}{(a)^2-(ib)^2} = \frac{a-ib}{a^2-i^2b^2} = \frac{a-ib}{a^2+b^2} = \frac{a}{a^2+b^2} - i\frac{b}{a^2+b^2}$$

$$= \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$$

Commutative Property

$\forall$ (a, b) , (c, d) $\in$ C

(a, b) . (c, d) = (c, d) . (a , b)

L.H.S.

(a, b) . (c, d) = (a + ib) (c + id) = ac + iad + ibc + i^2bd

= (ac – bd) + i (ad + bc)

R.H.S.

(c, d) . (a, b) = (c + id) (a + ib) = ca + icb + ida + i^2db

= (ac – bd) + i (bc + ad)

$\Rightarrow$ L.H.S. = R.H.S

Q.3 Verify the distributive law of complex numbers.

(a, b) [(c, d) + (e, f)] = (a, b) (c, d) + (a, b) (e, f)

Solution:

To show

(a, b) [(c, d) + e, f)] = (a, b) (c, d) + (a, b) (e, f)

L.H.S.

(a, b) [(c, d) + (e, f)]

= (a + ib) [c + id + e + if] = ac + iad + ae + iaf + ibc + i^2bd + ibe + i^2bf $\quad \because i^2 = -1$

= ac + ae – bd – bf + i (ad + af + bc + be)

R.H.S.

$(a, b)(c, d) + (a, b)(e, f)$

$= (a + ib)(c + id) + (a + ib)(e + if)$

$= ac + iad + ibc + i^2bd + ae + iaf + ibe + i^2bf \qquad \because \ i^2 = -1$

$= ac + ae - bd - bf + i(ad + bc + af + be)$

L. H. S. = R.H.S.

Hence Proved.

Q.4 Simplify the following:

(i) i^9

Solution:

$i^9 = i^8 . i = (i^2)^4 . i = (-1)^4 . i = (1)(i) = i \qquad \because \ i^2 = -1$

(ii) i^{14}

Solution:

$i^{14} = (i^2)^7 = (-1)^7 = -1 \qquad ¤ \ i^2 = -1$

(iii) $(-i)^{19}$ **(Lahore Board 2004)**

Solution:

$(-i)^{19} = (-1)\, i^{19} = -(1)\, i^{18} . i = -(i^2)^9 . i = -(-1)^9 i = (-1) . i = i \qquad ¤ \ i^2 = -1$

(iv) $(-1)^{-21/2}$ **(Lahore Board 2007)**

Solution:

$(-1)^{-21/2} = [(-1)^{1/2}]^{-21} = [(i^2)^{1/2}]^{-21} = i^{-21}$

$= \frac{1}{i^{21}} = \frac{1}{i^{20} . i} = \frac{1}{(i^2)^{10} . i}$

$= \frac{1}{(-1)^{10} i} = \frac{1}{i} \cdot \frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i \qquad \because \ i^2 = -1$

Q.5 Write in terms of i

Solution:

(i) $\sqrt{-1}\, b$

Solution:

$\sqrt{-1}\, b = ib \qquad ¤ \sqrt{-1} = i$

(ii) $\sqrt{-5}$

Solution:

$= \sqrt{-1} \cdot \sqrt{5} = i\sqrt{5} \quad \because \sqrt{-1} = i$

(iii) $\sqrt{\dfrac{-16}{25}}$

Solution:

$$\sqrt{\frac{-16}{25}} = \sqrt{-1}\sqrt{\frac{16}{25}} = i\left(\frac{4}{5}\right) = \frac{4}{5}i \qquad \because \quad \sqrt{-1} = i$$

(iv) $\sqrt{\dfrac{1}{-4}}$

Solution:

$$\sqrt{\frac{1}{-4}} = \frac{1}{\sqrt{-1}}\sqrt{\frac{1}{4}} = \frac{1}{i} \cdot \frac{1}{2} = \frac{i}{i \cdot i^2}\frac{1}{2} = \frac{i}{i^2 \cdot 2} = \frac{+i}{-2} = \frac{i}{2} \qquad \because \quad i^2 = -1$$

Q.6 Simplify the following:

(7, 9) + (3, – 5)

Solution:

$(7, 9) + (3, -5) = 7 + 9i + 3 - 5i = 10 + 4i = (10, 4)$

Q.7 (8, – 5) – (– 7, 4)

Solution:

$(8, -5) - (-7, 4) = 8 - 5i - (-7 + 4i) = 8 - 5i + 7 - 4i = 15 - 9i = (15, -9)$

Q.8 (2, 6) (3, 7)

Solution:

$$(2, 6)(3, 7) = (2 + 6i) \cdot (3 + 7i) = 6 + 14i + 18i + 42\,i^2$$
$$= 6 + 32i - 42 = -36 + 32i = (-36, 32) \qquad \because \; i^2 = -1$$

Q.9 (5, – 4) (– 3, – 2)

Solution:

$$(5, -4)(-3, -2) = (5 - 4i)(-3 - 2i)$$
$$= -15 - 10i + 12i + 8i^2 = -15 + 2i - 8 = -23 + 2i = (-23, 2)$$

Q.10 (0, 3) (0, 5)

Solution:

$$(0, 3)(0, 5) = (0 + 3i)(0 + 5i)$$
$$= 0 + 0 + 0 + 15i^2 = 0 + 15(-1) = -15 = -15 + 0i = (-15, 0)$$

Q.11 (2, 6) ÷ (3, 7)

Solution:

$$(2, 6) \div (3, 7) = (2 + 6i) \div (3 + 7i) = \frac{(2 + 6i)}{(3 + 7i)} \times \frac{(3 - 7i)}{(3 - 7i)} \qquad \text{Rationalizing}$$

$$= \frac{(2 + 6i)(3 - 7i)}{(3 + 7i)(3 - 7i)} = \frac{6 - 14i + 18i - 42\,i^2}{(3)^2 - (7i)^2} \qquad \because \; i^2 = -1$$

$$= \frac{6 + 4i + 42}{9 - 49i^2} = \frac{48 + 4i}{9 + 49} = \frac{48 + 4i}{58}$$

= = = = =

$$= \left(\frac{48}{58}, \frac{4i}{58}\right) = \left(\frac{24}{29}, \frac{2i}{29}\right)$$

Q.12 (5, – 4) ÷ (– 3, – 8)

Solution:

$$(5, -4) \div (-3, -8) = (5 - 4i) \div (-3 - 8i)$$

$$= \frac{5 - 4i}{-3 - 8i} \times \frac{-3 + 8i}{-3 + 8i} \quad \text{Rationalizing}$$

$$= \frac{(5 - 4i)(-3 + 8i)}{(-3 - 8i)(-3 + 8i)} = \frac{-15 + 40i + 12i - 32\,i^2}{(-3)^2 - (8i)^2}$$

$$= \frac{-15 + 52i + 32}{9 - 64i^2} = \frac{17 + 52i}{9 + 64} = \frac{17 + 52i}{73}$$

$$= \frac{17}{73} + \frac{52}{73}i = \left(\frac{17}{73}, \frac{52}{73}\right)$$

Q.13 Prove that the sum as well as product of any two conjugate complex numbers is a real number.

Solution:

Let $z = a + bi$ is a complex number then its conjugate is $\bar{z} = \overline{a + ib} = a - ib$

Sum $= z + \bar{z} = a + ib + a - ib = 2a$ (real number)

Product $= z \cdot \bar{z} = (a + ib)(a - ib) = (a)^2 - (ib)^2 = a^2 - i^2 b^2 \quad \because i^2 = -1$

$= a^2 + b^2$ (real number)

Hence the sum as well as the product of any two conjugate complex numbers is a real number.

Q.14 Find the multiplicative inverse of each of the following numbers.

(i) (– 4, 7) **(Lahore Board 2007)**

Solution:

(i) (– 4, 7)

As multiplicative inverse of $(a, b) = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2}\right)$

So multiplicative inverse of $(-4, 7) = \left(\frac{-4}{(-4)^2 + (7)^2}, \frac{-7}{(-4)^2 + (7)^2}\right)$

$$= \left(\frac{-4}{16 + 49}, \frac{-7}{16 + 49}\right) = \left(\frac{-4}{65}, \frac{-7}{65}\right)$$

(ii) $(\sqrt{2}, -\sqrt{5})$

Solution:

As multiplicative inverse of $(a, b) = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2}\right)$

So multiplicative inverse of $(\sqrt{2}, -\sqrt{5}) = \left(\frac{\sqrt{2}}{(\sqrt{2})^2 + (-\sqrt{5})^2}, \frac{+\sqrt{5}}{(\sqrt{2})^2 + (-\sqrt{5})^2}\right)$

$$= \left(\frac{\sqrt{2}}{2+5}, \frac{\sqrt{5}}{2+5}\right) = \left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right)$$

(iii) (1, 0)

As multiplicative inverse of (a, b) $= \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$

So multiplicative inverse of (1, 0) $= \left(\frac{1}{(1)^2+(0)^2}, \frac{-0}{(1)^2+(0)^2}\right)$

$$= \left(\frac{1}{1+0}, 0\right) = (1, 0)$$

Q.15 Factorize the following:

(i) $a^2 + 4b^2$

Solution:

$a^2 + 4b^2$

$= a^2 - (-4b^2) = (a^2) - (i^2 4b^2)$ ¤ $i^2 = -1$

$= (a)^2 - (2bi)^2 = (a + 2bi)(a - 2bi)$

(ii) $9a^2 + 16b^2$ **(Lahore Board 2006)**

Solution:

$9a^2 + 16b^2 = 9a^2 - (-16b^2)$

$= 9a^2 - (i^2 16b^2)$ $\because i^2 = -1$

$= (3a)^2 - (4bi)^2 = (3a + 4bi)(3a - 4bi)$

(iii) $3x^2 + 3y^2$ **(Gujranwala Board 2007)**

Solution:

$3x^2 + 3y^2 = 3[x^2 - (-y)^2] = 3[x^2 - (i^2 y^2)]$

$= 3[(x)^2 - (iy)^2] = 3(x + iy)(x - iy)$

Q.16 Separate into real and imaginary parts (write as a simple complex number)

(i) $\frac{2 - 7i}{4 + 5i}$ **(Lahore Board 2011)**

Solution:

$\frac{2-7i}{4+5i} = \frac{2-7i}{4+5i} \times \frac{4-5i}{4-5i}$ Rationalizing

$= \frac{(2-7i)(4-5i)}{(4+5i)(4-5i)} = \frac{8 - 10i - 28i + 35i^2}{(4)^2 - (5i)^2}$ $\because i^2 = -1$

$$= \frac{8 - 38i - 35}{16 - 25i^2} = \frac{-27 - 38i}{16 + 25}$$

$$= \frac{-27 - 38i}{41} = \frac{-27}{41} - \frac{38}{41}i$$

(ii) $\dfrac{(-2 + 3i)^2}{(1 + i)}$ **(Lahore Board 2003)**

Solution:

$$\frac{(-2 + 3i)^2}{(1 + i)} = \frac{(-2)^2 + (3i)^2 + 2(-2)(3i)}{1 + i}$$

$$= \frac{4 + 9i^2 - 12i}{1 + i} = \frac{4 - 9 - 12i}{1 + i} = \frac{-5 - 12i}{1 + i}$$

$$= \frac{-5 - 12i}{1 + i} \times \frac{1 - i}{1 - i}$$

$$= \frac{(-5 - 12i)(1 - i)}{(1 + i)(1 - i)} = \frac{-5 + 5i - 12i + 12i^2}{(1)^2 - (i)^2}$$

$$= \frac{-5 + 5i - 12i - 12}{1 - i^2} = \frac{-5 - 7i - 12}{1 - (-1)}$$

$$= \frac{-17 - 7i}{2} = -\frac{17}{2} - \frac{7}{2}i$$

(iii) $\dfrac{\mathbf{i}}{\mathbf{1 + i}}$ **(Gujranwala Board 2007)**

Solution:

$$\frac{i}{1 + i}$$

$$= \frac{i}{1 + i} \times \frac{1 - i}{1 - i} = \frac{i(1 - i)}{(1 + i)(1 - i)} = \frac{i - i^2}{(1)^2 - i^2} = \frac{i - (-1)}{1 - (-1)}$$

$$= \frac{i + 1}{1 + 1} = \frac{i + 1}{2}$$

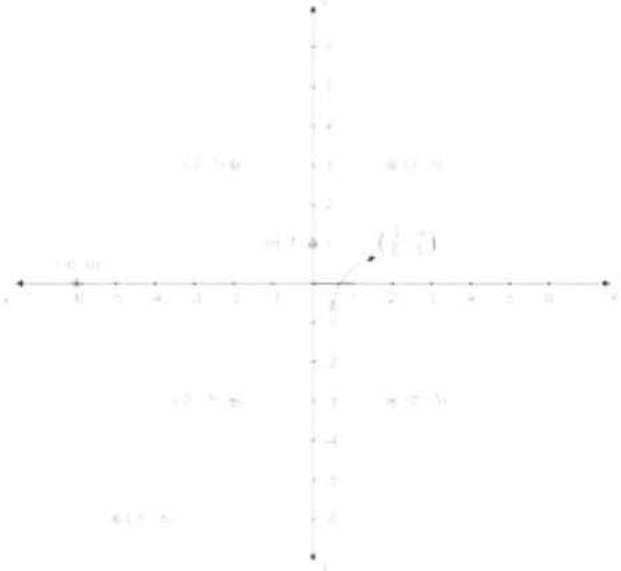
$$= \frac{i}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2}i$$

GEOMETRICAL REPRESENTATION OF COMPLEX NUMBERS

The Complex Plane:

EXERCISE 1.3

Q.1 Graph the following numbers on the complex plane:



(i) $2 + 3i = (2, 3)$

(ii) $2 - 3i = (2, -3)$

(iii) $-2 - 3i = (-2, -3)$

(iv) $-2 + 3i = (-2, 3)$

(v) $-6 = (-6, 0)$

(vi) $i = (0, 1)$

(vii) $\frac{3}{5} - \frac{4}{5}i = \left(\frac{3}{5}, -\frac{4}{5}\right) = (0.6, -0.8)$

(viii) $-5 - 6i = (-5, -6)$

Q.2 Find the multiplicative inverse of the following numbers:

(i) $-3i$

Solution:

$-3i = (0 - 3)$

As multiplicative inverse of $(a, b) = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2}\right)$

So multiplicative inverse of $(0, -3) = \left(\frac{0}{(0)^2 + (-3)^2}, \frac{3}{(0)^2 + (-3)^2}\right)$

$= \left(0, \frac{3}{9}\right) = \left(0, \frac{1}{3}\right) = 0 + \frac{1}{3}i = \frac{1}{3}i$

(ii) $1 - 2i$ **(Lahore Board 2006)**

Solution:

$1 - 2i = (1, -2)$

As multiplicative inverse of $(a, b) = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2}\right)$

So multiplicative inverse of $(1, -2) = \left(\frac{1}{(1)^2 + (-2)^2}, \frac{-(-2)}{(1)^2 + (-2)^2}\right)$

$= \left(\frac{1}{1+4}, \frac{2}{1+4}\right)$

$= \left(\frac{1}{5}, \frac{2}{5}\right) = \frac{1}{5} + \frac{2}{5}i$

(iii) $-3-5i$ **(Gujranwala Board 2004)**

Solution:

$-3-5i = (-3, -5)$

As multiplicative inverse of $(a, b) = \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$

So multiplicative inverse of $(-3, -5) = \left(\frac{-3}{(-3)^2 + (-5)^2}, \frac{-(-5)}{(-3)^2 + (-5)^2}\right)$

$= \left(\frac{-3}{9+25}, \frac{5}{9+25}\right)$

$= \left(\frac{-3}{34}, \frac{5}{34}\right) = \frac{-3}{34} + \frac{5}{34}i$

(iv) (1, 2)

Solution:

As multiplicative inverse of $(a, b) = \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$

So multiplicative inverse of $(1, 2) = \left(\frac{1}{(1)^2 + (2)^2}, \frac{-2}{(1)^2 + (+2)^2}\right)$

$= \left(\frac{1}{1+4}, \frac{-2}{1+4}\right)$

$= \left(\frac{1}{5}, \frac{-2}{5}\right)$

Q.3 Simplify

(i) i^{101} **(Gujranwala Board 2007)**

Solution:

$i^{101} = i^{100} \cdot i = (i^2)^{50} \cdot i = (-1)^{50} \cdot i = i$ $\because i^2 = -1$

(ii) $(-ai)^4$, $a \in R$

Solution:

$(-ai)^4 = (-1)^4 \cdot a^4 i^4 = a^4 (i^2)^2 = a^4 (-1)^2 = a^4$ $\because i^2 = -1$

(iii) i^{-3}

Solution:

$$i^{-3} = \frac{1}{i^3} = \frac{1}{i^2 \cdot i} = \frac{1}{-i} \cdot \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{-(-1)} = i$$

(iv) $\mathbf{i^{-10}}$

Solution:

$$i^{10} = \frac{1}{i^{10}} = \frac{1}{(i^2)^5} = \frac{1}{(-1)^5} = \frac{1}{-1} = -1$$

Q.4 Prove that $\bar{z} = z$ iff z is real. **(Lahore Board 2009)**

Solution:

Let $z = a + ib$ then $\bar{z} = a - ib$

Suppose $z = \bar{z}$

$\Rightarrow$ $a + ib = a - ib$

$\Rightarrow$ $a + ib - a + ib = 0$

$\Rightarrow$ $2ib = 0 \Rightarrow b = 0$ $\because 2i \neq 0$

$\Rightarrow$ $z = a + i0$

$\Rightarrow$ $z = a$

$\Rightarrow$ z is real.

Now conversely;

Suppose that z is real

Such that $z = a$

then $\bar{z} = a$

$\Rightarrow$ $z = \bar{z}$

Hence proved.

Q.5 Simplify by expressing in form a + bi

(i) $\mathbf{5 + 2\sqrt{-4}}$

Solution:

$5 + 2\sqrt{-4} = 5 + 2\sqrt{-1} \cdot \sqrt{4} = 5 + 2i\,(2) = 5 + 4i$ $\because \sqrt{-1} = 1$

(ii) $\mathbf{(2 + \sqrt{-3})(3 + \sqrt{-3})}$

Solution:

$$\begin{aligned}(2 + \sqrt{-3})(3 + \sqrt{-3}) &= 6 + 2\sqrt{-3} + 3\sqrt{-3} + (\sqrt{-3})^2 \\ &= 6 + 2\sqrt{-1} \cdot \sqrt{3} + 3\sqrt{-1} \cdot \sqrt{3} + (-3) \\ &= 6 + 2i\sqrt{3} + 3i\sqrt{3} - 3 \\ &= 3 + 5\sqrt{3}\,i\end{aligned}$$

(iii) $\mathbf{\dfrac{2}{\sqrt{5} + \sqrt{-8}}}$

$$\frac{2}{\sqrt{5}+\sqrt{-8}} = \frac{2}{\sqrt{5}+\sqrt{-8}} \cdot \frac{\sqrt{5}-\sqrt{-8}}{\sqrt{5}-\sqrt{-8}} \quad \text{Rationalizing}$$

$$= \frac{2\left(\sqrt{5}-\sqrt{-8}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{-8}\right)^2} = \frac{2\sqrt{5}-2\sqrt{-8}}{5-(-8)}$$

$$= \frac{2\sqrt{5}-2\sqrt{-1}\,.\,\sqrt{8}}{5+8} = \frac{2\sqrt{5}-2i\sqrt{8}}{13}$$

$$= \frac{2\sqrt{5}}{13} - \frac{4i\sqrt{2}}{13}$$

(iv) $\dfrac{3}{\sqrt{6}-\sqrt{-12}}$

Solution:

$$\frac{3}{\sqrt{6}-\sqrt{-12}} = \frac{3}{\sqrt{6}-\sqrt{12}} \times \frac{\sqrt{6}+\sqrt{-12}}{\sqrt{6}+\sqrt{-12}} \quad \text{Rationalizing}$$

$$= \frac{3\left(\sqrt{6}+\sqrt{12}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{-12}\right)^2}$$

$$= \frac{3\sqrt{6}+3\sqrt{-12}}{6-(-12)} = \frac{3\sqrt{6}+3\sqrt{-1}\,.\,\sqrt{12}}{6+12}$$

$$= \frac{3\sqrt{6}+3i\sqrt{12}}{18} = \frac{3\sqrt{6}}{18} + \frac{3i\,.\,2\sqrt{3}}{18}$$

$$= \frac{\sqrt{6}}{6} + \frac{\sqrt{3}}{3}i = \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{3}}i$$

Q.6 Show that $\forall\ z \in C$

(i) $z^2 + \bar{z}^2$ is real number

Solution:

Let $z = a + bi$

Then $\bar{z} = a - bi$

Now

$$z^2 + \bar{z}^2 = (a+bi)^2 + (a-bi)^2$$

$$= (a)^2 + (ib)^2 + 2abi + a^2 + (ib)^2 - 2abi$$

$$= a^2 + i^2b^2 + a^2 + i^2b^2 \qquad \because\ i^2 = -1$$

$$= a^2 - b^2 + a^2 - b^2 = 2a^2 - 2b^2 = 2(a^2 - b^2)$$

which is real number

Hence proved.

(ii) $(z - \bar{z})^2$ is real number **(Lahore Board 2009)**

Solution:

Let $z = a + bi$

then $\bar{z} = a - ib$

$\Rightarrow \quad (z - \bar{z})^2 = [(a + ib) - (a - ib)]^2 \qquad \because \ i^2 = -1$

$= [a + ib - a + ib]^2 = (2ib)^2 = 4i^2b^2 = -4b^2$

which is a real number.

Hence proved.

Q.7 Simplify the followings

(i) $\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3$ **(Gujranwala Board 2003, 2005)**

Solution:

$\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3 \qquad \because \ (a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$\left(\frac{-1 + \sqrt{3}\,i}{2}\right)^3 = \frac{(-1 + \sqrt{3\,i})^3}{(2)^3}$

$= \frac{1}{8}\left[(-1)^3 + (\sqrt{3i})^3 + 3(-1)^2(\sqrt{3i}) + 3(-1)(\sqrt{3i})^2\right]$

$= \frac{1}{8}\left[-1 + (\sqrt{3})^3 i^3 + 3(1)\sqrt{3}\,i - 3(3i^2)\right]$

$= \frac{1}{8}\left[-1 + \sqrt{3^3}\,i^2 . i + 3\sqrt{3}\,i - 3(3)(-1)\right]$

$= \frac{1}{8}\left[-1 + \sqrt{27}(-1)i + 3\sqrt{3}\,i + 9\right]$

$= \frac{1}{8}\left[-1 - 3\sqrt{3}\,i + 3\sqrt{3}\,i + 9\right]$

$= \frac{1}{8}(8) = 1$

(ii) $\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^3$

Solution:

$\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^3 = \left(\frac{-1 - \sqrt{3}}{2}\right)^3 = \left(\frac{(-1)(1 + \sqrt{3}i)}{2}\right)^3 = \frac{(-1)^3(1 + \sqrt{3}\,i)^3}{2^3}$

$= -\frac{1}{8}\left[(1)^3 + (\sqrt{3i})^3 + 3(1)^2(\sqrt{3i}) + 3(1)(\sqrt{3i})^2\right]$

$$= -\frac{1}{8}\left[1 + \left(\sqrt{3}\right)^3 i^3 + 3\sqrt{3}\, i + 3\,(3i^2)\right]$$

$$= -\frac{1}{8}\left[1 + \sqrt{3^3}\, i^2 \,.\, i + 3\sqrt{3}\, i + 9\,(-1)\right]$$

$$= -\frac{1}{8}\left[1 + \sqrt{27}\,(-1)\, i + 3\sqrt{3}\, i - 9\right]$$

$$= -\frac{1}{8}\left[1 - 3\sqrt{3}\, i + 3\sqrt{3}\, i - 9\right]$$

$$= -\frac{1}{8}[-8] = 1$$

(iii) $\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{-2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

Solution:

$$\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{-2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{-1}$$

$$= \left(\frac{-1 - \sqrt{3i}}{2}\right)^{-1}$$

$$= \frac{2}{-1 - \sqrt{3i}} \cdot \frac{-1 + \sqrt{3i}}{-1 + \sqrt{3i}} \qquad \text{Rationalizing}$$

$$= \frac{2\left(-1 + \sqrt{3}\, i\right)}{(-1)^2 - \left(\sqrt{3}\, i\right)^2} = \frac{-2 + 2\sqrt{3}\, i}{1 - 3i^2} = \frac{-2 + 2\sqrt{3}\, i}{1 + 3} \qquad \because i^2 = 1$$

$$= \frac{-2 + 2\sqrt{3}\, i}{4} = \frac{-2}{4} + \frac{2\sqrt{3}\, i}{4} = -\frac{1}{2} + \frac{\sqrt{3}\, i}{2}$$

(iv) $\mathbf{(a + bi)^2}$

Solution:

$$(a + bi)^2 = (a)^2 + 2\,(a)\,(bi) + (bi)^2 = a^2 + b^2 i^2 + 2abi$$

(v) $\mathbf{(a + bi)^2}$

Solution:

$$(a + bi)^2 = \frac{1}{(a + bi)^2} = \left(\frac{1}{a + bi} \cdot \frac{a - bi}{a - bi}\right)^2 \qquad \text{Rationalizing}$$

$$= \left(\frac{a - bi}{a^2 - b^2 i^2}\right)^2 = \left(\frac{a - bi}{a^2 + b^2}\right)^2 \qquad \because i^2 = -1$$

$$= \frac{(a - bi)^2}{(a^2 + b^2)^2}$$

$$= \frac{a^2 + b^2 i^2 - 2abi}{(a^2 + b^2)^2}$$

$$= \frac{a^2 - b^2 - 2abi}{(a^2 + b^2)^2} \qquad \because\ i^2 = -1$$

$$= \frac{a^2 - b^2}{(a^2 + b^2)^2} - \frac{2abi}{(a^2 + b^2)^2}$$

(vi) $(a + bi)^3$

Solution:

$$(a + bi)^3 = a^3 + b^3 i^3 + 3a^2 (bi) + 3a (bi)^2$$
$$= a^3 + b^3 . i^2 . i + 3a^2 bi + 3ab^2 i^2$$
$$= a^3 - b^3 i + 3a^2 bi - 3ab^2 \qquad \because\ i^2 = -1$$
$$= a^3 - 3ab^2 + i (3a^2 b - b^3)$$

(vii) $(a - bi)^3$

Solution:

$$(a - bi)^3 = (a)^3 - (bi)^3 - 3 (a)^2 (bi) + 3 (a) (bi)^2$$
$$= a^3 - b^3 . i^3 - 3a^2 bi + 3ab^2 i^2 \qquad \because (a-b)^3 = a^3 - b^3 - 3a^2 b + 3ab^2$$
$$= a^3 - b^3 i^2 . i - 3a^2 bi + 3ab^2 (-1)$$
$$= a^3 + b^3 i - 3a^2 bi - 3ab^2$$
$$= a^3 - 3ab^2 + i (b^3 - 3a^2 b)$$

(viii) $\left(3 - \sqrt{-4}\right)^{-3}$

Solution:

$$\left(3 - \sqrt{-4}\right)^{-3} = \left(3 - \sqrt{4} . \sqrt{-1}\right)^{-3}$$
$$= (3 - i\,2)^{-3} = (3 - 2i)^{-3}$$
$$= \frac{1}{(3 - 2i)^3} = \left(\frac{1}{3 - 2i} \cdot \frac{3 + 2i}{3 + 2i}\right)^3 \qquad \text{Rationalizing}$$
$$= \left(\frac{3 + 2i}{(3)^2 - (2i)^2}\right)^3$$
$$= \frac{(3 + 2i)^3}{(9 - 4i^2)^3} = \frac{(3 + 2i)^3}{(9 + 4)^3}$$
$$= \frac{1}{(13)^3} (3 + 2i)^3$$
$$= \quad (\ (\ (.$$

$$= \frac{1}{2197}\left[(3)^3 + (2i)^3 + 3\,(3)^2\,(2i) + 3\,(3)\,(2i)^2\right]$$

$$= \frac{1}{2197}\left[27 + 8i^3 + 54i + 9\,.\,4\,i^2\right]$$

$$= \frac{1}{2197}\left[27 + 8i^2\,.\,i + 54i + 36\,(-1)\right] = \frac{1}{2197}\left[27 - 8i + 54i - 36\right]$$

$$= \frac{1}{2197}\left[-9 + 46i\right]$$

$$= \frac{-9}{2197} + \frac{46}{2197}\,i$$



NUMBER SYSTEMS

MULTIPLE CHOICE QUESTIONS

(1) Modulus of $a + bi$ is:

(a) $a^2 + b^2$ (b) $\sqrt{a^2 + b^2}$

(c) $a^2 - b^2$ (d) $a - ib$

[Lahore Board 2005]

(2) $\left\{x|x = \frac{p}{q},\ p, q \in Z \wedge q \neq 0\right\}$ is set of all:

(a) Natural numbers (b) Integers

(c) Irrational numbers (d) Rational numbers

[Lahore Board 2005]

(3) $|2 - 3i| =$ ———

(a) $\sqrt{2}$ (b) $\sqrt{3}$

(c) $\sqrt{13}$ (d) $\sqrt{17}$

[Lahore Board 2005]

(4) $9a^2 + 64b^2 =$ ———

(a) $3a + 8b$ (b) $(3a + 8b)(3a - 8b)$

(c) $(3a + 8bi)(3a - 8bi)$ (d) None of these

[Lahore Board 2005][Lahore Board 2015]

(5) $|a + ib|$ is equal to:

(a) $\sqrt{a^2 + b^2}$ (b) $\sqrt{a^2 - b^2}$

(c) $\sqrt{-a^2 - b^2}$ (d) None of these

[Gujranwala Board 2005]

(6) The property of inequality used in $a > b$ and $b > c \Rightarrow a > c$ is:

(a) Additive property (b) Multiplicative property

(c) Transitive property (d) Additive inverse

[Gujranwala Board 2005]

(7) Which set has closure property with respect to addition:

(a) $\{-1, 1\}$ (b) $\{-1\}$

(c) $\{1\}$ (d) $\{0\}$

[Gujranwala Board 2006]

(8) $a, b \in R \; a = b \Rightarrow b = a$ is called:

(a) Reflexive property (b) Symmetric property

(c) Multiplicative property (d) None of these

[Gujranwala Board 2006]

(9) $(i)^{-10}$ is equal to:

(a) i (b) $-i$

(c) -1 (d) None of these

[Gujranwala Board 2006]

(10) If 'n' is a prime number then $\sqrt{n}$ is:

(a) An irrational number (b) A rational number

(c) Prime number (d) None of these

[Lahore Board 2006-07]

(11) $\forall a, b, c \in R \; , \; c > 0$

$a > b \Rightarrow ac > bc$ is:

(a) Reflexive property (b) Symmetric property

(c) Additive property (d) Multiplicative property

[Lahore Board 2006]

(12) The number of the form $\frac{1}{7}$ is called a/an:

(a) Rational number (b) Irrational number

(c) Integer (d) None of these

[Lahore Board 2006]

(13) $9a^2 + 16b^2 =$ _____

(a) $(3a + 4bi)(3a - 4bi)$ (b) $(3a + 4b)(3a - 4b)$

(c) $(3a + 4bi)(3a + 4bi)$ (d) None of these

[Lahore Board 2006]

(14) Geometrically the modulus of a complex number represents its distance from the point:

(a) $(1, 0)$ (b) $(0, 1)$

(c) $(0, 0)$ (d) $(1, 1)$

[Gujranwala Board 2007]

(15) The simplified form of $(i)^{101}$ is:

(a) -1 (b) 1

(c) i (d) $-i$

[Gujranwala Board 2007]

(16) $(-1)^{-21/2}$ is equal to:

(a) $-i$ (b) i

(c) 1 (d) -1

[Lahore Board 2007]

(17) The number 1 is a:

(a) Prime number (b) Irrational

(c) Even number (d) Odd number

[Lahore Board 2008]

(18) $\sqrt{3}$ is:

(a) Odd number (b) Natural number

(c) Rational (d) Irrational number

[Lahore Board 2008]

(19) Multiplicative property of order of real numbers is that $\forall$ a, b, c $\in$ R:

(a) $a > b \wedge c > 0 \Rightarrow ac > bc$ (b) $a > b \wedge c > 0 \Rightarrow ac < bc$

(c) $a > b \wedge c > 0 \Rightarrow ac = bc$ (d) $a > b \wedge c < 0 \Rightarrow ac > bc$

[Gujranwala Board 2008]

(20) $\overline{-a - ib}$ is equal to:

(a) $a + ib$ (b) $-a + ib$

(c) $a - ib$ (d) $-a - ib$

[Gujranwala Board 2008]

(21) Multiplicative inverse of (a, b) is:

(a) $\left(\frac{1}{a}, \frac{1}{b}\right)$ (b) $\left(\frac{a}{a^2+b^2}, \frac{b}{a^2+b^2}\right)$

(c) $\left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$ (d) $\left(\frac{1}{a^2+b^2}, \frac{b}{a^2+b^2}\right)$

[Gujranwala Board 2009]

(22) If $Z = x + iy$, $x, y \in R$ then $|Z|$ equals:

(a) $x^2 + y^2$ (b) $\sqrt{x^2+y^2}$

(c) $\sqrt{x^2-y^2}$ (d) $\sqrt{x+y}$

[Lahore Board 2009]

(23) $\sqrt{\frac{5}{16}}$ is:

(a) Rational number (b) Irrational number

(c) Prime number (d) Whole number

[Lahore Board 2009]

(24) Trichotomy is property of:

(a) In-equality (b) Equality

(c) Division (d) Subtraction

[Lahore Board 2010]

(25) The multiplicative inverse of $(\sqrt{2}\ ,\ -\sqrt{5})$ is:

(a) $\left(\frac{\sqrt{2}}{\sqrt{7}}, \frac{\sqrt{5}}{\sqrt{7}}\right)$ (b) $\left(\frac{\sqrt{2}}{7}, \frac{-\sqrt{5}}{7}\right)$

(c) $\left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right)$ (d) $\left(\frac{-\sqrt{2}}{7}, \frac{-\sqrt{5}}{7}\right)$

[Lahore Board 2010]

(26) The property used in $4 + 9 = 9 + 4$:

(a) Commutative property w.r.t multiplication

(b) Commutative property w.r.t addition.

(c) Associative property w.r.t addition.

(d) Associative property w.r.t multiplication.

[Lahore Board 2011]

(27) If $Z = 3 + 4i$, then $|z| =$ ————

(a) 25 (b) 5

(c) −25 (d) −5

[Lahore Board 2012]

(28) $\sqrt{3}$ is a ———— number.

(a) Rational (b) Irrational

(c) Prime (d) Even

[Lahore Board 2012]

(29) $\frac{22}{7}$ is:

(a) Natural number (b) Whole number

(c) Rational number (d) Irrational number

[Lahore Board 2012]

(30) Multiplicative inverse of $-3i$ is:

(a) $3i$ (b) $\frac{1}{3}i$

(c) $-\frac{1}{3}i$ (d) $-3i$

(31) $a > 0 \Rightarrow$

(a) $-a > 0$ (b) $2a < 0$

(c) $\frac{1}{a} > 0$ (d) $\frac{-1}{a} < 0$

[Lahore Board 2013]

(32) Multiplication inverse of (1, 0) is:

(a) (−1, 0) (b) (0, −1)

(c) (0, 1) (d) (1, 0)

[Lahore Board 2013]
[Lahore Board 2012]

(33) $(7, 9) + (3, -5) =$

(a) (7, 9) (b) (3, −5)

(c) (10, 4) (d) (4, 10)

(34) Inverse of b^{-1} is:

(a) b (b) b^{1}

(c) $\frac{1}{b}$ (d) None of these

(35) Tracheotomy property does not exists in:

(a) R (b) Q

(c) C (d) W

(36) $\sqrt{-i}$ = ————

(a) $\frac{1}{\sqrt{2}}(1-i)$ (b) $i\sqrt{-i}$

(c) $i\sqrt{i}$ (d) $1+i$

(37) $\sqrt{x+y}$ is modulus of:

(a) $x+iy$ (b) $(\sqrt{x}\ ,\ -\sqrt{y})$

(c) $(x^2\ ,\ y^2)$ (d) $(x\ ,\ y)$

(38) The number $1+i\sqrt{3}$ in polar form is:

(a) $2\ \text{cis}\ \frac{\pi}{6}$ (b) $2\ \text{cis}\ \frac{\pi}{3}$

(c) $\text{cis}\ \frac{\pi}{3}$ (d) None of these

(39) $\left(\frac{b^2}{b^4+b^2}\ ,\ \frac{-b}{b^4+b^2}\right)$ is multiplicative inverse of:

(a) (b^3, b) (b) (b^2, b)

(c) $\left(b, \frac{1}{b}\right)$ (d) (b, b^2)

(40) If $x > 0$ then:

(a) $\frac{1}{x} > 0$ (b) $-x > 0$

(c) $2x < 0$ (d) $-x < 0$

(41) If $Z = a + ib$ and $a = r\cos\theta$, $b = r\sin\theta$ then:

(a) $r = Z$ (b) $r = -Z$

(c) $r = -|Z|$ (d) $r = |Z|$

(42) Multiplicative inverse of $(0, 1)(-1, 0)$ is:

(a) $(0, 1)$ (b) $(1, 0)$

(c) $(0, 0)$ (d) $(1, 1)$

(43) $a(x, y)$ = ———— where $a \in R$:

(a) (ax, y) (b) (x, ay)

(c) (ax, ay) (d) (x, y)

(44) Imaginary part of a real number 'a' is:

(a) 0 (b) 1

(c) i (d) None of these

(45) In the set $C - \{0\}$, associative law does not holds w.r.t.

(a) x (b) +

(c) − (d) None of these

(46) $|Z_1 + Z_2|$ ———

(a) $< |Z_1 + Z_2|$ (b) $> |Z_1| > |Z_2|$

(c) $\leq |Z_1| + |Z_2|$ (d) $\geq |Z_1| + |Z_2|$

(47) Modulus of two conjugate complex numbers are ———

(a) Same (b) Different

(c) Not exists (d) None of these

(48) $(\cos\theta - i \sin\theta)^{-n} =$ ———

(a) $\cos n\theta - i \sin n\theta$ (b) $\cos(-n)\theta + i \sin(-n)\theta$

(c) $\cos\theta + i \sin\theta$ (d) $\cos n\theta + i \sin n\theta$

(49) The members of cartesian product $A \times B$ are called:

(a) Ordered pairs (b) Sets

(c) Subsets (d) Elements

(50) If $Z = 3 - 2i$ then $\overline{\overline{Z}} =$

(a) $3 + 2i$ (b) $3 - 2i$

(c) $-3 - 2i$ (d) $-3 + 2i$

(51) π is a/an:

(a) Rational number (b) Irrational number

(c) Even number (d) Whole number

(52) Better approximate value of π is:

(a) $\frac{22}{7}$ (b) $\frac{355}{113}$

(c) 3.14159 (d) All of these

(53) $Q \cup Q' =$ ———

(a) C (b) R

(c) $C - \{0\}$ (d) $R - \{0\}$

(54) If $Z = 4 - 3i$ then imaginary part of Z is:

(a) $3i$ (b) $-3i$

(c) 3 (d) -3

(55) Which set does not satisfy the order axioms:

(a) Q (b) C

(c) N (d) Q′

(56) $\forall\ Z_1, Z_2 \in C\ |Z_1| - |Z_2|$ ————

(a) $= |Z_1 - Z_2|$ (b) $\leq |Z_1 . Z_2|$

(c) $\leq |Z_1 + Z_2|$ (d) $\geq |Z_1| + |Z_2|$

(57) $(-1)^{-55\left|\frac{7}{3}-\frac{15}{6}\right|} =$

(a) i (b) $-i$

(c) 1 (d) -1

(58) Which number is not an imaginary number:

(a) $\sqrt{-9}$ (b) $(-1)^{-1/2}$

(c) $7 + 6i$ (d) $(1 + i)^2$

(59) $(-1, 0)^{2n+\frac{1}{2}} =$

(a) i (b) $-i$

(c) 1 (d) -1

(60) Solution of the equation $\frac{x^2}{2} + a = b$ belongs to set of complex numbers if:

(a) $a = 0$ (b) $a > b$

(c) $a < b$ (d) $a = b$

(61) $\sqrt{-5}$ belongs to set of:

(a) Real numbers (b) Prime numbers

(c) Even numbers (d) Complex numbers

(62) Multiplicative inverse of $-i$ is: **[Lahore Board 2014]**

(a) i (b) $-i$

(c) 1 (d) -1

(63) Types of ratios fractions are: **[Lahore Board 2015]**

(a) 3 (b) 2

(c) 4 (d) 1

(64) $(-i)^{19} =$ ————

(a) $-i$ (b) i

(c) 1 (d) -1

(65) An improper fraction can be reduced to proper fraction by an addition:

(a) Addition (b) Subtraction

(c) Multiplication (d) Division

Answers

(1)	b	*(2)*	d	*(3)*	c	*(4)*	c	*(5)*	a
(6)	c	*(7)*	d	*(8)*	b	*(9)*	c	*(10)*	a
(11)	d	*(12)*	b	*(13)*	a	*(14)*	c	*(15)*	c
(16)	a	*(17)*	d	*(18)*	d	*(19)*	a	*(20)*	b
(21)	c	*(22)*	b	*(23)*	b	*(24)*	a	*(25)*	c
(26)	b	*(27)*	b	*(28)*	b	*(29)*	c	*(30)*	b
(31)	d	*(32)*	d	*(33)*	c	*(34)*	a	*(35)*	c
(36)	c	*(37)*	b	*(38)*	b	*(39)*	b	*(40)*	d
(41)	d	*(42)*	a	*(43)*	c	*(44)*	a	*(45)*	c
(46)	c	*(47)*	a	*(48)*	d	*(49)*	a	*(50)*	b
(51)	b	*(52)*	c	*(53)*	b	*(54)*	d	*(55)*	b
(56)	c	*(57)*	b	*(58)*	c	*(59)*	a	*(60)*	b
(61)	d	*(62)*	a	*(63)*	b	*(64)*	b	*(65)*	d

SOLUTION

Q.1 Modulus of $a + ib = |a + ib|$

$= \sqrt{(a)^2 + (b)^2}$

$= \sqrt{a^2 + b^2}$

Q.2 As we know that:

The number which can be written in the form of $\frac{p}{q}$ where $p, q \in Z$ and $q \neq 0$ is called a rational number so the given set is a set of rational numbers.

Q.3 $|2 - 3i| = \sqrt{(2)^2 + (-3)^2} = \sqrt{4 + 9} = \sqrt{13}$

Q.4 $9a^2 + 64b^2 = 9a^2 - (-64b^2) = (3a)^2 - (8bi)^2$

$= (3a + 8bi)(3a - 8bi)$

Q.5 $|a + ib|$

By definition of modulus, $|a + ib| = \sqrt{a^2 + b^2}$.

Q.7 As $0 + 0 = 0 \in \{0\}$.

$\Rightarrow$ Only set $\{0\}$ has closure property with respect to addition.

Q.9 $(i)^{-10} = \frac{1}{i^{10}} = \frac{1}{(i^2)^5} = \frac{1}{(-1)^5} = \frac{1}{-1} = -1.$

Q.10 If 'n' is a prime number then $\sqrt{n}$ is an irrational number. Its proof can be seen at pg. 5 (Textbook).

Q.12 As $\frac{1}{7}$ is in $\frac{p}{q}$ form where $1, 7 \in Z$ and $7 \neq 0 \Rightarrow \frac{1}{7}$ is a rational number.

Q.13 $9a^2 + 16b^2$

$= 9a^2 - (-16b^2)$

$= (3a)^2 - (4ib)^2$

$= (3a + 4ib)(3a - 4ib)$

Q.14 Geometrically, the modulus of a complex number represents its distance from origin (0, 0) so correct option is (0, 0).

Q.15 $(i)^{101} = (i^2)^{50} \cdot i = (-1)^{50} \cdot i = i.$

Q.16 $(-1)^{-\frac{21}{2}} = [(-1)^{1/2}]^{-21} = (i)^{-21} = \frac{1}{i^{21}} = \frac{1}{(i^2)^{10} \cdot i}$

$= \frac{1}{(-1)^{10} \cdot i} = \frac{1}{i} = \frac{1}{i} \times \frac{i}{i} = -i$

Q.17 1 is an odd number.

Q.18 As $\sqrt{n}$ (where 'n' is a prime number) is an irrational number.

$\Rightarrow$ $\sqrt{3}$ is also irrational number.

Q.20 $\overline{-a - ib}$

$= -a + ib$ (by definition of conjugate)

Q.21 As multiplicative inverse of (a, b) is $\frac{1}{(a\ ,\ b)} = \frac{1}{a + ib}$

$$= \frac{1}{a + ib} \times \frac{a - ib}{a - ib}$$

$$= \frac{a - ib}{(a)^2 - (ib)^2}$$

$$= \frac{a - ib}{a^2 + b^2}$$

$$= \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2} i = \left(\frac{a}{a^2 + b^2}\ ,\ \frac{-b}{a^2 + b^2}\right)$$

Q.22 $|Z| = \sqrt{(x)^2 + (y)^2}$

$= \sqrt{x^2 + y^2}$ (by def of modulus)

Q.23 $\sqrt{\frac{5}{16}}$

$= \frac{\sqrt{5}}{\sqrt{16}} = \frac{\sqrt{5}}{4}$

As $\sqrt{5} \notin Z \Rightarrow \frac{\sqrt{5}}{4}$ is not a rational number.

$\Rightarrow$ $\frac{\sqrt{5}}{4}$ is an irrational number.

Q.25 As we know that:

Multiplicative inverse of (a, b) $= \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2}\right)$

Multiplicative inverse of $\left(\sqrt{2}, -\sqrt{5}\right) = \left(\frac{\sqrt{2}}{2 + 5}, \frac{\sqrt{5}}{2 + 5}\right)$

$= \left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right)$

Q.32 Inverse of $b^{-1} = (b^{-1})^{-1} = b$.

Q.33 Trichotomy property does not exists in C.

Q.34 $\sqrt{-i}$

$= \sqrt{(-1)i} = \sqrt{-1}\ \sqrt{i} = i\sqrt{i}$

Q.35 As, $(\sqrt{x}, -\sqrt{y})$

$= \sqrt{x} - \sqrt{y}\, i$

and $|\sqrt{x} - \sqrt{y}\, i| = \sqrt{(\sqrt{x})^2 + (-\sqrt{y})^2}$

$= \sqrt{x+y}$

$\Rightarrow$ $\sqrt{x+y}$ is modulus of $(\sqrt{x}, -\sqrt{y})$

Q.36 Let, $Z = 1 + i\sqrt{3}$

Let, $1 = r\cos\theta$ and $\sqrt{3} = r\sin\theta$

$r^2\cos^2\theta + r^2\sin^2\theta = 1 + (\sqrt{3})^2$

$r^2 = 1 + 3 = 4$

$r = 2$

And $\theta = \tan^{-1}\dfrac{\sqrt{3}}{1} = \tan^{-1}\sqrt{3} = 60^\circ = \dfrac{\pi}{3}$

So,

$$1 + i\sqrt{3} = r(\cos\theta + i\sin\theta) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$= 2\,\text{cis}\,\frac{\pi}{3}$$

Q.37 As, Multiplicative inverse of $(a,b) = \left(\dfrac{a}{a^2+b^2}, \dfrac{-b}{a^2+b^2}\right)$

Multiplicative inverse of $(b^2, b) = \left(\dfrac{b^2}{(b^2)^2+(b)^2}, \dfrac{-b}{(b^2)^2+(b)^2}\right)$

$= \left(\dfrac{b^2}{b^4+b^2}, \dfrac{-b}{b^4+b^2}\right)$

Q.38 If $x > 0$

$(-1)x < (-1)0$

$-x < 0$

Q.39 As,

$$Z = a + ib$$
$$Z = r\cos\theta + i\, r\sin\theta$$
$$|Z| = \sqrt{(r\cos\theta)^2 + (r\sin\theta)^2}$$
$$= \sqrt{r^2\cos^2\theta + r^2\sin^2\theta}$$
$$= \sqrt{r^2(\cos^2\theta + \sin^2\theta)}$$
$$= \sqrt{r^2(1)}$$
$$= \sqrt{r^2}$$
$$|Z| = r$$

Q.40 As,

$$(0, 1)(-1, 0) = (0 + i)(-1 + 0i)$$
$$= (i)(-1)$$
$$= -i = (0, -1)$$

$$\text{Multiplicative inverse of } (0, -1) = \left(\frac{0}{(0)^2 + (-1)^2}, \frac{-(-1)}{(0)^2 + (-1)^2}\right)$$
$$= \left(0, \frac{1}{0 + 1}\right)$$
$$= (0, 1)$$

Q.41 $a(x, y) = a(x + iy) = ax + iay = (ax, ay)$

Q.42 As,

$$a = a + 0i$$

$\Rightarrow$ Imaginary part of a is 0.

Q.43 Associative law does not holds w.r.t '–'.

Q.44 $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$

Q.45 Let,

$$Z = a + ib \text{ and its conjugate } \overline{Z} = a - ib$$

$$|Z| = \sqrt{a^2 + b^2} \quad \text{and} \quad |\overline{Z}| = \sqrt{a^2 + b^2} \text{ (same)}$$

Q.46 As,

$$(\cos\theta - i\sin\theta)^{-n}$$
$$= \cos(-n)\theta - i\sin(-n)\theta \qquad \cos(-\theta) = \cos\theta$$
$$= \cos n\theta + i\sin n\theta \qquad \sin(-\theta) = -\sin\theta$$

Q.47 The members of cartesian product are called ordered pairs.

Q.48 If

$$Z = 3 - 2i$$

$$\bar{Z} = 3 + 2i$$

$$\bar{\bar{Z}} = 3 - 2i$$

Q.49 π is an irrational number.

Q.50 Better approximate value of Π is 3.14159.

Q.51 $Q \cup Q' = R$

Q.52 If $Z = 4 - 3i$

then imaginary part of Z is -3.

Q.53 Set of complex numbers 'C' does not satisfy the order axioms.

Q.54 $|Z_1| - |Z_2| \leq |Z_1 + Z_2|$

Q.55 $(-1)^{-55\left|\frac{7}{3}-\frac{15}{6}\right|} = (-1)^{-55\left|\frac{14-15}{6}\right|} = (-1)^{-55\left|-\frac{1}{6}\right|}$

$$= (-1)^{-55 \cdot \frac{1}{6}}$$

$$= (-1)^{-\frac{55}{6}}$$

$$= [(-1)^{\frac{1}{2}}]^{-\frac{55}{3}}$$

$$= (i)^{-\frac{55}{3}}$$

$$= \frac{1}{(i)^{\frac{55}{3}}}$$

$$= \frac{1}{[i^{55}]^{1/3}}$$

$$= \frac{1}{[(i^2)^{27}.i]^{1/3}}$$

$$= \frac{1}{((-1)^{27}.i)^{1/3}}$$

$$= \frac{1}{(-i)^{1/3}}$$

$$= \frac{1}{(-1)^{1/3}.(i)^{1/3}}$$

$$= \frac{1}{(-1)^{1/3}.\left(\sqrt{-1}\right)^{1/3}} = \frac{1}{(-1)^{1/3}(-1)^{1/6}}$$

$$= \frac{1}{(-1)^{\frac{1}{3}+\frac{1}{6}}} = \frac{1}{(-1)^{3/6}} = \frac{1}{(-1)^{1/2}} = \frac{1}{i} = \frac{1}{i} \times \frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$$

Q.57 $(-1, 0)^{2n+\frac{1}{2}} = (-1)^{2n+\frac{1}{2}} = (-1)^{2n}.(-1)^{1/2} = (-1)^{2n}.i = i$ $\because$ 2n is even

Q.58 Let, a = 2 , b = 1 then $\frac{x^2}{2} + 2 = 1$.

$\Rightarrow$ $\frac{x^2}{2} = -1$ $\Rightarrow$ $x^2 = -2$ $\Rightarrow$ $x = \sqrt{-2} \in C$

$\Rightarrow$ If a > b then solution belongs to C.

Q.61 $\sqrt{5i} \in C$

Q.62 $\frac{1}{-i} \times \frac{i}{i}$

$= \frac{i}{-i^2} = \frac{i}{-(-1)} = i$

Q.64 $(-i)^{19} = -i^{19} = -i^{16}.i^3 = -(1)(-i) = i$