14. **The Rational Root Theorem is a tool used to find all possible ..... of a polynomial equation with integer coefficients.**
(A) irrational roots (B) rational roots (C) integral roots (D) real roots

Exercise 1.4

15. **If 'ω' be the cube root of unity, then ω^2 = ...**
(A) $\frac{-1-\sqrt{3}i}{2}$ (B) $\frac{1-\sqrt{3}i}{2}$ (C) 1 (D) $\frac{1+\sqrt{3}i}{2}$

16. **Three cube roots of -27 are .....**
(A) $3, \omega, \omega^2$ (B) $-3, -3\omega, -3\omega^2$ (C) $3, 3\omega, 3\omega^2$ (D) $1, \omega, \omega^2$

17. **If ω is the imaginary cube roots of unity then ω^3 = ...**
(A) ω (B) ω^{-1} (C) $\frac{1}{\omega^2}$ (D) ω^2

18. **For any $n \in Z$, ω^n is equivalent to .....**
(A) 1 or ω or ω^2 (B) $2, 2\omega, 2\omega^2$ (C) $3, 3\omega, 3\omega^2$ (D) $\pm i, 2$

19. **Product of all the four 4^{th} roots of unity is .....**
(A) -1 (B) 1 (C) zero (D) 2

Exercise 1.5

20. **Polar form of a complex number $1+i\sqrt{3}$ is .....**
(A) $2\cos30° + i2\sin30°$ (B) $2\sin60° + i2\cos60°$ (C) $2\cos60° + i2\sin60°$ (D) $2\cos60° - i2\sin60°$

21. **The range for principal argument θ of a complex number $z = a + bi$ is .....**
(A) $[-\pi, \pi]$ (B) $(-\pi, \pi]$ (C) $[-\pi, \pi)$ (D) $(-\pi, \pi)$

22. **If $\arg(z_1) = \theta_1$ and $\arg(z_2) = \theta_2$, then $\arg(z_1 \cdot z_2)$ = ...**
(A) $\theta_1 - \theta_2$ (B) $\theta_1 + \theta_2$ (C) $\theta_1 \theta_2$ (D) $\frac{\theta_1}{\theta_2}$

23. **If $z_1 = 5\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ and $z_2 = 4\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$ then $\arg(z_1 \cdot z_2)$ = ...**
(A) $\frac{5\pi}{6}$ (B) $\frac{3\pi}{2}$ (C) $\frac{5\pi}{3}$ (D) $\frac{10\pi}{3}$

24. **The encrypted form of letter T from word MATH by using encryption key $k = 2 + 3i$ is .....**
(A) $13 + 0i$ (B) $20 + 0i$ (C) $8 + 0i$ (D) $1 + 0i$

ANSWER KEY

1	B	2	C	3	C	4	C	5	A	6	C	7	A	8	A	9	C	10	C
11	C	12	B	13	B	14	B	15	A	16	B	17	B	18	A	19	A	20	B
21	B	22	B	23	C	24	B												

Unit 2 Functions and Graphs

Introduction

Functions are fundamental in mathematics, describing relationships between inputs and outputs through a rule of correspondence. Understanding key concepts such as domain, co-domain and range is essential for analyzing different types of functions, including one-to-one, onto and bijective functions. Graphical representation helps in identifying intersecting points, such as where a linear function meets the coordinate axes, where two linear functions intersect or where a linear and a quadratic function cross. These intersections provide valuable insights into solving equations visually. Additionally, exploring square root and cube root function graphs allows for a deeper understanding of their unique properties and behaviour. This unit will enhance problem-solving skills by combining algebraic and graphical approaches to functions.

Concept of Function

The term function was recognized by a German Mathematician Leibniz (1646- 1716) to describe the dependence of one quantity on another. The following examples illustrate how this term is used:

1. Area "A" of a square depends on one of its sides 'x'
Formula: $A = x^2$
We say that A is a function of x.



2. Volume "V" of a sphere depends on its radius 'r'
Formula: $V = \frac{4}{3}\pi r^3$
We say that V is a function of r.



A **function** is a rule or correspondence, relating two sets in such a way that each element in the first set corresponds to one and only one element in the second set.

Thus in, (1) above, a square of a given side has only one area and in, (2) above, a sphere of a given radius has only one volume.

Now we have a formal definition:

Definition (Function, Domain, Codomain, Range)

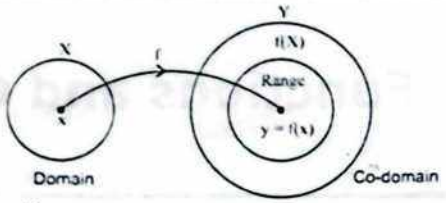
A **function** f from a set X to a set Y is a rule or a correspondence that assigns to each element x in X a unique element y in Y. The set X is called the **domain** of f.

The set of corresponding elements y in Y is called the **range** of f. While the **codomain** of a function is the set Y in which function's output values (range) lie.

Unless stated to the contrary, we shall assume hereafter that the set X and Y consist of real numbers.

Note:
Co-domain is the set of all possible outputs but the range is the actual set of outputs produced by the function under the given domain that is range set is always a subset of co-domain.

Conceptual Diagram

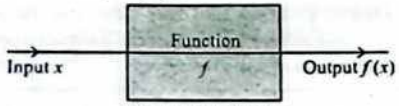


Notation and Value of a Function

If a variable y depends on a variable x in such a way that each value of x determines exactly one value of y, then we say that "y is a function of x".

Swiss mathematician Euler (1707-1783) invented a symbolic way to write the statement "y is a function of x" as $y = f(x)$, which is read as "y is equal to f of x".

A function can be thought as a computing machine f that takes an input x, operates on it in some way and produces exactly one output $f(x)$. This output $f(x)$ is called the value of f at x or image of x under f. The output $f(x)$ is denoted by a single letter, say y and we write $y = f(x)$.



Computing Machine

Independent variable: If $y = f(x)$, then the variable x is called the independent variable of f.

Dependent variable: If $y = f(x)$, then the variable y is called the dependent variable of f.

Note:
For now onward we shall only consider the function in which the variables are real numbers and we say that f is a real valued function of real numbers.

Example 1: Given $f(x) = x^3 - 2x^2 + 4x - 1$, find:

(i) $f(0)$ (ii) $f(1)$ (iii) $f(-2)$ (iv) $f(1+x)$ (v) $f\left(\frac{1}{x}\right), x \neq 0$

Solution:

Given: $f(x) = x^3 - 2x^2 + 4x - 1$

(i) $f(0) = ?$
Replace x by 0 in $f(x)$
$f(0) = (0)^3 - 2(0)^2 + 4(0) - 1$
$= 0 - 0 + 0 - 1$
$= -1$

(ii) $f(1) = ?$
Replace x by 1 in $f(x)$
$f(1) = (1)^3 - 2(1)^2 + 4(1) - 1$
$= 1 - 2 + 4 - 1$
$= 2$

(iii) $f(-2) = ?$
Replace x by -2 in $f(x)$
$f(-2) = (-2)^3 - 2(-2)^2 + 4(-2) - 1$
$= -8 - 8 - 8 - 1$
$= -25$

(iv) $f(1+x) = ?$
Replace x by $(1+x)$ in $f(x)$
$f(1+x) = (1+x)^3 - 2(1+x)^2 + 4(1+x) - 1$
$= 1 + 3x + 3x^2 + x^3 - 2 - 4x - 2x^2 + 4 + 4x - 1$
$= x^3 + x^2 + 3x + 2$

(v) $f\left(\frac{1}{x}\right) = ?$
Replace x by in $f(x)$

$$f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 - 2\left(\frac{1}{x}\right)^2 + 4\left(\frac{1}{x}\right) - 1 = \frac{1}{x^3} - \frac{2}{x^2} + \frac{4}{x} - 1, \; x \neq 0$$

Example 2: Find the domain and range of $f(x) = x^2$.

Solution:

$f(x) = x^2$
Since $f(x)$ is defined for every real number x, therefore
Domain f = set of all real numbers = $(-\infty, \infty)$
For every real number x, $f(x) = x^2$ is a non-negative real number. So,
Range f = set of all non-negative real numbers = $[0, \infty)$

Example 3: Find the domain and range of $f(x) = \frac{x}{x^2 - 4}$.

Solution:

At $x = 2$ and $x = -2$, $f(x) = \frac{x}{x^2 - 4}$ is not defined. So,
Domain f = set of all real numbers except -2 and $2 = R - \{-2, 2\}$

Let $y = \frac{x}{x^2 - 4} \Rightarrow y(x^2 - 4) = x \Rightarrow x^2 y - 4y = x$

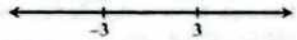
$yx^2 - 1.x - 4y = 0$ (Quadratic equation in x)
Here $a = y, b = -1, c = -4y$
For real values of x, Disc ≥ 0
$b^2 - 4ac \geq 0$
$(-1)^2 - 4(y)(-4y) \geq 0$
$1 + 16y^2 \geq 0$ which is true for all $y \in R$, so
Range f = set of all real numbers = $(-\infty, \infty)$

Remember!
There are two types of intervals known as open interval and closed interval. In an open interval (a, b), the endpoints are not included. In a closed interval $[a, b]$, the endpoints are included.

Example 4: Find the domain and range of $f(x) = \sqrt{x^2 - 9}$.

Solution:

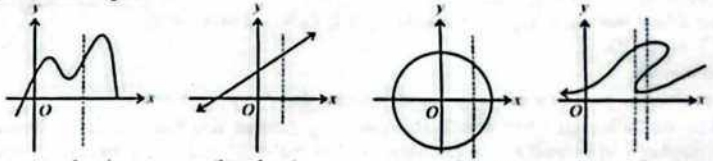
For real values of $f(x) = \sqrt{x^2 - 9}$,
$x^2 - 9 \geq 0 \Rightarrow x^2 \geq 9 \Rightarrow |x| \geq 3 \Rightarrow \pm x \geq 3$
Either, $x \geq 3$ or $-x \geq 3$
$x \geq 3$ or $x \leq -3$
Domain $f = (-\infty, -3] \cup [3, \infty)$
Put $x = \pm 3$ in $f(x)$, we have
$f(\pm 3) = \sqrt{(\pm 3)^2 - 9} = \sqrt{9 - 9} = 0$
and $f(x)$ is positive for all other values of x in domain of f, so
Range $f = [0, \infty)$



Vertical Line Test

The vertical line test is a method used to determine whether a graph represents a function. A graph represents a function if and only if no vertical line intersects the graph more than once. If any vertical line passes through the graph more than once, it is not a function.

Explanation is given in the figure.



(a) a function (b) a function (c) not a function (d) not a function

Types of Function

(i) One-to-One (Injective) Function

A function $f: A \to A$ is one-to-one if different inputs produce different outputs, that is if $f(x_1) = f(x_2)$ implies $x_1 = x_2$. This means that no two different elements of the domain map to the same element of the co-domain.

For example, $f(x) = 5x + 7$ is one-to-one because if $5x_1 + 7 = 5x_2 + 7 \Rightarrow 5x_1 = 5x_2 \Rightarrow x_1 = x_2$

(ii) Onto (Surjective) Function

A function $f: A \to B$ is called onto (or surjective) function if every element in the co-domain B has at least one pre-image in the domain A. In other words, for every y in B, there exists an x in A such that $f(x) = y$.

That is, Range $f = B$

For example, $f(x) = 2x + 3$, where the domain and co-domain are both real numbers.

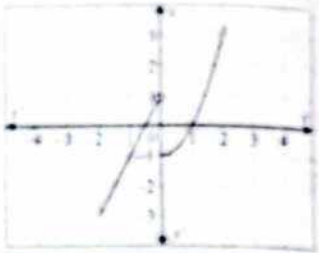
Here, $y = 2x + 3 \Rightarrow x = \frac{y-3}{2}$

Here, for each y in R, there exists $x = \frac{y-3}{2}$ in R such that $f\left(\frac{y-3}{2}\right) = y$

Hence f is an onto function.

(iii) Bijective Function

A function $f: A \to B$ is called bijective if it is both one-to-one and onto.



Piecewise Function

A piecewise function is a function that is defined by different expressions (or "pieces") over different intervals of its domain. Each piece applies to a specific part of the domain.

For example, $f(x) = \begin{cases} 2x + 1 & \text{if } x < 0 \\ x^2 - 1 & \text{if } x \geq 0 \end{cases}$

For $x < 0$, the function behaves as $2x + 1$ and for $x \geq 0$, it behaves as $x^2 - 1$.

Example 5: Show that the function $f(x) = x + 1$, where the domain and co-domain are all real numbers, is bijective.

Solution:

A function is bijective if it is both one-to-one and onto.

A function is one-to-one if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for $f(x) = x + 1$

Suppose $f(x_1) = f(x_2)$

$x_1 + 1 = x_2 + 1 \quad \because f(x) = x + 1$

$\Rightarrow x_1 = x_2$

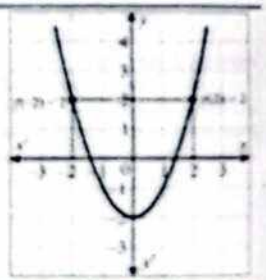
So, the given function is one-to-one.

It is also onto because for every real number y, there is a real number x (specifically $x = y - 1$) such that $f(y - 1) = y - 1 + 1 = y$

Hence, $f(x)$ is bijective.

Example 6: Show that the function $f(x) = x^2 - 2$, where the domain and co-domain are all real numbers, is neither one-to-one nor onto.

Solution:



As $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 - 2 = x_2^2 - 2 \quad \because f(x) = x^2 - 2$

$\Rightarrow x_1^2 = x_2^2$

Taking square root, we get

$x_1 = x_2$ or $x_1 = -x_2$

This does not imply that $x_1 = x_2$, for example $x_1 = 2, x_2 = -2 \Rightarrow x_1 \neq x_2$ and $f(2) = 2 = f(-2)$.

Thus, f is not one-to-one.

Also, the element -2 in the co-domain R is the smallest value that $f(x) = x^2 - 2$ can attain, and it is only achieved when $x = 0$. However, any number less than -2 (in co-domain R) is not the image of any real number x in domain R. For example, $f(x) = -3 \Rightarrow x^2 - 2 = -3$ has no real root. Hence, $f(x)$ is neither one-to-one nor onto.

Exercise 2.1

1. Given that

(a) $f(x) = x^2 - 1$ (b) $f(x) = \sqrt{2x + 3}$

Find

(i) $f(-3)$ (ii) $f(0)$

(iii) $f(x - 2)$ (iv) $f(x^2 + 3)$

Solution: (a)

Given that

$f(x) = x^2 - 1$

(i) $f(-3) = ?$

Replace x by -3 in $f(x)$

$f(-3) = (-3)^2 - 1$

$= 9 - 1$

$= 8$

(ii) $f(0) = ?$

Replace x by 0 in $f(x)$

$f(0) = (0)^2 - 1$

$= 0 - 1$

$= -1$

(iii) $f(x - 2) = ?$

Replace x by $x - 2$ in $f(x)$

$f(x - 2) = (x - 2)^2 - 1$

$= x^2 + 4 - 4x - 1$

$= x^2 - 4x + 3$

(iv) $f(x^2 + 3) = ?$

Replace x by $x^2 + 3$ in $f(x)$

$f(x^2 + 3) = (x^2 + 3)^2 - 1$

$= x^4 + 9 + 6x^2 - 1$

$= x^4 + 6x^2 + 8$

Solution: (b)

Given that $f(x) = \sqrt{2x + 3}$

(i) $f(-3) = ?$

Replace x by -3 in $f(x)$

$f(-3) = \sqrt{2(-3) + 3}$

$= \sqrt{-6 + 3}$

$= \sqrt{-3}$

(ii) $f(0) = ?$

Replace x by 0 in $f(x)$

$f(0) = \sqrt{2(0) + 3}$

$= \sqrt{0 + 3}$

$= \sqrt{3}$

(iii) $f(x - 2) = ?$

Replace x by $x - 2$ in $f(x)$

$f(x - 2) = \sqrt{2(x - 2) + 3}$

$= \sqrt{2x - 4 + 3}$

$= \sqrt{2x - 1}$

(iv) $f(x^2 + 3) = ?$

Replace x by $x^2 + 3$ in $f(x)$

$f(x^2 + 3) = \sqrt{2(x^2 + 3) + 3}$

$= \sqrt{2x^2 + 6 + 3}$

$= \sqrt{2x^2 + 9}$

2. Find $\frac{f(a + h) - f(a)}{h}$ and simplify where,

(i) $f(x) = 4x + 7$

Solution:

As $f(x) = 4x + 7$

$f(a + h) = 4(a + h) + 7$

$= 4a + 4h + 7$

and $f(a) = 4a + 7$

Now,

$\frac{f(a + h) - f(a)}{h} = ?$

$= \frac{(4a + 4h + 7) - (4a + 7)}{h}$

$= \frac{4a + 4h + 7 - 4a - 7}{h}$

$= \frac{4h}{h}$

$= 4$

(ii) $f(x) = \sin x$

Solution:

As $f(x) = \sin x$

$f(a + h) = \sin(a + h)$

and $f(a) = \sin a$

Now,

$\frac{f(a + h) - f(a)}{h} = ?$

$= \frac{\sin(a + h) - \sin a}{h}$

Using $\sin P - \sin Q = 2\cos\left(\frac{P + Q}{2}\right)\sin\left(\frac{P - Q}{2}\right)$

$= \frac{2\cos\left(\frac{a + h + a}{2}\right)\sin\left(\frac{a + h - a}{2}\right)}{h}$

$= \frac{2\cos\left(\frac{2a + h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

$= \frac{2}{h}\cos\left(a + \frac{h}{2}\right)\sin\left(\frac{h}{2}\right)$

(iii) $f(x) = x^3 + x^2 - 1$

Solution:

As $f(x) = x^3 + x^2 - 1$

$f(x+h) = (x+h)^3 + (x+h)^2 - 1$

$= x^3 + h^3 + 3xh(x+h) + x^2 + h^2 + 2xh - 1$

$= x^3 + h^3 + 3x^2h + 3xh^2 + x^2 + h^2 + 2xh - 1$

and $f(x) = x^3 + x^2 - 1$

Now, $\frac{f(x+h) - f(x)}{h}$ =

$= \frac{(x^3 + x^2 + 3x^2h + 3xh^2 + x^2 + h^2 + 2xh - 1) - (x^3 + x^2 - 1)}{h}$

$= \frac{x^3 + h^3 + 3x^2h + 3xh^2 + x^2 + h^2 + 2xh - 1 - x^3 - x^2 + 1}{h}$

$= \frac{h^3 + 3xh^2 + h^2 + 3x^2h + 2xh}{h}$

$= \frac{h(h^2 + 3xh + h + 3x^2 + 2x)}{h}$

$= h^2 + 3xh + h + 3x^2 + 2x$

(iv) $f(x) = \tan x$

Solution:

As $f(x) = \tan x$

$f(x+h) = \tan(x+h)$

and $f(x) = \tan x$

Now, $\frac{f(x+h) - f(x)}{h}$ =

$= \frac{\tan(x+h) - \tan x}{h}$

$= \frac{1}{h}\left[\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}\right]$

$= \frac{1}{h}\left[\frac{\sin(x+h)\cos x - \cos(x+h)\sin x}{\cos x \cos(x+h)}\right]$

Using $\sin\alpha\cos\beta - \cos\alpha\sin\beta = \sin(\alpha - \beta)$,

$= \frac{1}{h}\left[\frac{\sin(x+h-x)}{\cos x \cos(x+h)}\right]$

$= \frac{\sin h}{h \cos x \cos(x+h)}$

---

3. Express the following:

(a) The area A of a square as a function of its perimeter P.

Solution:

Let the length of each side of square be x unit, then



Area of square = $x \cdot x$

$A = x^2$ ... (1)

Perimeter of square = $x + x + x + x$

$P = 4x$

$x = \frac{P}{4}$ Put this in eq. (1)

$A = \left(\frac{P}{4}\right)^2$

$A = \frac{P^2}{16}$

which represents the area A of square as a function of its perimeter P.

(b) The circumference C of a circle as a function of its area A.

Solution:

Let the radius of circle be r unit, then



Circumference of circle = $2\pi r$

$C = 2\pi r$ ... (1) $r = ?$

Area of circle = πr^2

$A = \pi r^2$ $r = ?$

$r^2 = \frac{A}{\pi}$

$r = \frac{\sqrt{A}}{\sqrt{\pi}}$ Length is always real

Put $r = \frac{\sqrt{A}}{\sqrt{\pi}}$ in eq. (1), we have

$C = 2\pi\left(\frac{\sqrt{A}}{\sqrt{\pi}}\right)$

$C = 2\sqrt{\pi}\sqrt{A}$

$C = 2\sqrt{\pi A}$

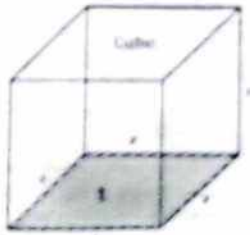
which represents the circumference C of circle as a function of its area A.

(c) The surface area S of a cube as a function of its volume V.

Solution:

Let the length of each edge of a cube be x unit, then

Surface area of cube = 6(area of each face)



$S = 6(x \cdot x)$

$S = 6x^2$ ... (1)

Volume of cube = length × breadth × height

$V = x \cdot x \cdot x$

$V = x^3$

$V^{\frac{1}{3}} = (x^3)^{\frac{1}{3}}$

$\Rightarrow x = V^{\frac{1}{3}}$ Put this in eq. (1)

$S = 6(V^{\frac{1}{3}})^2$

$S = 6V^{\frac{2}{3}}$

---

4. Find the domain and the range of the function g defined below

(i) $g(x) = 5 - x$

Solution:

$g(x) = 5 - x$

As $g(x)$ is defined for all $x \in R$, so

D_g = Set of all real numbers = $(-\infty, \infty)$

R_g = Set of all real numbers = $(-\infty, \infty)$

(ii) $g(x) = \sqrt{x+2}$

Solution:

$g(x) = \sqrt{x+2}$

For real values of g

$x + 2 \geq 0$

$x \geq -2$

$\Rightarrow D_g = [-2, \infty)$

As $g(-2) = \sqrt{-2+2} = 0$

and $g(x) > 0$ for all other values of x in D_g, so

$R_g = [0, \infty)$

(iii) $g(x) = \begin{cases} 6x + 7, & x \leq -2 \\ 4 - 3x, & x > -2 \end{cases}$

Solution:

$D_g = (-\infty, -2] \cup (-2, \infty) = (-\infty, \infty)$

$R_g = (-\infty, -5] \cup (-\infty, 10) = (-\infty, 10)$

$(-\infty, -5) \cup (-\infty, 10)$

(iv) $g(x) = |x - 5|$

Solution:

$g(x) = |x - 5|$

$= \begin{cases} +(x-5) & \text{if } x - 5 \geq 0 \Rightarrow x \geq 5 \\ -(x-5) & \text{if } x - 5 < 0 \Rightarrow x < 5 \end{cases}$

$D_g = (-\infty, 5) \cup [5, \infty) = (-\infty, \infty)$

As $g(5) = 5 - 5 = 0$

and $g(x) > 0$ for all other values of x in D_g, so

$R_g = [0, \infty)$

(v) $g(x) = \frac{x+2}{3-x}$

Solution:

Since $g(x)$ is not defined at $x = 3$, so

$D_g = R - \{3\} = (-\infty, 3) \cup (3, \infty)$

For range of $g(x)$:

Let $y = \frac{x+2}{3-x}, x \neq 3$

$y(3 - x) = x + 2$

$3y - xy = x + 2$

$3y - 2 = x + xy$

$x(1 + y) = 3y - 2$

$x = \frac{3y - 2}{y + 1}$

For real value of x, $y \neq -1$

$\Rightarrow R_g = R - \{-1\} = (-\infty, -1) \cup (-1, \infty)$

---

5. Given $f(x) = x^3 - ax^2 + bx + 1$. If $f(2) = -3$ and $f(-1) = 0$. Find the values of a and b.

Solution:

Given $f(x) = x^3 - ax^2 + bx + 1$

and $f(2) = -3$, $f(-1) = 0$

Put $x = 2$ in $f(x)$

$(2)^3 - a(2)^2 + b(2) + 1 = f(2)$

$8 - 4a + 2b + 1 = -3$ $\therefore f(2) = -3$

$-4a + 2b + 3 - 9$

$-4a + 2b - 12$

$-2a + b = 6$ ... (1)

$f(-1) = 0$

Put $x = -1$ in $f(x)$

$(-1)^3 - a(-1)^2 + b(-1) + 1 = f(-1)$

$-1 - a - b + 1 = 0$ $\therefore f(-1) = 0$

$-a - b = 0$

$a + b = 0$ ... (2)

Equation (1) + Equation (2)

$2a - b = 6$

$a + b = 0$

$3a = 6 \Rightarrow a = 2$

Put $a = 2$ in equation (2), we have

$2 + b = 0 \Rightarrow b = -2$

Hence $a = 2, b = -2$

6. A stone falls from a height of 60 m on the ground, the height h after x seconds is approximately given by $h(x) = 40 - 10x^2$.

(i) What is the height of stone when:

(a) $x = 1$sec ? **(b) $x = 1.5$sec ?**

(c) $x = 1.7$sec?

Solution:

(a) $h(1) = ?$

Put $x = 1$ in $h(x)$

$h(1) = 40 - 10(1)^2$

$= 40 - 10$

$h(1) = 30$m

(b) $h(1.5) = ?$

Put $x = 1.5$ in $h(x)$

$h(1.5) = 40 - 10(1.5)^2$

$= 40 - 10(2.25)$

$= 40 - 22.5$

$h(1.5) = 17.5$m

(c) $h(1.7) = ?$

Put $x = 1.7$ in $h(x)$

$h(1.7) = 40 - 10(1.7)^2$

$= 40 - 10(2.89)$

$= 40 - 28.9$

$h(1.7) = 11.1$m

(ii) When does the stone strike the ground?

Solution:

When the stone strikes the ground height is zero. i.e.,

$h(x) = 0$

$40 - 10x^2 = 0$

$-10x^2 = -40$

$10x^2 = 40$

$x^2 = \frac{40}{10}$

$x^2 = 4$

$x = 2$ sec (time is always +ve)

Thus the stone strikes the ground after 2 seconds.

7. Consider the function $f(x) = 3x - 5$.

(i) Determine the domain and range of $f(x)$.

Solution:

$f(x) = 3x - 5$

As $f(x)$ is defined for all $x \in R$, so

D_f = Set of all real numbers = $(-\infty, \infty)$

R_f = Set of all real numbers = $(-\infty, \infty)$

(ii) Is the function f one-to-one? Justify your answer.

Solution:

$f(x) = 3x - 5$

A function $f(x)$ is one-to-one if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

Suppose $f(x_1) = f(x_2)$

$3x_1 - 5 = 3x_2 - 5$

$3x_1 = 3x_2$

$\Rightarrow x_1 = x_2$

Thus, $f(x)$ is one-to-one function.

(iii) Is the function f onto if the co-domain is all real numbers? Explain.

Solution:

Given that the co-domain is the set of all real numbers.

R_f = Set of real numbers (From part (i))

As R_f = Co-domain of 'f'

So, the function $f(x)$ is onto.

8. Let $f: R \to R$ be defined by $f(x) = \frac{2x-3}{x+1}$.

(i) Find the domain and range of $f(x)$.

Solution:

$f(x) = \frac{2x-3}{x+1}$, where $f: R \to R$

Since $f(x)$ is not defined for $x = -1$, so

$D_f = R - \{-1\}$

For range of $f(x)$:

Let $y = \frac{2x-3}{x+1}, \quad x \neq -1$

$y(x + 1) = 2x - 3$

$xy + y = 2x - 3$

$y + 3 = 2x - xy$

$x(2 - y) = y + 3$

$x = \frac{y+3}{2-y}$

For real value of x, $y \neq 2$

$\Rightarrow R_f = R - \{2\}$

(ii) Determine whether $f(x)$ is onto.

Solution:

Since $f: R \to R$, therefore

$D_f = R$

Co-domain of 'f' $= R$

As Co-domain of 'f' $\neq R_f$, so the function $f(x)$ is not onto.

(iii) Prove that $f(x)$ is one-to-one.

Solution:

$f(x) = \frac{2x-3}{x+1}$

A function $f(x)$ is one-to-one if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

Suppose $f(x_1) = f(x_2)$

$\Rightarrow \frac{2x_1 - 3}{x_1 + 1} = \frac{2x_2 - 3}{x_2 + 1}$

$(2x_1 - 3)(x_2 + 1) = (x_1 + 1)(2x_2 - 3)$

$2x_1x_2 + 2x_1 - 3x_2 - 3 = 2x_1x_2 - 3x_1 + 2x_2 - 3$

$2x_1 + 3x_1 = 2x_2 + 3x_2$

$5x_1 = 5x_2$

$\Rightarrow x_1 = x_2$

Thus, the function $f(x)$ is one-to-one.

9. Consider the function $f: R \to R^+$ defined by $f(x) = e^{-x}$. Show that $f(x)$ is a bijective.

Solution:

$f(x) = e^{-x}$ and $f: R \to R^+$

A function $f(x)$ is one-to-one if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

Suppose $f(x_1) = f(x_2)$

$\Rightarrow e^{-x_1} = e^{-x_2}$

By taking '$\ln$' on both sides

$\ln(e^{-x_1}) = \ln(e^{-x_2})$

$-x_1 \cdot \ln e = -x_2 \cdot \ln e$

$-x_1 = -x_2 \quad \because \ln e = 1$

$\Rightarrow x_1 = x_2$

Thus, the function $f(x)$ is one-to-one.

Range of $f(x)$:

Let $y = e^{-x}$

$y = \frac{1}{e^x}$

$\Rightarrow e^x = \frac{1}{y}$

Taking '$\ln$' on both sides

$x \cdot \ln e = \ln\frac{1}{y} \quad \because \ln(a)^b = b \ln a$

$x = \ln\frac{1}{y} \quad \ln a, a \neq 1$

$x = \log_e \frac{1}{y}, \ y > 0$ and $y \neq 0$

$\Rightarrow R_f = (0, \infty) = R^+$

As $f: R \to R^+$, so co-domain $(f) = R^+$

Since R_f = co-domain (f), therefore the function $f(x)$ is onto.

Thus, the function $f(x)$ is bijective.

10. Let $g: R \to R$ be given by $g(x) = x^3 - 3x$. Determine if $g(x)$ is injective and/or surjective.

Solution:

$g(x) = x^3 - 3x$ and $g: R \to R$

A function $g(x)$ is not one-to-one, if $g(x_1) = g(x_2) \Rightarrow x_1 \neq x_2$

Given function $g(x)$ is not a one-to-one function, because

$g(0) = g(\sqrt{3})$, but $0 \neq \sqrt{3}$, that is

$0^3 - 3(0) = (\sqrt{3})^3 - 3\sqrt{3}$

$0 = 3\sqrt{3} - 3\sqrt{3}$

$0 = 0$, but $0 \neq \sqrt{3}$

For Onto:

To check for surjectivity, we need to see that for every $y \in R$, there exists $x \in R$, such that

$g(x) = y$

Since $g(x) = x^3 - 3x$ is a polynomial, therefore $g(x)$ is continuous everywhere.

As $x \to \infty, g(x) \to \infty$

Also $x \to -\infty, g(x) \to -\infty$

$\therefore -\infty < g(x) < \infty$

$\Rightarrow$ Range $= R$

Since R_g = co-domain (g), therefore the function $g(x)$ is onto.

Thus, the function $g(x)$ is not injective, but is surjective.

Finding the Intersecting Point(s) Graphically

The point of intersection is a point where two or more graphs meet on the coordinate plane. This point represents the solution to the equation of the given functions.

Intersection of a Linear Function and Coordinate Axes

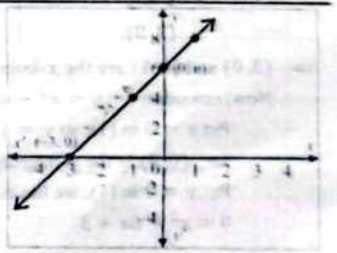
As we know that linear function is a function in which the highest power of the variable is one. While the coordinate axes refers to x-axis and y-axis in the Cartesian coordinate system.

Example 7: Find the points of intersection of a linear function $y = 2x + 6$ and coordinate axes graphically.

Solution:

Table of values of $y = 2x + 6$ is given below:

x	$y = 2x + 6$
-1	4
0	6
1	8



Hence, from the above graph, the points $(-3, 0)$ and $(0, 6)$ are the points of intersections of $y = 2x + 6$ and coordinate axes.

Intersection of Two Linear Functions

The point of intersection of two linear functions is the point where their graphs cross each other. This means the two functions have the same x and y values at that point.

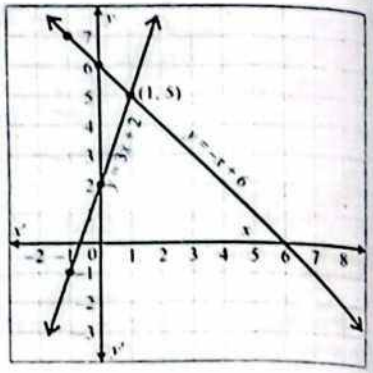
Example 8: Find the point of intersection of $y = 3x + 2$ and $y = -x + 6$ graphically.

Solution:

Table of different values of x and y is given below:

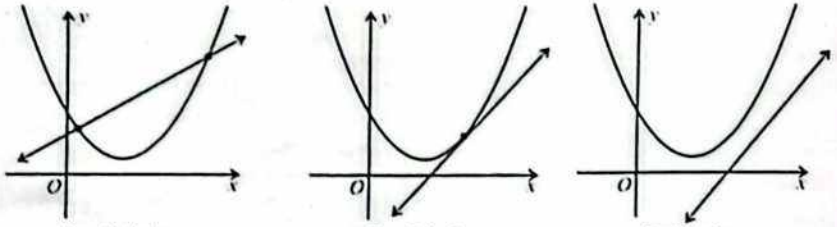
x	$y = 3x + 2$	$y = -x + 6$
−1	−1	7
0	2	6
1	5	5

By plotting the above points, we see that $(1, 5)$ is the point of intersection of both the straight lines as shown in figure.



Intersection of a Linear Function and a Quadratic Function

A line and a parabola can either intersect at two points, one point or not intersect at all. If there are two solutions, the system has two points of intersection. A single solution indicates that there is only one intersection point, suggesting that the line may be tangent to the parabola. If no solution exists, it means the line and the parabola do not intersect.



Example 9: Solve the linear function $y = -x + 3$ and quadratic function $y = x^2 - 6x + 3$ graphically.

Solution:

Given Functions: $y = -x + 3$ and $y = x^2 - 6x + 3$

Consider: $y = -x + 3$

Put $y = 0$, we have	Put $x = 0$, we have
$\Rightarrow 0 = -x + 3$	$\Rightarrow y = -0 + 3$
$x = 3$	$y = 3$
$(3, 0)$	$(0, 3)$

$\Rightarrow$ $(3, 0)$ and $(0, 3)$ are the x-intercept and y-intercept respectively of $y = -x + 3$.

Now, consider: $y = x^2 - 6x + 3$ ... (1)

Put $x = 0$ in (1), so $y = 3$

$\Rightarrow$ $(0, 3)$ is the y-intercept.

Put $y = 0$ in (1), we have

$0 = x^2 - 6x + 3$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 12}}{2} = \frac{6 \pm \sqrt{24}}{2}$$

$$x = \frac{6 \pm 2\sqrt{6}}{2}$$

$$x = 3 \pm \sqrt{6}$$

$$x = 3 - \sqrt{6}, 3 + \sqrt{6}$$

$$x = 0.6, 5.4$$

So, $(0.6, 0)$ and $(5.4, 0)$ are the x-intercepts.

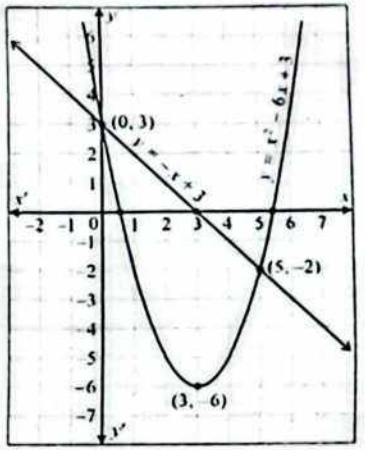
Now, we find vertex (h, k) of the parabola

$$h = -\frac{b}{2a} = -\frac{-6}{2(1)} = 3$$

$$k = (3)^2 - 6(3) + 3 = -6$$

So, the vertex is $(3, -6)$

Hence $(0,3)$ and $(5, -2)$ are the solutions (points of intersection) of the given functions.



Graph of the Square Root Function

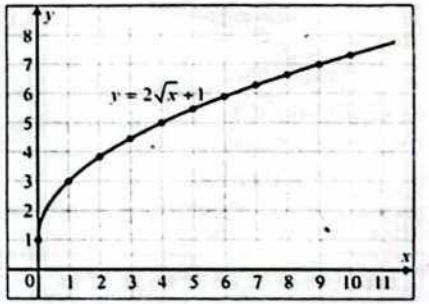
Example 10: Graph the square root function $y = 2\sqrt{x} + 1$

Solution:

Clearly the domain of $y = 2\sqrt{x} + 1$ is $x \geq 0$, as the square root of a negative number is not a real number. The range of $y = 2\sqrt{x} + 1$ is $y \geq 1$.

Table of values and the graph of the function are given below:

x	$y = 2\sqrt{x} + 1$
0	1
1	3
2	3.8
3	4.5
4	5
5	5.5
6	5.9
7	6.3
8	6.7
9	7
10	7.3



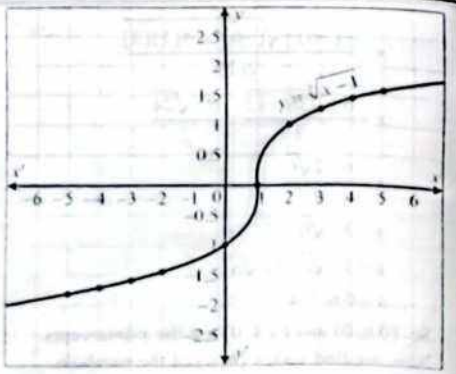
Graph of the Cube Root Function

Example 11: Graph the cube root function $y = \sqrt[3]{x-1}$

Solution:

Table values and the graph of the function are given belo .

x	$y = \sqrt{x-1}$
-5	-1.8
-4	-1.7
-3	-1.6
-2	-1.4
-1	-1.3
0	-1
1	0
2	1
3	1.3
4	1.4
5	1.6



Real Life Applications

Growth and Decay in Finance (Predicting Long-Term Stock Prices)

When something increases in quantity or size over time, it is called **growth**. For example, money in a bank account earning interest (it grows larger), a population of rabbits is increasing over months.

When something decreases in quantity or size over time, it is called decay. For example, a radioactive substance is losing its strength over years, a cup of hot coffee is cooling down over time.

Example 12: The value of a stock follows the exponential growth model $P(t) = P_0 e^{rt}$, where P_0 is the initial stock price, r is the growth rate per year and t is the time in years. Suppose a stock is currently valued at Rs. 5,000, and it is expected to grow at a rate of 5% per year.

(i) Find the value of the stock after 10 years. (ii) After how many years will the stock double in value?

Solution:

(i) The formula for the exponential growth is:

$$P(t) = P_0 e^{rt}$$

Given $P_0 = 5{,}000, r = 0.05$ (5% growth rate), and $t = 10$ years.

$$P(10) = 5{,}000e^{0.05 \cdot 10} = 5{,}000e^{0.5}$$

Using $e^{0.5} \approx 1.6487$

$$P(10) = 5{,}000 \times 1.6487$$

$$= 8244$$

So, the value of the stock after 10 years is approximately Rs. 8244.

(ii) We want to find t when the stock doubles, i.e., when $P(t) = 2P_0$.

Using the equation: $P(t) = P_0 e^{rt}$

$$2P_0 = P_0 e^{rt}$$

Dividing both sides by P_0, we have

$$2 = e^{rt}$$

Taking the natural logarithm on both sides:

$$\ln 2 = \ln e^{rt} \Rightarrow \ln 2 = rt \ln e \quad \because \ln a^b = b \ln a$$

$$\ln 2 = rt \quad \because \ln e = 1$$

$$\Rightarrow t = \frac{\ln 2}{r} = \frac{0.6931}{0.05} = 13.86 \approx 14$$

So, the stock will double in value in approximately 14 years.

Example 13: The concentration of a pollutant in a lake, in parts per million (ppm), decays over time according to the function $C(t) = \frac{100}{\sqrt{t+1}}$, where t is the time in days since the pollutant was introduced.

(i) What is the concentration of the pollutant after 4 days?

(ii) After how many days will the concentration drop below 10 ppm?

Solution:

(i) The pollutant concentration function is $C(t) = \frac{100}{\sqrt{t+1}}$, where t is the time in days.

Concentration after 4 days:

$$C(4) = \frac{100}{\sqrt{4+1}}$$

$$= \frac{100}{\sqrt{5}} \approx 44.72 \text{ppm}$$

The concentration after 4 days is about 44.72 ppm.

(ii) When will the concentration drop below 10 ppm?

Set $C(t) = 10$ and $t = ?$

$$\Rightarrow \frac{100}{\sqrt{t+1}} = 10$$

$$\sqrt{t+1} = \frac{100}{10} \Rightarrow \sqrt{t+1} = 10$$

Taking square on both sides

$$t + 1 = 100 \Rightarrow t = 99$$

After 99 days, the concentration will drop below 10 ppm.

Exercise 2.2

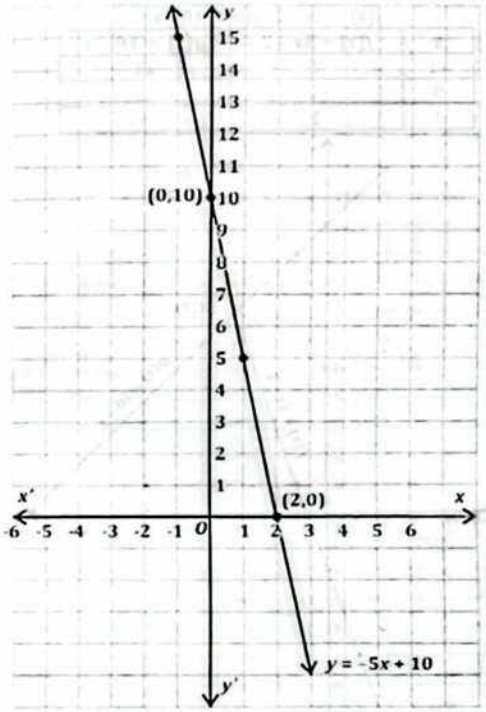
1. Find the point of intersection of the coordinate axes and the following linear functions graphically:

(i) $y = -5x + 10$

Solution:

Table values and the graph of $y = -5x + 10$ is given below:

x	$y = -5x + 10$
-1	15
0	10
1	5



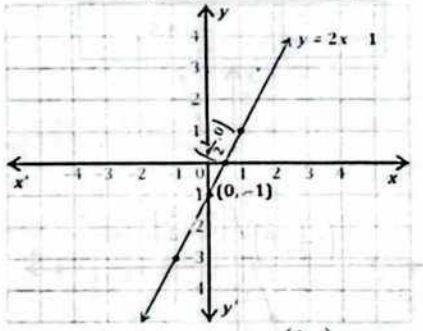
Hence, from the above graph, the points (2, 0) and (0, 10) are the points of intersections of $y = -5x + 10$ and coordinate axes.

(ii) $y = 2x - 1$

Solution:

Table values and the graph of $y = 2x - 1$ is given below:

x	$y = 2x - 1$
-1	-3
0	-1
1	1



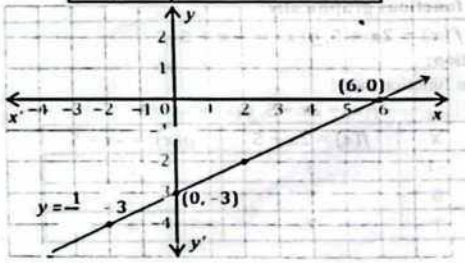
Hence, from the above graph, the points $\left(\frac{1}{2}, 0\right)$ and (0, −1) are the points of intersections of $y = 2x - 1$ and coordinate axes.

(iii) $y = \frac{1}{2}x - 3$

Solution:

Table values and the graph of $y = \frac{1}{2}x - 3$ is given below:

x	$y = \frac{1}{2}x - 1$
-2	-4
0	-3
2	-2



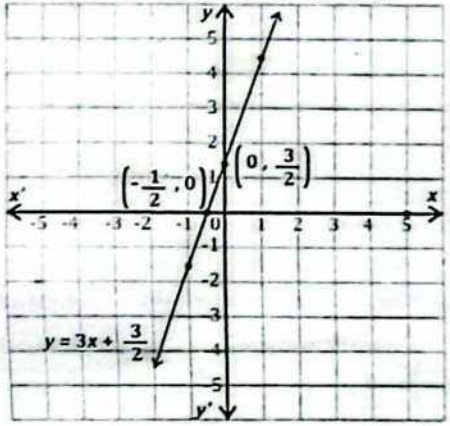
Hence, from the above graph, the points (6, 0) and (0, −3) are the points of intersections of $y=\frac{1}{2}x-3$ and coordinate axes.

(iv) $y=3x+\frac{3}{2}$

Solution:

Table values and the graph of $y=3x+\frac{3}{2}$ is given below:

x	$y=3x+\frac{3}{2}$
−1	$\frac{-3}{2}=-1.5$
0	$\frac{3}{2}=1.5$
1	$\frac{9}{2}=4.5$



Hence, from the above graph, the points $\left(-\frac{1}{2},0\right)$ and $\left(0,\frac{3}{2}\right)$ are the points of intersections of $y=3x+\frac{3}{2}$ and coordinate axes.

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2. Find the point(s) of intersection of the following functions graphically:

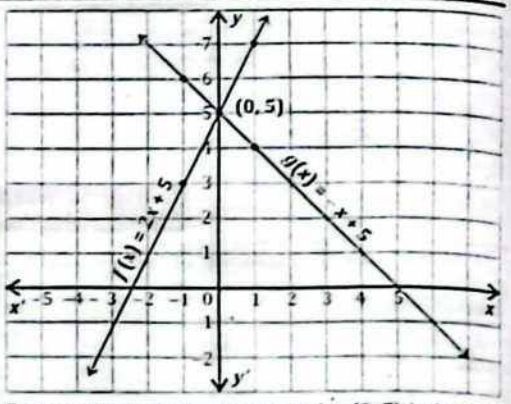
(i) $f(x)=2x+5, g(x)=-x+5$

Solution:

Given functions are

$f(x)=2x+5, g(x)=-x+5$

x	$f(x)=2x+5$	$g(x)=-x+5$
−1	3	6
0	5	5
1	7	4



From the above graph, we can see that (0, 5) is the point of intersection of both the functions.

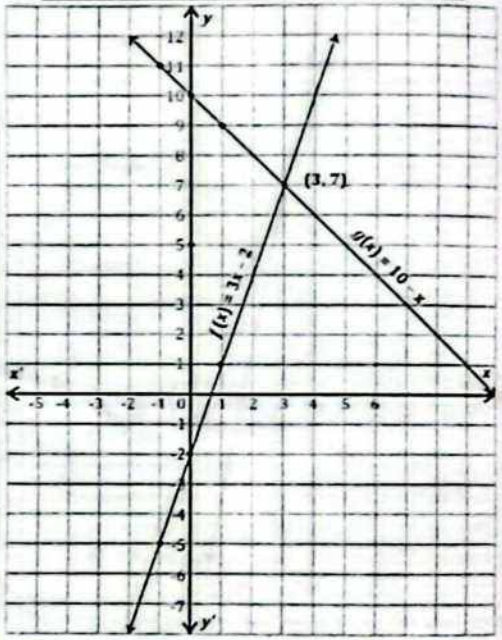
(ii) $f(x)=3x-2, g(x)=10-x$

Solution:

Given functions are

$f(x)=3x-2, g(x)=10-x$

x	$f(x)=3x-2$	$g(x)=10-x$
−1	−5	11
0	−2	10
1	1	9



From the above graph, we can see that (3, 7) is the point of intersection of both the functions.

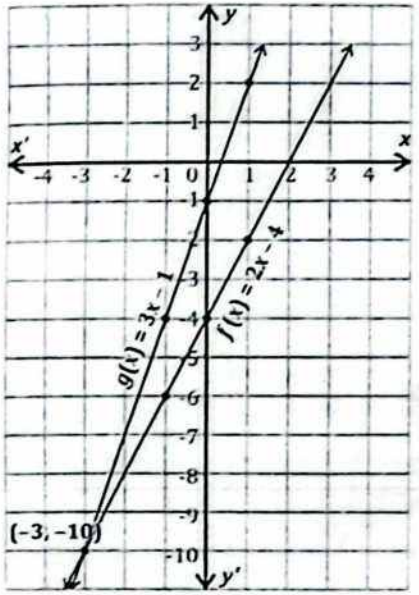
(iii) $f(x)=2x-4, g(x)=3x-1$

Solution:

Given functions are

$f(x)=2x-4, g(x)=3x-1$

x	$f(x)=2x-4$	$g(x)=3x-1$
−1	−6	−4
0	−4	−1
1	−2	2



From the above graph, we can see that (−3, −10) is the point of intersection of both the functions.

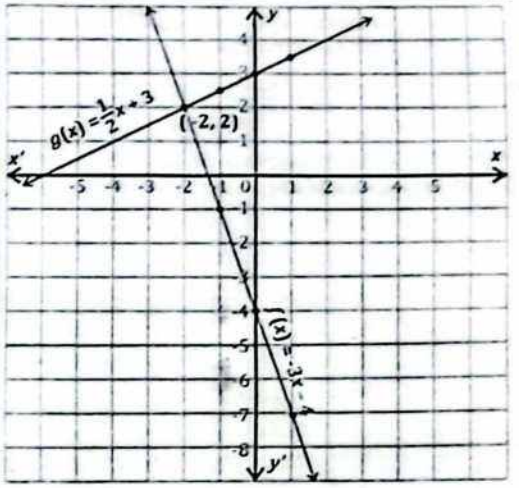
(iv) $f(x)=-3x-4, g(x)=\frac{1}{2}x+3$

Solution:

Given functions are

$f(x)=-3x-4, g(x)=\frac{1}{2}x+3$

x	$f(x)=-3x-4$	$g(x)=\frac{1}{2}x+3$
−1	−1	$\frac{5}{2}=2.5$
0	−4	3
1	−7	$\frac{7}{2}=3.5$



From the above graph, we can see that (−2, 2) is the point of intersection of both the functions.

(v) $f(x)=x-1, g(x)=x^2-4x+3$

Solution:

Given functions are

$f(x)=x-1, g(x)=x^2-4x+3$

Let $y=x-1$

Put	$y=0$	Put	$x=0$
⇒	$0=x-1$	⇒	$y=0-1$
	$x=1$		$y=-1$
	(1, 0)		(0, −1)

⇒ (1, 0) and (0, −1) are the x-intercept and y-intercept respectively of $y=x-1$

Let $y=x^2-4x+3$ ...(1)

Put $x=0$ in (1), so $y=3$

⇒ (0, 3) is the y-intercept.

Put $y=0$ in (1), we have

$1x^2-4x+3=0$

By using quadratic formula

$$x=\frac{4\pm\sqrt{(-4)^2-4(1)(3)}}{2(1)}$$

$$=\frac{4\pm\sqrt{16-12}}{2}$$

$$x=\frac{4\pm\sqrt{4}}{2}=\frac{4\pm2}{2}$$

$$x=\frac{4+2}{2},\frac{4-2}{2}$$

$x=3, 1$

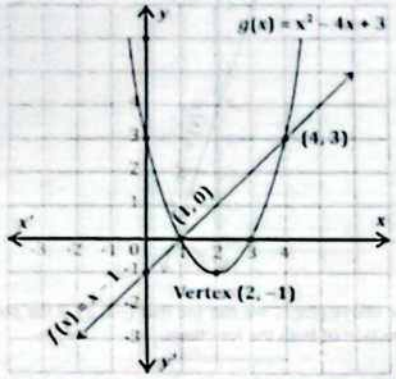
⇒ (3, 0), (1, 0) are the x-intercepts.

Now, we find vertex (h, k) of the parabola

$$h = -\frac{b}{2a} = -\frac{(-4)}{2(1)} = 2$$

$$k = g(h) = g(2)$$
$$= (2)^2 - 4(2) + 3 = 4 - 8 + 3 = -1$$

⇒ Vertex $= (h, k) = (2, -1)$



Hence (1, 0) and (4, 3) are the solutions (points of intersection) of the given functions.

(vi) $f(x) = 3x + 4, g(x) = x^2 + 2x - 8$

Solution:

Given functions are

$$f(x) = 3x + 4, g(x) = x^2 + 2x - 8$$

Let $y = 3x + 4$

Put $y = 0$	Put $x = 0$
$\Rightarrow 0 = 3x + 4$	$\Rightarrow y = 3(0) + 4$
$x = \frac{-4}{3}$	$y = 4$
$\left(\frac{-4}{3}, 0\right)$	$(0, 4)$

⇒ $\left(\frac{-4}{3}, 0\right)$ and $(0, 4)$ are the x-intercept and y-intercept respectively of $y = 3x + 4$

Let $y = x^2 + 2x - 8$ ...(1)

Put $x = 0$ in (1), so $y = -8$

⇒ $(0, -8)$ is the y-intercept.

Put $y = 0$ in (1), we have

$$x^2 + 2x - 8 = 0$$
$$x^2 + 4x - 2x - 8 = 0$$
$$x(x + 4) - 2(x + 4) = 0$$
$$(x + 4)(x - 2) = 0$$

⇒ $x + 4 = 0, x - 2 = 0$

$x = -4, x = 2$

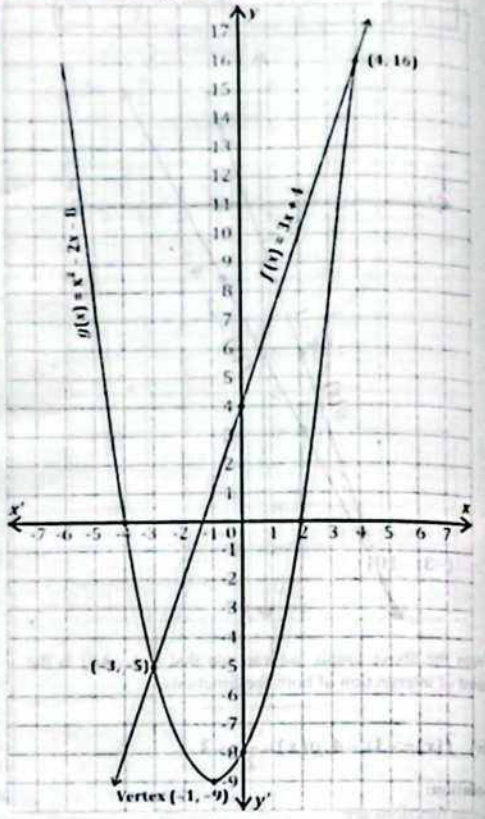
⇒ $(-4, 0), (2, 0)$ are the x-intercepts

Now, we find The vertex (h, k) of the parabola

$$h = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$$

$$k = g(h) = g(-1)$$
$$= (-1)^2 + 2(-1) - 8$$
$$= 1 - 2 - 8 = -9$$

⇒ Vertex $= (h, k) = (-1, -9)$



(vii) $f(x) = -2x - 1, g(x) = x^2 - 4x$

Solution:

Given functions are

$$f(x) = -2x - 1, g(x) = x^2 - 4x$$

Let $y = -2x - 1$

Put $y = 0$	Put $x = 0$
$\Rightarrow 0 = -2x - 1$	$\Rightarrow y = -2(0) - 1$
$x = \frac{1}{2}$	$y = -1$
$\left(-\frac{1}{2}, 0\right)$	$(0, -1)$

⇒ $\left(-\frac{1}{2}, 0\right)$ and $(0, -1)$ are the x-intercept and y-intercept respectively of $y = -2x - 1$

Let $y = x^2 - 4x$ ...(1)

Put $x = 0$ in (1), so $y = 0$

⇒ $(0, 0)$ is the y-intercept

Put $y = 0$ in (1), we have

$$x^2 - 4x = 0$$
$$x(x - 4) = 0$$

⇒ $x = 0, x - 4 = 0 \Rightarrow x = 4$

⇒ $(0, 0), (4, 0)$ are the x-intercepts.

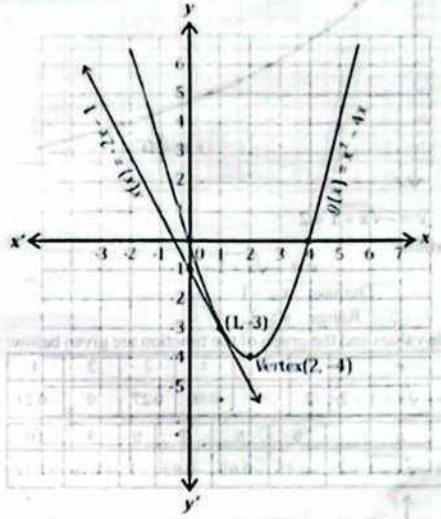
Now, we find the vertex (h, k) of the parabola

$$h = -\frac{b}{2a} = \frac{-(-4)}{2(1)} = 2$$

$$k = g(h) = g(2)$$
$$= (2)^2 - 4(2) = 4 - 8 = -4$$

⇒ Vertex $= (h, k) = (2, -4)$

Hence $(1, -3)$ is the solution (point of intersection) of the given functions.



(viii) $f(x) = -x^2 - 3x + 2, g(x) = x + 6$

Solution:

Given functions are

$$f(x) = -x^2 - 3x + 2, g(x) = x + 6 \quad ...(1)$$

Let $y = -x^2 - 3x + 2$

Put $x = 0$ in (1), so $y = 2$

⇒ $(0, 2)$ is the y-intercept

Put $y = 0$ in (1), we have

$$-x^2 - 3x + 2 = 0$$

Multiply by -1

$$x^2 + 3x - 2 = 0$$

By using quadratic formula

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-2)}}{2(1)} = \frac{-3 \pm \sqrt{9 + 8}}{2}$$

$$x = \frac{-3 \pm \sqrt{17}}{2}$$

⇒ $x = \frac{-3 + \sqrt{17}}{2}$, $x = \frac{-3 - \sqrt{17}}{2}$

$x = 0.56$, $x = -3.56$

⇒ $(0.56, 0), (-3.56, 0)$ are the x-intercepts. Now, we find the vertex (h, k) of the parabola.

Now, we find the vertex

$$h = -\frac{b}{2a} = \frac{-(-3)}{2(-1)} = -\frac{3}{2}$$

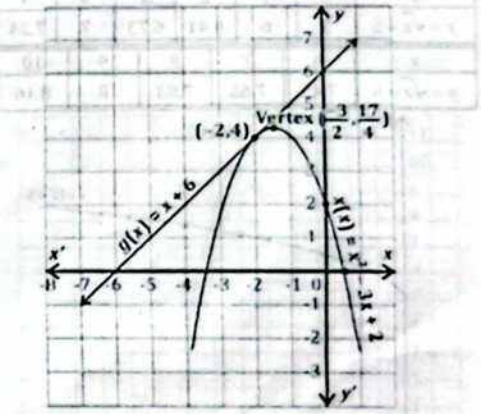
$$k = g(h) = g\left(-\frac{3}{2}\right)$$
$$= -\left(\frac{-3}{2}\right)^2 - 3\left(-\frac{3}{2}\right) + 2$$
$$= \frac{-9}{4} + \frac{9}{2} + 2 = \frac{-9 + 18 + 8}{4} = \frac{17}{4}$$

⇒ Vertex $= (h, k) = \left(-\frac{3}{2}, \frac{17}{4}\right)$

Let $y = x + 6$

Put $y = 0$	Put $x = 0$
$\Rightarrow 0 = x + 6$	$\Rightarrow y = 0 + 6$
$x = -6$	$y = 6$
$(-6, 0)$	$(0, 6)$

⇒ $(-6, 0)$ and $(0, 6)$ are the x-intercept and y-intercept respectively of $y = x + 6$



Hence $(-2, 4)$ is the solution (point of intersection) of the given functions.

3. Graph the following functions:

(i) $y = \sqrt{3x}$

Solution:

$y = \sqrt{3x}$

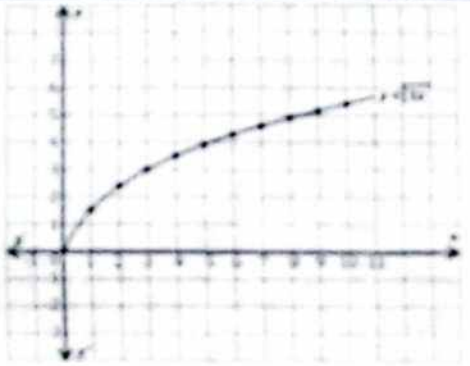
Domain $x \geq 0$

Range $y \geq 0$

Table values and the graph of the function are given below:

x	0	1	2	3	4	5
$y = \sqrt{3x}$	0	1.73	2.45	3	3.46	3.87

x	6	7	8	9	10
$y = \sqrt{3x}$	4.24	4.58	4.90	5.20	5.47



(ii) $y = \sqrt{x} + 5$

Solution:

$y = \sqrt{x} + 5$

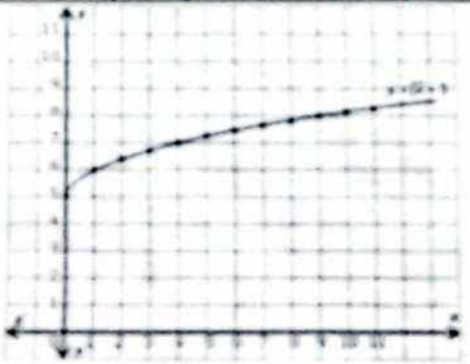
Domain $x \geq 0$

Range $y \geq 5$

Table values and the graph of the function are given below:

x	0	1	2	3	4	5
$y = \sqrt{x} + 5$	5	6	6.41	6.73	7	7.24

x	6	7	8	9	10
$y = \sqrt{x} + 5$	7.45	7.65	7.83	8	8.16



(iii) $y = \frac{1}{2}\sqrt{x}$

Solution:

$y = \frac{1}{2}\sqrt{x}$

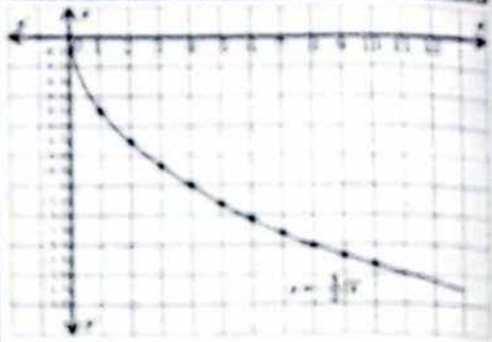
Domain $x \geq 0$

Range $y \leq 0$

Table values and the graph of the function are given below:

x	0	1	2	3	4	5
$y = \frac{-1}{2}\sqrt{x}$	0	-0.50	-0.71	-0.87	-1	-1.12

x	6	7	8	9	10
$y = \frac{-1}{2}\sqrt{x}$	-1.22	-1.32	-1.41	-1.5	-1.58



(iv) $y = -\sqrt{x+1} + 2$

Solution:

$y = -\sqrt{x+1} + 2$

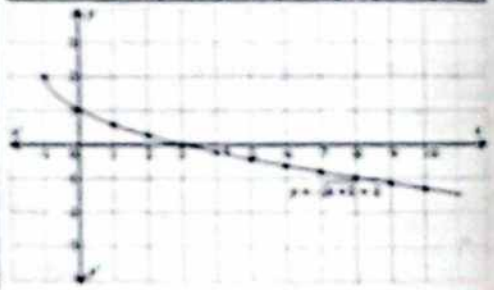
Domain $x \geq -1$

Range $y \leq 2$

Table values and the graph of the function are given below:

x	-1	0	1	2	3	4
$y = -\sqrt{x+1} + 2$	2	1	0.59	0.27	0	-0.24

x	5	6	7	8	9	10
$y = -\sqrt{x+1} + 2$	-0.45	-0.65	-0.83	-1	-1.16	-1.32



(v) $y = \sqrt[3]{2x+1}$

Solution:

$y = \sqrt[3]{2x+1}$

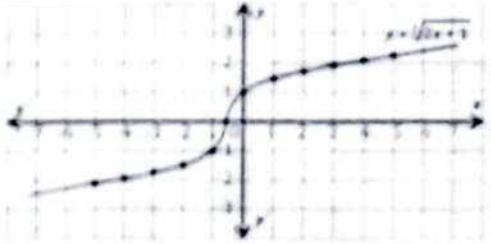
Domain $R = -\infty < x < \infty$

Range $R = -\infty < y < \infty$

Table values and the graph of the function are given below:

x	-5	-4	-3	-2	-1	0
$y = \sqrt[3]{2x+1}$	-2.08	-1.91	-1.71	-1.44	-1	1

x	1	2	3	4	5
$y = \sqrt[3]{2x+1}$	1.44	1.71	1.91	2.08	2.22



(vi) $y = 2\sqrt[3]{x} - 3$

Solution:

$y = 2\sqrt[3]{x} - 3$

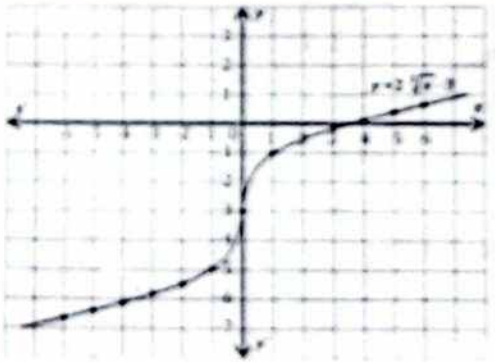
Domain $R = -\infty < x < \infty$

Range $R = -\infty < y < \infty$

Table values and the graph of the function are given below:

x	-5	-4	-3	-2	-1	0
$y = 2\sqrt[3]{x} - 3$	-6.42	-6.18	-5.88	-5.52	-5	-3

x	1	2	3	4	5
$y = 2\sqrt[3]{x} - 3$	-1	-0.48	-0.12	0.18	0.42



(vii) $y = \sqrt[3]{x^2 + x - 2}$

Solution:

$y = \sqrt[3]{x^2 + x - 2}$

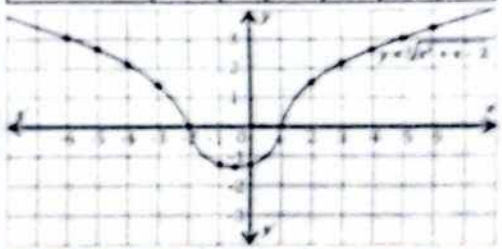
Domain $R = -\infty < x < \infty$

Range $R = -\infty < y < \infty$

Table values and the graph of the function are given below:

x	-5	-4	-3	-2	-1
$y = \sqrt[3]{x^2 + x - 2}$	2.62	2.15	1.59	0	-1.26

x	0	1	2	3	4	5
$y = \sqrt[3]{x^2 + x - 2}$	-1.26	0	1.59	2.15	2.62	3.04



4. A building's height over time is modeled by $H(t) = 100 + 20t$ which is in metres and t is the time in months. The height of a growing tree nearby is given by $T(t) = 50 + 10t + t^2$.

(i) At what time will the building and tree have the same height?

(ii) What will that height be?

Sketch the graphs of both functions and determine the time when the tree will overtake the height of the building.

Solution:

Building's height $H(t) = 100 + 20t$

Tree's height $T(t) = 50 + 10t + t^2$

where t is time in months and height is in meters.

(i) If the building and tree have the same height, Then

$H(t) = T(t)$

$\Rightarrow 100 + 20t = 50 + 10t + t^2$

$0 = 50 + 10t + t^2 - 100 - 20t$

$t^2 - 10t - 50 = 0$

Here $a = 1, b = -10, c = -50$

By using the quadratic formula

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{10 \pm \sqrt{(-10)^2 - 4(1)(-50)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{100 + 200}}{2}$$

$$= \frac{10 \pm \sqrt{100}}{2} = \frac{10 \pm 10\sqrt{3}}{2}$$

$t = 5 \pm 5\sqrt{3}$

We take $t = 5 + 5\sqrt{3}$ ($\because$ time is always +ve)

$t = 13.66$ months

$\Rightarrow$ $t = 14$ months (approximately)

(ii) Put $t = 13.66$ in $H(t)$, we have

$H(13.66) = 100 + 20(13.66)$

$= 100 + 273.2 = 373.2\ m$

Let $y = 100 + 20t$ (Linear function)

Put	$y = 0$	Put	$t = 0$
$\Rightarrow$	$0 = 100 + 20t$	$\Rightarrow$	$y = 100 + 20(0)$
	$t = -5$		$y = 100$
	$(-5, 0)$		$(0, 100)$

$\Rightarrow$ $(-5, 0)$ and $(0, 100)$ are the x-intercept and y-intercept respectively of $y = 100 + 20t$

Let $y = 50 + 10t + t^2$ ...(1) (Parabola)

Put $t = 0$ in eq. (1), so $y = 50$

$\Rightarrow$ $(0, 50)$ is the y-intercept.

Put $y = 0$ in (1), we have

$t^2 + 10t + 50 = 0$

Here $a = 1, b = 10, c = 50$

Since, $disc = b^2 - 4ac = (10)^2 - 4(1)(50) = -100 < 0$

Therefore, roots will be imaginary.

Thus, the graph does not intersect the x-axis.

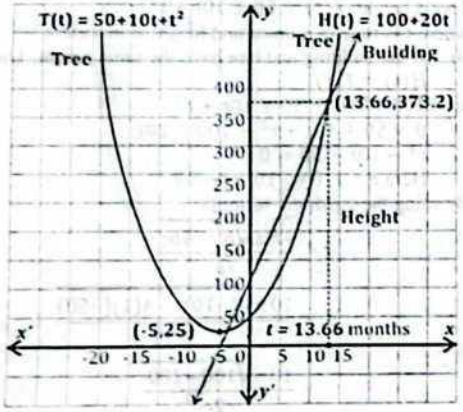
Now, we find vertex (h, k) of the parabola.

$h = \frac{-b}{2a} = \frac{-10}{2(1)} = -5$

$k = T(-5) = 50 + 10(-5) + (-5)^2$

$= 50 - 50 + 25 = 25$

$\Rightarrow$ Vertex $= (h, k) = (-5, 25)$



The tree overtakes the building after the time, when their heights are equal, so, the tree overtakes the building at $t = 13.66$.

5. A radioactive substance has a half-life of 2 years. If the initial quantity is 200 grams and the exponential decay function is $Q(t) = Q\left(\frac{1}{2}\right)^{\frac{t}{h}}$, then find the remaining quantity after 6 years graphically?

Solution:

Exponential decay function: $Q(t) = Q_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$

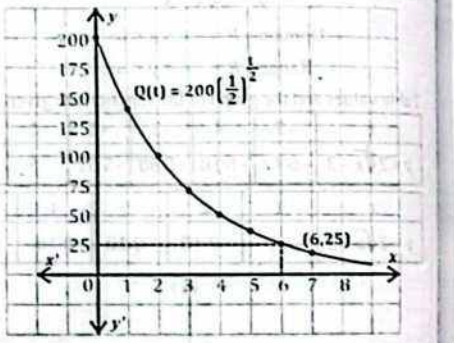
Half life: $h = 2$ years

Initial quantity: $Q_0 = 200$ grams.

$\Rightarrow$ $Q(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{2}}$

Table values and the graph of decay function are given below:

x	0	1	2	3	4	5	6	7
$Q(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{2}}$	200	141.42	100	70.71	50	35.35	25	17.68



From above graph, it is clear that the remaining quantity of the radioactive substance after 6 years is 25 grams.

Multiple Choice Questions (MCQs)

Exercise 2.1

1. A rule that assigns to each elements x in X a unique element y in Y is called a - - - - -
   (A) domain (B) range (C) function (D) value
2. If at least one vertical line meets the curve at more than one points then curve is - - -
   (A) a function (B) not a function (C) one-to-one function (D) onto function
3. If $f: x \longrightarrow x^2 - 2x + 1$ is a function, then $f(0)$ equals:
   (A) -1 (B) 0 (C) 1 (D) 2
4. Domain of $f(x) = \sqrt{x+1}$ is - - - - -
   (A) $(0, \infty)$ (B) $(1, \infty)$ (C) $[-1, \infty)$ (D) $(-\infty, \infty)$
5. Range of $f(x) = x^2 + 1$ is - - - - -
   (A) R (B) R - {1} (C) R - {-1} (D) $[1, \infty)$
6. The range of $f(x) = \sqrt{x^2 - 9}$ is:
   (A) $(-\infty, 0]$ (B) $[0, +\infty)$ (C) $(0, +\infty)$ (D) $(-\infty, \infty)$
7. A function $f: X \to Y$ is one-to-one if different inputs produce - - - - - outputs.
   (A) same (B) different (C) equal (D) unique
8. A function $f: X \to Y$ is called bijective if it is - - - - -
   (A) one-to-one (B) both one-to-one and into
   (C) onto (D) both one-to-one and onto
9. If $f(x) = 2x + 3$, where the domain and co-domain are both real numbers, then f is - - - - -
   (A) one-to-one (B) into (C) onto (D) both (A) & (C)

Exorcise 2.2

10. The point of intersection is a point where - - - - - graphs meet on the coordinate plane
    (A) two (B) three (C) four (D) two or more
11. The point of intersection of a linear function $y = 2x + 6$ and x-axis is:
    (A) (0, 6) (B) (0, -3) (C) (-3, 0) (D) (6, 0)
12. The point of intersection of $y = 3x + 2$ and $y = -x + 6$ is - - - - -
    (A) (5, 1) (B) (1, 6) (C) (2, 5) (D) (1, 5)
13. A line and a parabola can intersect at - - - - - point/s.
    (A) one (B) two (C) three (D) no
14. The x-intercept/s of parabola $y = x^2 - 6x + 3$ is/are - - - - -
    (A) $3 - \sqrt{6}, 3 + \sqrt{6}$ (B) $-3 - \sqrt{6}, 3 + \sqrt{6}$ (C) $3 - \sqrt{6}, -3 + \sqrt{6}$ (D) $3 - \sqrt{6}, -3 - \sqrt{6}$
15. The vertex of the parabola $y = x^2 - 6x + 3$ is - - - - -
    (A) (3, 6) (B) (3, -6) (C) (-3, -6) (D) (-3, 6)

ANSWER KEY

1.	C	2.	B	3.	C	4.	C	5.	D	6.	B	7.	B	8.	D	9.	D	10.	D
11.	C	12	D	13.	B	14.	A	15.	B										