Chapter 9

FUNDAMENTALS OF TRIGONOMETRY

TRIGONOMETRY

Trigonometry means measurement of triangle.

ANGLE

Two rays with a common starting point form an angle. Angle is measured positive if rotation is anticlockwise and negative if rotation is clockwise.

MEASUREMENT OF ANGLES

1. Sexagesimal System

In this system an angle is measured in degrees, minutes and seconds.

2. Radian or Circular Measure

Radian **(Gujranwala Board 2007)**

Radian is the measure of the angle subtended at the centre of the circle by an arc whose length is equal to radius of the circle.

Degree

If the circumference of circle is divided into 360 arcs of equal lengths, then central angle subtended by each arc is of one degree measure and is denoted by 1^{o}.

NOTE

$$\pi \text{ radians} = 180^{o}$$

$$1 \text{ radian} = \frac{180^{o}}{\pi}$$

$$\Rightarrow \quad 1 \text{ radian} = \frac{180^{o}}{3.1416} = 57.296^{o}$$

$$1^{o} = \frac{\pi}{180} \text{ radians}$$

$= \frac{3.1416}{180}$

$1^{\circ} = 0.0175$ radian

Conterminal Angles (General Angles)

The angles with the same initial and terminal sides are called co–terminal angles i.e.,

θ , $\theta \pm 2\pi$, $\theta \pm 4\pi$..... are co–terminal angles.

$\theta + 2k\pi$, $k \in z$ or $\theta + k\,(360^{\circ})$ $k \in Z$ is called general angle where $0 \leq \theta \leq 2\pi$

Angle in standard Position

An angle is said to be in standard position if its vertex lie at the origin and its initial side along the positive x–axis.

Quadrantal Angle

If the terminal side of an angle falls on x–axis or y–axis it is called quadrantal angle.

i.e., θ° , $\pm 90^{\circ}$, $\pm 180^{\circ}$, $\pm 270^{\circ}$, $\pm 360^{\circ}$

Allied Angles

The angles associated with basic angles of measure "θ" to a right angle or its multiple are called Allied Angles.

So the angles of measure $90^{\circ} \pm \theta$, $180^{\circ} \pm \theta$, $270^{\circ} \pm \theta$, $360^{\circ} \pm \theta$ are known as Allied Angles.

Basic Angles OR Reference Angles:

Def. A +ve acute angle with x-axis is called basic angle or reference angle.

EXERCISE 9.1

Q.1 Express the following sexagesimal measures of angle in radians

(i) 0° (ii) 45° (iii) 60° (iv) 75° (v) 90°

(vi) 105° (vii) 120° (viii) 135° (ix) 150°

(x) $10^{\circ}\,15'$ (xi) $35^{\circ}\,20'$ (xii) $75^{\circ}\,6'\,30'$

(xiii) $120^{\circ}\,40''$ (xiv) $154^{\circ}\,20''$ (xv) 0°

(xvi) $3''$

Solution:

(i) $30^{\circ} = 30^{\circ} \times \frac{\pi}{180}$ radian $= \frac{\pi}{6}$ radian

(ii) $45^{\circ} = 45^{\circ} \cdot \frac{\pi}{180}$ radian $= \frac{\pi}{4}$ radian

(iii) $60^{\circ} = 60 \times \frac{\pi}{180}$ radian $= \frac{\pi}{3}$ radian

(iv) $75^{\circ} = 75^{\circ} \times \frac{\pi}{180}$ radian $= 5\frac{\pi}{12}$ radian

(v) $90^{\circ} = 90^{\circ} \times \frac{\pi}{180}$ radian $= \frac{\pi}{2}$ radian

(vi) $105^{\circ} = 105^{\circ} \times \frac{\pi}{180}$ radian $= \frac{7\pi}{12}$ radian

(vii) $120^{\circ} = 120^{\circ} \times \frac{\pi}{180}$ radian $= \frac{2\pi}{3}$ radian

(viii) $135^{\circ} = 135^{\circ} \times \frac{\pi}{180}$ radian $= \frac{3\pi}{4}$ radian

(ix) $150^{\circ} = 150^{\circ} \times \frac{\pi}{180}$ radian $= \frac{5\pi}{6}$ radian

(x) $10^{\circ}\,15' = 10^{\circ} + \frac{15^{\circ}}{60}$

$= 10^{\circ} + \frac{1}{4}$

$= \frac{40^{\circ} + 1^{\circ}}{4} = \frac{41^{\circ}}{4}$

$= \frac{41}{4} \times \frac{\pi}{180}$ radian

$= \frac{41\,\pi}{720}$ radian

(xi) $35^{\circ}\,20' = 35^{\circ} + \frac{20^{\circ}}{60} = 35^{\circ} + \frac{1^{\circ}}{3}$

$= \frac{105^{\circ} + 1^{\circ}}{3} = \frac{106^{\circ}}{3}$

$= \frac{106}{3} \times \frac{\pi}{180}$ radian $= \frac{53\pi}{270}$ radian

(xii) $75^{\circ}\,6'\,30'' = 75^{\circ} + \frac{6^{\circ}}{60} + \frac{30^{\circ}}{3600}$

$= 75^{\circ} + \frac{1^{\circ}}{10} + \frac{1^{\circ}}{120}$

$= \left(75 + \frac{1}{10} + \frac{1}{120}\right)^{\circ}$

$= \left(\frac{9000 + 12 + 1}{120}\right)^{\circ}$

$= \left(\frac{9013}{120}\right)^{\circ}$

$= \frac{9013}{120} \times \frac{\pi}{180}$ radian $= \frac{9013\,\pi}{21600}$ radian

(xiii) $120' 40'' = \frac{120^\circ}{60} + \frac{40^\circ}{3600}$

$= \left(\frac{120}{60} + \frac{40}{3600} \right)^\circ$

$= \left(2 + \frac{1}{90} \right)^\circ$

$= \left(\frac{180 + 1}{90} \right)^\circ$

$= \frac{181^\circ}{90}$

$= \frac{181}{90} \times \frac{\pi}{180}$ radian $= \frac{181}{16200} \pi$ radian

(xiv) $154^\circ 20'' = 154^\circ + \frac{20^\circ}{3600}$

$= 154^\circ + \frac{20^\circ}{3600}$

$= \left(154 + \frac{1}{180} \right)^\circ$

$= \left(\frac{27720 + 1}{180} \right)^\circ$

$= \frac{27721^\circ}{180}$

$= \frac{27721}{180} \times \frac{\pi}{180}$ radian

$= \frac{27721}{32400}$ radian

(xv) $0^\circ = 0 \times \frac{\pi}{180}$ radian

$= 0$ radian

(xvi) $3'' = \frac{3^\circ}{3600}$

$= \frac{1^\circ}{1200}$

$= \frac{1}{1200} \times \frac{\pi}{180}$ radian

$= \frac{\pi}{216000}$ radian

Q.2 Convert the following radian measures of angles into the measures of sexagesimal system.

(i) $\frac{\pi}{8}$ **(ii)** $\frac{\pi}{6}$ **(iii)** $\frac{\pi}{4}$ **(iv)** $\frac{\pi}{3}$ **(v)** $\frac{\pi}{2}$ **(vi)** $\frac{2\pi}{3}$ **(vii)** $\frac{3\pi}{4}$ **(viii)** $\frac{5\pi}{6}$

(ix) $\frac{7\pi}{12}$ **(x)** $\frac{9\pi}{5}$ **(xi)** $\frac{11\pi}{27}$ **(xii)** $\frac{13\pi}{16}$ **(xiii)** $\frac{17\pi}{24}$ **(xiv)** $\frac{25\pi}{36}$ **(xv)** $\frac{19\pi}{32}$

Solution:

(i) $\frac{\pi}{8} = \frac{\pi}{8} \times \frac{180}{\pi}$ degree

$= 22.5$ degrees

$= 22^{\circ}\, 30'$

(ii) $\frac{\pi}{6} = \frac{\pi}{6} \cdot \frac{180}{\pi}$ degrees

$= 30^{\circ}$

(iii) $\frac{\pi}{4} = \frac{\pi}{4} \times \frac{180}{\pi}$ degrees

$= 45^{\circ}$

(iv) $\frac{\pi}{3} = \frac{\pi}{3} \times \frac{180}{\pi}$ degrees

$= 60^{\circ}$

(v) $\frac{\pi}{2} = \frac{\pi}{2} \times \frac{180}{\pi} = 90^{\circ}$

(vi) $\frac{2\pi}{3} = \frac{2\pi}{3} \times \frac{180}{\pi}$ degrees

$= 120^{\circ}$

(vii) $\frac{3\pi}{4} = \frac{3\pi}{4} \times \frac{180}{\pi}$ degrees

$= 135^{\circ}$

(viii) $\frac{5\pi}{6} = \frac{5\pi}{6} \times \frac{180}{\pi}$ degrees

$= 150^{\circ}$

(ix) $\frac{7\pi}{12} = \frac{7\pi}{12} \times \frac{180}{\pi}$ degrees

$= 105^{\circ}$

(x) $\frac{9\pi}{5} = \frac{9\pi}{5} \times \frac{180}{\pi}$ degrees

$= 324^{\circ}$

(xi) $\frac{11\pi}{27} = \frac{11\pi}{27} \times \frac{180}{\pi}$ degrees

$= 73^{\circ}\, 20'$

(xii) $\frac{13\pi}{16} = \frac{13\pi}{16} \times \frac{180}{\pi}$ degrees

$= 146^{\circ}\, 15'$

(xiii) $\frac{17\pi}{24} = \frac{17\pi}{24} \times \frac{180}{\pi}$ degrees

$= 127^{\circ}\, 30'$

(xiv) $\frac{25\pi}{36} = \frac{25\pi}{36} \times \frac{180}{\pi}$ degrees

$= 125^{\circ}$

(xv) $\frac{19\pi}{32} = \frac{19\pi}{32} \times \frac{180}{\pi}$ degrees

$= 106^{\circ}\, 52'\, 30''$

Q.3 What is the circular measure of angle between the hands of a watch at 4'O clock?

Solution:

Circular measure of angle for one complete revolution $= 2\pi$

Circular measure of angle between hands of a watch at 4'O clock $= 2\pi \times \frac{4}{12}$

$= \frac{2\pi}{3}$ radians



Q.4 Find θ when

(i) l = 1.5cm r = 2.5cm **(Lahore Board 2011)**

(ii) l = 3.2m r = 2m

Solution:

(i) l = 1.5cm r = 2.5cm

we know $l = r\theta$

$\Rightarrow \quad \theta = \frac{l}{r}$

$\theta = \frac{1.5}{2.5} = 0.6$ radian.

(ii) $l = 3.2m \quad r = 2m$

we know $l = r\theta$

$\Rightarrow \quad \theta = \frac{l}{r}$

$\theta = \frac{3.2}{2} = 1.6$ radians.

Q.5 Find l when

(i) $\theta = \pi$ **radian** $r = 6cm$

(ii) $\theta = 65^o\,20'$ $r = 18mm$

Solution:

(i) $\theta = \pi$ **radian** $r = 6cm$

We know

$l = r\theta$

$= 6\pi = 6 \times 3.1416 = 18.85$ cm

(ii) $\theta = 65^o\,20'$ $r = 18mm$

$\theta = = 65^o + \frac{20^o}{60} = \left(65 + \frac{20}{60}\right)^o = \frac{196^o}{3}$

$\theta = \frac{196}{3} \times \frac{\pi}{180}$ radian $= \frac{49}{3 \times 45}(3.1415)$

$l = r\theta$

$= 18 \times \frac{49}{3 \times 45}(3.1415) = 20.53$ mm

Q.6 Find r when

(i) $l = 5cm \quad \theta = \frac{1}{2}$ **radian** **(Gujranwala Board 2007, Lahore Board 2010)**

(ii) $l = 56cm \quad \theta = 45^o$ **(Lahore Board 2004 , 2007)**

Solution:

(i) $l = 5cm \quad \theta = \frac{1}{2}$ **radian**

we know that $l = r\theta$

$r = \frac{l}{\theta} = \frac{5}{\frac{1}{2}} = 10$cm

(ii) $l = 56\text{cm} \quad \theta = 45^\circ$

First we convert 45° into radian measure

$$\theta = 45^\circ \Rightarrow 45 \times \frac{\pi}{180} \text{ radian}$$

we know that $l = r\theta$

$$\Rightarrow \quad r = \frac{l}{\theta} \Rightarrow \frac{56}{45 \times \frac{\pi}{180} \text{ radian}} = \frac{56}{0.785} = 71.33 \text{ cm}$$

Q.7 What is the length of the arc intercepted on a circle of radius 14cm by the arms of central angle of 45°? (Gujranwala Board 2006)

Solution:

$l = ? \quad r = 14\text{cm} \quad \theta = 45^\circ$

$$\theta = 45 \times \frac{\pi}{180} \text{ radian}$$

$$\theta = 0.785$$

we know that $l = r\theta$

$l = 14 \times 0.785 = 10.99$ or 11 cm approximately.

Q.8 Find the radius of the circle, in which the arms of a central angle of measure 1 radian cut off an arc of length 35cm.

Solution:

$r = ? \quad \theta = 1 \text{ radian} \quad l = 35\text{cm}$

we know that $l = r\theta$

$$\Rightarrow \quad r = \frac{l}{\theta} = \frac{35}{1} = 35\text{cm}$$

Q.9 A railway train is running on a circular track of radius 500 meters at the rate of 30 km/h. Through what angle will it turn in 10 sec.

Solution:

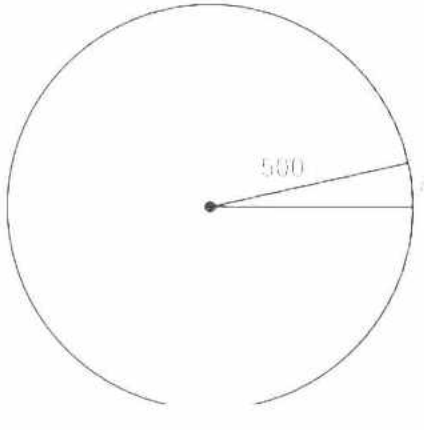
$\theta = ?, \quad r = 500 \text{ m}, \quad \text{Speed} = 30\text{km/h}$

$$\text{Speed} = \frac{30 \times 1000}{3600} = \frac{25}{3} \text{ m/sec}$$

Distance covered in 10 sec

Distance = speed × time

$$l = \frac{25}{3} \times 10 \text{ m}$$



$= \frac{250}{3}$ m

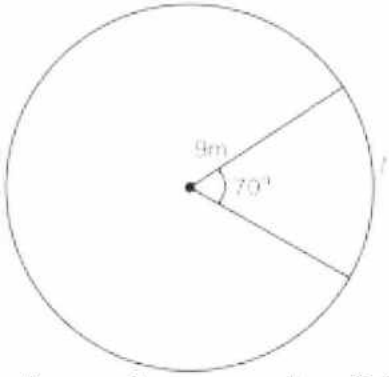
We know that $l = r\theta$

$\Rightarrow \quad \theta = \frac{l}{r}$

$= \frac{250}{3} \times \frac{1}{500} = \frac{1}{6}$ radian

Q.10 A horse is tethered to a peg by a rope of 9m in length and it can move in a circle with the peg as centre. If the horse moves along the circumference of the circle, keeping the rope tight, how far will it have gone when the rope has turned through an angle of 70°?

Solution:



$r = 9m, \quad \theta = 70^\circ = 70 \times \frac{\pi}{180}$ radians, $\quad l = ?$

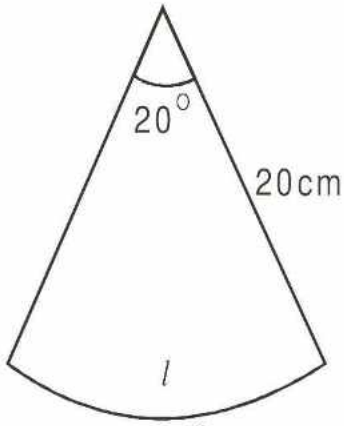
we know that

$l = r\theta$

$= 9 \times 70 \times \frac{3.1416}{180} = 10.99m$

Q.11 The pendulum of a clock is 20cm long and it swings through an angle of 20° each second. How far does the tip of the pendulum moves in 1 second?

Solution:



$r = 20\,cm, \quad \theta = 20^\circ = 20 \times \frac{\pi}{180}$ radian, $\quad l = ?$

we know that

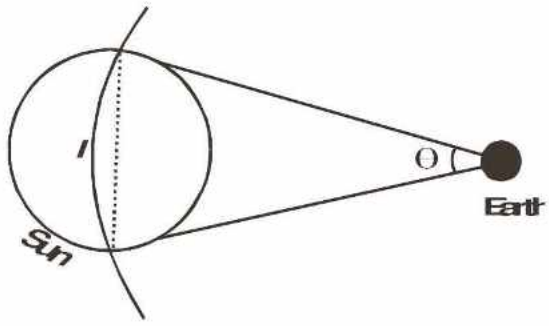
$l = r\theta$

$l = 20 \times 20 \times \frac{3.1416}{180}$

$l = 6.89$ cm

Q.12 Assuming average distance of the earth from the sun to be 148×10^6 km and angle subtended by the sun at the eye of the person on the earth measure 9.3×10^{-3} radian. Find the diameter of the sun.

Solution:



$l = ?, \quad r = 148 \times 10^6$ km, $\quad \theta = 9.3 \times 10^{-3}$ radian

we know that

$l = r\theta$

$l = 148 \times 10^6 \times 9.3 \times 10^{-3}$

$l = 1376.4 \times 10^3$ km.

$l = 1.3764 \times 10^3 \times 10^3$ km

$l = 1.3764 \times 10^6$km

Q.13 A circular wire of radius 6cm is cut straightened and then bent so as to lie along the circumference of hoop of radius 24cm. Find the measure of the angle which it subtends at the centre of hoop.

Solution:

θ = ? Length of circular wire of radius 6 cm

$l = 2\pi r = 2\pi (6)$

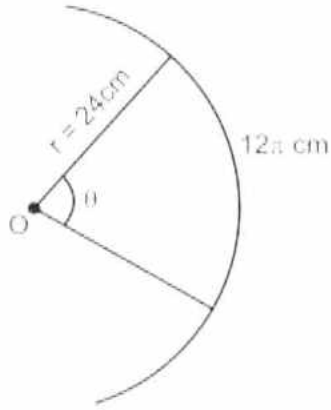
$l = 12\pi = 12\,(3.1416) = 37.7$ cm $\quad r = 24$ cm

we know that $l = r\theta$

$$\Rightarrow \quad \theta = \frac{l}{r}$$

$$\theta = \frac{37.7}{24}$$

$\theta = 1.57$ radian



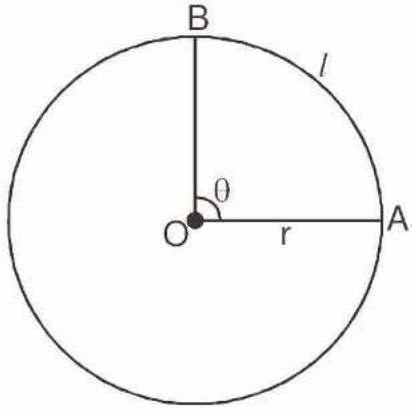
Q.14 Show that the area of a sector of a circular region of radius r is $\frac{1}{2} r^2 \theta$, where θ is the circular measure of the central angle of the sector.

Solution:

Let us consider a circle with centre at O. Let r be radius of circle. Let l be length of arc AB.

Let θ be central angle in radian. We know that

Area of sector of circular region : Area of circle = Angle of sector : Angle of circle



$$\Rightarrow \quad \frac{\text{Area of sector of circular region}}{\text{area of circle}} = \frac{\text{Angle of sector}}{\text{Angle of circle}}$$

$$\frac{\text{Area of sector of circular region}}{\pi r^2} = \frac{\theta}{2\pi}$$

$$\text{Area of sector of circular region} = \frac{\theta}{2\pi} \cdot \pi r^2$$

$$= \frac{1}{2} r^2 \theta$$

Hence proved.

Q.15 Two cities A and B lie on the equator such that their longitudes are 45^{o} E and 25^{o} W respectively. Find the distance between the two cities, taking radius of the earth as 6400 kms.

Solution:

θ = Total angle between two cities

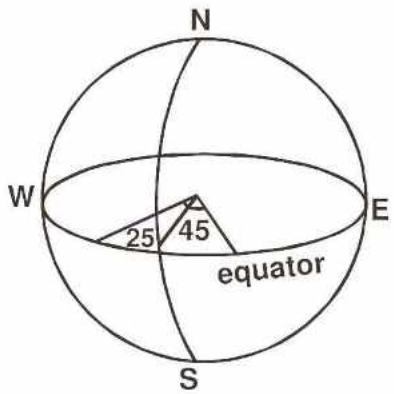
$\theta = 45^{\circ} + 25^{\circ} = 70^{\circ}$

$\theta = 70 \times \frac{\pi}{180}$ radian

r = 6400 km, l = ?

we know that $l = r\theta$

$= 6400 \times 70 \times \frac{3.1416}{180} = 7819.09$ km



Q.16 The moon subtends an angle of 0.5^{o} at the eye of an observer on earth. The distance of the moon from the earth is 3.844×10^{5} km approx what is the length of diameter of the moon?

Solution:

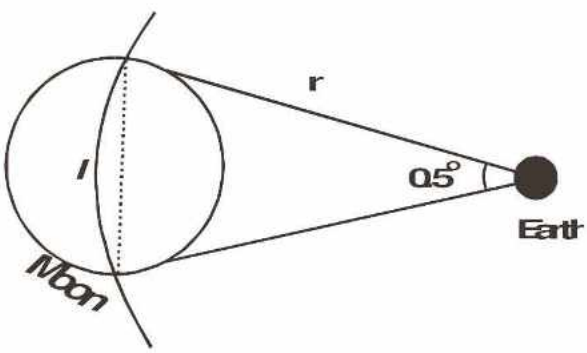
l = ? $\theta = 0.5^{\circ} = 0.5 \frac{\pi}{180}$ radian

$r = 3.844 \times 10^{5}$ km

we know that $l = r\theta$

$l = 3.844 \times 10^{5} \times 0.5 \times \frac{\pi}{180}$

$l = 3354.52$ km



Q.17 The angle subtended by the earth at an eye of spaceman, landed on the moon, is $1^{o}\,54'$. Find the radius of the earth is 6400km. Find the approximate distance between the moon and the earth.

Solution:

r = ?

θ = Angle subtended the spaceman on the moon

$\theta = 1^{o}\,54'$

$\theta = 1^{\circ} + \frac{54^{\circ}}{60} = \frac{114^{\circ}}{60} = \frac{114}{60} \times \frac{\pi}{180}$ radian

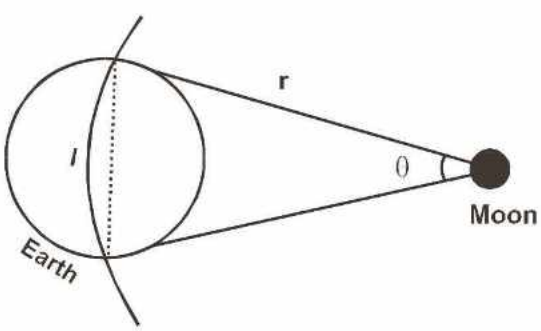
l = diameter of earth

$l = r + r = 6400 + 6400 = 12800$ km

we know that $l = r\theta$

$\Rightarrow \quad r = \frac{l}{\theta} = \frac{12800}{\frac{114}{60} \times \frac{3.1416}{180}}$

$r = \frac{12800}{0.03316} = 386007.23$ km



EXERCISE 9.2

Q.1 Find the signs of the following:

(i) sin 160 (ii) cos 190 (iii) tan 115

(iv) sec 245 (v) cot 80° (vi) cosec 297°

(i) sin 160°

Since sin 160° lies in the IInd quadrant, so its sign is +ve.

(ii) cos 190°

Since cos 190° lies in the III quadrant, so its sign is – ve.

(iii) tan 115°

Since tan 115° lies in the II quadrant, so its sign is – ve.

(iv) sec 245°

Since sec 245° lies in III quadrant, so its sign is – ve.

(v) cot 80°

Since cot 80° lies in the I quadrant, so its sign is + ve.

(vi) cosec 297°

Since cosec 297° lies in the IV quadrant, so its sign is – ve.

Q.2 Fill in the blanks:

(i) sin (–310°) = _____ sin 310°

(ii) cosec (–75°) = _____ cos 75°

(iii) tan (–182°) = _____ tan 182°

(iv) cot (–137°) = _____ cot 137°

(v) sec (–216°) = _____ sec 216°

(vi) cosec (–15) = _____ cosec 15°

Solution:

(i) $\sin(-310^{\circ}) = \frac{-\text{ve}}{\quad} \sin 310^{\circ}$

(ii) $\cos(-75^{\circ}) = \frac{+\text{ve}}{\quad} \cos 75^{\circ}$

(iii) $\tan(-182^{\circ}) = \frac{-\text{ve}}{\quad} \tan 182^{\circ}$

(iv) $\cot(-137^{\circ}) = \frac{-\text{ve}}{\quad} \cot 137^{\circ}$

(v) $\sec(-216^{\circ}) = \frac{+\text{ve}}{\quad} \sec 216^{\circ}$

(vi) $\operatorname{cosec}(-15^{\circ}) = \frac{-\text{ve}}{\quad} \operatorname{cosec} 15^{\circ}$

Q.3 In which quadrant are the terminal arms of the angle lies when

(i) $\sin\theta < 0$ **and** $\cos\theta > 0$

(ii) $\cot\theta > 0$ **and** $\operatorname{cosec} 0 > 0$

(iii) $\tan\theta < 0$ **and** $\cos\theta > 0$

(iv) $\sec\theta < 0$ **and** $\sin\theta < 0$

(v) $\cot\theta > 0$ **and** $\sin\theta < 0$

(vi) $\cos\theta < 0$ **and** $\tan\theta < 0$

Solution:

(i) $\sin\theta < 0$ **and** $\cos\theta > 0$

$\Rightarrow$ $\sin\theta$ is – ve & $\cos\theta$ is + ve so they lie in IV Quadrant.

(ii) $\cot\theta > 0$ **and** $\operatorname{cosec}\theta > 0$ **(Lahore Board 2005)**

As cot is + ve and cosec θ is + ve they lie in I Quadrant.

(iii) $\tan\theta < 0$ **and** $\cos\theta > 0$

Since $\tan\theta$ is – ve and $\cos\theta$ is + ve so they lie in IV Quadrant.

(iv) $\sec\theta < 0$ **and** $\sin\theta < 0$ **(Gujranwala Board 2007)**

As $\sec\theta$ is – ve , $\sin\theta$ is – ve so they lie in III Quadrant.

(v) $\cot\theta > 0$ **and** $\sin\theta < 0$

Since $\cot\theta$ is + ve and $\sin\theta$ is – ve so they lie in III Quadrant.

(vi) $\cos\theta < 0$ **and** $\tan\theta < 0$

Since $\cos\theta$ is – ve and $\tan\theta$ is – ve so they lie in II Quadrant.

Q.4 Find the values of the remaining trigonometric functions.

(i) $\sin\theta = \frac{12}{13}$ **and the terminal arm of the angle is in quadrant I.**

(ii) $\cos\theta = \frac{9}{41}$ **and the terminal arm of the angle is in quadrant IV.**

(Lahore Board 2007)

(iii) $\cos\theta = -\frac{\sqrt{3}}{2}$ **and the terminal arm of the angle is in quadrant III.**

(iv) $\tan\theta = -\frac{1}{3}$ **and the terminal arm of the angle is in quadrant II.**

(Gujranwala Board 2004)

(v) $\sin\theta = -\frac{1}{\sqrt{2}}$ **and the terminal arm of the angle is not in quadrant III**

(Lahore Board 2007)

Solution:

(i) $\sin\theta = \frac{12}{13}$ **and the terminal arm of the angle is in quad. I.**

$$\sin\theta = \frac{12}{13} \Rightarrow \boxed{\operatorname{cosec}\theta = \frac{13}{12}}$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{169-144}{169}} = \sqrt{\frac{25}{169}}$$

$\boxed{\cos\theta = \frac{5}{13}}$ (since terminal arm of the angle is in I quadrant. So all trigonometric functions will be +ve.)

$\boxed{\sec\theta = \frac{13}{5}}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12}{13} \div \frac{5}{13} = \frac{12}{13} \times \frac{13}{5} = \boxed{\frac{12}{5} = \tan\theta}$$

$\boxed{\cot\theta = \frac{5}{12}}$

(ii) $\cos\theta = \frac{9}{41}$ **and the terminal arm of the angle is in quadrant IV.**

(Lahore Board 2007)

$$\cos\theta = \frac{9}{41} \Rightarrow \boxed{\sec\theta = \frac{41}{9}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{9}{41}\right)^2} = \sqrt{1-\frac{81}{1681}} = \sqrt{\frac{1681-81}{1681}} = \sqrt{\frac{1600}{1681}}$$

$\sin\theta = \pm\frac{40}{41}$ (since terminal arm of the angle is in quadrant IV)

$\sin\theta = -\frac{40}{41}$ (so only $\cos\theta$ & $\sec\theta$ are +ve, other four trigonometric ratios will be –ve)

$\Rightarrow \quad \operatorname{cosec}\theta = \frac{-41}{40}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-40}{41}}{\frac{9}{41}} = \frac{-40}{9}$$

$$\cot\theta = \frac{-9}{40}$$

(iii) $\cos\theta = -\frac{\sqrt{3}}{2}$ **and the terminal arm of the angle is in quadrant III.**

$$\cos\theta = \frac{-\sqrt{3}}{2}$$

$$\Rightarrow \quad \sec\theta = \frac{-2}{\sqrt{3}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{-\sqrt{3}}{2}\right)^2} = \sqrt{1-\frac{3}{4}} = \sqrt{\frac{4-3}{4}} = \sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$

$\sin\theta = \frac{-1}{2}$ (Since terminal arm of the angle is in quad. III. So only tanθ and cotθ will be +ve, other four will be −ve)

$\text{cosec}\theta = -2$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-1}{2}}{\frac{-\sqrt{3}}{2}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \cot\theta = \sqrt{3}$$

(iv) $\tan\theta = -\frac{1}{3}$ **and the terminal arm of the angle is in quadrant II.**

(Gujranwala Board 2004)

$$\tan\theta = \frac{-1}{3} \quad \Rightarrow \cot\theta = -3$$

$$\Rightarrow \quad \sec\theta = \sqrt{1+\tan^2\theta}$$

$$\sec\theta = \sqrt{1+\frac{1}{9}} = \sqrt{\frac{9+1}{9}} = \sqrt{\frac{10}{9}} = \pm\frac{\sqrt{10}}{3}$$

$\sec\theta = -\frac{\sqrt{10}}{3}$ since terminal arm of angle is in quadrant II, so $\sin\theta$, $\text{cosec}\,\theta$ will be +ve and $\sec\theta$ and $\cos\theta$ will be − ve.

$$\cos\theta = \frac{-3}{\sqrt{10}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{9}{10}} = \sqrt{\frac{10-9}{10}} = \sqrt{\frac{1}{10}} = \frac{1}{\sqrt{10}}$$

$$\text{cosec}\,\theta = \sqrt{10}$$

(v) $\sin\theta = -\frac{1}{\sqrt{2}}$ **and the terminal arm of the angle is not in quadrant III**

(Lahore Board 2007)

$$\sin\theta = \frac{-1}{\sqrt{2}}$$

$\text{cosec}\ \theta = -\sqrt{2}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{1}{2}} = \sqrt{\frac{2-1}{2}} = \frac{1}{\sqrt{2}}$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

Since terminal arm of angle is not in quadrant III, $\sin\theta$ is $-$ ve, therefore quadrant will be IV, in IV quadrant $\cos\theta$ & $\sec\theta$ will be + ve.

$\cos\theta = \frac{1}{\sqrt{2}} \qquad \sec\theta = \sqrt{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$

$\tan\theta = -1 \Rightarrow \cot\theta = -1$

Q.5 If $\cot\theta = \frac{15}{8}$ and the terminal arm of angle is not in I quadrant find values find values of $\cos\theta$ & $\text{cosec}\ \theta$. (Gujranwala Board 2005)

Solution:

$\cot\theta = \frac{15}{8}$

$\Rightarrow \quad \tan\theta = \frac{8}{15}$

$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\frac{64}{225}} = \sqrt{\frac{225+64}{225}} = \frac{289}{225}$

$\sec\theta = \pm\frac{17}{15}$

Since terminal arm is not in I quadrant so it is in III quadrant.
In III quadrant $\tan\theta$ and $\cot\theta$ is +ve. All other will be $-$ ve.

$\text{Sec}\ \theta = \frac{-17}{15}$

$\Rightarrow \quad \cos\theta = \frac{-15}{17}$

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{225}{289}} = \sqrt{\frac{289-225}{289}} = \sqrt{\frac{64}{289}}$

$\sec\theta = \pm\frac{8}{17}$

$\sin\theta = \frac{-8}{17}$ (terminal arm in in III quadrant)

$\text{cosec}\ \theta = \frac{-17}{18}$

Q.6 **If $\operatorname{cosec}\theta = \dfrac{m^2+1}{2m}$ and $m > 0$ $\left(0 < \theta < \dfrac{\pi}{2}\right)$, find the value of the remaining trigonometric ratios.** **(Lahore Board 2005)**

Solution:

$$\operatorname{cosec}\theta = \frac{m^2+1}{2m}$$

$$\Rightarrow \quad \sin\theta = \frac{2m}{m^2+1}$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{2m}{m^2+1}\right)^2} = \sqrt{1-\frac{4m^2}{m^4+1+2m^2}}$$

$$= \sqrt{\frac{m^4+1+2m^2-4m^2}{m^4+1+2m^2}}$$

$$\cos\theta = \sqrt{\frac{m^4-2m^2+1}{m^4+1+2m^2}} = \sqrt{\frac{(m^2-1)^2}{(m^2+1)^2}}$$

$$\cos\theta = \frac{m^2-1}{m^2+1} \quad \left(\sin\theta \;\; 0 < \theta < \frac{\pi}{2} \text{ so quadrant is I, all trigonometric ratios are } +\text{ve}\right)$$

$$\sec\theta = \frac{m^2+1}{m^2-1}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\dfrac{2m}{m^2+1}}{\dfrac{(m^2-1)}{(m^2+1)}} = \frac{2m}{m^2-1}$$

$$\cot\theta = \frac{m^2-1}{2m}$$

Q.7 **If $\tan\theta = \dfrac{1}{\sqrt{7}}$ and terminal arm of the angle is not the III quadrant, find the value of $\dfrac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$.** **(Lahore Board 2010)**

Solution:

$$\tan\theta = \frac{1}{\sqrt{7}}$$

$$\Rightarrow \quad \cot\theta = \sqrt{7}$$

$$\sec^2\theta = 1+\tan^2\theta = 1+\frac{1}{7} = \frac{7+1}{7} = \frac{8}{7}$$

$$\sec^2\theta = \frac{8}{7}$$

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + 7$$

$$\operatorname{cosec}^2\theta = 8$$

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}} = \frac{\frac{56-8}{7}}{\frac{56+8}{7}} = \frac{48}{64}$$

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$$

Q.8 If $\cot\theta = \frac{5}{2}$ and the terminal arm of the angle is in the I quadrant, find the value of $\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$. (Lahore Board 2009)

Solution:

$$\cot\theta = \frac{5}{2}$$

$$\Rightarrow \quad \tan\theta = \frac{2}{5}$$

$$\operatorname{cosec}\theta = \sqrt{1 + \cot^2\theta} = \sqrt{1 + \frac{25}{4}} = \sqrt{\frac{4+25}{4}} = \frac{\sqrt{29}}{2}$$

$$\Rightarrow \quad \sin\theta = \frac{2}{\sqrt{29}}$$ (since terminal arm is in I quad, so all trigonometric functions are +ve)

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{4}{29}} = \sqrt{\frac{29-4}{29}} = \sqrt{\frac{25}{29}}$$

$$\cos\theta = \frac{5}{\sqrt{29}}$$

$$\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta} = \frac{3\left(\frac{2}{\sqrt{29}}\right) + 4\left(\frac{5}{\sqrt{29}}\right)}{\frac{5}{\sqrt{29}} - \frac{2}{\sqrt{29}}} = \frac{\frac{6}{\sqrt{29}} + \frac{20}{\sqrt{29}}}{\frac{3}{\sqrt{29}}} = \frac{\frac{26}{\sqrt{29}}}{\frac{3}{\sqrt{29}}}$$

$$\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta} = \frac{26}{3}$$

EXERCISE 9.3

Q.1 Verify that following:

(i) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$

(ii) $\sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{3} + \tan^2 \frac{\pi}{4} = 2$ **(Lahore Board 2010)**

(iii) $2 \sin 45^\circ + \frac{1}{2} \operatorname{cosec} 45^\circ = \frac{3}{\sqrt{2}}$

(iv) $\sin^2 \frac{\pi}{6} : \sin^2 \frac{\pi}{4} : \sin^2 \frac{\pi}{3} : \sin^2 \frac{\pi}{2} = 1 : 2 : 3 : 4$

Solution:

(i) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$

L.H.S $= \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{3-1}{4}$

$= \frac{1}{2}$

R.H.S $= \sin 30^\circ$

$= \frac{1}{2}$

$\therefore$ L.H.S. = R.H.S. Hence proved.

(ii) $\sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{3} + \tan^2 \frac{\pi}{4} = 2$

L.H.S. $= \sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{3} + \tan^2 \frac{\pi}{4}$

$= \sin^2 30^\circ + \sin^2 60^\circ + \tan^2 45^\circ$

$= (\sin 30^\circ)^2 + (\sin 60^\circ)^2 + (\tan 45^\circ)^2$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$

$= \frac{1}{4} + \frac{3}{4} + 1$

$= \frac{1+3+4}{4} = \frac{8}{4} = 2$

= R.H.S. Hence proved.

(iii) $\mathbf{2 \sin 45^o + \frac{1}{2} \operatorname{cosec} 45^o = \frac{3}{\sqrt{2}}}$

L.H.S. $= 2 \sin 45^o + \frac{1}{2} \operatorname{cosec} 45^o$

$= 2\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2} \frac{1}{\frac{1}{\sqrt{2}}}$

$= \frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{2}$

$= \frac{4+2}{2\sqrt{2}} = \frac{6}{2\sqrt{2}} = \frac{3}{\sqrt{2}}$

= R.H.S. Hence proved.

(iv) $\mathbf{\sin^2 \frac{\pi}{6} : \sin^2 \frac{\pi}{4} : \sin^2 \frac{\pi}{3} : \sin^2 \frac{\pi}{2} = 1 : 2 : 3 : 4}$

L.H.S. $= \sin^2 \frac{\pi}{6} : \sin^2 \frac{\pi}{4} : \sin^2 \frac{\pi}{3} : \sin^2 \frac{\pi}{2}$

$= \sin^2 30^o : \sin^2 45^o : \sin 2\, 60^o : \sin^2 90^o$

$= (\sin 30^o)^2 : (\sin 45^o)^2 : (\sin 60^o)^2 : (\sin 90^o)^2$

$= \left(\frac{1}{2}\right)^2 : \left(\frac{1}{\sqrt{2}}\right)^2 : \left(\frac{\sqrt{3}}{2}\right)^2 : (1)^2$

$= \frac{1}{4} : \frac{1}{2} : \frac{3}{4} : 1$

Multiplying throughout by 4

$= \frac{1}{4} \times 4 : \frac{1}{2} \times 4 : \frac{3}{4} \times 4 : 1 \times 4$

$= 1 : 2 : 3 : 4$

= R.H.S. Hence proved.

Q.2 Evaluate the following:

(i) $\dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}}$ (ii) $\dfrac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}}$

Solution:

(i) $\dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}} = \dfrac{\tan 60^\circ-\tan 30^\circ}{1+\tan 60^\circ\tan 30^\circ}$

$= \dfrac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}\cdot\frac{1}{\sqrt{3}}}$

$= \dfrac{\frac{(\sqrt{3})^2-1}{\sqrt{3}}}{1+1}$

$= \dfrac{\frac{3-1}{\sqrt{3}}}{2}$

$= \dfrac{2}{\sqrt{3}} \times \dfrac{1}{2}$

$\dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}} = \dfrac{1}{\sqrt{3}}$ Ans.

(ii) $\dfrac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}} = \dfrac{1-\tan^2 60^\circ}{1+\tan^2 60^\circ}$

$= \dfrac{1-(\tan 60^\circ)^2}{1+(\tan 60^\circ)^2}$

$= \dfrac{1-(\sqrt{3})^2}{1+(\sqrt{3})^2}$

$= \dfrac{1-3}{1+3} = \dfrac{-2}{4}$

$\dfrac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}} = \dfrac{-1}{2}$

Q.3 Verify the following, when $\theta = 30^\circ, 45^\circ$

(i) $\sin 2\theta = 2 \sin\theta \cos\theta$ **(Lahore Board 2008)**

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

(iii) $\cos 2\theta = 2\cos^2\theta - 1$

(iv) $\cos 2\theta = 1 - 2\sin^2\theta$

(v) $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Solution:

(i) $\sin 2\theta = 2 \sin\theta \cos\theta$

at $\theta = 30^\circ$

$\sin 2(30^\circ) = 2 \sin 30^\circ \cos 30^\circ$

$\sin 60^\circ = 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

L.H.S. = R.H.S.

at $\theta = 45^\circ$

$\sin 2(45^\circ) = 2 \sin 45^\circ \cos 45^\circ$

$\sin 90^\circ = 2 \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}$

$1 = 2 \cdot \frac{1}{2}$

$1 = 1$

L.H.S. = R.H.S.

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

at $\theta = 30^\circ$

$\cos 2(30^\circ) = \cos^2 30^\circ - \sin^2 30^\circ$

$\cos 60^\circ = (\cos 30^\circ)^2 - (\sin 30^\circ)^2$

$\frac{1}{2} = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$\frac{1}{2} = \frac{3}{4} - \frac{1}{4}$

$\frac{1}{2} = \frac{3-1}{4}$

$\frac{1}{2} = \frac{2}{4}$

$\frac{1}{2} = \frac{1}{2}$

L.H.S. = R.H.S.

at $\theta = 45^\circ$

$\cos 2(45) = \cos^2 45 - \sin^2 45$

$\cos 90^\circ = (\cos 45)^2 - (\sin 45^\circ)^2$

$0 = \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$0 = \frac{1}{2} - \frac{1}{2}$

$0 = 0$

L.H.S. = R.H.S.

(iii) $\cos 2\theta = 2\cos^2\theta - 1$

at $\theta = 30^\circ$

$\cos 2(30^\circ) = 2\cos^2 30^\circ - 1$

$\cos 60^\circ = 2(\cos 30^\circ)^2 - 1$

$\frac{1}{2} = 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$\frac{1}{2} = 2 \times \frac{3}{4} - 1$

$\frac{1}{2} = \frac{3}{2} - 1$

$\frac{1}{2} = \frac{3-2}{2}$

$\frac{1}{2} = \frac{1}{2}$

L.H.S. = R.H.S.

at $\theta = 45^\circ$

$\cos 2(45^\circ) = 2\cos^2 45^\circ - 1$

$\cos 90^\circ = 2(\cos 45^\circ)^2 - 1$

$0 = 2\left(\frac{1}{\sqrt{2}}\right)^2 - 1$

$= 2\left(\frac{1}{2}\right) - 1$

$0 = 1 - 1$

$0 = 0$

L.H.S. = R.H.S.

(iv) $\cos 2\theta = 1 - 2\sin^2\theta$

at $\theta = 30^\circ$

$\cos 2(30^\circ) = 1 - 2\sin^2 30^\circ$

$\cos 60^\circ = 1 - 2(\sin 30^\circ)^2$

$\frac{1}{2} = 1 - 2\left(\frac{1}{2}\right)^2$

$\frac{1}{2} = 1 - 2\left(\frac{1}{4}\right)$

$\frac{1}{2} = 1 - \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$

L.H.S. = R.H.S.

at $\theta = 45^\circ$

$\cos 2(45^\circ) = 1 - 2\sin^2 45^\circ$

$\cos 90^\circ = 1 - 2(\sin 45^\circ)^2$

$0 = 1 - 2\left(\frac{1}{\sqrt{2}}\right)^2$

$0 = 1 - 2 \cdot \frac{1}{2}$

$0 = 1 - 1$

$0 = 0$

L.H.S. = R.H.S.

(v) $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

at $\theta = 30^\circ$

$\tan 2(30^\circ) = \frac{2\tan 30^\circ}{1 - \tan^2 30^\circ}$

at $\theta = 45^\circ$

$\tan 2(45^\circ) = \frac{2\tan 45^\circ}{1 - \tan^2 45^\circ}$

$\tan 60^\circ = \dfrac{2\tan 30^\circ}{1-(\tan 30^\circ)^2}$ $\qquad$ $\tan 90^\circ = \dfrac{2\tan 45^\circ}{1-(\tan 45^\circ)^2}$

$\sqrt{3} = \dfrac{2\dfrac{1}{\sqrt{3}}}{1-\left(\dfrac{1}{\sqrt{3}}\right)^2}$ $\qquad$ $\infty = \dfrac{2(1)}{1-(1)^2}$

$\sqrt{3} = \dfrac{\dfrac{2}{\sqrt{3}}}{1-\dfrac{1}{3}}$ $\qquad$ $\infty = \dfrac{2}{1-1}$

$\sqrt{3} = \dfrac{2}{\sqrt{3}} \times \dfrac{3}{2}$ $\qquad$ $\infty = \dfrac{2}{0}$

$\sqrt{3} = \sqrt{3}$ $\qquad$ $\infty = \infty$

L.H.S. = R.H.S. $\qquad$ L.H.S. = R.H.S.

Q.4 Find x, if $\tan^2 45^\circ - \cos^2 60^\circ = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$.

Solution:

$\tan^2 45^\circ - \cos^2 60^\circ = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$

$(\tan 45^\circ)^2 - (\cos 60^\circ)^2 = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$

$(1)^2 - \left(\dfrac{1}{2}\right)^2 = x\left(\dfrac{1}{\sqrt{2}}\right).\left(\dfrac{1}{\sqrt{2}}\right).\sqrt{3}$

$1 - \dfrac{1}{4} = x\dfrac{\sqrt{3}}{2}$

$\dfrac{3}{4} = \dfrac{\sqrt{3}}{2}x$

$x = \dfrac{2\times 3}{4\times\sqrt{3}}$

$x = \dfrac{\sqrt{3}}{2}$

Q.5 Find the values of the trigonometric functions of the following quadrantal angles.

(i) $-\pi$ (ii) -3π (iii) $\dfrac{5}{2}\pi$ (iv) $-\dfrac{9}{2}\pi$

(v) -15π (vi) 1530° (vii) -2430 (viii) $\dfrac{235}{2}\pi$ (ix) $\dfrac{407}{2}\pi$

Solution:

Remember

if angle is in the form of π then the formula is $2k\pi + \theta$ and if angle is in the form of degree then use the formula $2k(180^\circ) + \theta$ or $k(360^\circ) + \theta$

(i) $-\pi = -2(\pi) + \pi$

$= \pi$

$\sin(-\pi) = \sin(\pi) = 0$

$\cos(-\pi) = \cos(\pi) = -1$

$\tan(-\pi) = \tan(\pi) = 0$

$\cot(-\pi) = \cot(\pi) = \infty$ (undefined)

$\sec(-\pi) = \sec(\pi) = -1$

$\operatorname{cosec}(-\pi) = \operatorname{cosec}(\pi) = \infty$ (undefined)

(ii) $-3\pi = -(2 \cdot 1 \cdot \pi + \pi)$ $\quad \boxed{-(2 \cdot k \cdot \pi + \theta)}$

$= -2 \cdot 1 \cdot \pi - \pi$

$\theta = -\pi$

$= \pi$ (which is same as π)

$\sin(-3\pi) = \sin(\pi) = 0$; $\operatorname{cosec}(-3\pi) = \operatorname{cosec}\pi = \infty$ (undefined)

$\cos(-3\pi) = \cos(\pi) = -1$; $\sec(-3\pi) = \sec(\pi) = -1$

$\tan(-3\pi) = \tan(\pi) = 0$; $\cot(-3\pi) = \cot\pi = \infty$ (undefined)

(iii)

$\frac{5}{2}\pi = 2 \cdot 1 \cdot \pi + \frac{\pi}{2}$ $\quad (2 \cdot k \cdot .\pi + \theta)$

$\Rightarrow \quad = \frac{\pi}{2}$

$\frac{5}{2} \times 180^\circ = \frac{450^\circ}{360^\circ} = 1.25 \quad k = 1$

$\theta = 450^\circ - 360^\circ$

$= 90^\circ = \frac{\pi}{2}$

$\sin\frac{5}{2}\pi = \sin\left(\frac{\pi}{2}\right) = 1$; $\operatorname{cosec}\frac{5}{2}\pi = \operatorname{cosec}\frac{\pi}{2} = 1$

$\cos\frac{5\pi}{2} = \cos\frac{\pi}{2} = 0$; $\sec\frac{5\pi}{2} = \sec\frac{\pi}{2} = \infty$ (undefined)

$\tan\frac{5\pi}{2} = \tan\frac{\pi}{2} = \infty$ undefined ; $\cot\frac{5\pi}{2} = \cot\frac{\pi}{2} = 0$

(iv) $-\frac{9}{2}\pi$

$-\frac{9\pi}{2} = -(3(2\pi) - 3\pi/2)$

$= -3(2\pi) + 3\pi/2$

$\frac{9}{2} \times 180 = \frac{810^\circ}{360^\circ} = 2.25 \quad k = 3$

$\theta = 1080^\circ - 270 = 810$

$\Rightarrow \quad = \frac{3\pi}{2}$ $\boxed{= 90^{o} = \frac{\pi}{2}}$

$= \frac{3\pi}{2}$ (which is same as $-\frac{\pi}{2}$)

$\sin\left(-\frac{9}{2}\pi\right) = \sin\left(\frac{3\pi}{2}\right) = -1$; $\text{cosec}\left(-\frac{9}{2}\pi\right) = \text{cosec}\,\frac{3\pi}{2} = -1$

$\cos\left(-\frac{9}{2}\pi\right) = \cos\left(\frac{3\pi}{2}\right) = 0$; $\sec\left(-\frac{9}{2}\pi\right) = \sec\frac{3\pi}{2} = \infty$ undefined

$\tan\left(-\frac{9}{2}\pi\right) = \tan\left(\frac{3\pi}{2}\right) = \infty$ (undefined) ; $\cot\left(-\frac{9}{2}\pi\right) = \cot\frac{3\pi}{2} = 0$

(v) $-15\,\pi$

$-(2 \,.\, k \,.\, \pi + \theta)$

$-15\,\pi = -(8 \,.\, 2 \,.\, \pi - \pi)$

$= -8 \,.\, 2 \,.\, \pi + \pi$

$\theta = \pi$

$15 \times 180^{o} = \frac{2700}{360} = 7.5$

$k = 8$

$\sin(-15\pi) = \sin(\pi) = 0$; $\text{cosec}(-15\pi) = \text{cosec}\,\pi = \infty$ (undefined)

$\cos(-15\pi) = \cos(\pi) = -1$; $\sec(-15\pi) = \sec\pi = -1$

$\tan(-15\pi) = \tan(\pi) = 0$; $\cot(-15\pi) = \cot\pi = \infty$ (undefined)

(vi) 1530^{o}

$2 \,.\, k \,.\, 180^{o} + \theta$

$1530^{o} = 2 \,.\, 4 \,.\, 180^{o} + 90^{o}$

$\theta = 90^{o}$

$\frac{1530^{o}}{360^{o}} = 4.25$

$k = 4$

$\theta = 1530^{o} - 4\,(360^{o}) = 90^{o}$

$\sin 1530^{o} = \sin 90^{o} = 1$; $\text{cosec}\,1530^{o} = \text{cosec}\,90^{o} = 1$

$\cos 1530^{o} = \cos 90^{o} = 0$; $\sec 1530^{o} = \sec 90^{o} = \infty$ (undefined)

$\tan 1530^{o} = \tan 90^{o} = \infty$ undefined ; $\cot 1530^{o} = \cot 90^{o} = 0$

(vii) -2430^{o}

$-(2 \,.\, k \,.\, 180^{o} + \theta)$

$\frac{2430^{o}}{360^{o}} = 6.75$

$-2430^\circ = -(7 \cdot 2 \cdot 180^\circ - 90^\circ)$

$-2430^\circ = -7 \cdot 2 \cdot 180^\circ + 90^\circ$

$\theta = 90^\circ$

$k = 7$

$\theta = 2430^\circ - 7 \times 360^\circ = -90^\circ$

$\sin(-2430^\circ) = \sin 90^\circ = 1$; $\operatorname{cosec}(-2430^\circ) = \operatorname{cosec} 90^\circ = 1$

$\cos(-2430) = \cos 90^\circ = 0$; $\sec(-2430^\circ) = \sec 90^\circ = \infty$ (undefined)

$\tan(-2430^\circ) = \tan 90^\circ = \infty$ (undefined) ; $\cot(-2430^\circ) = \cot 90^\circ = 0$

(viii) $\mathbf{\frac{235}{2}\pi}$

$2k \cdot \pi + \theta$

$\frac{235}{2}\pi = 2.58 \cdot \pi + \frac{3\pi}{2}$

$\Rightarrow \quad \theta = \frac{3\pi}{2}$

$\frac{235}{2} \times 180^\circ = \frac{21150^\circ}{360^\circ} = 58.75$

$k = 58$

$\theta = 21150^\circ - 58\,(360^\circ) = 270^\circ$

$\sin 235\frac{\pi}{2} = \sin\frac{3\pi}{2} = -1$; $\operatorname{cosec} 235\frac{\pi}{2} = \operatorname{cosec}\frac{3\pi}{2} = -1$

$\cos 235\frac{\pi}{2} = \cos\frac{3\pi}{2} = 0$; $\sec 235\frac{\pi}{2} = \sec\frac{3\pi}{2} = \infty$

$\tan 235\frac{\pi}{2} = \tan\frac{3\pi}{2} = \infty$; $\cot 235\frac{\pi}{2} = \cot\frac{3\pi}{2} = 0$

(ix) $\mathbf{\frac{407}{2}\pi}$

$2K \cdot \pi + \theta$

$\frac{407}{2}\pi = 2 \cdot 101\pi + \frac{3\pi}{2}$

$\theta = \frac{3\pi}{2}$

$407 \times 90^\circ = \frac{36630}{360^\circ} = 101.75$

$k = 101$

$\theta = 36630 - 101\,(360^\circ) = \frac{3\pi}{2}$

$\sin\frac{407}{2}\pi = \sin\frac{3\pi}{2} = -1$; $\operatorname{cosec}\frac{407}{2}\pi = \operatorname{cosec}\frac{3\pi}{2} = -1$

$\cos\frac{407}{2}\pi = \cos\frac{3\pi}{2} = 0$; $\sec\frac{407}{2}\pi = \sec\frac{3\pi}{2} = \infty$

$\tan\frac{407}{2}\pi = \tan\frac{3\pi}{2} = \infty$; $\cot\frac{407}{2}\pi = \cot\frac{3\pi}{2} = 0$

Q.6 Find the values of the trigonometric functions of the following angles.

(i) 390 **(ii) -330** **(iii) 765** **(iv) -675**

(v) $\frac{-17}{3}\pi$ **(vi)** $\frac{13}{3}\pi$ **(vii)** $\frac{25}{6}\pi$

(viii) $-\frac{71}{6}\pi$ **(ix) -1035**

Solution:

(i) 390°

$2 \cdot k \cdot 180^\circ + \theta$

$390^\circ = 2 \cdot 1 \cdot 180^\circ + 30^\circ$

$\theta = 30^\circ$

$\frac{390^\circ}{360^\circ} = 1.08$

$k = 1$

$\sin 390^\circ = \sin 30^\circ = \frac{1}{2}$; $\operatorname{cosec} 390^\circ = \operatorname{cosec} 30^\circ = 2$

$\cos 390^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$; $\sec 390^\circ = \sec 30^\circ = \frac{2}{\sqrt{3}}$

$\tan 390^\circ = \tan 30^\circ = \frac{1}{\sqrt{3}}$; $\cot 390^\circ = \cot 30^\circ = \sqrt{3}$

(ii) -330°

$-(2 \cdot k \cdot 180^\circ + \theta)$

$-330^\circ = (2 \cdot 1 \cdot 180^\circ - 30^\circ)$

$-330^\circ = -2 \cdot 1 \cdot 180^\circ + 30^\circ$

$\theta = 30^\circ$

$\sin(-330^\circ) = \sin 30^\circ = \frac{1}{2}$; $\operatorname{cosec}(-330^\circ) = \operatorname{cosec} 30^\circ = 2$

$\cos(-330^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$; $\sec(-330^\circ) = \sec 30^\circ = \frac{2}{\sqrt{3}}$

$\tan(-330^\circ) = \tan 30^\circ = \frac{1}{\sqrt{3}}$; $\cot(-330^\circ) = \cot 30^\circ = \sqrt{3}$

(iii) 765°

$2k \cdot 180^\circ + \theta$

$765^\circ = 2 \cdot 2 \cdot 180^\circ + 45^\circ$

$\frac{765^\circ}{360^\circ} = 2.125$, $k = 2$

$\theta = 765 - 2(360^\circ) = 45^\circ$

$\Rightarrow \quad \theta = 45^o$

$\sin 765^o = \sin 45^o = \frac{1}{\sqrt{2}}$; $\operatorname{cosec} 765^o = \operatorname{cosec} 45^o = \sqrt{2}$

$\cos 765^o = \cos 45^o = \frac{1}{\sqrt{2}}$; $\sec 765^o = \sec 45^o = \sqrt{2}$

$\tan 765^o = \tan 45^o = 1$; $\cot 765^o = \cot 45^o = 1$

(iv) $\mathbf{-675^o}$

$-(2k \cdot 180^o + \theta)$

$\frac{675}{360^o} = 1.875$

$k = 2$

$-675^o = -(2 \cdot 2 \cdot 180^o - 45^o)$

$-675^o = -2 \cdot 2 \cdot 180^o + 45^o$

$\Rightarrow \quad \theta = 45^o$

$\sin(-675^o) = \sin 45^o = \frac{1}{\sqrt{2}}$; $\operatorname{cosec}(-675^o) = \operatorname{cosec} 45^o = \sqrt{2}$

$\cos(-675^o) = \cos 45^o = \frac{1}{\sqrt{2}}$; $\sec(-675^o) = \sec 45^o = \sqrt{2}$

$\tan(-675^o) = \tan 45^o = 1$; $\cot(-675^o) = \cot 45^o = 1$

(v) $\mathbf{-17\frac{\pi}{3}}$

$\frac{17}{3} \times 180^o = \frac{1020^o}{360^o} = 2.8$

$k = 3$

$-(2k \cdot 180^o + \theta)$

$-17\frac{\pi}{3} = -(2 \cdot 3 \cdot 180^o - 60^o)$

$1020 - 3(360^o) = 60^o$

$\theta = 60^o$

$\sin\left(\frac{-17}{3}\pi\right) = \sin 60^o = \frac{\sqrt{3}}{2}$; $\sec\left(\frac{-17}{3}\pi\right) = \sec 60^o = \frac{2}{\sqrt{3}}$

$\cos\left(\frac{-17}{3}\pi\right) = \cos 60^o = \frac{1}{2}$; $\sec\left(\frac{-17}{3}\pi\right) = \sec 60^o = 2$

$\tan\left(\frac{-17}{3}\pi\right) = \tan 60^o = \sqrt{3}$; $\cot\left(\frac{-17}{3}\pi\right) = \cot 60^o = \frac{1}{\sqrt{3}}$

(vi) $\mathbf{13\,\frac{\pi}{3}}$

$2\,k\,.\,180^o + \theta$

$13\,\frac{\pi}{3} = 2\,.\,2\,.\,180^o + 60^o$

$\theta = 60^o$

$\frac{13}{3} \times 180^o = \frac{780^o}{360^o} = 2.16$

$k = 2$

$\theta = 780^o - 2\,(360^o) = 60^o$

$\sin \frac{13}{3}\pi = \sin 60^o = \frac{\sqrt{3}}{2}$; $\operatorname{cosec} \frac{13}{3}\pi = \operatorname{cosec} 60^o = \frac{2}{\sqrt{3}}$

$\cos \frac{13}{3}\pi = \cos 60^o = \frac{1}{2}$; $\sec \frac{13}{3}\pi = \sec 60^o = \frac{2}{1}$

$\tan \frac{13}{3}\pi = \tan 60^o = \sqrt{3}$; $\cot \frac{13}{3}\pi = \cot 60^o = \frac{1}{\sqrt{3}}$

(vii) $\mathbf{25\,\frac{\pi}{6}}$

$2\,k\,.\,180^o + \theta$

$25\,\frac{\pi}{6} = 2\,.\,2\,.\,180^o + 30^o$

$\Rightarrow \quad \theta = 30^o$

$\frac{25}{6} \times 180^o = \frac{750^o}{360^o} = 2.08$

$k = 2$

$750^o - 2\,(360^o) = 30^o$

$\sin \frac{25}{6}\pi = \sin 30^o = \frac{1}{2}$; $\operatorname{cosec} \frac{25}{6}\pi = \operatorname{cosec} 30^o = 2$

$\cos \frac{25}{6}\pi = \cos 30^o = \frac{\sqrt{3}}{2}$; $\sec \frac{25}{6}\pi = \sec 30^o = \frac{2}{\sqrt{3}}$

$\tan \frac{25}{6}\pi = \tan 30^o = \frac{1}{\sqrt{3}}$; $\cot \frac{25}{6}\pi = \cot 30^o = \sqrt{3}$

(viii) $\mathbf{-71\,\frac{\pi}{6}}$

$-(2\,k\,.\,180^o + \theta)$

$-71\,\frac{\pi}{6} = -(2\,.\,6\,.\,180^o - 30^o)$

$= -2\,.\,6\,.\,180^o + 30^o$

$\Rightarrow \quad \theta = 30^o$

$\frac{71}{6} \times 180^o = \frac{2130^o}{360^o} = 5.91$

$k = 6$

$\theta = 2130^o - 6\,(360^o)$

$\theta = -30^o$

$\sin\left(-\frac{71}{6}\pi\right) = \sin 30^\circ = \frac{1}{2}$; $\operatorname{cosec}\left(-\frac{71}{6}\pi\right) = \operatorname{cosec} 30^\circ = 2$

$\cos\left(-\frac{71}{6}\pi\right) = \cos 30^\circ = \frac{\sqrt{3}}{2}$; $\sec\left(-\frac{71}{6}\pi\right) = \sec 30^\circ = \frac{2}{\sqrt{3}}$

$\tan\left(-\frac{71}{6}\pi\right) = \tan 30^\circ = \frac{1}{\sqrt{3}}$; $\cot\left(-\frac{71}{6}\pi\right) = \cot 30^\circ = \sqrt{3}$

(ix) $-1035°$

$-(2k \cdot 180^\circ + \theta)$

$-1035^\circ = -(2 \cdot 3 \cdot 180^\circ - 45^\circ)$

$= -2 \cdot 3 \cdot 180^\circ + 45^\circ$

$\Rightarrow \quad \theta = 45^\circ$

$\frac{1035^\circ}{360^\circ} = 2.875$

$k = 3$

$\theta = 1035^\circ - 3(360^\circ)$

$\theta = -45^\circ$

$\sin(-1035^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$ $\operatorname{cosec}(-1035^\circ) = \operatorname{cosec} 45^\circ = \sqrt{2}$

$\cos(-1035^\circ) = \cos 45^\circ = \frac{1}{\sqrt{2}}$ $\sec(-1035^\circ) = \sec 45^\circ = \sqrt{2}$

$\tan(-1035^\circ) = \tan 45^\circ = 1$ $\cot(-1035^\circ) = \cot 45^\circ = 1$

EXERCISE 9.4

Q.1 Prove the identity, state the domain of θ in each case.

$\tan\theta + \cot\theta = \operatorname{cosec}\theta \sec\theta$. **(Gujranwala Board 2005)**

Solution:

$$\text{L.H.S.} = \tan\theta + \cot\theta$$
$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \operatorname{cosec}\theta \cdot \sec\theta \quad \text{R.H.S.}$$

Domain of $\theta : \theta \in \Re$, but $\theta \neq n\frac{\pi}{2}$, $n \in Z$.

Q.2

$\sec\theta \operatorname{cosec}\theta \sin\theta \cos\theta = 1$

Solution:

$$\text{L.H.S.} = \sec\theta \operatorname{cosec}\theta \sin\theta \cos\theta$$
$$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} \cdot \sin\theta \cdot \cos\theta$$
$$= 1 = \text{R.H.S.}$$

Domain of $\theta : \theta \in \Re$, but $\theta \neq n\frac{\pi}{2}$, $n \in Z$

Q.3 $\cos\theta + \tan\theta \sin\theta = \sec\theta$

Solution:

$$\text{L.H.S.} = \cos\theta + \tan\theta \,.\, \sin\theta$$

$$= \cos\theta + \frac{\sin\theta}{\cos\theta} \,.\, \sin\theta$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} = \sec\theta = \text{R.H.S.}$$

Domain of $\theta : \theta \in \Re$, but $\theta \neq (2n+1)\frac{\pi}{2}$, $n \in z$

Q.4 $\text{cosec}\,\theta + \tan\theta \,.\, \sec\theta = \text{cosec}\,\theta \sec^2\theta.$

Solution:

$$\text{L.H.S.} = \text{cosec}\,\theta + \tan\theta \sec\theta$$

$$= \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta \cos^2\theta}$$

$$= \frac{1}{\cos^2\theta \sin\theta}$$

$$= \text{cosec}\,\theta \sec^2\theta = \text{R.H.S.}$$

Domain of $\theta \quad \theta \in \Re$, but $\theta \neq n\frac{\pi}{2}$, $n \in Z$

Q.5 $\sec^2\theta - \text{cosec}^2\theta = \tan^2\theta - \cot^2\theta$

Solution:

$$\text{L.H.S.} = \sec^2\theta - \text{cosec}^2\theta$$

$$= 1 + \tan^2\theta - (1 + \cot^2\theta)$$

$$= 1 + \tan^2\theta - 1 - \cot^2\theta$$

$$= \tan^2\theta - \cot^2\theta$$

$$= \text{R.H.S.}$$

Domain of $\theta : \theta \in R$ but $\phi \neq \frac{n\pi}{2}$, $n \in z$

Q.6 $\cot^2\theta - \cos^2\theta = \cot^2\theta \cos^2\theta$

Solution:

$$\text{L.H.S.} = \cot^2\theta - \cos^2\theta$$

$$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{\cos^2\theta}{1}$$

$$= \frac{\cos^2\theta - \sin^2\theta \cos^2\theta}{\sin^2\theta}$$

$$= \frac{\cos^2\theta (1 - \sin^2\theta)}{\sin^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta} \cos^2\theta$$

$$= \cot^2\theta \cos^2\theta$$

$$= \text{R.H.S.}$$

Domain of θ, $\theta \in \Re$ but $\theta \neq n\pi$, $n \in z$

Q.7 $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

Solution:

$$\text{L.H.S.} = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$$

$$= (\sec\theta)^2 - (\tan\theta)^2 = \sec^2\theta - \tan^2\theta$$

$$= 1 + \tan^2\theta - \tan^2\theta$$

$$= 1 = \text{R.H.S.}$$

Domain of $\theta = \Re$.

Q.8 $2\cos^2\theta - 1 = 1 - 2\sin^2\theta.$

Solution:

$$\text{L.H.S.} = 2\cos^2\theta - 1$$

$$= 2(1 - \sin^2\theta) - 1$$

$$= 2 - 2\sin^2\theta - 1$$

$$= 1 - 2\sin^2\theta = \text{R.H.S.}$$

Domain of $\theta = \Re$

Q.9 $\cos^2\theta - \sin^2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}.$

Solution:

$$\text{R.H.S.} = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}.$$

$$= \frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{1 + \frac{\sin^2\theta}{\cos\theta^2}} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \cos^2\theta - \sin^2\theta = \text{L.H.S.}$$

Domain of θ : but $\theta \neq (2n+1)\frac{\pi}{2}$, $n \in Z$

Q.10 $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \dfrac{\cot\theta - 1}{\cot\theta + 1}$

Solution:

$$\text{R.H.S.} = \frac{\cot\theta - 1}{\cot\theta + 1}$$

$$= \frac{\frac{\cos\theta}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} + 1} = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \text{L.H.S.}$$

Domain of θ ; $\theta \in \Re$ but $\theta \neq n\pi + \frac{3\pi}{4}$, $n \in Z$

Q.11 $\dfrac{\sin\theta}{1 + \cos\theta} + \cot\theta = \operatorname{cosec}\theta$

Solution:

$$\text{L.H.S.} = \frac{\sin\theta}{1 + \cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos\theta\,(1 + \cos\theta)}{\sin\theta\,(1 + \cos\theta)}$$

$$= \frac{\sin^2\theta + \cos^2\theta + \cos\theta}{\sin\theta\,(1 + \cos\theta)}$$

$$= \frac{(1 + \cos\theta)}{\sin\theta\,(1 + \cos\theta)} = \frac{1}{\sin\theta} = \operatorname{cosec}\theta$$

$$= \text{R.H.S.}$$

Domain of $\theta : \theta \in \Re$ but $\theta \neq n\pi$, $n \in Z$

Q.12 $\dfrac{\cot^2\theta - 1}{1 + \cot^2\theta} = 2\cos^2\theta - 1$

Solution:

$$\text{L.H.S.} = \frac{\cot^2\theta - 1}{1 + \cot^2\theta}$$

$$= \frac{\frac{\cos^2\theta}{\sin^2\theta} - 1}{1 + \frac{\cos^2\theta}{\sin^2\theta}} = \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta + \cos^2\theta}$$

$$= \frac{\cos^2\theta - (1 - \cos^2\theta)}{1}$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$$= 2\cos^2\theta - 1 = \text{R.H.S.}$$

Domain of θ : $\theta \in \Re$ but $\theta \neq n\pi$, $n \in Z$

Q.13 $\dfrac{1 + \cos\theta}{1 - \cos\theta} = (\text{cosec}\,\theta + \cot\theta)^2$

Solution:

$$\text{R.H.S.} = (\text{cosec}\,\theta + \cot\theta)^2$$

$$= \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1 + \cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1 + \cos\theta)^2}{\sin^2\theta} = \frac{(1 + \cos\theta)^2}{1 - \cos^2\theta}$$

$$= \frac{(1 + \cos\theta)^2}{(1 - \cos\theta)(1 + \cos\theta)}$$

$$= \frac{1 + \cos\theta}{1 - \cos\theta} = \text{L.H.S.}$$

Domain of θ : $\theta \in \Re$ but $\theta \neq n\pi$, $n \in Z$

Q.14 $(\sec\theta - \tan\theta)^2 = \dfrac{1 - \sin\theta}{1 + \sin\theta}$

Solution:

$$\text{L.H.S.} = (\sec\theta - \tan\theta)^2$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1 - \sin\theta}{\cos\theta}\right)^2 = \frac{(1 - \sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1 - \sin\theta)^2}{1 - \sin^2\theta} = \frac{(1 - \sin\theta)^2}{(1 - \sin\theta)(1 + \sin\theta)}$$

$$= \frac{1 - \sin\theta}{1 + \sin\theta} = \text{R.H.S.}$$

Domain of θ , $\theta \in \Re$ but $\theta \neq (2n + 1)\frac{\pi}{2}$, $n \in Z$

Q.15 $\dfrac{\mathbf{2 \tan\theta}}{\mathbf{1 + \tan^2\theta}} = \mathbf{2 \sin\theta \cos\theta}$

Solution:

$$\text{L.H.S.} = \frac{2 \tan\theta}{1 + \tan^2\theta}$$

$$= \frac{2\dfrac{\sin\theta}{\cos\theta}}{1 + \dfrac{\sin^2\theta}{\cos^2\theta}} = \frac{2 \sin\theta}{\cos\theta} \div \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$$

$$= \frac{2 \sin\theta}{\cos\theta} \times \frac{\cos^2\theta}{1} = 2 \sin\theta \cos\theta$$

$$= \text{R.H.S.}$$

Domain of θ , $\theta \in \Re$ but $\theta \neq (2n + 1)\frac{\pi}{2}$, $n \in Z$

Q.16 $\dfrac{\mathbf{1 - \sin\theta}}{\mathbf{\cos\theta}} = \dfrac{\mathbf{\cos\theta}}{\mathbf{1 + \sin\theta}}$

Solution:

$$\text{L.H.S.} = \frac{1 - \sin\theta}{\cos\theta}$$

$$= \frac{1 - \sin\theta}{\cos\theta} \times \frac{1 + \sin\theta}{1 + \sin\theta}$$

$$= \frac{1 - \sin^2\theta}{\cos\theta(1 + \sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1 + \sin\theta)}$$

$$= \frac{\cos\theta}{1 + \sin\theta} = \text{R.H.S.}$$

Domain of θ : $\theta \in \Re$ but $\theta \neq (2n + 1)\frac{\pi}{2}$, $n \in Z$

Q.17 $(\tan\theta + \cot\theta)^2 = \sec^2\theta \operatorname{cosec}^2\theta$

Solution:

$$\text{L.H.S.} = (\tan\theta + \cot\theta)^2$$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}\right)^2$$

$$= \frac{1}{\cos^2\theta \sin^2\theta} = \sec^2\theta \operatorname{cosec}^2\theta$$

$$= \text{R.H.S.}$$

Domain of $\theta : \theta \in \Re$ but $\theta \neq n\frac{\pi}{2}$, $n \in Z$

Q.18 $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$ **(Lahore Board 2008)**

Solution:

$$\text{L.H.S.} = \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1} \qquad \because\ 1 + \tan^2\theta = \sec^2\theta$$

$$= \frac{\tan\theta + \sec\theta - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1} \qquad 1 = \sec^2\theta - \tan^2\theta$$

$$= \frac{(\sec\theta + \tan\theta)\left(1 - (\sec\theta - \tan\theta)\right)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \tan\theta + \sec\theta = \text{R.H.S.}$$

Domain of $\theta : \theta \in \Re$ but $\theta \neq (2n+1)\frac{\pi}{2}$

Q.19 $\dfrac{1}{\operatorname{cosec}\theta - \cot\theta} - \dfrac{1}{\sin\theta} = \dfrac{1}{\sin\theta} - \dfrac{1}{\operatorname{cosec}\theta + \cot\theta}$ **(Gujranwala Board 2005)**

Solution:

$$\text{L.H.S.} = \frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta}$$

$$= \frac{1}{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}} - \frac{1}{\sin\theta}$$

$$= \frac{\sin\theta}{1 - \cos\theta} - \frac{1}{\sin\theta} = \frac{\sin^2\theta - 1 + \cos\theta}{(1 - \cos\theta)\sin\theta}$$

$$= \frac{(1-\cos^2\theta)-(1-\cos\theta)}{(1-\cos\theta)\sin\theta}$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)-(1-\cos\theta)}{(1-\cos\theta)\sin\theta}$$

$$= \frac{(1-\cos\theta)(1+\cos\theta-1)}{(1-\cos\theta)\sin\theta}$$

$$= \frac{\cos\theta}{\sin\theta} = \cot\theta$$

R.H.S. $= \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

$$= \frac{1}{\sin\theta} - \frac{1}{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}}$$

$$= \frac{1}{\sin\theta} - \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{1+\cos\theta-\sin^2\theta}{(\sin\theta)(1+\cos\theta)}$$

$$= \frac{(1+\cos\theta)-(1-\cos\theta)(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{(1+\cos\theta)(1-1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\cos\theta}{\sin\theta} = \cot\theta$$

Hence L.H.S. = R.H.S.

Domain of $\theta : \theta \in \Re$ but $\theta \neq n\pi$, $n \in Z$

Q.20 $\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$ (Lahore Board 2004)

Solution:

L.H.S. $= \sin^3\theta - \cos^3\theta$

$= (\sin\theta)^3 - (\cos\theta)^3$

$\because\ a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$= (\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)$

$= (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$

$=$ R.H.S.

Domain of $\theta = \Re$.

Q.21 $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

Solution:

L.H.S. $= \sin^6\theta - \cos^6\theta$

$= (\sin^2\theta)^3 - (\cos^2\theta)^3$ $\qquad \boxed{|a^2 - b^2| = (a+b)(a-b)}$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + (\cos^2\theta)^2 + \sin^2\theta\cos^2\theta]$

Adding and subtracting $2\sin^2\theta\cos^2\theta$.

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta + \sin^2\theta\cos^2\theta]$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta]$

$= (\sin^2\theta - \cos^2\theta)\left((1)^2 - (\sin\theta\cos\theta)^2\right)$

$= (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

$=$ R.H.S.

Domain of $\theta = \Re$.

Q.22 $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$. **(Gujranwala, Lahore Board 2007)**

Solution:

L.H.S. $= \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$\boxed{a^3 + b^3 = (a+b)(a^2 + b^2 - ab)}$

$= (\sin^2\theta + \cos^2\theta)\left((\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta\cos^2\theta\right)$

$= 1\,(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

Adding & subtracting $2\sin^2\theta\cos^2\theta$

$= (\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta$

$=$ R.H.S.

Domain of $\theta = \Re$.

Q.23 $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} = 2\sec^2\theta$

Solution:

L.H.S. $= \dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta}$

$= \dfrac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

$= \dfrac{2}{1-\sin^2\theta} = \dfrac{2}{\cos^2\theta}$

$= 2\sec^2\theta$ = R.H.S.

Domain of $\theta : \theta \in \Re$ but $\theta \neq (2n+1)\dfrac{\pi}{2}$, $n \in Z$

Q.24 $\dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} + \dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \dfrac{2}{1-2\sin^2\theta}$

Solution:

L.H.S. $= \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} + \dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

$= \dfrac{(\cos\theta+\sin\theta)^2 + (\cos\theta-\sin\theta)^2}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}$

$= \dfrac{\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$

$= \dfrac{2\cos^2\theta+2\sin^2\theta}{1-\sin^2\theta-\sin^2\theta} = \dfrac{2(\cos^2\theta+\sin^2\theta)}{1-2\sin^2\theta}$

$= \dfrac{2}{1-2\sin^2\theta}$ = R.H.S.

Domain of θ : $\theta \in R$ but $\theta \neq (2n+1)\dfrac{\pi}{4}$, $n \in Z$



FUNDAMENTALS OF TRIGONOMETRY

MULTIPLE CHOICE QUESTIONS

(1) The value of $2\sin 45° + \frac{1}{2}\operatorname{cosec} 45°$ =

(a) $\frac{3}{\sqrt{2}}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\sqrt{\frac{3}{2}}$ (d) $-\frac{1}{2}$

[Lahore Board 2005]

(2) $\left(22\ \frac{1}{2}\right)^0$ =

(a) $\frac{\pi}{4}$ rad (b) $\frac{\pi}{8}$ rad

(c) $\frac{\pi}{2}$ rad (d) π rad

[Lahore Board 2005]

(3) If $\cos\theta = \frac{1}{\sqrt{2}}$ then θ is equal to:

(a) 30° (b) 45°

(c) 60° (d) 90°

[Lahore Board 2005]

(4) One angle of a right angle triangle is:

(a) 90° (b) 180°

(c) 360° (d) 225°

[Lahore Board 2005]

(12) $\frac{1-\sin\theta}{\cos\theta} =$

(a) $\frac{\cos 0}{1+\sin\theta}$ (b) $\frac{\cos 0}{1-\sin\theta}$

(c) $\frac{1+\sin\theta}{\cos 0}$ (d) None of these

[Gujranwala Board 2006]

(13) $\frac{\sec 0}{\operatorname{cosec} 0} =$

(a) $\cos\theta$ (b) $\tan\theta$

(c) $\cot\theta$ (d) $\sin\theta$

[Gujranwala Board 2006]

(14) One degree $\approx$ ———— radian:

(a) 0.157 (b) 0.751

(c) 0.175 (d) 0.0175

[Lahore Board 2006]

(15) $\cos\left(\frac{3\pi}{2}\right) =$

(a) 0 (b) 1

(c) -1 (d) None of these

[Lahore Board 2006]

(16) $\sin(-0) =$

(a) $\sin\theta$ (b) $-\sin\theta$

(c) $\cos 0$ (d) $-\cos 0$

[Lahore Board 2006]

(17) An arc of 8cm, subtends an angle of 1 radian at the centre of a circle, its radius is:

(a) 4cm (b) 8cm

(c) $\frac{1}{8}$ cm (d) None of these

[Lahore Board 2006]

(18) If $\sin 0 < 0$ and $\cos 0 > 0$ then 0 is in:

(a) 1st quad (b) 2nd quad

(c) 3rd quad (d) 4th quad

[Lahore Board 2006]

[Gujranwala Board 2007]

(19) 1° is equal to:

(a) 0.1745 rad (b) 1.1745 rad

(c) 0.0175 rad (d) 0.010745 rad

[Gujranwala Board 2007, Lahore Board 2015]

(20) One radian is equal to:

(a) 45° (b) 57.296°

(c) 58.296° (d) 50°

[Lahore Board 2007]

(21) θ° is measured in:

(a) Circular system (b) Sexagesimal system

(c) Radian measure (d) Rotation measure

[Lahore Board 2007]

(22) Sum of all angles in a triangle is equal to:

(a) 90° (b) 180°

(c) 270° (d) 360°

[Lahore Board 2007]

(23) $\sin^2\alpha + \cos^2\beta$ equals:

(a) 0 (b) 1

(c) -1 (d) None of these

[Lahore Board 2008]

(24) 1° equals:

(a) $\pi/180$ rad (b) $180/\pi$ rad

(c) $\pi/90$ rad (d) $\pi/360$ rad

[Lahore Board 2008]

(25) The vertex of an angle in standard form is at:

(a) (1, 0) (b) (0, 1)

(c) (1, 1) (d) (0, 0)

[Gujranwala Board 2008]

(26) If $\tan\theta > 0$, $\sin\theta > 0$, then the terminal arm of the angle lies in the quadrant:

(a) I (b) II

(c) III (d) IV

[Gujranwala Board 2008]

(27) Which angle is quadrantal angle?

(a) $120°$ (b) $270°$

(c) $60°$ (d) $45°$

[Gujranwala Board 2008]

(28) $\sin 390° =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 1 (d) $\frac{1}{\sqrt{2}}$

[Gujranwala Board 2009]

(29) $\operatorname{cosec}^2\theta - \cot^2\theta =$

(a) 1 (b) 0

(c) 2 (d) -1

[Lahore Board 2009]

(30) Which one is true:

(a) $1 \text{ rad} < 1°$ (b) $1 \text{ rad} > 1°$

(c) $1 \text{ rad} = 1°$ (d) $5 \text{ rad} = 2°$

[Lahore Board 2009]

(31) 3 radians is equal to in degrees:

(a) $169.78°$ (b) $171.888°$

(c) $170.889°$ (d) $171.5°$

[Lahore Board 2010]

(32) $1 - \sec^2\theta =$ ————

(a) $\tan^2\theta$ (b) $-\tan^2\theta$

(c) $\tan^2\theta - 1$ (d) $1 - \tan^2\theta$

[Lahore Board 2010]

(33) If length of arc is equal to radius of circle, then angle subtended at the centre of circle is equal to:

(a) One degree (b) One radian

(c) $180°$ (d) π radian

[Gujranwala Board 2010]

(34) π radians =

(a) $180°$ (b) $180'$

(c) $360°$ (d) $360'$

[Lahore Board 2011]

(35) Arc length of a circle of radius 'r' central angle 'θ' rad is ———

(a) $\frac{1}{2}r^2\theta$ (b) πr^2

(c) $2\pi r$ (d) θr

[Lahore Board 2012]

(36) If the length of arc is equal to radius of circle then angle subtended at the centre is equal to:

(a) One degree (b)

(c) $180°$ (d) π radian

[Lahore Board 2012]

(37) A right triangle in which one angle =

(a) $45°$ (b) $90°$

(c) $270°$ (d) $360°$

[Lahore Board 2013]

(38) $\frac{5\pi}{4}$ rad. =

(a) $360°$ (b) $335°$

(c) $270°$ (d) $225°$

[Lahore Board 2013]

(39) Angle of 30 degree is equal to:

(a) $\frac{\pi}{3}$ radian (b) $\frac{\pi}{30}$ radian

(c) $\frac{\pi}{6}$ radian (d) $\frac{\pi}{4}$ radian

[Lahore Board 2013]

(40) $[(3600)^{-1}]^0$ =

(a) $1'$ (b) $1''$

(c) $60°$ (d) $1°$

(41) Which one is conterminal angle with 60°:

(a) $-360°$ (b) $240°$

(c) $410°$ (d) $-300°$

(42) Which trigonometric function cannot be undefined for some quadrantal angle:

(a) tan x (b) sec x

(c) cot x (d) cos x

(43) $\cot 390° : \tan 390° =$

(a) 3 : 1 (b) $\sqrt{3} : 1$

(c) $1 : \sqrt{3}$ (d) 2 : 3

(44) If $\cos 2\theta = a \sin^2\theta - \frac{2}{a}$ then a = ?

(a) 1 (b) 2

(c) -2 (d) -1

(45) $\sec\left(-\frac{7\pi}{4}\right) =$

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

(46) Length of circumference of the circle is:

(a) 2π (b) $\frac{l}{r}$

(c) $r\theta$ (d) $2\pi r$

(47) If 'θ' lies in ———— quadrant then a point p(x, y) on its terminal side has negative x-coordinate and negative y-coordinate:

(a) I (b) II

(c) III (d) IV

(48) Trigonometry has been derived from ———— Greek Words:

(a) 2 (b) 3

(c) 4 (d) 5

(49) $45.25° =$

(a) $45°15'$ (b) $45°25'$

(c) $45°30°$ (d) $45°$

(50) Circular measure of the angle between the hands of a watch at 4 O' clock is:

(a) $90°$ (b) $120°$

(c) $150°$ (d) None of these

(51) If r = 3cm and l = 6cm then $\theta =$

(a) 2π (b) 3 radian

(c) 2 (d) None of these

(52) $\theta + 360k$ where $k \in Z$ is coterminal with:

(a) 20 (b) $2k\pi$

(c) θ (d) $\pi\theta$

(53) $\operatorname{cosec}(-15^\circ) =$ ————

(a) $\operatorname{cosec}15^\circ$ (b) $-\operatorname{cosec}15^\circ$

(c) $\sin15^\circ$ (d) $-\sin15^\circ$

(54) $\sec\left(-\frac{9}{2}\pi\right) =$

(a) 0 (b) 1

(c) ∞ (d) None of these

(55) Domain of $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ is:

(a) $\forall\ \theta \in R$

(b) $\forall\ \theta \in R$ but $\theta \neq n\pi$, $n \in Z$

(c) $\forall\ \theta \in R$ but $\theta \neq (2n + 1)\frac{\pi}{2}$, $n \in Z$

(d) None of these

(56) $\cos^4\theta - \sin^4\theta =$ ————

(a) 0 (b) 1

(c) $\cos^2\theta - \sin^2\theta$ (d) $\sin^2\theta - \cos^2\theta$

(57) $\cot^4\theta + \cot^2\theta =$ ————

(a) $\operatorname{cosec}^4\theta - \operatorname{cosec}^2\theta$ (b) $\operatorname{cosec}^4\theta + \operatorname{cosec}^2\theta$

(c) $\operatorname{cosec}^2\theta + \cot^2\theta$ (d) None of these

(58) Domain of $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ is:

(a) R (b) $R - \left\{x \middle| x = (2n + 1)\frac{\pi}{2}, n \in Z\right\}$

(c) $R - \{x|x = n\pi , n \in Z\}$ (d) None of these

(59) If $\cos\theta = \frac{12}{13}$ and the terminal side of the angle is not in the I quadrant then $\operatorname{cosec}\theta =$

(a) $\frac{13}{5}$ (b) $-\frac{13}{5}$

(c) $\frac{5}{13}$ (d) $-\frac{5}{13}$

(60) $\frac{1}{4}$ rotation anticlockwise = ————

(a) 45° (b) 90°

(c) 180° (d) 360°

(61) $\sin^2\theta + \cos^2\theta =$ _____

(a) -1 (b) 1

(c) 2 (d) 0

(62) If $\sin\theta = \frac{\sqrt{3}}{2}$, the θ = _____ **[Lahore Board 2015]**

(a) $30°$ (b) $45°$

(c) $90°$ (d) $60°$

Answers

(1)	a	*(2)*	b	*(3)*	b	*(4)*	a	*(5)*	b
(6)	c	*(7)*	c	*(8)*	d	*(9)*	c	*(10)*	d
(11)	c	*(12)*	a	*(13)*	b	*(14)*	d	*(15)*	a
(16)	b	*(17)*	b	*(18)*	d	*(19)*	c	*(20)*	b
(21)	b	*(22)*	b	*(23)*	d	*(24)*	a	*(25)*	d
(26)	a	*(27)*	b	*(28)*	a	*(29)*	a	*(30)*	b
(31)	b	*(32)*	b	*(33)*	b	*(34)*	a	*(35)*	d
(36)	b	*(37)*	b	*(38)*	d	*(39)*	c	*(40)*	b
(41)	d	*(42)*	d	*(43)*	a	*(44)*	c	*(45)*	b
(46)	d	*(47)*	d	*(48)*	b	*(49)*	a	*(50)*	d
(51)	c	*(52)*	c	*(53)*	b	*(54)*	c	*(55)*	b
(56)	c	*(57)*	a	*(58)*	c	*(59)*	b	*(60)*	b
(61)	b	*(62)*	d						

SOLUTION

Q.1 $2\sin 45^\circ + \frac{1}{2}\operatorname{cosec} 45^\circ$

$= 2 \cdot \frac{1}{\sqrt{2}} + \frac{1}{2}\sqrt{2}$

$= \frac{4+2}{2\sqrt{2}} = \frac{6}{2\sqrt{2}} = \frac{3}{\sqrt{2}}$

Q.2 $\left(22\ \frac{1}{2}\right)^0 = \left(\frac{45}{2}\right)^0 = \frac{45}{2} \times \frac{\pi}{180} = \frac{\pi}{8}$ rad

Q.3 As $\cos 45^\circ = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$

Q.5 $\sec^2\theta - \tan^2\theta = 1 + \tan^2\theta - \tan^2\theta = 1$

Q.6 $150^\circ = 150 \times \frac{\pi}{180} = \frac{5}{6}\pi$ rad

Q.8 $\frac{\pi}{12}$ rad $= \frac{\pi}{12} \cdot \frac{180}{\pi} = 15^\circ$

Q.9 $\tan\frac{\pi}{6} = \tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

Q.12 $\frac{1-\sin\theta}{\cos\theta} = \frac{1-\sin\theta}{\cos\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$

$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$

$= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{\cos\theta}{1+\sin\theta}$

Q.13 $\frac{\sec\theta}{\operatorname{cosec}\theta} = \frac{1/\cos\theta}{1/\sin\theta} = \frac{1}{\cos\theta} \times \sin\theta = \tan\theta.$

Q.17 $l = 8\text{cm}$, $\theta = 1$ rad

$r = ?$

As, $l = r\theta \Rightarrow r = \frac{l}{\theta} = \frac{8}{1} = 8\text{cm}$

Q.28 $\sin 390^\circ = \sin(360^\circ + 30^\circ)$

$= \sin 30^\circ$

$= \frac{1}{2}$

Q.29 $\operatorname{cosec}^2\theta - \cot^2\theta = 1 + \cot^2\theta - \cot^2\theta = 1$

Q.30 As,

$1 \text{ rad} = 57.296°$

$\Rightarrow \quad 1 \text{ rad} > 1°$

Q.31 As $\quad 1 \text{ rad} = 57.296°$

$3 \text{ rad} = 3(57.296°)$

$= 171.888°$

Q.32 $1 - \sec^2\theta = 1 - (1 + \tan^2\theta) = 1 - 1 - \tan^2\theta$

$= -\tan^2\theta$

Q.37 $[(3600)^{-1}]^0$

$= \left(\frac{1}{3600}\right)^0 = 1''$

Q.39 As,

$\cos 90° = 0 \;, \quad \cos 180° = -1 \;, \quad \cos 270° = 0$

$\cos 360° = 1 \Rightarrow$ cos cannot be undefined for some quadrantal angle.

Q.40

$\cot 390° : \tan 390°$

$= \cot(30°) : \tan 30°$

$= \frac{\cot 30°}{\tan 30°}$

$= \frac{\sqrt{3}}{1/\sqrt{3}}$

$= \frac{\sqrt{3} \cdot \sqrt{3}}{1} = \frac{3}{1} = 3 : 1$

Q.41 $\cos 2\theta = a \sin^2\theta - \frac{2}{a}$

Put, $\quad a = -2$

$\cos 2\theta = -2\sin^2\theta - \frac{2}{-2}$

$= -2\sin^2\theta + 1$

$= 1 - 2\sin^2\theta$ which is true

Q.42 As $-\frac{7\pi}{4} = \frac{\pi}{4} + (-1)\,2\pi \qquad \because (k = -1)$

$$\sec\left(-\frac{7\pi}{4}\right) = \sec\left(\frac{\pi}{4} + (-2\pi)\right)$$

$$= \sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$

Q.46 $45.25°$

As,

$$0.25° = \frac{25°}{100} = \frac{1°}{4} = \frac{60'}{4} = 15'$$

$$\Rightarrow \quad 45.25° = 45°15'$$

Q.48 As,

$$\theta = \frac{l}{r} = \frac{6}{3} = 2$$

Q.51 $-\frac{9}{2}\pi = \frac{3\pi}{2} + (-1)6\pi$

$$\sec\left(-\frac{9}{2}\pi\right) = \sec\left(\frac{3\pi}{2}\right) = \frac{1}{\cos\left(\frac{3\pi}{2}\right)} = \frac{1}{0} = \infty$$

Q.53

$$\cos^4\theta - \sin^4\theta$$
$$= (\cos^2\theta)^2 - (\sin^2\theta)^2$$
$$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$$
$$= (1)(\cos^2\theta - \sin^2\theta)$$
$$= \cos^2\theta - \sin^2\theta$$

Q.54

$$\cot^4\theta + \cot^2\theta$$
$$= \cot^2\theta\,(\cot^2\theta + 1)$$
$$= (\operatorname{cosec}^2\theta - 1)\operatorname{cosec}^2\theta$$
$$= \operatorname{cosec}^4\theta - \operatorname{cosec}^2\theta$$

Q.56 $\cos\theta = \frac{12}{13} \quad \Rightarrow \quad \sin\theta = \pm\sqrt{1 - \cos^2\theta} = \pm\sqrt{1 - \left(\frac{12}{13}\right)^2} = -\frac{5}{13}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = -\frac{13}{5} \qquad \because$ 'θ' is in IV quad.

Q.61 $\sin^2\theta + \cos^2\theta = 1$

Q.62 $\frac{1}{4} \times 360° = 90°$