

Unit 4

WORK AND ENERGY

Major Concepts

(17 PERIODS)

Conceptual Linkage

- Work done by a constant force
- Work as scalar product of force and displacement
- Work against gravity
- Work done by variable force
- Gravitational Potential at a point
- Escape velocity
- Power as scalar product of force and velocity
- Work energy principle in resistive medium
- Sources and uses of energy
 - (i) Conventional sources of energy
 - (ii) Non-conventional sources of energy

This chapter is built on
Work and energy
Physics IX
Gravitation Physics IX

Students Learning Outcomes

After studying this unit, the students will be able to:

- describe the concept of work in terms of the product of force F and displacement d in the direction of force (Work as scalar product of F and d).
- distinguish between positive, negative and zero work with suitable examples.
- describe that work can be calculated from the area under the force-displacement graph.
- explain gravitational field as an example of field of force and define gravitational field strength as force per unit mass at a given point.
- prove that gravitational field is a conservative field.
- compute and show that the work done by gravity as a mass ' m ' is moved from one given point to another does not depend on the path followed.
- describe that the gravitational PE is measured from a reference level and can be positive or negative, to denote the orientation from the reference level.
- define potential at a point as work done in bringing unit mass from infinity to that point.
- explain the concept of escape velocity in term of gravitational constant G , mass m and radius of planet r .
- differentiate conservative and non-conservative forces giving examples of each.
- express power as scalar product of force and velocity.
- explain that work done against friction is dissipated as heat in the environment.

- state the implications of energy losses in practical devices and the concept of efficiency.
- utilize work – energy theorem in a resistive medium to solve problems.
- discuss and make a list of limitations of some conventional sources of energy.
- describe the potentials of some nonconventional sources of energy.

INTRODUCTION

Work and energy are not two different things, but they are correlated to each other. When work is done by one system on another, indeed energy is transferred between the two systems. For example, when an engine pulls a train along a horizontal track, the engine does work. The engine transfers energy to the train. If we assume that there is no loss of energy, then the amount of energy of the moving train is equal to the work done. Similarly, when we walk upstairs, we do work; our work is equal to gain in gravitational Potential Energy (P.E).

Energy is present in the universe in various forms. It can be converted from one form into other form but neither be created nor be destroyed. This is the principle of conservation of energy. For example, a heat engine converts heat energy into mechanical energy. A fan converts electrical energy into mechanical energy, a bulb converts electrical energy into light and heat energy etc. All forms of energy can be explained in terms of kinetic energy or potential energy. These two are the most important types of energies. The Kinetic Energy (K.E) is due to motion while P.E is a stored energy.

Here a question arises, how much work is done or how much energy is consumed? It is measured in terms of rate of doing work or rate of consumption of energy which is called power. For example, a boy may carry a box upstairs in 3 minutes while a man may do it in 1 minute. Obviously, the power of the man is more than the power of the boy. Thus, time factor is important for power. A body which has the capacity to do work is said to possess energy. The greater the capacity of the body to do work, the greater is the energy possessed by it. Thus work, energy and power are related to each other. In this unit, we shall deal with these three most important parameters of physics.

4.1 WORK

In our daily life, we use the meaning of work in terms of any physical or mental activity. For example, reading a book, cooking, shopping etc. all are regarded as work but in physics, the word work has a different meaning. The work is only done when the force acting on an object produces a displacement in it in the direction of force. Thus, for work to be done by a force on an object, the two aspects must be considered:

- (i) Displacement of the object.
- (ii) Component of force in the direction of the displacement.

4.1.1 Work done by a constant force

The work done on an object by a constant force is defined as the product of the magnitude of displacement and the component of force in the direction of the displacement. It is explained as;

Consider an object which is pulled by an applied constant force $\vec{F}$ at an angle θ with a horizontal axis and the body is displaced through a displacement $\vec{d}$ as shown in Fig. 4.1.

The horizontal component of force $\vec{F}$ in the direction of displacement is $F \cos \theta$.

Thus, according to the definition, work is done on the body. This work done by a constant force is expressed as:

Work = (Horizontal component of force) (Displacement)

$$\text{Work} = F_x d$$

$$\text{Work} = F \cos \theta d$$

$$\text{Work} = Fd \cos \theta \quad \dots\dots(4.1)$$

$$\text{Work} = \vec{F} \cdot \vec{d} \quad \dots\dots(4.2)$$

This is a work done in terms of the scalar product of force and displacement. Therefore, work is a scalar quantity. It has magnitude but no direction. The SI unit of work is joule and its dimensional formula is $[ML^2T^{-2}]$.

Joule

One joule is defined as the amount of work done when a force of one newton displaces a body through one metre in its direction. Hence

$$1 \text{ Joule} = 1 \text{ newton} \times 1 \text{ metre}$$

$$1J = 1Nm$$

When the applied force remains constant during the whole path then graphically, there is a straight horizontal line in force and displacement graph as shown in Fig.4.2. The area under this straight horizontal line is equal to the work done on the body under a constant force.

Equation (4.1) shows that work done on the body depends upon force, displacement and angle ' θ ' between force and displacement. The

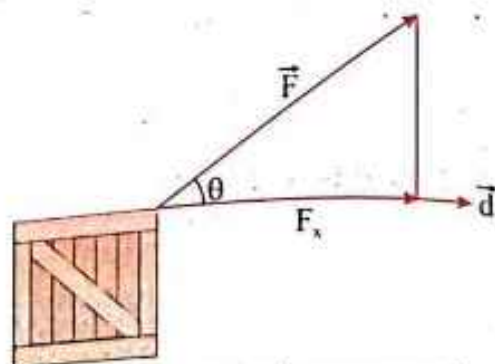


Fig.4.1: The horizontal component of force (F_x) in the direction of displacement (d) does work on the body.

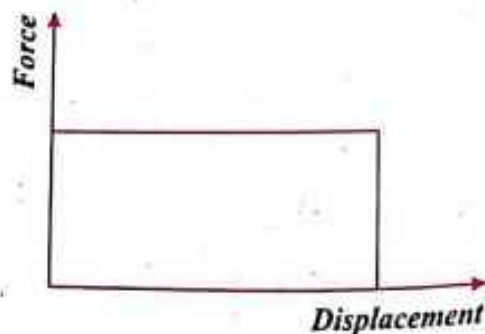


Fig.4.2: The area under a straight Horizontal line in Force-displacement graph showing work done by a constant force

term $\cos\theta$ in this equation indicates that the work done may be positive, negative or zero depending upon the value of θ . Therefore, the work done can be studied under the following three cases;

I. Positive Work (Maximum)

If $\theta < 90^\circ$, $\cos \theta$ is positive so work done is also positive. Under this condition there is a component of force in the direction of the displacement. If the angle between force and displacement is zero i.e. $\theta = 0^\circ$ then;

$$\text{Work} = Fd \cos 0^\circ = Fd$$

This result shows that work done is maximum when the applied force is parallel to the displacement.

Examples

- When a man pushes a cart on horizontal smooth surface, as shown in Fig. 4.3, the force and displacement are in the same direction.
- When a body falls freely under gravity, then gravitational force and displacement are in same direction as shown in Fig.4.4.

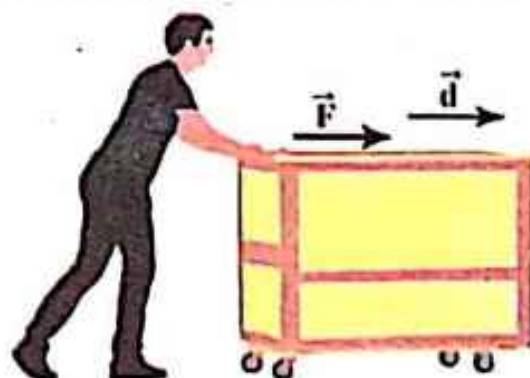


Fig.4.3: Positive work done by a man on cart



Fig.4.4: Positive work on a rolling ball



Fig.4.5: Zero work done by a man on wall

II. Zero Work

If $\theta = 90^\circ$, then $\cos \theta = 0$ and no work is done by the force on the body. Note that work done is also zero when either force or displacement or both are zero.

$$\text{Work} = Fd \cos 90^\circ = Fd (0) = 0 \because \cos 90^\circ = 0$$

Examples

- When a man pushing a rigid wall with a force and fails to move it, then work done is zero as shown in Fig. 4.5.
- When a man holding a pail in his hand while moving on a horizontal level surface as shown in Fig. 4.6 then angle between force and displacement is 90° . Work done by the man is zero.



Fig.4.6: Zero work done on a pail.

III. Negative Work (-ve Maximum)

If $\theta > 90^\circ$, then $\cos\theta$ is negative so work done is also negative. If the component of force is opposite to the direction of the displacement then the angle between force and displacement is 180° .

$$\text{Work} = Fd \cos 180^\circ = Fd(-1) = -Fd$$

This result shows that when the applied force is anti parallel to the displacement and angle ' θ ' between them is 180° then the work done will be negative maximum.

Examples

- When a body makes to slide over the a rough horizontal surface as shown in Fig. 4.7, the frictional force is opposite to the displacement and hence work obtained is -ve (displacement).
- When a body is thrown up, the gravitational force is vertically downward while the displacement is vertically upward. The work done by the body is negative and against the gravitational field.
- Two masses m_1 and m_2 connected by a string which is suspended from a frictionless pulley, as shown in Fig. 4.8. Gravitational force and displacement are opposite to each other. So the work done in this case will be negative if m_1 is lifted in the upward direction.

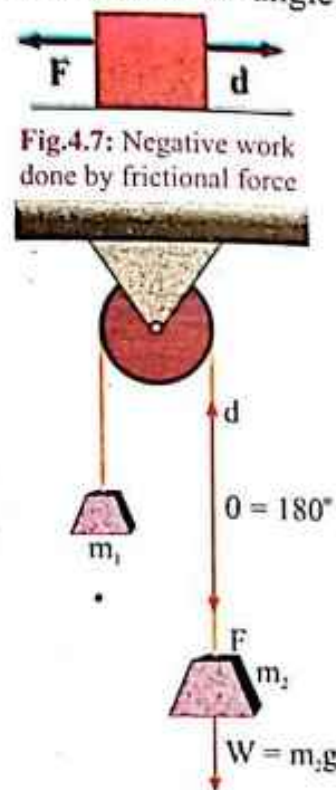


Fig.4.7: Negative work done by frictional force

Fig.4.8: Negative work done by gravitational force

4.1.2 Work done by a variable force

When the applied force on a body remains constant in terms of magnitude and direction then its work done can be calculated by the following equation;

$$\text{Work} = Fd \cos\theta \dots\dots(4.3)$$

So far we have considered the work done by constant forces. But sometimes the applied forces in terms of magnitude or direction, are not constant. For example, when a rocket moves away from the earth, the work is done against the gravitational force which varies as the square of the distance from the centre of the Earth, thus in case of variable force, the work done of an object cannot be determined by using eq.4.3 directly. It requires another relation which is developed as under;

Consider an object being displaced along the x-axis in a straight line under the action of a variable force as shown in Fig. 4.9. To find the total work done, we

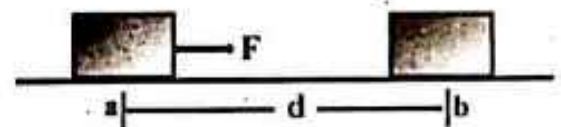


Fig.4.9: A body is displaced by a variable force from point a to point b.

divide the total displacement (whole path) along x-axis into 'n' number of very short segments; $\Delta d_1, \Delta d_2, \Delta d_3, \dots, \Delta d_n$ such that for each segment (displacement) the forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots, \vec{F}_n$ respectively may be treated as constant as shown in Fig.4.10.

Thus when the object moves through the small distance Δd_1 under the action of approximately constant force $\vec{F}_1$, the small amount of work done W_1 is given by;

$$W_1 = F_1 \cos \theta_1 \Delta d_1$$

Similarly, when the object moves through displacement Δd_2 under the action of force F_2 then the work done W_2 is given as;

$$W_2 = F_2 \cos \theta_2 \Delta d_2$$

Since the motion of the object from point 'a' to point 'b' is divided into 'n' number of small and equal segments therefore the total work done on the body is given as,

$$W_{\text{Total}} = F_1 \cos \theta_1 \Delta d_1 + F_2 \cos \theta_2 \Delta d_2 + F_3 \cos \theta_3 \Delta d_3 + \dots + F_n \cos \theta_n \Delta d_n \dots (4.4)$$

$$W_{\text{Total}} = \sum_{i=1}^n F_i \Delta d_i \cos \theta_i \dots (4.5)$$

In the limit then, we have,

$$\text{Work}_{\text{Total}} = \lim_{\Delta d \rightarrow 0} \sum_{i=1}^n F_i \Delta d_i \cos \theta_i \dots (4.6)$$

This is a resultant work done by a variable force and it shows that the total work done by a variable force is equal to the sum of the areas of all the segments (rectangles) from point 'a' to point 'b'. Graphically, when it is plotted on a variable force-displacement graph then we have a curved path as shown in Fig.4.11 and the area under this curved path is equal to the work done by a variable force on a body.

Example 4.1

A man pushes a lawn roller through a distance of 40 m under the action of force of 50 N which makes an angle of 60° with the direction of motion. Calculate the work done.

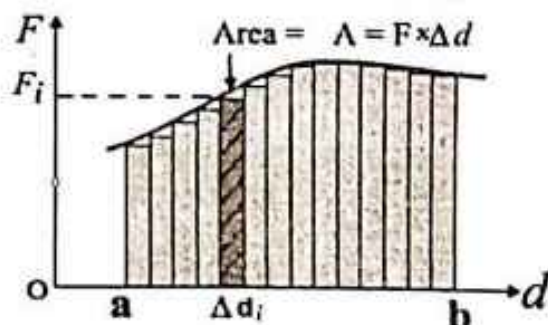


Fig.4.10: The path which is divided into 'n' number of small and equal displacements

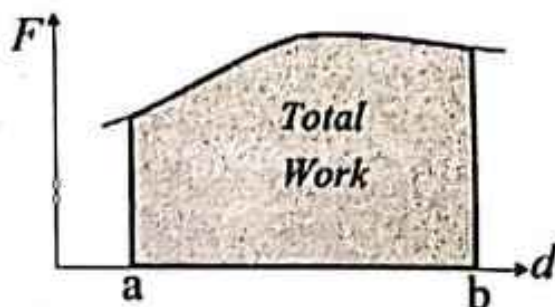


Fig.4.11: Area under curved path in the F-d graph is equal to the total work done by variable force.

Solution:

$$F = 50 \text{ N}$$

$$d = 40 \text{ m}$$

$$\theta = 60^\circ$$

$$\text{Work} = ?$$

$$\text{Work} = Fd \cos \theta$$

$$\text{Work} = (50)(40) \cos 60^\circ$$

$$\text{Work} = 2000 (0.5)$$

$$\text{Work} = 1000 \text{ J}$$

The work is done 100% when the applied force is acting at angle ' θ ' in the direction of displacement, what would be the angle of applied force when work is done 50%.

4.2 WORK DONE IN A GRAVITATIONAL FIELD

The space around the Earth in which the Earth can attract a body toward its centre is known as a gravitational field. Similarly, the gravitational force per unit mass on a body is called gravitational field strength and its SI unit is N kg^{-1} .

Consider a force F which is applied on a body of mass ' m ' and the body is raised from the surface of the Earth with uniform velocity in a gravitational field through a height ' h ' as shown in Fig. 4.12. It means that work is done on the body against the force of gravity and it is given as;

$$\text{Work} = F d \cos \theta$$

As the body moves under the force of gravity;

so $F = W = mg$ and $d = h$, and angle θ between weight (force) and displacement is 180° .

$$\text{Thus, } \text{Work} = W h \cos 180^\circ$$

$$\text{Work} = mg h (-1) = -mgh$$

The gravitational field is a conservative and it has the following properties;

- The work done in a field does not depend upon its path but it depends upon its initial and final points.
- The work done in a field along a closed path is zero.
- The work done on a body against the direction of gravitational field is stored in terms of its P.E.

4.2.1 Work done in a gravitational field is independent of path

Consider an object of mass ' m ' which can be displaced in a gravitational field with a constant velocity from point 'A' to 'C' along the following two different paths. The first path is the direct path from 'A' to 'C' and the other path is from 'A' to 'B' and then 'B' to 'C' as shown in Fig. 4.13.

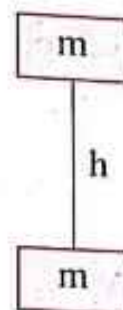


Fig.4.12: Work done on a body in a gravitational field

Let us consider the first path which is direct path from 'A' to 'C', the work done along this path is calculated as,

$$\text{Work}_{A \rightarrow C} = Fd \cos \theta$$

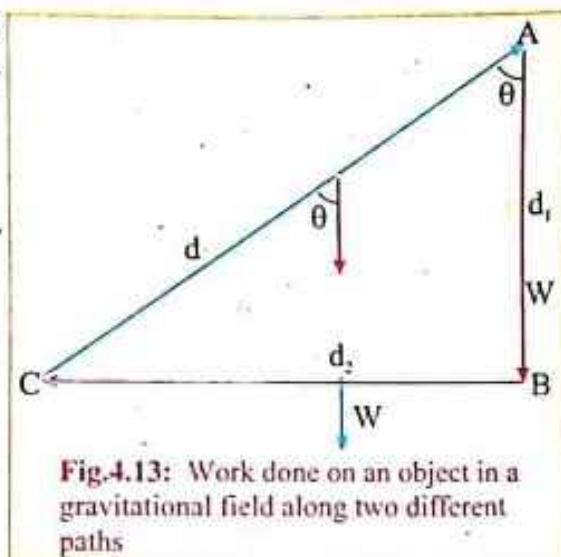
$$\text{Work}_{A \rightarrow C} = Wd \cos \theta \quad \because F = W$$

Using triangle ABC

$$\frac{d_1}{d} = \cos \theta$$

$$d_1 = d \cos \theta$$

$$\therefore \text{Work}_{A \rightarrow C} = W d_1 \dots\dots(4.7)$$



Let us now consider the second path i.e. path AB and path BC. Thus, the work done along AB is give as;

$$\text{Work}_{A \rightarrow B} = Fd_1 \cos \theta$$

$$\text{Work}_{A \rightarrow B} = Wd_1 \cos \theta \quad \because F = W$$

As the angle ' θ ' between W and d_1 is 0°

$$\text{So, } \text{Work}_{A \rightarrow B} = Wd_1 \cos 0 \quad (\text{because } \cos 0 = 1)$$

$$\text{Work}_{A \rightarrow B} = Wd_1 \dots\dots(4.8)$$

Similarly, the work done from B to C is,

$$\text{Work}_{B \rightarrow C} = Fd_2 \cos \theta$$

$$\text{Work}_{B \rightarrow C} = Wd_2 \cos \theta$$

As the angel ' θ ' between W and d_2 is 90°

$$\text{So, } \text{Work}_{B \rightarrow C} = Wd_2 \cos 90^\circ \quad \because \cos 90^\circ = 0$$

$$\text{Work}_{B \rightarrow C} = Wd_2 (0)$$

$$\text{Work}_{B \rightarrow C} = 0 \dots\dots(4.9)$$

From eq. (4.8) and eq. (4.9), we get

$$\text{Work}_{A \rightarrow B \rightarrow C} = Wd_1 + 0$$

$$\text{Work}_{A \rightarrow B \rightarrow C} = Wd_1 \dots\dots(4.10)$$

By comparing eq.4.7 and eq.4.10, we conclude that work done in a gravitational field is independent of path followed.

We can calculate the work done by a force on an object, but that force is not necessarily the cause of the displacement. For example, if you lift a body, work is done on the object by the gravitational force, although gravity is not the cause of the object moving upward.

The change in the gravitational potential energy of an object does not depend on the path it takes.

4.2.2 Work done in a gravitational field along a closed path is zero

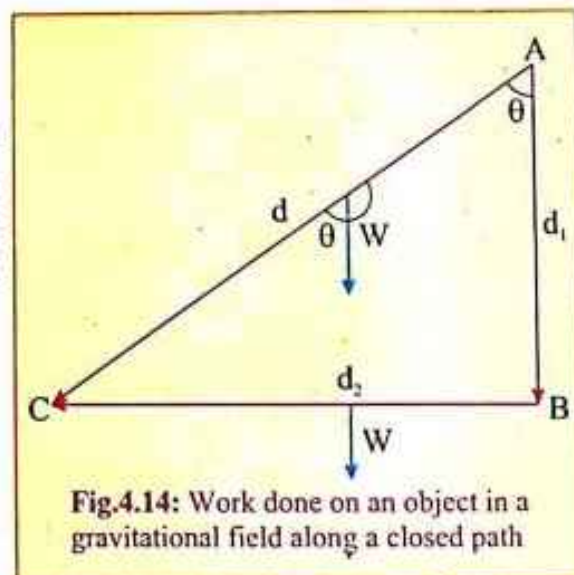
Let us find the work done in a gravitational field along a closed path ABCA. We have already calculated work along the path AC, AB and BC in previous section and these are as under.

$$\text{Work}_{A \rightarrow B} = Wd_1$$

$$\text{Work}_{B \rightarrow C} = 0$$

$$\text{Work}_{A \rightarrow C} = Wd_1$$

$$\text{Work}_{C \rightarrow A} = -Wd_1$$



It is noted that work from A to C and work from C to A are same in magnitude but in opposite direction.

$$\text{Total work done in a closed path ABCA} = \text{Work}_{A \rightarrow B} + \text{Work}_{B \rightarrow C} + \text{Work}_{C \rightarrow A}$$

$$\text{Work done in a closed path ABCA} = Wd_1 + 0 + (-Wd_1)$$

$$\text{Work done in a closed path ABCA} = 0 \quad \dots\dots(4.11)$$

Equation (4.11) shows that work done along a closed path in a gravitational field is zero.

From the above discussion, it is concluded that the work done on the body in a gravitational field is independent of path followed and the work done on the body in a gravitational field along a closed path is zero. Thus, the gravitational field is a conservative field.

4.3 ENERGY

The word energy is derived from the Greek word "Energeia" which means work. Hence energy is defined as the ability (or capacity) of a body to do work. This implies that energy is associated with the performance of work because the more work is done; the greater the quantity of energy is needed. Work is always done by a force. It means that a body possessing energy can exert force on any other body to do work. In other words, when a work is done on a body, an equal amount of energy is stored in it.

Energy is a scalar quantity. The SI unit of energy is same as that of the work, i.e., joule (J) and its dimensional formula is $[ML^2T^{-2}]$.

There are two basic form of energies.

(i) Kinetic Energy

(ii) Potential Energy

4.3.1 Kinetic Energy

The energy possessed by a body due to its motion is called kinetic energy. For example, a moving ball can break a window glass and a hammer can drive a nail into wood. These examples show that the K.E of a body depends upon its motion. The faster a body moves, the greater is its K.E. The mathematical expression of K.E of a body of mass 'm' moving with velocity 'v' is given as;

$$K.E = \frac{1}{2}mv^2 \dots\dots(4.12)$$

4.3.2 Potential Energy

The potential energy of a body is defined as the energy possessed by the body by virtue of its position or configuration.

A body can gain potential energy only when work is done on it. For example,

- I. When a body is lifted to some height against the gravitational force, then there is increase in its gravitational P.E.
- II. The water at the top of a water fall, or water stored in a dam possess gravitational P.E.
- III. If a spring is compressed, an elastic P.E is stored in it because a work is done in compressing the spring against the elastic force.

The mathematical relation for gravitational P.E. can be expressed as;

Consider a force is applied vertically upward on a body of mass 'm'. The body is raised up to a certain height 'h' above the Earth's surface as shown in Fig.4.15. It means there is a work done on the body against the direction of gravitational field and this work is stored in the body in terms of its potential energy. The value of such P.E is calculated as; by definition of work done.

$$\text{Work} = F.d$$

As the body is under gravity, so $F = W = mg$ and $d = h$.

$$\text{Work} = mgh$$

This work is stored in the body in the form of gravitational potential energy.

$$\text{Thus, } P.E = mgh \dots\dots(4.13)$$

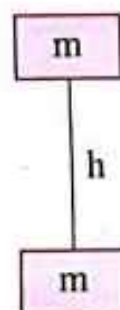
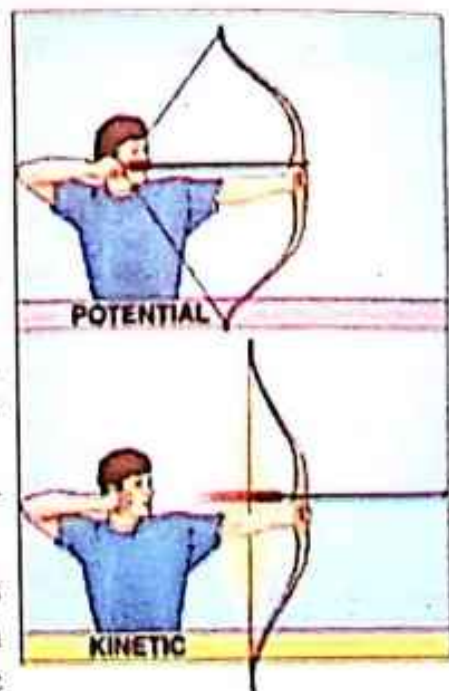


Fig.4.15: P.E due to the work done on the body at height h against the gravitational field.

The eq. (4.13) shows that the P.E. of a body depends upon height. That is, higher a body is above the Earth, the greater will be its P.E. If the height of object from the surface of Earth is zero then its P.E is also zero. In this expression, the surface of Earth is considered as reference point for zero potential energy.

Example 4.2

A neutron of mass 1.67×10^{-27} kg travels a distance of 12 m in 3.6×10^{-4} s. If its speed remains constant, then what is its K.E?

Solution: Mass = $m = 1.67 \times 10^{-27}$ kg
 Distance = $S = 12$ m
 Time = $t = 3.6 \times 10^{-4}$ sec
 Kinetic Energy = K.E = ?

$$v = \frac{S}{t}$$

$$v = \frac{12}{3.6 \times 10^{-4}} = 3.33 \times 10^4 \text{ m s}^{-1}$$

$$\text{K.E} = \frac{1}{2}mv^2$$

$$\text{K.E} = \frac{1}{2}(1.67 \times 10^{-27})(3.33 \times 10^4)^2$$

$$\text{K.E} = 9.26 \times 10^{-19} \text{ J}$$

FOR YOUR INFORMATION

The matter in 0.453 kg of anything, when it is completely converted into energy according to, $E = mc^2$, will produce 11400 million kilowatt-hours of energy.

$$1 \text{ kWh} = 3.6 \times 10^6 \text{ joules}$$

4.4 WORK - ENERGY THEOREM

This theorem is stated as; "The work done by an applied force on a body is equal to the change in its energy either K.E or P.E". It is explained as;

When a force is applied on a body in the direction of motion of the body, the speed and hence kinetic energy of the body increases. According to work-energy theorem, the increase in kinetic energy of the body is equal to the work done by the force on the body.

Similarly, if a force is applied on a body in the direction opposite to its motion, kinetic energy of the body decreases. This decrease in kinetic energy is equal to the work done by the body against the retarding force. In either case, the change in kinetic energy of the body is equal to the work done (positive or negative work done). A mathematical relation for work-energy theorem is derived as under.

Consider a force 'F' which is applied on a body of mass 'm' moving with initial velocity ' v_i ' and after some time it covers a displacement 'd' and its final velocity becomes ' v_f ' as shown in Fig. 4.16. Now work done on the body is given as;

$$\text{Work} = W = Fd \dots\dots(4.14)$$

According to Newton's second law of motion.

$$F = ma \dots\dots(4.15)$$

According to 3rd equation of motion.

$$2ad = v_f^2 - v_i^2$$

or

$$d = \frac{v_f^2 - v_i^2}{2a} \dots\dots(4.16)$$

Substituting eq. (4.15) and eq. (4.16) in eq. (4.14), we have

$$W = ma \times \left(\frac{v_f^2 - v_i^2}{2a} \right)$$

$$W = \frac{mv_f^2 - mv_i^2}{2}$$

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \dots\dots(4.17)$$

$$W = K.E_f - K.E_i$$

$$W = \Delta K.E$$

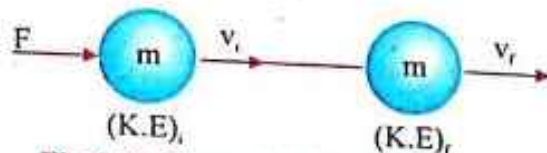


Fig.4.16: Change in K.E of a body due to Work done

This is a mathematical form of work-energy theorem. Here work is expressed in terms of change in K.E. Similarly, the same principle can be used for P.E i.e.,

$$\text{Work} = (P.E.)_f - (P.E)_i$$

$$\text{Work} = \Delta P.E.$$

Example 4.3

A force of 1500 N is acting horizontally on a vehicle of mass 200 kg and the vehicle starts its motion from rest. What will be the speed of the vehicle after covering a distance of 30 m?

Solution:

$$F = 1500 \text{ N}$$

$$m = 200 \text{ kg}$$

$$\theta = 0^\circ$$

$$v_i = 0 \text{ ms}^{-1}$$

$$v_f = ?$$

$$d = 30 \text{ m}$$

$$W = K.E_f - K.E_i$$

$$Fd \cos \theta = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$Fd \cos \theta = \frac{1}{2}mv_f^2 - \frac{1}{2}m(0)^2$$

$$Fd \cos \theta = \frac{1}{2} m v_f^2$$

$$v_f^2 = \frac{2Fd \cos \theta}{m}$$

$$v_f^2 = \frac{2(1500)(30) \cos 0^\circ}{200} = \frac{90000}{200}$$

$$v_f^2 = 450 \text{ m}^2 \text{ s}^{-2}$$

$$v_f = 21.2 \text{ ms}^{-1}$$

4.5 GRAVITATIONAL POTENTIAL ENERGY

When a body is raised to certain height 'h' from the surface of Earth in a gravitational field, then work is said to be done on the body against the gravitational field. This work is stored in the body in the form of its gravitational potential energy. Its value is given as,

$$\text{Gravitational P.E} = mgh \quad \dots\dots(4.18)$$

This shows that the gravitational P.E depends upon height. That is, when the body gains height, its P.E increases while the value of 'g' decreases. Now at very large height where the value of 'g' becomes zero and the P.E due to the work done on a body from the surface of Earth to above stated point is called absolute gravitational potential energy whose value can be calculated as;

Consider a body of mass 'm' which is raised above the surface of Earth at a distance 'r' from the centre of Earth. Then the gravitational force between the body and the Earth is given by;

$$F = G \frac{mM}{r^2} \quad \dots\dots(4.19)$$

where 'G' is a gravitational constant whose value is $6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$ and 'M' is mass of the Earth. Equation (4.19) shows that the gravitational force is inversely proportional to the square of the distance. Therefore, this relation cannot be used directly to calculate the total work on the body. For this we divide the whole path into 'N' number of points (1,2,3 ...N) at distances ($r_1, r_2, r_3, r_4, \dots, r_N$) respectively from the centre of the Earth such that the force in each step almost remains constant

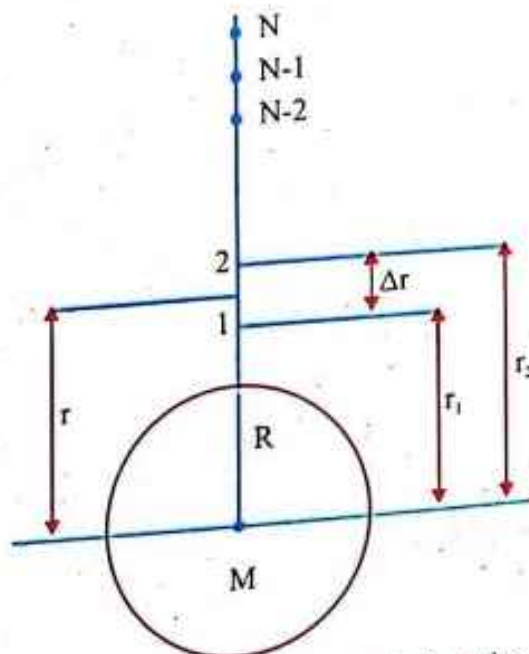


Fig.4.17: Work done on a body from the surface of earth to the point 'N'.

because the distance between every two consecutive points is ' Δr ' and it remains the same.

First we consider the points 1 and 2 at distances r_1 and r_2 respectively from the center of the earth, to get an average force, we consider the midpoint between 1 and 2 at a distance ' r ' from centre of earth such that;

$$r = \frac{r_1 + r_2}{2}$$

As

$$\Delta r = r_2 - r_1$$

and

$$r_2 = r_1 + \Delta r$$

Thus

$$r = \frac{r_1 + r_1 + \Delta r}{2}$$

$$r = \frac{2r_1 + \Delta r}{2}$$

Squaring both sides

$$r^2 = \left(\frac{2r_1 + \Delta r}{2} \right)^2$$

$$r^2 = \frac{4r_1^2 + \Delta r^2 + 4r_1\Delta r}{4}$$

Δr^2 is very small, so this term can be neglected.

$$r^2 = r_1^2 + r_1\Delta r$$

Substituting the value of Δr in above equation

$$r^2 = r_1^2 + r_1(r_2 - r_1) = r_1^2 + r_1r_2 - r_1^2$$

$$r^2 = r_1r_2$$

Hence, Eq. (4.19) becomes

$$F = G \frac{mM}{r_1r_2}$$

Work from 1 to 2 we have,

$$W_{1 \rightarrow 2} = Fd \cos \theta$$

$$W_{1 \rightarrow 2} = F\Delta r \cos 180^\circ$$

$$W_{1 \rightarrow 2} = F\Delta r(-1)$$

$$W_{1 \rightarrow 2} = -F\Delta r$$

Negative sign shows that the work is done against the gravitational field.
Substituting the values of F and Δr



A boy bounces on a trampoline. The boy moves upward with an initial speed v and reaches maximum height with a final speed of zero. So energy changes from elastic P.E to K.E and then into gravitational P.E

$$W_{1 \rightarrow 2} = -G \frac{mM}{r_1 r_2} (r_2 - r_1)$$

$$W_{1 \rightarrow 2} = -GmM \left(\frac{r_2 - r_1}{r_1 r_2} \right)$$

$$W_{1 \rightarrow 2} = -GmM \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \dots\dots(4.20)$$

Similarly, the work done from 2 to 3 is

$$W_{2 \rightarrow 3} = -GmM \left(\frac{1}{r_2} - \frac{1}{r_3} \right) \dots\dots(4.21)$$

Finally the work done from r_{N-1} to r_N is

$$W_{N-1 \rightarrow N} = -GmM \left(\frac{1}{r_{N-1}} - \frac{1}{r_N} \right) \dots\dots(4.22)$$

Hence, the total work done from point 1 to N is

$$\begin{aligned} W_{\text{Total}} &= W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 4} + \dots\dots + W_{N-1 \rightarrow N} \\ W_{\text{Total}} &= -GmM \left(\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2} - \frac{1}{r_3} + \dots\dots + \frac{1}{r_{N-1}} - \frac{1}{r_N} \right) \\ W_{\text{Total}} &= -GmM \left(\frac{1}{r_1} - \frac{1}{r_N} \right) \dots\dots(4.23) \end{aligned}$$

This work cause of P.E of the body from point (1) to point (N). It is represented by Δu . Thus according to work energy theorem.

$$\Delta u = -W_{\text{Total}} \quad u_N - u_1 = GmM \left(\frac{1}{r_1} - \frac{1}{r_N} \right) \dots\dots(4.24)$$

If the r_N is very large distance say at infinity ($r_N = \infty$), at that point $u_N = 0$ and u_1 at distance r_1 from the centre of the earth is known as absolute potential energy. Thus, eq. 4.24 becomes;

$$0 - u_1 = GmM \left(\frac{1}{r_1} - \frac{1}{\infty} \right) \quad \because \left(\frac{1}{\infty} = 0 \right)$$

$$u_1 = -GmM \left(\frac{1}{r_1} \right)$$

or

$$u_1 = -\frac{GMm}{r_1} \dots\dots(4.25)$$

The gravitational potential energy is called absolute gravitational potential energy on the surface of Earth. If $r_1 = R$ and $u_1 = u$. Then, eq.4.25 becomes;

$$U = -\frac{GMm}{R}$$

This is the absolute potential energy on the surface of the Earth.

4.6 ESCAPE VELOCITY

When a body is projected vertically upward from the surface of Earth then due to the gravitational force of attraction, the velocity of the body decreases and finally becomes zero at some height and the body returns to the ground. If we keep on increasing the initial projection velocity of the object, a stage will be reached such that its final velocity becomes zero at the point where the gravitational field becomes zero as shown in Fig.4.18. The body escapes the gravitational field of the Earth or any other planet and it will not return back. This projected velocity is called its escape velocity. Its value can be calculated as;

When a body is projected upward with maximum initial velocity, then it loses its K.E and gains P.E. Thus the escape velocity can be calculated by using the law of exchange of energy.

Loss of initial K.E. = Gain in absolute P.E.

$$\begin{aligned}\frac{1}{2}mv_{esc}^2 &= \frac{GMm}{R} \\ v_{esc}^2 &= \frac{2GM}{R} \\ v_{esc} &= \sqrt{\frac{2GM}{R}} \dots\dots(4.26)\end{aligned}$$

We know that

$$g = \frac{GM}{R^2}$$

or

$$gR = \frac{GM}{R}$$

Hence,

$$v_{esc} = \sqrt{2gR}$$

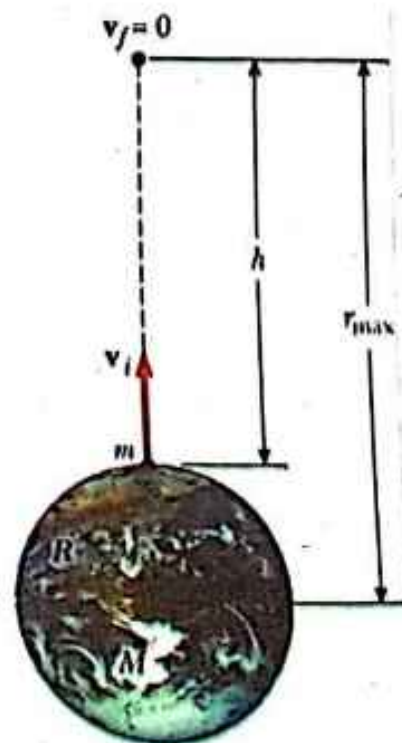


Fig.4.18: Escape velocity of a body from surface of the earth

Escape Velocities for Planets		
Planets	Velocity m/s	Velocity $k.m/s$
Sun	618033.60	618.03
Mercury	4247.56	4.25
Venus	10360.79	10.36
Earth	11174.36	11.17
Mars	5021.09	5.02
Jupiter	59542.35	59.54
Saturn	35457.55	35.46
Uranus	21284.62	21.28
Neptune	23439.59	23.44
Moon	2375.18	2.38

$$v_{\text{esc}} = \sqrt{2(9.8 \text{ m s}^{-2})(6.4 \times 10^6 \text{ m})}$$

$$v_{\text{esc}} = 11200 \text{ m s}^{-1}$$

$$v_{\text{esc}} = 11.2 \times 10^3 \text{ m s}^{-1}$$

$$v_{\text{esc}} = 11.2 \text{ km s}^{-1}$$

This result shows that if a body is thrown upward from the surface of Earth with a velocity of 11.2 km s^{-1} or more, then it will never return to the Earth.

It is worth noting that the escape velocity does not depend on the mass of the body. It is the same for all masses for a given planet.

4.7 POWER

The rate at which energy is transferred or work is done by a body is a sign of power of that body. For example, a boy may carry a box upstairs in 3 minutes while a man may do it in 1 min. Obviously, the power of the man is more than the power of the boy. This example also shows that the time factor is important for the power. That is, power is defined in terms of the ratio between work and time. Thus, it is stated as **the rate of doing work of a body or rate of transfer of energy of a system** is called its power.

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Energy}}{\text{Time}} \dots\dots(4.27)$$

We can also find another expression for power. Suppose a force ' $\vec{F}$ ' acts on a body so that it moves with velocity ' $\vec{v}$ ' and it covers a displacement $\vec{d}$ then by definition of work

$$W = \vec{F} \cdot \vec{d}$$

Eq. 4.27 becomes,

$$P = \frac{\text{Work}}{\text{Time}} = \frac{\vec{F} \cdot \vec{d}}{t} = \vec{F} \cdot \frac{\vec{d}}{t}$$

$$P = \vec{F} \cdot \vec{v} \dots\dots(4.28) \quad \because \vec{v} = \frac{\vec{d}}{t}$$

This is a power may be defined as dot product of the force and the velocity of the body. It is also written as;

$$P = \vec{F} \cdot \vec{v} = Fv \cos \theta$$

Since power is the dot product of $\vec{F}$ and $\vec{v}$, so it is a scalar quantity i.e., power has magnitude but no direction. The dimensional formula of power is $[ML^2T^{-3}]$.

Unit of Power

The SI unit of power is watt.

Watt

The power of a body is 1 watt if it is doing 1 J of work in 1s, or One watt is equal to work of one joule per second.

When the rate of doing work is different, then we introduce average power, if ΔW be the total amount of work which is done in time Δt . Then the average power of the body is given as;

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \dots\dots(4.29)$$

Now when the rate of doing work of a body is for very short interval of time which approaches to zero then the power is called instantaneous power.

$$P_{\text{inst}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t} \dots\dots(4.30)$$

Watt is SI unit of power and its use is very common in electrical engineering. However, in mechanical engineering, horse power is the practical unit of power. The relation between horse power (hp) and watt (W) is as under;

$$1 \text{ h.p} = 746 \text{ W}$$

Example 4.4

What is the power of an electric motor when it performs work of $6.45 \times 10^7 \text{ J}$ in 12 hours?

Solution:

$$\text{Power} = P = ?$$

$$\text{Work} = W = 6.45 \times 10^7 \text{ J}$$

$$\text{Time} = t = 12 \text{ hours}$$

$$\text{Time} = t = 12 \times 3600 \text{ s}$$

$$t = 4.32 \times 10^4 \text{ s}$$

$$P = \frac{\text{Work}}{\text{Time}}$$

$$P = \frac{6.45 \times 10^7 \text{ J}}{4.32 \times 10^4 \text{ s}}$$

$$P = 1.49 \times 10^3 \text{ W}$$

4.8 WORK DONE AGAINST FRICTION

When a body does work then there is a resistive force against the motion of the body called friction. This friction dissipates the kinetic energy of the body and it causes decreasing the efficiency of the body in performing work. But its role in the

working of a body cannot be neglected. Thus in the presence of friction, the work of the body is called frictional work. To calculate the frictional work, we consider a block of mass 'm' lying on a rough horizontal surface such that there is a coefficient of friction between the two surfaces. In order to move the block, we applied a constant force F in the horizontal direction against the kinetic friction force f_k . Thus, the resultant force acting on the block is given by;

Resultant force = applied force – friction force

$$ma = F - f_k \quad \dots\dots (4.31)$$

As the applied force causes change in velocity of the body from v_i to v_f through a horizontal displacement 'd'. So according to 3rd equation of motion;

$$2ad = v_f^2 - v_i^2$$

$$a = \frac{v_f^2 - v_i^2}{2d}$$

Hence equation (4.31) becomes;

$$m \left(\frac{v_f^2 - v_i^2}{2d} \right) = F - f_k$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = Fd - f_k d$$

$$Fd = \Delta K.E + f_k d$$

$$\text{Work} = \Delta K.E + f_k d \quad \dots\dots (4.32)$$

This is the work done on a block which is sliding on a horizontal surface. This shows that a part of work is used for change in K.E. of the block. The remaining part is used against the friction between the two surfaces.

On the other hand, the work done against the friction is definitely converted into heat or thermal energy. As a result, both the surfaces warm up and their temperature is raised. The gain in thermal energy by the surfaces will then transfer into the environment. The same process of transfer of heat into the environment will be observed, when the work is done on a body in a gravitational field or work is done on a machine. Thus, equation (4.32) can also be expressed as;

$$\text{Work} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}} \quad \dots\dots (4.33)$$

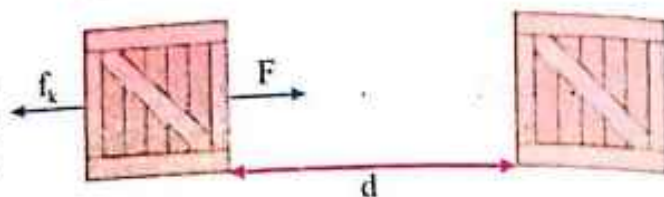
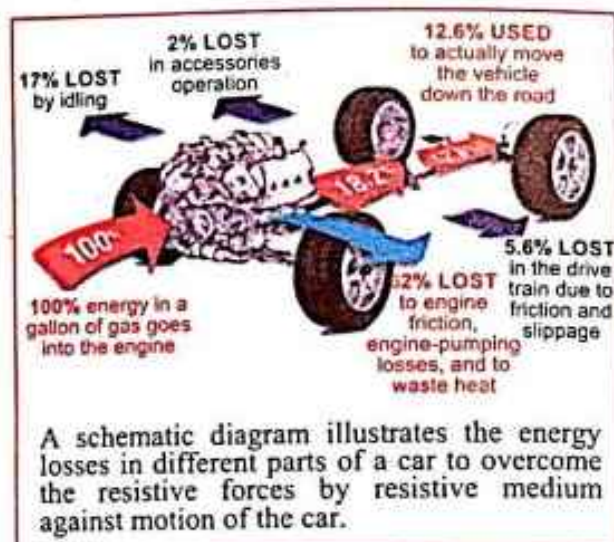


Fig.4.19: A block is sliding over the horizontal surface under the action of applied constant force against the friction force.



4.9 IMPLICATION OF ENERGY LOSSES IN PRACTICAL DEVICES AND EFFICIENCY:

According to law of conservation of energy, the energy can transform from one form to another form through a system or a device but the total energy remains constant. It is possible only when a system or a device is free from friction. In real life situation, frictional forces are always present and a device cannot do any work without friction. Due to presence of frictional forces, it is not possible to convert the available energy completely into useful work. Only a fraction of energy is converted into useful work and the rest is wasted in form of heat. How much energy is utilized for the useful work by the system or device and how much energy is wasted?

In this regard, we can determine the efficiency of any device or system in terms of ratio using the following relation;

$$\text{Efficiency} = \frac{\text{Utilised output energy}}{\text{Total input energy}} \times 100 \quad \text{..... (4.33)}$$

An ideal machine or engine (Carnot engine) is a theoretical machine or engine whose efficiency is 100%. Because, there is no loss of energy in an ideal machine. i.e., their output is equal to their input. Practically, it is not possible to design a machine which will have 100% efficiency because frictional forces are always present. These frictional forces can be minimized but cannot be eliminated completely.

4.10 CONSERVATIVE AND NON-CONSERVATIVE FORCES

All forces can be classified into two classes on the basis of their different properties i.e., conservative and non-conservative forces.

4.10.1 Conservative Force

A force is said to be a conservative force, if the work done in moving a body between any two points is independent of path followed but it depends on the initial and final positions of the body. In other words, we can also say that a force is conservative if the work done on a body is zero when the body moves around any closed path returning to its initial position.

This definition shows an important feature of conservative force i.e., work done by a conservative force is recoverable.

The work done by a conservative force is always stored in a body in the form of potential energy and in the presence of a conservative force; law of conservation of energy of an isolated system is valid.

Some common examples of conservative forces are:

- (i) The gravitational force.
- (ii) The force exerted by a spring.
- (iii) The electrostatic force between two charges.

4.10.2 Non-Conservative Force

A force is said to be a non-conservative if the work done by that force in moving a body between two points depends on the path followed. Similarly, the work done by a non-conservative force in moving a body along a closed path is not zero. In other words, work done by a non-conservative force cannot be represented by potential energy and the law of conservation of energy is not valid in the presence of non-conservative forces.

Some common examples of non-conservative forces are:

- (i) The frictional force.
- (ii) The resistance force exerted by resistive mediums force.
- (iii) The tension.

Let us explain a non-conservative force with an example. Suppose you have to displace a book between two points A and B on a rough horizontal surface such as; a table as shown in Fig.4.20.

If the book is displaced in a straight line between the two points, then the work done by friction force is given as;

$$W = Fd$$

However, if the book is moved along a semi-circular path between the two points, the work done by the frictional force would be greater than the work in a straight line. Finally, if the book is moved around a closed path, the work done by the frictional force is not zero. Thus the work done by a non-conservative force is not recoverable.

4.11 NATURAL SOURCES OF ENERGY

In this technological age, the demand of energy is increasing rapidly day by day. Energy is one of the most important factor of the economic infrastructure and it is the basic input source to maintain economic growth. Thus, one can say that all the developments of the countries directly or indirectly depend upon energy. This is a reason that why over the last few decades; the scientists are

FOR YOUR INFORMATION

Conservative Forces	Non Conservative Forces
Gravitational	Friction
Elastic	Air Resistance
Electric	Tension in cord
	Motor or rocket Propulsion
	Push or pull by a person

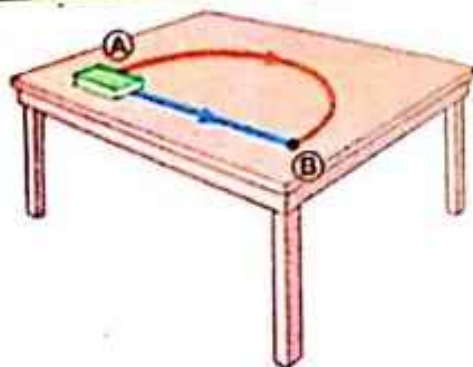
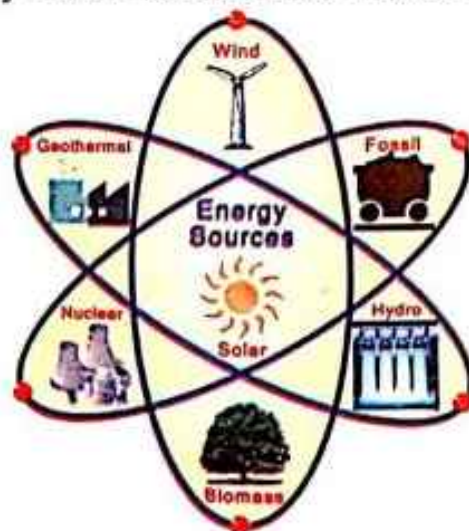


Fig.4.20: Work done by a non-conservative force (Frictional force)



paying attention towards the exploring of new energy resources in order to fulfill the energy demands. The energy sources are classified into two groups.

- Conventional sources or Non-renewable sources of energy
- Non-conventional or Renewable source of energy

4.11.1 Conventional or non-renewable sources of energy

Conventional or non-renewable sources of energy are those which can be used for a long time. These sources are exhaustible and they cannot replenish easily. Coal, oil and natural gas called fossil fuels are the examples of non-renewable sources of energy. All these are remnants of plants and animals and their formation took billion of years. For example, plants and animals store energy under process of photosynthesis. This stored energy remains with them when they die. Therefore, it has been estimated that the fossil fuels were formed by natural process over millions of years ago when decomposed plants and animals matter was buried in earth's crust.

Although the world's major energy sources are fossil fuels but they are hydro-carbons. That is they contain high percentage of carbon. So, when these fossil fuels are burnt they release carbon dioxide, methane and nitrogen into the atmosphere. This causes the pollution, which leads to smog, acid rain and a greenhouse effect.

The greenhouse effect refers to the rising temperature caused by the sun's energy being trapped in our atmosphere by these extra gases. This raises the earth's temperature. It is estimated that 71% of the energy is being used in the world from these conventional sources. The pie chart about the contribution of various energy sources is shown in Fig.4.21. On the other hand, uranium and hydro are also conventional sources of energy but they are neither fossil fuels, nor exhaustible and they also make no pollution.

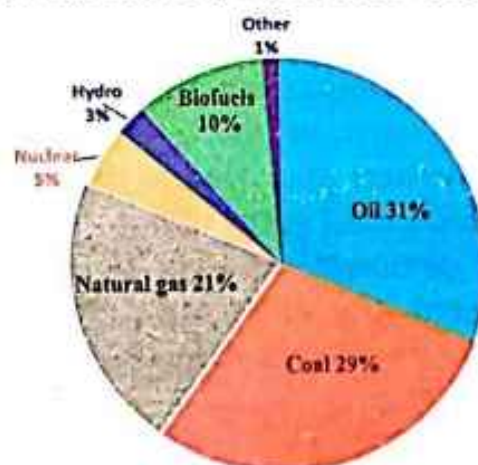


Fig.4.21: Pie chart natural sources of energy

I. Coal:

Coal is also known as black diamond which is the most abundant solid form of fossil fuel on the Earth. The world total coal reserves are estimated around in trillion metric tons, which is sufficient for more than 200 years for the energy generation. Pakistan has the 7th largest coal reserves in the world.



Fig.4.22: Coal thermal power plant

Thar coal reserves in Sindh contain 175-billion tonnes like; Shahrag, Marwaar, Duki, Much, Chamalung etc. also contain coal at large scale. The usage of coal at the large scale played a vital role in the industrial revolution in the 19th and 20th centuries and it still remains essential for the industrial sector in the 21st century.

Coal is used for electric power generation and also used directly in heavy industries like steel making. A typical coal thermal power plant is shown in Fig.4.22.

II. Oil:

Oil is a liquid fossil fuel which is found underground in form of crude oil through drilling. This crude oil is refined into various products such as, gasoline, diesel fuel, jet fuel, etc. through distillation process.

Oil is not only being used for transportation but by products of crude oil are also used in the production of plastic tyres, polyesters etc. Due to the process of burning of oil, harmful gases like carbon dioxide are emitted in atmosphere and it is a major cause of greenhouse effect and global warming. An oil power station is shown in Fig.4.23.

III. Natural Gas:

Natural gas is a 3rd form of Fossil fuels. It is found in oil deposits. It is mainly composed of methane and ethane. It burns completely and leaves no ashes. It emits less carbon dioxide than coal and oil and is a friendly fuel. Natural gas is used for cooking, heating and transporting as well as in industries.

Natural gas is the 2nd largest energy source in Pakistan. The first gas field in Pakistan was discovered in 1952 at Sui in Balochistan.

Sui gas field is contributing 46 percent of the total installed gas capacity of the country. A Natural gas power plant is shown in Fig.4.24.

million-ton coal. Some other areas

Interesting Information

Fuel	Approximate Time Scale of Formation (million years)	Approximate Energy per mass of fuel (kWh/Kg)
Residual fuel oil	165	12.5
Coal	325	8
Diesel	165	12.9
Natural gas	165	10.8
Uranium	6000	22500000

derground and it is obtained in the oil is refined into various energy heating oil etc. Using fractional



Fig.4.23: Oil Power Plant

s found under the oceans and near with small amount of propane and This is the reason that Natural gas and it is known as environmental

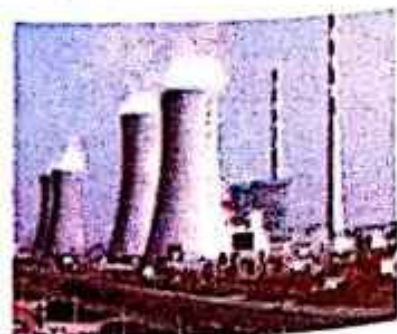


Fig.4.24: Natural Gas Power Plant

IV. Nuclear Power:

Nuclear power is not a fossil fuel. It is a conventional and non-renewable wonderful source of energy. It adds up 12% of the world's total installed electric capacity. In nuclear power plant, fission process is being used to get the nuclear energy.

Nuclear fission is the most common technique to harness nuclear energy in which

uranium, sometimes plutonium is used as a fuel like fossil fuels. The fission of 1 kg of uranium has the capacity to produce the same amount of energy as one million ton of coal. Nuclear power plant does not exhaust any greenhouse gases in atmosphere, but it has its own drawbacks along with benefits. The radioactive waste products remain dangerous for thousands of years and must be safely locked away so that they cannot get into the environment. In Pakistan, there are five functional nuclear power plants i.e. KANUPP in Karachi, CHASNUPP-I, CHASNUPP-II, CHASNUPP-III, CHASNUPP-IV Miawali district. The total installed capacity of these plants is 1430 MW while 2500MW is under construction.

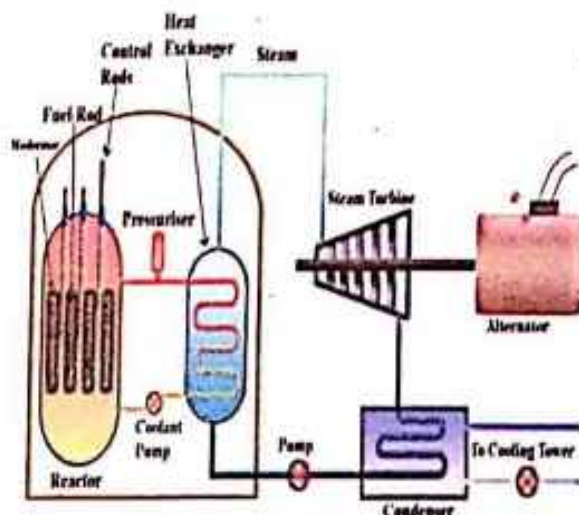


Fig.4.25: Nuclear Power Plant

V. Hydro Power:

Hydro power is conventional and renewable form of energy and it is obtained from hydro power generator. Such generator operates on running water or water falling from high potential to low potential. In this connection a large hydroelectric dam reservoirs are being constructed.

Hydro power is generating 17% of the world's total installed electricity. In the field of renewable energy, it adds up 70% of the total installed capacity. In Pakistan the total hydro power stations are generating about 6700 MW.

The main sources of hydro power of the country are Tarbela dam, Ghazi-Brotha hydro plant, Mangla dam, Neelum-Jhelum hydro plant, and Warsak dam.



Fig.4.26: Hydro Power Plant

4.12.2 Non-Conventional or renewable sources of energy

Non-conventional sources of energy are still under development. These sources are inexhaustible and they are replenished quickly. On the other hand, renewable sources of energy are inexpensive and pollution free. The range of non-conventional sources is limited and these can be used at domestic level, some non-conventional energy sources are explained below:

I. Solar Energy:

Sun is a tremendous source of energy. It is 150 million kilometres away from our Earth and we receive solar energy in the form of heat and light from it.

On a clear and cloudless day the incident solar power at the Earth's surface may be up to 10^7 joules per second or 1.4 kW m^{-2} . There are several techniques to harness solar energy. A direct conversion of solar energy into electrical energy can be done by using semiconductor devices called photocells or photovoltaic cells or silicon solar cells. These solar cells can also be connected to rechargeable batteries which store the energy collected, so that it can be used during the darkness or in cloudy weather.

Similarly, solar energy can also be converted into thermal energy by using thermal collectors to heat water in order to produce steam for running turbines to produce electricity.

Solar thermal power systems concentrate solar radiation by mirrors. In this method solar radiation heat a fluid such as molten salt to much higher temperatures, $>450^\circ\text{C}$. Thermal energy from a source at high

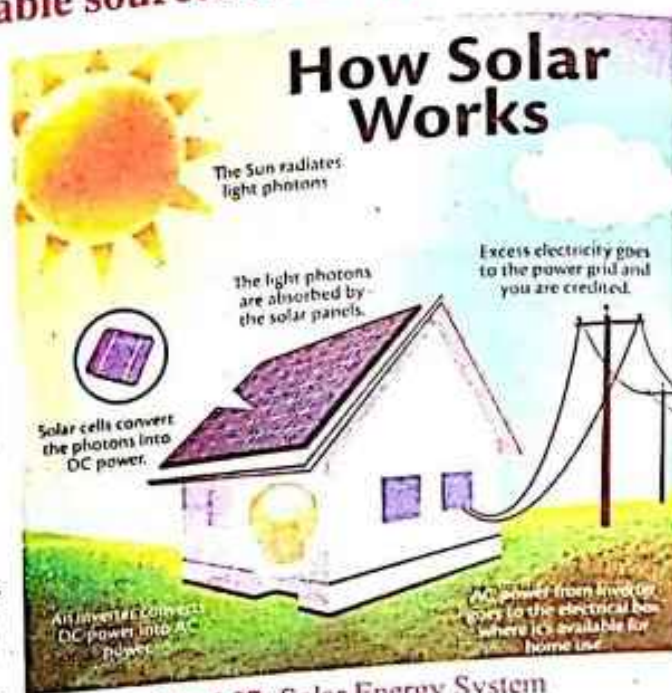


Fig.4.27: Solar Energy System



Fig.4.28: Solar Thermal Power Plant



Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar power aircraft

temperature can be converted into mechanical energy (to drive a turbine). Solar water heaters are a common sight on roofs of the houses and their use is becoming more widespread.

II. Wind Energy:

Wind energy is pollution free and cheap source but it requires a vast area of land and a method of storing electricity for use when the wind drops. Due to the uneven heating and cooling of the atmosphere by the Sun as well as the rotation of earth, wind blows from areas of high pressure to areas of low pressure. Wind mills are being installed in such areas to rotate turbines and produce electricity. Wind in coastal and high altitude areas can be harnessed up to 5MW power from a single turbine. It is estimated that the capacity of wind energy is 2% of the total energy produced in the world.



Wind Energy Farm: The Gansu wind farm in China is the biggest wind farm in the world. It generates up 7900 MW There are 7000 wind turbines in Gobi Desert.

III. Biomass Energy:

Biomass is an organic material which originates from plants, trees, crops, cattle dung, sewages, agricultural and urban wastes and so many other things. Biomass energy is the conversion of biomass into heat, electricity and liquid fuels. Biomass energy can be achieved under various processes.

A biomass fuel can be achieved using a biological method. According to this method, trees and plants store energy from the Sun in the form of carbohydrate through the process of photosynthesis. The carbohydrates are then converted into ethanol or methanol which can be used as a liquid fuel in vehicles.

Similarly, direct burning of biomass such as wood, agriculture residue etc., can be used for heating and cooking purpose.

Energy can also be extracted from the waste biomasses such as animal dung, household waste, urban waste. Ethane and other biogases are produced from the bacterial decomposition of these wastes. As a result, heat energy is produced by burning biogas which can be used to generate steam and operate turbines.

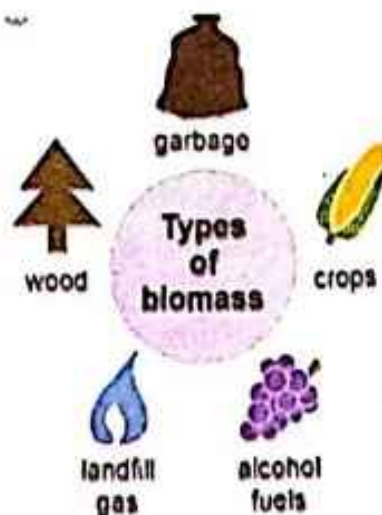


Fig.4.29: Bio Mass Energy

IV. Geothermal Energy

Geothermal energy is the natural heat present inside the Earth, which is available in the depth of 10 km and it is present inside the earth due to the following three reasons,

- (a) Ancient heat stored in the core of Earth at temperature 4000°C ,
- (b) Friction of Earth plates
- (c) Decay of radioactive elements which occur naturally.

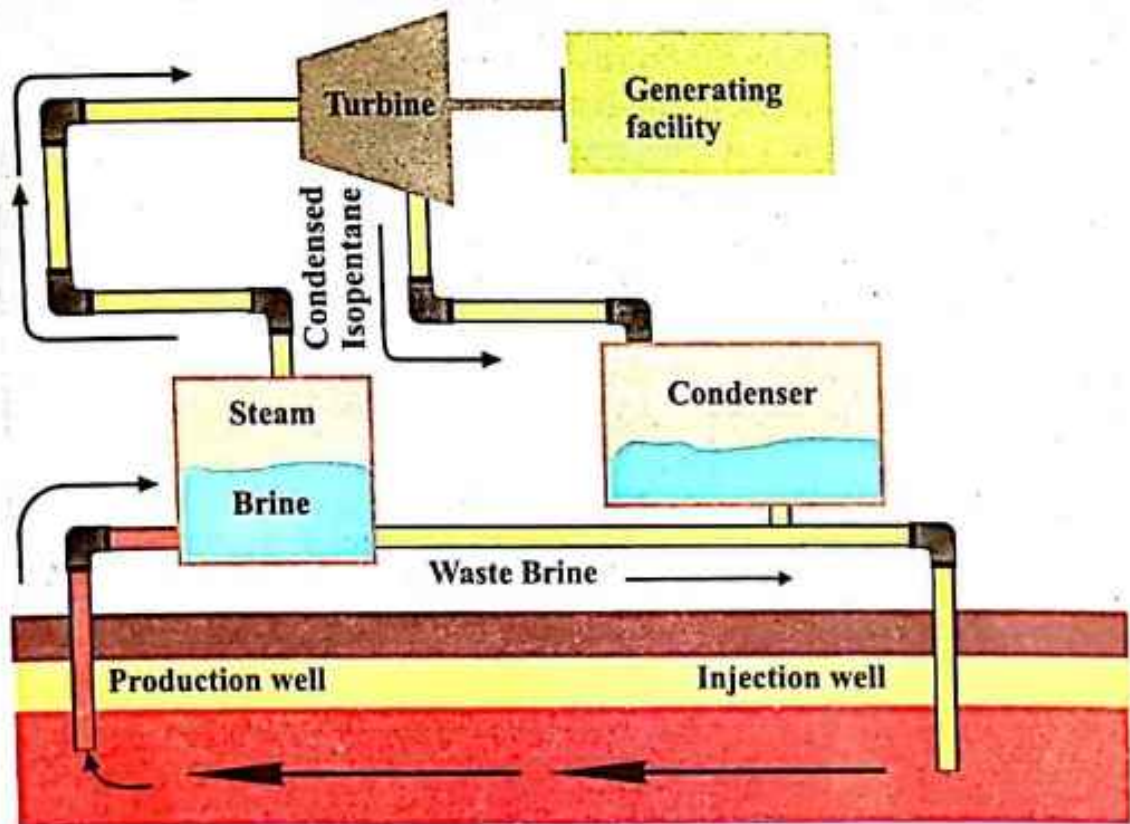


Fig.4.30: Geothermal Energy

The Geothermal energy can be extracted from inside of the Earth in the form of steam and hot water under the following process.

In the first process, underground hot steam and water which are emitted from natural existing springs, can operate turbines to produce electricity as shown in Fig.4.30.

In the second process, holes are drilled into the Earth's crust to put cool water in and pump out the steam. This steam can be used to rotate the electrical turbines for the production of electricity. The amount of geothermal energy is enormous. It has been estimated that only one percent of heat contains in the upper most 10km of Earth's crust is equivalent to 500 times of the present energy obtained from oil and gas sources.

V. Tidal Energy

Tidal energy is also known as gravitational energy which is obtained by the tides of the oceans. Tides in ocean are due to the gravitational force of moon as well as the rotation of the earth. These tides can operate turbines to produce electricity. A tidal barrage system is developed to store water in a tidal basin behind a large dam; by this system the tides are trapped in the basin to operate the turbine as shown in Fig.4.31. Similarly, oceans tidal streams and tidal current below the surface of the sea can also be used for the generation of electricity.

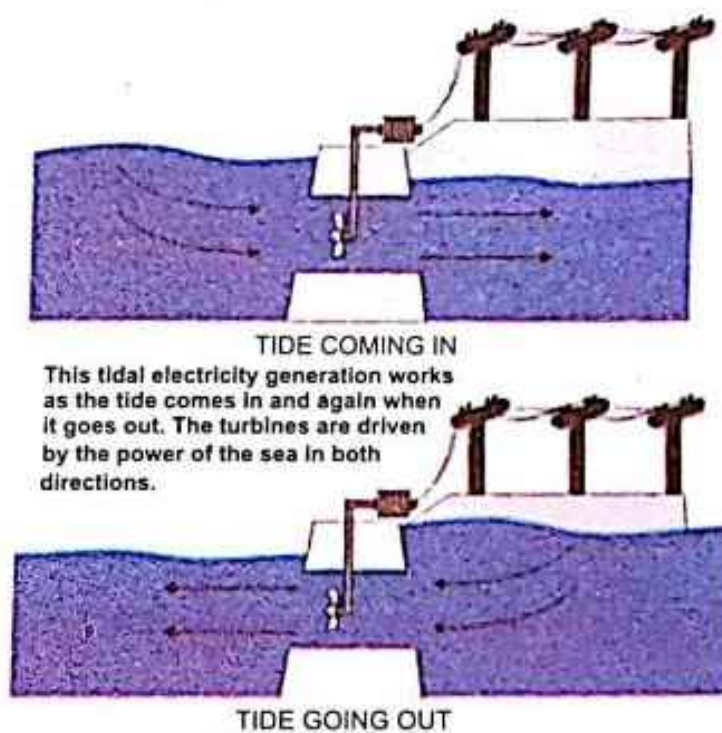


Fig.4.32: Tidal Energy

SUMMARY

- **Work:** The scalar product of force and displacement is called work. The work under a *constant force* = $Fd \cos\theta$ and work under a *variable force* = $\sum F_i \Delta d_i \cos\theta_i$.
- Work done in *gravitational field* does not depend upon path. Work done in a *gravitational field along a closed path* is zero.
- **Energy:** The ability of a body to do work is called energy.
- **Kinetic and Potential energies:** Kinetic energy is an energy of a body due to its motion while potential energy is the energy of a body due to its position.
- **Work-energy theorem:** Work has always changed the energy (K.E, P.E) of a body. This is work-energy theorem.
- **Absolute P.E.:** Work on a body from the surface of earth to the infinity where $g = 0$ is called absolute P.E.
- $$U = \frac{-GmM}{R}$$
- **Escape velocity:** The projected initial velocity of a body, such that it gets out of the gravitational field is called escape velocity. $V_{esc} = \sqrt{2gR}$
- **Power:** The rate of doing work is called power.

- **Law of conservation of energy:** This law is stated as, "the energy in an isolated system can be transformed from one form to another or transformed from one body to another; but the total amount of energy remains constant".
- **Conservative and non-conservative forces:** If work done by a force does not depend upon path followed, then such force is called conservative force. If the work done by a force depends on path followed then such force is called non-conservative force.
- **Conventional sources of energy:** Sources of energy which are exhaustible and have been in use for long time are called conventional sources of energy such as coal, oil, gas, hydro and uranium.
- **Non-conventional sources of energy:** Sources of energy which are inexhaustible and they exhaust no greenhouse gases in environment are called non-conventional. Solar power, wind powered, Tidal power, geothermal power, biomass power.

EXERCISE

○ Multiple choice questions.

1. When the applied force and covered displacement are parallel to each other then work done of the body is;
(a) Zero (b) Negative (c) Maximum (d) Minimum
2. A man pulling a bag with force of 15N at angle 60° with horizontal plane. If bag covers a distance of 10 m, then work done by the man is
(a) 50 J (b) 75 J (c) 100 J (d) 150 J
3. The area under a curved in a force and displacement graph shows that
(a) Work under a constant force (b) Work under a variable force
(c) Work under a maximum force (d) Work under a minimum force
4. If the velocity is doubled then the K.E of the body will be;
(a) Remain same (b) Double (c) Three times (d) Four times
5. A bullet of mass 20 g is fired with velocity of 2000 ms^{-1} , the K.E of the bullet is;
(a) 2000 J (b) 4000 J (c) 20000 J (d) 40000 J
6. A body of mass 100 g is raised vertically from surface of Earth in a gravitational field. The P.E of the body at height 100 m is;
(a) 0.98 J (b) 9.8 J (c) 98 J (d) 980 J
7. What is the power of an electric motor when it consumes energy of $9 \times 10^3 \text{ J}$ in 3 s?
(a) 1 hp (b) 2 hp (c) 3 hp (d) 4 hp

8. Absolute potential energy of a body at the surface of the earth is
 (a) Gm/R (b) Gm/R^2 (c) GmM/R (d) GmM/R^2
9. What is the value of potential energy of a body at the height where the value of 'g' is zero?
 (a) Zero (b) Negative (c) Maximum (d) mgh
10. When velocity of body of mass 10 kg is increased from 2 m s^{-1} to 8 m s^{-1} then the work done of the body is
 (a) 100 J (b) 200 J (c) 300 J (d) 400 J
11. Which force is a non-conservative?
 (a) Gravitational force (b) Friction force
 (c) Electrostatic force (d) Magnetic force
12. Which the following source of energy is not a fossil fuel?
 (a) Coal (b) Uranium (c) Oil (d) Gas
13. Conventional method of energy extracted is
 (a) Hydro power (b) Wind power (c) Tidal power (d) Biomass power
14. One megawatt hour is equal to:
 (a) $3.6 \times 10^7 \text{ J}$ (b) $3.6 \times 10^9 \text{ J}$ (c) $3.6 \times 10^{12} \text{ J}$ (d) $3.6 \times 10^{18} \text{ J}$
15. If the speed of an object is tripled, its kinetic energy is increased by;
 (a) $\frac{1}{9}$ times (b) $\frac{1}{3}$ times (c) 6 times (d) 9 times

SHORT QUESTIONS

1. Using work formula, at what angle, the work will be negative? Give example of negative work.
2. Calculate the work of a body when it is moving with uniform velocity.
3. Why the work done of a body in a gravitational field along a closed path is zero?
4. Why the value of K.E is always positive?
5. How can you prove mathematically that power is a scalar quantity?
6. How can absolute potential energy be achieved?
7. What would be the value of P.E of a body when it gets out the gravitational field?
8. Does the escape velocity of a body depend upon its mass?
9. Is there any conversion of energy when law of conservation is not valid?
10. What are the three properties of a conservative force?
11. What are the differences between conventional and non-conventional sources of energy?

12. How many conventional and non-conventional power plants are working in Pakistan?
13. A meteor burns into ashes when it enters into outer atmosphere of Earth. Why?
14. Can a centripetal force do any work? If yes then explain it.
15. What are the essential conditions for conservative field?
16. A bucket is taken to the bottom of a well, does the bucket possess any potential energy.
17. A boy drops a glass from a certain height, which breaks into pieces. What energy changes are involved?
18. Does the kinetic energy of a car changes more when it speeds up from 10 m s^{-1} to 15 m s^{-1} or from 15 m s^{-1} to 20 m s^{-1} explain.

COMPREHENSIVE QUESTIONS

1. Define work done by a constant force and explain with examples of positive work, negative work and zero work.
2. State and explain the work done by a variable force with its graphical representation.
3. What is gravitational field? Verify that; (i) the work done in a gravitational does not depend upon its path, (ii) the work done in a gravitational field along a closed path is zero.
4. State and explain energy with its two forms such as; kinetic energy and potential energy. Also describe the work-energy theorem.
5. What is gravitational potential energy? Derive an expression for the absolute gravitational potential energy.
6. Define escape velocity and derive its mathematical relation.
7. What do you know about power? Define power in terms of the scalar product of force and velocity.
8. Study the work done against friction and show that energy is lost due to friction.
9. State and explain conventional and non-conventional sources of energy.

NUMERICAL PROBLEMS

1. A man pulls a bag along the ground with a force of 80 N at an angle of 30° from the ground. How much work done the man do in pulling the bag 10 m ?
(693 J)
2. A 250 kg cart is pushed up on an inclined surface. How much work does the pushing force when the cart moves up and at 3 m above the ground, friction is neglected?
(7350 J)

3. A pump lifts water from a well to a tank 30 m above the well. If there are 100 m³ water stored in tank, then how much work against the gravity is done by the pump. Density of water is 1000 kg m⁻³. (3 × 10⁷ J)
4. A force of 6000 N is applied horizontal on a van of mass 2500 kg. The van starts its motion from rest and if it has traveled a distance of 110 m. What will be its speed and it's K.E.? (23 ms⁻¹, 660 kJ)
5. A proton of mass 1.67×10^{-27} kg is being accelerated along a straight line with acceleration of 3.6×10^{15} m s⁻². If the proton's initial velocity is 2.4×10^7 m s⁻¹ and travels a distance of 250 m, what is its final velocity and increase in its K.E.? (5.56×10^7 m s⁻¹, 2.6×10^{-12} J)
6. A car of mass 1500 kg is accelerated from rest to a speed of 30 ms⁻¹ in a time of 10 sec. What is the power of car in hp when friction is neglected? (90.5 hp)
7. What power is required to raise a block of mass 500 kg vertical to height of 15 m in a time of 50 s? Express your answer in hp. (2 hp)
8. How much work is required to accelerate a body of mass 200 kg from 5m s⁻¹ to 15 m s⁻¹? If its covers a distance of 150 m what is the net force acting on it. (2×10^4 J, 133 N)
9. A body of mass 'm' is dropped from a tower 100 m above the ground. What will be the height from ground to the point at which the velocity of body becomes 30 m s⁻¹. Air resistance is neglected. (54 m)
10. A block starts from rest at the top of an inclined surface of height 10 m above the ground, what is its speed when it reaches at the ground and friction is neglected. Now by including friction and when it reaches at the ground with a speed of 10 m s⁻¹ then what is its energy loses in percent. (14 m s⁻¹, 49%)