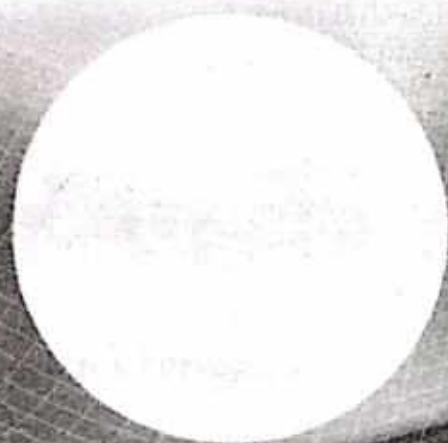


RELATIVITY

UNIT

13



Student Learning Outcomes (SLOs)

The students will:

- distinguish between inertial and non-inertial frames of reference.
- describe the significance of Einstein's assumption of the constancy of the speed of light.
- Describe that if c is constant then space and time become relative.
- State the postulates of special relativity.
- Explain qualitatively and quantitatively the consequences of special relativity.
- [Specifically in the case of:
 - a- the relativity of simultaneity.
 - b- the equivalence between mass and energy.
 - c- length contraction.
 - d- time dilation.
 - e- mass increase.]
- State that spacetime is a mathematical model in relativity that treats time as a fourth dimension of the traditional three dimensions of space. (It can be thought of as a metaphorical sheet of paper that can bend, and when it bends it can cause effects such as stretching and compression seen when gravitational waves pass through objects).

Albert Einstein's theory of relativity transformed the theoretical physics and astronomy during the 20th century by superseding a 200-years-old theory of mechanics created by Isaac Newton. The ideas behind relativity might seem mysterious, as they introduced new concepts such as four-dimensional spacetime, relativity of simultaneity, time dilation, and length contraction.

The theory of relativity, given by Albert Einstein, includes two theories: special relativity and general relativity. Albert Einstein published special theory of relativity and general theory of relativity in 1905 and 1915, respectively. Special relativity applies to all physical phenomena in the absence of gravity. General relativity explains the law of gravitation and its relation to the forces of nature. Einstein explained the situations in which Newtonian physics might fail to deal successfully with phenomena, and in doing so proposed revolutionary changes in human concepts of time, space and gravity.

Over the last century, many experiments have confirmed the validity of both special relativity and general relativity. It includes the observation that light deflect from distant stars as the starlight passed by our sun, proving that gravity distort or curve space. Instead of being an invisible force that attracts objects to one another, gravity is a curving or warping of space. The more massive an object, the more it warps the space around it, as shown in the cover picture of this unit.

The special theory of relativity became an important and necessary tool in the new fields of nuclear physics, atomic physics and quantum mechanics. Nuclear power plants and nuclear weapons etc., would be impossible without the knowledge that matter can be transformed into energy. So, the theory has an enormous impact on the modern world. In this unit, we shall discuss some important aspects of special relativity.

13.1 FRAME OF REFERENCE

A frame of reference is a coordinate system that can be used to determine positions and velocities of objects with in that frame.

For example, when a ball rolls in a street, you can say that the ball is moving because the frame of reference is the streets, whatever may be on the side of the street or the Earth itself. The origin, orientation and scale of a reference frame are specified by a set of geometric points whose positions are identified both mathematically and physically. All measurements of motion are made relative compared to a frame of reference. Different frames of reference can move relative to one another.

Consider two persons A and B are seated in a bus and the bus is moving with a velocity v , as shown in Fig. 13.1. If we ask to A about the velocity of B, he will say that B is at rest. But if we ask the same question to a person C standing on ground, road side. He will say that B is moving with a velocity v in the positive X direction. So, before specifying the velocity we have to specify the frame of reference.

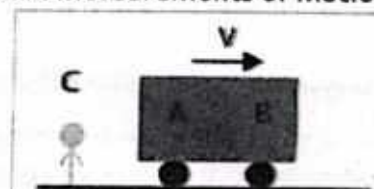


Figure 13.1: two persons A and B are seated in a bus and the bus is moving with a velocity v .

Frame of reference are of two types: Inertial frame of reference and non-inertial frame of reference.

Inertial Reference Frame

An inertial reference frame is a frame of reference in which Newton's law holds true: i.e., an object at rest remains at rest, and object in motion remains in motion with a constant velocity on a straight line, unless acted upon by an external force.

A frame of reference that is at rest or moving with a uniform velocity along a straight line is called an inertial frame of reference.

An inertial frame of reference does not accelerate. Any reference frame that moves with constant velocity relative to an inertial frame is itself inertial. The Earth can be considered an inertial reference frame for many experiments. The interior of a car moving along a road at constant velocity and the interior of a stationary house are examples of inertial reference frames.

Non-inertial Reference Frame

In a non-inertial frame of reference, objects experience acceleration even in the absence of applied forces because the reference frame itself is accelerating relative to an inertial frame. Newton's laws do not hold in a non-inertial frame of reference.

A frame of reference that is accelerating is called a non-inertial frame of reference.

An accelerating car is example of non-inertial reference frame.

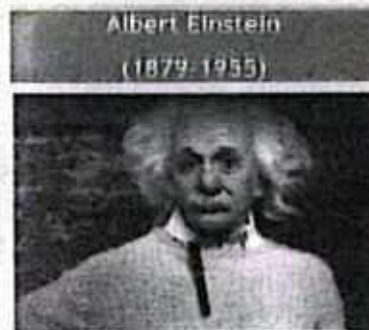
13.2 SPECIAL THEORY OF RELATIVITY

In 1905, Albert Einstein introduced his famous theory of relativity. This theory deals with the structure of space-time. Einstein's special theory of relativity is based on two postulates:

1) The Principle of Relativity

The laws of physics are the same for all observers in any inertial frames of reference. As long as an object is moving in a straight line at a constant speed (i.e., with zero acceleration), the laws of physics are the same for every observer.

This implies that the experiments performed in stationary and moving inertial frames of reference yield the same results. For example, it is impossible to determine experimentally whether an inertial reference frame is stationary or moving without observing it from an external frame of reference.



Albert Einstein was a German-born theoretical physicist. He was one of the greatest and most influential scientists of all time.

Consider two individuals; one standing in a stationary train and the other is standing in a train moving at constant velocity v , as shown in Fig. 13.2. Both are observing the free fall motion of a ball under the action of gravity.

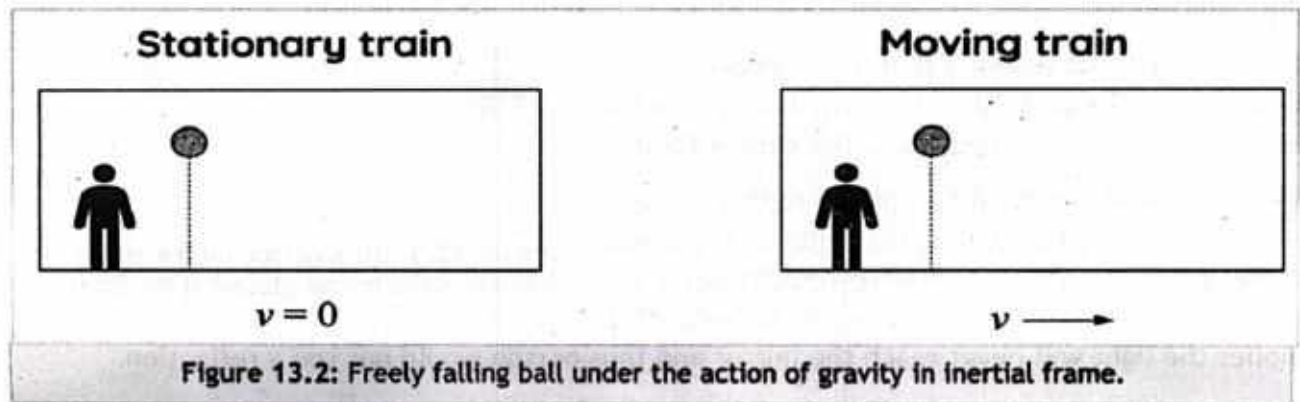


Figure 13.2: Freely falling ball under the action of gravity in inertial frame.

Both the individuals are in inertial frames of reference, where Newton's laws apply equally for both of them. Both persons will observe the ball falling in a vertical path. If the trains are windowless, the passengers will have no way of knowing whether their train is stationary or moving at constant velocity.

2) The Principle of the Constancy of the Speed of Light

The speed of light in vacuum is always constant for all observers, regardless of the relative velocity between the source of light and the observer measuring it. No matter how fast an observer or the light source is moving, measured speed of light (c) always yields the same value i.e., $c = 299,792,458 \text{ m s}^{-1}$.

This postulate has profound implications, particularly as it contradicts Newtonian mechanics. For instant, you were travelling at $0.5c$ in the same direction as the beam of light, Newtonian mechanics predicts that you would observe the speed of the light beam to be $0.5c$. However, according to special relativity, you would still observe the speed of light to be at c . This leads to a range of fascinating effects.

When a stationary person measures the speed of light emitted from a train at rest, the speed is simply c . However, the speed of light remains c even when the train is moving relative to the person, as shown in the Fig. 13.3 (a).

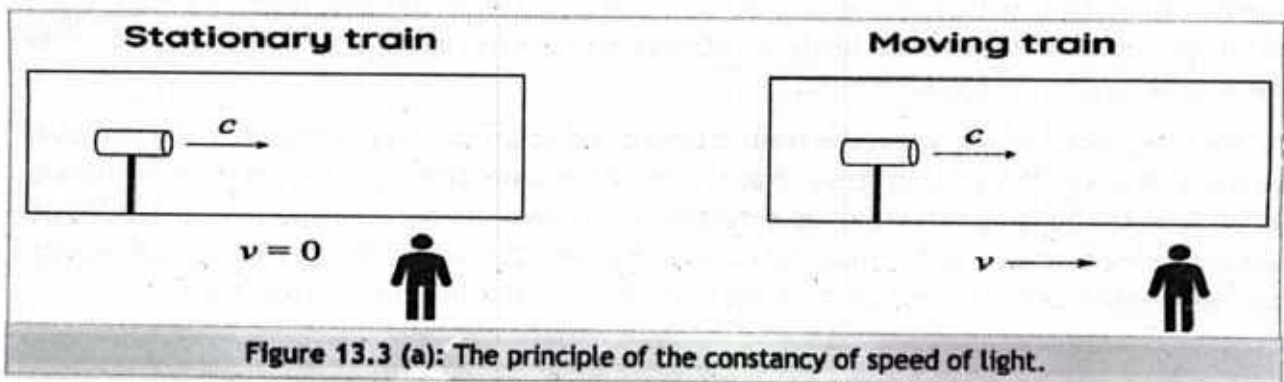
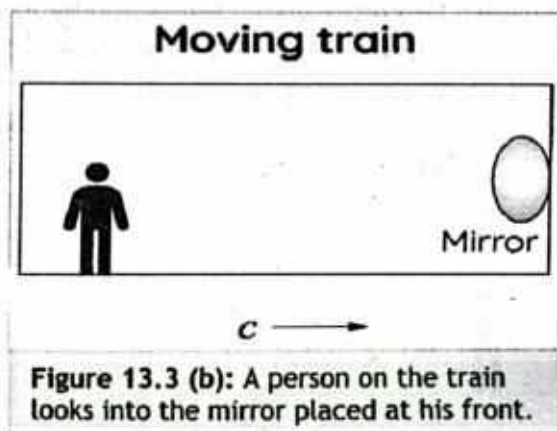


Figure 13.3 (a): The principle of the constancy of speed of light.

If a person travelling on this moving train looks into the mirror placed at his front, as shown in Fig. 13.3 (b). Will this person see a reflection of themselves in the mirror?

Newtonian Physics suggests that no reflection would be seen by a person when a light ray travels from a person to a mirror and returns to the person's eyes.

When Newtonian relativity is applied to this thought experiment as depicted in figure, a mirror is moving at the speed of light, the relative velocity between light from the person and the mirror is zero. This implies the light will never reach the mirror and thus person would not see a reflection.



However, this outcome contradicts the first postulate of special relativity which states that a person in an inertial frame of reference, such as this one, will not be able to identify whether the frame of reference is stationary or moving at a constant speed. If the person cannot see their reflection in the mirror, it would suggest that the train must not be stationary, as a reflection would be visible if were the case.

Special relativity, however, asserts that a reflection is indeed seen. The contradiction derived from Newtonian mechanics in this thought experiment supports the constancy of light's speed. The second postulate of special relativity states that light's speed is constant in a vacuum for all inertial frames of reference. This means that from the perspective of the person and mirror in the moving train, light's relative velocity is still c . as a result, the light will reach the mirror and be reflected back to the person's eyes. Therefore, the person will see a reflection of themselves in the mirror.

13.3 CONSEQUENCES OF THE SPECIAL THEORY OF RELATIVITY

There are other surprising consequences of the special theory of relativity. These consequences of the special theory of relativity are summarized in the following.

Relativity of simultaneity

Two events that are simultaneous for one observer, may not be simultaneous for another observer in relative motion. For example, consider a person is standing next to a train track and comparing observations of a lightning storm with a person inside a moving train. The train is moving at nearly the speed of light.

Einstein imagined scenario when the train is positioned at a point on the track equally between two trees. If a bolt of lightning strikes both trees at the same time, the person standing beside the track would receive the strike as simultaneous. However, because the person inside the train is moving toward one lightning bolt and away from the other, the person on the moving train would see the bolt ahead of the train first, and the bolt behind the train later.



Einstein concluded that simultaneity is not absolute. In other words, events that appear simultaneous to one observer may occur at different times for another observer. He realized that it is not light speed that changes, but time itself that is relative. Time moves differently for objects in motion compared to those at rest. However, the speed of light remains constant and is observed to be the same by anyone, anywhere in the universe, regardless of their motion.

Mass-Energy Equivalence

Energy and mass are equivalent and transmutable, expressed by the equation:

$$E = mc^2 \quad (13.1)$$

where E is Energy, m stands for mass and c for the velocity of light. Many scientists observed that the object's mass increases with the velocity but never knew how to calculate it. This equation is the answer to their problem, which explains that the increased relativistic weight of the object is equal to the kinetic energy divided by the square of the speed of light. As, the speed of light is too high, so a tiny amount of mass is equivalent to a very large amount of energy. That's why atomic and hydrogen bombs are so powerful. The concept of mass defect in atomic nucleus is also justified by the expression of mass-energy equivalence.

Length Contraction

Objects appear shorter in the direction of their motion relative to the observer. The length of an object measured in its rest frame is called the *proper length* (L_0). Other observers in different reference frames, which are in relative motion will always measure the length (L) to be shorter. This phenomenon is described by the following equation:

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} \quad (13.2)$$

This effect, known as *length contraction* occurs only in direction of motion.

Time Dilation

One of the key consequences of Einstein's special relativity is that time is experienced differently by moving objects. An object in motion undergoes time dilation, meaning that it experiences time more slowly compared to when it is at rest. Moving clocks are observed to tick more slowly than clocks that are stationary from the observer's perspective.

The time taken for an event to occur within its rest frame is called *proper time* (t_0). Observers in different reference frames in a relative motion will always measure the lapsed time taken (t) to be longer. This is expressed by the equation:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (13.3)$$

Mass Variation

The mass of a moving object increases as its velocity increases. This phenomenon is known as mass variation, is another expression of mass-energy equivalence. It is represented mathematically as:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (13.4)$$

Where, m is the relativistic mass, m_0 is the rest mass of particle, c is the speed of light and v is the velocity of the particle relative to a stationary observer.

This effect becomes noticeable only at relativistic speeds (speed close to the speed of light). As an object is accelerated closer to the speed of light, its mass increases. The more massive it becomes, the more energy is required to achieve the same acceleration, making further acceleration more and more difficult. The energy that is put into attempting acceleration is instead converted into mass. The total energy of an object is kinetic energy plus the energy embodied in its mass. To accelerate even the smallest object to the speed of light would require an infinite amount of energy. Therefore, material objects are restricted to speeds less than the speed of light.

Example 13.1: If a 0.5 kg body is moving at a speed of equivalent to 90 % of the speed of light. What will be its mass in this situation?

Given: $v = 90\% \text{ of } c = 0.9c$ $m_0 = 0.5 \text{ kg}$

To Find: $m = ?$

Solution: Using the relation:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Putting values, we get:

$$m = \frac{0.5}{\sqrt{1 - \frac{(0.9c)^2}{c^2}}} = 1.15 \text{ kg}$$

Example 13.2: At what speed would the mass of proton be tripled? The rest mass of a proton is $1.673 \times 10^{-27} \text{ kg}$.

Given: $m_0 = 1.673 \times 10^{-27} \text{ kg}$

To Find: $v = ?$

Solution: Using the relation:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Putting the given values, we get:

$$3 m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$



$$3 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{3}$$

squaring both sides, we get:

$$1 - \frac{v^2}{c^2} = \frac{1}{9}$$

$$\text{or } \frac{v^2}{c^2} = 1 - \frac{1}{9} = \frac{8}{9}$$

$$v^2 = \frac{8}{9}c^2$$

$$\text{or } v = \frac{\sqrt{8}}{3}c = 0.9428c$$

Assignment 13.1

- 1) What is the speed of a rod relative to the observer, if the length of rod is measure to be half of its proper length?
- 2) The time period of a pendulum is measured to be 3 s in the inertial frame. What is the period when measured by an observer moving with a speed of $0.95c$ with respect to the pendulum?

13.4 TIME AS A FOURTH DIMENSION IN THE SPACETIME MODEL

Until the 20th century, it was thought that the three-dimensional geometry of the universe (its description in terms of locations, shapes, distances, and directions) was distinct from time (the measurement of when events occur within the universe). However, according to Einstein, you need to describe the location of an object not only in three-dimensional space (length, width and height) but also in time. Hence time is the fourth dimension.

To know where you are, you have to know what time it is. The history of an object's location through time traces out a line or curve on a spacetime diagram. Each point in a spacetime diagram represents a unique position in space and time.

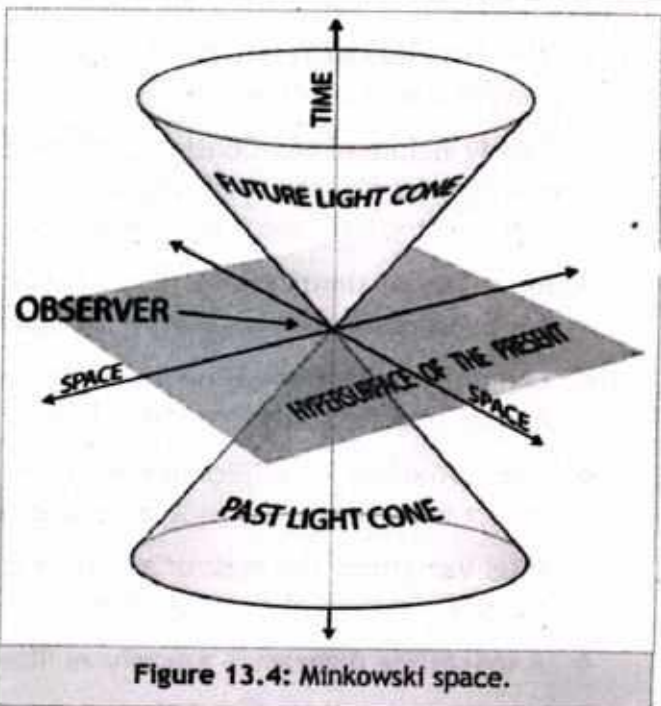


Figure 13.4: Minkowski space.

Spacetime is a mathematical model, as shown in the Fig. 13.4, that unites the three dimensions of space with the single dimension of time into a single four-dimensional continuum. Spacetime diagrams are useful in visualizing and understanding relativistic effects such as how different observers perceive where and when events occur.

A spacetime diagram is a graphical representation of locations in space at various times, as shown in figure 13.4. Spacetime diagrams can depict the geometry underlying phenomena like time dilation and length contraction without any mathematical equations.

For Your Information

In 1908, Hermann Minkowski presented a geometric interpretation of special relativity that combined time and the three spatial dimensions of space into a single four-dimensional continuum now known as Minkowski space. This interpretation proved vital to the general theory of relativity, wherein spacetime is curved by mass and energy.

SUMMARY

- ❖ A **frame of reference** is a coordinate system that can be used to determine the positions and velocities of objects in that frame.
- ❖ A frame of reference which is accelerating is called a **non-inertial frame of reference**.
- ❖ A frame of reference which is at rest or moving with a uniform velocity along a straight line is called an **inertial frame of reference**.
- ❖ **The principle of relativity:** The laws of physics are the same for all observers in any inertial frame of reference.
- ❖ **The Principle of the Constancy of the Speed of Light:** The speed of light in vacuum is constant for all observers, regardless of the relative velocity between the source of light and the observer who is measuring its velocity.
- ❖ **Relativity of simultaneity:** Two actions that appears simultaneous for one person may not be simultaneous for another person in relative motion.
- ❖ **Length Contraction:** Objects are measured and appear shorter in the direction of motion relative to the observer. This effect of relativity is called length contraction.
- ❖ **Time Dilation:** An object in motion experiences time dilation, meaning that when an object is moving very fast it experiences time more slowly than when it is at rest.
- ❖ **Mass Variation:** The mass of a moving objects increases as its velocity increases. This is the phenomenon of mass variation.
- ❖ A **spacetime diagram** is a graphical illustration of locations in space and times.
- ❖ **Minkowski space** is a geometric interpretation of special relativity that combines time and the three spatial dimensions of space into a four-dimensional continuum.



EXERCISE

Multiple Choice Questions

Encircle the correct option.

1) Theory of Relativity was formulated by:

- A. Isaac Newton
- B. Stephen William Hawking
- C. Albert Einstein
- D. Ernest Rutherford

2) A frame of reference is:

- A. a graph plotted between distance and time.
- B. a graph plotted between speed and time.
- C. the velocity of an object through empty space without regard to its surroundings.
- D. an arbitrarily fixed point with respect to which motion of other points is measured.

3) A pendulum has a time period of 5.0 s relative to the pendulum's frame of reference. What is the speed of an observer relative to the pendulum's frame of reference if the period measured by the observer is 10.0 s? (here, c is the speed of light).

- A. $0.606c$
- B. $0.779c$
- C. $0.866c$
- D. $0.693c$

4) An airliner traveling at 200 m s^{-1} emits light from the front of the plane. Which statement describes the speed of the light?

- A. It travels at a speed of $c + 200 \text{ m s}^{-1}$.
- B. It travels at a speed of $c - 200 \text{ m s}^{-1}$.
- C. It travels at a speed c , like all light.
- D. It travels at a speed slightly less than c .

5) An inertial frame of reference is the one:

- A. which is at rest.
- B. which is moving with a uniform velocity along a straight line.
- C. which has zero acceleration.
- D. All of these.

Short Questions

Give short answers of the following questions.

13.1 What is the difference between inertial and non-inertial frames of reference?

13.2 State the postulates of special theory of relativity?

13.3 Explain why is it impossible for a particle with mass to move faster than the speed of light.

13.4 Imagine a train travelling at the speed of light. A person on this train looks into the mirror placed in front of him. Will this person see a reflection of himself in the mirror?

13.5 What happens to the density of an object as its speed increases?

13.6 What is meant by the relativity of simultaneity?

Comprehensive Questions

Answer the following questions in detail.

13.1 What is meant by frames of reference? Explain with the help of examples. Also discuss its types.

13.2 Discuss the postulate of the special theory of relativity.

13.3 Explain the following consequences of the special theory of relativity:

a) Relativity of simultaneity b) Length Contraction

c) Time Dilation

d) Mass Variation

e) Mass-Energy Equivalence.

13.4 State and explain how time is considered the fourth dimension alongside the traditional three dimensions of space.

Numerical Problems

13.1 Calculate the equivalent energy of an electron with rest mass 9.11×10^{-31} kg.

(Ans: 0.512 MeV)

13.2 The length of a spaceship is 100 m. What will be its length if the spaceship moves at a speed of $0.99c$?

(Ans: 14 m)

13.3 Particles called π -mesons are produced by accelerator beams. If these particles travel at 2.70×10^8 m s⁻¹ and live 2.60×10^{-8} s when at rest relative to an observer. How long do they live as viewed in the laboratory?

(Ans: 5.96×10^{-8} s)

13.4 A neutral π -meson lives 1.40×10^{-16} s as measured in the laboratory, and 0.840×10^{-16} s when at rest relative to an observer. What is its velocity relative to the laboratory?

(Ans: 0.800 c)