

Based on National Curriculum of Pakistan 2022-23

Model Textbook of
Physics
Grade
11

National Curriculum Council
Ministry of Federal Education and Professional Training



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A Model Textbook of Physics for Grade 11

based on National Curriculum of Pakistan (NCP) 2022-23

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Note

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Preface

This Model Textbook for Physics Grade 11 has been developed by NBF according to the National Curriculum of Pakistan 2022-2023. The aim of this textbook is to enhance learning abilities through inculcation of logical thinking in learners, and to develop higher order thinking processes by systematically building the foundation of learning from the previous grades. A key emphasis of the present textbook is creating real life linkage of the concepts and methods introduced. This approach was devised with the intent of enabling students to solve daily life problems as they grow up in the learning curve and also to fully grasp the conceptual basis that will be built in subsequent grades.

After amalgamation of the efforts of experts and experienced authors, this book was reviewed and finalized after extensive reviews by professional educationists. Efforts were made to make the contents student friendly and to develop the concepts in interesting ways.

The National Book Foundation is always striving for improvement in the quality of its textbooks. The present textbook features an improved design, better illustration and interesting activities relating to real life to make it attractive for young learners. However, there is always room for improvement, the suggestions and feedback of students, teachers and the community are most welcome for further enriching the subsequent editions of this textbook.

May Allah guide and help us (Ameen).

Dr. Kamran Jahangir
Managing Director

Practical Applications of Physics-XI in Everyday Life.

Physics is the foundation of our understanding of the natural world, therefore learning physics empowers students to:

- Comprehend the underlying principles governing our universe.
- Develop critical thinking and problem-solving skills.
- Foster curiosity, creativity, and innovation.
- Address real-world challenges and technological advancements.
- Prepare for innovative careers in science, technology, engineering, and mathematics (STEM).

Unit wise relevance of this book to our natural world is briefly given below:

Unit 1: Physical Quantities and Measurements

Understanding physical quantities and measurements is crucial in everyday life. Accurate measurements are essential in medicine, engineering, and architecture. This unit lays the foundation for problem-solving and critical thinking, skills vital in professions like science, technology, engineering, and mathematics (STEM).

Unit 2: Vectors

Vectors play a significant role in navigation systems (GPS), video games, and weather forecasting. Mastering vectors helps engineers design safer buildings, optimize traffic flow, and predict natural disasters. You'll see vectors in action in fields like aviation, oceanography, and computer graphics.

Unit 3: Translatory Motion

Translatory motion principles govern vehicle movement, projectile trajectories, and sports performance. This unit's concepts are applied in transportation systems, aerospace engineering, and athletic training. Understanding translatory motion enhances safety, efficiency, and innovation.

Unit 4: Rotational and Circular Motion

Rotational motion is integral to machinery, gears, and engines. This unit's concepts are crucial in designing amusement park rides, bicycle gears, and satellite orbits. You'll find applications in mechanical engineering, robotics, and renewable energy systems.

Unit 5: Work and Kinetic Energy

Understanding work and kinetic energy helps optimize energy consumption in industries like manufacturing, transportation, power generation and construction. This unit's principles are applied in designing more efficient machines and renewable energy systems. and

Unit 6: Fluid Mechanics

Fluid mechanics governs water supply systems, ocean currents, blood circulation and atmospheric circulation. Mastering fluid mechanics improves irrigation systems, water treatment processes, and weather forecasting. Applications extend to aviation, chemical engineering, naval architecture, and environmental science.

Unit 7: Physics of Solids

The physics of solids underlies material science, structural engineering, and architecture. This Unit's concepts help develop stronger, lighter materials for construction, aerospace, and biomedical applications.

Unit 8: Heat and Thermodynamics

Thermodynamics principles govern heating and cooling systems, refrigeration, engines. Understanding heat transfer enhances energy efficiency in buildings, industries, and transportation systems.

Unit 9: Waves

Wave phenomena are essential in music, telecommunications, and medical imaging. This unit's concepts are applied in sonar technology, wireless communication, and radar sensation.

Unit 10: Electrostatics

Electrostatics principles govern lightning protection, static electricity safety, and high-voltage transmission. Mastering electrostatics enhances electrical engineering, materials science, and nanotechnology.

Unit 11: Electricity

Electricity powers our daily lives. This unit's concepts underlie electrical circuits, electronic devices, and power distribution systems. Understanding electricity enables innovation in fields like renewable energy, electronics, and telecommunications.

Unit 12: Magnetism

Magnetism is crucial in electric motors, generators, and medical imaging (MRI). This unit's principles are applied in materials science, electromagnetism, and advanced technologies like magnetic levitation trains.

Unit 13: Relativity

Einstein's relativity revolutionized our understanding of space, time, and gravity. Relativity's implications extend to GPS technology, particle physics, and cosmology, inspiring breakthroughs in fields like astrophysics and quantum mechanics.

Unit 14: Particle Physics

Particle physics reveals the fundamental nature of matter and energy. This unit's concepts underlie cutting-edge technologies like particle accelerators, quantum computing, medical imaging, and radiation therapy.

By learning these fundamental physics concepts, you'll gain a deeper understanding of the world around you and develop problem-solving skills essential for innovative careers in science, technology, engineering, and mathematics (STEM).

Managing Author

Physics-XI

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PHYSICAL QUANTITIES AND MEASUREMENTS

UNIT

1

Student Learning Outcomes (SLOs)

The students will:

- Make reasonable estimates of physical quantities [of those quantities that are discussed in the topics of this grade].
- Express derived units as products or quotients of the SI base units.
- Analyze the homogeneity of physical equations [Through dimensional analysis].
- Derive formulae in simple cases [Through using dimensional analysis].
- Analyze and critique the accuracy and precision of data collected by measuring instruments.
- Assess the uncertainty in a derived quantity [By simple addition of absolute, fractional or percentage uncertainties].
- Justify why all measurements contain some uncertainty.

Physics is based on experimental observations. Observations may be qualitative or quantitative. Qualitative observations have no associated numbers. These deal with facts that can be observed with our five senses: sight, smell, taste, touch and hearing. Colors, shapes and textures of objects are examples of qualitative observations. Observations, like 'water keeps its level', is also an example of qualitative observation. A quantitative observation includes numbers, and is also called a measurement. We can measure mass, time, distance, speed, pressure, force, torque, momentum, and energy. Quantitative observations are useful to a scientist.

1.1 ESTIMATION OF PHYSICAL QUANTITIES

In our daily life, we may face some situations like: What will be the height of this building? Will the piece of equipment fit in the back of our car, or do we need to rent a truck? How long will this download take? How large a current will be there in this circuit? How many houses could a proposed power plant actually power if it is built? Usually, we solve such problems by making estimations. In many circumstances, scientists and engineers also need to make estimates of some specific physical quantity with the help of little or no actual data.

An estimation is a rough educated guess of the value of a physical quantity by using prior experience and sound physical reasoning.

An estimation usually includes the identification of correct physical principles and a good guess about the relevant variables. Estimation is very useful in developing a physical sense. Estimation does not mean guessing a formula or a number at random. Some of the following strategies may help to improve our skill of estimation:

Estimation of length: When estimating lengths, remember that anything can be a ruler. For breaking a big thing into smaller things, first estimate the length of one of the smaller things and then multiply this value to the number of smaller things to obtain the length of the big thing. For example, to estimate the height of building, we first count the number of floors it has. Then, estimate the height of a single floor by imagining how many people would have to stand on each other's shoulders to reach the ceiling. In the last, we estimate the height of a person. These estimates give you the height of the building.

Sometimes it also helps to do this in reverse, i.e., to estimate the length of a small thing which in bulk making up a bigger thing. For example, to estimate the thickness of a sheet of paper, estimate the thickness of a stack of paper and then divide it by the number of pages in the stack. These same strategies of breaking big things into smaller things or aggregating smaller things into a bigger thing can sometimes be used to estimate other physical quantities, such as mass and time. In such situations, some of the length, mass and time scales, as shown in Table 1.1 may be helpful.

Estimate Areas and Volumes from Lengths: While dealing with area or volume of a complex object, introduce a simple model of the object, such as a sphere or a box. Estimate the linear dimensions (such as the radius of the sphere or the length, width, and height of the box) and then use standard geometric formulas to find the area or volume. If you have an estimate of



area or volume, you can also do the reverse; that is, use standard geometric formulas to get an estimate of its linear dimensions.

Table 1.1: The estimation of some physical quantities.

Length (m)	Mass (kg)	Time (s)
Diameter of proton = 10^{-15}	Mass of electron = 10^{-30}	Mean lifetime of unstable nucleus = 10^{-22}
Diameter of large nucleus = 10^{-14}	Mass of proton = 10^{-27}	Time for single floating-point operating in a supercomputer = 10^{-17}
Diameter of H-tom = 10^{-10}	Mass of bacterium = 10^{-15}	Time period of visible light = 10^{-15}
Diameter of typical virus = 10^{-9}	Mass of mosquito = 10^{-6}	Time period of an atom in solid = 10^{-13}
Width of pinky fingernail = 10^{-2}	Mass of hummingbird = 10^{-2}	Time period of nerve impulse = 10^{-3}
Height of 4-years old child = 10^0	Mass of 1 liter water = 10^0	Time for 1 heartbeat = 10^0
Length of football ground = 10^2	Mass of a Motorcycle = 10^2	One day = 10^5
Diameter of Earth = 10^7	Mass of atmosphere = 10^{19}	One year = 10^7
Diameter of solar system = 10^{12}	Mass of Moon = 10^{22}	Human lifetime = 10^9
1 light-year = 10^{16}	Mass of Earth = 10^{25}	Recorded human history = 10^{11}
Diameter of Milky-Way = 10^{21}	Mass of Sun = 10^{30}	Age of Earth = 10^{17}
Distance between edges of observable universe = 10^{26}	Mass of known universe = 10^{53}	Age of universe = 10^{18}

Estimate Mass from Volume and Density: To estimate the mass of an object, it is helpful first to estimate its volume, and then to determine its mass using an estimate of its average density (recall, that density has dimension of mass/volume, so mass = density $\times$ volume). For this estimation, it helps to remember that the density of air is about 1 kg m^{-3} , the density of water is 10^3 kg m^{-3} , and the densest everyday solids has a maximum value around 10^4 kg m^{-3} . Asking yourself whether an object floats or sinks in either air or water can give you a rough estimate of its density. You can also do the reverse: if you have an estimate of an object's mass and its density, you can use them to get an estimate of its volume.

Example 1.1: Estimate the energy required for an adult man to walk up through stairs from ground floor to 1st floor?

Solution:

As, the energy required = $m g h$

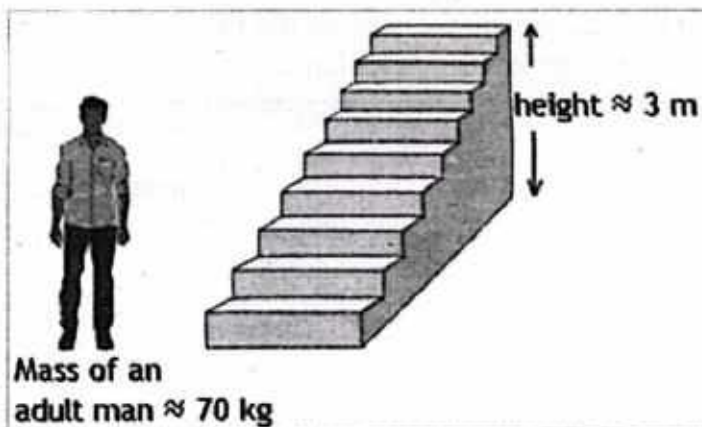
We have to take the following estimations:

Mass of an adult = $m = 70 \text{ kg}$

Distance between 2 floors = 3 m

So,

$$\begin{aligned}\text{Energy required} &= 70 \text{ kg} \times 10 \text{ m s}^{-2} \times 3 \text{ m} \\ &= 2100 \text{ kg m}^2 \text{ s}^{-2} \\ &= 2100 \text{ J}\end{aligned}$$



Assignment 1.1

Estimate that how many floating-point operations can a supercomputer do in 1 day? Time for single floating-point operating in a supercomputer is 10^{-17} s .

1.2 DERIVED UNITS IN TERMS OF BASE UNITS

In Grade 9, we have studied about base and derived physical quantities and their units. We know that derived units can be expressed in terms of base units and are obtained by multiplying or dividing base units with each other. Here, we will express some more derived units as products or quotients of the SI base units. Let us first take force as an example:

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$\text{Force} = \text{mass} \times \frac{\text{velocity}}{\text{time}}$$

$$\text{Force} = \text{mass} \times \frac{\text{displacement}}{\text{time}^2}$$

Now, we put SI units for each physical quantity i.e., N for force, kg for mass, m for displacement and s for time, so we get:

$$\text{N} = \text{kg} \times \text{m s}^{-2}$$

For work we proceed as:

$$\text{Work} = \text{Force} \times \text{displacement}$$

$$\text{Work} = \text{mass} \times \text{acceleration} \times \text{displacement}$$

$$\text{Work} = \text{mass} \times \frac{\text{velocity}}{\text{time}} \times \text{displacement}$$

$$\text{Work} = \text{mass} \times \frac{\text{displacement}}{\text{time}^2} \times \text{displacement}$$

$$\text{Work} = \text{mass} \times \frac{\text{displacement}^2}{\text{time}^2}$$



Now, we put SI units for each physical quantity i.e., J for work, kg for mass, m for displacement and s for time, so we get:

$$J = \text{kg} \times \frac{\text{m}^2}{\text{s}^2}$$

Similarly, we can express other derived units as products or quotients of the SI base units. Some examples are shown in the Table 1.2.

Table 1.2: Some derived units as products or quotients of the SI base units.

Name of Derived Quantity	SI Unit	Symbol	In terms of base units
Force	newton	N	kg m s^{-2}
Work	joule	J	$\text{N m} = \text{kg m}^2 \text{s}^{-2}$
Power	watt	W	$\text{J s}^{-1} = \text{kg m}^2 \text{s}^{-3}$
Pressure	pascal	Pa	$\text{N m}^{-2} = \text{kg m}^{-1} \text{s}^{-2}$
Electric Charge	coulomb	C	A s

1.3 DIMENSIONS OF PHYSICAL QUANTITIES

Dimension denotes the qualitative nature of a physical quantity. For example, length, width, height, distance, displacement, radius etc. all are measured in meters because they have the same nature and thus share the same dimensions.

Dimension of a physical quantity is often represented by capital letter enclosed in square brackets []. Dimensions for base quantities are given in the Table 1.3.

Table 1.3: Dimensions of Base Quantities.

Sr. No	Physical Quantity	Dimensions
1	mass	[M]
2	length	[L]
3	time	[T]
4	electric current	[I]
5	temperature	[θ]
6	intensity of light	[J]
7	amount of substance	[N]

Dimensions of derived quantities are obtained by multiplication or division of the dimensions of base quantities, from which these quantities are derived. For example, the dimension for area, volume, velocity and acceleration are $[L^2]$, $[L^3]$, $[LT^{-1}]$ and $[LT^{-2}]$ respectively.

Thus, dimensions give the relation of a given physical quantity with base quantities i.e. mass, length, time etc. There are the following essential terms used in dimensional analysis:

Dimensional Variables: Those physical quantities that have dimensions and variable in magnitude are called dimensional variables. Some dimensional variables are length, velocity, acceleration, force, energy and acceleration etc.

Dimensional Constants: Those physical quantities that have dimensions and a constant magnitude are called dimensional constants. Some examples of dimensional constants are Planck's constant (h), gravitational constant (G), speed of light in vacuum (c) and ideal gas constant (R) etc.

Dimensionless Variables: Those physical quantities that have no dimensions and have variable magnitudes are called dimensionless variables. Some examples of dimensionless variables are plane angle, solid angle, strain and coefficient of friction etc.

Dimensionless Constants: Those physical quantities that have no dimensions and have constant magnitude are called dimensionless constants. The pure numbers (1, 2, 3, ...), the exponential constant ($e = 2.718$) and π are some examples of dimensionless constants.

1.3.1 Advantages of Dimensions

Using the method of dimensions (called dimensional analysis), we can check the homogeneity of an equation, derive a possible formula and determine units of physical quantities. Dimensional analysis makes use of the fact that dimensions can be treated as algebraic quantities. That is, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions.

(i) The Homogeneity of an Equation

In order to check the correctness of an equation, we must ensure that both sides of the equation have the same dimensions: otherwise, the equation cannot be considered as a physically correct equation. This is called the principle of homogeneity of dimensions.

Let us check whether the equation $v_f = v_i + at$ is dimensionally correct.

$$\text{Dimensions of L.H.S.} = [LT^{-1}]$$

$$\begin{aligned}\text{Dimensions of R.H.S.} &= [LT^{-1}] + [LT^{-2}][T] \\ &= [LT^{-1}] + [LT^{-1}] \\ &= 2[LT^{-1}]\end{aligned}$$

As 2 is dimensionless constant, therefore

$$\text{Dimensions of L.H.S} = \text{Dimensions of R.H.S}$$

Hence, the equation is dimensionally correct.

(ii) To Derive a Possible Formula

Deriving a relation for a physical quantity depends on the correctly guessing various factors on which the physical quantity depends. Let us derive the formula for wavelength of matter waves using dimensional analysis.

As wavelength (λ) of matter waves may depend upon Planck's constant (h), velocity (v) and mass (m) of the particle.

So, the relation for the wavelength (λ) will be of the form:

$$\lambda \propto h^a m^b v^c$$



$$\lambda = (\text{constant}) h^a m^b v^c \quad \text{_____ (1)}$$

We have to find the values of powers i.e. a, b and c.

Using dimension on both sides, we get:

$$[L] = \text{constant} [M L^2 T^{-1}]^a [M]^b [LT^{-1}]^c$$

$$\text{OR} \quad [M^0 L^1 T^0] = \text{constant} [M]^{a+b} [L]^{2a+c} [T]^{-a-c} \quad \text{_____ (2)}$$

Equating the powers of M on both sides of equation (2), we get:

$$a + b = 0 \quad \text{_____ (3)}$$

Equating the powers of L on both sides of equation (2), we get:

$$2a + c = 1 \quad \text{_____ (4)}$$

Equating the powers of T on both sides of equation (2), we get:

$$-a - c = 0 \quad \text{_____ (5)}$$

On solving equations (3), (4) and (5), we get:

$$a = 1, b = -1 \text{ and } c = -1$$

Put the values of a, b, and c in (1), we get:

$$\lambda = (\text{constant}) h^1 m^{-1} v^{-1}$$

$$\text{OR} \quad \lambda = (\text{constant}) \times \frac{h}{mv}$$

1.3.2 Limitations of Dimensional Analysis

Some limitations of dimensional analysis are:

- 1) Dimensional analysis does not distinguish between the physical quantities having same dimensions. For example, if the dimensional formula of a physical quantity is $[ML^2T^{-2}]$ it may represent work, or energy, or torque.
- 2) Dimensional analysis cannot be used to derive formulas containing trigonometric functions, exponential functions, logarithmic functions, etc.
- 3) Dimensional analysis cannot determine the dimensionless constant when deriving a possible formula.
- 4) Dimensional analysis doesn't always prove that a relation is physically correct although relation is dimensionally correct. However, a dimensionally wrong equation is always wrong.

Example 1.2: Derive the formula for the time period of a simple pendulum using dimensional analysis.

Solution: The time period of a simple pendulum is possibly depending on the mass of the bob (m), the length of the pendulum (l), the angle which the string makes with vertical (θ) and the acceleration due to gravity (g). So, the relation for the time period T will be of the form:

$$T \propto m^a l^b \theta^c g^d$$

$$T = (\text{constant}) m^a l^b \theta^c g^d \quad \text{_____ (1)}$$

We have to find the values of powers i.e. a, b, c and d:

Using the dimension on both sides, we get:

$$[M^0 L^0 T] = \text{constant} [M]^a [L]^b [L L^{-1}]^c [LT^{-2}]^d$$

$$[M^0 L^0 T] = \text{constant} [M]^a [L]^{b+d} [T]^{-2d} \quad \text{_____ (2)}$$

Equating the powers of M on both sides of equation (2), we get:

$$a = 0 \quad \text{_____ (3)}$$

Equating the powers of L on both sides of equation (2), we get:

$$b + d = 0 \quad \text{_____ (4)}$$

Equating the powers of T on both sides of equation (2), we get:

$$-2d = 1$$

or

$$d = -1/2 \quad \text{_____ (5)}$$

Put $d = -1/2$, in (4), we get:

$$b = 1/2$$

Put the values of a, b, c and d in (1):

$$T = (\text{constant}) m^0 l^{1/2} g^{-1/2}$$

$$T = (\text{constant}) \times \sqrt{\frac{l}{g}}$$

Where the constant, found by experiment, is 2π .

Assignment 1.2

Which of the following relationships is dimensionally consistent with an expression yielding a value for acceleration? In these equations, x is a distance, t is time, and v is velocity.

- (a) v/t^2 (b) v/x^2 (c) v^2/t (d) v^2/x

1.4 PRECISION AND ACCURACY

Science is based on observations and experiments, which involve measurements. Precision is measurements of the same physical quantity agree with each other. Accuracy refers to how closely a measurement agrees with a standard or true value. Hence:

Precision refers to the closeness of measured values to each other, while accuracy refers to the closeness of a measured value to a standard or true value.

For Your Information



(a) precise and accurate



(b) precise but not accurate



(c) not precise but accurate



(d) neither precise nor accurate

Several independent trials of shooting at a bullseye target illustrate the difference between being accurate and being precise. Accuracy is how close an arrow gets to the bull's-eye center. Precision is how close a second arrow is to the first one (regardless of closeness to the target).

To understand the concept of precision and accuracy, consider a person who weighs exactly 160.0 pounds and he weighs himself three times on three different scales. Results of the scales are:

Scale A: 170.1, 169.9 and 170.0 pounds.

Scale B: 161, 162 and 158 pounds.

Scale C: 159.9, 160.0 and 160.1 pounds.



In this case, the weight measured by scale A is very precise, but not accurate. The weight measured by scale B is fairly accurate but not precise. The weight measured by scale C is both precise and accurate.

The precision of a measurement is associated with least count of the measuring instrument. The smaller the least count of the measuring instrument, the greater its precision. Precision is indicated by the absolute uncertainty in measurement. Accuracy is indicated by the fractional or percentage uncertainty or error in measurement. The smaller the magnitude of fractional or percentage uncertainty or error, the greater its accuracy.

1.5 UNCERTAINTIES

In Grade 9, we have studied about the sources of human error, systematic error and random error in experiments. The difference between the true value and observed value of a measurement is called error. i.e.,

$$\text{Error} = \text{true value} - \text{observed value}$$

In measurement the error may occur due to:

- Negligence or inexperience of a person.
- Using a faulty apparatus.
- Inappropriate method or technique.

Errors can be divided into the following three types:

- Personal Error
- Systematic Error
- Random Error

Here we will study about uncertainty.

Tip for Solving Numerical Problems Symbolic Solutions!

When solving problems, it is very useful to perform the solution completely in algebraic form and wait until the very end to enter numerical values into the final symbolic expression. This method will save many calculator keystrokes, especially if some quantities cancel so that you never have to enter their values into your calculator! In addition, you will only need to round once, on the final result.

Uncertainty is the range of possible values within which the true value of the measurement lies.

For example, a measurement of $3.06 \text{ mm} \pm 0.02 \text{ mm}$ means that the experimenter is confident that the actual value for the quantity being measured lies between 3.04 mm and 3.08 mm .

Uncertainty is a quantitative measurement of variability in the data.

All measurements have a degree of uncertainty. This is caused by two factors, the limitations of the measuring instrument (systematic error) and the skills of the experimenter.

Absolute uncertainty is equal to the least count of a measuring instrument, for example the length of a glass slab measured with a meter rod is 37.5 cm . The least count of meter rod is $1 \text{ mm} = 0.1 \text{ cm}$, then the absolute uncertainty in the measured value will be $\pm 0.1 \text{ cm}$ i.e., $\pm 0.05 \text{ cm}$ uncertainty develops at each end. For example, if one end of the slab coincides with 20.5 cm mark and the other end with 58.0 cm mark of meter rule, the length of the slab along with uncertainty is given by:

$$(58.0 \pm 0.05) \text{ cm} - (20.5 \pm 0.05) \text{ cm} = (37.5 \pm 0.1) \text{ cm}$$

It means that the length of slab is between 37.4 cm and 37.6 cm.

In the above measurement, precision is ± 0.1 cm, which is equal to the magnitude of absolute uncertainty.

The accuracy in the measurement is indicated by the magnitude of fractional error. Here:

$$\text{Fractional Uncertainty} = \frac{\text{Absolute Uncertainty}}{\text{Measured Value}}$$

$$\text{Fractional Uncertainty} = \frac{\pm 0.1}{37.5} = \pm 0.003$$

$$\text{Percentage uncertainty} = \text{fractional uncertainty} \times 100 \%$$

The smaller the magnitude of fractional (relative) uncertainty or error, the greater will be the accuracy of measurement.

1.5.1 Rules for Calculating Uncertainties in Final Result

There are some rules for calculating uncertainties in different cases but we need to be very careful whether we use the absolute or percentage uncertainty in each case.

Let x and y are two different physical quantities with uncertainties Δx and Δy respectively. If z is a physical quantity which is obtained by operating x and y , then the propagated uncertainty Δz in the result can be calculated by using the following rules.

a) Rule for Addition and Subtraction

If two or more than two measured quantities are added or subtracted, then their absolute uncertainties are added to get uncertainty in the result.

$$\text{If: } z = x + y \quad \text{or} \quad z = x - y$$

$$\text{then; } \Delta z = \pm (\Delta x + \Delta y)$$

$$\text{For example, if: } x \pm \Delta x = (24.0 \pm 0.1) \text{ cm}$$

$$\text{and } y \pm \Delta y = (30.0 \pm 0.1) \text{ cm}$$

$$\text{then; } \Delta z = \pm 0.2 \text{ cm}$$

b) Rule for Multiplication and Division

If two or more than two quantities are multiplied or divided, then their percentage uncertainties are added to get uncertainty in the result.

$$\text{If: } z = xy \quad \text{or} \quad z = x/y,$$

$$\text{then; } \% \text{ uncertainty in } z = \% \text{ uncertainty in } x + \% \text{ uncertainty in } y$$

c) Rule for Power of a Quantity

The total uncertainty in power of a quantity is equal to the percentage uncertainty multiplied with that power.

$$\text{If: } z = x^3,$$

$$\text{then; } \% \text{ uncertainty in } z = \pm 3 (\% \text{ uncertainty in } x)$$

d) Uncertainties in Average Values of Many Measurements

The uncertainty in the average value is calculated by adopting the following steps:

- Find the average of measured values.
- Find the deviation of each value from the average.

Remember!

If $x \pm \Delta x = (2.0 \pm 0.1) \text{ mm}$, then
Actual/Absolute uncertainty is

$$\Delta x = \pm 0.1 \text{ mm}$$

Fractional uncertainty is:

$$\frac{\Delta x}{x} = 0.05$$

Percentage uncertainty is:

$$\frac{\Delta x}{x} \times 100\% = 5 \%$$



iii) The mean deviation is the uncertainty in the average.

For example, three readings are recorded for the radius of a small cylinder as:

$$r_1 = 1.50 \text{ cm}, r_2 = 1.51 \text{ cm} \text{ and } r_3 = 1.52 \text{ cm}$$

The uncertainty in the average radius is calculated as:

Finding average:
$$\bar{r} = \frac{r_1 + r_2 + r_3}{3}$$
$$= \frac{1.50 \text{ cm} + 1.51 \text{ cm} + 1.52 \text{ cm}}{3} = 1.51 \text{ cm}$$

Finding deviation:
$$\Delta r_1 = \bar{r} - r_1 = 1.51 - 1.50 = 0.01 \text{ cm}$$
$$\Delta r_2 = \bar{r} - r_2 = 1.51 - 1.51 = 0 \text{ cm}$$
$$\Delta r_3 = \bar{r} - r_3 = 1.51 - 1.52 = 0.01 \text{ cm}$$

Finding mean deviation:
$$\overline{\Delta r} = \frac{\Delta r_1 + \Delta r_2 + \Delta r_3}{3}$$
$$= \frac{0.01 \text{ cm} + 0 \text{ cm} + 0.01 \text{ cm}}{3} = 0.0067 \text{ cm} = 0.007 \text{ cm}$$

e) Uncertainty in Timing Experiment.

The time period T of a vibrating body can be found by dividing time of multiple vibrations by the number of vibrations.

$$T = \frac{\text{Time of multiple vibrations}}{\text{No. of vibrations}}$$

The uncertainty in time period ΔT is found by dividing the least count (L.C) of the time recording device by the number of vibrations.

$$\Delta T = \frac{\text{L.C}}{\text{No. of vibrations}}$$

For example, the time recorded for 20 vibrations of a pendulum is $t = 35.2 \text{ s}$. Let the least count of stop watch used is 0.1 s ($1/10 \text{ s}$). So, the uncertainty in measured time is $(35.2 \text{ s} \pm 0.1 \text{ s})$.

Then the time period of the pendulum is obtained as:

$$T = 35.2/20 = 1.76 \text{ s}$$

Uncertainty in time period is $\Delta T = 0.1/20 = 0.005 \text{ s}$

So, $T \pm \Delta T = (1.76 \pm 0.005) \text{ s}$

Example 1.3: If voltage measured across a conductor is $V \pm \Delta V = (7.3 \pm 0.1) \text{ volts}$ and current is $I \pm \Delta I = (2.73 \pm 0.051) \text{ ampere}$. Find the resistance and uncertainty in it.

Given: $V \pm \Delta V = (7.3 \pm 0.1) \text{ volts}$ $I \pm \Delta I = (2.73 \pm 0.051) \text{ ampere}$

To Find: $R \pm \Delta R = ?$

Solution: According to ohm's law, R is calculated as:

$$R = V/I = 7.3/2.73 = 2.7 \Omega$$

Percentage uncertainty in V is:

$$= \frac{\Delta V}{V} \times 100\% = \frac{0.1}{7.3} \times 100\% = 1.37\% = 1\%$$

Percentage uncertainty in I is:

$$= \frac{\Delta I}{I} \times 100\% = \frac{0.05}{2.73} \times 100\% = 1.83\% = 2\%$$

Thus, the total uncertainty in R is:

$$= 1\% + 2\% = 3\%$$

So, $R \pm \Delta R = 2.7 \pm 3\%$

$$= 2.7 \Omega \pm \left(\frac{3}{100} \times 2.7 \right) \Omega$$

$$= (2.7 \pm 0.08) \Omega$$

Example 1.4: If radius of a circular disc is measured as 2.25 cm with uncertainty ± 0.01 cm. Find its surface area with uncertainty in it.

Given: $r = 2.25$ cm, $\Delta r = \pm 0.01$ cm

To Find: $A \pm \Delta A = ?$

Solution: As, $A = \pi r^2 = 3.14 \times 2.25^2 = 15.90 \text{ cm}^2$

Percentage uncertainty in $r = \frac{\Delta r}{r} \times 100\% = \frac{0.01}{2.25} \times 100\% = 0.4\%$

Percentage uncertainty in area is $= 2 \times 0.4\% = 0.8\%$

So, $\Delta A = 0.8\% \times 15.90 \text{ cm}^2 = 0.13 \text{ cm}^2$

Thus $A \pm \Delta A = (15.90 \pm 0.13) \text{ cm}^2$

Assignment 1.3

The radius of a circle is measured to be (10.5 ± 0.2) m. Calculate (a) the area and (b) the circumference of the circle, also give the uncertainty in each value.

SUMMARY

- ❖ **Estimation** does not mean guessing a formula or a number at random. An estimation is a rough educated guess to the value of a physical quantity by using prior experience and sound physical reasoning.
- ❖ **Derived units** can be expressed in terms of base units and are obtained by multiplying or dividing base units.
- ❖ **Dimension** denotes the qualitative nature of a physical quantity.
- ❖ In order to check the correctness of an equation, we have to show that both sides of an equation have the same dimensions; otherwise, the equation cannot be physically correct. This is called the **principle of homogeneity of dimensions**.
- ❖ **Uncertainty** is the range of possible values within which the true value of the measurement lies.
- ❖ **Absolute uncertainty** is equal to the least count of a measuring instrument.
- ❖ **Precision** refers to the closeness of measured values to each other.
- ❖ **Accuracy** refers to the closeness of a measured value to a standard or true value.



EXERCISE

Multiple Choice Questions

Encircle the correct option.

- 1) The mean diameter of a wire is found to be (0.50 ± 0.02) mm. The percentage uncertainty in the diameter is:
 A. 2 % B. 4 % C. 6 % D. 8 %
- 2) A reaction takes place that is expected to yield 171.9 g of product, but it only yields 154.8 g. What is the percent error for this experiment?
 A. 17.1 % B. 90.1 % C. 111.0 % D. 9.9 %
- 3) Three different people weigh a standard mass of 2.00 g on the same balance. Each person obtains a reading of exactly 7.32 g for the mass of the standard. These results imply that the balance is:
 A. both accurate and precise B. neither accurate nor precise
 C. accurate but not precise D. precise but not accurate.
- 4) Dimension of universal gravitational constant (G) is:
 A. $[M^{-2}L^3T^{-2}]$ B. $[M^3L^{-1}T^{-2}]$ C. $[M^{-1}L^3T^{-2}]$ D. $[M^{-3}L^3T^{-2}]$
- 5) A measurement, which on, repetition gives same or nearly same result is called:
 A. accurate B. average C. precise D. estimated
- 6) A student is measuring the time of an event by using stopwatch. He takes 5 measurements as: 3.0 s, 3.2 s, 3.4 s, 2.8 s, 3.1 s. What is the uncertainty in the results?
 A. ± 0.3 s B. ± 0.6 s C. ± 3.1 s D. ± 7.75 s
- 7) Which of the following quantity has a different dimension?
 A. force B. weight C. modulus of elasticity D. tension
- 8) If the dimensions of a physical quantity are given by $[L^a M^b T^c]$, then the physical quantity will be:
 A. force, if $a = -1, b = 0, c = -2$ B. pressure, if $a = -1, b = 1, c = -2$
 C. velocity, if $a = 1, b = 0, c = 1$ D. acceleration, if $a = 1, b = 1, c = -2$
- 9) Order of magnitude of $(10^6 + 10^3)$ is:
 A. 10^{18} B. 10^9 C. 10^6 D. 10^3
- 10) Which of the following may be used as a valid formula to calculate the speed of ocean waves? [v = speed, g = acceleration due to gravity, λ = wavelength, ρ = density, h = depth].
 A. $v = \sqrt{\lambda g}$ C. $v = \rho gh$ C. $v = gh/\lambda$ D. $v = \lambda gh$



PHYSICAL QUANTITIES AND MEASUREMENTS

Short Questions

Give short answers of the following questions.

- 1.1 Draw a table to show a reasonable estimate of some physical quantities.
- 1.2 Express the units of the following derived quantities in terms of base units. (a) Force (b) Work (c) Power (d) Pressure (e) Electric charge.
- 1.3 Why is it important to use an instrument with the smallest resolution?
- 1.4 What is the importance of increasing the number of readings in an experiment?
- 1.5 What is the difference between precision and accuracy?
- 1.6 What is the principle of homogeneity of dimensions?
- 1.7 A ball is thrown in the air and 5 different students are individually measuring the time it takes to fall back down using stopwatches. The times obtained by each student are the following: 6.2 s, 6.0 s, 6.4 s, 6.1 s, 5.8 s. (i) What is the uncertainty of the results? (ii) How should the resulting time be expressed?
- 1.8 The energy of a photon is given by $E = hf$, where f is frequency. Find the dimensions of Planck's constant h .
- 1.9 Justify why all measurements contain some uncertainty.

Comprehensive Questions

Answer the following questions in detail.

- 1.1 Define and explain the term uncertainty.
- 1.2 Discuss the rules for calculating uncertainty propagation in the final results in different cases.
- 1.3 What does the dimension of a physical quantity mean? What are its advantages? Explain with examples?
- 1.4 What is meant by estimation of a physical quantity? Explain with examples.

Numerical Problems

- 1.1 Estimate number of heartbeats in a lifetime of 60-years? (Ans: 10^9)
- 1.2 Determine the dimensions of each of the following quantities.
a) $\frac{v^2}{ax}$ b) $\frac{at^2}{2}$ (Ans: (a) No, (b) [L])
- 1.3 If $A = \frac{X^2}{Y^2Z}$, then find the percentage uncertainty in A. The percentage uncertainties in X, Y and Z are 1 %, 1 % and 2 % respectively. (Ans: 6 %)
- 1.4 A spherical ball of radius r experiences a resistive force F due to the air as it moves through it at speed v . The resistive force F is given by the expression

$$F = c r v$$



Where c is constant. By using dimensions, derive the SI base unit of the constant c .

(Ans: $\text{kg m}^{-1}\text{s}^{-1}$)

1.5 The pressure (P) at a depth (h) in an incompressible fluid of density (ρ) is given by

$$P = \rho g h$$

Where g is acceleration due to gravity. Check the homogeneity of this equation.

1.6 Estimate how many protons are there in a bacterium? (Take mass of bacterium as 10^{-15} kg and mass of proton 10^{-27} kg).

Ans: 10^{12} protons)

1.7 Estimate how many hydrogen atoms does it take to stretch across the diameter of the Sun? (Take diameter of the Sun as 10^6 km and diameter of proton 10^{-14} km).

(Ans: 10^{19} hydrogen atoms)

1.8 The current passing through a resistor $R = (13 \pm 0.5) \Omega$ is $I = (3 \pm 0.1) \text{ A}$.

- Calculate the power consumed (correct to one significant figure).
- Find the percentage uncertainty of the current passing through the resistor.
- Find the percentage uncertainty of the resistance.
- Find the absolute uncertainty of the electrical power.

(Ans: 117 W, 3 %, 3.84 %, 11.7 W)