

Students' Learning Outcomes

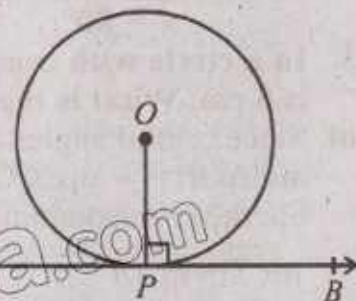
After completing this unit, the students will be able to:

- ▶ Solve problems by using the tangent and angle properties of a circle:
 - o If a line is drawn perpendicular to a radial segment of a circle at its outer end point, it is tangent to the circle at that point.
 - o The tangent to a circle and the radial segment joining the point of contact and the centre are perpendicular to each other.
 - o The two tangents drawn to a circle from a point outside it, are equal in length.
 - o If two circles touch externally or internally, then the distance between their centres is respectively equal to the sum or difference of their radii.
 - o The measure of a central angle of a minor arc of a circle is double that of the angle subtended by the corresponding major arc.
 - o Any two angles in the same segment of a circle are equal.
 - o The angle in a semi-circle is a right angle, in a segment greater than a semi-circle is less than a right angle, in a segment less than a semi-circle is greater than a right angle.
 - o The opposite angles of a quadrilateral inscribed in a circle are supplementary.
- ▶ Find the arc length and area of sector of a circle using angle in both degrees and radians.
- ▶ Apply concepts of tangents and angles of a circle to real life world problems (such as architecture, monuments, Pyramids).
- ▶ Apply trigonometry to solve real life problems in arc length and area of the sector of circle.

9.1

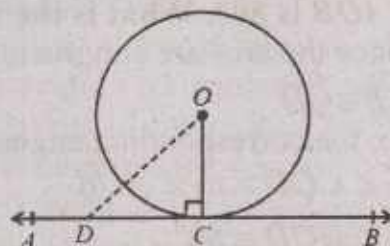
Tangent to a Circle

In a plane, a line is a tangent to a circle that touches only one point on the circumference of a circle. Here $\overline{AB}$ is a tangent at point P on the circle. Point P is called **point of tangency** or **point of contact**.



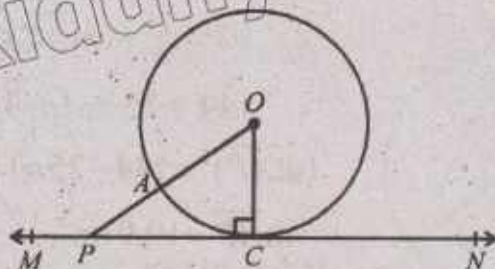
(a) If a Line is Drawn Perpendicular to a Radial segment of a Circle at its Outer End Point, it is Tangent to the Circle at that Point

If a circle with centre O and $\overline{OC}$ is its radial segment. $\overline{AB} \perp \overline{OC}$ at C , then $\overline{AB}$ is tangent to the circle at point C .



b The Tangent to a Circle and the Radial Segment Joining the Point of Contact and the Centre, are Perpendicular to Each Other.

If a circle with centre O , $\overline{OC}$ is its radial segment. $\overline{MN}$ is a tangent at point C , then $\overline{OC} \perp \overline{MN}$



Example 1: $\overline{QR}$ is a tangent to a circle with centre P at point Q of radius 4 cm. It meets the line segment PR such that $m\overline{PR} = 8\text{ cm}$. What is the length of $\overline{QR}$?

Solution: Since $\overline{QR}$ is tangent to the circle at Q and it is perpendicular to the $\overline{PQ}$.

$\triangle PQR$ is a right-angled triangle and $\overline{PR}$ is its hypotenuse. By using Pythagoras theorem,

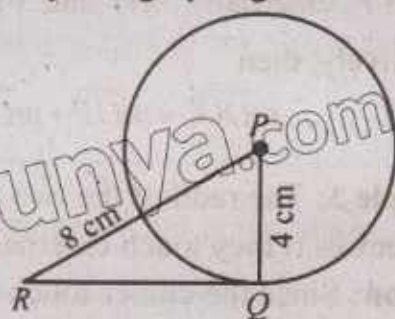
$$(m\overline{PR})^2 = (m\overline{PQ})^2 + (m\overline{QR})^2$$

$$8^2 = 4^2 + (m\overline{QR})^2$$

$$64 = 16 + (m\overline{QR})^2$$

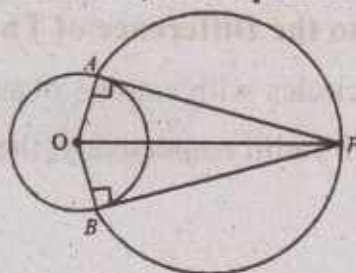
$$(m\overline{QR})^2 = 64 - 16 = 48$$

$$m\overline{QR} = 6.93\text{ cm}$$



The Two Tangents Drawn to a Circle. From a Point Outside it, are Equal in Length.

If $\overline{PA}$ and $\overline{PB}$ are two tangents drawn from an external point P to a circle having centre at O , where A and B are points of contact, then $m\overline{PA} = m\overline{PB}$.

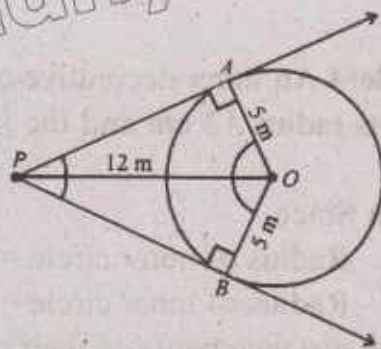


Example 2: An architect designs a circular dome on a building. A lighting rod is placed 12 m away from the centre of the dome which has a radius of 5 m. Support cables are attached from the rod to the dome, touching the dome at two points. What the length of each cable?

Solution: Suppose two tangents (cables) $\overline{PA}$ and $\overline{PB}$ are drawn to the circumference of the circle (dome) from the point P (on the rod). Here,

$$m\overline{OP} = 12\text{ m}, m\overline{OA} = m\overline{OB} = 5\text{ m}$$

In the right triangle OAP , by Pythagoras theorem, we have



$$(\overline{OP})^2 = (\overline{OA})^2 + (\overline{AP})^2$$

$$12^2 = 5^2 + (\overline{AP})^2$$

$$144 = 25 + (\overline{AP})^2$$

$$(\overline{AP})^2 = 144 - 25 = 119$$

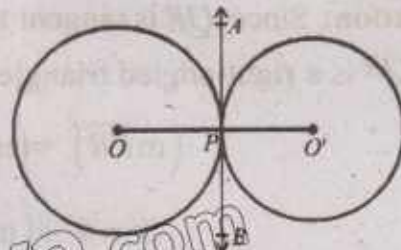
$$\overline{AP} = 10.91\text{m}$$

So, the length of each cable is 10.91 m

(a) If Two Circles Touch Externally, then the Distance Between Their Centres is Equal to the Sum of their Radii.

If two circles with centres O and O' touch each other at point P , externally. $\overline{PO}$ and $\overline{PO'}$ are their radii respectively, then

$$\overline{OO'} = \overline{OP} + \overline{O'P}$$



Example 3: The radii of the two circles are 4 cm and 6 cm. What is the distance between their centres if they touch externally?

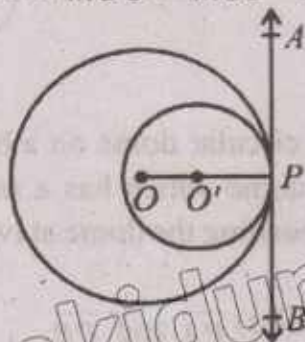
Solution: Since the circles touch externally,

So, the distance between their centres = sum of their radii

$$= 4 + 6 = 10\text{cm}$$

(b) If Two Circles Touch Internally, Then the Distance Between Their Centres is Equal to the Difference of Their Radii.

If two circles with centres O and O' touch each other at point P internally, $\overline{PO}$ and $\overline{PO'}$ are their radii respectively, then $\overline{OO'} = \overline{OP} - \overline{O'P}$.



Example:4 An inner decorative circle touches the outer clock face internally. If the outer circle has radius 12 cm and the inner has radius 7.5 cm, find the distance between their centres.

Solution Since,

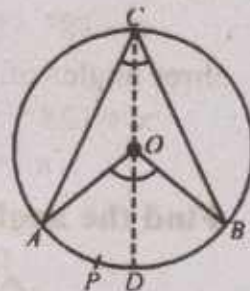
Radius of outer circle = 12 cm

Radius of inner circle = 7.5 cm

Thus, the distance between their centres = $12 - 7.5 = 4.5$ cm

The Measure of a Central Angle of a Minor Arc of a Circle is Double that of the Angle Subtended by the Corresponding Major Arc.

In the circle with centre O , APB is a minor arc. It subtends an angle AOB at the centre and an angle ACB is subtended by the major arc.



then $m\angle AOB = 2m\angle ACB$

Example 5: A curved running track forms an arc that subtends 60° at the centre of the circular ground. What angle does it subtend at a point on the boundary?

Solution Since curved running track forms an arc that subtends 60° at the centre of the circular ground

As we know that the measure of an angle which an arc of a circle subtends at the centre is double that which it subtends at any point on the remaining part of the circumference.

Therefore, angle at circumference = $\frac{60^\circ}{2} = 30^\circ$

Skilled practice!

A radar antenna scans an arc subtending 90° at the centre. What is the angle subtended by the arc at the edge of its circular path?

Sol. Angle at the centre = 90°

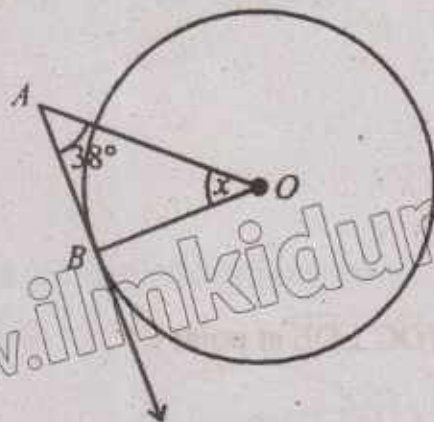
2 (angle at circumference) = Angle at the centre (inscribed angle theorem) angle at

circumference = $\frac{90^\circ}{2} = 45^\circ$

So, angle at the edge = 45°

Solved Exercise 9.1

1. Find the value of x .



Sol. In a triangle

Measure of the angle outside the circle = 38°

Measure of the angle on the centre of circle = x

Measure of the third angle of triangle = 90° (Radial segment and Tangent are

perpendicular)

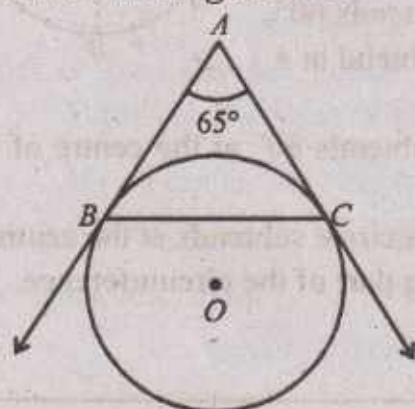
Thus,

$x + 38^\circ + 90^\circ = 180^\circ$ (sum of the three angles of triangle)

$$x + 128^\circ = 180^\circ$$

$$x = 180^\circ - 128^\circ = 52^\circ$$

2. Find the angle ABC .



Sol. Tangents from a point (A) are equal (Theorem)

$$m\overline{AB} = m\overline{AC}$$

3. A, B and C are points on the circumference of a circle with centre at O. $\overline{AC}$ is the diameter of the circle and $\overline{DE}$ is the tangent to the circle at the point C and $m\angle BCD = 62^\circ$. Find

(i) $m\angle BCA$

(ii) $m\angle BAC$

Sol. Given

Diameter of circle = $\overline{AC}$

Tangent on point C = $\overline{DE}$

$$m\angle BCD = 62^\circ$$

$$m\angle BCA + m\angle BCD = 90^\circ \text{ (OC} \perp \overline{DE} \text{ at point C)}$$

$$m\angle BCA + 62^\circ = 90^\circ$$

$$m\angle BCA = 90^\circ - 62^\circ$$

$$m\angle BCA = 28^\circ \quad \text{_____ (i)}$$

In triangle ABC

So $\triangle ABC$ is an isosceles triangle

In $\triangle ABC$, the sum of the angles is 180°

$$\text{Since } m\overline{AB} = m\overline{AC}$$

then

$$m\angle ABC = m\angle ACB$$

$$\text{Let } m\angle ABC = m\angle ACB = x$$

We know

$$x + x + 65^\circ = 180^\circ \text{ (Sum of the angles of a triangle)}$$

$$2x + 65^\circ = 180^\circ$$

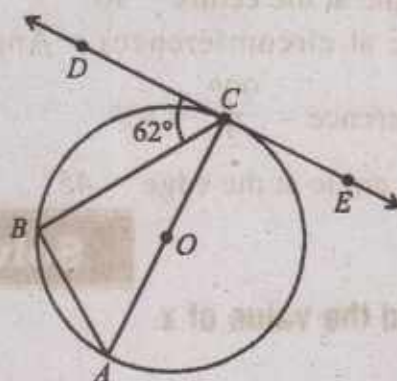
$$2x = 180^\circ - 65^\circ$$

$$2x = 115^\circ$$

$$x = \frac{115}{2}$$

$$x = 57.5^\circ$$

$$\text{So, } m\angle ABC = 57.5^\circ$$



$m\angle ABC = 90^\circ$ [The angle on circumference made by diameter is a right angle (90°)]

$$m\angle BCA = 28^\circ$$

We know

$$m\angle BAC + m\angle ABC + m\angle BCA = 180^\circ$$

$$m\angle BAC + 90^\circ + 28^\circ = 180^\circ$$

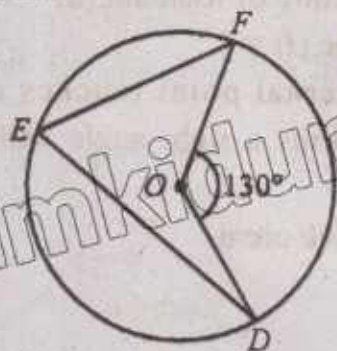
$$m\angle BAC + 118^\circ = 180^\circ$$

$$m\angle BAC = 180^\circ - 118^\circ$$

$$= 62^\circ \quad \text{--- (ii)}$$

So, $m\angle BCA = 28^\circ$, $m\angle BAC = 62^\circ$ (From (i) and (ii))

4. D, E and F are points on the circumference of a circle with centre at O and $m\angle DOF = 130^\circ$. Find $m\angle DEF$.



Sol. In circle with centre O, $\widehat{DF}$ is minor arc.

It subtends an angle DOF at the centre and an angle DEF is subtended by the major arc.

As by theorem, the measure of a central angle of a minor arc of a circle is double that of the angle subtended by the corresponding major arc.

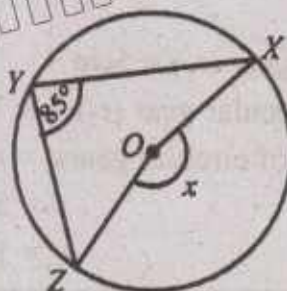
$$2 m\angle DEF = m\angle DOF \quad (\text{By the Theorem})$$

$$2m\angle DEF = 130^\circ$$

$$m\angle DEF = \frac{130^\circ}{2}$$

$$m\angle DEF = 65^\circ$$

5. X, Y and Z are points on the circumference of a circle with centre at O and $m\angle XYZ = 85^\circ$. Find x .



Sol. In the circle with centre O, $\widehat{ZX}$ is the minor arc.

It subtends an angle ZO at the centre, and angle ZYX is subtended by the major arc. As by Theorem, the measure of a central angle of a minor arc of circle is double that of the angle subtended by the corresponding major arc.

$$m\angle ZOX = 2m\angle XYZ$$

$$x = 2(85^\circ)$$

$$x = 170^\circ$$

- 6.** In a historical monument, a circular fountain with a radius of 3 m is built. A flagpole is erected 7 m away from the centre of the fountain. Two ropes from the pole are tied to the edge of the fountain, just touching it. Find the length of each rope.

Sol. Radius of circular fountain (r) = 3m

Distance of the pole from centre of fountain (d) = 7m

length of the rope (l) = ?

When a line from an external point touches a circle's edge (tangent) it forms a right angle with the radius at that point

We can use the pythagoras the orem

$$d^2 = l^2 + r^2$$

$$7^2 = l^2 + 3^2$$

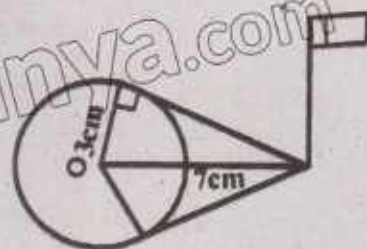
$$49 = l^2 + 9$$

$$49 - 9 = l^2$$

$$40 = l^2$$

$$\sqrt{l^2} = \sqrt{40}$$

$$l = 6.32\text{m}$$



As two tangents to a circle from a point outside are equal. So, the length of each rope is equal.

So, length of each rope = 6.32m

- 7.** Two circular gears touch each other externally for proper rotation in a machine. The radii of the two circular gears are 5cm and 7cm. What is the distance between their centres if they touch externally?

Sol. When two circles touch externally, the distance between their centres is the sum of their radii.

Radius of first circular gear (r_1) = 5cm

Radius of the second circular gear (r_2) = 7cm

Distance between centres of circular gears = $r_1 + r_2$

$$= 5\text{cm} + 7\text{cm}$$

$$= 12\text{cm}$$

8. A small sensor lies inside a satellite dish and touches its wall internally. If the dish has radius 15 cm and the sensor has radius 2.5 cm, find the distance between their centres.

Sol. When two circles touch internally, the distance between their centres is the difference between their radii.

$$\text{Radius of the satellite dish (R)} = 15\text{cm}$$

$$\text{Radius of the small sensor (r)} = 2.5\text{cm}$$

$$\text{Distance between their centers} = R - r$$

$$= 15\text{cm} - 2.5\text{cm}$$

$$= 12.5\text{cm}$$

9. An inner holder touches the outer cylindrical container internally. If the outer container has radius 8 cm and the inner holder has radius 6 cm, find the distance between their centres.

Sol. When two circles internally, the distance between their centres is the difference between their radii.

$$\text{Radius of outer cylindrical holder (R)} = 8\text{cm}$$

$$\text{Radius of inner holder (r)} = 6\text{cm}$$

$$\text{Distance between their centres} = R - r$$

$$= 8\text{cm} - 6\text{cm}$$

$$= 2\text{cm}$$

10. A pyramid-shaped sculpture is placed in the center of a circular plaza with a radius of 10 m. A decorative pole stands 26 m from the center of the circle. Two guide wires are attached from the pole to the plaza edge, just touching the circle. Find the length of each wire.

Sol. Radius of the circular plaza (r) = 10m

$$\text{Distance of the pole from centre of plaza (d)} = 26\text{m}$$

$$\text{Length of the wire (l)} = ?$$

When a line from an external point touches a circle's edge (a tangent), it forms a right angle with the radius at that point.

No, we can use Pythagoras theorem

$$d^2 = l^2 + r^2$$

$$(26)^2 = l^2 + (10)^2$$

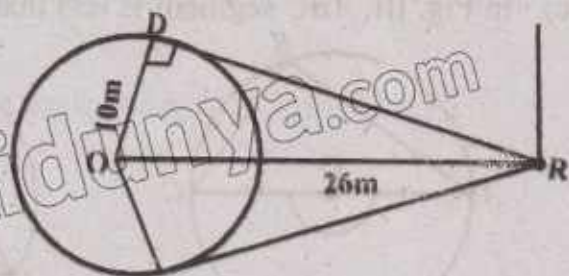
$$676 = l^2 + 100$$

$$676 - 100 = l^2$$

$$576 = l^2$$

$$\sqrt{l^2} = \sqrt{576}$$

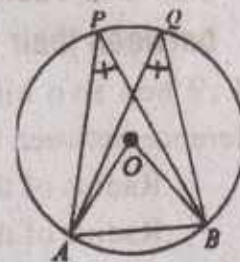
$$l = 24\text{m}$$



As two tangents to a circle from a point outside are equal. So, the length of two wires is equal. So, the length of each wire = 24m

Any Two Angles in the Same Segment of a Circle are Equal.

If $\angle APB$ and $\angle AQB$ be angles in the same segment APQB, of a circle whose centre is O , then $m\angle APB = m\angle AQB$



Example 6 In a large circular stained-glass window of a library, two decorative beams are drawn from the same chord AB to two points P and Q on the arc opposite to $\widehat{AB}$ such that $\angle APB$ and $\angle AQB$ are formed. If $\angle APB$ is measured to be 40° , what is $m\angle AQB$?

Sol. According to the article 9.1.5, angles in the same segment are equal.

Since $m\angle APB = 40^\circ$

Therefore, $m\angle AQB = 40^\circ$

Skilled practice!

A projector casts an image over an arc on a circular wall. Two observers on the arc segment report one observing a 50° angle. What is the other angle in the same segment?

Sol. Observer 1 reports an angle of 50°

Since observer 2 is in the same

Segment, their observed angle must be identical to the first.

Therefore, the other angle is also of 50°

(a) The Angle in a Semi-Circle is a Right Angle

(b) The Angle in a Segment Greater than a Semi-Circle is Less than a Right-Angle.

(c) The Angle in a Segment Less than a Semi-Circle is Greater than a Right-Angle.

A circle with centre at O and $\widehat{AC}$ is a chord of the circle.

(a) In Fig. I, ABC segment is a semi-circle, then $m\angle ABC = 90^\circ$.

(b) In Fig. II, ABC segment is greater than a semi-circle, then $m\angle ABC < 90^\circ$.

(c) In Fig. III, ABC segment is less than a semi-circle, then $m\angle ABC > 90^\circ$.

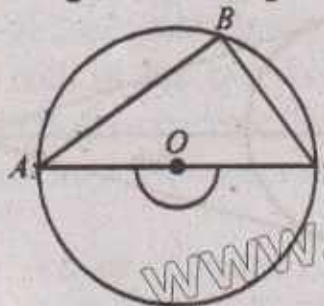


Figure I



Figure II

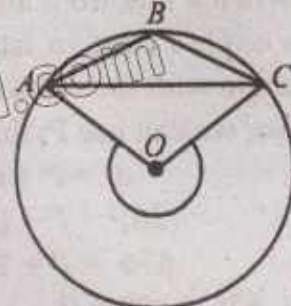


Figure III

Example 7 In the diagram P , Q and R are points on the circumference of a circle and the line segment PQ is the diameter. Find the angles x and y .

Solution. Since angle in semi-circle is 90°

Therefore, $x = 90^\circ$

As we know that the sum of the interior angles of any triangle is 180°

$$46^\circ + 90^\circ + y = 180^\circ$$

i.e., $136^\circ + y = 180^\circ$

$$y = 180^\circ - 136^\circ = 44^\circ$$

Skilled practice!

A cable in a semicircular bridge forms a triangle with the diameter of the arch. If the cable meets the endpoints of the diameter and the top of the arch, what is the angle at the top?

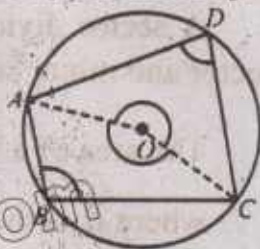
Sol. Because the bridge arch is semi-circular and the triangle is formed using the diameter, the angle at the top is a right angle of 90° .

The Opposite Angles of Any Quadrilateral Inscribed In a Circle are Supplementary.

If a circle with centre at O and $ABCD$ is an inscribed quadrilateral, then

$$m\angle CBA + m\angle ADC = 180^\circ$$

$$m\angle BAD + m\angle BCD = 180^\circ$$



Example 8 S, T, U and V are points on the circumference of a circle. Find the angles x and y .

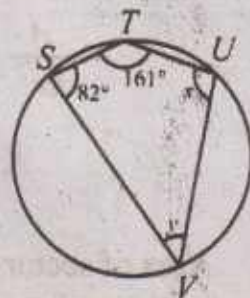
Solution: As we know that opposite angles of any quadrilateral inscribed in a circle are supplementary. Therefore,

$$x + 82^\circ = 180^\circ$$

$$x = 98^\circ$$

$$y + 161^\circ = 180^\circ$$

$$y = 19^\circ$$



Skilled practice!

A round table has a decorative quadrilateral pattern inscribed on its surface. If one angle of the quadrilateral is 105° , what is the measure of the opposite angle?

Sol. Let the given angle be $A = 105^\circ$

Let the opposite angle be B

Since they are supplementary. [opposite angles of cyclic Quadrilateral]

$$m\angle A + m\angle B = 180^\circ$$

$$105^\circ + m\angle B = 180^\circ$$

$$m\angle B = 180^\circ - 105^\circ$$

$$m\angle B = 75^\circ$$

9.2

Area of Sector and Arc Length of a Circle

Arc Length and Area of Sector

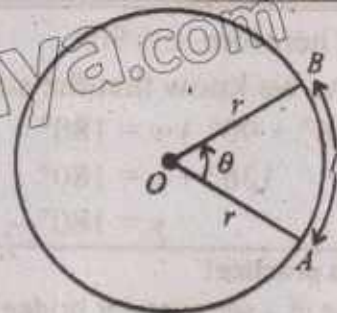
Let ℓ be the length of a circular arc AB of a circle of radius r and θ be its central angle measure in radians. Then Ratio of ℓ to the circumference $2\pi r$ = Ratio of θ to 2π

or $\ell : 2\pi r = \theta : 2\pi$

or $\frac{\ell}{2\pi r} = \frac{\theta}{2\pi}$

$$\ell = \frac{\theta}{2\pi} \times 2\pi r$$

$$\ell = r\theta, \text{ where } \theta \text{ is in radian.}$$



Area of a Sector

A sector of a circle is a region bounded by two radii and corresponding arc.

A slice of pizza and region between the hands of a watch are examples of a sector of circle.

A sector divides the circle into two regions namely major sector and minor sector.



The area of a sector of a circular region of radius r is $\frac{1}{2} r^2 \theta$

where θ is the central angle and it is measured in radians.

Proof: As shown in adjoining figure,

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Central angle of sector}}{2\pi}$$

$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

$$A = \frac{\pi r^2 \theta}{2\pi}$$

$$\Rightarrow \text{Area of sector} = \frac{1}{2} r^2 \theta$$

Note

$$A = \frac{1}{2} r^2 \theta$$

$$\therefore \ell = r\theta$$

$$\text{So, } A = \frac{1}{2} r(r\theta)$$

$$A = \frac{1}{2} r\ell$$

Example 9 Find the area of sector of a circle having central angle 60° and radius 7 cm. Also find the length of the arc AB.

Solution: Here $\theta = 60^\circ = \frac{60 \times \pi}{180} = 1.047 \text{ rad}, r = 7 \text{ cm}$

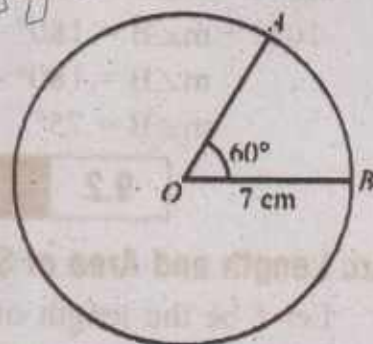
$$\text{Area of sector} = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} (7)^2 (1.047)$$

$$= \frac{1}{2} (49) (1.047) = 25.65 \text{ cm}^2$$

$$\text{Length of arc } AB = r\theta$$

$$= (7)(1.047) = 7.33 \text{ cm}$$



Example 10 The length of an arc and area of sector of a circle are 5 cm and 25 cm^2 respectively. Find radius and the central angle of sector.

Solution: $\ell = 5 \text{ cm}$, $A = 25 \text{ cm}^2$, $r = ?$, $\theta = ?$

As $\ell = r\theta \Rightarrow 5 = r\theta$

$$\theta = \frac{5}{r} \quad \text{(i)}$$

and $A = \frac{1}{2} r^2 \theta$

$$25 = \frac{1}{2} r^2 \left(\frac{5}{r} \right) \quad \text{using eq. (i)}$$

$$\frac{25 \times 2}{5} = r \Rightarrow \boxed{r = 10 \text{ cm}}$$

Put value of r in (i), we have

$$\theta = \frac{5}{10}$$

$$\theta = \frac{1}{2} \text{ radian}$$

Skilled Practise!

If ℓ is the arc length and r is the radius of the circle where $\ell = 14 \text{ m}$, $r = 4 \text{ m}$, then find the central angle. Also find the area of the sector of the circle.

Sol. $\ell = 14 \text{ cm}$, $r = 4 \text{ cm}$, $\theta = ?$

we know $\ell = r\theta$

$$\theta = \frac{\ell}{r}$$

$$\theta = \frac{14}{4}$$

$$\theta = 3.5 \text{ radians}$$

Area of sector of the circle

$$A = \frac{1}{2} r^2 \theta$$

$$A = \frac{1}{2} \times (4)^2 (3.5)$$

$$A = \frac{1}{2} \times 16 \times 3.5$$

$$A = 8 \times 3.5$$

$$A = 28 \text{ cm}^2$$

Solved Exercise 9.2

1. A, B, C and D are points on circumference of a circle with centre at O as shown in figure. Find the angle x .

Sol. measure of exterior angle at point $C = 114^\circ$

$$m\angle BCD + \text{exterior angle} = 180^\circ \text{ (angles of straight line)}$$

$$m\angle BCD + 114^\circ = 180^\circ$$

$$\begin{aligned} m\angle BCD &= 180^\circ - 114^\circ \\ &= 66^\circ \end{aligned}$$

In cyclic quadrilateral

$$m\angle BAD + m\angle BCD = 180^\circ$$

$$m\angle BAD + 66^\circ = 180^\circ$$

$$\begin{aligned} m\angle BAD &= 180^\circ - 66^\circ \\ &= 114^\circ \end{aligned}$$

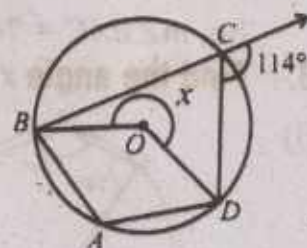
The reflex angle is exactly double the angle subtended by the same arc at the circumference on the opposite side

So, $m\angle BOD$ is reflex angle

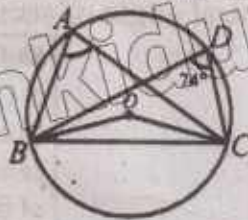
$$m\angle BOD = 2m\angle BAD$$

$$x = 2(114^\circ)$$

$$x = 228^\circ$$



2. In the adjoining figure $m\angle BDC = 74^\circ$, find $m\angle BAC$, $m\angle BOC$ and $m\angle OBC$.



Sol. Given $m\angle BDC = 74^\circ$

$$m\angle BAC = m\angle BDC$$

$$m\angle BAC = 74^\circ \quad \text{---(i)}$$

$$m\angle BOC = 2(m\angle BDC)$$

$$m\angle BOC = 2(74^\circ)$$

$$m\angle BOC = 148^\circ \quad \text{---(ii)}$$

In $\triangle OBC$

$$m\overline{OB} = m\overline{OC}$$

$$m\angle OBC = m\angle OCB$$

$$m\angle OBC + m\angle OCB + m\angle BOC = 180^\circ$$

$$m\angle OBC + m\angle OBC + m\angle BOC = 180^\circ$$

$$2(m\angle OBC) + 148^\circ = 180^\circ$$

$$2(m\angle OBC) = 180^\circ - 148^\circ$$

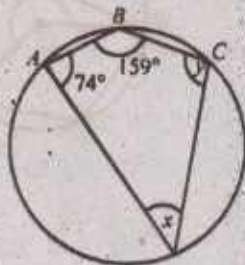
$$2(m\angle OBC) = 32^\circ$$

$$m\angle OBC = \frac{32^\circ}{2} = 16^\circ \quad \text{---(iii)}$$

So, $m\angle BAC = 74^\circ$, $m\angle BOC = 148^\circ$, $m\angle OBC = 16^\circ$ (from (i), (ii) and (iii))

3. Find the angle x and y in the following figures.

(i)



Sol. In cyclic quadrilateral

$$m\angle ABC + m\angle ADC = 180^\circ \quad \text{---(i)}$$

$$159^\circ + x = 180^\circ$$

$$x = 180^\circ - 159^\circ$$

$$x = 21^\circ \quad \text{---(i)}$$

$$m\angle BAD + m\angle BCD = 180^\circ \quad \text{(as in (i))}$$

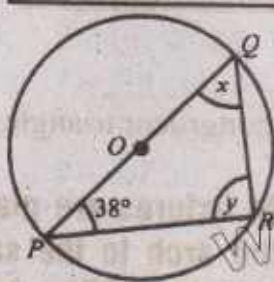
$$74^\circ + y = 180^\circ$$

$$y = 180^\circ - 74^\circ$$

$$y = 106^\circ \quad \text{---(ii)}$$

So, $x = 21^\circ$, $y = 106^\circ$ from (i) and (ii)

(ii)



Sol.

In triangle PQR

$$m\angle PRQ = 90^\circ$$

$$y = 90^\circ \quad \text{---(i)}$$

$$m\angle QPR + m\angle PRQ + m\angle PQR = 180^\circ$$

$$38^\circ + y + x = 180^\circ$$

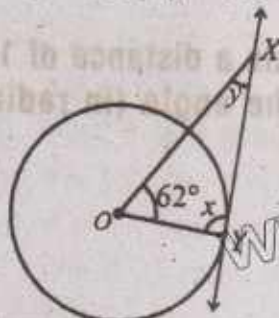
$$38^\circ + 90^\circ + x = 180^\circ$$

$$128^\circ + x = 180^\circ = 180^\circ - 128^\circ$$

$$x = 52^\circ \quad \text{---(ii)}$$

So, $x = 52^\circ$, $y = 90^\circ$ from (i) and (ii)

(iii)

Sol. In $\triangle XOY$

$$m\angle XOY = 62^\circ \quad \text{(Given)}$$

$$m\angle OYX = 90^\circ \quad \left[\text{Radius } \overline{OY} \perp \text{Tangent } \overline{XY} \right]$$

$$x = 90^\circ \quad \text{---(i)}$$

$$m\angle XOY + m\angle OYX + m\angle OXY = 180^\circ$$

$$62^\circ + x + y = 180^\circ$$

$$62^\circ + 90^\circ + y = 180^\circ$$

$$152^\circ + y = 180^\circ = 180^\circ - 152^\circ$$

$$y = 28^\circ \quad \text{---(ii)}$$

So, $x = 90^\circ$, $y = 28^\circ$ from (i) and (ii)4. Find the angle x in the given figure. Fig page 165

Sol. In the given figure

$$m\angle BCE = 70^\circ \quad \text{(Given)}$$

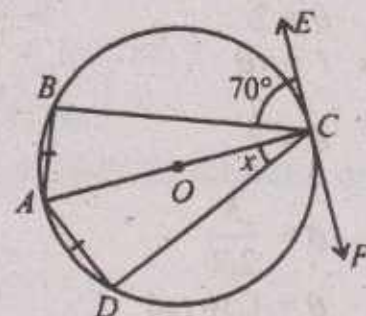
$$m\angle CBA = m\angle BCE \quad \text{(Alternate Segment Theorem)}$$

$$m\angle CBA = 70^\circ \quad \text{---(i)}$$

In $\triangle ABC$

$$m\angle ABC = 90^\circ \quad \text{(The angle in the semi-circle)}$$

$$m\angle CBA = 70^\circ \quad \text{(by (i))}$$



$$m\angle BCA = 180^\circ - 90^\circ - 70^\circ = 20^\circ$$

$$\triangle ACD \cong \triangle ABC \text{ (R.H.S.)}$$

$$m\angle ACD = m\angle BCA$$

$$x = 20^\circ$$

(corresponding angle of congruent triangles)

- 5** In a circular arch over a monument entrance, two spotlight fixtures are placed such that they each shine from two different points on the arch to the same chord PQ on the base of the arch. If the angle formed at one light is 55° , what is the angle at the second light on the same side of chord PQ ?

Sol. Angles subtended by the same chord at the circumference in the same segment are equal.

$$\text{Angle formed by one light} = 55^\circ$$

$$\text{Angle formed by other light} = 55^\circ \quad (\text{Property stated above})$$

- 6.** A circular garden has a walking path forming a quadrilateral inscribed in it. If one angle is 87° , what is opposite angle?

Sol. Measure of one angle in a inscribed quadrilatera = 87°

$$\text{Sum of the opposite angles of inscribed quadrilateral} = 180^\circ$$

(opposite angles are supplementary in a cyclid quadrilateral)

$$\text{Sum of the opposite angle} = 180^\circ - 87^\circ = 93^\circ$$

- 7.** A ferris wheel has a radius of 12m. If a passenger travels a distance of 18 m along the circumference of the ferris wheel, what is the angle (in radians) swept by the passenger's position from the starting point?

Sol. Radius of the ferris wheel (r) = 12 m

$$\text{Distance traveled by a passenger } (\ell) = 18 \text{ m}$$

$$\text{Measure of angle from thje starting point } (\theta) = ?$$

$$\text{As } \ell = r\theta$$

$$18 = 12\theta$$

$$\frac{18}{12} = \theta$$

$$\frac{3}{2} = \theta$$

$$1.5 = \theta$$

$$\text{So, } \theta = 1.5 \text{ rad}$$

- 8. Find θ , when**

(i) $\ell = 3\text{cm}, r = 2.2 \text{ cm}$

Sol. $\ell = 3 \text{ cm}$

$$r = 2.2 \text{ cm}$$

$$\theta = ?$$

$$\text{As } \ell = r\theta$$

$$\theta = \frac{\ell}{r}$$

$$\theta = \frac{3}{2.2}$$

$$\theta = 1.36 \text{ rad}$$

(ii) $\ell = 5.6\text{cm}, r = 2 \text{ cm}$

Sol. $\ell = 5.6 \text{ cm}$

$$r = 2 \text{ cm}$$

$$\theta = ?$$

$$\text{As } \ell = r\theta$$

$$\theta = \frac{\ell}{r}$$

$$\theta = \frac{5.6}{2}$$

$$\theta = 2.8 \text{ rad}$$

9. Find r , when

(i) $\ell = 5.5 \text{ cm}$, $\theta = 40^\circ 20'$

Sol. $\ell = 5.5 \text{ cm}$

$$\theta = 40^\circ 20'$$

$$r = ?$$

As $\theta = 40^\circ + \left(\frac{20}{60}\right)^\circ$

$$= 40^\circ + (0.33)^\circ$$

$$= 40.33^\circ$$

$$= 40.33 \times \frac{\pi}{180}$$

$$= 0.704 \text{ rad}$$

As $\ell = r\theta$

$$r = \frac{\ell}{\theta}$$

$$r = \frac{5.5}{0.704}$$

$$r = 7.8 \text{ cm}$$

10. Find ℓ and area of sector, when

(i) $r = 1.7 \text{ cm}$, $\theta = 0.25 \text{ radian}$

Sol. $r = 1.7 \text{ cm}$

$$\theta = 0.25 \text{ rad}$$

$$\ell = ?$$

$$\text{Area of sector (A)} = ?$$

As $\ell = r\theta$

$$\ell = (1.7)(0.25)$$

$$\ell = 0.425 \text{ cm}$$

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} (1.7)^2 (0.25)$$

$$= \frac{1}{2} (2.89)(0.25)$$

$$= 0.36125 \text{ cm}^2$$

(ii) $\ell = 13 \text{ cm}$, $\theta = 70^\circ$

Sol. $\ell = 13 \text{ cm}$

$$\theta = 70^\circ$$

$$r = ?$$

$$\theta = 70 \times \frac{\pi}{180}$$

$$= 1.222 \text{ rad}$$

As $\ell = r\theta$

$$r = \frac{\ell}{\theta}$$

$$r = \frac{13}{1.222}$$

$$r = 10.64 \text{ cm}$$

(ii) $r = 3 \text{ cm}$, $\theta = 45^\circ$

Sol. $r = 3 \text{ cm}$

$$\theta = 45^\circ$$

$$\ell = ?$$

$$\text{Area of sector (A)} = ?$$

$$\theta = 45 \times \frac{\pi}{180}$$

$$\theta = 0.785 \text{ rad}$$

$$\ell = r\theta$$

$$\ell = 3(0.785)$$

$$\ell = 2.36 \text{ cm}$$

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} (3)^2 (0.785)$$

$$= \frac{1}{2} \times 9(0.785)$$

$$= 3.53 \text{ cm}^2$$

11. Uzma cut a pizza of radius 14 cm into 8 equal slices. What is the area of one slice (sector).

Sol. Radius of pizza (r) = 14 cm

Total equal slices of pizza = 8

$$\text{Central angle of each slice } (\theta) = \frac{2\pi}{8}$$

$$= \frac{\pi}{4} \text{ rad}$$

Area of one slice (sector) (A) = ?

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} (14)^2 \left(\frac{\pi}{4} \right)$$

$$= \frac{1}{2} (196) \left(\frac{\pi}{4} \right)$$

$$= 76.97 \text{ cm}^2$$

- 12. The perimeter and area of a sector are 14 cm and 10 cm² respectively. Find the radius of the circle and the central angle of the sector.**

Sol. Perimeter of the sector (P) = 14 cm

Area of the sector (A) = 10 cm²

The perimeter of a sector consists of two radii (r) and the arc length (ℓ).

$$P = 2r + \ell$$

$$14 = 2r + \ell$$

$$14 - 2r = \ell$$

$$\ell = 14 - 2r$$

$$A = \frac{1}{2} r \ell$$

$$10 = \frac{1}{2} r \ell$$

$$10 \times 2 = r \ell$$

$$20 = r \ell$$

$$20 = r(14 - 2r)$$

$$20 = 14r - 2r^2$$

$$2r^2 - 14r + 20 = 0$$

Divide by 2

$$r^2 - 7r + 10 = 0$$

$$r^2 - 5r - 2r + 10 = 0$$

$$r(r - 5) - 2(r - 5) = 0$$

$$(r - 5)(r - 2) = 0$$

$$r - 5 = 0$$

$$r = 5$$

$$\theta = \frac{\ell}{r}$$

$$\theta = \frac{14 - 2r}{r}$$

$$\theta = \frac{14 - 2(5)}{5}$$

$$\theta = \frac{14 - 10}{5}$$

$$\theta = \frac{4}{5}$$

$$\theta = 0.8 \text{ rad}$$

$$r - 2 = 0$$

$$r = 2$$

$$\theta = \frac{\ell}{r}$$

$$\theta = \frac{14 - 2r}{r}$$

$$\theta = \frac{14 - 2(2)}{2}$$

$$\theta = \frac{14 - 4}{2}$$

$$\theta = \frac{10}{2}$$

$$\theta = 5 \text{ rad}$$

So, $r = 5 \text{ cm}$, $\theta = 0.8 \text{ rad}$ or $r = 2 \text{ cm}$, $\theta = 5 \text{ rad}$

Solved Review Exercise 9

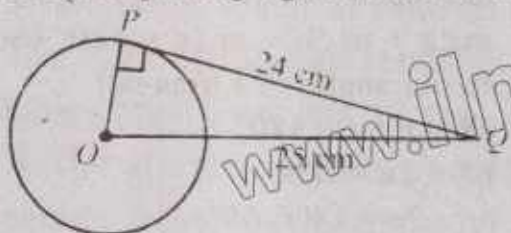
1. Four possible answers are given for the following questions. Choose the correct answer.

- (i) Tangent is a line that touches the circumference of the circle at:
 (a) many points (b) three points (c) two points (d) one point
- (ii) _____ tangents can be drawn to a circle from a point outside the circle:
 (a) one (b) two (c) three (d) many
- (iii) _____ tangents can be drawn to a circle from centre of the circle:
 (a) zero (b) one (c) three (d) many
- (iv) In circle, radius and tangent are:
 (a) parallel (b) equal (c) perpendicular (d) zero
- (v) An angle in a segment greater than a semicircle is _____ angle:
 (a) an acute (b) right (c) an obtuse (d) straight
- (vi) The angle subtended by the arc at the centre of the circle is called:
 (a) acute angle (b) central angle (c) right angle (d) complete angle
- (vii) What is the measure of an angle inscribed in a semi-circle?
 (a) 45° (b) 60° (c) 90° (d) 120°
- (viii) Any two angles in the same segment of the circle are:
 (a) supplementary (b) complementary (c) equal (d) zero
- (ix) Area of the sector of the circle = _____.
 (a) $\frac{1}{2} r\theta$ (b) $\frac{3}{2} r^2\theta$ (c) $\frac{1}{2} r^2\theta$ (d) $\frac{3}{2} r^2\theta$
- (x) If $r = 6\text{cm}$, $\theta = 2$ radians, then arc length is
 (a) 12 cm (b) 8 cm (c) 6 cm (d) 3 cm

ANSWERS:

- (i) one point (ii) two (iii) zero (iv) perpendicular (v) an acute
 (vi) central angle (vii) 90° (viii) equal (ix) $\frac{1}{2} r^2\theta$ (x) 12 cm

2. In the adjoining figure, find the radius of the circle.



Sol: $m \overline{PQ} = 24 \text{ cm}$
 $m \overline{OQ} = 25 \text{ cm}$
 $m \angle OPQ = 90^\circ$ (radius $\overline{OQ} \perp$ tangent $\overline{PQ}$)

Because of that 90° angle, $\triangle OPQ$ is a right angled triangle.

By pythagorean theorem

$$(m\overline{OQ})^2 = (m\overline{PO})^2 + (m\overline{OP})^2$$

$$(25 \text{ cm})^2 = (24 \text{ cm})^2 + (m\overline{OP})^2$$

$$625 \text{ cm}^2 = 576 + (m\overline{OP})^2$$

$$(m\overline{OP})^2 = 625 - 576$$

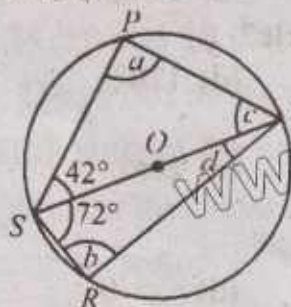
$$(m\overline{OP})^2 = 49$$

$$\sqrt{(m\overline{OP})^2} = \sqrt{49}$$

$$(m\overline{OP}) = 7$$

This the radius of circle is 7 cm

3. Find the angles a , b , c and d in the given figure, if O is the centre of the circle.



Sol. In $\triangle PQS$

$$m\angle PSQ = 42^\circ \text{ - (Given)}$$

$m\angle a = 90^\circ$ (The angle in semi-circle is a right angle)

$$m\angle PSQ + m\angle a + m\angle c = 180^\circ \text{ (sum of the angle of a triangle)}$$

$$42^\circ + 90^\circ + m\angle c = 180^\circ$$

$$132^\circ + m\angle c = 180^\circ$$

$$m\angle c = 180^\circ - 132^\circ$$

$$m\angle c = 48^\circ$$

In $\triangle RQS$

$$m\angle RSQ = 72^\circ$$

$$m\angle b = 90^\circ$$

[The angle in semi-circle is a right angle]

$$m\angle RSQ + m\angle b + m\angle d = 180^\circ \text{ (Sum of the interior angles of a triangles)}$$

$$72^\circ + 90^\circ + m\angle d = 180^\circ$$

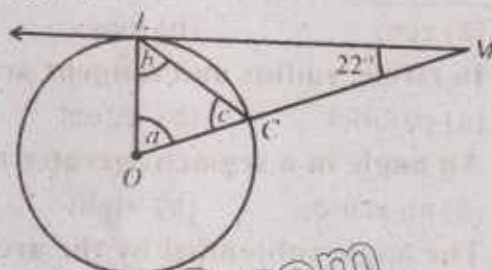
$$162^\circ + m\angle d = 180^\circ$$

$$m\angle d = 180^\circ - 162^\circ$$

$$m\angle d = 18^\circ$$

$$\text{So, } m\angle a = 90^\circ, m\angle b = 90^\circ - m\angle c = 48^\circ, m\angle d = 18^\circ$$

4. Find the angles a , b and c , if $\overline{ML}$ is a tangent line and O is the centre of the circle.



Sol: In $\triangle OLM$

$$m\angle M = 22^\circ \text{ (Given)}$$

$$m\angle OLM = 90^\circ \text{ (radius } \overline{OL} \perp \text{ tangent } \overline{ML})$$

$$m\angle a + m\angle M + m\angle OLM = 180^\circ \text{ [sum of the angles of a triangle]}$$

$$m\angle a + 22^\circ + 90^\circ = 180^\circ$$

$$m\angle a + 112^\circ = 180^\circ$$

$$m\angle a = 180^\circ - 112^\circ$$

$$m\angle a = 68^\circ$$

In $\triangle LOC$

$$m\angle OL = m\angle OC \text{ (radii of the same circle)}$$

So, $\triangle LOC$ is an isosceles triangle

$$m\angle b = m\angle c$$

[angles opposite sides are equal]

$$\text{Let } m\angle b = m\angle c = x$$

$$m\angle a + m\angle b + m\angle c = 180^\circ \text{ [Sum of the interior angles of a triangle]}$$

$$68 + x + x = 180^\circ$$

$$68 + 2x = 180^\circ$$

$$2x = 180^\circ - 68^\circ$$

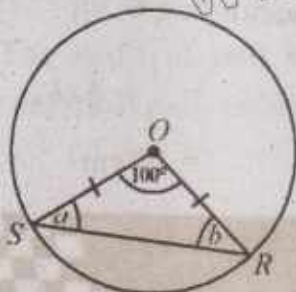
$$2x = 112^\circ$$

$$x = \frac{112^\circ}{2}$$

$$x = 56^\circ = m\angle b = m\angle c$$

$$\text{So, } \angle a = 68^\circ, \angle b = 56^\circ, \angle c = 56^\circ$$

5. Find the angles marked in the given figure, where O is the centre of the circle.



Sol: In $\triangle SOR$ $m\angle SOR = 100^\circ$ (Given)

$m\overline{OS} = m\overline{OR}$ (radii of the same circle)

So, $\triangle SOR$ is an isosceles triangle

Thus

$m\angle a = m\angle b$ [Angles opposite sides are of equal length]

$$\text{Let } m\angle a = m\angle b = x$$

$m\angle SOR + m\angle a + m\angle b = 180^\circ$ [Sum of the angles of a triangle]

$$100^\circ + x + x = 180^\circ$$

$$100^\circ + 2x = 180^\circ$$

$$2x = 180^\circ - 100^\circ$$

$$2x = 80^\circ$$

$$2x = \frac{80^\circ}{2}$$

$$x = 40^\circ = m\angle a = m\angle c$$

$$\text{so } \angle a = 40^\circ, \angle c = 40^\circ$$

6. Two round dining tables are arranged to touch at their edges. The radii of the two round dining tables are 10 cm and 12 cm. What is the distance between their centres if they touch externally?

Sol: Radius of one dining table $r_1 = 10$ cm

Radius of second dining table $r_2 = 12$ cm

When two circles touch externally, the

distance between their centres is the sum of their radii.

$$\begin{aligned} \text{Distance between the centres of both dining tables} &= r_1 + r_2 \\ &= 10 \text{ cm} + 12 \text{ cm} \\ &= 22 \text{ cm} \end{aligned}$$

7. A circular mosaic pattern is laid on the floor beneath a modern pyramid structure. Two tiles are placed such that they both connect to the same chord EF and reach two points G and H on the arc above EF . If $m\angle EGF = 65^\circ$, find $m\angle EHF$.

Sol: Theorem angles subtended by the same chord at the circumference of circle are equal.

Both $\angle EGF$ and $\angle EHF$ are subtended by the same chord EF .

Same chord EF .

So, $m\angle EHF = m\angle EGF$

Since $m\angle EGF = 65^\circ$ (Given)

Then $m\angle EHF = 65^\circ$

8. A triangular frame within a semicircular gate joins the ends of the diameter to the peak. What is the angle formed at the peak?

Sol: Any angle inscribed in a semicircle is always a right angle.

Thus, the angle formed at the peak is of 90° .

9. A circular birthday cake has a radius of 15 cm. A slice has an arc length of 10 cm. What is the area of this slice?

Sol: Radius of circular cake (r) = 15 cm

Arc length of a slice (l) = 10 cm

$$\text{Area of the slice} = \frac{1}{2}rl$$

$$= \frac{1}{2} \times 15 \times 10$$

$$= 75 \text{ cm}^2$$

10. A sector cut from a circle of radius 5cm has a perimeter of 16cm. Find area of this sector.

Sol: Radius of the circle (r) = 5 cm

Perimeter of the sector (p) = 16 cm

$$P = r + r\ell + \ell$$

$$16 = 5 + 5 + \ell$$

$$16 = 10 + \ell$$

$$16 - 10 = \ell$$

$$6 = \ell$$

$$\ell = 6 \text{ cm}$$

$$\text{Area of the sector} = \frac{1}{2} r \ell$$

$$= \frac{1}{2} \times 5 \times 6$$

$$= 15 \text{ cm}^2$$

Objective Type Questions

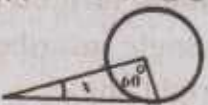
Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

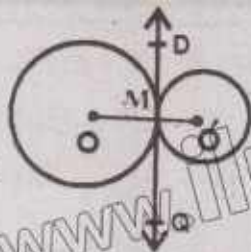
9.1

Tangent to a Circle

- Encircle the correct option.

- The point where a tangent meets the circle is known as:
 - Point of union
 - End point
 - Point of tangency
 - Origin
- What is angle formed between the tangent and the radius at point between the tangent and the radius at point of tangency.
 - 45°
 - 90°
 - 180°
 - 60°
- If $\overline{OC}$ is the radial segment in a circle, $\overline{MN}$ is a tangent, then
 - $\overline{OC} \perp \overline{MN}$
 - $\overline{OC} > \overline{MN}$
 - $\overline{OC} < \overline{MN}$
 - $\overline{OC} = \overline{MN}$
- In the figure  $m \angle x$ is:
 - 12°
 - 30°
 - 90°
 - 40°
- Two tangents are drawn to a circle. If the length of one tangent is 10 cm. then the length of other tangent is:
 - $\frac{10}{2}$ cm
 - 15 cm
 - 3 cm
 - 10 cm
- Tangent PA touches the circle at A. if $\triangle PAO$ is an isosceles right triangle, $\angle APO$ must be:
 - 30°
 - 45°
 - 60°
 - 90°
- Point p is 13 cm from the centre of a circle, if the radius is 4cm, what is length of tangent from p to circle.
 - 8 cm
 - 10 cm
 - 12 cm
 - 18 cm

8. In the figure



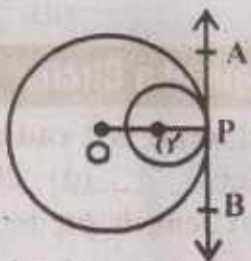
$m\overline{OO'}$ is:

- (a) $m\overline{OP} + m\overline{OM}$ (b) $m\overline{MQ} + m\overline{MP}$ (c) $m\overline{O'O} + m\overline{OP}$ (d) $m\overline{OM} + m\overline{MO'}$

9. The radii of two circles are 8cm, 10cm. What is the distance between their centres if they touch externally?

- (a) 18 cm (b) 180 cm (c) 2 cm (d) 80 cm

10. In the figure



the $m\overline{OO'}$ = _____.

- (a) $m\overline{OP} + m\overline{OO'}$ (b) $m\overline{OP} - m\overline{OP'}$ (c) $m\overline{O'P} - m\overline{OP}$ (d) $m\overline{AP} - \overline{BP}$

11. The radii of two circles are 15cm, 9cm, what is the distance between their centres they touch internally.

- (a) 24 cm (b) 35 cm (c) 6 cm (d) 135 cm

12. If the measure of the central angle of a circle is 80° the measure of the angle subtended by the corresponding major arc is:

- (a) 80° (b) 100° (c) 60° (d) 40°

13. In the figure



if $m\angle PRQ = 35^\circ$ then $m\angle POQ$ is:

- (a) 70° (b) 60° (c) 180° (d) 100°

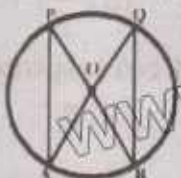
14. A circular running track subtends angle of 65° at the centre. What angle does it subtend at a point on the boundary.

- (a) 30 (b) 90 (c) 32.5° (d) 32°

15. In a cyclic quadrilateral, the sum of the opposite angles is:

- (a) 90° (b) 180° (c) 270° (d) 36°

16. In figure



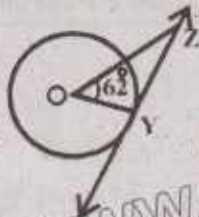
if the $m\angle APB = 30^\circ$ then $m\angle AQB$ will be of:

- (a) 70° (b) 90° (c) 60° (d) 30°

17. The angle in a segment less than a semi-circle is greater than a:

- (a) Right angle (b) Equal angle (c) Acute angle (d) obtuse angle

18. In the figure



$m\angle OZY$ is:

- (a) 22° (b) 27° (c) 28° (d) 26°

19. In the figure



then $m\angle BCD$ is:

- (a) 100° (b) 180° (c) 175° (d) 360°

9.2

Area of Sector and Arc Length of a Circle.

20. Find the arc length if radius is 10cm and central angle is 2 radians.

- (a) 5cm (b) 10cm (c) 20cm (d) 20π cm

21. What is the radius of a circle when an arc of length 6π is subtended by a central angle of π radians.

- (a) 3 (b) 6 (c) 12 (d) 6π

22. Find the central angle if $\ell = 15$ m and $r = 5$ m.

- (a) 3rad (b) 0.33 rad (c) 3π rad (d) 30 rad

23. If arc length is 10π and the $\theta = 120^\circ$ what is radius?

- (a) 10 (b) 15 (c) 20 (d) 30

24. What will be the area of a sector with a radius of 6cm and central angle is 1 radian.

- (a) 3 cm^2 (b) 6 cm^2 (c) 18 cm^2 (d) 36 cm^2

25. If area of a sector is 25π and $r = 10$ what the central angle in radians?

- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) 2π

26. Find the radius of a sector with an area of 50 units² and θ is 4 radian.

- (a) 5 units (b) 50 units (c) 25 units (d) 50 units

27. Find the radius if arc length is 24 and the area of the sector is 48.

- (a) 2 (b) 4 (c) 8 (d) 12

28. Which is the formula of area of a sector of a circle.

- (a) $\frac{1}{2}r(r\theta)$ (b) $\frac{1}{2}r\theta$ (c) $\frac{1}{2}\pi r^2$ (d) $\frac{1}{2}r^2\theta$

29. A sector of a circle is a region bounded by two radii and corresponding.

- (a) θ (b) arc (c) arc (d) π

30. For area of a sector of a circle θ is measured in:

- (a) π (b) degree (c) d (d) radians

ANSWERS

1. Point of tangency 2. 90° 3. $\overline{OC} \perp \overline{MN}$ 4. 30° 5. 10cm

6. 45° 7. 12cm 8. $m\overline{OM} + m\overline{MO'}$ 9. 18cm 10. $m\overline{OP} - m\overline{O'P}$
 11. 6cm 12. 40° 13. 70° 14. 32.5° 15. 180° 16. 30°
 17. Right angle 18. 28° 19. 100° 20. 20cm 21. 6 22. 3 rad
 23. 15 24. 18cm 25. $\frac{\pi}{2}$ 26. 5 units 27. 4 28. $\frac{1}{2}r$ ($r\theta$)
 29. arc 30. radians

Additional Short Question

Short Answer Questions prepared in the light of new Examination Techniques
 (Knowledge, Understanding, Application, Analytical & Conceptual)

9.1

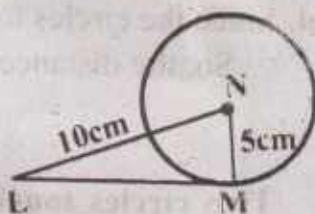
Tangent to a Circle

○ Give short answers.

1. Define the tangent to a circle:

Ans. In a plane, a line is a tangent to a circle that touches only one point on the circumference of a circle.

2. $\overline{LM}$ is the tangent to a circle with centre N at point M of radius 5cm. $m\overline{LN} = 10$ cm. What is the length of $\overline{LM}$.



Ans. Since $\overline{LM}$ is tangent to the circle at M and it is perpendicular to $\overline{NM}$.

$\triangle LMN$ is a right angled triangle.

By using Pythagoras theorem.

$$(m\overline{LN})^2 = (m\overline{LM})^2 + (m\overline{NM})^2$$

$$(10)^2 = (m\overline{LM})^2 + (5)^2$$

$$100 = (m\overline{LM})^2 + 25$$

$$(m\overline{LM})^2 = 100 - 25$$

$$(m\overline{LM})^2 = 75$$

$$\sqrt{(m\overline{LM})^2} = \sqrt{75}$$

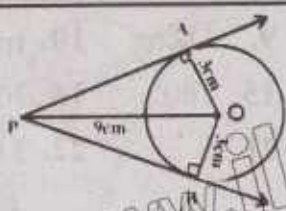
$$m\overline{LM} = 8.7 \text{ cm}$$

3. If two tangents, $\overline{PA}$ and $\overline{PB}$ are drawn from a point P to the circle with centre O. What can we concluded about their lengths.

Ans. The lengths of both tangents are equal

$$\text{So, } m\overline{PA} = m\overline{PB}$$

4. In figure



Find the length of PA.

Sol. $\triangle AOP$ is a right angled triangle because

$\overline{OA} \perp \text{tangent } \overline{PA}$ thus $\angle OAP = 90^\circ$

By Pythagoras theorem

$$(m \overline{PO})^2 = (m \overline{PA})^2 + (m \overline{OP})^2$$

$$(9\text{cm})^2 = (m \overline{PA})^2 + (3\text{cm})^2$$

$$81\text{cm}^2 = (m \overline{PA})^2 + 9\text{cm}^2$$

$$81\text{cm}^2 - 9\text{cm}^2 = (m \overline{PA})^2$$

$$72\text{cm}^2 = (m \overline{PA})^2$$

$$\sqrt{(m \overline{PA})^2} = \sqrt{72\text{cm}^2}$$

$$m \overline{PA} = 8.5\text{cm}$$

5. The radii of two circles are 7cm, 9cm. What is the distance between their centres if they touch externally.

Sol. Since the circles touch externally

So, the distance between their centres = sum of their radii

$$= 7\text{cm} + 9\text{cm}$$

$$= 16\text{cm}$$

6. Two circles touch externally. Radius of one circle is 10cm. Find the radius of other circle if the distances between their centers is 22 cm.

Sol. Since, the circles touch externally, so, the distance between their centres is equal to sum of their radii.

$$r_1 = 10\text{ cm}, d = 22\text{cm}$$

$$r_1 + r_2 = d$$

$$10\text{cm} + r_2 = 22\text{ cm}$$

$$r_2 = 22\text{ cm} - 10\text{cm}$$

$$r_2 = 12\text{ cm.}$$

7. If two circles of radii $r_1 = 11$ and $r_2 = 7$ touch internally, what is the distance of their centres.

Sol. $r_1 = 11$

$$r_2 = 7$$

$$\text{Distance between their centres} = 11 - 7$$

$$= 4$$

8. Two identical circles touch externally if the distances their centres is 18 cm. What is the diameter of one of these circles.

Sol. $r_1 = r_2$ (identical circles)

$$\text{Distance between their centres} = 18\text{ cm}$$

$$r_1 + r_2 = 18$$

$$r_1 + r_1 = 18 \text{ as } r_1 = r_2$$

$$2r_1 = 18$$

$$r = \frac{18}{2}$$

$$r = 9 \text{ cm}$$

So, diameter of the circle = $2r$

$$= 2(9)$$

$$= 18 \text{ cm}$$



9. In the figure Find the $m\angle APC$:

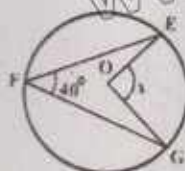
Sol. As we know that the measure of an angle which an arc of a circle subtends at the centre is double that which subtends at any point on the remaining part of circumference. therefore

$$2(m\angle APC) = m\angle AOC$$

$$2 m\angle APC = 100^\circ$$

$$m\angle APC = \frac{100^\circ}{2}$$

$$m\angle APC = 50^\circ$$



10. In figure find x.

Sol. The measure of a central angle of a minor arc of a circle is double that of the angle subtended by the corresponding major arc the

$$m\angle GOE = 2m\angle GFE$$

$$x = 2(40^\circ)$$

$$x = 80^\circ$$

Angles in the same segment of a circle



11. In figure . If is $m\angle ACB = 35^\circ$. What is $m\angle ADB$

Sol. $m\angle ADB = 35^\circ$ (Given) [any two angles in the same segment of a circle are equal]

$$m\angle ADB = m\angle ACB$$

$$m\angle ADB = 35^\circ$$

12. What is the measure of an angle subtended in a semi-circle?

Ans. The angle subtended in a semi-circle is right angle of 90° .

13. What is the measure of the angle in a segment greater than a semi-circle.

Ans. The measure in a segment greater than a semi-circle is less than a right angle ($< 90^\circ$).

14. In figure  Why is the angle x equal to 90° ?

Sol. Because PQ is a diameter, making x an angle in a semi-circle. The angle in a semi-circle is a right angle of 90° .

9.2 Area of sector and Arc Length of a Circle

15. A circle has a radius of 10cm and a central angle of 60° . Find the arc length.

Sol.

$$r = 10\text{cm}$$

$$\theta = 60^\circ$$

$$\ell =$$

$$\theta = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$$

we know

$$\ell = \theta r$$

$$\ell = \frac{\pi}{3} \times 10$$

$$\ell = 10.47\text{ cm}$$

16. Calculate the area of sector with a radius of 8cm and $\theta = 90^\circ$

Sol.

$$r = 8\text{ cm}$$

$$\theta = 90^\circ$$

$$\theta = 90 \times \frac{\pi}{180}$$

$$\theta = \frac{\pi}{2} = 1.57\text{ radian}$$

$$\text{Area of sector} = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} (8)^2 (1.57)$$

$$= \frac{1}{2} \times 64 (1.57)$$

$$= 50.24\text{ cm}^2$$

17. Find the central angle θ of a circle when $\ell = 12\text{ cm}$ and $r = 4\text{ cm}$.

Sol.

$$\ell = 12\text{ cm}$$

$$r = 4\text{ cm}$$

$$\theta = ?$$

we know

$$\ell = r\theta$$

$$\theta = \frac{\ell}{r}$$

$$\theta = \frac{12}{4} = 3\text{ radian}$$

18. A circular clock face has radius of 12cm. What is the area of sector formed between the number 12 and 3 on the clock?

Sol.

$$r = 12 \text{ cm}$$

$$\theta = 90^\circ$$

$$A = ?$$

$$\theta = 90 \times \frac{\pi}{180}$$

$$= \frac{\pi}{2} = 1.57 \text{ radian}$$

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} (12)^2 (1.57)$$

$$= \frac{1}{2} (144) (1.57) = 113.04 \text{ cm}^2$$

19. A circular cake with a diameter of 20cm is cut into 10 equal slices. Calculate the area of a single slice.

Sol.

$$\text{Diameter} = 20 \text{ cm}$$

$$r = \frac{20}{2} = 10 \text{ cm}$$

The cake is cut into 10 equal parts

$$\theta = \frac{2\pi}{10} = \frac{\pi}{5} \text{ rad}$$

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} (10)^2 \times \frac{\pi}{5}$$

$$= \frac{1}{2} \times 100 \times \frac{\pi}{5}$$

$$= 10\pi$$

$$= 31.42 \text{ cm}^2$$

20. The perimeter of a sector is 25 cm and its arc length is 9cm. Find the radius of circle.

Sol.

$$l = 9 \text{ cm}$$

$$P = 25 \quad (\text{Given})$$

$$P = l + 2r$$

$$25 = 9 + 2r$$

$$25 - 9 = 2r$$

$$16 = 2r$$

$$\frac{16}{2} = r$$

$$r = 8 \text{ cm}$$

