

Students' Learning Outcomes

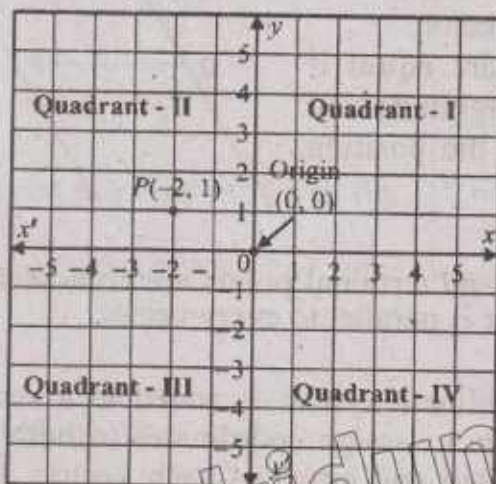
After completing this unit, the students will be able to:

- ▶ Introduce a rectangular coordinate system in the plane.
- ▶ Represent vectors as directed line segment.
- ▶ Express a vector in terms of two non-zero and non-parallel coplanar vectors.
- ▶ Express a vector in terms of a position vector.
- ▶ Express translation by a vector.
- ▶ Find the magnitude of a vector.
- ▶ Add and subtract vectors.
- ▶ Multiply a vector by a scalar.
- ▶ Solve geometrical problems involving the use of vectors.
- ▶ Apply concepts from geometrical problems involving the use of vectors (such as parallel and perpendicular lines in geometrical shapes, vector projectile motion, crosswinds aviation, military usage and designing roller coasters).

6.1 Rectangular Coordinate Plane

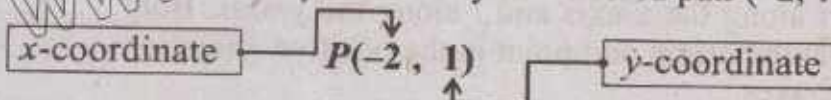
A rectangular coordinate plane is formed by the intersection of a horizontal number line called the **axis of x** or shortly **x-axis** and a vertical number line called the **axis of y** or shortly **y-axis**.

The axes meet at a point called the **origin** and divide the coordinate plane into four **quadrants**.



Points in a coordinate plane are represented by **ordered pairs**. The first element is the **x-coordinate** and the second element is the **y-coordinate**.

In the given figure, point **P** is represented by the ordered pair **(-2, 1)**.

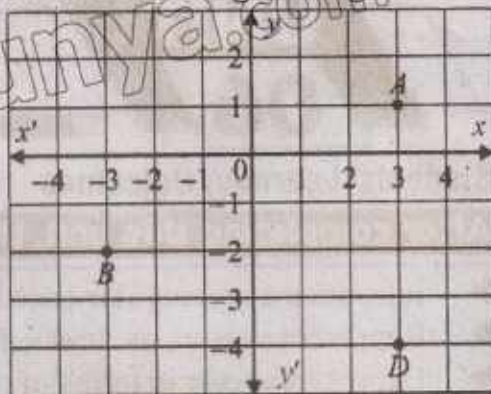


Example 1: Write the coordinates of the following points from the given figure:

- (i) A (ii) B (iii) D

Solution:

- (i) Point A is 3 units to the right of the origin and 1 unit up. So, the x-coordinate is 3 and the y-coordinate is 1. The coordinates of A are (3, 1).
- (ii) Point B is 3 units to the left of the origin and 2 units down. So, the x-coordinate is -3 and the y-coordinate are -2. The coordinates of B are (-3, -2).
- (iii) Point D is 3 units to the right of the origin and 4 units down. So, the x-coordinate is 3 and the y-coordinate is -4. The coordinates of D are (3, -4).



6.2

Scalars and Vectors

There are two types of physical quantities that are used in Physics and Mathematics. **Scalar:** A scalar is a quantity having magnitude but no direction. Mass, time, volume and power are the examples of scalar quantities.

Vector: A vector is a quantity having both magnitude and direction. Displacement, velocity, acceleration, force and momentum are examples of vector quantities.

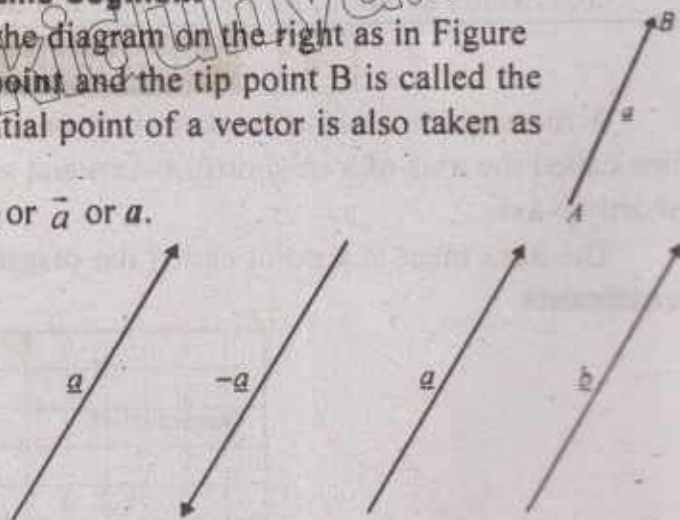
Representation of a Vector as Directed Line Segment

A vector has a tail and a tip. Consider the diagram on the right as in Figure

The tail point A is called the **initial point** and the tip point B is called the **terminal point** of the vector $\vec{AB}$. The initial point of a vector is also taken as **origin** of the vector.

We can also represent a vector with $\underline{a}$ or $\bar{a}$ or a .

- (i) A vector which has same magnitude but opposite in direction is called negative of a vector.
- (ii) Two vectors $\underline{a}$ and $\underline{b}$ are **equal** if they have the same magnitude and direction regardless of the position of their initial points.

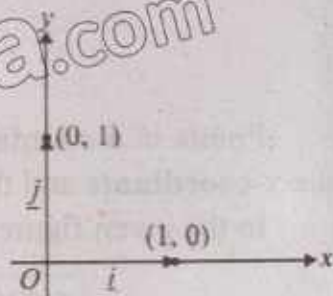


Thus $\underline{a} = \underline{b}$ in Figure.

- (iii) A vector whose initial and terminal points are coincident is called a **zero vector** or **null vector**. Zero vector is parallel to every vector.

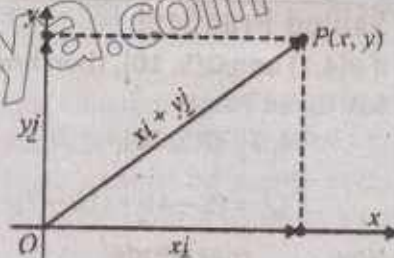
Vectors and Coordinates

When a Cartesian coordinate system is added to a vector diagram, it becomes possible to assign coordinates to both the initial and terminal points (tail and head) of each vector. This allows vectors to be expressed in terms of their coordinates, which can then be used effectively in calculations. To simplify this process, we introduce two special vectors along the coordinate axes: $\hat{i}$ along the x-axis and $\hat{j}$ along the y-axis. Both vectors originate at the origin and point in the positive direction of their respective axes.



Position Vectors

Let $O(0, 0)$ be the origin and $P(x, y)$ be any point in the plane, then the vector $\overrightarrow{OP} = (x-0)\underline{i} + (y-0)\underline{j} = x\underline{i} + y\underline{j}$ is called the **position vector** of the point P with respect to the origin O . x and y are horizontal and vertical components respectively.



Magnitude of a Vector

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the given points as shown in fig.

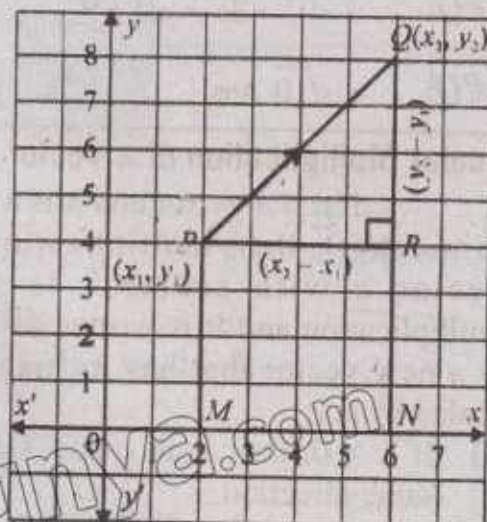
Draw $\overline{PM}$ and $\overline{QN}$ perpendicular on x -axis.

Draw $\overline{PR} \perp \overline{QN}$

Now, $|\overline{PR}| = |x_2 - x_1|$

$|\overline{QR}| = |y_2 - y_1|$

PQR is a right angled triangle.



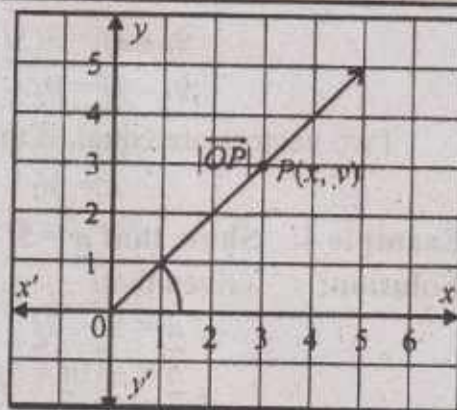
Note

Magnitude of a vector is also known as norm or length of a vector.

$$\begin{aligned} \therefore m\overline{PQ} &= \sqrt{(\overline{PR})^2 + (\overline{QR})^2} \\ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{aligned}$$

or $m\overline{PQ} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

If $P(x, y)$ be any point in the plane as shown in fig. then the magnitude of position vector $\overrightarrow{OP} = x\underline{i} + y\underline{j}$ from origin is $|\overrightarrow{OP}| = \sqrt{x^2 + y^2}$.



Example 2: If $P(2, 4)$ and $Q(-5, 3)$, then find

(i) $|\overline{PQ}|$ = in the form $x\underline{i} + y\underline{j}$ (ii) $|\overline{PQ}|$

Solution: (i) $\overline{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (-5-2)\underline{i} + (3-4)\underline{j} = -7\underline{i} - \underline{j}$

(ii) $|\overline{PQ}| = \sqrt{(-7)^2 + (-1)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$

Skilled Practice!

If $P(4,7)$ and $Q(5, 10)$, then find the magnitude of the vector $\overrightarrow{PQ}$.

Sol. Given Points

$$P(4, 7), Q(5, 10)$$

$$\overrightarrow{PQ} = (5-4)\underline{i} + (10-7)\underline{j} \Rightarrow \overrightarrow{PQ} = \underline{i} + 3\underline{j}$$

Now magnitude:

$$|\overrightarrow{PQ}| = \sqrt{1^2 + 3^2} = \sqrt{1+9}$$

$$|\overrightarrow{PQ}| = \sqrt{10} \text{ Ans.}$$

Scalar Multiplication of a Vector

If $\underline{a}$ is a vector and λ is a scalar (a number), then multiplication of a vector $\underline{a}$ with scalar λ is scalar multiplication and it is written as $\lambda \underline{a}$. $\lambda \underline{a}$ is a new vector that has its magnitude scaled by $|\lambda|$.

(i) If $\lambda > 0$, then both $\underline{a}$ and $\lambda \underline{a}$ are in same direction.

(ii) If $\lambda < 0$, then both $\underline{a}$ and $\lambda \underline{a}$ are in opposite direction.

Example 3: If $\underline{a} = 3\underline{b}$, where $\underline{a} = 9\underline{i} - k\underline{j}$ and $\underline{b} = 3\underline{i} - 12\underline{j}$, then find the value of k .

Solution: Given that $\underline{a} = 3\underline{b}$

$$9\underline{i} - k\underline{j} = 3(3\underline{i} - 12\underline{j})$$
$$9\underline{i} - k\underline{j} = 9\underline{i} - 36\underline{j}$$

Two vectors are equal, if their horizontal and vertical components are equal.

$$\therefore k = 36$$

Example 4: Show that $\underline{a} = 5\underline{i} - 2\underline{j}$ and $\underline{b} = -10\underline{i} + 4\underline{j}$ are parallel.

Solution: Given that:

$$\underline{a} = 5\underline{i} - 2\underline{j}$$
$$\underline{b} = -10\underline{i} + 4\underline{j}$$
$$\underline{b} = -2(5\underline{i} - 2\underline{j})$$
$$\underline{b} = -2\underline{a}$$

Which shows $\underline{a}$ and $\underline{b}$ are parallel with $\lambda = -2$

Unit Vector

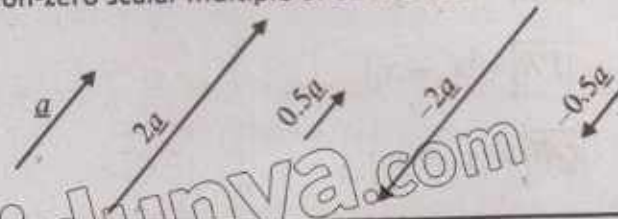
A unit vector is a vector whose magnitude is 1. If $\underline{a}$ is any vector, then its unit vector is represented as $\hat{\underline{a}}$ (read as $\underline{a}$ hat). It is defined as $\hat{\underline{a}} = \frac{\underline{a}}{|\underline{a}|}$.

Example 5: Find a unit vector in the direction of $\underline{a} = -5\underline{i} + 2\underline{j}$.

Solution: Given that $\underline{a} = -5\underline{i} + 2\underline{j}$

Note

Two vectors are said to be parallel, if they are non-zero scalar multiple of each other.



Note

- (i) If $\underline{a} = \underline{b}$, then $\underline{a}$ and $\underline{b}$ have the same direction and $|\underline{a}| = |\underline{b}|$.
- (ii) If $\underline{a}$ is zero vector, then $|\underline{a}| = 0$

$$|a| = \sqrt{(-5)^2 + (2)^2} = \sqrt{25 + 4} = \sqrt{29}$$

$$\hat{a} = \frac{a}{|a|} = \frac{-5i + 2j}{\sqrt{29}} \\ = \frac{-5}{\sqrt{29}}i + \frac{2}{\sqrt{29}}j$$

Challenge!

(i) Can a zero vector be a unit vector? Explain.

(ii) If $\hat{a}$ is a unit vector, what is the value of $|\hat{a}|$.

Sol. (i) A zero vector cannot be a unit vector because its magnitude is zero, whereas a unit vector has a magnitude of 1.

(ii) If $\hat{a}$ is a unit vector, the value of its magnitude $|\hat{a}|$ is 1.

Solved Exercise 6.1

1. Name the quadrant in which each point lies.

(i) (4, 3)

(ii) (5, -4)

(iii) (-6, 2)

(iv) (-4, -4)

Sol. Quadrant-I

Quadrant-IV

Quadrant-II

Quadrant-III

2. Plot the following points on the coordinate plane:

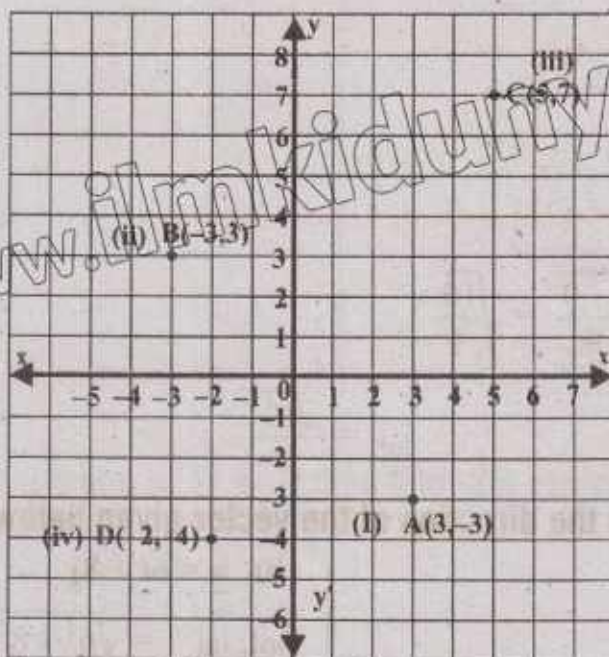
(i) A(3, -3)

(ii) B(-3, 3)

(iii) C(5, 7)

(iv) D(-2, -4)

Sol.



3. Name the tail and tip of the following vectors:

(i)



(ii)



(iii)



(iv)



(v)



(vi)



Tail: B, Tip: A

Tail: Q, Tip: P

Tail: S, Tip: R

Tail: N, Tip: M

Tail: A, Tip: B

Tail: D, Tip: C

4. Write the vector $\underline{AB}$ in the form of $x\underline{i} + y\underline{j}$:

(i) $A(1, -7), B(-2, 4)$

$$\begin{aligned}\text{Sol. } \therefore \underline{AB} &= (x_2 - x_1)\underline{i} + (y_2 - y_1)\underline{j} \\ &= (-2 - 1)\underline{i} + [4 - (-7)]\underline{j} \\ \underline{AB} &= -3\underline{i} + 11\underline{j}\end{aligned}$$

(ii) $A(8, 9), B(12, 3)$

$$\begin{aligned}\text{Sol. } \therefore \underline{AB} &= (x_2 - x_1)\underline{i} + (y_2 - y_1)\underline{j} \\ &= (12 - 8)\underline{i} + (3 - 9)\underline{j} \\ \underline{AB} &= 4\underline{i} - 6\underline{j}\end{aligned}$$

5. Find the magnitude of the $\underline{a}$:

(i) $\underline{a} = -3\underline{i} + 2\underline{j}$

$$\begin{aligned}\text{Sol. } \therefore |\underline{a}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-3)^2 + (2)^2} \\ &= \sqrt{9 + 4} \\ |\underline{a}| &= \sqrt{13}\end{aligned}$$

(ii) $\underline{a} = 4\underline{i} - 3\underline{j}$

$$\begin{aligned}\text{Sol. } \therefore |\underline{a}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{4^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \quad |\underline{a}| = 5\end{aligned}$$

(iii) $\underline{a} = \frac{1}{2}\underline{i} + \frac{3}{2}\underline{j}$

$$\begin{aligned}\text{Sol. } \therefore |\underline{a}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2} \\ &= \sqrt{\frac{1}{4} + \frac{9}{4}} = \sqrt{\frac{1+9}{4}} = \sqrt{\frac{10}{4}} \\ |\underline{a}| &= \frac{\sqrt{10}}{2}\end{aligned}$$

6. Find a unit vector in the direction of the vector given below:

(i) $\underline{a} = -4\underline{i} + 5\underline{j}$

$$\begin{aligned}\text{Sol. } \therefore |\underline{a}| &= \sqrt{(-4)^2 + (5)^2} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \\ \therefore \hat{\underline{a}} &= \frac{\underline{a}}{|\underline{a}|} \\ &= \frac{-4\underline{i} + 5\underline{j}}{\sqrt{41}} \\ \hat{\underline{a}} &= -\frac{4}{\sqrt{41}}\underline{i} + \frac{5}{\sqrt{41}}\underline{j}\end{aligned}$$

(ii) $\underline{a} = 6\underline{i} + 8\underline{j}$

$$\begin{aligned}\text{Sol. } |\underline{a}| &= \sqrt{6^2 + 8^2} \\ &= \sqrt{36 + 64} = \sqrt{100} \\ |\underline{a}| &= 10 \\ \hat{\underline{a}} &= \frac{\underline{a}}{|\underline{a}|} \\ &= \frac{6\underline{i} + 8\underline{j}}{10} = \frac{6}{10}\underline{i} + \frac{8}{10}\underline{j} \\ \hat{\underline{a}} &= \frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}\end{aligned}$$

$$(iii) \underline{a} = \frac{1}{\sqrt{6}}\underline{i} + \frac{1}{\sqrt{6}}\underline{j}$$

$$\begin{aligned} \text{Sol. } |\underline{a}| &= \sqrt{\left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2} \\ &= \sqrt{\frac{1}{6} + \frac{1}{6}} = \sqrt{\frac{1+1}{6}} \\ &= \sqrt{\frac{2}{6}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \therefore \underline{\hat{a}} &= \frac{\underline{a}}{|\underline{a}|} \\ &= \frac{\frac{1}{\sqrt{6}}\underline{i} + \frac{1}{\sqrt{6}}\underline{j}}{\frac{1}{\sqrt{3}}} \end{aligned}$$

$$= \sqrt{3} \left(\frac{1}{\sqrt{6}}\underline{i} + \frac{1}{\sqrt{6}}\underline{j} \right)$$

$$= \frac{\sqrt{3}}{\sqrt{6}}\underline{i} + \frac{\sqrt{3}}{\sqrt{6}}\underline{j}$$

$$= \sqrt{\frac{3}{6}}\underline{i} + \sqrt{\frac{3}{6}}\underline{j}$$

$$\underline{\hat{a}} = \frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}$$

$$(iv) \underline{a} = \frac{1}{2}\underline{i} - \frac{3}{4}\underline{j}$$

$$\begin{aligned} \text{Sol. } |\underline{a}| &= \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{3}{4}\right)^2} \\ &= \sqrt{\frac{1}{4} + \frac{9}{16}} = \sqrt{\frac{4 \times 1 + 1 \times 9}{16}} \\ &= \sqrt{\frac{4+9}{16}} = \sqrt{\frac{13}{16}} \\ &= \frac{\sqrt{13}}{4} \end{aligned}$$

$$\begin{aligned} \therefore \underline{\hat{a}} &= \frac{\underline{a}}{|\underline{a}|} \\ &= \frac{\frac{1}{2}\underline{i} - \frac{3}{4}\underline{j}}{\frac{\sqrt{13}}{4}} \end{aligned}$$

$$= \frac{4}{\sqrt{13}} \left(\frac{1}{2}\underline{i} - \frac{3}{4}\underline{j} \right)$$

$$= \frac{4}{\sqrt{13}} \times \frac{1}{2}\underline{i} - \frac{4}{\sqrt{13}} \times \frac{3}{4}\underline{j}$$

$$\underline{\hat{a}} = \frac{2}{\sqrt{13}}\underline{i} - \frac{3}{\sqrt{13}}\underline{j}$$

7. If $\underline{a} = 5\underline{i} - 7\underline{j}$, $\underline{b} = -\underline{i} - \underline{j}$ and $\underline{c} = 2\underline{i} + 3\underline{j}$, then find unit vector parallel to $\underline{a} + \underline{b} - 3\underline{c}$.

$$\begin{aligned} \text{Sol. } \underline{a} + \underline{b} - 3\underline{c} &= 5\underline{i} - 7\underline{j} + (-\underline{i} - \underline{j}) - 3(2\underline{i} + 3\underline{j}) \\ &= 5\underline{i} - 7\underline{j} - \underline{i} - \underline{j} - 6\underline{i} - 9\underline{j} \\ &= (5 - 1 - 6)\underline{i} - (7 + 1 + 9)\underline{j} \end{aligned}$$

$$\underline{a} + \underline{b} - 3\underline{c} = -2\underline{i} - 17\underline{j}$$

$$\text{Let } \underline{u} = -2\underline{i} - 17\underline{j}$$

$$\Rightarrow |\underline{u}| = \sqrt{(-2)^2 + (-17)^2}$$

$$= \sqrt{4 + 289}$$

$$|\underline{u}| = \sqrt{293}$$

Required unit vector

$$\begin{aligned} &= \frac{\underline{u}}{|\underline{u}|} \\ &= \frac{-2\underline{i} - 17\underline{j}}{\sqrt{293}} \end{aligned}$$

$$= -\frac{2}{\sqrt{293}}\underline{i} - \frac{17}{\sqrt{293}}\underline{j}$$

8. If $\underline{a} = 3\hat{i} - \hat{j}$, $\underline{b} = -2\hat{i} + 4\hat{j}$ and $\underline{c} = \hat{i} + 2\hat{j}$, then find unit vector parallel to $3\underline{a} - 2\underline{c} + 4\underline{b}$.

Sol.
$$\begin{aligned} 3\underline{a} - 2\underline{c} + 4\underline{b} &= 3(3\hat{i} - \hat{j}) - 2(\hat{i} + 2\hat{j}) + 4(-2\hat{i} + 4\hat{j}) \\ &= 9\hat{i} - 3\hat{j} - 2\hat{i} - 4\hat{j} - 8\hat{i} + 16\hat{j} \\ &= (9 - 2 - 8)\hat{i} - (3 + 4 - 16)\hat{j} \\ &= -\hat{i} + 9\hat{j} \end{aligned}$$

Let $\underline{u} = -\hat{i} + 9\hat{j}$

$$\Rightarrow |\underline{u}| = \sqrt{(-1)^2 + 9^2}$$

$$= \sqrt{1 + 81}$$

$$|\underline{u}| = \sqrt{82}$$

Required unit vector $= \frac{\underline{u}}{|\underline{u}|}$

$$= \frac{-\hat{i} + 9\hat{j}}{\sqrt{82}}$$

$$= -\frac{1}{\sqrt{82}}\hat{i} + \frac{9}{\sqrt{82}}\hat{j}$$

9. Which of the following vectors are parallel?

(i) $\underline{a} = 6\hat{i} + \hat{j}$, $\underline{b} = 12\hat{i} + 2\hat{j}$

Sol. Given that:

$$\underline{b} = 12\hat{i} + 2\hat{j}$$

$$\underline{b} = 2(6\hat{i} + \hat{j})$$

$$\underline{b} = 2\underline{a}$$

Which shows that $\underline{a}$ and $\underline{b}$ are parallel

with $\lambda = 2$

(ii) $\underline{a} = -2\hat{i} + 3\hat{j}$, $\underline{b} = 6\hat{i} - 9\hat{j}$

Sol. Given that:

$$\underline{b} = 6\hat{i} - 9\hat{j}$$

$$= -3(-2\hat{i} + 3\hat{j})$$

$$\underline{b} = -3\underline{a}$$

Which shows that $\underline{a}$ and $\underline{b}$ are parallel

with $\lambda = -3$

(iii) $\underline{a} = 5\hat{i} - 4\hat{j}$, $\underline{b} = 6\hat{i} - 3\hat{j}$

Sol. We see that there is no such constant λ such that $\underline{b} = \lambda \underline{a}$.

So these vectors are non parallel.

(iv) $\underline{a} = 3\hat{i} - 7\hat{j}$, $\underline{b} = 6\hat{i} - 14\hat{j}$

Sol. Given that

$$\underline{b} = 6\hat{i} - 14\hat{j}$$

$$= 2(3\hat{i} - 7\hat{j})$$

$$\Rightarrow \underline{b} = 2\underline{a}$$

Which shows that $\underline{a}$ and $\underline{b}$ are parallel with $\lambda = 2$

10. Find a vector thrice in length of $3\hat{i} - 2\hat{j}$, but opposite in direction.

Sol. Required vector $= -3(3\hat{i} + 2\hat{j})$

$$= -9\hat{i} + 6\hat{j}$$

11. Find two vectors that are double in magnitude of $3\hat{i} - 5\hat{j}$, one in the same direction of it and other in its opposite direction.

Sol. Vector in same direction with double magnitude:

$$2(3\hat{i} - 5\hat{j})$$

$$= 6\hat{i} - 10\hat{j}$$

Vector in opposite direction with double magnitude:

$$-2(3\hat{i} - 5\hat{j})$$

$$= -6\hat{i} + 10\hat{j}$$

6.3

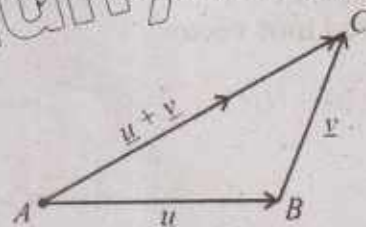
Addition of Vectors

- (i) Triangle Law of Addition:

If two vectors $\underline{u}$ and $\underline{v}$ are represented by the two sides $\overline{AB}$ and $\overline{BC}$ of a triangle such that the terminal point of $\underline{u}$ coincides with the initial point of $\underline{v}$, then the third side $\overline{AC}$ of the triangle gives vector sum $\underline{u} + \underline{v}$ that is

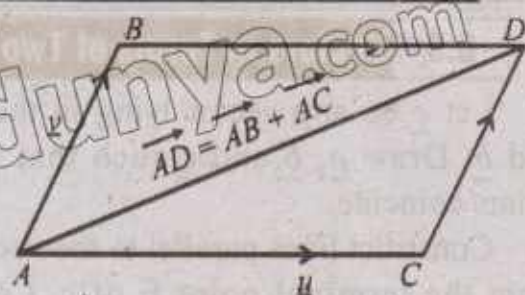
$$\overline{AB} + \overline{BC} = \overline{AC} \Rightarrow \underline{u} + \underline{v} = \underline{u} + \underline{v}$$

By the head to tail rule



(ii) **Parallelogram Law of Addition:**

If two vectors $\underline{u}$ and $\underline{v}$ are represented by two adjacent sides $\overline{AB}$ and $\overline{AC}$ of a parallelogram as shown in the figure 6.10, then diagonal $\overline{AD}$ gives the sum or resultant of $\overline{AB}$ and $\overline{AC}$.



$$\overline{AD} = \overline{AB} + \overline{AC}$$

$$\overline{AD} = \underline{u} + \underline{v}$$

Example 6: If $\underline{a} = 2\underline{i} + 3\underline{j}$, $\underline{b} = 4\underline{i} - 6\underline{j}$, then find $\underline{a} + \underline{b}$

Sol. $\underline{a} + \underline{b} = (2\underline{i} + 3\underline{j}) + (4\underline{i} - 6\underline{j})$
 $= (2 + 4)\underline{i} + (3 - 6)\underline{j}$
 $= 6\underline{i} - 3\underline{j}$

Skilled Practice!

If $\underline{a} = 10\underline{i} - \frac{1}{2}\underline{j}$, $\underline{b} = \frac{3}{4}\underline{i} - \frac{7}{6}\underline{j}$, then find $\underline{a} + \underline{b}$.

Sol. Given that

$$\underline{a} = 10\underline{i} - \frac{1}{2}\underline{j}, \quad \underline{b} = \frac{3}{4}\underline{i} - \frac{7}{6}\underline{j}$$

Now,

$$\begin{aligned}\underline{a} + \underline{b} &= \left(10\underline{i} - \frac{1}{2}\underline{j}\right) + \left(\frac{3}{4}\underline{i} - \frac{7}{6}\underline{j}\right) \\ &= \left(10 + \frac{3}{4}\right)\underline{i} - \left(\frac{1}{2} + \frac{7}{6}\right)\underline{j} = \left(\frac{40+3}{4}\right)\underline{i} - \left(\frac{3+7}{6}\right)\underline{j} \\ &= \frac{43}{4}\underline{i} - \frac{10}{6}\underline{j} \\ \underline{a} + \underline{b} &= \frac{43}{4}\underline{i} - \frac{5}{3}\underline{j}\end{aligned}$$

6.4

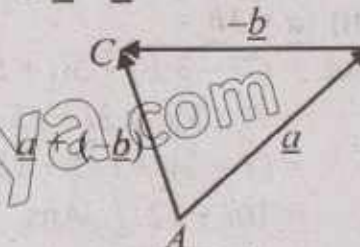
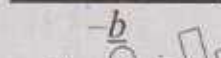
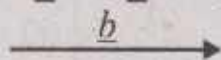
Difference of Two Vectors

The vector subtraction of two vectors $\underline{a}$ and $\underline{b}$ is represented by $\underline{a} - \underline{b}$ and defined as

$$\underline{a} + (-\underline{b}).$$

$$\text{If } \underline{a} = x_1\underline{i} + y_1\underline{j}, \underline{b} = x_2\underline{i} + y_2\underline{j}$$

$$\begin{aligned}\text{then } \underline{a} + (-\underline{b}) &= (x_1\underline{i} + y_1\underline{j}) + (-x_2\underline{i} - y_2\underline{j}) \\ &= (x_1 - x_2)\underline{i} + (y_1 - y_2)\underline{j}\end{aligned}$$



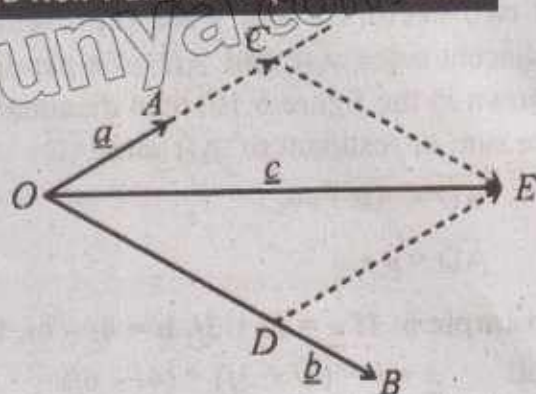
Example 7: If $\underline{a} = 6\underline{i} + 13\underline{j}$ and $\underline{b} = -4\underline{i} + 7\underline{j}$, then find $\underline{a} - \underline{b}$.

Sol. $\underline{a} - \underline{b} = (6\underline{i} + 13\underline{j}) - (-4\underline{i} + 7\underline{j})$
 $= (6 + 4)\underline{i} + (13 - 7)\underline{j} = 10\underline{i} + 6\underline{j}$

6.5 Vectors in Terms of Two Non-Zero and Non-Parallel Coplanar Vectors

Let $\underline{c}$ be any vector lying in the plane of $\underline{a}$ and $\underline{b}$. Draw $\underline{a}$, $\underline{b}$ and $\underline{c}$ such that their initial points coincide.

Construct lines parallel to the vectors $\underline{a}$ and $\underline{b}$ from the terminal point E of $\underline{c}$. Complete the parallelogram ODEC by extension of $\underline{a}$ and $\underline{b}$ if necessary.



$$\overline{OC} = m\overline{OA} = m\underline{a}, \quad \text{where } m \text{ is a scalar.}$$

$$\overline{OD} = n\overline{OB} = n\underline{b},$$

where n is a scalar.

By the parallelogram law of vector addition

$$\overline{OE} = \overline{OC} + \overline{OD} \quad \text{or } \underline{c} = m\underline{a} + n\underline{b}$$

Where $m\underline{a}$ and $n\underline{b}$ are vector components of $\underline{c}$.

Remember!

Two or more vectors are said to be coplanar vectors if they lie in the same plane.

Example 8: If $\underline{a} = 2\underline{i} - 3\underline{j}$ and $\underline{b} = 3\underline{i} + 4\underline{j}$, then find the vector components of $3\underline{a} + 4\underline{b}$.

Sol. $3\underline{a} + 4\underline{b}$
 $= 3(2\underline{i} - 3\underline{j}) + 4(3\underline{i} + 4\underline{j})$
 $= 6\underline{i} - 9\underline{j} + 12\underline{i} + 16\underline{j} = 18\underline{i} + 7\underline{j}$
 $18\underline{i}$ and $7\underline{j}$ are the components of $3\underline{a} + 4\underline{b}$.

Solved Exercise 6.2

1. If $\underline{a} = 7\underline{i} - 3\underline{j}$ and $\underline{b} = \underline{i} + 5\underline{j}$, then find the following vectors:

(i) $\underline{a} + \underline{b}$

Sol. $(7\underline{i} - 3\underline{j}) + (\underline{i} + 5\underline{j})$
 $= (7 + 1)\underline{i} + (-3 + 5)\underline{j}$
 $= 8\underline{i} + 2\underline{j}$ Ans

(ii) $\underline{a} + 3\underline{b}$

Sol. $(7\underline{i} - 3\underline{j}) + 3(\underline{i} + 5\underline{j})$
 $= 7\underline{i} - 3\underline{j} + 3\underline{i} + 15\underline{j}$
 $= (7 + 3)\underline{i} + (-3 + 15)\underline{j}$
 $= 10\underline{i} + 12\underline{j}$ Ans

(iii) $3\underline{a} + \frac{1}{2}\underline{b}$

Sol. $3(7\underline{i} - 3\underline{j}) + \frac{1}{2}(\underline{i} + 5\underline{j})$

$$= 21\underline{i} - 9\underline{j} + \frac{1}{2}\underline{i} + \frac{5}{2}\underline{j}$$

$$= \left(21 + \frac{1}{2}\right)\underline{i} + \left(-9 + \frac{5}{2}\right)\underline{j}$$

$$= \left(\frac{42+1}{2}\right)\underline{i} + \left(\frac{-18+5}{2}\right)\underline{j}$$

$$= \frac{43}{2}\underline{i} - \frac{13}{2}\underline{j}$$

$$= \frac{43}{2}\underline{i} - \frac{13}{2}\underline{j} \text{ Ans}$$

(iv) $\underline{b} - \underline{a}$

$$\begin{aligned}\text{Sol. } \underline{b} - \underline{a} &= (\underline{i} + 5\underline{j}) - (7\underline{i} - 3\underline{j}) \\ &= \underline{i} + 5\underline{j} - 7\underline{i} + 3\underline{j} \\ &= (1 - 7)\underline{i} + (5 + 3)\underline{j} \\ &= -6\underline{i} + 8\underline{j} \quad \text{Ans}\end{aligned}$$

(v) $4\underline{b} - 5\underline{a}$

$$\begin{aligned}\text{Sol. } 4\underline{b} - 5\underline{a} &= 4(\underline{i} + 5\underline{j}) - 5(7\underline{i} - 3\underline{j}) \\ &= 4\underline{i} + 20\underline{j} - 35\underline{i} + 15\underline{j} \\ &= (4 - 35)\underline{i} + (20 + 15)\underline{j} \\ &= -31\underline{i} + 35\underline{j} \quad \text{Ans}\end{aligned}$$

2. If $\underline{a} = 6\underline{i} - \underline{j}$, $\underline{b} = \underline{i} + 5\underline{j}$ and $\underline{c} = 3\underline{i} + 5\underline{j}$, then find the magnitudes of the following vectors:

(i) $\underline{b} - \underline{c}$

$$\begin{aligned}\text{Sol. } \underline{b} - \underline{c} &= (\underline{i} + 5\underline{j}) - (3\underline{i} + 5\underline{j}) \\ &= \underline{i} + 5\underline{j} - 3\underline{i} - 5\underline{j} \\ &= (1 - 3)\underline{i} + (5 - 5)\underline{j} \\ &= -2\underline{i} + 0\underline{j}\end{aligned}$$

Now

$$\begin{aligned}|\underline{b} - \underline{c}| &= |-2\underline{i} + 0\underline{j}| \\ &= \sqrt{(-2)^2 + (0)^2} \\ |\underline{b} - \underline{c}| &= \sqrt{4 + 0} \\ &= \sqrt{4}\end{aligned}$$

$$|\underline{b} - \underline{c}| = 2 \quad \text{Ans}$$

(iii) $\underline{c} - \underline{b} - \underline{a}$

$$\begin{aligned}\text{Sol. } \underline{c} - \underline{b} - \underline{a} &= (3\underline{i} - 5\underline{j}) - (\underline{i} + 5\underline{j}) - (6\underline{i} - \underline{j}) \\ &= 3\underline{i} + 5\underline{j} - \underline{i} - 5\underline{j} - 6\underline{i} + \underline{j} \\ &= (3 - 1 - 6)\underline{i} + (5 - 5 + 1)\underline{j} \\ &= -4\underline{i} + \underline{j}\end{aligned}$$

Now

$$\begin{aligned}|\underline{c} - \underline{b} - \underline{a}| &= \sqrt{(-4)^2 + 1^2} \\ &= \sqrt{16 + 1}\end{aligned}$$

$$|\underline{c} - \underline{b} - \underline{a}| = \sqrt{17} \quad \text{Ans}$$

(vi) $\frac{3}{2}\underline{a} - \underline{b}$

$$\begin{aligned}\text{Sol. } \frac{3}{2}\underline{a} - \underline{b} &= \frac{3}{2}(7\underline{i} - 3\underline{j}) - (\underline{i} + 5\underline{j}) \\ &= \frac{21}{2}\underline{i} - \frac{9}{2}\underline{j} - \underline{i} - 5\underline{j} \\ &= \left(\frac{21}{2} - 1\right)\underline{i} - \left(\frac{9}{2} + 5\right)\underline{j} \\ &= \left(\frac{21 - 2}{2}\right)\underline{i} - \left(\frac{9 + 10}{2}\right)\underline{j} \\ &= \frac{19}{2}\underline{i} - \frac{19}{2}\underline{j} \quad \text{Ans.}\end{aligned}$$

(ii) $\underline{a} - 2\underline{b} + \underline{c}$

$$\begin{aligned}\text{Sol. } \underline{a} - 2\underline{b} + \underline{c} &= (6\underline{i} - \underline{j}) - 2(\underline{i} + 5\underline{j}) + 3\underline{i} + 5\underline{j} \\ &= 6\underline{i} - \underline{j} - 2\underline{i} - 10\underline{j} + 3\underline{i} + 5\underline{j} \\ &= (6 - 2 + 3)\underline{i} - (1 + 10 - 5)\underline{j} \\ &= 7\underline{i} - 6\underline{j}\end{aligned}$$

Now

$$\begin{aligned}|\underline{a} - 2\underline{b} + \underline{c}| &= \sqrt{7^2 + (-6)^2} \\ &= \sqrt{49 + 36}\end{aligned}$$

$$|\underline{a} - 2\underline{b} + \underline{c}| = \sqrt{85} \quad \text{Ans}$$

3. Find the values of x and y in each of the following equations:

(i) $(x\mathbf{i} + y\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) = 7\mathbf{i} + 6\mathbf{j}$

Sol.

$$\Rightarrow (x+2)\mathbf{i} + (y+3)\mathbf{j} = 7\mathbf{i} + 6\mathbf{j}$$

Equating the coefficients of $\mathbf{i}$ & $\mathbf{j}$ on both sides

$$x+2=7, \quad y+3=6$$

$$x=7-2, \quad y=6-3.$$

$$\boxed{x=5}$$

$$\boxed{y=3}$$

(ii) $(x\mathbf{i} - 5\mathbf{j}) + (3\mathbf{i} + 5\mathbf{j}) = -8\mathbf{i} + y\mathbf{j}$

Sol.

$$\Rightarrow (x+3)\mathbf{i} - (5-5)\mathbf{j} = -8\mathbf{i} + y\mathbf{j}$$

$$(x+3)\mathbf{i} - 0\mathbf{j} = -8\mathbf{i} + y\mathbf{j}$$

Equating the coefficients of $\mathbf{i}$ & $\mathbf{j}$ on both sides

$$x+3=-8, \quad 0=y$$

$$x=-8-3, \quad \boxed{y=0}$$

$$\boxed{x=-11}$$

(iii) $(y\mathbf{i} + 3\mathbf{j}) + (-5\mathbf{i} + 2x\mathbf{j}) = 9\mathbf{i} + 7\mathbf{j}$

Sol.

$$(y-5)\mathbf{i} + (3+2x)\mathbf{j} = 9\mathbf{i} + 7\mathbf{j}$$

Equating the coefficients of $\mathbf{i}$ & $\mathbf{j}$ on both sides

$$y-5=9, \quad 3+2x=7$$

$$y=9+5, \quad 2x=7-3$$

$$\boxed{y=14}, \quad 2x=4$$

$$x=\frac{4}{2}$$

$$\boxed{x=2}$$

4. If $\underline{a} = \mathbf{i} + 3\mathbf{j}$, $\underline{c} = 2\mathbf{i} + \mathbf{j}$ and $\underline{a} + 2\underline{b} = \underline{c}$ then find $|\underline{b}|$.

Sol. $\underline{a} + 2\underline{b} = \underline{c}$

$$\mathbf{i} + 3\mathbf{j} + 2\underline{b} = 2\mathbf{i} + \mathbf{j}$$

$$2\underline{b} = 2\mathbf{i} + \mathbf{j} - \mathbf{i} - 3\mathbf{j}$$

$$= (2-1)\mathbf{i} + (1-3)\mathbf{j}$$

$$2\underline{b} = \mathbf{i} - 2\mathbf{j}$$

$$\underline{b} = \frac{1}{2}\mathbf{i} - \frac{2}{2}\mathbf{j}$$

$$\underline{b} = \frac{1}{2}\underline{i} - \underline{j}$$

$$|\underline{b}| = \left| \frac{1}{2}\underline{i} - \underline{j} \right|$$

$$= \sqrt{\left(\frac{1}{2}\right)^2 + (-1)^2}$$

$$= \sqrt{\frac{1}{4} + 1} = \sqrt{\frac{1+4}{4}} = \sqrt{\frac{5}{4}}$$

$$|\underline{b}| = \frac{\sqrt{5}}{2} \text{ Ans}$$

5. If $\underline{a} = -2\underline{i} + 7\underline{j}$ and $\underline{b} = 3\underline{i} - 5\underline{j}$, then find the vector components of $\underline{a} + 5\underline{b}$.

Sol. $\underline{a} + 5\underline{b} = -2\underline{i} + 7\underline{j} + 5(3\underline{i} - 5\underline{j})$
 $= -2\underline{i} + 7\underline{j} + 15\underline{i} - 25\underline{j}$
 $= (-2 + 15)\underline{i} + (7 - 25)\underline{j}$

$$\underline{a} + 5\underline{b} = 13\underline{i} - 18\underline{j}$$

So, the vector components $\underline{a} + 5\underline{b}$ of are 13 and -18.

6. If $5\underline{i} - 3\underline{j} = m(\underline{i} - 10\underline{j}) + n(4\underline{i} - 3\underline{j})$, then find the value of m and n.

Sol. Given eq. is

$$5\underline{i} - 3\underline{j} = m(\underline{i} - 10\underline{j}) + n(4\underline{i} - 3\underline{j})$$

$$5\underline{i} - 3\underline{j} = m\underline{i} - 10m\underline{j} + 4n\underline{i} - 3n\underline{j}$$

$$5\underline{i} - 3\underline{j} = (m + 4n)\underline{i} - (10m + 3n)\underline{j}$$

Equating the coefficients of $\underline{i}$ and $\underline{j}$ on both sides

$$m + 4n = 5 \dots\dots(i) \quad -(10m + 3n) = -3$$

$$\Rightarrow 10m + 3n = 3 \dots\dots(ii)$$

Multiplying eq. (i) by 10 and sub. from eq. (ii)

$$\begin{array}{r} 10m + 3n = 3 \\ +10m + 40n = +50 \\ \hline -37n = -47 \end{array}$$

$$\boxed{n = \frac{47}{37}}$$

Put value of 'n' in eq. (i)

$$m + 4\left(\frac{47}{37}\right) = 5$$

$$m + \frac{188}{37} = 5$$

$$\begin{aligned} m &= 5 - \frac{188}{37} \\ m &= \frac{185 - 188}{37} \end{aligned}$$

$$\boxed{m = \frac{-3}{37}}$$

A translation vector is a vector that describes the movement of a point, a body, a system from one position to another without rotation or deformation. The translated figure is called the image of original figure. The image is congruent to the original figure. Let us translate point $P(2, 1)$ by translation vectors $-2\mathbf{j}$, $3\mathbf{i}$, $-3\mathbf{j}$ and $-5\mathbf{i}$.

$$P_1 = (2\mathbf{i} + \mathbf{j}) + (0\mathbf{i} + 2\mathbf{j})$$

$$= 2\mathbf{i} + 3\mathbf{j}$$

$$P_2 = (2\mathbf{i} + \mathbf{j}) + (3\mathbf{i} + 0\mathbf{j})$$

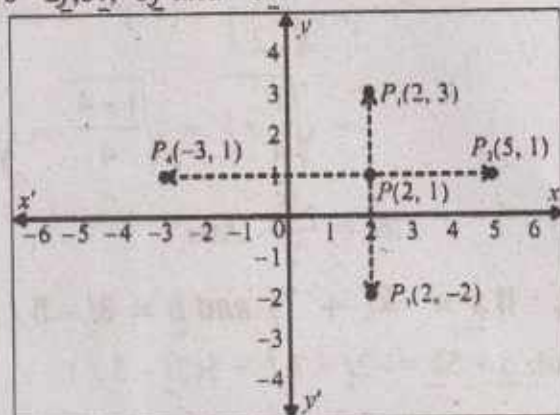
$$= 5\mathbf{i} + \mathbf{j}$$

$$P_3 = (2\mathbf{i} + \mathbf{j}) + (0\mathbf{i} - 3\mathbf{j})$$

$$= 2\mathbf{i} - 2\mathbf{j}$$

$$P_4 = (2\mathbf{i} + \mathbf{j}) + (-5\mathbf{i} + 0\mathbf{j})$$

$$= -3\mathbf{i} + \mathbf{j}$$



Example 9: Plot $A(-4, 2)$, $B(-1, 2)$ and $C(-3, 4)$ to form a triangle ABC . Also translate $\triangle ABC$ to $\triangle A'B'C'$ by the translation vector $7\mathbf{i} + 2\mathbf{j}$.

Solution:

$$\text{New vertex } A' = (-4\mathbf{i} + 2\mathbf{j}) + (7\mathbf{i} + 2\mathbf{j})$$

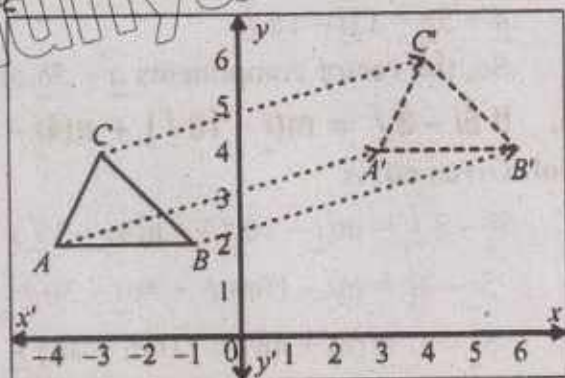
$$= 3\mathbf{i} + 4\mathbf{j} = (3, 4)$$

$$\text{New vertex } B' = (-1\mathbf{i} + 2\mathbf{j}) + (7\mathbf{i} + 2\mathbf{j})$$

$$= 6\mathbf{i} + 4\mathbf{j} = (6, 4)$$

$$\text{New vertex } C' = (-3\mathbf{i} + 4\mathbf{j}) + (7\mathbf{i} + 2\mathbf{j})$$

$$= 4\mathbf{i} + 6\mathbf{j} = (4, 6)$$



6.7

Solution and Application of Geometric problems involving Vectors

Example 10: If $A(1, 2)$, $B(4, 6)$ and $C(7, 2)$ are the vertices of triangle. Check whether triangle ABC is an isosceles.

$$\text{Solution: } \overline{AB} = (4 - 1)\mathbf{i} + (6 - 2)\mathbf{j} = 3\mathbf{i} + 4\mathbf{j}$$

$$\overline{BC} = (7 - 4)\mathbf{i} + (2 - 6)\mathbf{j} = 3\mathbf{i} - 4\mathbf{j}$$

$$\overline{AC} = (7 - 1)\mathbf{i} + (2 - 2)\mathbf{j} = 6\mathbf{i} - 0\mathbf{j}$$

$$\text{Now } |\overline{AB}| = \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$|\overline{BC}| = \sqrt{(3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\text{Shows } |\overline{AB}| = |\overline{BC}| = 5$$

So, $\triangle ABC$ is an isosceles.

Remember!

A triangle having two sides equal in length is called an isosceles triangle.

Example 11: Use vector method, show that the points $A(1, 2)$, $B(4, 6)$, $C(7, 4)$ and $D(4, 0)$ form a parallelogram.

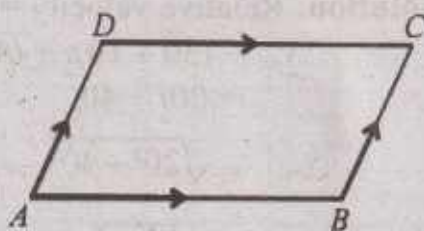
Solution: $\overrightarrow{AB} = (4-1)\underline{i} + (6-2)\underline{j} = 3\underline{i} + 4\underline{j}$

$$\overrightarrow{DC} = (7-4)\underline{i} + (4-0)\underline{j} = 3\underline{i} + 4\underline{j}$$

$$\overrightarrow{BC} = (7-4)\underline{i} + (4-6)\underline{j} = 3\underline{i} - 2\underline{j}$$

$$\overrightarrow{AD} = (4-1)\underline{i} + (0-2)\underline{j} = 3\underline{i} - 2\underline{j}$$

$\therefore \overrightarrow{AB} \parallel \overrightarrow{DC}$ and $\overrightarrow{BC} \parallel \overrightarrow{AD}$



Shows the given points form a parallelogram.

Note

Equations of motions are formulas used to describe the movement of objects under uniform acceleration. They help us to calculate the relationship between initial velocity ($\underline{v}_i$), final velocity ($\underline{v}_f$), acceleration ($\underline{a}$), time (t) and displacement ($\underline{s}$). These are very useful in problems involving moving cars, falling objects or thrown balls.

First Equation of Motion $\underline{v}_f = \underline{v}_i + \underline{a}t$

It tells us the final velocity ($\underline{v}_f$) after time t when an object starts with initial velocity $\underline{v}_i$ and moves with acceleration $\underline{a}$.

Second Equation of Motion $\underline{s} = \underline{v}_i t + \frac{1}{2} \underline{a}t^2$

It tells us the distance ($\underline{s}$) travelled by the object in time t .

Third Equation of Motion $\underline{v}_f^2 = \underline{v}_i^2 + 2\underline{a}\underline{s}$

It relates the initial and final velocity, acceleration and displacement.

Example 12: A projectile is launched from ground with initial velocity vector $12\underline{i} + 16\underline{j}$. After how long does it hit a target located 24 metres away horizontally?

Solution: Give that $\underline{v}_i = 12\underline{i} + 16\underline{j}$

$\therefore$ Horizontal velocity = $v_x = 12$

Vertical velocity = $v_y = 16$

Horizontal velocity is constant, therefore

$$x = v_x t$$

$$t = \frac{x}{v_x} = \frac{24}{12} = 2s$$

Note

Equation for horizontal motion is $x = \underline{v}_x t$

Equation for vertical motion is $y = \underline{v}_y t - \frac{1}{2} g t^2$

Example 13:

A tank fires a shell with velocity vector $\underline{v}_1 = 30\mathbf{i} + 40\mathbf{j}$ at a moving target heading east at $\underline{v}_2 = 10\mathbf{i} + 0\mathbf{j}$. Find relative velocity and its magnitude.

Solution: Relative velocity = Shell velocity - Target velocity

$$\begin{aligned}\underline{v}_{\text{rel}} &= (30 - 10)\mathbf{i} + (40 - 0)\mathbf{j} \\ &= 20\mathbf{i} + 40\mathbf{j}\end{aligned}$$

$$|\underline{v}_{\text{rel}}| = \sqrt{20^2 + 40^2} = \sqrt{400 + 1600}$$

$$|\underline{v}_{\text{rel}}| = \sqrt{2000} = 44.72 \text{ m/s}$$

Note

Relative velocity is the velocity of an object as observed from another moving object. It tells us how fast and in what direction one object is moving relative to another.

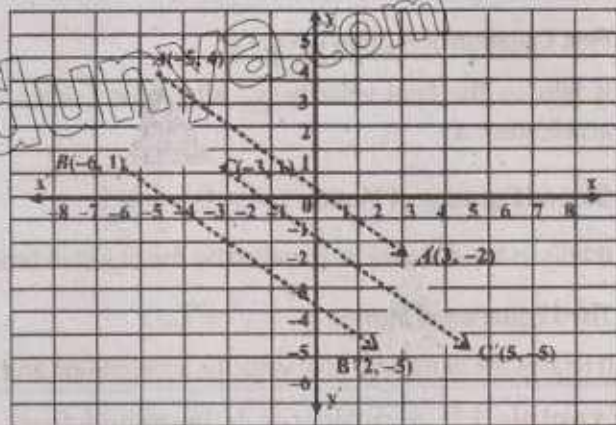
SOLVED EXERCISE 6.3

1. Plot $A(-5,4)$, $B(-6,1)$ and $C(-3,1)$ to form a $\triangle ABC$. Also translate $\triangle ABC$ to $\triangle A'B'C'$ by translation vector $8\mathbf{i} - 6\mathbf{j}$.

Sol. New vertex $A' = (-5\mathbf{i} + 4\mathbf{j}) + (8\mathbf{i} - 6\mathbf{j})$
 $= (-5 + 8)\mathbf{i} + (4 - 6)\mathbf{j}$
 $= 3\mathbf{i} - 2\mathbf{j} = (3, -2)$

New vertex $B' = (-6\mathbf{i} + \mathbf{j}) + (8\mathbf{i} - 6\mathbf{j})$
 $= (-6 + 8)\mathbf{i} + (1 - 6)\mathbf{j}$
 $= 2\mathbf{i} - 5\mathbf{j} = (2, -5)$
 $= (-6 + 8)\mathbf{i} + (1 - 6)\mathbf{j}$

New vertex $C' = (-3\mathbf{i} + \mathbf{j}) + (8\mathbf{i} - 6\mathbf{j})$
 $= (-3 + 8)\mathbf{i} + (1 - 6)\mathbf{j}$
 $= 5\mathbf{i} - 5\mathbf{j} = (5, -5)$



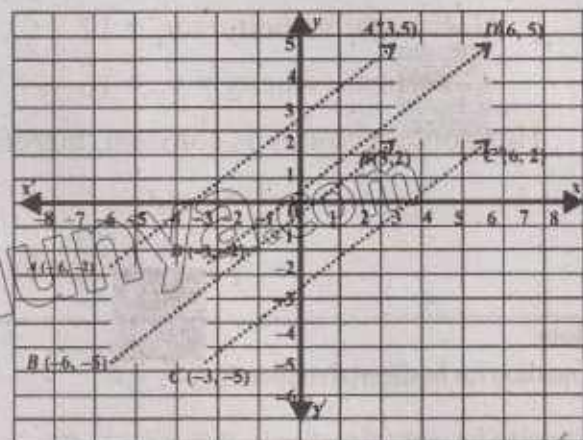
2. Plot $A(-6, -2)$, $B(-6, -5)$, $C(-3, -5)$ and $D(-3, -2)$ to form a square $ABCD$. Also translate square $ABCD$ to square $A'B'C'D'$ by translation vector $9\mathbf{i} + 7\mathbf{j}$.

Sol. New vertex $A' = (-6\mathbf{i} - 2\mathbf{j}) + (9\mathbf{i} + 7\mathbf{j})$
 $= (-6 + 9)\mathbf{i} + (-2 + 7)\mathbf{j}$
 $= 3\mathbf{i} + 5\mathbf{j} = (3, 5)$

New vertex $B' = (-6\mathbf{i} - 5\mathbf{j}) + (9\mathbf{i} + 7\mathbf{j})$
 $= (-6 + 9)\mathbf{i} + (-5 + 7)\mathbf{j}$
 $= 3\mathbf{i} + 2\mathbf{j} = (3, 2)$

New vertex $C' = (-3\mathbf{i} - 5\mathbf{j}) + (9\mathbf{i} + 7\mathbf{j})$
 $= (-3 + 9)\mathbf{i} + (-5 + 7)\mathbf{j}$
 $= 6\mathbf{i} + 2\mathbf{j} = (6, 2)$

New vertex $D' = (-3\mathbf{i} - 2\mathbf{j}) + (9\mathbf{i} + 7\mathbf{j})$
 $= (-3 + 9)\mathbf{i} + (-2 + 7)\mathbf{j}$
 $= 6\mathbf{i} + 5\mathbf{j} = (6, 5)$



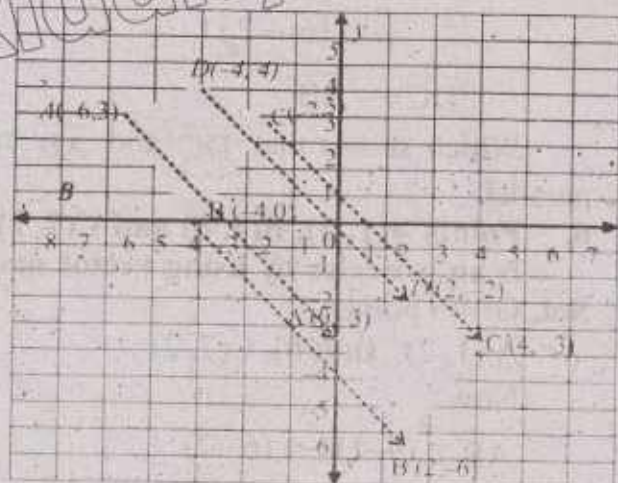
3. Plot $A(-6, 3)$, $B(-4, 0)$, $C(-2, 3)$ and $D(-4, 4)$ to form a kite $ABCD$. Also translate kite $ABCD$ to kite $A'B'C'D'$ by translation vectors $6\mathbf{i} - 6\mathbf{j}$.

Sol. New vertex $A' = (-6\mathbf{i} + 3\mathbf{j}) + (6\mathbf{i} - 6\mathbf{j})$
 $= (-6 + 6)\mathbf{i} + (3 - 6)\mathbf{j}$
 $= 0\mathbf{i} - 3\mathbf{j} = (0, -3)$

New vertex $B' = (-4\mathbf{i} + 0\mathbf{j}) + (6\mathbf{i} - 6\mathbf{j})$
 $= (-4 + 6)\mathbf{i} + (0 - 6)\mathbf{j}$
 $= 2\mathbf{i} - 6\mathbf{j} = (2, -6)$

New vertex $C' = (-2\mathbf{i} + 3\mathbf{j}) + (6\mathbf{i} - 6\mathbf{j})$
 $= (-2 + 6)\mathbf{i} + (3 - 6)\mathbf{j}$
 $= 4\mathbf{i} - 3\mathbf{j} = (4, -3)$

New vertex $D' = (-4\mathbf{i} + 4\mathbf{j}) + (6\mathbf{i} - 6\mathbf{j})$
 $= (-4 + 6)\mathbf{i} + (4 - 6)\mathbf{j}$
 $= 2\mathbf{i} - 2\mathbf{j} = (2, -2)$



4. The coordinates of A, B and D are $(1, 2)$, $(6, 3)$ and $(2, 8)$ respectively. Find the coordinates of C by using vector method if $ABCD$ is a parallelogram.

Ans. Given vertices:

$A(1, 2)$, $B(6, 3)$, $D(2, 8)$

Let $C(x, y)$ be the fourth vertex of parallelogram $ABCD$.

Since in a parallelogram

$$\overrightarrow{AB} = \overrightarrow{DC}$$

$$(6-1)\mathbf{i} + (3-2)\mathbf{j} = (x-2)\mathbf{i} + (y-8)\mathbf{j}$$

$$5\mathbf{i} + \mathbf{j} = (x-2)\mathbf{i} + (y-8)\mathbf{j}$$

Comparing the coefficients of $\mathbf{i}$ and $\mathbf{j}$ on both sides

$$x-2=5, \quad y-8=1$$

$$x=5+2, \quad y=1+8$$

$$\boxed{x=7}$$

$$\boxed{y=9}$$

So required coordinates of C are $(7, 9)$

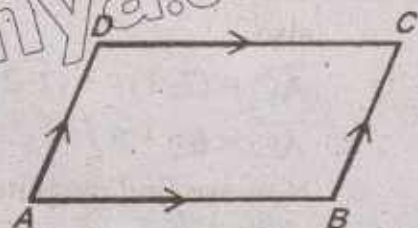
5. In parallelogram $ABCD$, the vectors representing two opposite sides are $\overrightarrow{AB} = 6\mathbf{i} + 2\mathbf{j}$, $\overrightarrow{DC} = -6\mathbf{i} - 2\mathbf{j}$. Show that the opposite sides are equal in magnitude and parallel by using vectors.

Sol. $|\overrightarrow{AB}| = \sqrt{6^2 + 2^2}$, $|\overrightarrow{DC}| = \sqrt{(-6)^2 + (-2)^2}$
 $= \sqrt{36+4}$
 $= \sqrt{40}$

$$|\overrightarrow{AB}| = \sqrt{40}$$

$$|\overrightarrow{DC}| = \sqrt{40}$$

$$|\overrightarrow{AB}| = |\overrightarrow{DC}|$$



Which shows that opposite sides are equal in magnitude.

$$\overrightarrow{DC} = -6\mathbf{i} - 2\mathbf{j}$$

$$= -(6\mathbf{i} + 2\mathbf{j})$$

$$\overrightarrow{DC} = -\overrightarrow{AB}$$

Which shows that $\overrightarrow{DC}$ and $\overrightarrow{AB}$ are scalar multiple of each other. Hence, they are parallel.

6. Points $A(1, 2)$, $B(4, 6)$ and $C(7, 2)$ form a triangle. Check whether triangle ABC is an isosceles by using vector magnitude.

Sol. Given points:

$$A(1, 2), B(4, 6), C(7, 2)$$

Now

$$\overrightarrow{AB} = (4-1)\mathbf{i} + (6-2)\mathbf{j}$$

$$\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j}$$

and

$$\overrightarrow{BC} = (7-4)\mathbf{i} + (2-6)\mathbf{j}$$

$$\overrightarrow{BC} = 3\mathbf{i} - 4\mathbf{j}$$

also

$$\overrightarrow{AC} = (7-1)\mathbf{i} + (2-2)\mathbf{j}$$

$$\overrightarrow{AC} = 6\mathbf{i} + 0\mathbf{j}$$

Now we find magnitude

$$|\overrightarrow{AB}| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$|\overrightarrow{BC}| = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\text{Since } |\overrightarrow{AB}| = |\overrightarrow{BC}|$$

So, points A, B & C form an isosceles triangle.

yes ABC is an isosceles triangle

7. Use vectors to show that $PQRS$ is a parallelogram, where the points P, Q, R and S have coordinates $(1, 2), (5, 2), (7, 6)$ and $(3, 6)$ respectively

Sol. Given points are

$$P(1, 2), Q(5, 2), R(7, 6), S(3, 6)$$

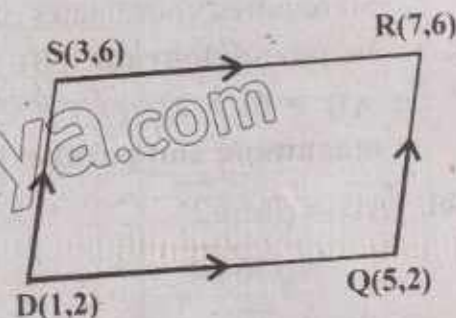
$$\overrightarrow{PQ} = (5-1)\mathbf{i} + (2-2)\mathbf{j} = 4\mathbf{i} + 0\mathbf{j}$$

$$\overrightarrow{SR} = (7-3)\mathbf{i} + (6-6)\mathbf{j} = 4\mathbf{i} + 0\mathbf{j}$$

$$\overrightarrow{PS} = (3-1)\mathbf{i} + (6-2)\mathbf{j} = 2\mathbf{i} + 4\mathbf{j}$$

$$\overrightarrow{QR} = (7-5)\mathbf{i} + (6-2)\mathbf{j} = 2\mathbf{i} + 4\mathbf{j}$$

$$\therefore \overrightarrow{PQ} \parallel \overrightarrow{SR} \text{ and } \overrightarrow{PS} \parallel \overrightarrow{QR}$$



Which shows that the given points form a parallelogram.

8. Use vectors to show that triangle XYZ is an isosceles, where the points X, Y, Z have the coordinates (0, 0), (2, 0) and (1, 3) respectively.

Sol. Given points:

$$X(0,0), Y(2,0), Z(1,3)$$

Now

$$\overrightarrow{XY} = (2-0)\underline{i} + (0-0)\underline{j}$$

$$\overrightarrow{XY} = 2\underline{i} + 0\underline{j}$$

$$\Rightarrow |\overrightarrow{XY}| = \sqrt{2^2 + 0^2} = \sqrt{4+0} = \sqrt{4}$$

$$\Rightarrow |\overrightarrow{XY}| = 2$$

and

$$\overrightarrow{YZ} = (1-2)\underline{i} + (3-0)\underline{j}$$

$$\overrightarrow{YZ} = -\underline{i} + 3\underline{j}$$

$$\Rightarrow |\overrightarrow{YZ}| = \sqrt{(-1)^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$$

also

$$|\overrightarrow{XZ}| = (1-0)\underline{i} + (3-0)\underline{j}$$

$$|\overrightarrow{XZ}| = \underline{i} + 3\underline{j}$$

$$\Rightarrow |\overrightarrow{XZ}| = \sqrt{1^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$$

$$\therefore |\overrightarrow{XZ}| = |\overrightarrow{YZ}|$$

Hence XYZ is an isosceles triangle.

9. A ball is projected with an initial velocity vector $\underline{v}_0 = 10\underline{i} + 20\underline{j}$. The horizontal component is in the x-direction and gravity is $\underline{g} = 0\underline{i} - 10\underline{j}$

Sol: initial velocity = $\underline{v}_0 = 10\underline{i} + 20\underline{j}$

$$\Rightarrow \underline{v}_x = 10, \underline{v}_y = 20$$

$$\text{and } \underline{g} = 0\underline{i} - 10\underline{j}$$

$$\Rightarrow \underline{g} = 10$$

$$\begin{aligned} \therefore \text{Maximum Height (H)} &= \frac{v_y^2}{2g} \\ &= \frac{(20)^2}{2(10)} = \frac{400}{20} \\ &= 20 \text{ m} \end{aligned}$$

$$\begin{aligned} \therefore \text{Horizontal Range (R)} &= \underline{v}_x \times T \\ &= 10 \times \frac{2v_y}{g} \quad \therefore T = \frac{2v_y}{g} \\ &= 10 \times \frac{2(20)}{10} \\ &= 40 \text{ m} \quad \text{Ans.} \end{aligned}$$

10. A car enters a loop with velocity-vector $\underline{v} = 30\hat{j}$ and exits with velocity $\underline{v} = 30\hat{i}$. What is the change in velocity vector?

Sol. initial velocity $\underline{v} = 30\hat{j}$
 final velocity ($\underline{v}'$) $= 30\hat{i}$
 $\Rightarrow$ Change in velocity $= \Delta \underline{v} = \underline{v}' - \underline{v}$
 $= 30\hat{i} - 30\hat{j}$ Ans.

11. An aeroplane has airspeed $\underline{v}_p = 20\hat{i}$ and there is a crosswind $\underline{v}_w = 50\hat{j}$. Find the resultant velocity and its magnitude.

Sol: airspeed $= (\underline{v}_p) = 20\hat{i}$
 cross wind ($\underline{v}_w$) $= 50\hat{j}$
 $\Rightarrow$ Resultant velocity $= \underline{v} = 20\hat{i} + 50\hat{j}$
 Resultant velocity $= |\underline{v}| = \sqrt{(20)^2 + (50)^2}$
 $= \sqrt{400 + 2500}$
 $= \sqrt{2900}$
 $= 10\sqrt{29}$
 $= 53.85$ units

Solved Review Exercise 6

- Four possible answers are given for the following questions. Choose the correct answer.
 - In how many parts the x-axis and y-axis divide a coordinate plane into _____ parts?
 (a) one (b) two (c) three (d) four
 - $P(4, -4)$ lies in _____ quadrant?
 (a) first (b) second (c) third (d) fourth
 - A vector having magnitude 1, is it called:
 (a) equal vector (b) parallel vector (c) unit vector (d) zero vector
 - What is the value of $|3\hat{i} + 4\hat{j}|$?
 (a) 3 (b) 4 (c) 5 (d) 7
 - If $\underline{a} = \lambda \underline{b}$, then what are $\underline{a}$ and $\underline{b}$?
 (a) equal (b) parallel (c) perpendicular (d) none of these
 - If $\underline{OA} = \underline{a}$, $\underline{OB} = \underline{b}$, then $\underline{AB}$:
 (a) $\underline{b} - \underline{a}$ (b) $\underline{a} - \underline{b}$ (c) $\underline{a} + \underline{b}$ (d) $\underline{b} + \underline{a}$
 - Translation vector show:
 (a) deformation (b) rotation (c) movement (d) enlargement
 - Sum of two vectors is;
 (a) a triangle (b) a vector (c) a scalar (d) a length

- (ix) The position vector of point P(3,-2) with respect to O?
 (a) $3\mathbf{i} + 4\mathbf{j}$ (b) $3\mathbf{i} - 2\mathbf{j}$ (c) $-3\mathbf{i} - 4\mathbf{j}$ (d) $3\mathbf{i} - 4\mathbf{j}$
 (x) Vector from point p(3,4) to origin is:
 (a) $3\mathbf{i} + 4\mathbf{j}$ (b) $-3\mathbf{i} + 4\mathbf{j}$ (c) $-3\mathbf{i} - 4\mathbf{j}$ (d) $3\mathbf{i} - 4\mathbf{j}$

ANSWERS:

- i. four ii. fourth iii. unit vector iv. 5 v. parallel
 vi. $\underline{b} - \underline{a}$ vii. movement viii. a vector ix. $3\mathbf{i} - 2\mathbf{j}$ x. $-3\mathbf{i} - 4\mathbf{j}$

2. Find magnitude of the $\overline{AB}$:

- (i) A (7,7), B(-12, 0)

Sol: Given points

$$A(7,7), B(-12,0)$$

$$\overline{AB} = (-12-7)\mathbf{i} + (0-7)\mathbf{j}$$

$$\overline{AB} = -19\mathbf{i} - 7\mathbf{j}$$

$$\Rightarrow |\overline{AB}| = \sqrt{(-19)^2 + (-7)^2}$$

$$= \sqrt{361 + 49}$$

$$|\overline{AB}| = \sqrt{410}$$

- (ii) A (9,3), B(2, 11)

Sol: Given points

$$A(9,3), B(2,11)$$

$$\overline{AB} = (2-9)\mathbf{i} + (11-3)\mathbf{j}$$

$$\overline{AB} = -7\mathbf{i} + 8\mathbf{j}$$

$$|\overline{AB}| = \sqrt{(-7)^2 + (8)^2}$$

$$= \sqrt{49 + 64}$$

$$|\overline{AB}| = \sqrt{113}$$

3. Find a unit vector in the direction of $\underline{a} = \frac{5}{3}\mathbf{i} + \frac{1}{3}\mathbf{j}$.

$$\text{Sol: } \underline{a} = \frac{5}{3}\mathbf{i} + \frac{1}{3}\mathbf{j}$$

$$|\underline{a}| = \sqrt{\left(\frac{5}{3}\right)^2 + \left(\frac{1}{3}\right)^2}$$

$$= \sqrt{\frac{25}{9} + \frac{1}{9}} = \sqrt{\frac{25+1}{9}}$$

$$= \frac{\sqrt{26}}{\sqrt{9}}$$

$$|\underline{a}| = \frac{\sqrt{26}}{3}$$

$$\text{Required Unit Vector} = \frac{\underline{a}}{|\underline{a}|}$$

$$= \frac{\frac{5}{3}\mathbf{i} + \frac{1}{3}\mathbf{j}}{\frac{\sqrt{26}}{3}}$$

$$= \frac{3}{\sqrt{26}} \times \frac{5}{3}\mathbf{i} + \frac{3}{\sqrt{26}} \times \frac{1}{3}\mathbf{j}$$

$$= \frac{5}{\sqrt{26}}\mathbf{i} + \frac{1}{\sqrt{26}}\mathbf{j} \text{ Ans.}$$

4. If $\underline{a} = 2\mathbf{i} - \mathbf{j}$, $\underline{b} = 3\mathbf{i} + \mathbf{j}$ and $\underline{c} = 4\mathbf{i} + \mathbf{j}$, then find following vectors:

- (i) $5\underline{b} - \underline{a} + \underline{c}$

$$\text{Sol: } = 5(3\mathbf{i} + \mathbf{j}) - (2\mathbf{i} - \mathbf{j}) + (4\mathbf{i} + \mathbf{j})$$

$$= 15\mathbf{i} - 2\mathbf{i} + 4\mathbf{i} + 5\mathbf{j} + \mathbf{j} + \mathbf{j}$$

$$= (15-2+4)\mathbf{i} + (5+1+1)\mathbf{j}$$

$$= 17\mathbf{i} + 7\mathbf{j} \text{ Ans.}$$

- (ii) $8\underline{a} + \underline{b} + 5\underline{c}$

$$\text{Sol: } = 8(2\mathbf{i} - \mathbf{j}) + (3\mathbf{i} + \mathbf{j}) + 5(4\mathbf{i} + \mathbf{j})$$

$$= 16\mathbf{i} + 3\mathbf{i} + 20\mathbf{i} - 8\mathbf{j} + \mathbf{j} + 5\mathbf{j}$$

$$= (16+3+20)\mathbf{i} - (8-1-5)\mathbf{j}$$

$$= 39\mathbf{i} - 2\mathbf{j} \text{ Ans.}$$

$$\begin{aligned}
 \text{(iii)} \quad \underline{c} + \underline{b} - 4\underline{a} &= 4\underline{i} + \underline{j} + 3\underline{i} + \underline{j} - 4(2\underline{i} - \underline{j}) \\
 &= 4\underline{i} + \underline{j} + 3\underline{i} + \underline{j} - 8\underline{i} + 4\underline{j} \\
 &= (4+3-8)\underline{i} + (1+1+4)\underline{j} \\
 &= -\underline{i} + 6\underline{j} \quad \text{Ans.}
 \end{aligned}$$

5. Find the values of x and y in the following equation.

$$\text{Sol: } (2x\underline{i} + y\underline{j}) + (-\underline{i} + 5\underline{j}) = \frac{1}{4}\underline{i} - 8\underline{j}$$

$$(2x\underline{i} + y\underline{j}) + (-\underline{i} + 5\underline{j}) = \frac{1}{4}\underline{i} - 8\underline{j}$$

$$(2x-1)\underline{i} + (y+5)\underline{j} = \frac{1}{4}\underline{i} - 8\underline{j}$$

Comparing coefficients of $\underline{i}$ and $\underline{j}$ on both sides

$$2x-1 = \frac{1}{4} \quad , \quad y+5 = -8$$

$$2x = \frac{1}{4} + 1 \quad , \quad y = -8 - 5$$

$$2x = \frac{1+4}{4} \quad , \quad \boxed{y = -13}$$

$$x = \frac{5}{2 \times 4} \Rightarrow \boxed{x = \frac{5}{8}}$$

6. Plot $A(-5, 3)$, $B(-2, 3)$ and $C(-4, 5)$ to form a triangle. $\triangle ABC$ Also translate $\triangle ABC$ to $\triangle A'B'C'$ by translation vectors $5\underline{i} - 2\underline{j}$.

$$\text{Sol: New vertex } A' = (-5\underline{i} + 3\underline{j}) + (5\underline{i} - 2\underline{j})$$

$$= (-5+5)\underline{i} + (3-2)\underline{j} = 0\underline{i} + \underline{j} = (0, 1)$$

$$\text{New vertex } B' = (-2\underline{i} + 3\underline{j}) + (5\underline{i} - 2\underline{j})$$

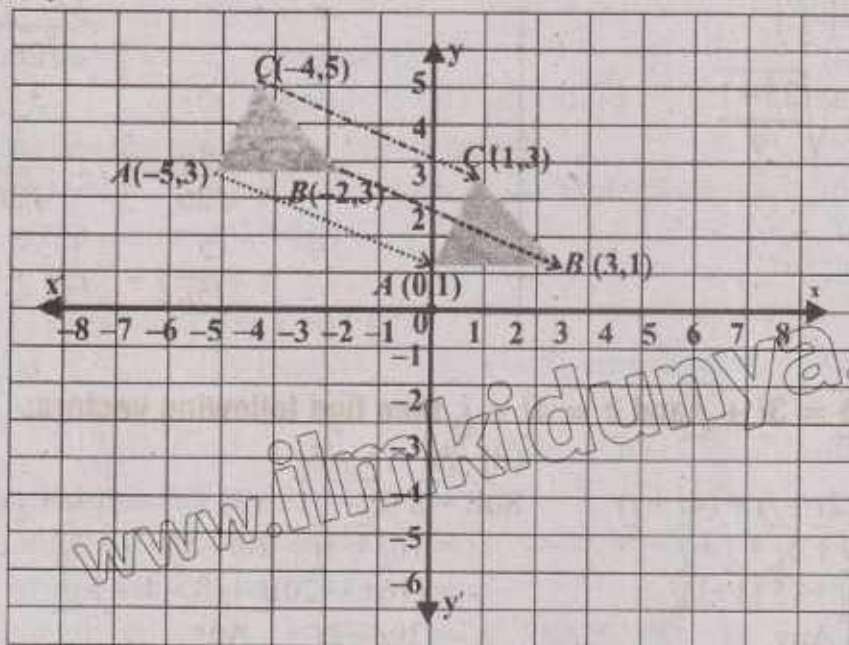
$$= (-2+5)\underline{i} + (3-2)\underline{j}$$

$$= 3\underline{i} + \underline{j} = (3, 1)$$

$$\text{New vertex } C' = (-4\underline{i} + 5\underline{j}) + (5\underline{i} - 2\underline{j})$$

$$= (-4+5)\underline{i} + (5-2)\underline{j}$$

$$= \underline{i} + 3\underline{j} = (1, 3)$$



7. Use vectors to show that $ABCD$ is a parallelogram. Where the points are: $A(2,3)$, $B(6,3)$, $C(7,6)$, $D(3,6)$.

Sol: $A(2,3)$, $B(6,3)$, $C(7,6)$, $D(3,6)$

$$\overrightarrow{AB} = (6-2)\underline{i} + (3-3)\underline{j}$$

$$\overrightarrow{AB} = 4\underline{i} + 0\underline{j}$$

$$\overrightarrow{DC} = (7-3)\underline{i} + (6-6)\underline{j}$$

$$\overrightarrow{DC} = 4\underline{i} + 0\underline{j}$$

$$\Rightarrow \overrightarrow{AB} \parallel \overrightarrow{DC}$$

and

$$\overrightarrow{AD} = (3-2)\underline{i} + (6-3)\underline{j}$$

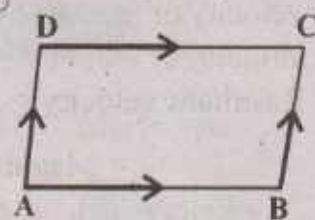
$$\overrightarrow{AD} = \underline{i} + 3\underline{j}$$

$$\overrightarrow{BC} = (7-6)\underline{i} + (6-3)\underline{j}$$

$$\overrightarrow{BC} = \underline{i} + 3\underline{j}$$

$$\Rightarrow \overrightarrow{AD} \parallel \overrightarrow{BC}$$

Which shows that given points form a parallelogram.



8. Use vectors to show that triangle ABC is an isosceles triangle. Where the points A , B and C have coordinates $(1, 2)$, $(4, 6)$ and $(7, 2)$ respectively.

Sol: $A(1,2)$, $B(4,6)$, $C(7,2)$

Now

$$\overrightarrow{AB} = (4-1)\underline{i} + (6-2)\underline{j}$$

$$\overrightarrow{AB} = 3\underline{i} + 4\underline{j}$$

and

$$\overrightarrow{BC} = (7-4)\underline{i} + (2-6)\underline{j}$$

$$\overrightarrow{BC} = 3\underline{i} - 4\underline{j}$$

also

$$\overrightarrow{AC} = (7-1)\underline{i} + (2-2)\underline{j}$$

$$\overrightarrow{AC} = 6\underline{i} + 0\underline{j}$$

Now

$$AB = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$BC = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\Rightarrow AB = BC$$

Which shows that given points form an isosceles triangle.

9. A ball is projected with velocity vector $\underline{v} = 6\underline{i} + 8\underline{j}$. What is the magnitude of velocity?

Sol: magnitude of velocity = $|\underline{v}| = \sqrt{6^2 + 8^2}$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100}$$

$$|\underline{v}| = 10 \quad \text{Ans.}$$

10. An aircraft is flying due east with an airspeed of 200 km/h. There is a wind blowing due north at 60 km/h. Find the resultant velocity and its magnitude.

Sol: velocity of aircraft (V_a) = 200i (East)

velocity of wind (V_w) = 60j (North)

Resultant velocity = 200i + 60j

$$\begin{aligned}\text{Magnitude} = |V| &= \sqrt{(200)^2 + (60)^2} = \sqrt{40000 + 3600} \\ &= \sqrt{43600} = 20\sqrt{109} \text{ km/h} \\ &= 208.8 \text{ km/h} \quad \text{Ans.}\end{aligned}$$

Activity

1. Suppose vectors a and b are equal.

(i) Can we say they originate from the same point? Why or why not?

Ans. No, they do not necessarily originate from the same point. Equal vectors have the same magnitude and direction regardless of the position of their initial points.

(ii) Do they have equal magnitudes? Explain.

Ans. Yes, they must have equal magnitudes. By definition, two vectors are equal if they have the same magnitude.

(iii) Do they have same direction? Why?

Ans. Yes, they have the same direction. Vectors are considered equal only when they have same direction and magnitude.

2. Suppose vectors a and b are opposite.

(i) Can we assume they begin at the same point? Give a reason.

Ans. No we cannot assume they begin at the same point. The relationship of two vectors is defined by their length and direction, not their starting point.

(ii) Do they have same magnitude? Why?

Ans. Yes, two opposite vectors have same magnitude. The two opposite vectors just differ by their direction only.

(iii) Do they have the same direction? Explain why or why not.

Ans. No, they do not have the same direction. They are directed exactly at 180° away from each other, which is why they are called opposite.

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

6.1

Rectangular Coordinate Plane

- The x-axis and y-axis meet at a point called:
 - Abscissa
 - Ordinate
 - Origin
 - Vertex
- The point P(-2,1) lies in:
 - Quad-I
 - Quad-II
 - Quad-III
 - Quad-IV
- A Point lying in Quad-III is:
 - (-2, 1)
 - (-4, -1)
 - (2, 1)
 - (3, -1)

4. The intersecting point of x-axis and y-axis is:
 (a) (0,0) (b) (0,1) (c) (1,0) (d) (1,1)

6.2

Scalars and Vectors

5. A quantity having magnitude only is called:
 (a) Scalar (b) Vector (c) Non-Physical (d) None
6. An example of scalar quantity is:
 (a) Mass (b) Force (c) Acceleration (d) Velocity
7. A quantity having both magnitude and direction is called:
 (a) Scalar (b) Vector (c) Non-physical (d) None
8. An example of vector quantity is:
 (a) Velocity (b) Mass (c) Time (d) Volume

6.2.1

Representation of a vector as Directed Line Segment

9. A vector equal in magnitude but in opposite direction to a given vector is known as:
 (a) Unit vector (b) Null vector
 (c) Negative of vector (d) Position vector
10. A vector having magnitude zero is:
 (a) Unit vector (b) Null vector (c) Negative vector (d) Position vector
11. Two vectors $\underline{a}$ and $\underline{b}$ having same length and direction are called:
 (a) Equivalent vectors (b) Equal vectors
 (c) Same vectors (d) Parallel vectors
12. A vector whose initial point is origin O (0,0) is:
 (a) Null vector (b) Unit vector (c) Position vector (d) None
13. The position vector of point P(-1,4) with respect to O is:
 (a) $-\underline{i} + 4\underline{j}$ (b) $\underline{i} - 4\underline{j}$ (c) $-\underline{i} - 4\underline{j}$ (d) $\underline{i} + 4\underline{j}$
14. Magnitude of a vector $\underline{a} = x\underline{i} + y\underline{j}$ is:
 (a) $\sqrt{x^2 - y^2}$ (b) $\sqrt{x^2 + y^2}$ (c) $x^2 - y^2$ (d) $\sqrt{x^2 \underline{i} + y^2 \underline{j}}$
15. Magnitude of a vector is also known as _____ of vector:
 (a) Norm (b) Length (c) Both (a) and (b) (d) None
16. If $\underline{b} = 3\underline{i} - 4\underline{j}$, the $|\underline{b}| =$ _____?
 (a) $\sqrt{-25}$ (b) $\sqrt{-7}$ (c) $\sqrt{7}$ (d) 5
17. Magnitude of a unit vector is:
 (a) 1 (b) 2 (c) 3 (d) 4
18. Magnitude of a Null vector is:
 (a) 0 (b) 1 (c) 2 (d) 3

6.2.5

Scalar Multiplication of a Vector

19. If $\underline{a} = \lambda \underline{b}$, then $\underline{a}$ & $\underline{b}$ are in same direction if:
 (a) $\lambda < 0$ (b) $\lambda > 0$ (c) $\lambda = 1$ (d) $\lambda = 0$
20. If $\underline{a} = \lambda \underline{b}$, then $\underline{a}$ and $\underline{b}$ are in opposite direction if:
 (a) $\lambda < 0$ (b) $\lambda > 0$ (c) $\lambda = 1$ (d) $\lambda = 0$

21. A vector whose magnitude is 1 is called _____ vector.
 (a) Null vector (b) Unit vector
 (c) Position vector (d) Negative vector
22. If $\underline{a}$ is a non-zero vector then its unit vector is represented by:
 (a) $\hat{a}$ (b) $|\underline{a}|$ (c) $\frac{\underline{a}}{|\underline{a}|}$ (d) $\bar{a}$
23. If $\underline{a}$ is a non-zero vector, then its unit vector is defined as:
 (a) $\underline{a} = \lambda \underline{a}$ (b) $\hat{a}$ (c) $\hat{a} = \frac{\underline{a}}{|\underline{a}|}$ (d) $\hat{a} = \frac{|\underline{a}|}{\underline{a}}$
24. If $\underline{a}$ is a unit vector then the value of $|\underline{a}|$ is:
 (a) 1 (b) 2 (c) 4 (d) 4

6.3

Addition of Vectors

25. If two sides of triangle represent vectors $\underline{u}$ and $\underline{v}$ in order, the third side represents:
 (a) $\underline{u} - \underline{v}$ (b) $\frac{\underline{u}}{\underline{v}}$ (c) $\underline{u} + \underline{v}$ (d) $|\underline{u} + \underline{v}|$
26. If $\underline{a} = -\underline{i} + \underline{j}$ and $\underline{b} = 2\underline{i} + 2\underline{j}$, then $\underline{a} + \underline{b} =$?
 (a) $-\underline{i} + 3\underline{j}$ (b) $\underline{i} + 3\underline{j}$ (c) $3\underline{i} + 3\underline{j}$ (d) $-3\underline{i} + 3\underline{j}$
27. Two vectors $\underline{a}$ and $\underline{b}$ can be added by:
 (a) Triangle Law of Addition (b) Head to tail Rule
 (c) Parallelogram Law of Addition (d) All of above
28. If two vectors $\underline{a}$ and $-\underline{a}$ are added, then resulting vector is:
 (a) Null vector (b) Unit vector
 (c) Position vector (d) Negative of vector

6.4

Difference of two Vectors

29. The difference of two vectors is a _____?
 (a) Scalar (b) Vector (c) Constant (d) Magnitude
30. If two vectors having equal x and y components are subtracted, then the result is a _____?
 (a) Scalar (b) Unit vector (c) Zero vector (d) Position vector

6.5

Vectors in terms of two non-zero and non-parallel coplanar vectors

31. Two or more vectors lying in the same plane are called:
 (a) Equal vectors (b) Coplanar vectors (c) Equivalent vectors (d) Parallel vectors
32. Translation of a vector gives us a _____?
 (a) Vector (b) Scalar (c) Null vector (d) Unit vector
33. The new vertex of point A(-4,2) by a translation vector $7\underline{i} + 2\underline{j}$ is:
 (a) (-3, -4) (b) (3,4) (c) (-4, 2) (d) (7,2)

34. First equation of motion in vector form is:

(a) $\underline{v}_f = \underline{v}_i + \underline{a}t$ (b) $\underline{a}t = \underline{v}_f + \underline{v}_i$ (c) $\underline{v}_f = \underline{v}_i + \underline{a}t$ (d) $\underline{v}_f - \underline{v}_i = 2\underline{a}S$

35. Equation for horizontal motion is:

(a) $\underline{x} = \underline{v}_x t$ (b) $\underline{x} = \underline{v}_x t$ (c) $\underline{y} = -\underline{v}_y t$ (d) $\underline{y} = \underline{v}_y t$

36. Equation for vertical motion is:

(a) $\underline{x} = \underline{v}_x t$ (b) $\underline{y} = \underline{v}_y t - \frac{1}{2}gt^2$ (c) $\underline{y} = \underline{v}_y t$ (d) None

ANSWERS:

1. Origin 2. Quad-II 3. $(-4, -1)$ 4. $(0,0)$ 5. Scalar
6. Mass 7. Vector 8. Velocity 9. Negative of vector
10. Null vector 11. Equal vectors 12. Position vector 13. $-i + 4j$
14. $\sqrt{x^2 + y^2}$ 15. Both (a) and (b) 16. 5 17. 1 18. 0
19. $\lambda > 0$ 20. $\lambda < 0$ 21. Unit vector 22. $\hat{a}$ 23. $\hat{a} = \frac{\underline{a}}{|\underline{a}|}$ 24. 1
25. $\underline{u} + \underline{v}$ 26. $i + 3j$ 27. All of above 28. Null vector 29. Vector
30. Zero vector 31. Coplanar vectors 32. Vector
33. $(3,4)$ 34. $\underline{v}_f = \underline{v}_i + \underline{a}t$ 35. $\underline{x} = \underline{v}_x t$ 36. $\underline{y} = \underline{v}_y t - \frac{1}{2}gt^2$

Additional Short Question

Short Answer Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

6.1

Rectangular Coordinate Plane

1. Name the quadrant in which each point lies.

- (i) $(-4,2)$ (ii) $(-3,-6)$ (iii) $(5,-3)$ (iv) $(1,2)$

Sol:

- (i) Quad-II (ii) Quad-III (iii) Quad-IV (iv) Quad-I

2. Write the vector $\underline{PQ}$ in the form of $x\underline{i} + y\underline{j}$, where $P(-3, -6)$ and $Q(1,2)$

Sol: $\underline{PQ} = (1 - (-3))\underline{i} + (2 - (-6))\underline{j}$

$\underline{PQ} = 4\underline{i} + 8\underline{j}$ Ans

3. If $\underline{u} = \frac{1}{4}\underline{i} + \frac{15}{4}\underline{j}$ then find $|\underline{u}|$.

Sol: $|\underline{u}| = \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{15}{4}\right)^2}$

$$= \sqrt{\frac{1}{16} + \frac{225}{16}} = \sqrt{\frac{1+225}{16}} = \frac{\sqrt{226}}{16}$$

$$|\underline{u}| = \frac{\sqrt{226}}{4}$$

4. Find unit vector of $\underline{v} = -3\underline{i} + 4\underline{j}$

Sol: $\underline{v} = -3\underline{i} + 4\underline{j}$

OR

$$\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|}$$

$$|\underline{v}| = \sqrt{(-3)^2 + 4^2}$$

$$= \sqrt{9+16}$$

$$= \sqrt{25}$$

$$|\underline{v}| = 5$$

$$\hat{\underline{v}} = \frac{|\underline{v}|}{\underline{v}} = \frac{-3\underline{i} + 4\underline{j}}{5}$$

$$\hat{\underline{v}} = -\frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}$$

$$= \frac{3\underline{i} + 4\underline{j}}{\sqrt{(-3)^2 + 4^2}}$$

$$= \frac{-3\underline{i} + 4\underline{j}}{\sqrt{9+16}}$$

$$= \frac{-3\underline{i} + 4\underline{j}}{\sqrt{25}} = \frac{-3\underline{i} + 4\underline{j}}{5}$$

$$\hat{\underline{v}} = -\frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}$$

5. Check that the vectors $\underline{u} = 8\underline{i} + 4\underline{j}$ and $\underline{v} = 2\underline{i} + \underline{j}$ are parallel or not.

Sol: $\underline{u} = 8\underline{i} + 4\underline{j}$

$$= 4(2\underline{i} + \underline{j})$$

$$\underline{u} = 4\underline{v}$$

which shows that $\underline{u}$ is parallel to $\underline{v}$ with $\lambda = 4$.

6. If $\underline{u} = 5\underline{v}$, where $\underline{u} = 10\underline{i} - \lambda\underline{j}$ and $\underline{v} = -2\underline{i} + 7\underline{j}$, then find.

Sol: $\underline{u} = 5\underline{v}$

$$10\underline{i} - \lambda\underline{j} = 5(-2\underline{i} + 7\underline{j})$$

$$\therefore \underline{u} = 10\underline{i} - \lambda\underline{j}$$

$$10\underline{i} - \lambda\underline{j} = 10\underline{i} - 35\underline{j}$$

$$\underline{v} = -2\underline{i} + 7\underline{j}$$

Comparing coefficients of $\underline{j}$

$$-\lambda = -35$$

$$\Rightarrow \boxed{\lambda = 35}$$

7. Find unit vector opposite in direction of $\underline{u} = 3\underline{i} - 2\underline{j}$.

Sol: $|\underline{u}| = \sqrt{3^2 + (-2)^2}$

$$= \sqrt{9+4}$$

$$|\underline{u}| = \sqrt{13}$$

$$\therefore \hat{\underline{u}} = \frac{\underline{u}}{|\underline{u}|}$$

Now a unit vector opposite in direction to $\underline{u}$ is:

Required unit vector $= -\frac{\underline{u}}{|\underline{u}|}$

$$= -\frac{3\underline{i} - 2\underline{j}}{\sqrt{13}}$$

$$= -\frac{3}{\sqrt{13}}\underline{i} + \frac{2}{\sqrt{13}}\underline{j}$$

Ans.

6.3

Addition of vectors

8. If $\underline{u} = 5\underline{i} - 4\underline{j}$ and $\underline{v} = 6\underline{i} - 3\underline{j}$, then find $\underline{u} + \underline{v}$.

Sol: $\underline{u} + \underline{v} = (5\underline{i} - 4\underline{j}) + (6\underline{i} - 3\underline{j})$
 $= (5+6)\underline{i} - (4+3)\underline{j}$

$$\underline{u} + \underline{v} = 11\underline{i} - 7\underline{j}$$

9. If $\underline{a} = 4\underline{i} - \underline{j}$ and $\underline{b} = -3\underline{i} + 5\underline{j}$, then find $3\underline{b} + 4\underline{a}$.

Sol: $3\underline{b} + 4\underline{a}$
 $= 3(-3\underline{i} + 5\underline{j}) + 4(4\underline{i} - \underline{j})$
 $= -9\underline{i} + 15\underline{j} + 16\underline{i} - 4\underline{j}$
 $= (-9+16)\underline{i} + (15-4)\underline{j}$
 $= 7\underline{i} + 11\underline{j}$ Ans.

10. If $\underline{a} = 4\underline{i} - 6\underline{j}$ and $\underline{b} = 9\underline{i} + 12\underline{j}$, then find $\frac{1}{2}\underline{a} + \frac{2}{3}\underline{b}$.

Sol: $\frac{1}{2}\underline{a} + \frac{2}{3}\underline{b} = \frac{1}{2}(4\underline{i} - 6\underline{j}) + \frac{2}{3}(9\underline{i} + 12\underline{j})$
 $= 2\underline{i} - 3\underline{j} + 2(3\underline{i} + 4\underline{j})$
 $= 2\underline{i} - 3\underline{j} + 6\underline{i} + 8\underline{j}$
 $= (2+6)\underline{i} - (3-8)\underline{j}$

$$\frac{1}{2}\underline{a} + \frac{2}{3}\underline{b} = 8\underline{i} + 5\underline{j} \quad \text{Ans.}$$

11. If $\underline{u} = \underline{i} - 2\underline{j}$, $\underline{w} = 3\underline{i} + 3\underline{j}$ and $\underline{u} + 4\underline{v} = \underline{w}$, then find $\underline{v}$.

Sol: $\underline{u} + 4\underline{v} = \underline{w}$
 $\underline{i} - 2\underline{j} + 4\underline{v} = 3\underline{i} + 3\underline{j}$
 $4\underline{v} = 3\underline{i} + 3\underline{j} - \underline{i} + 2\underline{j}$
 $4\underline{v} = (3-1)\underline{i} + (3+2)\underline{j}$
 $4\underline{v} = 2\underline{i} + 5\underline{j}$

$$\Rightarrow \underline{v} = \frac{2}{4}\underline{i} + \frac{5}{4}\underline{j} \Rightarrow \underline{v} = \frac{1}{2}\underline{i} + \frac{5}{4}\underline{j} \quad \text{Ans.}$$

6.4

Difference of two vectors

12. If $\underline{u} = \underline{i} + 5\underline{j}$ and $\underline{v} = 6\underline{i} - \underline{j}$, then find $\underline{v} - 2\underline{u}$.

Sol: $\underline{v} - 2\underline{u} = 6\underline{i} - \underline{j} - 2(\underline{i} + 5\underline{j})$

$$= 6\mathbf{i} - \mathbf{j} - 2\mathbf{i} - 10\mathbf{j}$$

$$= (6-2)\mathbf{i} - (1+10)\mathbf{j}$$

$$\mathbf{v} - 2\mathbf{u} = 4\mathbf{i} - 11\mathbf{j} \quad \text{Ans.}$$

13. If $\mathbf{a} = 7\mathbf{i} - 3\mathbf{j}$, $\mathbf{b} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{c} = 2\mathbf{i} - \mathbf{j}$, then find $2\mathbf{a} - 3\mathbf{b} - \mathbf{c}$.

$$\begin{aligned} \text{Sol: } 2\mathbf{a} - 3\mathbf{b} - \mathbf{c} &= 2(7\mathbf{i} - 3\mathbf{j}) - 3(3\mathbf{i} + 4\mathbf{j}) - (2\mathbf{i} - \mathbf{j}) \\ &= 14\mathbf{i} - 6\mathbf{j} - 9\mathbf{i} - 12\mathbf{j} - 2\mathbf{i} + \mathbf{j} \\ &= (14-9-2)\mathbf{i} - (6+12-1)\mathbf{j} \end{aligned}$$

$$2\mathbf{a} - 3\mathbf{b} - \mathbf{c} = 3\mathbf{j} - 7\mathbf{i} \quad \text{Ans.}$$

14. If $(x\mathbf{i} + y\mathbf{j}) - (2\mathbf{i} - 4\mathbf{j}) = -5\mathbf{i} + 3\mathbf{j}$, then find the values of x and y .

$$\text{Sol: } (x\mathbf{i} + y\mathbf{j}) - (2\mathbf{i} - 4\mathbf{j}) = -5\mathbf{i} + 3\mathbf{j}$$

$$(x-2)\mathbf{i} + (y+4)\mathbf{j} = -5\mathbf{i} + 3\mathbf{j}$$

Comparing coefficients of x and y .

$$\Rightarrow x-2 = -5, \quad y+4 = 3$$

$$x = -5+2, \quad y = 3-4$$

$$\boxed{x = -3}, \quad \boxed{y = -1}$$

6.6

Translation by a vector

15. Define a translation vector.

Ans: A translation vector is a vector that describes the movement of a point, a body, a system from one position to another without rotation or deformation.

16. Plot A (4,3), B(-1,-2), C(5,8) to form a ΔABC . Also translate ΔABC to $\Delta A'B'C'$ by the translation vector $3\mathbf{i} + 5\mathbf{j}$.

$$\begin{aligned} \text{Sol: New vertex } A' &= (4\mathbf{i} + 3\mathbf{j}) + (3\mathbf{i} + 5\mathbf{j}) \\ &= (4+3)\mathbf{i} + (3+5)\mathbf{j} \\ &= 7\mathbf{i} + 8\mathbf{j} = (7,8) \end{aligned}$$

$$\begin{aligned} \text{New vertex } B' &= (-\mathbf{i} - 2\mathbf{j}) + (3\mathbf{i} + 5\mathbf{j}) \\ &= (-1+3)\mathbf{i} - (2-5)\mathbf{j} \\ &= 2\mathbf{i} + 3\mathbf{j} = (2,3) \end{aligned}$$

$$\begin{aligned} \text{New vertex } C' &= (5\mathbf{i} + 8\mathbf{j}) + (3\mathbf{i} + 5\mathbf{j}) \\ &= (5+3)\mathbf{i} + (8+5)\mathbf{j} \\ &= 8\mathbf{i} + 13\mathbf{j} = (8,13) \end{aligned}$$

17. If A(7, -3), B(-2, 4) and C(5,4) are the vertices of a ΔABC , then find the new vertices by a translated vector $4\mathbf{i} - \mathbf{j}$.

$$\begin{aligned} \text{Sol: New vertex } A' &= (7\mathbf{i} - 3\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) & \text{New vertex } C' &= (5\mathbf{i} + 4\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) \\ &= (7+4)\mathbf{i} - (3+1)\mathbf{j} & &= (5+4)\mathbf{i} + (4-1)\mathbf{j} \\ &= 11\mathbf{i} - 4\mathbf{j} & &= 9\mathbf{i} + 3\mathbf{j} = (9,3) \end{aligned}$$

$$\begin{aligned} \text{New vertex } B' &= (-2\mathbf{i} + 4\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) \\ &= (-2+4)\mathbf{i} + (4-1)\mathbf{j} \\ &= 2\mathbf{i} + 3\mathbf{j} = (2,3) \end{aligned}$$

18. If A (3,1) , B(-2, -3) and C(2,2) are the vertices of a triangle. Check whether $\triangle ABC$ is an isosceles.

Sol: $\overrightarrow{AB} = (-2-3)\underline{i} + (-3-1)\underline{j} = -5\underline{i} - 4\underline{j}$
 $\overrightarrow{BC} = (2-(-2))\underline{i} + (2-(-3))\underline{j} = 4\underline{i} + 5\underline{j}$
 $\overrightarrow{AC} = (2-3)\underline{i} + (2-1)\underline{j} = -\underline{i} + \underline{j}$

Now

$$|\overrightarrow{AB}| = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$$

$$|\overrightarrow{BC}| = \sqrt{4^2 + 5^2} = \sqrt{16+25} = \sqrt{41}$$

Since $|\overrightarrow{AB}| = |\overrightarrow{BC}|$

So $\triangle ABC$ is an isosceles.

19. If $\overrightarrow{AB} = x\underline{i} + 2\underline{j}$ and $\overrightarrow{DC} = -3\underline{i} + 2\underline{j}$ are two opposite sides of a parallelogram, then find the value of x.

Sol: Since $\overrightarrow{AB} = x\underline{i} + 2\underline{j}$ and $\overrightarrow{DC} = -3\underline{i} + 2\underline{j}$ are opposite sides of a parallelogram, then

$$\overrightarrow{AB} = \overrightarrow{DC}$$

$$x\underline{i} + 2\underline{j} = -3\underline{i} + 2\underline{j}$$

$$\boxed{x = -3}$$

20. If $\overrightarrow{AB} = 4\underline{i} - 5\underline{j}$ and $\overrightarrow{DC} = 4\underline{i} - y\underline{j}$ are two Parallel and opposite sides a parallelogram, then what is the value of y?

Sol: For a parallelogram

$$\overrightarrow{AB} = \overrightarrow{DC}$$

$$4\underline{i} - 5\underline{j} = 4\underline{i} - y\underline{j}$$

$$-5 = -y$$

$$\boxed{y = 5}$$

21. If A,B and C are respectively the points (2, -4), (4,0) and (1,6). Use vectors to find coordinates of point D if ABCD is a parallelogram.

Sol: Let D(x,y) be the required vertex. Since ABCD is a parallelogram.

So $\overrightarrow{AB} = \overrightarrow{DC}$

$$(4-2)\underline{i} + (0-(-4))\underline{j} = (1-x)\underline{i} + (6-y)\underline{j}$$

$$2\underline{i} + 4\underline{j} = (1-x)\underline{i} + (6-y)\underline{j}$$

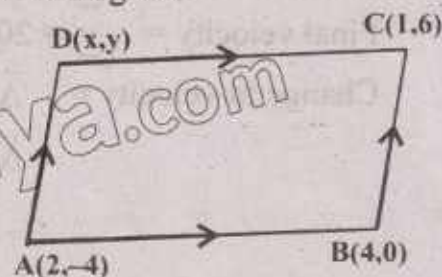
$$1-x = 2, \quad 6-y = 4$$

$$-x = 2-1, \quad -6y = 4-6$$

$$-x = 1, \quad -y = -2$$

$$\boxed{x = -1}, \quad \boxed{y = 2}$$

So required vertex is D (-1, 2)

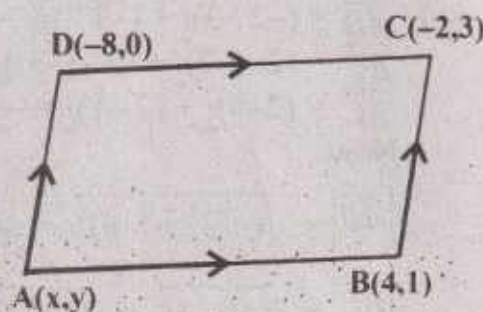


22. If $B(4,1)$, $C(-2,3)$ and $D(-8,0)$ are the vertices of a parallelogram ABCD, then find coordinates of the vertex A.

Sol: Let $A(x,y)$ be the required vertex.

Since ABCD is a parallelogram.

$$\begin{aligned}\text{So } \overline{AB} &= \overline{DC} \\ (4-x)\underline{i} + (1-y)\underline{j} &= (-2-(-8))\underline{i} + (3-0)\underline{j} \\ (4-x)\underline{i} + (1-y)\underline{j} &= 6\underline{i} + 3\underline{j} \\ 4-x &= 6 & 1-y &= 3 \\ -x &= 6-4 & -y &= 3-1 \\ -x &= 2 & -y &= 2 \\ \boxed{x = -2} & & \boxed{y = -2}\end{aligned}$$



So required vertex is $A(-2, -2)$

23. A projectile is launched from ground with initial velocity vector $3\underline{i} + 5\underline{j}$. After how long does it hit a target located 15 meters away horizontally?

Sol: Given that

$$\underline{v}_i = 3\underline{i} + 5\underline{j}$$

$$\text{Horizontal Velocity} = v_x = 3$$

$$\text{Vertical Velocity} = v_y = 5$$

$$\text{Distance} = x = 15\text{m}$$

$$\therefore x = v_x t$$

$$t = \frac{x}{v_x}$$

$$t = \frac{15}{3}$$

$$\boxed{t = 5 \text{ sec}}$$

24. A vehicle enters a loop with velocity vector $\underline{v}_i = 17\underline{i}$ and exits with velocity vector $\underline{v}_f = 20\underline{j}$. Find the magnitude of change in velocity.

Sol: Given that

$$\text{Initial velocity} = \underline{v}_i = 17\underline{i}$$

$$\text{Final velocity} = \underline{v}_f = 20\underline{j}$$

$$\therefore \text{Change in velocity} = \underline{\Delta v} = \underline{v}_f - \underline{v}_i = 20\underline{j} - 17\underline{i}$$

$$\underline{\Delta v} = 20\underline{j} - 17\underline{i}$$

$$= \sqrt{20^2 + (-17)^2}$$

$$= \sqrt{400 + 289}$$

$$\boxed{|\underline{\Delta v}| = \sqrt{689}} \quad \text{Ans.}$$