UNIT 12 Probability

Students' Learning Qutiones.

After completing this unit, the students will be able to:

- Calculate the probability of combined events using, where appropriate: space diagrams, possibility diagram, tree diagrams and Venn diagrams.
- Apply addition law of probability to solve problems involving mutually exclusive events (such as left and right-hand turns, tossing a coin, even and odd numbers on a dice, winning and losing a game)
- Apply the Multiplication law of probability to solve problems involving independent and dependent events (trading, flipping a coin, such as 2 cards being drawn one by one with replacement and without replacement etc.)

12.1 Probability

Probability helps us understand how likely something is to happen. Simply, probability is useful whenever we deal with uncertainty.

> **Do you know?**
> A statistical experiment is a type of random experiment where the outcomes are numerically measured and analyzed using statistical methods.
> All statistical experiments are random experiments, but all random experiments are not statistical experiments.

Random Experiment

A random experiment is an experiment in which:

(i) The set of all possible outcomes are known.

(ii) Exact outcome is not known.

For example, tossing a coin, rolling a dice.

Outcomes and Sample Space

The possible result of a random experiment is known as **outcome**.

The set of all possible outcomes in a random experiment is called a **sample space.** It is generally denoted by S.

For example, when we roll a dice, the possible outcomes are the face numbers 1, 2, 3, 4, 5, 6 of the dice. Therefore the sample space is $S = \{1, 2, 3, 4, 5, 6\}$.

Each element of a sample space is called a **sample point.**

Event

In a random experiment, each possible outcome is called an event. Thus, an event will be a subset of the sample space.

For example, getting two heads is an event when we toss two coins.

> **Remember!**
> Performing an experiment once is called a **trial.**
> When we toss a coin thrice, then each toss of a coin is a trial.

Events	Explanation	Example
Equally likely events	Two or more events are said to be equally likely if each one of them has an equal chance of occurring.	Numbers 1, 2, 3, 4, 5, 6 are equally likely events, when we roll a dice.
Certain events	In an experiment, the event which surely occurs is called certain event.	When we roll a dice, the event of getting any natural number from 1 to 6 is a certain event.
Impossible events	An impossible event is an event that cannot happen under any circumstances in a given situation.	When we toss two coins, the event of getting three heads is an impossible event.
Mutually exclusive events	Two or more events are said to be mutually exclusive if they don't have common sample points. i.e., events A and B are said to be mutually exclusive if $A \cap B = \phi$	When we roll a dice the events of getting odd numbers and even numbers are mutually exclusive events.
Exhaustive events	The collection of events whose union is the whole sample space is called exhaustive events.	When we toss a coin twice, the collection of events of getting two heads, exactly one head and no head are exhaustive events.
Complementary events	The complement of an event A is the event representing a collection of sample points not in A. It is denoted A' or A^c or $\overline{A}$. The event A and its complement A' are mutually exclusive and exhaustive.	When we toss a coin, the event 'Head' and the event 'Tail' are complementary events.

Combined Events

A combined event, (or compound event) is an event that includes two or more simple events, happening together. Remember simple event is an event that has a single point of the sample space.

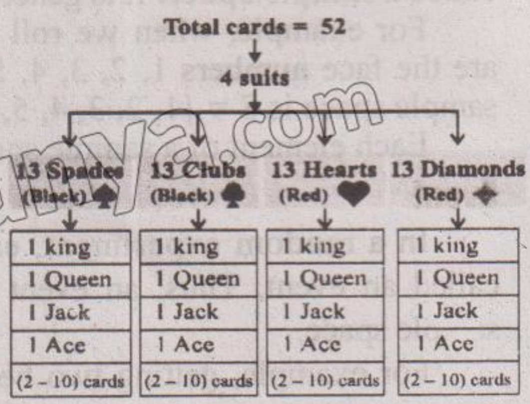
For example, flipping a coin twice and getting at least one head is a combined event.

Types of Combined Events

a Independent Events:

Two or more events are said to be independent, if the outcome of one does not affect the outcome of the other.

For example, flipping a coin multiple times, each flip is independent of the others.



b Dependent Events

Two or more events are said to be dependent, if the outcome of one does affect the outcome of the other.

For example, drawing cards from a deck without replacement; the outcome of the first draw changes the probabilities for subsequent draws.

e. Mutually Exclusive Events

Two or more events are called mutually exclusive events if they cannot happen at the same time, If one event occurs, the other cannot.

For example, when we roll a single dice, getting a 3 and 5 at the same time is impossible.

d. Non-Mutually Exclusive EventsL

Two or more events are called non-mutually exclusive events if they can happen at the same time. These events may overlap.

For example, drawing a card that is red or a king from a deck

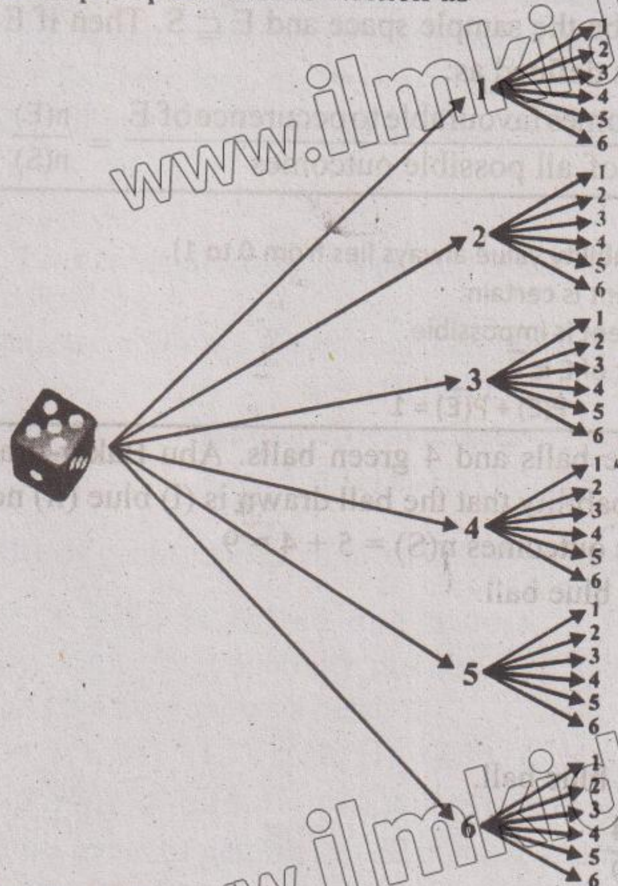
Tree Diagram

A tree diagram is a visual representation used in probability to show all possible outcomes of one or more events. Each branch in a tree diagram represents a possible outcome.

Example 1: Express the sample space for rolling two dice using tree diagram.

Solution: When we roll two dice, since each dice contains 6 faces marked with 1, 2, 3, 4, 5, 6, the tree diagram will look like.

Hence, the sample space can be written as



$$S = \begin{Bmatrix} (1,1), (1,2), (1,3) \\ (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3) \\ (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3) \\ (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3) \\ (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3) \\ (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3) \\ (6,4), (6,5), (6,6) \end{Bmatrix}$$

Sample Space Diagram

Sample space diagram is a representation that shows all possible outcomes of an experiment in list or table form.

When we toss two coins, then the sample space can be written as

$$S = \{HH, HT, TH, TT\}$$

Possibility Diagram

Possibility diagram is a representation in grid or table that shows all possible combinations of two events.

When we toss two coins, then the possibility diagram will look like

Coin 2 \ Coin 1	*H*	*T*
H	*HH*	*HT*
T	*TH*	*TT*

Example 2: Hashim rolled a dice and tossed a coin simultaneously. Draw a possibility diagram.

Solution:

Coin \ Dice	1	2	3	4	5	6
H	$(H,1)$	$(H,2)$	$(H,3)$	$(H,4)$	$(H,5)$	$(H,6)$
T	$(T,1)$	$(T,2)$	$(T,3)$	$(T,4)$	$(T,5)$	$(T,6)$

Probability of an Event

In a random experiment, let S be the sample space and $E \subseteq S$. Then if E is an event, the probability of occurrence of E is defined as:

$$P(E) = \frac{\text{Number of outcomes favourable to occurence of E}}{\text{Number of all possible outcomes}} = \frac{n(E)}{n(S)}$$

Remember!
- $0 \leq P(E) \leq 1$ (The probability value always lies from 0 to 1).
- If $P(E) = 1$, then the event is certain.
- If $P(E) = 0$, then the event is impossible.
- The complement event of E is $\overline{E}$.

$$P(E) + P(\overline{E}) = 1$$

Example 3: A bag contains 5 blue balls and 4 green balls. Abu Bakar draws a ball at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Solution: Total number of possible outcomes $n(S) = 5 + 4 = 9$

(i) Let A be the event of getting a blue ball.

$n(A) = 5$

Therefore, $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{5}{9}$

(ii) $\overline{A}$ be the event of not getting a blue ball.

So $P(\overline{A}) = 1 - P(A) = 1 - \dfrac{5}{9} = \dfrac{4}{9}$

Example 4: Two dice are rolled. Find the probability that the sum of outcomes is **(i)** equal to 4 **(ii)** greater than 10 **(iii)** less than 13.

Solution: When we roll two dice, the sample space is given by:

$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$
$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$
$(3, 1), (3, 2), (3, 3), (3, 4), (4, 5), (5, 6)$
$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$
$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$
$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$; $n(S) = 36$

(i) Let A be the event of getting the sum of outcomes equal to 4. Then $A = \{(1,3), (2,2), (3,1)\}$; $n(A) = 3$

Probability of getting the sum of outcomes equal to 4 is

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

Challenge!
When is the complement event of an impossible event?
Sol: Let A be the event of impossible $n(A) = 0$, $P(A) = 0$
We know
$P(A) + P'(A) = 1$
$0 + P'(A) = 1$
$P'(A) = 1$

(ii) Let B be the event of getting the sum of outcomes greater than 10.

The $B = \{(5,6), (6,5), (6,6)\}$; $n(B) = 3$

Probability of getting the sum of outcomes greater than 10 is

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

(iii) Let C be the event of getting the sum outcomes less than 13.

Here all the 36 outcomes have the sam value less than 13.

Hence C = S. There fore, $n(c) = n(s) = 36$

Probability of getting the total value less than 13 is

$$P(C) = \frac{n(c)}{n(s)} = \frac{36}{36} = 1$$

Example 5: Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution: When two coins are tossed together, the sample space is

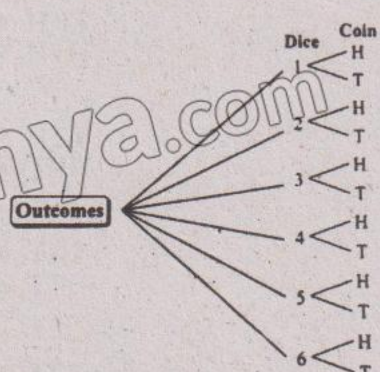
$$S = \{HH, HT, TH, TT\}$$

Let A be the event of getting different faces on the coins.

$$A = \{HT, TH\} \qquad n(A) = 2$$

Probability of getting different faces on the coins is $P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

Example 6: A dice is rolled and a coin is tossed simultaneously. Find the probability that the dice shows an odd number and the coin shows a head.



Solution: $S = \{(1, H), (1, T), (2, H), (2,T), (3,H), (3,T), (4,H), (4,T), (5, H), (5, T), (6, H), (6,T)\}$; $n(S) = 12$

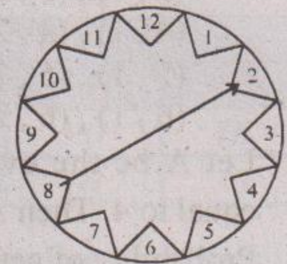
Let A be the event of getting an odd number and a head.

$$A = \{(1, H), (3,H), (5,H)\}; n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

Example 7: A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1, 2, 3, ..., 12. What is the probability that it will point to:

(i) 7 (ii) a prime number (iii) a composite number?



Solution: Sample space S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12};
n(S) = 12

(i) Let A be the event of resting in 7. n(A) = 1

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{12} = \frac{1}{2}$$

(ii) Let *B* be the event that the arrow will come to rest in a prime number.

B = {2, 3, 5, 7, 11} ; *n(B)* = 5

$$P(B) = P(B) = \frac{n(B)}{n(S)} = \frac{5}{12}$$

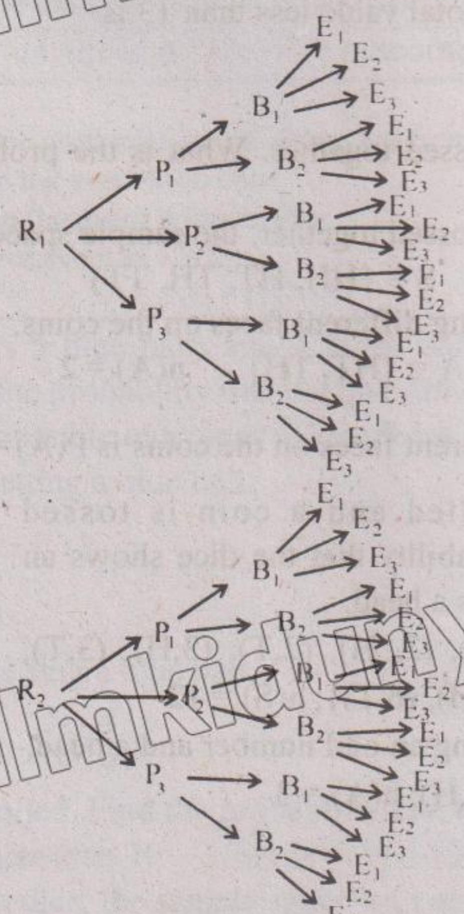
(iii) Let *C* be the event that arrow will come to rest in a composite number.

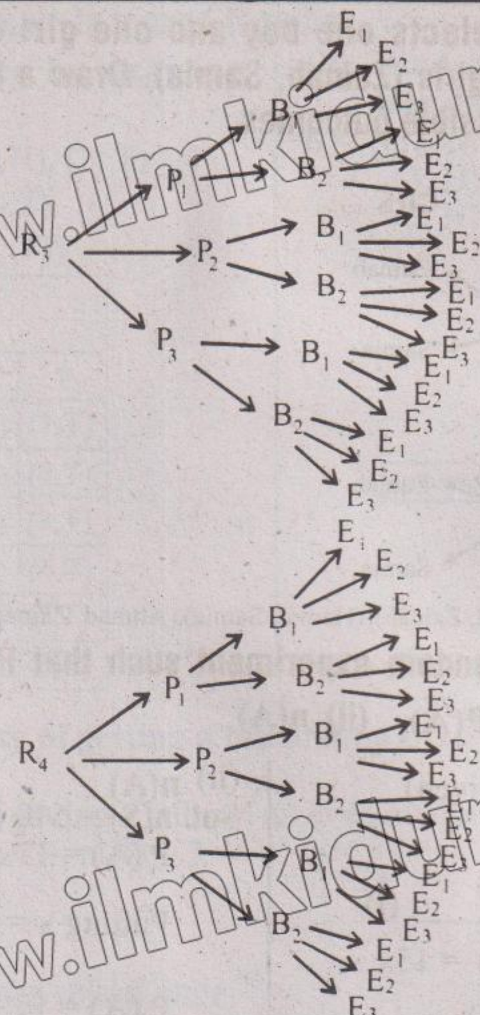
C = {4, 6, 8, 9, 10, 12} ; n(C) = 6

$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{12} = \frac{1}{2}$$

Activity!
There are four routes R_1, R_2, R_3, and R_4 from Huria's home to her place of work. There are three parking lots P_1, P_2, P_3, and two entrances B_1, B_2 into the office building. There are three elevators E_1, E_2, E_3, to her floor. Using the tree diagram explain how many ways can she reach her office.

Solution:



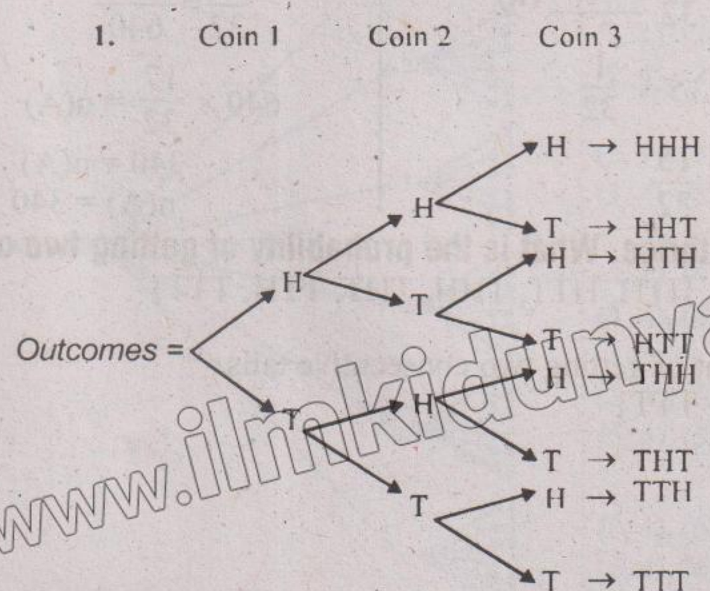


So total ways = 72

Solved Exercise 12.1

1. Find the sample space for tossing three coins using tree diagram.

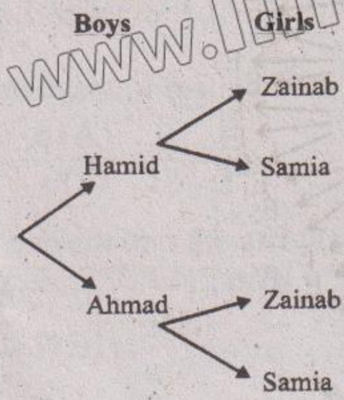
Sol.



S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

2. A teacher randomly selects one boy and one girl from a group of 2 boys (Hamid, Ahmad) and 2 girls (Zainab, Samia). Draw a tree diagram and list the sample space for all possible outcomes.

Sol.



Sample Space = {(Hamid, Zainab), (Hamid, Samia) (Ahmad, Zainab), (Ahmad, Samia)}

3. If A is an event of a random experiment such that $P(A) : P(\overline{A}) = 17 : 15$ and $n(S) = 640$ then find (i) $P(\overline{A})$ (ii) $n(A)$

Sol. $P(A) : P(\overline{A}) = 17 : 15$ (Given)

We know

$P(A) + P(\overline{A}) = 1$ ______ (i)

Let $P(A) = 17x$ and $P(\overline{A}) = 15x$

Put these values in (i)

$17x + 15x = 1$

$32x = 1$

$x = \frac{1}{32}$ ______ (ii)

$P(\overline{A}) = 15 \times \frac{1}{32}$

$= \frac{15}{32}$

(ii) n(A)

Sol. $n(S) = 640$ (given)

$P(A) = 17x$

Putting $x = \frac{1}{32}$ from (ii)

$P(A) = 17 \times \frac{1}{32} = \frac{17}{32}$

we know

$P(A) = \frac{n(A)}{n(S)}$

$\frac{17}{32} = \frac{n(A)}{640}$

$640 \times \frac{17}{32} = n(A)$

$340 = n(A)$

$n(A) = 340$

4. A coin is tossed thrice. What is the probability of getting two consecutive tails?

Sol. S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

$n(S) = 8$

Let A be the event of getting two consecutive tails.

A = {HTT, TTH, TTT}

$n(A) = 3$

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{3}{8}$

5. A dice is rolled and coin is tossed together.

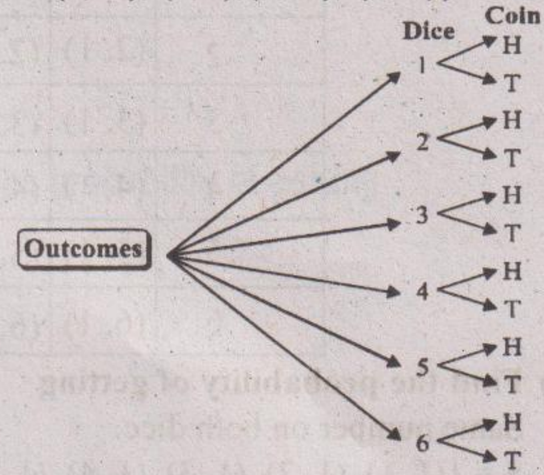
(i) Find sample space by drawing possibility diagram.

Sol. S = {(1, H),(1,T), (2,H), (2, T)
(3,H), (3,T), (4,H), (4,T)
(5,H), (5,T), (6,H), (6,T)}

Dice \ Coin	H	T
1	(1,H)	(1,T)
2	(2,H)	(2,T)
3	(3,H)	(3,T)
4	(4,H)	(4,T)
5	(5,H)	(5,T)
6	(6,H)	(6,T)

(ii) Find sample space by sketching tree diagram.

S = {(1, H), (1,T), (2, H), (2, T)
(3, H), (3,T), (4, H), (4, T)
(5, H), (5,T),(6, H), (6, T)}



(iii) What is a probability of getting a tail and an even number?

Sol. n(S) = 12

Let B be the event of getting a tail and an even number.

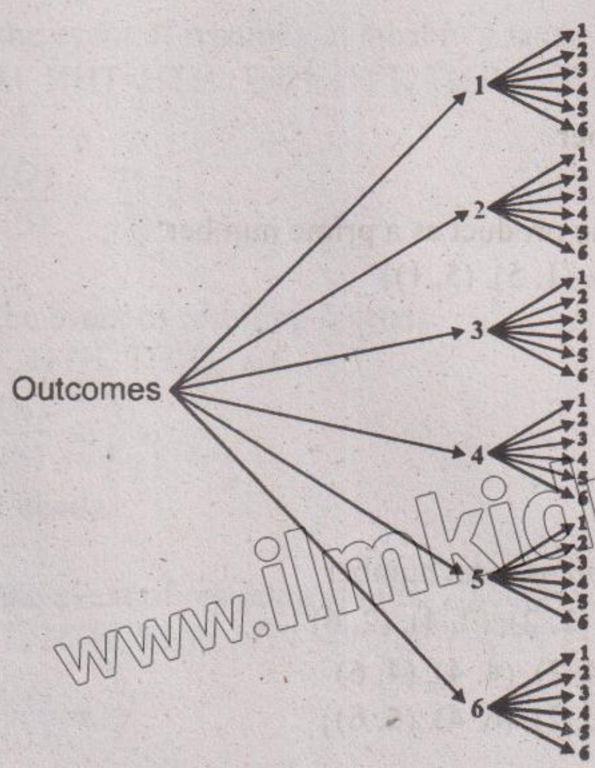
B = {(T,2), (T,4), (T, 6)}, n(B) = 3

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

6. Two unbiased dice are rolled once.

(i) Find sample space by sketching tree diagram.

Sol.



S = {(1, 1), (1, 2), (1, 3)
(1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3)
(2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3)
(3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3)
(4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3)
(5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3)
(6, 4), (6, 5), (6, 6)}

(ii) Find sample space by drawing possibility diagram.

Sol.

Dice2 \ Dices	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

(iii) Find the probability of getting

(a) Same number on both dice.

Sol. S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Let A be the event of getting same number on both dice.

A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

n (A) = 6

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(b) the product as a prime number.

Sol. n(S) = 36

Let B be the event of getting the product as a prime number

B = {(1, 2), (2, 1), (1, 3), (3, 1), (1, 5), (5, 1)}

n (B) = 6

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(c) the sum as an even number.

Sol. n(S) = 36

Let C be the event of getting sum as even number

C = {(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6)
(3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6)
(5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)}

n(C) = 18

$P(C) = \frac{n(C)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

(d) the sum as 13.

Sol. Let D the event of getting the sum as 13 the maximum sum is $6 + 6 = 12$

So, sum 13 is impossible

$n(D) = 0$

$P(D) = \frac{n(D)}{n(S)} = \frac{0}{36} = 0$

7. Three fair coins are tossed together. Find the probability of getting

(i) all tails

Sol. {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Let A be the event of resulting all tails

A = {TTT}, $n(A) = 1$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$

(ii) at least one head

Sol. $n(S) = 8$

Let B be the event of resulting at least one head.

B = {HHH, HHT, HTH, THH, HTT, THT, TTH}, $n(B) = 7$

$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$

(iii) at most two tails.

Sol. $n(S) = 8$

Let D be the event of resulting at most two tails.

D = {HHH, HHT, HTH, THH, HTT, THT, TTH]

$n(D) = 7$

$p(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$

(iv) 2 heads.

Sol. Let E be the event of resulting 2 heads.

E = [HHT, HTH, THH}, $n(E) = 3$

$P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$

(v) at most 2 heads.

Sol. $n(S) = 8$

Let G be the event of resulting at most 2 heads

G = {HHT, HTH, THH, HTT, THT, TTH, TTT}, $n(G) = 7$

$P(G) = \frac{n(G)}{n(S)} = \frac{7}{8}$

(viii) no head.

Sol. $n(S) = 8$

Let H be the event of resulting no head

$H = \{TTT\}$, $n(H) = 1$

$$P(H) = \frac{n(H)}{n(S)} = \frac{1}{8}$$

8. A bag contains 4 red balls, 5 white balls, 6 green balls and 3 black balls. Ali draws a ball at random from the bag. Find the probability that the ball drawn is:

Total number of balls = 4+5+6+3 = 18

$n(S) = 18$

(i) white

Sol. Let W be the event of resulting a white ball

$n(W) = 5$

$$p(W) = \frac{n(W)}{n(S)} = \frac{5}{18}$$

(ii) red

Sol. Let B the event of resulting red ball.

$n(B) = 4$, $n(S) = 18$ (Total number of balls)

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{18} = \frac{\cancel{4}^{2}}{\cancel{18}_{9}} = \frac{2}{9}$$

(iii) not white

Sol. $n(S) = 18$ (Total number of balls)

Let W be the event of resulting a white ball

$n(W) = 5$

$$P(W) = \frac{n(W)}{n(S)} = \frac{5}{18} = \frac{5}{18}$$

Let $\overline{W}$ be the event of resulting not white ball

$$P(\overline{W}) = 1 - P(W)$$

$$= 1 - \frac{5}{18}$$

$$= \frac{18-5}{18} = \frac{13}{18}$$

(iv) not black

Sol. $n(S) = 18$ (Total number of event)

Let C be the event of resulting black ball.

$n(C) = 3$

$$P(C) = \frac{n(C)}{n(S)} = \frac{\cancel{3}^{1}}{\cancel{18}_{6}} = \frac{1}{6}$$

let C' be the erent resulting not Black ball

$$P(C')=1-P(C)$$

$$=1-\frac{1}{6}$$

$$=\frac{6-1}{6}=\frac{5}{6}$$

9. A number is selected at random from the set of whole numbers 1 to 15, both inclusive. Find the probability that the number selected is:

Sol. $S=\{1,2,3,4,\ldots,15\}$

$n(S)=15$

(i) odd

Sol. $n(S)=15$

Let A be the event of resulting an odd number.

$A=\{1,3,5,7,9,11,13,15\}$, $n(A)=8$

$$P(A)=\frac{n(A)}{n(S)}=\frac{8}{15}$$

(ii) a multiple of 5

Sol. $n(S)=15$

Let B be the event of resulting a multiple of 5.

$B=\{5,10,15\}$, $n(B)=3$

$$P(B)=\frac{n(B)}{n(S)}=\frac{3}{15}=\frac{1}{5}$$

(iii) the square of 2

Sol. $n(S)=15$

Let C be the event of resulting the square of 2

$C=\{4\}$, $n(C)=1$

$$P(C)=\frac{n(C)}{n(S)}=\frac{1}{15}$$

(iv) prime

Sol. $n(S)=15$

Let D be the event of resulting a prime number.

$D=\{2,3,5,7,11,13\}$, $n(D)=6$

$$P(D)=\frac{n(D)}{n(S)}=\frac{6}{15}=\frac{2}{5}$$

(v) 20

Sol. $n(S)=15$

Let N be event of getting number 20.

Since 20 is not included in the set from 1 to 15.

$N = \{\ \}, n(N) = 0$

$$P(N) = \frac{n(N)}{n(S)} = \frac{0}{15} = 0$$

10. If the probability of an event A is $\frac{7}{10}$, then find the probability of the event "not A".

Sol. $P(A) = \frac{7}{10}$ (Given)

$$P(\overline{A}) = 1 - P(A)$$
$$= 1 - \frac{7}{10}$$
$$= \frac{10-7}{10} = \frac{3}{10}$$

11. A dice is rolled twice. Find the probability of having a number greater than 4 on each roll.

Sol. $S = \{1, 2, 3, 4, 5, 6\}$ n, $n(S) = 6$

Number of greater than 4 on a single dice are $\{5, 6\}$, $n = 2$

$$P(\text{First roll} > 4) = \frac{2}{6} = \frac{1}{3}$$

$$P(\text{second roll} > 4) = \frac{2}{6} = \frac{1}{3}$$

$$P(\text{both roll} > 4) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

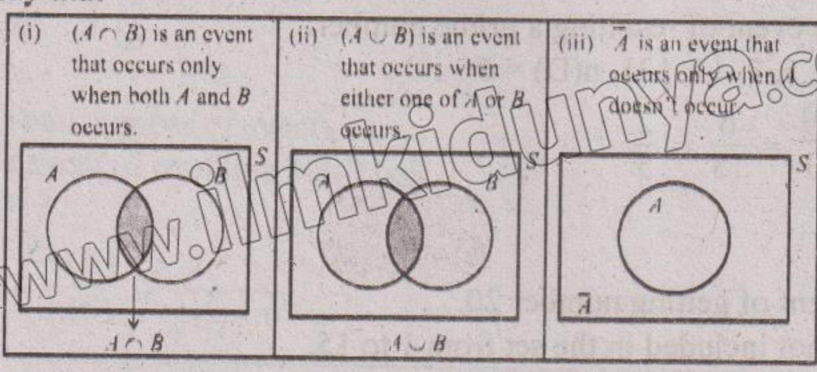
12.2 Addition Law of Probability

(i) If A and B are any two non mutually exclusive events then
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(ii) If A and B are any two mutually exclusive events then
$P(A \cup B) = P(A) + P(B)$

Venn Diagram

In a random experiment, let S be the sample space. Let $A \subseteq S$ and $B \subseteq S$ be the events in S. we say that



Example 8: If $P(A) = 0.37$, $P(B) = 0.42$, $P(A \cap B) = 0.09$, then find $P(A \cup B)$.

Solution: Given that: $P(A) = 0.37$, $P(B) = 0.42$, $P(A \cap B) = 0.09$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B) = 0.37 + 0.42 - 0.09 = 0.7$$

Example 9: A flower is selected at random from a basket containing 50 yellow, 70 red and 80 white flowers. Find the probability of selecting a yellow or red flower?

Challenge!

What is addition of $P(A \cup B)$ and $P(A \cap B)$?

Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Add $P(A \cap B)$ on both sides

$P(A \cup B) + P(A \cap B) = P(A) + P(B) - P(A \cap B) + P(A \cap B)$

$P(A \cup B) + P(A \cap B) = P(A) + P(B)$

Solution: Total number of flowers, $n(S) = 50 + 70 + 80 = 200$

No. of yellow flowers, $n(Y) = 50$

$$\therefore P(Y) = \frac{n(Y)}{n(S)} = \frac{50}{200}$$

No. of red flowers, $n(R) = 70$

$$\therefore P(R) = \frac{n(R)}{n(S)} = \frac{70}{200}$$

Y and R are mutually exclusive events, so probability of drawing either a yellow or red flower is

$$P(Y \cup R) = P(Y) + P(R)$$
$$\therefore P(Y \cup R) = \frac{50}{200} + \frac{70}{200} = \frac{120}{200} = \frac{3}{5}$$

Example 10: Two dice are rolled together. Find the probability of getting a same number on both dice or sum of faces as 10.

Solution: Let S be the sample space, then $n(S) = 36$.

Let A be the event of getting a same number and B be the event of getting face sum 10.

Then $A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$B = \{(4, 6), (5, 5), (6, 4)\}$

$\therefore A \cap B = \{(5, 5)\}$

$n(A) = 6$, $n(B) = 3$, $n(A \cap B) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$
$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

$\therefore$ P(getting a same number or a total of 10) = $P(A\cup B)$

$$= P(A) + P(B) - P(A\cap B)$$

$$= \frac{6}{36}+\frac{3}{36}-\frac{1}{36}=\frac{8}{36}=\frac{2}{9}$$

Hence, the required probability is $\frac{2}{9}$.

Solved Exercise 12.2

1. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A\cap B) = \frac{1}{8}$, find $P(A\cup B)$.

Sol. Given that: $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(A\cap B) = \frac{1}{8}$

(i) P(A or B)

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(A\cup B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$$

$$P(A\cup B) = \frac{2+4-1}{8} = \frac{5}{8}$$

2. In an apartment, in selecting a house from door numbers 1 to 50 randomly, find the probability of getting the door number of the house to be an even number or a perfect square number.

Sol. $S = \{1, 2, 3, 4, \ldots\ldots, 50\}$, $n(S) = 50$

Let A be the event of resulting an even number and B be the event of getting a perfect square

$$A = \{2, 4, 6, \ldots., 50\},\ n(A) = 25$$

$$B = \{1, 4, 9, 16, 25, 36, 49\},\ n(B) = 7$$

$$A\cap B = \{4, 16, 36\},\ n(A\cap B) = 3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{25}{50} = \frac{1}{2}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{50}$$

$$P(A\cap B) = \frac{n(A\cap B)}{n(S)} = \frac{3}{50}$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{1}{2} + \frac{7}{50} - \frac{3}{50}$$

$$= \frac{25+7-3}{50} = \frac{29}{50}$$

3. The probability of a team winning any match is $\frac{3}{10}$ and the probability of losing any match is $\frac{2}{10}$. What is the probability that

(i) The team wins or loses a particular match.

(ii) The team neither wins nor loses a match.

Sol. Let W be the event of winning and L be the event of losing.

Given that: $P(W) = \frac{3}{10}$, $P(L) = \frac{2}{10}$

(i) the team wins or loses a particular match.

Sol. Winning and losing are mutully exclusive events.

$$P(W \cup L) = P(W) + P(L)$$
$$= \frac{3}{10} + \frac{2}{10} = \frac{5}{10} = \frac{1}{2}$$

(ii) the team neither wins nor loses a match.

P(the can neither wins or losses) = $P(\overline{W \cup L})$

Sol.
$$P(\overline{W \cup L}) = 1 - P(W \cup L)$$
$$= 1 - \frac{1}{2}$$
$$= \frac{2-1}{2} = \frac{1}{2}$$

4. In a single throw of two dice, find the probability of having sum of 7 or 11.

Sol. Total out comes $n(S) = 6 \times 6 = 36$

Let A be the event of having sum of 7 and let B to the event of having sum of 11.

$A = \{(1,6), (2, 5), (3, 4), (4, 3), (5, 2), (6,1)\}$, $n(A) = 6$

$B = \{(5, 6), (6, 5)\}$, $n(B) = 2$

$A \cap B = \phi$

So, A and B are mutully exclusive events

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

$$P(A \cup B) = P(A) + P(B)$$
$$= \frac{1}{6} + \frac{1}{18}$$
$$= \frac{3+1}{18} = \frac{4}{18} = \frac{2}{9}$$

5. Find the probability of getting a sum of 5 or 7 in a throw of two dice.

Sol. Total out comes n(S) = 6 × 6 = 36

Let A be the event of getting a sum of 5 and B be the event of getting a sum of 7.

$$A = \{(1,4), (2,3), (3,2), (4,1)\}, n(A) = 4$$

$$B = \{(1,6), (2,5), (3,4), (4,3), (5,2), 16,1)\}, n(B) = 6$$

$$A \cap B = \phi$$

So, A and B are mutully exclusive events

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{1}{9} + \frac{1}{6}$$

$$= \frac{2+3}{18} = \frac{5}{18}$$

6. A card is taken out at random from a standard pack of 52 cards. Find the probability of taking out.

(i) A king or a Jack.

Sol. Total cards = 52, n(s) = 52

Number of kings = 4, Number of Jacks = 4

Let A be the event of taking out a king and B be the event of taking out a Jack.

$$n(A) = 4, n(B) = 4$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$$

(ii) Neither a king nor a Jack.

Sol. P(neither a king nor a Jack) =(P $(\overline{A \cup B})$

$$P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$= 1 - \frac{2}{13}$$

$$= \frac{13-2}{13} = \frac{11}{13}$$

7. A dice is thrown twice. What is the probability that at least one of the two throws comes up with number 3.

Sol. Total outcomes $n(S) = 6 \times 6 = 36$

Let C be the event of getting at least one of the two throws comes up with number 3.

$C = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (1, 3), (2, 3) (4, 3), (5, 3), (6, 3)\}$

$n(C) = 11$

$$p(C) = \frac{n(C)}{n(S)} = \frac{11}{36}$$

8. There are 15 cards in a bag marked as 1, 2, 3, ..., 15. Find the probability of picking a card at random, the number written on which is a multiple of 5 or of 7.

Sol. $S = \{1, 2, 3, 4, ..., 15\}$, $n(S) = 15$

Let A be the event picking a card which is multiple of 5 and B be the event picking a card which is multiple of 7.

$A = \{5, 10, 15\}$, $n(A) = 3$

$B = \{7, 14\}$, $n(B) = 2$

$A \cap B = \phi$

So, A and B are mutually exclusive events

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{15} = \frac{1}{5}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{15}$$

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{1}{5} + \frac{2}{15}$$

$$= \frac{3+2}{15} = \frac{5}{15} = \frac{1}{3}$$

9. Two fair coins are tossed once. What is the probability of getting at least one head or two heads.

Sol. $S = \{HH, HT, TH, TT\}$, $n(S) = 4$

Let A be the event of getting at least one head and B be the event of getting at least two heads.

$A = \{HH, HT, TH\} = n(A) = 3$

$B = \{HH\}$, $n(B) = 1$

$A \cap B = \{HH\}$, $n(A \cap B) = 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{4}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{4}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{4} + \frac{1}{4} - \frac{1}{4}$$

$$= \frac{3+1-1}{4} = \frac{3}{4}$$

10. At a busy intersection, 50% of vehicles turn right, 30% turn left and 20% go straight. What is the probability that a randomly selected vehicle turn left or right?

Sol. Let R be event of turning right, L be the event of turning left and T be the event of going straight.

$$P(R) = 50\% = \frac{50}{100} = 0.50$$

$$P(L) = 30\% = \frac{30}{100} = 0.30$$

$$P(T) = 20\% = \frac{20}{100} = 0.20$$

Since turning left or right are mutully enclusine.

$$P(R \cup L) = P(R) + P(L)$$

$$= 0.50 + 0.30 = 0.80$$

11. Two fair coins are tossed. What is the probability of getting either two heads or two tails?

Sol. $S = \{HH, HT, TH, TT\}$, $n(S) = 4$

Let A be the event of getting two heads and B be the event of getting two tails.

$A = \{HH\}, n(A) = 1$

$B = \{TT\}, n(B) = 1$

$A \cap B = \phi$

So, the events are mutually exclusive.

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{4}$$

$P(A \cup B) = P(A) + P(B)$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$$

12. If A and B are two mutually exclusive events of a random experiment and P(not A) = 0.45, P(A∪B) = 0.65, then find P(B)

Sol. Given that

$$P(\text{not } A) = 0.45,$$
$$P(A) = 1 - P(\text{not } A)$$
$$= 1 - 0.45 = 0.55$$
$$P(A \cup B) = P(A) + P(B)$$
$$0.65 = 0.55 + P(B)$$
$$P(B) = 0.65 - 0.55$$
$$P(B) = 0.10$$

13. If $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(A\cup B) = \frac{1}{3}$ then find $P(A\cap B)$.

Sol. Given that:

$$P(A) = \frac{2}{3},\ P(B) = \frac{2}{5},\ P(A\cup B) = \frac{1}{3}$$
$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$
$$\frac{1}{3} = \frac{2}{3} + \frac{2}{5} - P(A\cap B)$$
$$P(A\cap B) = \frac{2}{3} + \frac{2}{5} - \frac{1}{3}$$
$$= \frac{10+6-5}{15} = \frac{11}{15}$$

14. A and B are two events such that, P(A)=0.42, P(B)= 0.48, and P(A∩B) = 0.16. Find: (i) $P(\overline{A})$ (ii) $P(\overline{B})$ (iii) $P(A\cup B)$

Sol. Given that: $P(A) = 0.42$, $P(B) = 0.48$, $P(A\cap B) = 0.16$

(i) Find: $P(\overline{A})$

$$P(\text{not } A) = 1 - P(A)$$
$$= 1 - 0.42 = 0.58$$

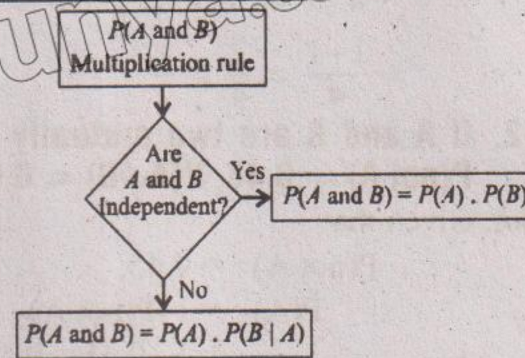
(ii) Find: $P(\overline{B})$

$$P(\text{not } B) = 1 - P(B)$$
$$= 1 - 0.48 = 0.52$$

(iii) Find: $P(A\cup B)$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$
$$= 0.42 + 0.48 - 0.16$$
$$= 0.74$$

12.3 Multiplication Law of probability



(i) If A and B are independent events, then p(A and B) = P ($A \cap B$) = P(A)×P(B)

(ii) Conditional probability

If A and B are dependent events, then P(A and B) P ($A \cap B$) = P(A)×P($B|A$)

Where $P(B|A)$ represents the probability of event B occurring after it is assumed that event A has already occurred (read B|A as "B given A").

Example 11: A bag contains 4 red and 3 green balls. Kabeer draw two balls one after the other with replacement (the second is drawn after the first is replaced). Find the probability that the first ball is red and the second ball is also red.

Solution: Let A be the event that first ball is red and B be the event that second ball is red.

Then $P(A)=\frac{4}{7}$ and $P(B)=\frac{4}{7}$ (Since the ball is replaced, the sample space is not affected)

$$p(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$

$$= \frac{4}{7} \times \frac{4}{7} = \frac{16}{49}$$

Example 12: A bag contains 3 white and 2 black marbles. Two marbles are drawn one after the other without replacement. Find the probability that the first marble is white and the second marble is also white.

Solution: Let A be the event that first marble is white and B be the event the second marble is white.

Total marbles = 3 + 2 =5

The $P(\text{A})=\frac{3}{5}$ and $P(\text{B|A})=\frac{2}{4}$ (Since the marble is not replaced, the sample space is affected).

$$p(A \text{ and } B) = P(A) \times P(B/A)$$

$$= \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$$

Example 13: Majid dealt two cards successively (without replacement) from a well shuffled deck of 52 playing cards. Find the probability that the first card is a king and the second card is a queen.

Solution: Let A be the event that drawing card is a king and B be the event that drawing card is a queen. Remember that we must take care for the sample space of event B, that event A has already occurred.

$$P(A) = \frac{4}{52} = \frac{1}{13} \;;\; P(B|A) = \frac{4}{51}$$

$$P(A \text{ and } B) = P(A) \times P(B/A)$$
$$= \frac{1}{13} \times \frac{4}{51} = \frac{4}{663}$$

Solved Exercise 12.3

1. Two numbers are randomly chosen from 1 to 10 with replacement. Find the probability that: (i) both are prime (iii) their product is even

Sol: $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $n(s) = 10$

(i) both are prime

Let A be the event of first drawn prime number and B be the event of second drawn prime number.

$$A = \{2, 3, 5, 7\}, n(A) = 4$$
$$P(A) = \frac{n(A)}{n(s)} = \frac{4}{10} = \frac{2}{5}$$

Since the number is replaced, the sample space is not affected $n(s) = 10$

$$B = \{2, 3, 5, 7\} = n(B) = 4$$
$$P(B) = \frac{n(B)}{n(s)} = \frac{4}{10} = \frac{2}{5}$$
$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$
$$= \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

(ii) Their product is even

When product of number is even the product is even if at least one number is even.

Let A be the event of first drawn odd number and B be the event of second drawn odd number.

$$A = \{1, 3, 5, 7, 9,\} = n(A) = 5$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{10} = \frac{1}{2}$$

When product of number is even the product is even if at least one number is even.
Since the number is replaced, the sample space is not affected, $n(s) = 10$

$$B = \{1, 3, 5, 7, 9\} \; n(B) = 5$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{10} = \frac{1}{2}$$
$$P(A \cap B) = P(A)\,P(B)$$
$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Now $1 - P(A \cap B)$ will be the Probability when the product of both numbers is even.

$P(\overline{A\cap B}) = 1 - P(A\cap B)$

$= 1 - \frac{1}{4}$

$= \frac{4-1}{4} = \frac{3}{4}$

2. One letter is chosen from the word PUNJAB and another from LAHORE. What is the probability that:

(i) both are vowels (ii) one is consonant and other is vowel

Sol. S = {P, U, N, J, A, B}, n(S) = 6

(i) both are vowels

Let A be the event for letter vowel

A = {U, A}, n (A) =2

$P(A) = = \frac{n(A)}{n(s)} = \frac{2}{6} = \frac{1}{3}$

S = {L, A, H, O, R, E}, $n(S) = 6$

Let B be the event for letter vowel

B = {A, O, E}, n(B) =3

$P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

$P(A\cap B) = P(A) \times P(B)$

$= \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

(ii) one is consonant and other is vowel

Calculation scenario A

Let C be event of chosen a consonant letter from PUNJAB

C = {P, N, J, B}, n (C) = 4

$P(C) = \frac{n(C)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

Let D be event of chosen a vowel Letter from LAHORE

D = {A, O, E} = n, (D) =3

$P(D) = \frac{n(D)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

$P(C\cap D) = P(C) \times P(D)$

$= \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

Calculation of scenario B

Let E be the event of chosen a vowel from PUNJAB and F be event of chosen a consonant from LAHORE

$n(E) = 2, n(F) = 3$

$$P(F) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

$$P(D) = \frac{n(F)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

$$P(E \cap F) = P(E) \times P(F)$$

$$= \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

P (Total) = P (Scenarioa) + P (Scenario B)

$$= \frac{1}{3} + \frac{1}{6}$$

$$= \frac{2+1}{6} = \frac{3}{6} = \frac{1}{2}$$

So, final answer $= \frac{1}{2}$

3. A single dice is rolled twice. Find the probability that one roll is a multiple of 3 and the other is a 5.

Sol: Total outcomes $= 6 \times 6 = 36$, $n(s) = 36$

Let A be the event of getting one roll is a multiple of 3 and the other is 5.

$A = \{(3, 5), (6,5), (5,3), (5,6)\}$, $n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

4. Two dice are rolled. Find the probability of getting an odd number on one and a multiple of 2 on other.

Sol: Total outcomes $n(s) = 6 \times 6 = 36$

Let A be the event of getting an odd number on one and a multiple of 2 on other.

$A = \{(1,2), (1,4), (1,6), (3,2), (3,4), (3,6), (5,2), (5,4), (5,6), (2,1), (4,1), (6,1), (2,3), (4,3), (6,3), (2,5), (4,5), (6,5)\}$

$n(A) = 18$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

5. From a pack of well shuffled cards, two cards are drawn at random one by one with replacement. Find the probability that the first is heart and second is king.

Sol: Total numbers of cards $n(s) = 52$

Let A be the event that first card is heart and B be the event that second card is king.

$n(A) = 13$

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

Since, the card is replaced, the sample space is not affected $n(s) = 52$

$n(B) = 4$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

$$P(A \cap B) = P(A) \times P(B)$$

$$= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

6. If two cards are selected from a standard deck of 52 cards without replacement, find the probability that.

(i) Both are black. (ii) Both are quecns. (iii) Both are spadcs.

(iv) Both are diamonds.

Sol: Total cards $n(S) = 52$

(i) Both are black

Let A be the event of selecting first black card and B be the event of selecting second black card.

$n(A) = 26$

$$P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

Since the card is not replaced, the sample space is affected $n(S)=51$

$n(B|A) = 25$

$$P(B|A) = \frac{25}{51}$$

$$P(A \text{ and } B) = P(A) \times P(B/A)$$

$$= \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

(ii) Both are queens.

$n(s) = 52$

Let A be the event of selecting first queen card and B be the event of selecting second queen card.

$n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

Since the card is not replaced, the sample space is affected $n(S) = 51$

$n(B|A) = 3$

$$P(B/A) = \frac{3}{51} = \frac{1}{17}$$

$$P(A \text{ and } B) = P(A) \times P(B/A)$$

$$= \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$$

(iii) Both are spades.

Sol: $n(s) = 52$

Let A be the event of selecting first spade card and B be the event of selecting second spade card.

$$n(A) = 13$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

Since the card is not replaced, the sample space is affected n(s) = 51

$$n(B|A) = 12$$

$$P(B|A) = \frac{12}{51} = \frac{4}{17}$$

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

$$= \frac{1}{4} \times \frac{4}{17} = \frac{1}{17}$$

(iv) Both are diamonds.

$n(s) = 52$

Let A be the event of selecting first diamond card and B be the event of selecting second diamond card.

$$n(A) = 13$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

Since the card is not replaced, the sample space is affected n(s) = 51

$$n(B|A) = 12$$

$$P(B|A) = \frac{12}{51} = \frac{4}{17}$$

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

$$= \frac{1}{4} \times \frac{4}{17} = \frac{1}{17}$$

7. Saleem draws two cards on by one without replacement from a well shuffled pack of 52 playing cards. What is the probability that first card is jack and the second card is queen.

Sol: Total cards $n(s) = 52$

Let A be the event of drawing first card of jack and B be the event of drawing second card of a queen.

$n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

Since the card is not replaced the sample space is affected

$n(s) = 51$

$n(B|A) = 4$

$$P(B|A) = \frac{n(B|A)}{n(s)} = \frac{4}{51}$$

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

$$= \frac{1}{13} \times \frac{4}{51} = \frac{4}{663}$$

Solved Review Exercise 12

1. Four possible answers are given for the following questions. Choose the correct option.

(i) The probability of an impossible event is:

(a) 0 (b) 1 (c) not defined (d) −1

(ii) What is the probability of getting a head when a fair coin is tossed once?

(a) 0 (b) 0.25 (c) 0.5 (d) 1

(iii) The sum of probabilities of all possible outcomes of an experiment is:

(a) 0.5 (b) 0.25 (c) 1 (d) 0.4

(iv) If P(A) = 0.6, then the probability of event A not happening is;

(a) 0.4 (b) 0.6 (c) 1 (d) 1.6

(v) Two events said to be mutually exclusive if:

(a) they can happen at the same time. (b) one affects the other.

(c) they cannot happen together. (d) they are always equal.

(vi) If one coin is tossed and one dice is rolled, then what is the number of sample points?

(a) 3 (b) 6 (c) 2 (d) 12

(vii) What is the probability of sure event?

(a) 1 (b) 0 (c) $\frac{2}{3}$ (d) $\frac{4}{5}$

(viii) The probability of getting a number greater than 6 on a dice is:

(a) 0.33 (b) 1 (c) 0 (d) 0.5

(ix) ______ outcomes are possible when we draw a card from deck of cards:

(a) 13 (b) 1 (c) 52 (d) 26

(x) If P(E) = 0.07, then what is the probability of 'not E' is:

(a) 0.95 (b) 0.89 (c) 0.93 (d) 0.90

ANSWERS:

(i)	0	(ii)	0.5	(iii)	1	(iv)	0.4	(v)	they cannot happen together
(vi)	12	(vii)	1	(viii)	0	(ix)	52	(x)	0.93

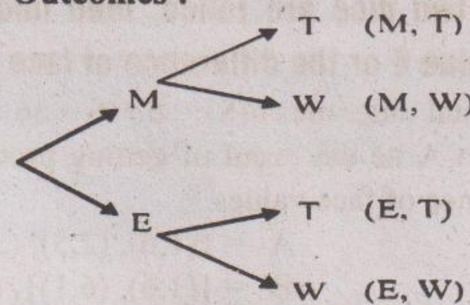
2. Arshia selects a day from (Tuesday, Wednesday) and a time from (Morning, Evening). Draw a possibility diagram. Also sketch a tree diagram.

Sol. **Possibility diagram**

	Tuesday	Wednesday
Morning	(M, T)	(M, W)
Evening	(E, T)	(E, W)

Tree diagram

Outcomes :



3. A card is drawn from a deck of cards. Find the probability of getting an ace or a spade card.

Sol. Total cards, n(S) = 52

Let A be the event of drawing an ace card and B be the event of drawing a spade card.

$n(A) = 4$

$n(B) = 13$

$n(A\cap B) = 1$ (Ace of spade is counted both events)

$$p(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

$$p(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

$$p(A\cap B) = \frac{n(A\cap B)}{n(S)} = \frac{1}{52}$$

$$p(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{1}{13} + \frac{1}{4} - \frac{1}{52}$$

$$= \frac{4+13-1}{52} = \frac{16}{52} = \frac{4}{13}$$

4. Fatima dealt two cards successively (without replacement) from a shuffled deck of 52 playing cards. Find the probability that both cards are red.

Sol. Total cards , n(S) = 52

Let A be the even of dealting first red card and B be the event of dealting second read card.

$n(A) = 26$

$$P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

Since the card is not replaced, the sample space is affected. n(S) = 51

$n(B/A) = 25$

$$p(B|A) = \frac{n(B/A)}{n(S)} = \frac{25}{51}$$

$$P(A \text{ and } B) = P(A) \times P(B|A)$$
$$= \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

5. If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.

Sol. Total outcomes $n(S) = 6 \times 6 = 36$

Let A be the event of getting product of face value 6 and B be the event of getting difference of face values 5.

$$A = \{(1,6), (2,3), (3,2), (6,1)\},\ n(A) = 4$$
$$B = \{(1,6), (6,1)\},\ n(B) = 2$$
$$A \cap B = \{(1,6), (6,1)\},\ n(A \cap B) = 2$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$
$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{1}{9} + \frac{1}{18} - \frac{1}{18}$$
$$= \frac{2+1-1}{18} = \frac{2}{18} = \frac{1}{9}$$

6. A bag contains 7 orange and 5 purple marbles. Two marbles are drawn one after the other without replacement. Find the probability that the first marble is orange and the second marble is also orange.

Sol. Total marbles = 7 orange + 5 purple = 12, $n(S) = 12$

Let A be the event of drawing the first marble is orange and B be the event of drawing the second marble is also orange

$$n(A) = 7$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{12}$$

Since the marble is not replaced the sample space is affected, $n(S) = 11$

$$n(B|A) = 6$$
$$p(B|A) = \frac{n(B|A)}{n(S)} = \frac{6}{11}$$
$$P(A \text{ and } B) = p(A) \times P(B|A)$$
$$= \frac{7}{12} \times \frac{6}{11} = \frac{42}{132} = \frac{7}{22}$$

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

12.1 Probabilty

Encircle the correct option.

1. **Which do help us understand how likely something is to happen:**
   (a) outcome (b) probability (c) event (d) square
2. **The set of all possible outcomes in a random experiment is called:**
   (a) sample space (b) trail (c) coin (d) point
3. **The sample space of roling a dice once:**
   (a) {2, 4, 6} (b) 1 – P(A) (c) {4, 5, 6} (d) {1, 3, 3, 4, 5, 6}
4. **Each element of a sample space is called:**
   (a) event (b) sample point (c) outcome (d) random

12.1.6 Events

5. **In a random experiment, each possible outcome is called:**
   (a) an event (b) sample space (c) probability (d) rolling a dice
6. **Events A and B are said to be matually exclusive if:**
   (a) $A \cup B = B$ (b) $A \cap B = A$ (c) $A \cap B = \phi$ (d) $A \cap B = U$
7. **In tossing a coin, the event "Head" and event "tail" are:**
   (a) same events (b) complementary events
   (c) real events (d) impossible events
8. **If an event E consists of only one outcome, then it is called:**
   (a) single event (b) an elementary event
   (c) an exclusive event (d) mutually event
9. **Flipping a coin multiple times each flip is __________ of others.**
   (a) Dependent (b) Mutually (c) Independent (d) Possible
10. **What is called a visual representation to show all possible outcomes:**
    (a) a free diagram (b) a tall diagram (c) sample diagram (d) a tree diagram

12.1.9 Probability of an Event

11. **The probabilityvalue always lies from:**
    (a) 0 to 1 (b) 1 to 2 (c) 1 to 1 (d) 2 to 6

12. **The complement event of E is:**

(a) $\underline{\underline{E}}$ (b) P(E) (c) $\bar{E}$ (d) $f^{-1}E$

13. **The event is certain if:**

(a) P(E) = 1 (b) P(E) = 0 (c) P(E) > 1 (d) P(E) < 0

14. **What is the complement event of an impossible event:**

(a) certain event (b) greater event (c) mutually event (d) no event

15. **If you toss a coin twice, how many sample points are in the sample space S?**

(a) 2 (b) 8 (c) 4 (d) 6

16. **Which equation is always true?**

(a) $P(A) = P(A')$ (b) $P(A) + P(A') = 0$

(c) $P(A) \times P(A') = 1$ (d) $P(A) + P(A') = 1$

17. **Rolling a dice, What is the probability of rolling a number greater than 4:**

(a) $\frac{3}{6}$ (b) $\frac{2}{6}$ (c) $\frac{4}{6}$ (d) $\frac{1}{6}$

18. **If the probability of rain is 0.3, what is the probability that it will no rain.**

(a) 0.7 (b) 0.0 (c) 0.1 (d) 1

19. **Which is not a valid probability value?**

(a) $\frac{2}{3}$ (b) 2.3 (c) 0.3 (d) 0.5

20. **Two dice are rolled, what is the probability of the sum of outcomes is 17:**

(a) 1 (b) $\frac{17}{36}$ (c) 0.17 (d) 0

21. **Two dice are rolled. What is the probability of the sum of outcomes is less than 13:**

(a) 1 (b) $\frac{12}{36}$ (c) $\frac{13}{36}$ (d) 0

22. **Two dice are rolled together, if the A be the event of getting same number, the n(A) is:**

(a) 36 (b) 6 (c) $\frac{1}{36}$ (d) $\frac{18}{36}$

23. **If $P(A) = \frac{1}{6}$ then $P(A')$ will be:**

(a) 1.6 (b) 0.6 (c) $\frac{5}{6}$ (d) $\frac{6}{5}$

24. Which formula is correct about addition law of probability?

(a) $P(A \cup B) = P(A) - P(B)$ (b) $P(A \cup B) = P(A) \times P(B)$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (d) $P(A \cup B) = P(A) - P(B) + P(A \cap B)$

25. If P(A) = 0.3, P(B) = 0.4 A and B are mutually exclusive, What is P(A∪B) = ?

(a) 0.2 (b) 0.7 (c) 1 – 0.3 (d) 1 – 0.4

26. If P(A) = 0.5, P(B) = 0.3 and P(A∩B) = 0.1 then find P(A∪B).

(a) 0.7 (b) 0.2 (c) 0.5 (d) 0.9

27. A card drawm from a deck of 52. What is the probability it is a King or Queen:

(a) $\frac{4}{52}$ (b) $\frac{1}{13}$ (c) $\frac{2}{13}$ (d) $\frac{8}{51}$

28. Out of 100 students, 60 like Math, 40 like science, 20 like both. What is probability a random student likes Math or Science:

(a) 0.8 (b) 0.6 (c) 0.4 (d) 0.2

29. Roll a dice, What is the probability of getting an even number or a number greater than 4:

(a) $\frac{3}{6}$ (b) $\frac{4}{6}$ (c) $\frac{5}{6}$ (d) $\frac{6}{6}$

30. If P(A∪B) = 0.9, P(A) = 0.6 and P(B) = 0.5. What must be P(A∩B) = _______?

(a) 1.1 (b) 0.3 (c) 0.4 (d) 0.2

31. Which pairs of events are mutually exclusive:

(a) During a red card and drawing an ace

(b) It is raining and carrying an umbrella

(c) Rolling a 3 and rolling an event number on a dice

(d) Being a student and being a math lover

32. A box contain 5 red balls, 3 blue balls and 2 green balls. What is probability of picking a red or a blue ball.

(a) 0.6 (b) 0.2 (c) 0.17 (d) 0.8

33. The P(A) = 0.4, P(B) = 0.5, If A and B are independent, What is P(A∪B)?

(a) 0.2 (b) 0.1 (c) 0.7 (d) 0.9

34. If P(A) = 0.6 and P(B) = 0.7, What is the minimum possible value for P(A∩B):

(a) 0.3 (b) 0.1 (c) 0 (d) 0.6

35. Which word is commonly associated with the Addition Law of Probability?

(a) Given (b) AND (c) NOT (d) OR

12.3 Multiplication Law of Probability

36. Which formula is correct for multiplication Law of probability of independent events A and B:

(a) $P(A|B) = P(A) \times P(B)$ (b) $P(A \cup B) = P(A) + P(B)$

(c) $P(A \cap B) = P(A) + P(B)$ (d) $P(A \cap B) = P(A) \times P(B)$

37. If events A an B are dependent, What is correct for multiplication law:

(a) $P(A \cap B) = P(A) \times P(B/A)$ (b) $P(A \cap B) = \frac{P(A)}{P(B)}$

(c) $P(A \cap B) = P(A) \times P(B)$ (d) $P(\frac{A}{B}) = P(A) \times P(B)$

38. A fair coin is flipped twice, What is the probability of getting heads on the first flip and heads on second flip?

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) 1 (d) $\frac{1}{2}$

39. If $P(A) = 0.6$ and $P(B) = 0.5$ What is $P(A \cap B)$?

(a) 0.3 (b) 0.83 (c) 0.1 (d) 1.1

40. Two fair dice are rolled. What is the probability of getting 6 on the first dice and 6 on the second dice?

(a) $\frac{2}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{36}$ (d) $\frac{1}{6}$

41. In a deck of 52 cards. You draw two cards without replecement. What is the probability that both are Aces?

(a) $\frac{4}{52}+\frac{3}{52}$ (b) $\frac{4}{52}\times\frac{3}{52}$ (c) $\frac{4}{52}\times\frac{3}{51}$ (d) $\frac{1}{13}\times\frac{1}{13}$

42. If $P(A \cap B) = 0.2$ and $P(A) = 0.5$, What is $P(B|A)$?

(a) 2.5 (b) 0.4 (c) 0.7 (d) 0.1

ANSWERS:

1. probability 2. sample space 3. $\{1, 2, 3, 4, 5, 6\}$ 4. Sample point
5. an event 6. $A \cap B = \phi$ 7. complementary events
8. an elementry event 9. independent
10. a tree diagram 11. 0 to 1 12. $\overline{E}$
13. $P(E) = 1$ 14. certain event 15. 4 16. $P(A) + P(A') = 1$
17. $\frac{2}{6}$ 18. 0.7 19. 2.3 20. 0 21. 1 22. 6 23. $\frac{5}{6}$
24. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ 25. 0.7 26. 0.7 27. $\frac{2}{13}$

28. 0.8 29. $\frac{4}{6}$ 30. 0.2 31. Roll a 3 and an even number on a dice

32. 0.8 33. 0.9 34. 0.3 35. OR 36. $P(A\cap B) = P(A) \times P(B)$

37. (a) $P(A\cap B) = P(A) \times P(B|A)$ 38. $\frac{1}{4}$ 39. 0.3 40. $\frac{1}{36}$

41. $\frac{4}{52}\times\frac{3}{51}$ 42. 0.4

Additional Short Question

Short Answer Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

12.1 Probability

Give short answers.

1. What is the basic definition of probability?

Ans. Probability helps us understand how likely something is to happen, especially when we are dealing with uncertainty.

2. What are two main characteristics of a random experiment?

Ans. A random experiment is one where

1. The set set of all possible outcomes are known.
2. Exact outcome is not known.

3. What is an outcome in probability?

Ans. An outcome is the possible result of a random experiment.

4. How is a sample space defined?

Ans. A sample space is the set of all possible outcomes in a random experiment. It is generally denoted by letter S.

5. Define a sample point.

Ans. Each element of a sample space is called a sample point.

12.1.3 Events

6. Define an event:

Ans. In a random experiment, each possible outcome is called an event.

7. Define equally likely events:

Ans. Two are more events are said to be equally likely if each one of them has equal chance of occurring.

8. What is an impossible event.

Ans. An impossible event is an event that cannot happen under any circumstances in a given situation.

9. Define complementary event:

Ans. The complenent of an event A is the event representing a collection of a sample points not in A.

10. Define an elementary event.

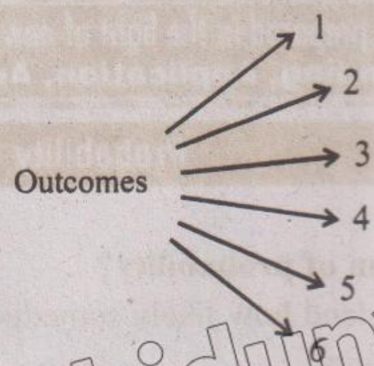
Ans. If an event E consists of only one outcome, then it is called an elementary event.

11. What are mutually exclusive events?

Ans. Two are more events are said to be mutually exclusive if they do not have common sample points.

12. Find the sample space for rolling a dice using tree diagram:

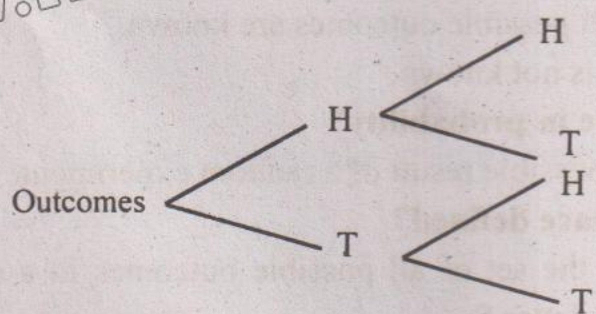
Sol:



$S = \{1, 2, 3, 4, 5, 6\}$

13. Find the sample space tossing two coins using tree diagram.

Sol:



$S = \{HH, HT, TH, TT\}$

12.1.9 Probability of an event

14. Three fair coins are tossed. Find the probability of getting at least one tail.

Sol: = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

$n(S) = 8$

Let A be the event to getting at least one tail.

A= {HHT, HTH, THH, HTT, THT, TTH, TTT}

$n(A) = 7$

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$$

15. A bag contains 5 white and 3 green balls. Hanzla Asad draw a ball random from the bag Find the probability that ball drawn is green.

Sol: Total balls = 5 white + 3 green = 8

$n(S) = 8$

Let A be the event of getting a gree ball.

$n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

16. If $P(A) = \frac{5}{9}$ find the probability of A'.

Sol: $P(A) = \frac{5}{9}$ (Given)

$$P(A') = 1 - P(A)$$

$$= 1 - \frac{5}{9}$$

$$= \frac{9-5}{9} = \frac{4}{9}$$

17. Find the probability of drawing a card of a spade from a deck of 52 card.

Sol: $n(S) = 52$

Let B be the exent of arawing a spade card

$n(B) = 13$

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

18. A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 3, 4, 5, ... 13.

What is the probability that it will point to multiple of 3.

Sol: $S = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$, $n(s) = 11$

Let A be the event that arrow will point to multiple of 3.

$A = \{3, 6, 9, 12\}$, $n(A) = 4$

$$P(A) = \frac{n(A)}{n(s)} = \frac{4}{11}$$

19. If the $p(\overline{A}) = \frac{7}{10}$, the find P (A).

Sol: $p(\overline{A}) = \frac{7}{10}$ (Given)

We know

$$P(\overline{A}) = 1 - P(A)$$

$$\frac{7}{10} = 1 - (PSA)$$

$$P(A) = 1 - \frac{7}{10}$$

$$P(A) = \frac{10-7}{10} = \frac{3}{10}$$

20. A dice is rolled twice. Find the probability to having a number greater than 6 on each roll.

Sol: Total outcomes n(s) = 6×6 = 36

Let A be event of having a number greater than 6 on each roll.

Since there is not simple point in sample space greater the six of each roll.

A = {} , n (A) = 0

$$P(A) = \frac{n(A)}{n(S)} = \frac{0}{36} = 0$$

12.2 Addition Law of probability

21. Write the addition law of probability for two non-mutually exclusive events.

Sol: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

22. Write the addition law of probability for two mutually exclusive events.

Sol: $P(A \cup B) = P(A) + P(B)$

23. If P(A) = 0.3, P (B) = 0.4, A and B are mutually exclusive, what is P (A∪B)

Ans. Given that: P (A) = 0.3, P (B) = 0.4

$$P(A \cup B) = P(A) + P(B)$$
$$= 0.3 + 0.4 = 0.7$$

24. If $P(A) = \frac{7}{10}$, $P(B) = \frac{5}{10}$ and $P(A \cup B) = \frac{8}{10}$ what is $(A \cap B)$

Sol: Given That

$$P(A) = \frac{7}{10}, P(B) = \frac{5}{10} \text{ and } P(A \cup B) = \frac{8}{10}$$

We know

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{8}{10} = \frac{7}{10} + \frac{5}{10} - P(A \cap B)$$

$$P(A \cap B) = \frac{7}{10} + \frac{5}{10} - \frac{8}{10}$$

$$= \frac{4}{10} = \frac{2}{5}$$

25. If P (A) = 0.43, P $(A\cup B)$ = 0.8 and P $(A\cap B)$ = 0.07 find the P (B).

Sol: Given that P (A) = 0.43, P $(A\cup B)$ = 0.8, P$(A\cap B)$ = 0.07

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$
$$0.8 = 0.43 + P(B) - 0.07$$
$$0.8 - 0.43 + 0.07 = P(B)$$
$$0.44 = P(B)$$
$$P(B) = 0.44$$

26. If P $(A\cup B)$ = 0.9 and P (B) = 0.4, find P (A).

Sol: Given that P $(A\cup B)$ = 0.9, P (B) = 0.4

$$P(A\cup B) = P(A) + P(B)$$
$$0.9 = P(A) + 0.4$$
$$0.9 - 0.4 = P(A)$$
$$0.5 = P(A)$$
$$P(A) = 0.5$$

(27) If P $(A\cup B)$ = 0.6 what is the probability that neither A or B occurs.

Sol: P $(A\cup B)$ = 0.6 (Given)

$$P(\text{Neither}) = 1 - p(A\cup B)$$
$$= 1 - 0.6$$
$$= 0.4$$

12.3 Multiplication law of probability

28. Write the multiplication law of probability of two independent events.

Sol: P (A an B) = P $(A\cup B)$ = P (A) × P (B)

29. Write the multiplication law of probability of two dependents event.

Sol: P (A an B) = P (A) × P (B|A)

30. Arshman rolls a fair dice twice. What is the probability of rolling 4 both time.

Sol: $S = \{1, 2, 3, 4, 5, 6,\}$, $n(S) = 6$

Let A be the event of rolling 4 at first roll and B be the event of rolling 4 at second time.

These are independent event

$n(A) = 1$

$$P(A) = \frac{n(A)}{n(s)} = \frac{1}{6}$$

$n(B) = 1$

$$P(B) = \frac{n(B)}{n(s)} = \frac{1}{6}$$

$$P(A\cap B) = P(A) \times P(B)$$
$$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

31. A bag contains 5 red and 3 marbles. Minhah picks two marbel one after other replacing the first what is the probability both are red.

Sol: $n(S) = 8$ (Total of marbles)

Let A be the event of first picking and B be the event of second picking.

$n(A) = 5$, $n(B) = 5$

$P(A) = \frac{5}{8}$, $P(B) = \frac{5}{8}$

$P(A \cap B) = P(A) \times P(P)$

$= \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$

32. A box has 6 black balls and 4 white balls. Ajwa takes two ball at random (without replacement). What is the probability that both are white balls.

Sol: $n(S) = 10$ (Total of balls)

Let A be the event that first ball is white and B be the event that second ball is also white (Since first ball is not replaced, the sample space is affected)

$n(A) = 4$, $n(B) = 3$

$P(A) = \frac{4}{10}$, $P(B|A) = \frac{3}{9}$

$P(A \cap B) = P(A) \times P(B|A)$

$= \frac{3}{10} \times \frac{4}{10} \times \frac{3}{9} = \frac{12}{90} = \frac{2}{15}$

$= \frac{3}{10} \times \frac{2}{9} = \frac{6}{90} = \frac{1}{15}$

33. The probability that Afzal passes math is 0.7 and the probability that urdu passes is 0.6. Both are independent. Find the probability he passes both.

Sol: $P(\text{Math}) = 0.7$, $P(\text{Urdu}) = 0.6$

$P(\text{Math} \cap \text{Urdu}) = P(\text{Math}) \times P(\text{Urdu})$

$= 0.7 \times 0.6$

$= 0.42$

Instructions for Preparation of Exam Paper of Mathematics for Grade-10

- The paper of Mathematics for Grade-10 will consist of 75 marks.
- Objective Type = 15 + Subjective Type = 60 marks.
- Timing of the paper will be 2:30 hours.
- Objective Type = 20 minutes + Subjective Type = 2:10 hours.
- The paper will be made as per following details:

Objective (MCQs)	Q-1: There will be 15 Multiple Choice Questions (MCQs) from the entire content of the textbook. The distribution is as follows: One MCQ each from chapters 1, 2, 3, 4, 5, 6, 7, 8, and 10, and two MCQs each from chapters 9, 11, and 12.	1×15=15
Part-1: Subjective:	This section contains three short questions. The details are as follows: Q-2: Students are required to attempt any 6 out of 9 short questions. The details are as follows: • At least two short questions will be drawn each from chapters 1,2,3, and 4.	2×6=12
	Q-3: Students are required to attempt any 6 out of 9 short questions. The details are as follows: • At least two short questions will be drawn each from Chapters 5,6,7, and 8.	2×6=12
	Q-4: Students are required to attempt any 6 out of 9 short questions. The details are as follows: • At least two short questions will be drawn each from Chapters 9 10 11 and 12.	2×6 = 12
Part-II Subjective	This section will contain three long questions bifurcated in two parts **a & b** (carrying 4 marks each) and students have to attempt two questions. The details are as follows: Q-5: (a) One long-answer question will be selected from chapter 1. (b) One long-answer question will be selected from chapter 2. Q-6: (a) One long-answer question will be selected from chapter 3. (b) One long-answer question will be selected from chapter 4. Q-7: (a) One long-answer question will be selected from chapter 5. (b) One long-answer question will be selected from chapter 7.	2×8= 16
Part-III Subjective	This section will contain two long questions bifurcated in two parts **a & b** (carrying 4 marks each) and students have to attempt one questions. The details are as follows: Q-8: (a) One long-answer question will be selected from chapter 6. (b) One long-answer question will be selected either from chapter 8 or chapter 11. Q-9: (a) One long-answerquestion will be selected from chapter 10. (b) One long-answer question will be selected either from chapter 9 or chapter 12.	1 × 8 = 8

Model Paper of Mathematics For Grade-10

Objective Type

Time Allowed: 20 Min. **Max. Marks: 15**

1-Note: Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with marker or ink pen in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

Sr.No	Questions	A	B	C	D
i.	$i^2 + i^4$	-1	0✓	1	2
ii.	What is the discriminant of $x^2 + 5x - 5 = 0$?	-20	20	25	45✓
iii.	If $A^t = -A$, then A is _____ matrix.	symmetric	row	rectangular	skew symmetric✓
iv.	A function f from X to Y is represented by:	$f: XY$	$f: Y \to X$	$f: X \to Y$✓	$f: \frac{X}{Y}$
v.	Lowest form of $\frac{15xyz}{10}$ is:	$\frac{3xyz}{2}$✓	$\frac{3}{2}$	xyz	$\frac{xyz}{2}$
vi.	P(4,–4) lies in _____ quadrant.	first	second	third	fourth✓
vii.	$\sin(90° + \theta) =$	$\sin\theta$	$-\sin\theta$	$\cos\theta$✓	$-\cos\theta$
viii	The sum of the measures of central angles of a circle is:	90°	180°	270°	360°✓
ix.	_____ tangents can be drawn to a circle from a point outside the circle.	one	two✓	three	many
x.	If $r = 6$cm, $\theta = 2$ radians, then arc length is:	12cm✓	8cm	6cm	3cm
xi.	At least how many chords are needed to locate the centre of the circle?	1	2✓	3	4
xii.	Second quartile represents:	mean	median✓	mode	variance
xiii	Positive square root of variance is called:	mean	median	standard deviation✓	range
xiv	If P(A) = 0.6, then the probability of event A not happening is:	0.4✓	0.6	1	1.6
xv.	_____ outcomes are possible when we draw a card from deck of cards.	13	1	52✓	26

Subjective Types

Time Allowed: 2:10 hrs. **Max. Marks: 60**

(Part-I)

Q.2: Write short answers to any six (06) questions: **$2 \times 6 = 12$**

(i) Simplify: i^{17}

(ii) Find the modulus of $-5-4i$

(iii) Define quadratic equation with example.

(iv) Write the quadratic equation $2x(x+1) = 4(2x+3)$ in standard form.

(v) Examine the nature of roots of equation $3x^2 - 9x - 2 = 0$

(vi) Find A – B and B – A. $A=\begin{bmatrix} 5 & 9 \\ 4 & 8 \end{bmatrix}, B=\begin{bmatrix} 3 & 6 \\ 2 & 1 \end{bmatrix}$

(vii) Find the multiplicative inverse of $\begin{bmatrix} -4 & 8 \\ 7 & 2 \end{bmatrix}$

(viii) Find $f(x)\,g(x)$, if $f(x) = x + 1$, $g(x) = 2x - 3$

(ix) Solve and express the solution on number line: $|4x - 9| = 3$

Q.3: Write short answers to any six (06) questions: **$2 \times 6 = 12$**

(i) Simplify $\dfrac{3}{x-y} + \dfrac{1}{y-x}$

(ii) Define reciprocal equation with example.

(iii) Reduce to the lowest form: $\dfrac{x^2-5x}{x^2-4x-5}$

(iv) Show that $\underline{a} = 5\underline{i} - 2\underline{j}$ and $\underline{b} = -10\underline{i} + 4\underline{j}$ are parallel.

(v) Find a unit vector in the direction of $\underline{a} = \dfrac{5}{3}\underline{i} + \dfrac{1}{3}\underline{j}$

(vi) State law of sines.

(vii) Calculate area of triangle ABC in which $a = 6.12$cm, $b = 8.34$ cm, $c = 7.12$ cm:

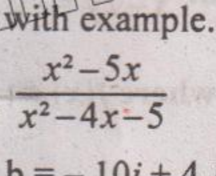
(viii) Define "Secant" and draw its figure.

(ix) Prove that a unique circle can pass through the points $A(1, 1)$, $B(4, 2)$ and $C(3, 5)$.

Q.4: Write short answers to any six (06) questions: **$2 \times 6 = 12$**

(i) Define tangent to a circle with Figure.

(ii) Find the value of x.



(iii) Find the area of sector of a circle having central angle 60° and radius 7cm. Also find the length of the arc.

(iv) Construct a circle with the help of radius = 2 cm and locate its centre by construction.

(v) Draw a circle of radius 1.3cm draw a tangent at point P, when P lies on its circumference.

(vi) Find median for the given data. 102, 98, 95, 100, 93, 110, 108, 104, 97, 96, 92, 101, 99, 105, 107

(vii) Find the range of the given data: 15, 22, 18, 30, 17, 25

(viii) Two coins are tossed together, what is probability of getting different faces on the coins.

(ix) Two numbers are randomly chosen from 1 to 10, with replacement, find the probability that both numbers are prime.

(Part-II)

Note: Attempt any two (02) questions. $2 \times 8 = 16$

Q.5: (a) Solve $3z + (2 + i)w = 11 - i$, $(2 - i)z - w = -1 + i$ with complex coefficient for w and z.

(b) If the equation $x^2 + 2(1 + k)x + k^2 = 0$ has equal roots, then find the value of k.

Q.6: (a) Solve by matrix inverse method: $\begin{array}{l} 3x+2y=2 \\ x-2y=-2 \end{array}$

(b) Verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$, where $f(x) = \dfrac{x-4}{x+2}, x \neq -2$.

Q.7: (a) Solve: $4.2^{2x+1} - 9.2^x + 1 = 0$

(b) Solve the triangle ABC. $c = 7\text{cm}$, $\beta = 34°$, $\gamma = 64°$

(Part-III)

Note: Attempt any one (01) question. $1 \times 8 = 8$

Q.8: (a) The coordinates of A,B and D are (1, 2), (6, 3), and (2, 8) respectively. Find the coordinates of C by using vector method if ABCD is a parallelogram.

(b) A factory produces rods with lengths (in cm). 50.1, 49.9, 50.0, 50.2, 49.8 Check consistency using standard deviation.

Q.9: (a) Draw a circle of radius 1.5 cm. Draw two tangents that meet an angle of 30°.

(b) Two dice are rolled togehter. Find the probability of getting a [illegible] number or sum of faces as 10.