UNIT 07 Trigonometry

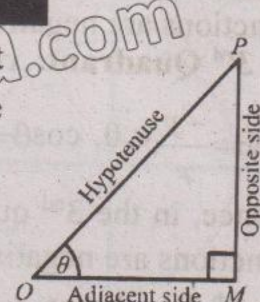
Students' Learning Outcomes

After completing this unit, the students will be able to:

- Extend sine and cosine functions to angles between 90° and 180°.
- Solve problems using the laws of sine, cosine and the area formulas for any triangle.
- Solve simple trigonometric problems in three dimensions.
- Apply concepts of trigonometry to real life world problems (such as video games, flight engineering, navigation and sound waves).
- Interpret and use three figure bearings.
- Solve real life problems involving bearing.
- Apply the concepts of bearing to real world problems.

7.1 Trigonometric Ratios or Functions

Consider an angle θ of right angled triangle POM, with right angle at M, where $\overline{OM}$ is called adjacent side, $\overline{PM}$ is called opposite side and $\overline{PO}$ is called hypotenuse. Then,



(i) sineθ (sin θ) $= \dfrac{\text{opp.}}{\text{hyp.}}$ (ii) cosine θ(cos θ) $= \dfrac{\text{adj.}}{\text{hyp.}}$

(iii) cosecantθ (cosee θ) $= \dfrac{1}{\sin\theta} = \dfrac{\text{hyp.}}{\text{opp.}}$ (vi) secantθ (sec θ) $\dfrac{1}{\cos\theta} = \dfrac{\text{hyp.}}{\text{opp.}}$

(v) tangentθ (tanθ) $= \dfrac{\text{opp.}}{\text{adj.}}$ (iv) cotangent θ(cot θ) $= \dfrac{1}{\tan\theta} = \dfrac{\text{adj.}}{\text{opp.}}$

The above six trigonometric ratios, are also called trigonometric functions.

Remember!
sin θ does not mean the product of sin and θ.
The sin θ is correctly read as sin of angle θ.

Note
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

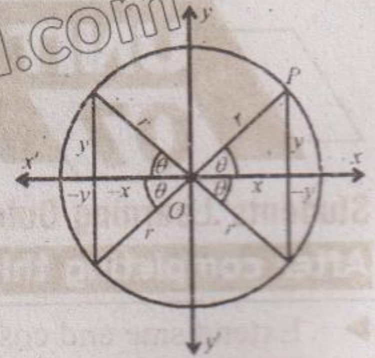
Trigonometric Values for Special Angles

Trigonometric values for 0°, 30°, 45°, 60°, 90° and 180 are given in the following table:

Ratios / Angleθ.	0°	30°	45°	60°	90°	180°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined	0

Signs of Trigonometric Functions

The signs depend on the quadrant in which terminal side of the angle lies.



Consider a circle having radius r and centre at point 0.

In 1[st] Quadrant: $x > 0, y > 0$

$\sin\theta = \frac{y}{r} > 0, \cos\theta = \frac{x}{r} > 0, \tan\theta = \frac{y}{x} > 0,$

$\operatorname{cosec}\theta = \frac{r}{y} > 0, \sec\theta = \frac{r}{x} > 0, \cot\theta = \frac{x}{y} > 0$

Hence, in the 1[st] quadrant all trigonometric functions are positive.

In 2[nd] Quadrant: $x < 0, y > 0$

$\sin\theta = \frac{y}{r} > 0, \cos\theta = \frac{-x}{r} < 0, \tan\theta = \frac{y}{-x} < 0, \operatorname{cosec}\theta = \frac{r}{y} > 0, \sec\theta = \frac{r}{-x} < 0\ \cot\theta = \frac{-x}{y} < 0$

Hence, in the 2[nd] quadrant sin and cosec functions are positive and all other trigonometric functions are negative.

In 3[rd] Quadrant: $x < 0, y < 0$

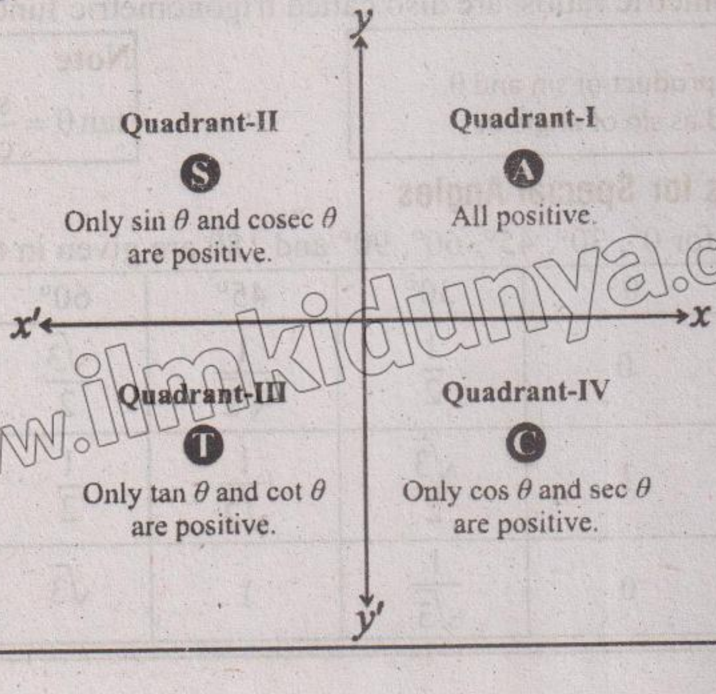
$\sin\theta = \frac{-y}{r} < 0, \cos\theta = \frac{-x}{r} < 0, \tan\theta = \frac{-y}{-x} > 0, \operatorname{cosec}\theta = \frac{r}{-y} < 0, \sec\theta = \frac{r}{-x} < 0, \cot\theta = \frac{-x}{-y} > 0$

Hence, in the 3[nd] quadrant tan and cot functions are positive and all other trigonometric functions are negative.

In 4[th] Quadrant: $x > 0, y < 0$

$\sin\theta = \frac{-y}{r} < 0, \cos\theta = \frac{x}{r} > 0, \tan\theta = \frac{-y}{x} < 0, \operatorname{cosec}\theta = \frac{r}{-y} < 0, \cot\theta = \frac{x}{-y} < 0, \sec\theta = \frac{r}{x} > 0$

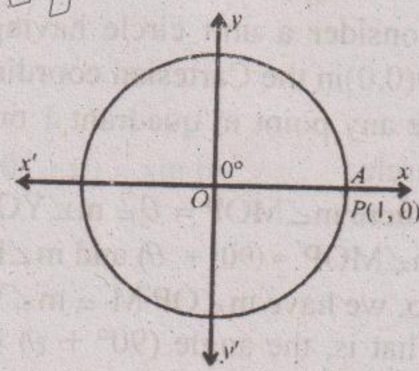
Hence, in the 4[th] quadrant cos and sec functions are positive and all other trigonometric functions are negative.



Trigonometric Values for Special Angles

i. Trigonometric Ratios of 0°

Consider a unit circle with centre at O and take any point P(x,y) on it such that m∠AOP = 0°.



As P(x,y) lies on x – axis,

So, $x = 1, y = 0$, r = 1

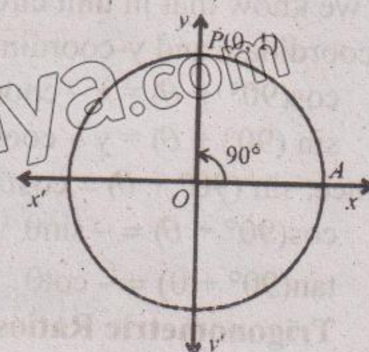
$\sin 0° = \dfrac{y}{r} = \dfrac{0}{1} = 0$, $\text{cosec } 0° = \dfrac{1}{\sin 0°} = \dfrac{1}{0}$ = undefined

$\cos 0° = \dfrac{x}{r} = \dfrac{1}{1} = 1$, $\sec 0° = \dfrac{1}{\cos 0°} = \dfrac{1}{1} = 1$

$\tan 0° = \dfrac{y}{x} = \dfrac{0}{1} = 0$, $\cot 0° = \dfrac{1}{\tan 0°} = \dfrac{1}{0}$ undefined

ii. Trigonometric Ratios of 90°

Consider a unit circle with centre at O and take any point P(x,y) on it such that m∠AOP = 90°.



As P(x,y) lies on y-axis,

So, $x = 0, y = 1$, r = 1

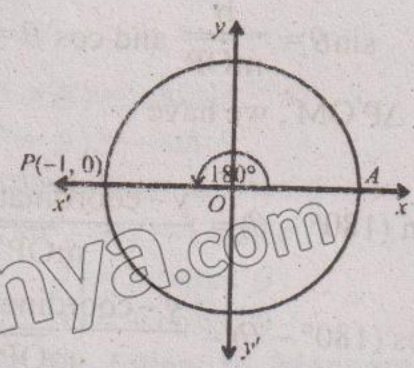
$\sin 90° = \dfrac{y}{r} = \dfrac{1}{1} = 1$, $\text{cosec } 90° = \dfrac{1}{\sin 90°} = \dfrac{1}{1} = 1$

$\cos 90° = \dfrac{x}{r} = \dfrac{0}{1} = 0$, $\sec 90° = \dfrac{1}{\cos 90°} = \dfrac{1}{0}$ = undefined

$\tan 90° = \dfrac{y}{x} = \dfrac{1}{0}$ undefined, $\cot 90° = \dfrac{0}{1} = 0$

iii. Trigonometric Ratios of 180°

Consider a unit circle with centre at O and take any point P(x,y) on it such that $m\angle AOP = 180°$ As P(x,y) lies on x' – axis.



so, $x = -1, y = 0$, r = 1

$\sin 180° = \dfrac{x}{y} = \dfrac{0}{1} = 0,$

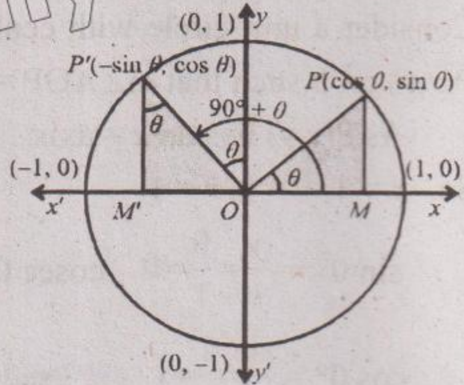
$\text{cosec } 180° = \dfrac{1}{\sin 180°} = \dfrac{1}{0}$ = undefined

$\cos 180° = \dfrac{x}{y} = \dfrac{-1}{1} = -1$, $\sec 180° = \dfrac{1}{\cos 180°} = \dfrac{1}{-1} = -1$

$\tan 180° = \dfrac{y}{x} = \dfrac{0}{-1} = 0$, $\cot 180° = \dfrac{1}{\tan 180°} = \dfrac{-1}{0}$ = undefined

Extending sine and cosine Functions to Angles Between 90° and 180°.

a. Trigonometric Ratios of an Angle of the Form $(90° + \theta)$



Consider a unit circle having radius 1 with centre at O(0,0)in the Cartesian coordinate system. $P(\cos\theta, \sin\theta)$ be any point in quadrant 1 on the circumference of the circle,

where $m\angle MOP = \theta = m\angle YOP'$

$m\angle MOP' = (90° + \theta)$ and $m\angle POP' = 90°$

so, we have $m\angle OP'M' = m\angle YOP'$ (Alternate angles)

That is, the angle $(90° + \theta)$ is obtained by rotating the angle θ counterclockwise by 90°

The coordinates of P' will be $(-\sin\theta, \cos\theta)$, i.e. the new x-coordinates of P' becomes the negative of the y-coordinate of the P and new y-coordinate of P' becomes the x-coordinate of the P.

As we know that in unit circle, cosine and sine of any angle are represented by x–coordinate and y-coordinate respectively. Therefore,

$$\cos(90° + \theta) = x - \text{coordinate of } (90° + \theta) = -\sin\theta$$

$$\sin(90° + \theta) = y - \text{coordinate of } (90° + \theta) = \cos\theta$$

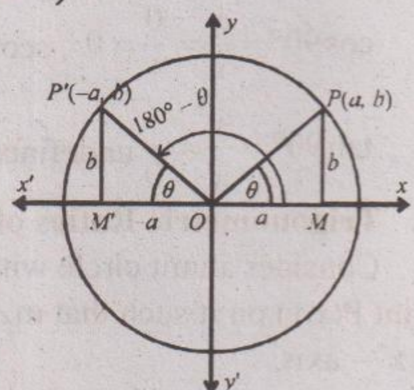
Hence, $\sin(90° + \theta) = \cos\theta$, $\operatorname{cosec}(90° + \theta) = \sec\theta$

$\cos(90° + \theta) = -\sin\theta$, $\sec(90° + \theta) = -\operatorname{cosec}\theta$

$\tan(90° + \theta) = -\cot\theta$, $\cot(90° + \theta) = -\tan\theta$

> **Note**
> $\sin(-\theta) = -\sin\theta$
> $\cos(-\theta) = \cos\theta$
> $\tan(-\theta) = -\tan\theta$

b. Trigonometric Ratios of an Angle of the Form $(180° - \theta)$



Let $m\angle MOP = \theta$ and $m\angle MOP' = (180° - \theta)$

Here ΔPOM and $\Delta P'OM'$ are congruent, such that $m\overline{OP'} = m\overline{OP}$, $m\overline{OM'} = m\overline{OM}$ and $m\overline{P'M'} = m\overline{PM}$

In ΔPOM, we have

$$\sin\theta = \frac{b}{m\overline{OP}} \text{ and } \cos\theta = \frac{a}{m\overline{OP}}$$

In $\Delta P'OM'$, we have

$$\sin(180° - \theta) = \frac{y - \text{coordinate of } P'}{m\overline{OP'}} = \frac{b}{m\overline{OP'}} = \sin\theta$$

$$\cos(180° - \theta) = \frac{y - \text{coordinate of } P'}{m\overline{OP'}} = \frac{-a}{m\overline{OP}} = -\cos\theta$$

Hence
$\sin(180° - \theta) = \sin\theta$, $\operatorname{cosec}(180° - \theta) = \operatorname{cosec}\theta$
$\cos(180° - \theta) = -\cos\theta$, $\sec(180° - \theta) = -\sec\theta$
$\tan(180° - \theta) = -\tan\theta$, $\cot(180° - \theta) = -\cot\theta$

Example:1 Find the signs of the following:

Sol. (i) $\cos 75°$ (ii) $\cot 250°$

(i) 75° lies in first quadrant, where cos is positive.

(ii) 250° lies in third quadrant, where cot is positive.

Example:2 Without using calculator, find the exact values of the following trigonometric functions: (i) sin 120° (ii) cot 150°

sol. (i) sin 120°

120° lies in quadrant II, where sin is positive.

$\therefore \quad \sin 120° = \sin(180° - 60°) = \sin 60° \qquad \therefore \quad \sin(180° - \theta) = \sin\theta$

$= \dfrac{\sqrt{3}}{2}$

(ii) cot 150°

150° lies in quadrant II, where cot is negative.

$\therefore \quad \cot 150° = \cot(180° - 30°) = -\cot 30° \quad \therefore \quad \cot(180° - \theta) = -\cot\theta$

$= -\sqrt{3}$

Solved Exercise 7.1

1. Find the signs of the following:

(i) sin 55°

Sol. sin55° lies in the first quadrant, where sine is positive.

(ii) cos 145°

Sol. cos 145° lies in the second quadrant, where cosine is negative.

(iii) tan 111°

Sol. tan 111° lies in the second quadrant, where tangent is negative.

(iv) sec 179°

Sol. sec179° lies in the second quadrant, where secant is negative.

(v) cosec 88°

Sol. cosec 88° lies in the first quadrant, where cosecant is positive.

(vi) sin 14°

Sol. sin 14° lies in the first quadrant, where sine is positive.

2. Fill in the blanks:

(i) $\tan(180° - \theta) = ...\tan\theta$

Sol. $\tan(180° - \theta) = -\tan\theta$

(ii) $\sin(180° - \theta) = ...\sin\theta$

Sol. $\sin(180° - \theta) = +\sin\theta$

(iii) $\tan(90° + \theta) = ...\cot\theta$

Sol. $\tan(90° + \theta) = -\cot\theta$

(iv) $\cos(90° + \theta) = ...\sin\theta$

Sol. $\cos(90° + \theta) = -\sin\theta$

(v) $\text{cosec}(180° - \theta) = ...\text{cosec}\,\theta$

Sol. $\text{cosec}(180° - \theta) = +\text{cosec}\,\theta$

(vi) $\sec(90° - \theta) = ...\text{cosec}\,\theta$

Sol. $\sec(90° - \theta) = +\text{cosec}\,\theta$

3. Without using calculator, find the exact value of the following trigonometric functions.

(i) sin 150°

Sol: $\sin 150° = \sin(180° - 30°) = +\sin 30 = \dfrac{1}{2} \quad \therefore \sin(180° - \theta) = \sin\theta$

(ii) tan 150°

Sol: $\tan 150° = \tan(180°-30°) = -\tan 30 = \dfrac{-1}{\sqrt{3}}$ $\quad \therefore \tan(180°-\theta) = -\tan\theta$

(iii) sec 150°

Sol: $\sec 150° = \sec(180°-30°) = -\sec 30° = -\dfrac{2}{\sqrt{3}}$ $\quad \therefore \sec(180°-\theta) = -\sec\theta$

(iv) cosec 120°

Sol: $\operatorname{cosec} 120° = \operatorname{cosec}(180°-60°) = +\operatorname{cosec} 60° = \dfrac{2}{\sqrt{3}}$ $\quad \therefore \operatorname{cosec}(180°-\theta) = -\operatorname{cosec}\theta$

(v) cos 120°

Sol: $\cos 120° = \cos(180°-60°) = -\cos 60° = -\dfrac{1}{2}$ $\quad \therefore \cos(180°-\theta) = -\cos\theta$

(vi) cot 120°

Sol: $\cot 120° = \cot(180°-60°) = -\cot 60° = -\dfrac{1}{\sqrt{3}}$ $\quad \therefore \cot(180°-\theta) = -\cot\theta$

(vii) sin 135°

Sol: $\sin 135° = \sin(180°-45°) = +\sin 45° = \dfrac{1}{\sqrt{2}}$ $\quad \therefore \sin(180°-\theta) = \sin\theta$

(viii) sec 135°

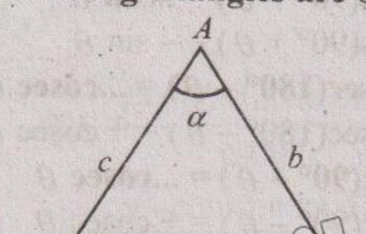
Sol: $\sec 135° = \sec(180°-45°) = -\sec 45° = -\sqrt{2}$ $\quad \therefore \sec(180°-\theta) = -\sec\theta$

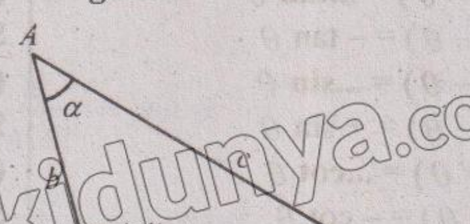
(ix) cot 135°

Sol: $\cot 135° = \cot(180°-45°) = -\cot 45° = -1$ $\quad \therefore \cot(180°-\theta) = -\cot\theta$

7.2 Oblique Triangles

An oblique triangle is any triangle that does not contain a right angle. It can be either acute triangle (all angles less than 90°) or obtuse triangle (one angle greater than 90°). Following triangles are oblique triangles:





The Law of sines

If a, b and c are the sides and α, β and γ are the angles of opposite sides of any triangle respectively, then the law of sines is:

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$$

The law of sines is a relationship between the angles and the sides of a triangle. While solving a triangle, the law of sines can be used to solve a triangle, if two angles and one side which is opposits to one of given angles, are given.

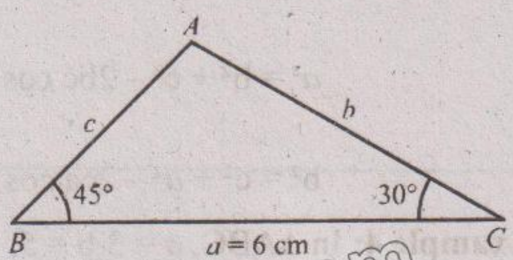
Example: 3

In a triangle ABC, $\beta = 45°$, $\gamma = 30°$, $a = 6$cm, calculate b and c.

sol: As we know that

$$\alpha+\beta+\gamma=180°$$
$$\alpha=180°-\beta-\gamma$$
$$=180°-45°-30°=105°$$

Now, by law of sines



$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$$

$$\frac{a}{\sin 150°}=\frac{b}{\sin 45°}=\frac{c}{\sin 30°}$$

$$\frac{a}{\sin 75°}=\frac{b}{\sin 45°}=\frac{c}{\sin 30°}$$

[sin 105° = sin(180° – 75°) = sin 75°]

Taking $\frac{a}{\sin 75°}=\frac{b}{\sin 45°}$

$$=\frac{a\sin 45°}{\sin 75°}=\frac{6\times(0.71)}{0.97}=\frac{4.26}{0.97}4.4cm$$

Now, $\frac{a}{\sin 75°}=\frac{c}{\sin 30°}$

$$c=\frac{a\times\sin 30°}{\sin 75°}.=\frac{6\times(0.5)}{0.97}=\frac{3.0}{0.97}=3.1cm$$

Skilled Practice!

Solved ΔABC in which

$\alpha = 77°$, $\beta = 66°$, $a = 2$cm

Sol: $a : 77°$, B = 66°, a = 2am

we know

$$\alpha+\beta+\gamma=180°$$
$$\gamma=180°-\alpha-\beta$$
$$=180°-77°-66°$$
$$=37°$$

By law of sines

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}$$

$$\frac{2}{\sin 77°}=\frac{b}{\sin 66°}$$

$$\sin 66°\times\frac{2}{\sin 77°}=b$$

$$\frac{0.9135\times 2}{0.9744}=b$$

$$1.87=b$$
$$b=1.87cm$$

$$\frac{a}{\sin a}=\frac{c}{\sin\gamma}$$

$$\frac{2}{\sin 77°}=\frac{c}{\sin 37°}$$

$$\frac{\sin 37°\times 2}{\sin 77}=c$$

$$\frac{0.6018\times 2}{0.9744}=c$$

$$1.24=c$$
$$c=1.24cm$$

So, $r = 37°$, b = 1.87cm, c = 1.24cm

Challenge!

Can we use law of sines, when two sides and included angle or the three sides of a triangle are given?

Sol. No, not directly if you have sides a and b and angle between them. You lack d for side α and γ for side. No pair is complete.

If you have only sides a,b L and C you do not have any angle to start the ratio

The Law of cosines

If a, b and c are the sides and α, β and γ are the angles opposite to the sides of any triangle respectively, then the law of cosines is:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma \text{ or } \cos\gamma = \frac{a^2+b^2-c^2}{2ab}$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \text{ or } \text{cas}\,\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$b^2 = c^2 + a^2 - 2ca\cos\beta \text{ or } \text{cas}\,\beta = \frac{c^2+a^2-b^2}{2ca}$$

Example 4: In ΔABC, $a = 3, b = 5, c = 7$. Find the values of α, β and γ.

Solution: Given that $a = 3, b = 5, c = 7$

By using the cosines law, we have

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc} = \frac{5^2+7^2-3^2}{2\times5\times7} = \frac{25+49-9}{70} = \frac{13}{14}$$

$$\alpha = \cos^{-1}\left(\frac{13}{14}\right) = 21.8^\circ$$

$$\cos\beta = \frac{a^2+c^2-b^2}{2ac} = \frac{3^2+7^2-5^2}{2\times3\times7} = \frac{9+49-25}{42} = \frac{11}{14}$$

$$\beta = \cos^{-1}\left(\frac{11}{14}\right) = 38.2^\circ$$

As $\alpha + \beta + \gamma = 180^\circ$

$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 21.8^\circ \text{ of} - 38.2^\circ = 120^\circ$

Area a Triangle

a. When two sides and their included angle of a triangle are given, then

$$\text{Area of } \Delta ABC = \frac{1}{2} bc\sin\alpha = \frac{1}{2} ac\sin\beta = \frac{1}{2} ab\sin\gamma$$

Example 5: Calculate area of the triangle ABC in which $a = 4.2$, $b = 3.2$ and $\gamma = 70^\circ$.

Solution: Given that $a = 4.2, b = 3.2, \gamma = 70^\circ$

$$\text{Area of } \Delta ABC = \frac{1}{2} ab\sin\gamma = \frac{1}{2}(4.2)(3.2)\sin 70^\circ$$

$$= \frac{1}{2}(4.2)(3.2)(0.94) = 6.31 \text{ square units}$$

b. When three sides of a triangle are given, then

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{a+b+c}{2}$

Example 6: Calculate area of ΔABC in which $a = 4$cm, b = 3cm, c = 5cm

Solution: Given that $a = 4$cm, b = 3cm, c = 5cm

$$s = \frac{a+b+c}{2} = \frac{4+3+5}{2} = \frac{12}{2} = 6$$
$$s - a = 6 - 4 = 2$$
$$s - b = 6 - 3 = 3$$
$$s - c = 6 - 5 = 1$$
$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{6(2)(3)(1)} = \sqrt{36}$$
$$= 6\text{cm}^2$$

Skilled Practice!

Find area of ΔABC, in which

(i) a = 8 cm, b = 9cm, c = 10cm **(ii) a = 6.1cm, b = 2.3 cm. γ = 80°**

Sol. (i) $s = \dfrac{a+b+c}{2} = \dfrac{8+9+10}{2} = \dfrac{27}{2} = 13.5cm$

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{13.5(13.5-8)(13.5-9)(13.5-10)}$$
$$= \sqrt{13.5(5.5)(4.5)(3.5)}$$
$$\sqrt{1169.4375} = 34.20\text{cm}^2$$

(ii) $a = 6.1$cm, b = 2. 3cm, $\gamma = 80°$

$$\text{Area} = \frac{1}{2}\, ab \sin\gamma$$
$$= \frac{1}{2} \times (6.1)(2.3)\sin(80°)$$
$$= (7.015)(0.984)$$
$$= 6.91\text{cm}^2$$

c. When two angles and one side of a triangle are given, then.

$$\text{Area of } \Delta ABC = \frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha} = \frac{b^2 \sin\alpha \sin\gamma}{2\sin\beta} = \frac{c^2 \sin\alpha \sin\beta}{2\sin\gamma}$$

Example 7: Calculate area of triangle ABC in which $\alpha = 35°$, $\beta = 65°$, $a = 2$cm

Solution: As we know that

$$\alpha + \beta + \gamma = 180°$$
$$35° + 65° + \gamma = 180°$$
$$\gamma = 180° - 100° = 80° = 80°$$

$$\text{Area of } \Delta ABC = \frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha}$$

$$= \frac{(2)^2 \sin 65^\circ \sin 80^\circ}{2\sin\alpha}$$

$$= \frac{4(0.91)(0.98)}{2\sin 35^\circ}$$

Skilled Practice!

Calculate area of ΔABC, in which

$\beta = 50^\circ, \gamma = 40^\circ$, c = 10cm

Sol. $\beta = 50^\circ$, $\gamma = 40^\circ$, c = 10cm

$$\alpha + \beta + \gamma = 180^\circ$$

$$\alpha = 180^\circ - \beta - \gamma$$

$$= 180 - 50^\circ - 40^\circ = 90^\circ$$

$$\text{Area} = \frac{c^2 \sin(\alpha)\sin(\beta)}{2\sin(\gamma)}$$

$$= \frac{10^2 \sin(90^\circ)\sin(50^\circ)}{2\sin(40^\circ)}$$

$$\text{Area} = \frac{100\times1\times0.766^\circ}{2\sin(40)}$$

$$= \frac{76.60}{1.2856} = 59.58\text{cm}^2$$

Solved Exercise 7.2

1. Solve the triangle ABC, in which

(i) $a = 6.1$ cm, b = 8.4 cm, $\alpha = 42^\circ$

Sol: $a = 6.1$ cm, b = 8.4 cm, $\alpha = 42^\circ$

use law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\frac{6.1}{\sin 42^\circ} = \frac{8.4}{\sin\beta}$$

$$\sin\beta = \frac{8.4\times\sin 42^\circ}{6.1}$$

$$\sin\beta = 0.92$$

$$\beta = \sin^{-1} 0.92$$

$$\beta = 67.1^\circ$$

$$\alpha + \beta + \gamma = 180^\circ$$

$$\gamma = 180^\circ - \alpha - \beta$$

$$= 180 - 42^\circ - 67.1^\circ = 70.9^\circ$$

use law of sines

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\sin\gamma \times \frac{b}{\sin\beta} = c$$

$$\frac{\sin 70.9^\circ \times 8.4}{\sin 67.1^\circ} = c$$

$$\frac{0.94\times8.4}{0.92} = c$$

$$8.b = c$$

So, c = 8.b cm, $\beta = 67.1^\circ$, $\gamma = 70.9^\circ$

(ii) $a = 12.2$ cm , $c = 15.8$ cm, $\gamma = 50$

Sol. $a = 12.2$ cm , $c = 15.8$ cm, $\gamma = 50$

use law of sines

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$\frac{12.2}{\sin\alpha} = \frac{15.8}{\sin 50}$$

$$\sin 50° \times \frac{12.2}{15.8} = \sin\alpha$$

$$\frac{0.76 \times 12.2}{15.8} = \sin\alpha$$

$$0.59 = \sin\alpha$$

$$\sin\alpha = 0.59$$

$$\alpha = \sin^{-1}(0.59)$$

$$\alpha = 36.3°$$

$$\alpha + \beta + \gamma = 180°$$

$$\beta = 180 - \alpha - \gamma$$

$$\beta = 180 - 36.3° - 50°$$

$$\beta = 93.7°$$

use law of sine

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\frac{b}{\sin 93.7} = \frac{15.8}{\sin 50}$$

$$b = \frac{15.8}{\sin 50°} \times \sin 93.8°$$

$$b = \frac{15.8}{0.76} \times 0.998$$

$$b = 20.6 \text{ cm}$$

So, $b = 20.6$ cm, $\alpha = 36.3°$, $\beta = 93.7°$

(iii) $b = 5.2$ cm, $c = 5$cm $\gamma = 48°$

Sol. $b = 5.2$ cm, $c = 5$cm, $\gamma = 48°$

use law of sines

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\frac{5.2}{\sin\beta} = \frac{5}{\sin 48°}$$

$$\frac{\sin 48° \times 5.2}{5} = \sin\beta$$

$$\sin\beta = \frac{\sin 48° \times 5.2}{5}$$

$$\sin\beta = \frac{0.74 \times 5.2}{5}$$

$$\sin\beta = 0.77$$

$$\beta = \sin^{-1} 0.77$$

$$\beta = 50.6°$$

As we know

$$\alpha + \beta + \gamma = 180°$$

$$\alpha = 180° - \beta - \gamma$$

$$= 180° - 50.4° - 48°$$

$$= 81.4°$$

By law of sines

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$\frac{a}{\sin 81.4} = \frac{5}{\sin 48}$$

$$a = \frac{5}{\sin 48} \times \sin 81.4$$

$$a = \frac{5}{0.74} \times 0.99$$

$$a = 6.7 \text{ cm}$$

So, $a = 6.7$ cm, $\alpha = 81.4°$, $\beta = 50.6°$

(iv) $b = 4.8$ cm, $a = 4$ cm , $\beta = 71°$

Sol: $b = 4.8$ cm, $a = 4$ cm , $\beta = 71°$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\frac{4}{\sin\alpha} = \frac{4.8}{\sin 71°}$$

$$\frac{\sin 71° \times 4}{4.8} = \sin\alpha$$

$$\frac{0.95 \times 4}{4.8} = \sin\alpha$$

$$0.79 = \sin\alpha$$

$$\sin\alpha = 0.79$$

$$\alpha = \sin^{-1} 0.79$$

$$\alpha = 52°$$

we know

$$\alpha + \beta + \gamma = 180°$$
$$\gamma = 180° - \alpha - \beta$$
$$= 180° - 52° - 71°$$
$$= 57°$$

By law of sine

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$
$$\frac{4.8}{\sin 71°} = \frac{c}{\sin 57°}$$
$$\sin 57° \times \frac{4.8}{\sin 71} = c$$
$$\frac{0.8 \times 4.8}{0.9} = c$$
$$4.3 = c$$

so, c = 4.3 cm , $\alpha = 52°$, $\gamma = 57°$

(v) $\beta = 70°$, b = 8cm , $\alpha = 100$

Sol. $\beta = 70°$, b = 8cm , $\alpha = 100$

use law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$
$$\frac{a}{\sin 100°} = \frac{8}{\sin 70°}$$
$$a = \frac{8}{\sin 70°} \times \sin 100°$$
$$a = \frac{8}{0.94} \times 0.99$$
$$a = 8.4 \text{ cm}$$

we know

$$\alpha + \beta + \gamma = 180°$$
$$\gamma = 180° - \alpha - \beta$$
$$\gamma = 180° - 100° - 70°$$
$$\gamma = 10°$$

use law of sines

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$
$$\frac{8}{\sin 70°} = \frac{c}{\sin 10°}$$
$$\sin 10° \times \frac{8}{\sin 70°} = c$$
$$0.17 \times \frac{8}{0.94} = c$$
$$1.5 = c$$
$$c = 1.5 \text{ cm}$$

So, So, a = 8.4 cm, c = 1.5 cm, $\gamma = 10°$

(vi) a = 6cm, $\alpha = 55°$, $\gamma = 60°$

Sol. a = 6cm, $\alpha = 55°$, $\gamma = 60°$

we know

$$\alpha + \beta + \gamma = 180°$$
$$\beta = 180° \alpha + \gamma$$
$$\beta = 180° - 55 - 60$$
$$\beta = 65°$$

By law of sine

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$
$$\frac{6}{\sin 55°} = \frac{6}{\sin 65°}$$
$$\sin 65° \times \frac{6}{\sin 55°} = b$$
$$0.91 \times \frac{6}{0.82} = b$$
$$6 = b$$
$$b = 6 \text{ cm}$$

By law of sines

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$
$$\frac{6.b}{\sin 65} = \frac{c}{\sin 60°}$$
$$\sin 60° \times \frac{6.b}{\sin 65} = c$$
$$\frac{0.87 \times 6.6}{0.91} = c$$
$$6.3 = c$$
$$c = 6.3 \text{ cm}$$

So, c = 6.4 cm, b = 6.7 cm, $\beta = 65°$

(vii) c = 7 cm, $\beta = 34°$, $\gamma = 64°$

Sol. c = 7 cm, $\beta = 34°$, $\gamma = 64°$

we know

$$\alpha+\beta+\gamma = 180°$$
$$\alpha = 180°-\beta-\gamma$$
$$\alpha = 180-34-64$$
$$\alpha = 82°$$

By law of sines

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$
$$\frac{b}{\sin 34°} = \frac{7}{\sin 64°}$$
$$b = \frac{7}{\sin 64°} \times \sin 34°$$
$$b = \frac{7}{0.89} \times 0.56$$
$$b = 4.4 \text{ cm}$$

By law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$
$$\frac{a}{\sin 82°} = \frac{4.4}{\sin 34°}$$
$$a = \frac{4.4}{\sin 34°} \times \sin 82°$$
$$a = \frac{4.4}{0.56} \times 0.99$$
$$a = 7.7 \text{ cm}$$

So, a = 7.7 cm, b = 4.4 cm, $\alpha = 82°$

(viii) b = 12 cm, $\alpha = 92°$, $\beta = 77°$

Sol. b = 12 cm, $\alpha = 92°$, $\beta = 77°$

We know

$$\alpha+\beta+\gamma = 180°$$
$$\gamma = 180°-92°-77$$
$$= 11°$$

By law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$
$$\frac{a}{\sin 92°} = \frac{12}{\sin 77°}$$
$$a = \frac{12}{\sin 77°} \times \sin 92°$$
$$a = \frac{12}{0.97} \times 0.999$$
$$a = 12.3 \text{ cm}$$

By law of sines

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$
$$\frac{12}{\sin 77°} = \frac{c}{\sin 11°}$$
$$\frac{\sin 11° \times 12}{\sin 77°} = c$$
$$\frac{0.19\times 12}{0.97} = c$$
$$2.4 = c$$
$$c = 2.4 \text{ cm}$$

So, a = 12.3 cm, c = 2.4 cm, $\gamma = 11°$

2. Calculate area of each triangle ABC.

(i) a = 7 cm, b = 8cm, $\gamma = 38°$

Sol. a = 7 cm, b = 8cm, $\gamma = 38°$

$$\text{Area} = \frac{1}{2}\, ab \sin\gamma$$
$$= \frac{1}{2}\,(7)(8)(\sin 38°)$$
$$= 28\,(0.6157)$$
$$= 17.24 \text{ cm}^2$$

(ii) a = 11cm, c = 14 cm, $\beta = 51°$

Sol. a = 11cm, c = 14 cm, $\beta = 51°$

$$\text{Area} = \frac{1}{2}\, a c \sin\beta$$
$$= \frac{1}{2}\,(11)(14)(\sin 51°)$$
$$= 77\,(0.7771)$$
$$= 59.84 \text{ cm}^2$$

(iii) b = 3m, c = 9 cm, $\alpha = 78°$

Sol. b = 3m, c = 9 cm, $\alpha = 78°$

$$\text{Area} = \frac{1}{2}\, b c \sin\alpha$$

$= \frac{1}{2}(3)(9)(\sin 78°)$

$= 13.5\,(0.9781)$

$= 13.20\text{ cm}^2$

(iv) a = 10 cm, α = 62°, β = 69°

Sol. $a = 10$ cm, $\alpha = 62°$, $\beta = 69°$

We know

$$\alpha + \beta + \gamma = 180°$$

$$\gamma = 180 - \alpha - \beta$$

$$= 180 - 62° - 69° = 49°$$

$$\text{Area} = \frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha}$$

$$= \frac{(10)^2(\sin 69°)(\sin 49)}{2\sin 62°}$$

$$= \frac{100(0.9336)(0.7547)}{2(0.8829)}$$

$$= \frac{70.4588}{1.7658} = 39.90\text{ cm}^2$$

(v) c = 4cm, β = 36°, γ = 80°

Sol. c = 4cm, β = 36°, γ = 80°

We know

$$\alpha + \beta + \gamma = 180°$$

$$\alpha = 180° - \beta - \gamma$$

$$= 180° - 36 - 80 = 64°$$

$$\text{Area} = \frac{c^2 \sin\alpha \sin\beta}{2\sin\gamma}$$

$$= \frac{(4)^2(\sin 64°)(\sin 36°)}{2(\sin 80°)}$$

$$= \frac{(16)(0.8988)(0.5878)}{2(0.9848)}$$

$$= \frac{8.4530}{1.9696} = 4.29\text{ cm}^2$$

(vi) c = 6.6 cm, α = 23°, γ = 89°

Sol. c = 6.6 cm, α = 23°, γ = 89°

We know

$$\alpha + \beta + \gamma = 180°$$

$$\beta = 180° - \alpha - \gamma$$

$$= 180° - 23° - 89° = 68°$$

$$\text{Area} = \frac{c^2 \sin\alpha \sin\beta}{2\sin\gamma}$$

$$= \frac{(6.6)^2(\sin 23°)(\sin 68°)}{2(\sin 89°)}$$

$$= \frac{43.56(0.3907)(0.9272)}{2(0.9998)}$$

$$= \frac{15.7799}{1.9996} = 7.89\text{ cm}^2$$

(vii) a = 5.3 cm, b = 4.7 cm, c = 8.2 cm

Sol. a = 5.3 cm, b = 4.7 cm, c = 8.2 cm

$a = 5.3$, $b = 4.7$, $c = 8.2$

$$S = \frac{a+b+c}{2}$$

$$= \frac{5.3+4.7+8.2}{2}$$

$$= \frac{18.2}{2} = 9.1$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{9.1(9.1-5.3)(9.1-4.7)(9.1-8.2)}$$

$$= \sqrt{9.1(3.8)(4.4)(0.9)}$$

$$= \sqrt{136.93}$$

$$= 11.70\text{ cm}^2$$

(viii) a = 6.12 cm, b = 8.34 cm, c = 7.12 cm

Sol. a = 6.12 cm, b = 8.34 cm, c = 7.12 cm

$$s = \frac{a+b+c}{2}$$

$$s = \frac{6.12+8.34+7.12}{2}$$

$$= \frac{21.58}{2} = 10.79$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{10.79(10.79-6.12)(10.79-8.34)(10.79-7.12)}$$
$$= \sqrt{10.79(4.67)(2.45)(3.67)}$$
$$= \sqrt{453.53} = 21.29cm^2$$

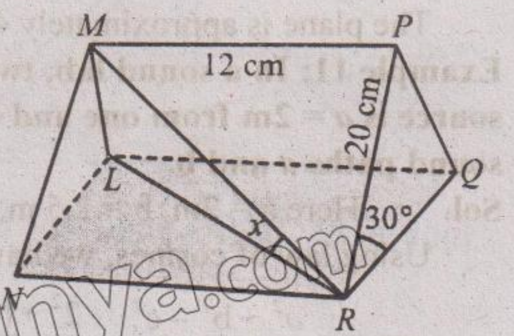
7.3 Solve Simple Trigonometric Problems in Three Dimensions

Example 8: The diagram shows a prism. Triangle PQR is a cross section of the prism, where

$m\overline{PR} = 20\text{cm}, \quad m\overline{MP} = 12\text{cm},$

$m\angle PRQ = 30°, \quad m\angle PQR = 90°$

Calculate the size of the angle that the line MR makes with the plane PQLN.



Sol. In right angled triangle PQR, we have

$$\frac{m\overline{PQ}}{20} = \sin 30°$$

$$m\overline{PQ} = 20 \times \frac{1}{2}$$

$$= 10 \text{ cm}$$

In right angled triangle MPR, we have

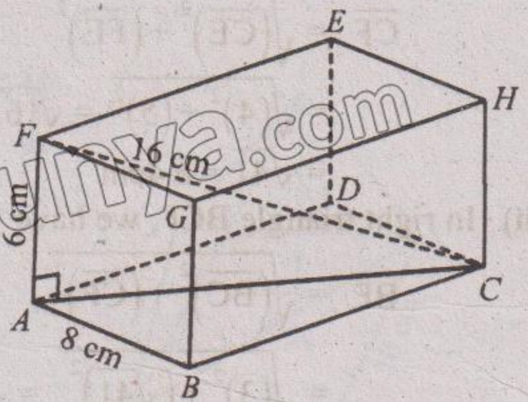
$$m\overline{MR} = \sqrt{\left(m\overline{PR}\right)^2 + \left(m\overline{MP}\right)^2} = \sqrt{(20)^2 + (12)^2} = \sqrt{400+144} = \sqrt{544}$$
$$= 23.3238 \text{ cm}$$

In right angled triangle MLR, we have

$$\sin x = \frac{\text{opp.}}{\text{hyp.}} = \frac{10}{23.3238} = 0.4287$$

$$\therefore \quad x = \sin^{-1}(0.4287)$$
$$= 25.4°$$

Example 9: The diagram shows a cuboid ABCDEFGH, $m\overline{AB}$ = 8cm, $m\overline{AF}$ = 6cm, $m\overline{FC}$=16 cm. Find the length of $m\overline{BC}$.



Sol. In right angled triangle AFC, we have

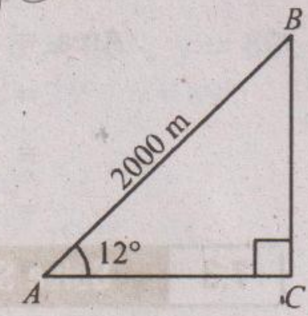
$$m\overline{AC} = \sqrt{(mFC)^2 - (mAF)^2} = \sqrt{(16)^2 - (6)^2}$$
$$= \sqrt{256-36} = \sqrt{220} = 14.83\text{cm}$$

In right angled triangle ABC, we have

$$m\overline{BC} = \sqrt{(14.83)^2 - (8)^2} = \sqrt{220-64}$$
$$= \sqrt{156} = 12.5\text{cm}$$

7.4 Concept of Trigonometry in Real Life

Example 10: A plane takes off at an angle of elevation of 12° and travels 2000 metres along this path. How high is the plane after traveling this distance?



Sol. In triangle ABC,

$$\sin 12° = \frac{m\overline{BC}}{2000}$$

$$(0.2079)2000 = m\overline{BC}$$

$$m\overline{BC} = 415.8 \text{ m}$$

The plane is approximately 415.8 m above the ground.

Example 11: In a sound lab, two microphones are placed c = 3 m apart and a sound source is *a* = 2m from one and b = 2.5 m from the other. Find the angle between the sound paths *a* and b.

Sol. Here a = 2m, b = 2.5 m, c = 3 m

Using law of cosines, we have

$$\cos\gamma = \frac{a^2+b^2-c^2}{2ab} = \frac{2^2+(2.5)^2-3^2}{2\times2\times2.5} = \frac{4+6.25-9}{10} = 0.125$$

$$\gamma = \cos^{-1}(0.125)$$

$$= 82.8°$$

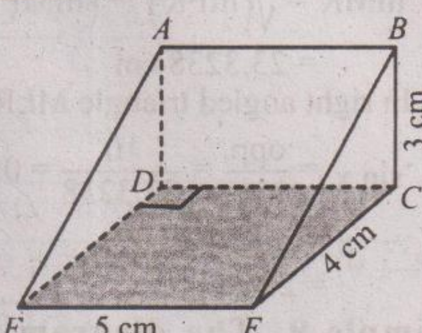
Solved Exercise 7.3

1. In the triangular prism, find

(i) the length $\overline{CF}$.

(ii) the length $\overline{BF}$.

(iii) the angle BFC, correct to one decimal place.



Sol.(i) $\overline{BC} = 3\text{cm}, \overline{CE} = 4\text{cm}, \overline{FE} = 5\text{cm}$

In right triangle CEF, we have

$$\overline{CF} = \sqrt{(\overline{CE})^2 + (\overline{FE})^2}$$

$$= \sqrt{(4)^2+(5)^2} = \sqrt{16+25}$$

$$= \sqrt{41} = 6.4\text{cm}$$

(ii) In right triangle BCF, we have

$$\overline{BF} = \sqrt{(\overline{BC})^2 + (\overline{CF})^2}$$

$$= \sqrt{(3)^2 + (\sqrt{41})^2} = \sqrt{9+41}$$

$$= \sqrt{50} = 7.1 \text{ cm}$$

(iii) In right triangle BCF, we have

$$\tan\theta = \frac{\text{opposite}}{\text{Adjecent}}$$

$$\tan\theta = \frac{\overline{BC}}{\overline{CF}}$$

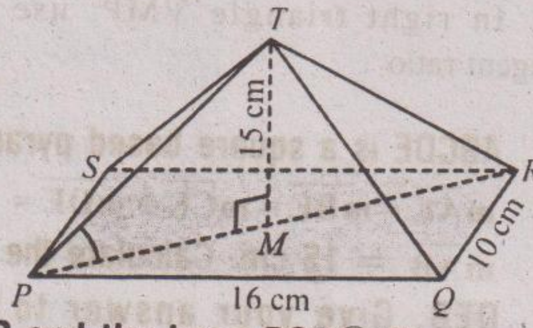
$$\tan\angle BFC = \frac{3}{\sqrt{41}}$$

$$\angle BFC = \tan^{-1}\left(\frac{3}{\sqrt{41}}\right)$$

$$\angle BFC = \tan^{-1}\left(\frac{3}{6.403}\right)$$

$$= 25.1^\circ$$

2. The diagram shows a pyramid with a horizontal rectangular base PQRS, in which $m\overline{PQ} = 16$ cm, $m\overline{QR} = 10$ cm. M is the midpoint of the line PR. The vertex, T, is vertically above M and $m\overline{MT} = 15$ cm. Calculate the size of the angle between TP and the base PQRS. Give your answer correct to 1 decimal place.



Sol. Rectangular base PQRS,

$$\overline{PQ} = 16 \text{ cm}$$

$$\overline{QR} = 10 \text{ cm}$$

vertical height $\overline{MT} = 15$ cm

M is the midpoint of diagonal PR

In right triangle PQR, we have

$$\overline{PR} = \sqrt{(\overline{PQ})^2 + (\overline{QR})^2}$$

$$= \sqrt{(16)^2 + (10)^2}$$

$$= \sqrt{256 + 100}$$

$$PR = \sqrt{356} = 18.868 \text{ cm}$$

M is the mid point of $\overline{PR}$

$$\overline{PM} = \frac{\overline{PR}}{2} = \frac{18.868}{2} = 9.434 \text{ cm}$$

In right triangle TMP, we have

$$\tan\theta = \frac{\text{opposite}}{\text{Adjacent}}$$

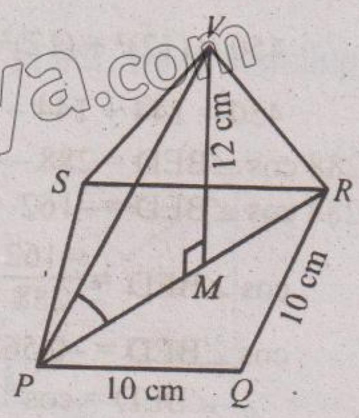
$$\tan\theta = \frac{\overline{TM}}{\overline{PM}}$$

$$\tan\angle TMP = \frac{15}{9.434}$$

$$\angle TMP = \tan^{-1}(1.590)$$

$$= 57.8^\circ$$

3. The diagram shows a pyramid. The base, PQRS, is a horizontal square of side 10 cm. The vertex, V, $m\overline{VM} = 12$ cm. Calculate the size of angle VPM.



Sol. In right triangle PQR we have

$$\overline{PR} = \sqrt{(\overline{PQ})^2 + (\overline{QR})^2}$$

$$= \sqrt{(10)^2 + (10)^2}$$

$$= \sqrt{100 + 100}$$

$$= \sqrt{200}$$
$$= \sqrt{4 \times 50}$$
$$= 2\sqrt{50}$$

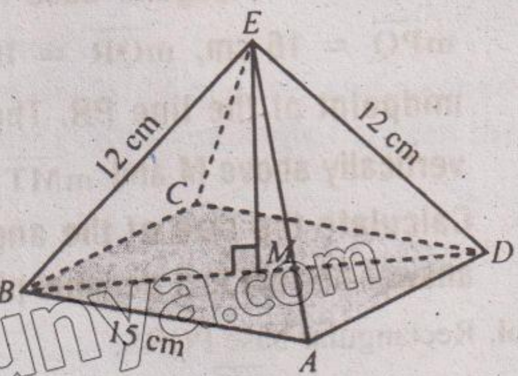
$\overline{PM}$ is the half of $\overline{PR}$

$$\overline{PM} = \frac{\overline{PR}}{2} = \frac{2\sqrt{50}}{2} = \sqrt{50}\text{ cm}$$

In right triangle VMP, use the tangent ratio

$$\tan\theta = \frac{\text{opposite}}{\text{Adjacent}}$$
$$\tan\angle VPM = \frac{\overline{VM}}{\overline{PM}}$$
$$\tan\angle VPM = \frac{12}{\sqrt{50}}$$
$$\angle VPM = \tan^{-1}\left(\frac{12}{7.071}\right)$$
$$\angle VPM = 59.5^\circ$$

4. ABCDE is a square based pyramid, in which $m\overline{AE} = m\overline{BE} = m\overline{CE} = m\overline{DE} =$ 12 cm and $m\overline{AB}$ = 15 cm. Calculate the size of angle DEB. Give your answer to the nearest degree.



Sol. In isosceles triangle DEB, we have

$m\overline{EB} = m\overline{ED} = 12\text{cm}$

$\overline{BD}$ is the diagonal of the square base

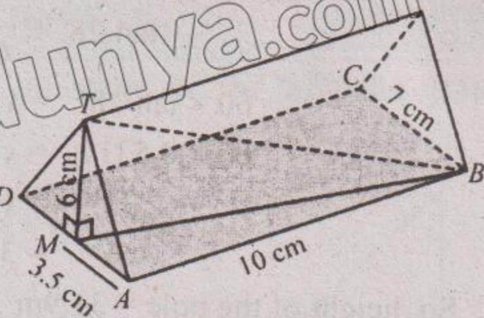
$m\overline{AB} = 15\text{cm}$

$$m\overline{BD} = \sqrt{(\overline{BA})^2 + (\overline{AD})^2}$$
$$= \sqrt{(15)^2 + (15)^2}$$
$$= \sqrt{225 + 225}$$
$$= \sqrt{450} = 21.21\text{ cm}$$

use law of cosine

$$(\overline{BD})^2 = (\overline{BE})^2 + (\overline{DE})^2 - 2(\overline{BE})(\overline{DE})\cos(\angle BED)$$
$$450 = (12)^2 + (12)^2 - 2\,(12)\,(12)\cos(\angle BED)$$
$$450 = 144 + 144 - 288\cos(\angle BED)$$
$$288\cos\angle BED = 288 - 450$$
$$288\cos\angle BED = -162$$
$$\cos\angle BED = \frac{-162}{288}$$
$$\cos\angle BED = -0.5625$$
$$\angle BED = \cos^{-1}(-0.5625)$$
$$\angle BED = 124^\circ$$

5. The diagram shows a triangular prism with a horizontal rectangular base ABCD. $m\overline{AB} = 10cm$, $m\overline{BC} = 7$ cm, M is the midpoint of AD. the vertex T is vertically above M and $m\overline{MT} = 6cm$. Calculate the size of the angle between $\overline{TB}$ and the base.



Sol. M is the mid point of $\overline{AD}$

$m\overline{MA} = 3.5$ cm, $m\overline{AB} = 10$ cm

In right triangle BMA, we have

$$m\overline{MB} = \sqrt{(\overline{MA})^2 + (AB)^2}$$
$$= \sqrt{(3.5)^2 + (10)^2}$$
$$= \sqrt{12.25 + 100}$$
$$= \sqrt{112.25} = 10.59 \text{ cm}$$

in right triangle TMB, we have

$$\tan\theta = \frac{\overline{TM}}{MB}$$
$$\tan\angle TBM = \frac{6}{10.59}$$
$$\angle TBM = \tan^{-1}\left(\frac{6}{10.59}\right)$$
$$\angle TBM = 29.5°$$

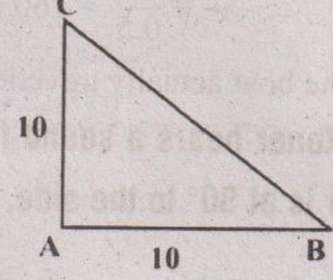
6. In an isometric game, the camera is placed at a 45° angle from the ground. If the player is 10 units in front and 10 units above, what is the direct line of sight distance?

Sol. Let front distance $\overline{AB} = 10$ units

Height from the ground $\overline{AC} = 10$ units

Let Direct line sight $\overline{BC} = ?$ (Hypotenuse)

By Pythagoreas theorem



$$m\overline{BC} = \sqrt{(m\overline{AB})^2 + (m\overline{AC})^2}$$
$$= \sqrt{(10)^2 + (10)^2}$$
$$= \sqrt{100 + 100} = \sqrt{200}$$
$$= 14.14 \text{ units}$$

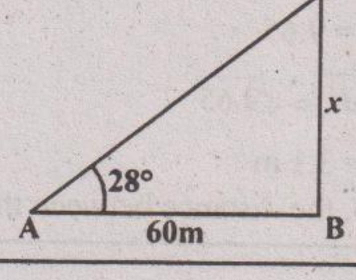
7. A surveyor spots the top of a tower at an elevation angle of 28°. He is standing 60 metres from the base. Find the height of the tower.

Sol.

Let Height of the tower $= x$ m

Distance of base $= 60$ m

Angle of elevation $= 28°$



$$\tan\theta = \frac{\text{opposite}}{\text{Adjacent}}$$

$$\tan 28^\circ = \frac{x}{60}$$

$$60 \times \tan 28^\circ = x$$

$$60 \times 0.5317 = x$$

$$31.9 = x$$

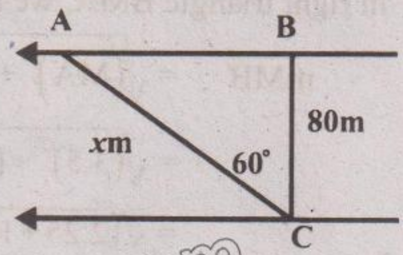
$$x = 31.9\text{m}$$

So, height of the pole = 31.9m

8. A boat crosses a river 80 m wide, at a 60° angle to the current. What distance does the boat actually travel?

Sol. Width of the river = 80 m

Angle to current = 60°



Let distance travelled by boat = x m (Hypotenuse)

$$\cos\theta = \frac{\text{Adjacent}}{\text{hypotenuse}}$$

$$\cos 60^\circ = \frac{80}{x}$$

$$0.5 = \frac{80}{x}$$

$$x = \frac{80}{0.5} = 160 \text{ m}$$

So, the boat actually travelled the distance = 160 m

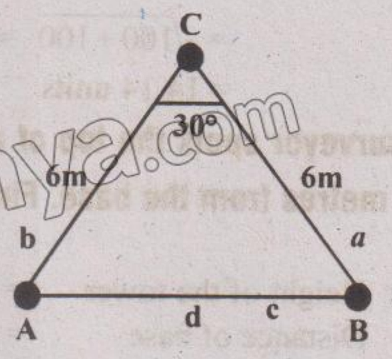
9. A listener hears a sound from two speakers. One is 6 m directly ahead and the other is at 30° to the side, 6 m away. Find the distance between the speakers.

Sol.

This forms a triangle with sides 6m, 6m and an included angle of 30°.

Let distance between speakers A and B = d

use Law of cosines



$$c^2 = a^2 + b^2 - 2ab\cos 30^\circ$$

$$d^2 = (6)^2 + (6)^2 - 2(6)(6)(0.866)$$

$$d^2 = 36 + 36 - 72(0.866)$$

$$d^2 = 72 - 62.35$$

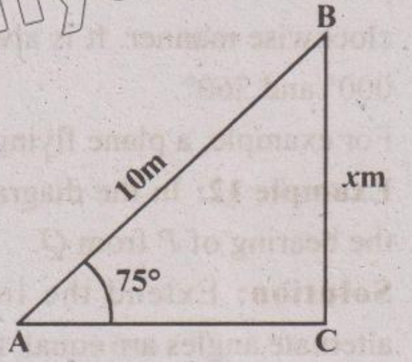
$$d^2 = 9.65$$

$$\sqrt{d^2} = \sqrt{9.65}$$

$$d = 3.1 \text{ m}$$

So, the distance between the speakers = 3.1 m

10. A 10 m ladder leans against a wall making an angle of 75° with the ground. How height does it reach up the wall?



Sol. Let the height of wall, where the ladder

Let the height where the ladder reach up the wall = x m

for right triangle use ratio of sine

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 75^\circ = \frac{x}{10}$$

$$0.966 = x$$

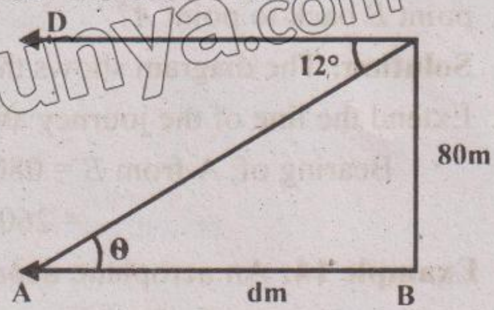
$$(0.966)(10) = x$$

$$9.66 = x$$

$$x = 9.66 \text{ m}$$

So, the ladder reaches up the wall at height of 9.66m

11. A lighthouse is located on a cliff 80 m above sea level. A ship is spotted at an angle of depression of 12°. How far is the ship from the base of the cliff?



Sol. Height of light house above the sea level=80 m

angle of depression = 12°

angle of depression = angle of elevation

So, angle of elevation = 12°

Let the distance of ship from the base of cliff = dm

use the trigonometric ratio tangent

$$\tan\theta = \frac{\text{opposite}}{\text{Adjacent}}$$

$$\tan(12)^\circ = \frac{80}{d}$$

$$0.21256 = \frac{80}{d}$$

$$d = \frac{80}{0.21256}$$

$$d = 376.4 \text{ m}$$

So, the distance of ship from the base of cliff = 376.4 m

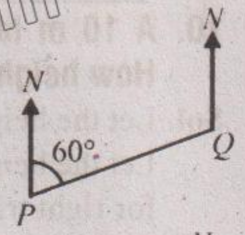
7.5 Three Figure Bearing and its Applications

Do you know?

Cardinal directions are the four main points on a compass: North, South, East and West.

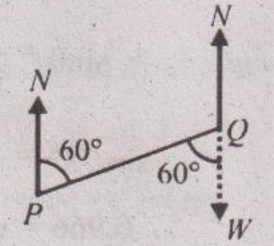
Ordinal directions are the four inter-cardinal points of a compass: NE, SE, SW, NW

A bearing is a way of describing the direction or angle from one point to another, measured in degrees from the north direction in a clockwise manner. It is always written as a three-digit angle between 000° and 360°.



For example, a plane flying east has a bearing of 090°.

Example 12: In the diagram, the bearing of Q from P is 060°. Find the bearing of P from Q.



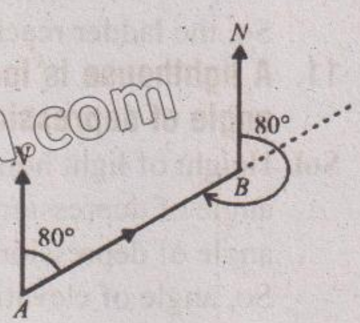
Solution: Extend the line NQ, so that angle NQW = 180° As alternate angles are equal, therefore

$$060° = m\angle PQW$$

So, the bearing of P from Q $= 180° + 060°$

$= 240°$

Example 13: Abdullah sails a ship from point A to point B on a bearing of 080°. What bearing should he follow to return from point B back to point A?



Solution: The diagram shows the journey from A to B.

Extend the line of the journey and we get an angle of 80° at B.

Bearing of, A from B = 080° + 180°

= 260°

Example 14: An aeroplane departs from an airport and travels 40 *km* due east, followed by 50 *km* due north. The pilot then returns to the airport along the shortest possible route. What is the bearing and the distance of the shortest route?

Solution:

To find θ

$$\tan\theta = \frac{40}{50} = 0.8$$

$$\theta = \tan^{-1}(0.8)$$

$$= 38.66°$$

Bearing $= 180° + 38.66° = 218.66°$

Suppose x is the shortest route

$$x^2 = 40^2 + 50^2$$

$$= 1600 + 2500$$

$$= 4100$$

$$x = \sqrt{4100}$$

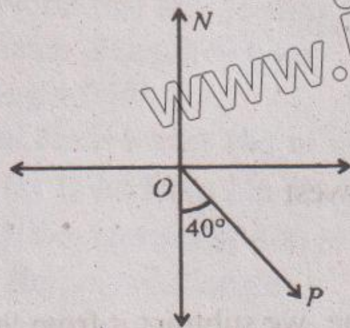
$$= 64.03 \text{ km}$$

The pilot will fly on a bearing of 218.66° for 64.03 km to return to the airport.

Solved Exercise 7.4

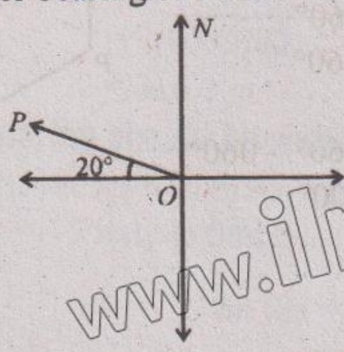
1. Find bearing of point P in each of the following:

(i)



Sol. The angle from North to South (a straight line) is exactly 180°.
Since P is 040° counter - clock wise fromt the south line,
we subtract that valve from 180°
So the bearing of Point $P = 180° - 040°$

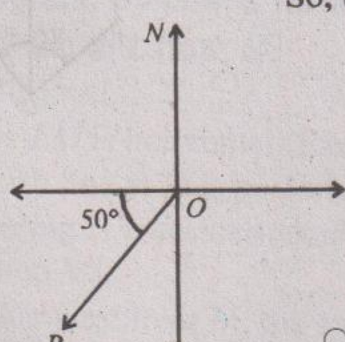
(ii)



Quadrant of Point P = Northwest

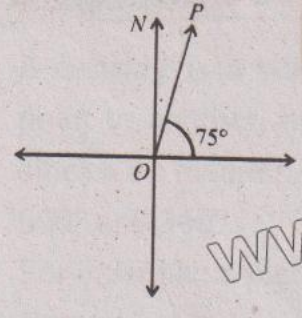
Sol. Measure of given angle from the west line = 020°
The angle from north clockwise to the west line = 270
Since P is 020° further clockwise = 270
from the west line, we add it to 270°
So, the bearing of Point P = 270° + 020° = 290°

(iii)



Quadrant of point P = Southwest

Sol. Measure of given angle from the west line = 050°
The angle from North clockwise to the West line = 270
Since P is behind the west line, we subtract it from 270°
So, the bearing of Point P = 270° – 050° = 220°

(iv) 

Quadrant of point P = Southwest

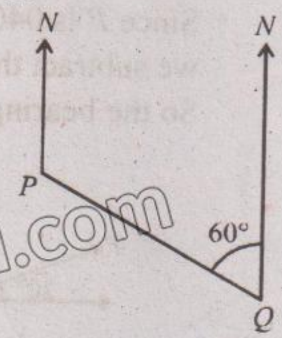
Sol. Measure of given angle from the west line = 075°

The angle from North clockwise to the East line = 90°

Since P is 075° back (counter-clockwise) from the east line, we subtract it from 90°

So, the bearing of Point P = 090° − 075° = 015°

2. The diagram shows the positions of two ships P and Q.



(i) What is the bearing of ship P from ship Q?

Sol. Measure of angle to the left of North = 060°

Measure of angle of full circle = 360°

Since, we clockwise:

Bearing of Ship P from Sift Q = 360° − 060°

= 300°

(ii) What is bearing of ship Q from ship P?

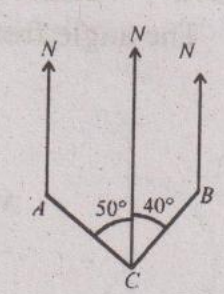
Sol. Since North lines are parallel = 060°

$m\angle P + m\angle Q = 180°$

$m\angle Q = 060°$

Bearing of Ship Q from ship P = 180° − 060° = 120°

3. The diagram shows 3 places A, B and C.



Find the bearing of:

(i) A from C

Sol. The line CA is 050° counter - clockwise from North.

Measure fo the angle of full circle = 360°

Bearing of A from C = 360° − 050° = 310°

(ii) C from A

Sol. A is now our starting point

The North line at A and North line at C are parallel.

So, angles are sum up to 180°.

Bearing of C from A = 180° ⊖ 050° = 130°

(iii) C from B

Sol. B is our starting point

The North line at B and North line at C are parallel so, interior angles are added up to 180.

Bearing of C from B = 180° + 040° = 220°

(iv) B from C

Sol. C is our starting point

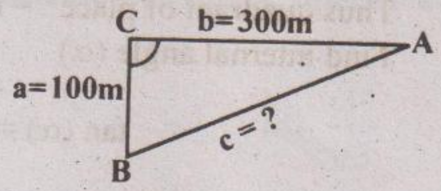
The North line at C leading to B is shown as being 040° to right (clockwise)

No match is required here,

Bearing = 040°

4. Abdul Hadi walks 100 m north and then 300 m east.

(i) How far is he from his starting position?



Sol. Walk100m North and 300 m to East forms a right-angled triangle

By Pythagores theorem

$$c^2 = a^2 + b^2$$

$$\text{Distance} = \sqrt{(100)^2+(300)^2}$$

$$= \sqrt{10000+90000}$$

$$= \sqrt{100,000}$$

$$= 316.2 \text{ m}$$

(ii) On what bearing should he walk to get back to his starting position?

Sol. To return, he must go from his current position (North-East of start) back to origin.

First we find θ

$$\tan(\theta) = \frac{300}{100}$$

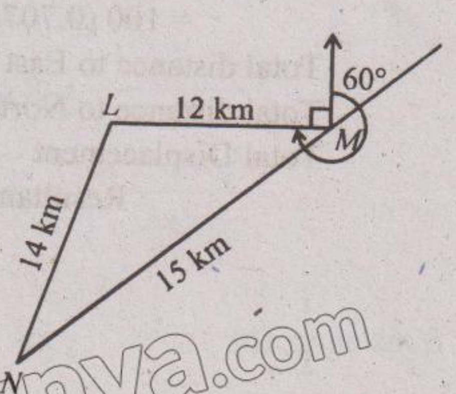
$$\theta = \tan^{-1}(3)$$

$$\theta = 71.6°$$

Current bearing from start = 71.6°

The return bearing = 71.6° + 180° = 251.6° or 252°

5. Three ships *L*, *M*, *N* are in the position shown in the diagram. Ship *M* is North East of ship *N*. Find the bearing of *L* from *M*.



Sol. From diagram

The line *LM* is horizontal (East=West)

The North line at *M* is perpendicular to *LM*

The bearing is measured clockwise from North line at *M* to the line *ML*.

LM is due west from M, the angle is simply 270°

Bearing of *L* from *M* = 270°.

6. A ship sails 20 km on a bearing of 120°. How far south has the ship moved from its original position.

Sol. Bearing of ship = 120°

Angle from South = 180° − 120°

= 060°

South ward distance $= 20 (\cos 060°)$

$= 20 \times 0.5$

$= 10$ km

7. Fatima walked south for 5.5 km and then turned west for 1.3 km. Calculate Huria's bearing from her starting point.

Sol. Walk to South = 5.5 km

Walk to West = 1.3 km

Thus quadrant of place = III

Find internal angle (α)

$$\tan(\alpha) = \frac{\text{opposite (West)}}{\text{Adjacent (South)}}$$

$$= \frac{1.3}{5.5}$$

$$a \tan(\alpha) = 0.236$$

$$\alpha = \tan^{-1}(0.236)$$

$$\alpha = 13.3$$

Bearing from her starting point $= 180° + 13.3°$

$= 193.3°$

8. An aircraft flies 150 km east, then 100 km northeast (45° from east). What is the total displacement?

Sol. Break the 100 km leg into components

East $= 100 (\cos 45°)$

$= 100 (0.7071) = 70.71$ km

North $= 100 (\sin 45°)$

$= 100 (0.7071) = 70.71$ km

Total distance to East $= 150 + 70.71 = 220.71$ km

Total distance to North $= 70.71$ km

Total Displacement

$$\text{Resultant} = \sqrt{(220.71)^2 + (70.7)^2}$$

$$= \sqrt{48712.9 + 4999.9}$$

$$= \sqrt{53712.8}$$

$= 231.76$ km or 231.8 km

9. A ship sails 100 km on a bearing of 045°, then changes course and sails 120 km on a bearing of 135°. Find the distance between the starting point and the final position.

Sol. Bearing for 100 km = 045°

Bearing for 120 km = 135°

Difference of bearin $= 135° - 045° = 090°$

This means path forms a right-angled triangle use Pythagoras

$$\text{Distance} = \sqrt{(100)^2 + (120)^2}$$

$$= \sqrt{10000+14400} = \sqrt{24400}$$

$$= 156.2 \text{ km}$$

Solved Review Exercise 7

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) What is the value of cot 60°?

(a) 0 (b) 1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

(ii) All trigonometric ratios positive?

(a) I–quadrant (b) II–quadrant (c) III–quadrant (d) IV–quadrant

(iii) cosesθ positive in:

(a) I & III–quadrants (b) I & IV–quadrants
(c) I & II–quadrants (d) I & IV–quadrants

(iv) sin (90° +θ) =

(a) $\sin\theta$ (b) $-\sin\theta$ (c) $\cos\theta$ (d) $-\cos\theta$

(v) tan (180 – θ) =

(a) $\tan\theta$ (b) $\cot\theta$ (c) $-\cot\theta$ (d) $-\tan\theta$

(vi) What is true about The law of sines is:

(a) $\frac{b}{sin\alpha} = \frac{\alpha}{sin\beta} = \frac{c}{sin\gamma}$ (b) $\frac{a}{sin\alpha} = \frac{c}{sin\beta} = \frac{b}{sin\gamma}$

(c) $\frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$ (d) $\frac{a}{sin\alpha} = \frac{a}{sin\beta} = \frac{a}{sin\gamma}$

(vii) The law of law of cosines?

(a) $\cos a = \frac{b^2+a^2-c^2}{2ab}$ (b) $\cos\beta = \frac{c^2+b^2-a^2}{2ab}$

(c) $\cos\gamma = \frac{a^2+b^2-c^2}{2ab}$ (d) $\cos\gamma = \frac{ca^2+b^2-c^2}{2ab}$

(viii) Area of ΔABC =

(a) $\frac{1}{2}ab\sin a$ (b) $\frac{1}{2}bc\sin B$ (c) $\frac{1}{2}\sin\gamma$ (d) $\frac{1}{2}ac\sin\beta$

(ix) Bearing is measured from?

(a) East (b) West (c) North (d) South

(x) Bearing is written as a:

(a) 1 figure (b) 2 figures (c) 3 figures (d) 4 figures

ANSWERS:

(i) $\frac{1}{\sqrt{3}}$ (ii) I–quadrant (iii) I & II–quadrants (iv) $\cos\theta$ (v) $-\tan\theta$

(vi) $\dfrac{b}{\sin\alpha} = \dfrac{b}{\sin B} = \dfrac{c}{\sin y}$ (vii) $\cos\gamma = \dfrac{a^2+b^2-c^2}{2ab}$ (viii) $\dfrac{1}{2}ac\sin\beta$ (ix) North

(x) 3 figures

2. Calculate the area of ΔABC, in which.

(i) $a = 4$ cm, b = 6 cm, c = 8 cm

Sol: $s = \dfrac{a+b+c}{2} = \dfrac{4+6+8}{2} = \dfrac{18}{2} = 9$

Area of ΔABC, $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{9(9-4)(9-6)(9-8)}$

$= \sqrt{9(5)(3)(1)} = \sqrt{135}$

$= 11.62\ cm^2$

(ii) $b = 2.1$ cm, c = 5 cm, $\gamma = 45°$

Sol: Area of ΔABC $= \dfrac{1}{2}\ b\,c\sin\gamma$

$= \dfrac{1}{2} \times 5 \times 2.1 \times \sin 45°$

$= 5.25 \times (0.7071)$

$= 3.71\ cm^2$

(iii) $c = 3.1$ cm, $\gamma = 44°$, $a = 36°$

Sol: We know

$\alpha + \beta + y = 180°$

$\beta = 180° - \alpha - \gamma$

$= 180° - 36° - 44°$

$= 100°$

Area of ΔABC $= \dfrac{c^2 \sin\alpha \sin\beta}{2\sin(\gamma)}$

$= \dfrac{(3.1)^2 \times \sin(36°) \times \sin(100°)}{2\sin(44°)}$

$= \dfrac{9.61 \times 0.5878 \times 0.9844}{2(0.6947)}$

$= \dfrac{5.5606}{1.3894} = 4.01$ cm

3. Solve the triangle ABC, in which

(i) $a = 5.4$ cm, b = 3.4 cm, $\alpha = 49°$

Sol. $a = 5.4$ cm, b = 3.4 cm, $\alpha = 49°$

use law of sines

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$

$\dfrac{5.4}{\sin 49°} = \dfrac{3.4}{\sin\beta}$

$\sin\beta = \dfrac{3.4}{5.4} \times \sin 49°$

$\sin\beta = 0.629 \times 0.7547$

$\sin\beta = 0.475$

$\beta = \sin^{-1}(0.475)$

$= 28.37°$

We know

$\alpha + \beta + \gamma = 180°$

$\gamma = 180° - \alpha - \beta$

$\gamma = 180° - 49° - 28.37°$

$\gamma = 102.63°$

$\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$

$\dfrac{5.4}{\sin 49°} = \dfrac{c}{\sin 102.63°}$

$\sin 102.63° \times \dfrac{5.4}{07547} = c$

$0.9758 \times \dfrac{5.4}{07547} = c$

$c = 6.98$ cm

So, $c = 6.98$ cm, $\beta = 28.37°$, $\gamma = 102.63°$

(ii) $\alpha = 32°$, $\gamma = 48°$, $c = 81$ cm

Sol: $\alpha = 32°$, $\gamma = 48°$, $c = 81$ cm

we know

$\alpha + \beta + \gamma = 180$

$\beta = 180° - \alpha - \gamma$

$= 180° - 32° - 48° = 100°$

$\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

$$\frac{b}{\sin 100^\circ} = \frac{81}{\sin 48^\circ}$$

$$b = \frac{81}{\sin 48^\circ} \times \sin 100^\circ$$

$$b = \frac{81}{0.743} \times 0.985$$

$b = 107.3\text{cm}$

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$\frac{a}{\sin 32^\circ} = \frac{81}{\sin 48^\circ}$$

$$a = \frac{81}{\sin 48} \times \sin 32$$

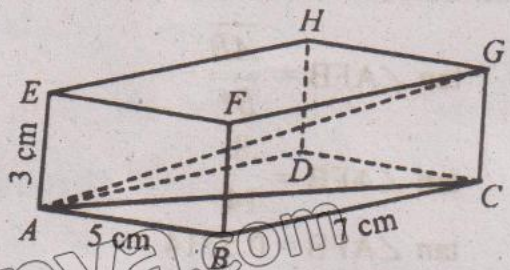
$$a = \frac{81}{0.743} \times 0.5299$$

$a = 57.8\text{cm}$

So, $\beta = 100^\circ$, a = 57.8 cm, b = 107.3cm

4. The diagram shows a cuboid *ABCDEFGH*.

$m\overline{AB} = 5$ cm, $m\overline{BC} = 7$ cm, $m\overline{AE} = 3$ cm



(i) Calculate the length of $\overline{AG}$. Give your answer correct to 3 significant figures.

Sol: AG is the space diagonal of cuboid.

$$AG = \sqrt{(\overline{AB})^2 + (\overline{BC})^2 + (\overline{AE})^2}$$

$$= \sqrt{5^2 + 7^2 + 3^2}$$

$$= \sqrt{83} \quad = 9.11\text{cm}$$

(ii) Calculate the size of the angle between $\overline{AG}$ and the plane *ABCD*. Give your answer correct to 1 decimal place.

Sol. The projection of $\overline{AG}$ onto the base is AC.

$$AC = \sqrt{5^2 + 7^2}$$

$$= \sqrt{25 + 49}$$

$$= \sqrt{74}$$

$= 8.60$ cm

$$\tan Q = \frac{\overline{AE}}{\overline{AC}}$$

$$= \frac{3}{8.60}$$

$\tan Q = 0.348$

$Q = \tan^{-1}(0.348)$

$Q = 19.2^\circ$

5. The diagram shows a triangular prism *ABCDEF*. $m\overline{AB} = 8$ cm, $m\overline{BF} = 14$ cm, $m\overline{EF} = 19$ cm.

(i) Calculate the distance between A and F

Sol: Looking at the right–angled triangle ABF

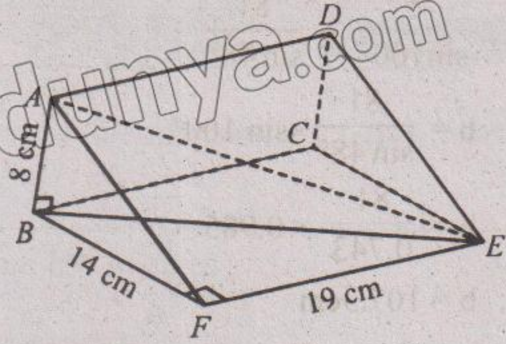
$$AF = \sqrt{(\overline{AB})^2 + (\overline{BF})^2}$$

$$= \sqrt{8^2 + 14^2}$$

$$= \sqrt{64 + 196}$$

$$= \sqrt{260}$$

$$= 16.1 \text{ cm}$$



(ii) Calculate the angle between $\overline{AF}$ and the plane *BCEF*.

Sol: Since a $\overline{AB}$ is vertical to the base plane BCEF, the angle is LAFB

$$\tan \angle AFB = \frac{\overline{AB}}{\overline{BF}}$$

$$\tan \angle AFB = \frac{8}{14}$$

$$\tan \angle AFB = 0.5714$$

$$\angle AFB = \tan^{-1}(0.5714)$$

$$\angle AFB = 29.7°$$

6. Hashim walks 4 km due North, then turns and walks 3 km due East. What is the bearing from the starting point to his final position?

Sol: walk to North = 4km

walk to East = 3km

This form a right triangle where the angle from North (θ) is.

$$\tan(\theta) = \frac{opposite}{Adjacant(North)}$$

$$\tan(\theta) = \frac{3}{4}$$

$$(\theta) = \tan^{-1}\left(\frac{3}{4}\right)$$

$$(\theta) = 036.9°$$

The bearing is written in three figures = 036.9°

7. A pilot flies 200 km on a bearing of 045°, then turns and flies 150 km on a bearing of 135°. How far is the plane from its original position?

Sol: Bearing of 200 km f flight = 045°

Bearing of 150 km k flight = 135°

The difference between bearing = 135°–045°

= 090°

This means two legs of journey are perpendicular

So Distance $= \sqrt{200^2 + 150^2}$

$= \sqrt{40000 + 22500}$

$= \sqrt{62500}$

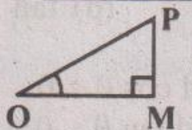
$= 250$ km

Objective Type Questions

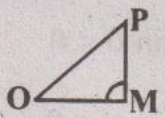
Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

7.1 Trigonometric Ratio or Functions

1. In ΔOPM  **$\overline{PM}$ which side is hypotenuse?**

(a) θ (b) $\overline{OM}$ (c) $\overline{OP}$ (d) $\overline{MP}$

2. In  **$\overline{PM}$ is opposite to θ. How is $\cos\theta$ calculated?**

(a) $\frac{\overline{OM}}{OP}$ (b) $\frac{\overline{PM}}{OP}$ (c) $\frac{\overline{OP}}{OM}$ (d) $\frac{\overline{OP}}{PM}$

3. $\sin\theta$ = ________?

(a) $\tan\theta$ (b) $\frac{1}{\cot\theta}$ (c) $\frac{1}{\text{cosec}\theta}$ (d) $\frac{1}{\cos\theta}$

4. $\cot\theta$ = ________?

(a) $\frac{1}{\sin\theta}$ (b) $\frac{1}{\cos\theta}$ (c) $\sin\theta$ $\frac{1}{\text{cosec}\theta}$ (d) $\frac{1}{\tan\theta}$

5. $\tan\theta$ = ________?

(a) $\frac{\sin\theta}{\cos\theta}$ (b) $\frac{\cos\theta}{\sin\theta}$ (c) $\frac{\sin\theta}{\tan\theta}$ (d) $\frac{1}{\cos\theta}$

6. $\frac{1}{\cos\theta}$ = ________?

(a) $\cot\theta$ (b) $\frac{\sin\theta}{\cos\theta}$ (c) $\sec\theta$ (d) $\tan\theta$

7. $\frac{\text{adj}}{\text{opp}}$ = ________.

(a) $\cot\theta$ (b) $\tan\theta$ (c) $\frac{1}{\cot}$ (d) $\frac{1}{\text{sim}\theta}$

7.1.1 Trigonometric values for special angles

8. What is the value of $\tan 45°$.

(a) $\sqrt{3}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{1}{2}$ (d) 1

9. **What is the value of sin 180°?**

(a) $\frac{1}{\sqrt{2}}$ (b) 0 (c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{2}$

10. $\frac{\sqrt{3}}{2}$ **is the value of:**

(a) sin60° (b) cot 60° (c) cos 45° (d) tan 180°

11. **For which the value is undefined?**

(a) tan 45° (b) tan 90° (c) tan 0° (d) tan 30°

12. **0 is the value of:**

(a) cos 45° (b) sin 30° (c) tan 90° (d) tan 180°

13. **In 2nd quadrant.**

(a) $\sin\theta < 0$ and $\cot\theta > 0$ (b) $\sin\theta > 0$ and $\cos\theta < 0$
(c) $\tan\theta > 0$ and $\cot\theta < 0$ (d) $\cos\theta > 0$ and $\tan\theta < 0$.

14. **In 3rd quadrant:**

(a) $\cot\theta < 0$ (b) $\sec\theta > 0$ (c) $\tan\theta < 0$ (d) $\sin\theta < 0$

15. **In 4th quadrant**

(a) $\sin\theta > 0$ (b) $\tan\theta < 0$ (c) $\cot\theta > 0$ (d) $\sec\theta < 0$

16. **In which quadrant only tan θ and cot θ are positive?**

(a) I (b) II (c) III (d) IV

17. **In which quadrants is cosθ negative**

(a) I and III (b) II and III (c) I and IV (d) III and IV

18. **What is the value of tan0°?**

(a) 1 (b) −1 (c) 0 (d) $\frac{1}{2}$

19. **What is the value of cosec 90°**

(a) 1 (b) −1 (c) 0 (d) $\frac{1}{2}$

20. **What is the value of sec 180°**

(a) 1 (b) 0 (c) $\sqrt{3}$ (d) −1

21. **Value of cot 180° is:**

(a) 1 (b) −1 (c) undefined (d) −2

22. **Value of sin 0° is:**

(a) 0 (b) 1 (c) −1 (d) $\frac{-1}{-2}$

7.1.4 Extending sine and cosine Functions to Angles Between 90° and 180°

23. **What is true about sec (90°+θ)?**

(a) −sin θ (b) −cosecθ (c) cosθ (d) cotθ

24. **What is true about tan (90° + θ)?**

(a) −cotθ (b) cot θ (c) −tan θ (d) −sinθ

25. **What is true about sin (180° − θ)?**

(a) −cosθ (b) cos θ (c) −sinθ (d) sin θ

26. **What is true about sec (180° − θ)?**

(a) sec θ (b) − sec θ (c) cot θ (d) sin θ

27. cosec (180° – θ) = __________?

(a) cosec θ (b) –sec θ (c) tan θ (d) tan θ

7.2.1 The Law of Sines

28. **The law of sines is useful for solving a triangle, when you are given:**

(a) Three sides only (b) Two angles and one side

(c) The area and one angle (d) Two sides and one angle

29. **Sin 105° = ________?**

(a) sin 75° (b) cos 50° (c) sin 95° (d) –2

30. **Sin 105° = ________?**

(a) cos 105° (b) cos 90° (c) cos 15° (d) 105°

31. **Which of the following is equivalent form of Law of sines?**

(a) $a \sin\alpha = b \sin\beta$ (b) $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

(c) $\frac{a}{b} = \frac{\sin a}{\sin\gamma}$ (d) $a^2 = b^2 + c^2 - 2bc\cos\beta$

32. **Which is not true about law of sines?**

(a) $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$ (b) $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$ (c) $\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ (d) $\frac{a}{\sin\beta} = \frac{b}{\sin\alpha}$

33. **Which is true about law of sines.**

(a) $a \sin\beta = bc$ (b) $a \sin\alpha = b$ (c) $b \sin\gamma = c \sin\beta$ (d) $c \sin d = b \sin\gamma$

7.2.2 The Law of cosines

34. **Which is true about law of cosines?**

(a) $\cos\beta = \frac{c^2 + a^2 - b^2}{2ca}$ (b) $\cos\beta = \frac{c^2 + a^2 - b^2}{2ab}$

(c) $\cos\beta = \frac{a^2 + b^2 - c^2}{2abc}$ (d) $\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$

35. **Which is true about law of cosines?**

(a) $\cos a = \frac{a^2 + b^2 - c^2}{2ca}$ (b) $\cos\beta = \frac{b^2 - c^2 + a^2}{2ca}$

(c) $\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$ (d) $\cos\gamma = \frac{a^2 - b^2 + c^2}{2ab}$

36. **$a^2 = b^2 + c^2 - 2bc\cos\alpha$ is called.**

(a) Law of sines (b) Law of cosines (c) Law of tangent (d) Law of cotagent

37. **Which is not true for law of cosines?**

(a) $b^2 = c^2 + a^2 - 2ca\cos B$ (b) $c^2 = a^2 + b^2 - 2ab\cos\gamma$

(c) $\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$ (d) $\cos\beta = \frac{c^2 - a^2 + b^2}{2ca}$

38. **If $\alpha = 70°$, $\beta = 40°$ then $\gamma = -$?**

(a) 80° (b) 70° (c) 60° (d) 90°

7.2.3 Area of a Triangle

39. Which is correct formula for area of a triangle when two sides and their included angles are given?

(a) $\frac{1}{2}$ bc sin α (b) $\frac{1}{2}$ bc sin β (c) $\frac{1}{2}$ ac sin α (d) $\frac{1}{2}$ ac sin γ

40. Which of the following not correct for area of a triangle

(a) $\frac{1}{2}$ bc sin α (b) $\frac{1}{2}$ ac sin β (c) $\frac{1}{2}$ ac sin α (d) $\frac{1}{2}$ ab sin γ

41. If a, b and c are the three sides of a triangle, then S for area is:

(a) $s=\frac{a+b-c}{2}$ (b) $s=\frac{a+b+c}{2}$ (c) 2s= a–b+c (d) $\frac{-a+b+c}{2}$

42. Area of ΔABC = – is:

(a) $\sqrt{s(s+a)(s+b)(s-c)}$ (b) $\sqrt{s(s+a)(s+b)(s+c)}$

(c) $\sqrt{s(s-a)(s-b)(s-c)}$ (d) $\sqrt{(a-b)(a-c)(a+b)}$

43. Which is true for area of a triangle?

(a) $\frac{b^2 \sin\alpha \sin\beta}{2\sin\beta}$ (b) $\frac{b^2 \sin\alpha \sin\gamma}{2\sin\beta}$ (c) $\frac{c^2 \sin\alpha \sin\beta}{2\sin\gamma}$ (d) $\frac{c^2 \sin\alpha \sin\beta}{2\sin\gamma}$

44. Which is not true for area of a triangle:

(a) $\frac{b^2 \sin\alpha \sin\beta}{2\sin\beta}$ (b) $\frac{b^2 \sin\alpha \sin\gamma}{2\sin\beta}$ (c) $\frac{c^2 \sin\alpha \sin\beta}{2\sin\gamma}$ (d) $\frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha}$

7.5 Three Figure Bearing and its Applications

45. In which direction is a bearing always measured?

(a) Counter – Clockwise (b) Towards nearest pole

(c) Clockwise (d) Depend on position

46. Which of the following is considered an "ordinal direction"?

(a) North (N) (b) South (s) (c) West (w) (d) Sout – west (SW)

47. Which of the following is considered a "cardinal direction"

(a) East (E) (b) NE (c) SE (d) NW

48. What is the maximum value a three- figure bearing can rach before resetting?

(a) 090° (b) 180° (c) 270° (d) 360°

49. If a point y is at a bearing of 200 from x, how would you calculate the bearing of x from y?

(a) 200°+180° (b) 200°–180° (c) 360°–180° (d) 360°–200°

ANSWERS:

1.	$\overline{OP}$	2.	$\frac{\overline{OM}}{OP}$	3.	$\frac{1.}{\text{cosec}\theta}$	4.	$\frac{1}{\tan\theta}$	5.	$\frac{\sin\theta}{\cos\theta}$	6.	sec θ

7. $\cot\theta$ 8. 1 9. 0 10. $\sin 60°$ 11. $\tan 90°$ 12. $\tan 180°$

13. $\sin\theta > 0$ and $\cos\theta < 0$ 14. $\sin\theta < 0$ 15. $\tan\theta < 0$

16. III 17. II and III 18. 0 19. 1 20. -1

21. undefined 22. -1 23. $-\text{cosec}\theta$ 24. $-\cot\theta$ 25. $\sin\theta$

26. $-\sec\theta$ 27. $\text{cosec}\,\theta$ 28. Two angles and one side 29. sim 75° 30. cos 15°

31. $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c}$ 32. $\dfrac{a}{\sin\beta} = \dfrac{b}{\sin\alpha}$ 33. $b\sin\gamma = c\sin\beta$

34. $\cos\beta = \dfrac{c^2 + a^2 - b^2}{2ca}$ 35. $\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$ 36. Law of cosines

37. $\cos\beta = \dfrac{c^2 - a^2 + b^2}{2ca}$ 38. 70° 39. $\dfrac{1}{2}bc\sin\alpha$ 40. $\dfrac{1}{2}bc\sin\alpha$

41. $s = \dfrac{a+b+c}{2}$ 42. $\sqrt{s(s-a)(s-b)(s-c)}$ 43. $\dfrac{b^2\sin\alpha\sin\gamma}{2\sin\beta}$

44. $\dfrac{b^2\sin\alpha\sin\beta}{2\sin\beta}$ 45. Clockwise 46. Sout – west (SW)

47. East 48. 360° 49. 200°–180°

Objective Type Questions

Additional Short Question

Short Answer Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

7.1 Trigonometric Ratio or Functions

1. Write the three primary trigonometric ratios for angle θ.

Sol. $\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$

$\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

$\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

2. In which trigonometric function all three primary ratios for θ are used?

Ans. The function for three primary ratios for θ is following:

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

3. **Write all six trigonometric functions.**

Sol. $\sin\theta = \dfrac{\text{opp.}}{\text{hyp.}}$, $\operatorname{cosec}\theta = \dfrac{hyp.}{opp.}$

$\cos\theta = \dfrac{\text{adj.}}{\text{hyp.}}$, $\sec\theta = \dfrac{\text{hyp.}}{\text{adj.}}$

$\tan\theta = \dfrac{\text{opp.}}{\text{adj.}}$, $\cot\theta = \dfrac{\text{adj.}}{\text{opp.}}$

4. **What are the trigonometric values of sin 30° and cos 60°.**

Ans. The value of sin 30° is $\dfrac{1}{2}$ and value of cos 60° is also $\dfrac{1}{2}$.

5. **Add values of sin 90°, cos 0° and tan 45°.**

Sol. $\sin 90° + \cos 0° + \tan 45°$.
$= 1 + 1 + 1$
$= 3$

6. **Subtract the value of cos 30° from the value of sin 180°.**

Sol. $\sin 180° - \cos 30°$
$= 0 - \dfrac{\sqrt{3}}{2} = -\dfrac{\sqrt{3}}{2}$

7. **Find the product sin (45°) cos (30°) (tan 45°).**

Sol. $\sin(45°) \times \cos(30°) \times \tan 45°$
$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} \times 1 = \dfrac{\sqrt{3}}{2\sqrt{2}}$

7.1.2 Signs of Trigonometric Functions.

8. **In which quadrant is sin $\theta > 0$ and cos < 0?**

Ans. $\sin\theta > 0$ and $\cos\theta < 0$ is in quadrant II. only sine is positive in 2nd quadrant.

9. **If tan $\theta > 0$ and sin $\theta < 0$, where is θ?**

Ans. $\tan\theta > 0$ means that $\tan\theta$ is positive and $\sin\theta < 0$ means that $\sin\theta$ is negative. So, it is located in quadrant III.

10. **Determine the sign of tan (315°).**

Ans. 315° lies in 4th quadrant, where only cosine is positive. So, tan (315°) has negetive sign.

11. **What is the sign of sin (210°).**

Ans. The sign of sin (210°) is negative as 210° lies in quadrant III where only $\tan\theta$ is positive.

12. **Determine the sign of cot (19°).**

Ans. The sign of cot (19°) is positive because 19° lies in first quadrant where all functions are positive.

7.1.3 Trigonometric Ratios of 0°, 90° and 180°

13. **What are the coordinates of point $P(x, y)$ on a unit circle when the angle is 90°?**

Ans. Since it lies on the positive y-axis, So $x = 0, y = 1$.

14. **Why is tan 90° considered "undefined"?**

Ans. Because $\tan\theta = \dfrac{y}{x}$. At 90°, $x = 0$ and division by zero is undefined.

15. Calculate the value of sin 90°+cos0°.

Ans. $\sin 90° + \cos 0°$
$= 1 + 1$
$= 2$

16. Calculate the value of $\cos 90° \times \tan 180° - \text{cosec } 90°$.

Sol. $\cos 90° \times \tan 180° - \text{cosec } 90°$
$= 0 \times 0 - 1 = 0 - 1$
$= -1$

7.1.4 Extending sine and cosine functions to angles between 90° and 180°

17. Without calculator, find the value of tan 120°.

Sol. $\tan 120° = \tan(180° - 60°) = -\tan 60° = -\sqrt{3}$.

18. Without calculator, find the value of cosec 135°.

Sol. $\text{cosec } 135° = \text{cosec}(180° - 45°) = \text{cosec } 45° = \sqrt{2}$.

19. Without calculator, find the value of sec 120°.

Sol. $\sec 120° = \sec(90° + 30°) = -\text{cosec } 30° = -2$

20. Without calculator, find the value of cos 150°.

Sol. $\cos 150° = \cos(90° + 60°) = -\sin 60 = -\frac{\sqrt{3}}{2}$.

7.2.1 The Law of sines

21. Define an oblique triangle.

Ans. An oblique triangle is any triangle that does not contain a right angle. It can be either acute triangle or obtuse triangle.

22. Write the formula for Law of sines.

Ans. If a, b and c are the sides and α, β, γ are angles of opposite sides of any triangle respectively, then the law of sine is:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

23. When law of sines can be used?

Ans. The law of sines can be used to solve a triangle, if two angles and one side which is opposite to one of the angles, are given.

24. If in a triangle side $a = 10$, angle $\alpha = 30°$, angle $\beta = 45°$, then find side b.

Sol. By law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\frac{10}{\sin 30°} = \frac{b}{\sin 45°}$$

$$\frac{10}{0.5} = \frac{b}{0.707}$$

$$0.707 \times \frac{10}{0.5} = b$$

$$14.14 = b$$

$$b = 14.14$$

25. In a triangle if c = 3 cm, $\beta = 60°$ and $\gamma = 30°$, then find b.

Sol. By law of sines

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\frac{b}{\sin 60°} = \frac{3}{\sin 30°}$$

$$\frac{b}{0.886} = \frac{3}{0.5}$$

$$b = \frac{3}{0.5} \times 0.866$$

$$b = 5.20 \text{ cm}$$

7.2.2 The Law of cosines

26. How do you arrange the law of cosines to solve for the cos one of angle α.

Sol. The arranged formula is:

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

27. If $a = 4$, $b = 7$, $c = 9$, then find the value of $\cos\beta$.

Sol. $\cos\beta = \dfrac{c^2 + a^2 - b^2}{2ca}$

$$= \frac{(9)^2 + (4)^2 - (7)^2}{2(9)(4)} = \frac{81 + 16 - 49}{72}$$

$$= \frac{48}{72} = 0.67$$

28. In a triangle if $b = 7$, $c = 5$ and $\cos\alpha = 22°$ then find a.

Sol. By law of cosine

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$a^2 = (7)^2 + (5)^2 - 2(7)(5)\cos(22°)$$

$$a^2 = 49 + 25 - 70(0.93)$$

$$a^2 = 74 - 65.1$$

$$a^2 = 8.9$$

$$\sqrt{a^2} = \sqrt{8.9}$$

$$a = 2.98 \approx 3\text{cm}$$

7.2.3 Area of a Triangle

29. What is the formula to find the area of a triangle when two sides and their included angle are given?

Ans. When two sides and their included angle are given, the formula to find area of Triangle is:

Area of $\Delta ABC = \frac{1}{2}\ bc \sin \alpha$

30. Calculate the area of a triangle ABC in which $a = 4$cm, $b = 3.5$cm and $\gamma = 60°$.

Sol. Area of $\Delta ABC = \frac{1}{2}\ ab \sin \gamma$

$$= \frac{1}{2}(4)(3.5) \sin 60°$$
$$= 2\ 3.5(0.866)$$
$$= 6.1 \text{ cm}^2$$

31. Write the formula to find the area of a triangle when three sides are given.

Ans. When three sides of a triangle are given, the formula to find the area of triangle is:

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

32. Find the area of a triangle ΔABC in which $a = 6$cm, $b = 9$cm and $c = 11$cm.

Sol. Given $a = 6$cm, $b = 9$cm, $c = 11$cm.

$$s = \frac{a+b+c}{2} = \frac{6+9+11}{2} = \frac{26}{2} = 13$$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{13(13-6)(13-9)(13-11)}$$
$$= \sqrt{13(7)(4)(2)}$$
$$= \sqrt{728}$$
$$= 26.98 \text{ cm}^2$$

33. Find the area of the triangle ABC if $b = 6$cm, $\beta = 75°$, $\alpha = 65°$.

Sol. We know

$$\alpha + \beta + \gamma = 180$$
$$\gamma = 180 - \alpha - \beta$$
$$= 180° - 75° - 65° = 40°$$

Area of $\Delta ABC = \frac{b^2 \sin \alpha \sin \gamma}{2 \sin \beta}$

$$= \frac{(6)^2 (\sin 65°)(\sin 40°)}{2(\sin 75°)}$$
$$= \frac{36(0.91)(0.65)}{2(0.97)}$$
$$= \frac{21.294}{1.94}$$
$$= 10.98 \text{ cm}^2$$

7.5 Three figure Bearing and its Application

34. What are the two primary rules for measuring a bearing?

Ans. The two rules are following:

i. It must be measured in degrees from the North direction.

ii. It must be measured in a clockwise manner.

35. How should a bearing always be written numerically?

Ans. It is always written as a three-digit angle between 000° to 360°.

36. What are cardinal directions?

Ans. Cardinal directions are four main joints on a compass North, South, East and West.

37. What are ordinal directions?

Ans. Ordinal directions are the four inter-cardinal points of compass. NE, SE, SW, NW.

38. In a diagram, the bearing of point *y* from point x is 045°. Find the bearing of *x* from *y*.

Ans. Since the bearing is less than 180°, we add 180° to find the return direction.

Bearing = 045° + 180° = 225

39. A hiker walks from point C to point D on a bearing of 290°. What bearing should the hiker follow to return from 0 to C.

Sol. Because the bearing is greater than 180°, we subtract 180° to find the reverse bearing

$$\text{Bearing} = 290° - 180° = 110°$$

40. A boat departs from a pier and travel 30 km due East, followed by 40 km due North. What is the distance and bearing of the shortest route back the pier.

Sol. Using Pythagoras theorem

$$d = \sqrt{30^2 + 40^2} = \sqrt{900 + 1600} = \sqrt{2500} = 50\,\text{km}$$

To return, the boat must travel South and West. The internal angle θ from the South line is:

$$\tan\theta = \frac{30}{40}$$

$$\theta = \tan^{-1}\left(\frac{30}{40}\right) = 36.9°$$

$$\text{Bearing} = 180° + 36.9° = 216.9°$$