UNIT 05 Algebraic Fractions

Students' Learning Outcomes

After completing this unit, the students will be able to:

- Describe rational expressions.
- Factorize and simplify rational expressions.
- Demonstrate manipulation of algebraic fractions.
- Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials).
- Solve different types of equations reducible to quadratic form of the type: o Exponential Equation
  - Reciprocal Equation
  - Reciprocal Equation
  - $ax^{2n} + bx^n + c = 0;\ a \neq 0,\ n = 2$
  - $(x+a)(x+b)(x+c)(x+d) = k$, where $a+b=c+d$
- Apply the concept of rational equations (limited to numerators and denominators that are monomials, binomials or trinomials) to real-world problems (such as the amount of work a person can do in certain amount of time, rates and work).

5.1 Algebraic Fraction

In Mathematics, an algebraic fraction is a fraction where the numerator, denominator or both contains algebraic expressions.

For example, $\frac{3}{x}, \frac{2x}{5y}, \frac{2x+9}{10x}, \frac{x^2+8}{x^2+x+3}$ are all algebraic fractions. The denominator is never zero.

Rational Expression

A rational expression is the quotient $\frac{P(x)}{Qx}$ Where $Q(x) \neq 0$ of two polynomials $P(x)$ and Q(x). For example

$\frac{5}{x-2}, \frac{x+5}{5}, x^2+5x+4, \frac{x+1}{x+2}$ are all rational algebraic expressions.

Simplification of Rational Expressions

Simplification of a rational expression means reducing the fraction to its lowest form by cancelling out the common factors of the numerator and the denominator.

Example 1: Reduce to lowest form $\frac{24a^3c^2x^2}{18a^3x^2-12a^2x^3}$

Solution : $\frac{24a^3c^2x^2}{18a^3x^2-12a^2x^3}$

$$= \frac{24a^3c^2x^2}{6a^2x^2(3a-2x)} = \frac{4a^{3-2}c^2x^{2-2}}{3a-2x}$$

Note:
Factorise where possible because there might be a factor that will be cancelled.

$$= \frac{4ac^2}{3a - 2x}$$

Example 2: Reduce to lowest form $\dfrac{6x^2 - 8xy}{9xy - 12y^2}$

Solution : $\dfrac{6x^2 - 8xy}{9xy - 12y^2} = \dfrac{2x(3x - 4y)}{3y\,(3x - 4y)} = \dfrac{2x}{3y}$

Solved Exercise 5.1

1. Reduce the following rational expressions to lowest forms:

(i) $\dfrac{3a^2 - 6ab}{2a^2b - 4ab^2}$

Sol: $= \dfrac{3a^2 - 6ab}{2a^2b - 4ab^2}$

$$= \frac{3a\,(a - 2b)}{2ab\,(a - 2b)}$$

$$= \frac{3a}{2ab} = \frac{3}{2b}$$

(ii) $\dfrac{abx + bx^2}{acx + cx^2}$

Sol: $= \dfrac{abx + bx^2}{acx + cx^2}$

$$= \frac{bx\,(a + x)}{cx\,(a + x)}$$

$$= \frac{bx}{cx} = \frac{b}{c}$$

(iii) $\dfrac{ac}{a^2x^2 - ax}$

Sol: $\dfrac{ac}{a^2x^2 - ax}$

$$= \frac{ac}{ax\,(ax - 1)}$$

$$= \frac{c}{x\,(ax - 1)}$$

(iv) $\dfrac{15a^2b^2c}{100(a^2 - a^2b)}$

Sol: $\dfrac{15a^2b^2c}{100(a^2 - a^2b)}$

$$= \frac{3a^2b^2c}{(20)a^2(1 - b)}$$

$$= \frac{3b^2c}{20(1 - b)}$$

(v) $\dfrac{4x^2 - 9y^2}{4x^2 + 6xy}$

Sol: $\dfrac{4x^2 - 9y^2}{4x^2 + 6xy}$

$$= \frac{(2x)^2 - (3y)^2}{2x\,(2x + 3y)}$$

$$= \frac{(2x - 3y)(2x + 3y)}{2x\,(2x + 3y)}$$

$$= \frac{2x - 3y}{2x}$$

(vi) $\dfrac{20(x^3 - y^3)}{5x^2 + 5xy + 5y^2}$

Sol: $= \dfrac{20(x^3 - y^3)}{5x^2 + 5xy + 5y^2}$

$$= \frac{{}^{4}\cancel{20}\,(x - y)(x^2 + xy + y^2)}{\cancel{5}\,(x^2 + xy + y^2)}$$

$$= 4\,(x - y)$$

(vii) $\dfrac{x(2a^2-3ax)}{a(4a^2x-9x^3)}$

Sol. $\dfrac{x(2a^2-3ax)}{a(4a^2x-9x^3)}$

$$= \frac{(x)(a)(2a-3x)}{(a)(x)(4a^2-9x^2)}$$

$$= \frac{ax(2a-3x)}{ax[(2a)^2-(3x)^2]}$$

$$= \frac{ax(2a-3x)}{ax[(2a-3x)(2a+3x)]}$$

$$= \frac{1}{2a+3x}$$

(ix) $\dfrac{3x^2+6x}{x^2+4x+4}$

Sol: $\dfrac{3x^2+6x}{x^2+4x+4}$

$$= \frac{3x(x+2)}{x^2+2x+2x+4}$$

$$= \frac{3x(x+2)}{x(x+2)+2(x+2)}$$

$$= \frac{3x(x+2)}{(x+2)(x+2)}$$

$$= \frac{3x}{x+2}$$

(viii) $\dfrac{x^2-5x}{x^2-4x-5}$

Sol: $\dfrac{x^2-5x}{x^2-4x-5}$

$$= \frac{x(x-5)}{x^2-5x+x-5}$$

$$= \frac{x(x-5)}{x(x-5)+1(x-5)}$$

$$= \frac{x(x-5)}{(x-5)(x+1)}$$

$$= \frac{x}{x+1}$$

(xi) $\dfrac{2x^2+17x+21}{3x^2+26x+35}$

Sol: $\dfrac{2x^2+17x+21}{3x^2+26x+35}$

$$= \frac{2x^2+14x+3x+21}{3x^2+21x+5x+35}$$

$$= \frac{2x(x+7)+3(x+7)}{3x(x+7)+5(x+7)}$$

$$= \frac{(x+7)(2x+3)}{(x+7)(3x+5)}$$

$$= \frac{2x+3}{3x+5}$$

(x) $\dfrac{x^2+xy-2y^2}{x^3-y^3}$

Sol: $\dfrac{x^2+xy-2y^2}{x^3-y^3}$

$$= \frac{x^2+2xy-xy-2y^2}{(x-y)(x^2+xy+y^2)} = \frac{x(x+2y)-y(x+2y)}{(x-y)(x^2+xy+y^2)}$$

$$= \frac{(x-y)(x+2y)}{(x-y)(x^2+xy+y^2)} = \frac{x+2y}{(x-y)(x^2+xy+y^2)}$$

5.2 Manipulation of Rational Expressions

Definition:

Manipulation is to apply operations like addition, subtraction, multiplication and division while maintaining the equality or value of the expression.

Addition and Subtraction of Rational Expressions with Unlike Denominators

Steps:

(i) Determine the Least Common Multiple (LCM) of the denominators.

(ii) Rewrite each fraction as an equivalent fraction with the LCM obtained in step (i)

(iii) Follow the same step for doing addition or subtraction of rational expression with like denominators.

Example 3: Simplify: $\frac{x-2y}{4}+\frac{x+y}{6}-\frac{2x-y}{15}$

Solution : LCM of 4, 6 and 15 = 60

$$\frac{x-2y}{4}+\frac{x+y}{6}-\frac{2x-y}{15}$$

$$=\frac{15(x-2y)+10(x+y)-4(2x-y)}{60}$$

$$=\frac{15x-30y+10x+10y-8x+4y}{60}=\frac{17x-16y}{60}$$

Example 4: Simplify: $\frac{4}{x^2-2xy-3y^2}+\frac{1}{x^2+3xy-2y^2}$

Solution : $\frac{4}{x^2-2xy-3y^2}+\frac{1}{x^2+3xy+2y^2}$

$$=\frac{4}{x^2+xy-3xy-3y^2}+\frac{1}{x^2+2xy+xy+2y^2}$$

$$=\frac{4}{x(x+y)-3y(x+y)}+\frac{1}{x(x+2y)+y(x+2y)}$$

$$=\frac{4}{(x-3y)(x+y)}+\frac{1}{(x+y)(x+2y)}$$

$$=\frac{4(x+2y)+1(x-3y)}{(x-3y)(x+y)(x+2y)} \text{ (Taking LCM)}$$

$$=\frac{4x+8y+x-3y}{(x-3y)(x+y)(x+2y)}=\frac{5x+5y}{(x-3y)(x+y)(x+2y)}$$

$$=\frac{5(x+y)}{(x-3y)(x+y)(x+2y)}=\frac{5}{(x-3y)(x+2y)}$$

Example 5: Simplify: $\frac{1}{x^2-5x+6}+\frac{1}{x^2-3x+2}-\frac{1}{x^2-8x+15}$

Solution : $\frac{1}{x^2-5x+6}+\frac{1}{x^2-3x+2}-\frac{1}{x^2-8x+15}$

$$= \frac{1}{(x-2)(x-3)} + \frac{1}{(x-2)(x-1)} - \frac{1}{(x-5)(x-3)}$$

$$= \frac{(x-1)(x-5)+(x-3)(x-5)-(x-1)(x-2)}{(x-1)(x-2)(x-3)(x-5)} \quad \text{(Taking LCM)}$$

$$= \frac{(x^2-6x+5)+(x^2-8x+15)-(x^2-3x+2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2-6x+5+x^2-8x+15-x^2+3x-2}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2-11x+18}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{(x-9)(x-2)}{(x-1)(x-2)(x-3)(x-5)} = \frac{x-9}{(x-1)(x-3)(x-5)}$$

Skilled Practice!

Simplify: (i) $\frac{4a}{a^2-1} - \frac{a+1}{a-1}$

Sol: $= \frac{4a}{(a+1)(a-1)} - \frac{a+1}{a-1}$

$$= \frac{4a-(a+1)^2}{(a+1)(a-1)} \quad \text{(Taking LCM)}$$

$$= \frac{4a-(a^2+2a+1)}{(a+1)(a-1)}$$

$$= \frac{4a-a^2-2a-1}{(a+1)(a-1)}$$

$$= \frac{-a^2+2a-1}{(a+1)(a-1)} = \frac{-(a^2-2a+1)}{(a+1)(a-1)}$$

$$= \frac{-(a-1)^2}{(a+1)(a-1)}$$

$$= \frac{-(a-1)}{a+1}$$

(ii) $\frac{a^3}{a-b} - \frac{b^3}{b-a}$

Sol: $\frac{a^3}{a-b} - \frac{b^3}{b-a}$

$$= \frac{a^3}{a-b} - \frac{b^3}{-a+b}$$

$$= \frac{a^3}{a-b} - \frac{b^3}{-(a-b)}$$

$$= \frac{a^3}{a-b} + \frac{b^3}{a-b}$$

$$= \frac{a^3+b^3}{a-b} \quad \text{(Taking LCM)}$$

$$= \frac{(a+b)(a^2-ab+b^2)}{a-b}$$

(iii) $\frac{4-x^2}{x} - \frac{x+2}{x+2} + 5$

Sol. $= \frac{4-x^2}{x} - \frac{x+2}{x+2} + 5$

$$= \frac{4-x^2}{x} - 1 + 5 = \frac{4-x^2}{x} + 4$$

$$= \frac{4-x^2+4x}{x} \quad \text{(Taking LCM)}$$

$$= \frac{-x^2+4x+4}{x}$$

$$= \frac{-(x^2-4x-4}{x}$$

Solved Exercise 5.2

Simplify:

(i) $\frac{3}{x-y}+\frac{1}{y-x}$

Sol: $\frac{3}{x-y}+\frac{1}{-x+y}$

$$=\frac{3}{x-y}+\frac{1}{-(x-y)}$$

$$=\frac{3}{x-y}-\frac{1}{(x-y)}$$

(L.C.M = x − y)

$$=\frac{3-1}{x-y}$$

$$=\frac{2}{x-y}$$

(ii) $\frac{4x}{x^2-3x+2}-\frac{4}{1-x}-\frac{5}{x-2}$

Sol: $\frac{4x}{x^2-3x+2}-\frac{4}{1-x}-\frac{5}{x-2}$

$$=\frac{4x}{x^2-2x-x+2}-\frac{4}{-x+1}-\frac{5}{x-2}$$

$$=\frac{4x}{x(x-2)-1(x-2)}-\frac{4}{-(x-1)}-\frac{5}{x-2}$$

$$=\frac{4x}{(x-2)(x-1)}+\frac{4}{x-1}-\frac{5}{x-2}$$

LCM $(x-2)(x-1)$

$$=\frac{4x+4(x-2)-5(x-1)}{(x-2)(x-1)}$$

$$=\frac{4x+4x-8-5x+5}{(x-2)(x-1)}$$

$$=\frac{3x-3}{(x-2)(x-1)}=\frac{3(x-1)}{(x-2)(x-1)}=\frac{3}{x-2}$$

(iii) $\frac{x+y}{12x-6y}+\frac{x-y}{18x-9y}$

Sol. $\frac{x+y}{12x-6y}+\frac{x-y}{18x-9y}$

$$=\frac{x+y}{6(2x-y)}+\frac{x-y}{9(2x-y)}$$

L.C.M = 18 (2x–y)

$$=\frac{3(x+y)+2(x-y)}{18(2x-y)}$$

$$=\frac{3x+3y+2x-2y}{18(2x-y)}$$

$$=\frac{5x+y}{18(2x-y)}$$

(iv) $\frac{2}{x+2y}-\frac{x-6y}{x^2-4y^2}$

Sol. $\frac{2}{x+2y}-\frac{x-6y}{x^2-4y^2}$

$$=\frac{2}{x+2y}-\frac{x-6y}{(x)^2-(2y)^2}$$

$$=\frac{2}{x+2y}-\frac{x-6y}{(x+2y)(x-2y)}$$

L.C.M = $(x+2y)(x-2y)$

$$=\frac{2(x-2y)-(x-6y)}{(x+2y)(x-2y)}$$

$$=\frac{2x-4y-x+6y}{(x+2y)(x-2y)}$$

$$=\frac{x+2y}{(x+2y)(x-2y)}=\frac{1}{x-2y}$$

(v) $\frac{1}{x+1}-\frac{2}{x+2}-\frac{2x+3}{x^2+3x+2}$

Sol: $\frac{1}{x+1}-\frac{2}{x+2}-\frac{2x+3}{x^2+3x+2}$

$=\frac{1}{x+1}-\frac{2}{x+2}-\frac{2x+3}{x^2+2x+x+2}$

$=\frac{1}{x+1}-\frac{2}{x+2}-\frac{2x+3}{x(x+2)+1(x+2)}$

$=\frac{1}{x+1}-\frac{2}{x+2}-\frac{2x+3}{(x+2)(x+1)}$

L.C.M = (x + 2)(x + 1)

$=\frac{1(x+2)-2(x+1)-(2x+3)}{(x+1)(x+2)}$

$=\frac{x+2-2x-2-2x-3}{(x+1)(x+2)}$

$=\frac{x-2x-2x+2-2-3}{(x+1)(x+2)}$

$=\frac{-3x-3}{(x+1)(x+2)}=\frac{-3(x+1)}{(x+1)(x+2)}$

$=\frac{-3}{x+2}$

(vi) $\frac{5}{x^2+x-6}-\frac{1}{2x^2-7x+6}$

Sol: $\frac{5}{x^2+x-6}-\frac{1}{2x^2-7x+6}$

$=\frac{5}{x^2+3x-2x-6}-\frac{1}{2x^2-4x-3x+6}$

$=\frac{5}{x(x+3)-2(x+3)}-\frac{1}{2x(x-2)-3(x-2)}$

$=\frac{5}{(x+3)(x-2)}-\frac{1}{(x-2)(2x-3)}$

LCM = (x+3) (x–2) (2x–3)

$=\frac{5(2x-3)-1(x+3)}{(x+3)(x-2)(2x-3)}$

$=\frac{10x-15-x-3}{(x+3)(x-2)(2x-3)}$

$=\frac{9x-18}{(x+3)(x-2)(2x-3)}$

$=\frac{9(x-2)}{(x+3)(x-2)(2x-3)}=\frac{9}{(x+3)(2x-3)}$

(vii) $\frac{2x}{x+y}-\frac{y}{x-y}-\frac{2y^2}{x^2-y^2}$

Sol. $\frac{2x}{x+y}-\frac{y}{x-y}-\frac{2y^2}{x^2-y^2}$

$=\frac{2x}{x+y}-\frac{y}{x-y}-\frac{2y^2}{(x+y)(x-y)}$

LCM = (x–y) (x–y)

$=\frac{2x(x-y)-y(x+y)-2y^2}{(x+y)(x-y)}$

$=\frac{2x^2-2xy-xy-y^2-2y^2}{(x+y)(x-y)}$

$=\frac{2x^2-3xy-3y^2}{(x-y)(n+y)}$

(viii) $\frac{7}{2x^2-x-6}-\frac{8}{3x^2-4x-4}$

Sol: $\frac{7}{2x^2-x-6}-\frac{8}{3x^2-4x+4}$

$=\frac{7}{2x^2-4x+3x-6}-\frac{8}{3x^2-6x+2x-4}$

$=\frac{7}{2x(x-2)+3(x-2)}-\frac{8}{3x(x-2)+2(x-2)}$

$$= \frac{7}{(x-2)(2x+3)} - \frac{8}{(x-2)(3x+2)}$$

LCM = (x − 2) (2x + 3) (3x + 2)

$$= \frac{7(3x+2)-8(2x+3)}{(x-2)(2x+3)(3x+2)} = \frac{21x+14-16x-24}{(x-2)(2x+3)(3x+2)}$$

$$= \frac{5x-10}{(x-2)(2x+3)(3x+2)} = \frac{5(x-2)}{(x-2)(2x+3)(3x+2)}$$

$$= \frac{5}{(2x+3)(3x+2)}$$

(ix) $\frac{x+2}{x^2-x-12} - \frac{x}{x^2+6x+9}$

Sol. $\frac{x+2}{x^2-x-12} - \frac{x}{x^2+6x+9}$

$$= \frac{x+2}{x^2-4x+3x-12} - \frac{x}{x^2+3x+3x+9}$$

$$= \frac{x+2}{x(x-4)+3(x-4)} - \frac{x}{x(x+3)+3(x+3)}$$

$$= \frac{x+2}{(x-4)(x+3)} - \frac{x}{(x+3)(x+3)}$$ LCM = (x−4) (x +3) (x+3)

$$= \frac{(x+2)(x+3)-x(x-4)}{(x-4)(x+3)(x+3)} = \frac{x^2+3x+2x+6-x^2+4x}{(x-4)(x+3)(x+3)}$$

$$= \frac{\cancel{x^2}+5x+6-\cancel{x^2}+4x}{(x-4)(x+3)^2} = \frac{9x+6}{(x-4)(x+3)^2}$$

$$= \frac{3(3x+2)}{(x-4)(x+3)^2}$$

(x) $\frac{1}{x^2-4x+3} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-5x+6}$

Sol. $\frac{1}{x^2-4x+3} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-5x+6}$

$$= \frac{1}{x^2-3x-x+3} + \frac{1}{x^2-2x-x+2} - \frac{1}{x^2-3x-2x+6}$$

$$= \frac{1}{x(x-3)-1(x-3)} + \frac{1}{x(x-2)-1(x-2)} - \frac{1}{x(x-3)-2(x-3)}$$

$$= \frac{1}{(x-3)(x-1)} + \frac{1}{(x-2)(x-1)} - \frac{1}{(x-3)(x-2)}$$

$$= \frac{(x-2)+(x-3)-(x-1)}{(x-1)(x-2)(x-3)} \qquad \text{LCM} = (x-1)(x-2)(x-3)$$

$$= \frac{x-2+x-3-x+1}{(x-1)(x-2)(x-3)} = \frac{x+x-x-2-3+1}{(x-1)(x-2)(x-3)}$$

$$= \frac{x-4}{(x-1)(x-2)(x-3)}$$

(xi) $\frac{x^3}{x-y}+\frac{y^3}{y-x}$

Sol. $\frac{x^3}{x-y}+\frac{y^3}{y-x}$

$$= \frac{x^3}{x-y}+\frac{y^3}{x-y} = \frac{x^3}{x-y}+\frac{y^3}{-1(x-y)}$$

$$= \frac{x^3}{x-y}-\frac{y^3}{x-y} \quad \text{LCM} = (x-y)$$

$$= \frac{x^3-y^3}{x-y} = \frac{(x-y)(x^2+xy+y^2)}{(x-y)}$$

$$= (x^2+xy+y^2)$$

2. Subtract $\frac{1}{x^2+2}$ **from** $\frac{2x^3+x^2+3}{(x^2+2)^2}$

Sol. $\frac{2x^3+x^2+3}{(x^2+2)^2}-\frac{1}{x^2+2}$

$$= \frac{2x^3+x^2+3-(x^2+2)}{(x^2+2)^2} \quad \text{LCM}=(x^2+2)^2$$

$$= \frac{2x^3+x^2+3-x^2-2}{(x^2+2)^2}$$

$$= \frac{2x^3+1}{(x^2+2)^2}$$

Multiplication and Division of Rational Expressions

(a) Multiplication

Example 6: Simplify: $\frac{x^2-5x+6}{x^2-25}\times\frac{x^2+5x}{2x-6}$

Solution : $\frac{x^2-5x+6}{x^2-25}\times\frac{x^2+5x}{2x-6} = \frac{x^2-2x-3x+6}{x^2-5^2}\times\frac{x(x+5)}{2(x-3)}$

$$= \frac{(x-2)(x-3)}{(x-5)(x+5)}\times\frac{x(x+5)}{2(x-3)} = \frac{x(x-2)}{2(x-5)}$$

Example 7: Simplify: $\frac{3a^2-6ab}{6ab}\times\frac{10b^2}{4ab-8b^2}$

Solution : $\frac{3a^2-6ab}{6ab}\times\frac{10b^2}{4ab-8b^2}$

$$= \frac{3a(a-2b)}{6ab}\times\frac{10b^2}{4b(a-2b)}$$

$$= \frac{30}{24}=\frac{5}{4}$$

Division : To divide by a rational expression, multiply by its reciprocal as in multiplication.

Example: $\dfrac{2a^2-2c^2}{15a^2c^2} \div \dfrac{3a+3c}{5ac}$

Solution : $\dfrac{2a^2-2c^2}{15a^2c^2} \div \dfrac{3a+3c}{5ac}$

$$= \frac{2a^2-2c^2}{15a^2c^2} \times \frac{5ac}{3a+3c} = \frac{2(a^2-c^2)}{15a^2c^2} \times \frac{5ac}{3(a+c)}$$

$$= \frac{2(a+c)(a-c)}{15a^2c^2} \times \frac{5ac}{3(a+c)} = \frac{2(a-c)}{9ac}$$

Example 9: Simplify: $\dfrac{x^3-a^3}{x^2-2bx+b^2} \times \dfrac{x^2-bx-cx+bc}{ax-x^2} \div \dfrac{x^2-cx}{x-b}$

Solution : $\dfrac{x^3-a^3}{x^2-2bx+b^2} \times \dfrac{x^2-bx-cx+bc}{ax-x^2} \div \dfrac{x^2-cx}{x-b}$

$$= \frac{x^3-a^3}{x^2-2bx+b^2} \times \frac{x^2-bx-cx+bc}{ax-x^2} \times \frac{x-b}{x^2-cx}$$

$$= \frac{(x-a)\,(x^2+ax+a^2)}{(x-b)\,(x-b)} \times \frac{(x-b)(x-c)}{x(a-x)} \times \frac{x-b}{x(x-c)}$$

$$= -(a-x)(x^2+ax+a^2) \times \frac{1}{x(a-x)} \times \frac{1}{x}$$

$$= \frac{-(x^2+ax+a^2)}{x^2}$$

Skilled Practice!

Simplify:

(i) $\dfrac{32x^2y^2z^5}{3x^3y^3} \cdot \dfrac{243x^6}{x^4y^2z}$

Sol. $\dfrac{32x^2y^2z^5}{3x^3y^3} \cdot \dfrac{243x^6}{x^4y^2z}$

$$= \frac{32 \times 243x^{2+6}y^2z^5}{3x^{3+4}\,y^{3+2}Z}$$

$$= \frac{32 \times 81x^8y^2z^5}{x^7y^5z}$$

$$= \frac{2592\,x^{8-7}z^{5-1}}{y^{5-2}}$$

$$= \frac{2592xz^4}{y^3}$$

(ii) $\dfrac{5a^2}{4a-8} \cdot \dfrac{6a-12}{10a}$

Sol. $\dfrac{5a^2}{4a-8} \cdot \dfrac{6a-12}{10a}$

$$= \frac{5a^2}{4(a-2)} \cdot \frac{6(a-2)}{10a}$$

$$= \frac{5 \times 6 \times a^2 \times (a-2)}{4 \times 10 \times a \times (a-2)}$$

$$= \frac{30a^{2-1}}{40}$$

$$= \frac{3a}{4}$$

Solved Exercise 5.3

1. Simplify:

(i) $\dfrac{24\ell m}{5\ell+10m}\times\dfrac{5\ell^2-20m^2}{16mn}$

Sol: $\dfrac{24\ell m}{5\ell+10m}\times\dfrac{5\ell^2-20m^2}{16mn}$

$=\dfrac{24\ell m}{5(\ell+2m)}\times\dfrac{5(\ell^2-4m^2)}{16mn}$

$=\dfrac{24\ell m}{5(\ell+2m)}\times\dfrac{5[(\ell)^2-(2m)^2]}{16mn}$

$=\dfrac{24\ell m}{5\cancel{(\ell+2m)}}\times\dfrac{5[\cancel{(\ell+2m)}\ (\ell-2m)]}{16mn}$

$=\dfrac{\overset{3}{\cancel{24}}\ell m\times\overset{3}{\cancel{5}}(\ell-2m)}{\cancel{5}\times\underset{2}{\cancel{16}}mn}$

$=\dfrac{3\ell(\ell-2m)}{2n}$

(ii) $\dfrac{x^2-3x+2}{x^2+3x-4}\times\dfrac{2x^2+8x}{3x+6}$

Sol: $\dfrac{x^2-3x+2}{x^2+3x-4}\times\dfrac{2x^2+8x}{3x+6}$

$=\dfrac{x^2-2x-x+2}{x^2+4x-x-4}\times\dfrac{2x(x+4)}{3(x+2)}$

$=\dfrac{x(x-2)-1(x-1)}{x(x+4)-1(x+4)}\times\dfrac{2x(x+4)}{3(x+2)}$

$=\dfrac{(x-2)(x-1)}{(x+4)(x-1)}\times\dfrac{2x(x+4)}{3(x+2)}$

$=\dfrac{2x(x-2)(x-1)(x+4)}{3(x+2)(x-1)(x+4)}$

$=\dfrac{2x(x-2)}{3(x+2)}$

(iii) $\dfrac{a^2-4b^2}{a^2+2ba}\times\dfrac{2a^2+10ab}{a^2+3ab-10b^2}$

Sol: $\dfrac{a^2-4b^2}{a^2+2ba}\times\dfrac{2a^2+10ab}{a^2+3ab-10b^2}$

$=\dfrac{(a)^2-(2b)^2}{a(a+2b)}\times\dfrac{2a(a+5b)}{a^2+5ab-2ab-10b^2}$

$=\dfrac{(a+2b)(a-2b)}{a(a+2b)}\times\dfrac{2a(a+5b)}{a(a+5b)-2b(a+5b)}$

$=\dfrac{a+2b}{a}\times\dfrac{2a(a+5b)}{(a+2b)(a+5b)}=\dfrac{(a+2b)\times2a}{a(a+2b)}$

$=\dfrac{2a}{a}=2$

(iv) $\dfrac{(a+b)^2-c^2}{(a+c)^2-b^2}\times\dfrac{a^2-(b-c)^2}{(a+c)^2-c^2}$

Sol: $\dfrac{(a+b)^2-c^2}{(a+c)^2-b^2}\times\dfrac{a^2-(b-c)^2}{(a+c)^2-c^2}$

$=\dfrac{(a+b+c)(a+b-c)}{(a+c+b)(a+c-b)}\times\dfrac{[a+(b-c)][a-(b-c)]}{(a+c+c)(a+c-c)}$

$=\dfrac{(a+b+c)(a+b-c)}{(a+b+c)(a-b-c)}\times\dfrac{(a+b-c)(a-b+c)}{(a+2c)(a)}$

$=\dfrac{a+b-c}{(a-b+c)}\times\dfrac{(a+b-c)(a-b+c)}{(a+2c)(a)}$

$=\dfrac{(a+b-c)(a+b-c)}{a(a+2c)}$

$=\dfrac{(a+b-c)^2}{a(a+2c)}$

(v) $\dfrac{x^3-8}{x^2-4}\div\dfrac{x^2+2x+4}{x^2+4x+4}$

Sol: $\dfrac{x^3-8}{x^2-4}\div\dfrac{x^2+2x+4}{x^2+4x+4}$

$=\dfrac{(x)^3-(2)^3}{(x)^2-(2)^2}\div\dfrac{x^2+2x+4}{x^2+2x+2x+4}$

$$= \frac{(x-2)[(x)^2+(x)(2)+(2)^2]}{(x+2)(x-2)} \div \frac{x^2+2x+4}{x(x+2)+2(x+2)}$$

$$= \frac{(x-2)(x^2+2x+4)}{(x+2)(x-2)} \div \frac{x^2+2x+4}{(x+2)(x+2)}$$

$$= \frac{x^2+2x+4}{x+2} \times \frac{(x+2)(x+2)}{x^2+2x+4} = \frac{(x^2+2x+4)(x+2)(x+2)}{(x^2+2x+4)(x+2)}$$

$$= x+2$$

(vi) $\dfrac{(a^2+ab)^2}{(a^2-ab)^2} \div \left(\dfrac{a+b}{a-b}\right)^2$

Sol: $\dfrac{(a^2+ab)^2}{(a^2-ab)^2} \div \left(\dfrac{a+b}{a-b}\right)^2$

$$= \frac{[a(a+b)]^2}{[a(a-b)]^2} \div \frac{(a+b)^2}{(a-b)^2} = \frac{a^2(a+b)^2}{a^2(a-b)^2} \times \frac{(a-b)^2}{(a+b)^2}$$

$$= \frac{a^2(a+b)^2(a-b)^2}{a^2(a+b)^2(a-b)^2} = 1$$

(vii) $\dfrac{x^2-x-6}{x^2-x-20} \div \left\{\dfrac{x^3-3x^2}{x^2+4x} \times \dfrac{x^3-5x^2-14x}{x^2-12x+35}\right\}$

Sol: $\dfrac{x^2-x-6}{x^2-x-20} \div \left\{\dfrac{x^3-3x^2}{x^2+4x} \times \dfrac{x^3-5x^2-14x}{x^2-12x+35}\right\}$

$$= \frac{x^2-3x+2x-6}{x^2-5x+4x-20} \div \left\{\frac{x^2(x-3)}{x(x+4)} \times \frac{x(x^2-5x-14)}{x^2-7x-5x+35}\right\}$$

$$= \frac{x(x-3)+2(x-3)}{x(x-5)+4(x-5)} \div \left\{\frac{x^2(x-3)}{x(x+4)} \times \frac{x(x^2-7x+2x-14)}{x(x-7)-5(x-7)}\right\}$$

$$= \frac{(x-3)(x+2)}{(x-5)(x+4)} \div \left\{\frac{x^2(x-3)}{x(x+4)} \times \frac{x[x(x-7)+2(x-7)]}{(x-7)(x-5)}\right\}$$

$$= \frac{(x-3)(x+2)}{(x-5)(x+4)} \div \left\{\frac{x^2(x-3)}{x(x+4)} \times \frac{x(x-7)(x+2)}{(x-7)(x-5)}\right\}$$

$$= \frac{(x-3)(x+2)}{(x-5)(x+4)} \div \left\{\frac{x(x-3)}{(x+4)} \times \frac{x(x+2)}{x-5}\right\}$$

$$= \frac{(x-3)(x+2)}{(x-5)(x+4)} \div \frac{x^2(x-3)(x+2)}{(x+4)(x-5)} = \frac{(x-3)(x+2)}{(x-5)(x+4)} \times \frac{(x+4)(x-5)}{x^2(x-3)(x+2)}$$

$$= \frac{(x-2)(x-3)(x+4)(x-5)}{x^2(x+2)(x-3)(x+4)(x-5)} = \frac{1}{x^2}$$

(viii) $\dfrac{3x^2-3xy}{10x^2+10xy-20y^2}\times\dfrac{5xy+10y^2}{6x^2+6xy+6y^2}\div\dfrac{2x^2-2y^2}{4x^3-4y^3}$

Sol. $\dfrac{3x^2-3xy}{10x^2+10xy-20y^2}\times\dfrac{5xy+10y^2}{6x^2+6xy+6y^2}\div\dfrac{2x^2-2y^2}{4x^3-4y^3}$

$$=\frac{3x(x-y)}{10(x^2+xy-2y^2)}\times\frac{5y(x+2y)}{6(x^2+xy+y^2)}\div\frac{2(x^2-y^2)}{4(x^3-y^3)}$$

$$=\frac{3x(x-y)}{10(x^2+2xy-xy-2y^2)}\times\frac{5y(x+2y)}{6(x^2+xy+y^2)}\div\frac{2(x-y)(x+y)}{4(x-y)(x^2+xy+y^2)}$$

$$=\frac{3x(x-y)}{10[x(x+2y)-y(x+2y)]}\times\frac{5y(x+2y)}{6(x^2+xy+y^2)}\times\frac{4(x-y)(x^2+xy+y^2)}{2(x-y)(x+y)}$$

$$=\frac{3x(x-y)}{10(x-y)(x+2y)}\times\frac{5y(x+2y)}{6(x^2+xy+y^2)}\times\frac{2(x^2+xy+y^2)}{x+y}$$

$$=\frac{3x}{10(x+2y)}\times\frac{5y(x+2y)}{6(x^2+xy+y^2)}\times\frac{2(x^2+xy+y^2)}{x+y}$$

$$=\frac{2\times3\times5\times x\times y(x+2y)(x^2+xy+y^2)}{10\times6(x+y)(x+2y)(x^2+xy+y^2)}=\frac{30xy}{60(x+y)}$$

$$=\frac{xy}{2(x+y)}$$

(ix) $\dfrac{2x^2-98}{x^3-125}\times\dfrac{x^2-3x-10}{3x-21}\div\dfrac{x^2+5x-14}{x^2+5x+25}$

Sol. $\dfrac{2x^2-98}{x^3-125}\times\dfrac{x^2-3x-10}{3x-21}\div\dfrac{x^2+5x-14}{x^2+5x+25}$

$$=\frac{2(x^2-49)}{(x)^3-(5)^3}\times\frac{x^2-5x+2x-10}{3(x-7)}\div\frac{x^2+7x-2x-14}{x^2+5x+25}$$

$$=\frac{2[(x)^2-(7)^2]}{(x-5)[(x)^2+(x)(5)+(5)^2]}\times\frac{x(x-5)+2(x-5)}{3(x-7)}\div\frac{x(x+7)-2(x+7)}{x^2+5x+25}$$

$$=\frac{2(x+7)(x-7)}{(x-5)(x^2+5x+25)}\times\frac{(x-5)(x+2)}{3(x-7)}\div\frac{(x+7)(x-2)}{x^2+5x+25}$$

$$=\frac{2(x+7)(x-7)}{(x-5)(x^2+5x+25)}\times\frac{(x-5)(x+2)}{3(x-7)}\times\frac{x^2+5x+25}{(x+7)(x-2)}$$

$$=\frac{2(x+2)(x+7)(x-7)(x-5)(x^2+5x+25)}{3(x-2)(x+7)(x-7)(x-5)(x^2+5x+25)}$$

$$=\frac{2(x+2)}{3(x-2)}$$

(x) $\dfrac{x^2-2x}{2x+6}\times\dfrac{x^2+x-6}{x^2-5x}\times\dfrac{6x-30}{x^2-2x}$

Sol. $\dfrac{x^2-2x}{2x+6}\times\dfrac{x^2+x-6}{x^2-5x}\times\dfrac{6x-30}{x^2-2x}$

$$=\frac{x(x-2)}{2(x+3)}\times\frac{x^2+3x-2x-6}{x(x-5)}\times\frac{6(x-5)}{x(x-2)}$$

$$=\frac{x(x-2)}{2(x+3)}\times\frac{x(x+3)-2(x+3)}{x(x-5)}\times\frac{6(x-5)}{x(x-2)}$$

$$=\frac{x(x-2)}{2(x+3)}\times\frac{(x+3)(x-2)}{x(x-5)}\times\frac{6(x-5)}{x(x-2)}$$

$$=\frac{6\times x(x-2)(x-2)(x+3)(x-5)}{2\times x\times x(x-2)(x+3)(x-5)}$$

$$=\frac{3(x-2)}{x}$$

5.3 Solution of Equations Reducible to Quadratic Equations

Some algebraic equations are not in standard quadratic form, but they can be reduced or transformed into a quadratic equation by suitable substitution. Such type of equations are called equations reducible to quadratic equations.

Type (i): $ax^{2n}+bx^n+c=0;\ a\neq 0,\ n=2$

Example 10: Solve: $2x^4-11x^2+12=0$

Solution:

$$2x^4-11x^2+12=0$$

Let $x^2=y$, so that

$$2y^2-11y+12=0$$
$$2y^2-3y-8y+12=0$$
$$y(2y-3)-4(2y-3)=0$$
$$(2y-3)(y-4)=0$$
$$2y-3=0,\ y-4=0$$
$$2y-3=0,\ y-4=0$$
$$y=\frac{3}{2},\ y=4$$

Put $x^2=y$

$$\therefore x^2=\frac{3}{2},\ x^2=4$$

$$x=\pm\sqrt{\frac{3}{2}}=\pm\frac{\sqrt{6}}{2},\ x=\pm 2\Rightarrow \text{S.S}=\left\{\pm 2,\pm\frac{\sqrt{6}}{2}\right\}$$

Type (ii): Exponential Equation

Equations in which variable occur in exponents are called exponential equations.

For example, $2^x - 3 \cdot 2^{2x} = -56$

Example 11: Solve: $4^x - 3 \cdot 2^{x+3} + 128 = 0$

Solution: $4^x - 3 \cdot 2^{x+3} + 128 = 0$

The equation can be written as

$$(2^2)^x - 3 \cdot 2^x \cdot 2^3 + 128 = 0$$

or $$2^{2x} - 24 \cdot 2^x + 128 = 0 \qquad \text{(i)}$$

Let $2^2 = y$, then $2^{2x} = y^2$

$\therefore$ (i) becomes

$$y^2 - 24y + 128 = 0$$

$$\Rightarrow y^2 - 16y - 8y + 128 = 0$$

$$y(y-16) - 8(y-16) = 0$$

$$(y-16)(y-8) = 0$$

$$y - 16 = 0,\ y - 8 = 0$$

$$y = 16,\ y = 8$$

But $y = 2^x$

$$2^x = 16,\ 2^x = 8$$

$$2^x = 2^4,\ 2^x = 8$$

$$x = 4,\ x = 3$$

$\therefore$ Solution set $= \{4, 3\}$

Type (iii) Reciprocal Equation

An equation which remains unchanged when variable is replaced by its reciprocal.

For example, $\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) + 4 = 0$

Example 12: Solve: $3\left(x^2 + \frac{1}{x^2}\right) + 4\left(x + \frac{1}{x}\right) - 14 = 0$

Solution: $3\left(x^2 + \frac{1}{x^2}\right) + 4\left(x + \frac{1}{x}\right) - 14 = 0$

Let $x + \frac{1}{x} = y$

So $\left(x + \frac{1}{x}\right)^2 = y^2$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 = y^2$$

or $$x^2 + \frac{1}{x^2} = y^2 - 2$$

The given equation becomes

$$3(y^2 - 2) + 4y - 14 = 0$$

$$3y^2 - 6 + 4y - 14 = 0$$

$$3y^2 + 4y - 20 = 0$$
$$3y^2 + 10y - 6y - 20 = 0$$
$$y(3y+10) - 2(3y+10) = 0$$
$$(3y+10)(y-2) = 0$$
$$3y + 10 = 0 \quad , \quad y - 2 = 0$$
$$y = -\frac{10}{3} \quad , \quad y = 2$$

Put $y = -\frac{10}{3}$ in (i). we get

$$x + \frac{1}{x} = -\frac{10}{3} \quad \text{[Multiplying by 3x]}$$
$$3x^2 + 3 = -10x$$
$$3x^2 + 10x + 3 = 0$$
$$3x^2 + 9x + x + 3 = 0$$
$$3x(x+3) + 1(x+3) = 0$$
$$(x+3)(3x+1) = 0$$
$$x + 3 = 0, 3x + 1 = 0$$
$$x = -3, -\frac{1}{3}$$

Now put $y = 2$ in (i), we get

$$x + \frac{1}{x} = 2 \quad \text{[Multiplying by x]}$$
$$x^2 + 1 = 2x$$
$$x^2 - 2x + 1 = 0 \quad \text{[Transposing]}$$
$$(x-1)^2 = 0$$
$$x - 1 = 0$$
$$\Rightarrow x = 1$$

$$\text{Solution set} = \left\{1, -3, -\frac{1}{3}\right\}$$

Type (iv) : $(x + a)(x + b)(x + c)(x + d) = k$, where $a + b = c + d$ and k is any constant.

Example 13: Solve: $(x + 2)(x + 3)(x - 5)(x - 6) = -12$

Solution: Here $2 - 5 = 3 - 6$

Re-arrange the factors, we get

$$[(x+2)(x-5)][(x+3)(x-6)] = -12$$
$$(x^2 - 3x - 10)(x^2 - 3x - 18) = -12 \quad \text{(i)}$$
$$\text{Let} \quad x^2 - 3x = y \quad \text{(ii)}$$
$$\text{(i) becomes: } (y-10)(y-18) = -12$$
$$y^2 - 28y + 180 = -12$$
$$y^2 - 28y + 192 = 0 \quad \text{[Transposing]}$$
$$y^2 - 12y - 16y + 192 = 0$$

$$y(y-12)-16(y-12)=0$$
$$(y-12)(y-16)=0$$
$$y-12=0,\ y-16=0$$
$$y=12,\ 16$$
Put $y = 12$ in (ii), we get
$$x^2-3x=12 \text{ or}$$
$$x^2-3x-12=0$$
$$x=\frac{3\pm\sqrt{(-3)^2-4\times1\times(-12)}}{2(1)}$$
$$=\frac{3\pm\sqrt{57}}{2}$$
Now put $y = 16$ in (ii), we get
$$x^2-3x=16$$
$$x^2-3x-16=0$$
$$x=\frac{3\pm\sqrt{(-3)^2-4\times1\times(-16)}}{2(1)}=\frac{3\pm\sqrt{73}}{2}$$
$$\text{Solution set}=\left\{\frac{3\pm\sqrt{57}}{2},\frac{3\pm\sqrt{73}}{2}\right\}$$

Skilled practiced

Sol: $(x+1)(x+2)(x-5)(x-6)=144$

$[(x+1)(x-5)][(x+2)(x-6)]=144$

$(x^2-5x+x-5)(x^2-6x+2x-12)=144$

$(x^2-4x-5)(x^2-4x-12)=144$

Let $x^2-4x=y$

$(y-5)(y-12)=144$

$y^2-12y-5y+60=144$

$y^2-17y+60-144=0$

$y^2-17y-84=0$

$y^2-21y+4y-84=0$

$y(y-21)+4(y-21)=0$

$(y-21)(y+4)=0$

$y=21,\ y=-4$

put $x^2-4x=y$

$x^2-4x=21$

$x^2-4x-21=0$

$x^2-7x+3x-21=0$

$x(x-7)+3(x-7)=0$

$(x-7)(x+3)=0$

$x-7=0\quad,\quad x+3=0$

$x=7\quad,\quad x=-3$

$x^2-4x=-4$

$x^2-4x+4=0$

$(x-2)^2=0$

$x-2=0$

$x=2$

S.S. $=\{-3, 2, 7\}$

Solved Exercise 5.4

1. Solve the following equations:

(i) $5x^4-19x^2+12=0$

Sol: $5x^4-19x^2+12=0$

Let $x^2=y$

$$(x^2)^2 = (y)^2$$
$$x^4 = y^2$$

So, that

$$5y^2 - 19y + 12 = 0$$
$$5y^2 - 15y - 4y + 12 = 0$$
$$5y(y-3) - 4(y-3) = 0$$
$$(y-3)(5y-4) = 0$$
$$y - 3 = 0 \quad , \quad 5y - 4 = 0$$
$$y = 3 \quad , \quad 5y = 4$$
$$y = \frac{4}{5}$$

Put $x^2 = y$

$$x^2 = 3$$
$$\sqrt{x^2} = \sqrt{3}$$
$$x = \pm\sqrt{3}$$
$$x^2 = \frac{4}{5}$$
$$\sqrt{x^2} = \sqrt{\frac{4}{5}}$$
$$x = \pm\frac{2}{\sqrt{5}} = \pm\frac{2\sqrt{5}}{5}$$

$$S.S = \left\{\pm\sqrt{3}, \pm\frac{2\sqrt{5}}{5}\right\}$$

(ii) $4x^4 - 27x^2 + 18 = 0$

Sol: $4x^4 - 27x^2 + 18 = 0$

Let $x^2 = y$

$$(x^2)^2 = y^2$$
$$x^4 = y^2$$

So, that

$$4y^2 - 27y + 18 = 0$$
$$4y^2 - 24y - 3y + 18 = 0$$
$$4y(y-6) - 3(y-6) = 0$$
$$(y-6)(4y-3) = 0$$
$$y - 6 = 0 \quad , \quad 4y - 3 = 0$$
$$y = 6 \quad , \quad 4y = 3$$
$$y = \frac{3}{4}$$

put $x^2 = y$

$$x^2 = 6$$
$$\sqrt{x^2} = \sqrt{6}$$
$$x = \pm\sqrt{6}$$
$$x^2 = \frac{3}{4}$$
$$\sqrt{x^2} = \sqrt{\frac{3}{4}}$$
$$x = \pm\frac{\sqrt{3}}{2}$$

$$S.S = \left\{\pm\sqrt{6}, \pm\frac{\sqrt{3}}{2}\right\}$$

(iii) $5x^4 - 22x^2 + 8 = 0$

Sol: $5x^4 - 22x^2 + 8 = 0$

Let $x^2 = y$

$$(x^2)^2 = y^2$$
$$x^4 = y^2$$

So, that

$$5y^2 - 22y + 8 = 0$$
$$5y^2 - 20y - 2y + 4 = 0$$
$$5y(y-4) - 2(y-4) = 0$$
$$(y-4)(5y-2) = 0$$
$$y - 4 = 0 \quad , \quad 5y - 2 = 0$$
$$y = 4 \quad , \quad 5y = 2$$
$$y = \frac{2}{5}$$

Put $x^2 = y$

$$x^2 = 4$$
$$\sqrt{x^2} = \sqrt{4}$$
$$x = \pm 2$$
$$x^2 = \frac{2}{5}$$
$$\sqrt{x^2} = \sqrt{\frac{10}{5}}$$
$$x = \pm\sqrt{\frac{2}{5}} = \pm\frac{\sqrt{10}}{5}$$

$$S.S = \left\{\pm 2, \pm\frac{\sqrt{10}}{5}\right\}$$

(iv) $4\cdot 2^{2x+1} - 9.2^x + 1 = 0$

Sol: $4\cdot 2^{2x+1} - 9\cdot 2^x + 1 = 0$

$4\cdot 2^{2x}.2 - 9\cdot 2^x + 1 = 0$

$8\cdot 2^{2x} - 9\cdot 2^x + 1 = 0$

Let $2^x = y$

$(2^x)^2 = y^2$

$2^{2x} = y^2$

So, that

$8y^2 - 9y + 1 = 0$

$8y^2 - 8y - y + 1 = 0$

$8y(y-1) - 1(y-1) = 0$

$(y-1)(8y-1) = 0$

$y-1 = 0 \ , \ 8y-1 = 0$

$y = 1 \ , \ 8y = 1$

$, y = \frac{1}{8}$

put $2^x = 1$

$2^x = 1$

$2^x = 20$

$x = 0$

$2^x = \frac{1}{8}$

$2^x = \frac{1}{2^3}$

$2^x = 2^{-3}$

$x = -3$

$S.S = \{0, -3\}$

(v) $4^{1+x} + 4^{1-x} - 10 = 0$

Sol: $4^{1+x} + 4^{1-x} - 10 = 0$

$4^1.4^x + 4^1.4^{-x} - 10 = 0$

$4.4^x + \frac{4}{4^x} - 10 = 0$

Let $4^x = y$

So, hat

$4y + \frac{4}{y} - 10 = 0$

$y(4y) + y\left(\frac{4}{y}\right) - y(10) = y(0)$

$4y^2 + 4 - 10y = 0$

$4y^2 - 10y + 4 = 0$

$4y^2 - 8y - 2y + 4 = 0$

$4y(y-2) - 2(y-2) = 0$

$(y-2)(4y-2) = 0$

$y - 2 = 0 \ , \ 4y - 2 = 0$

$y = 2 \ , \ 4y = 2$

$y = \frac{2}{4}$

$y = \frac{1}{2}$

put $4^x = y$

$4^x = 2$

$(2^2)^x = 2$

$2^{2x} = 2$

$2x = 1$

$x = \frac{1}{2}$

$4^x = 2^{-1}$

$(2^2)^x = 2^{-1}$

$2^{2x} = 2^{-1}$

$2x = -1$

$x = -\frac{1}{2}$

$S.S = \left\{\frac{1}{2}, -\frac{1}{2}\right\}$

(vi) $3^x + 3^{3-x} - 12 = 0$

Sol: $3^x + 3^{3-x} - 12 = 0$

$3^x + 3^3.3^{-x} - 12 = 0$

$3^x + \frac{27}{3^x} - 12 = 0$

Let $3^x = y$

$y + \frac{27}{y} - 12 = 0$

$y(y) + y\left(\frac{27}{y}\right) - y(12) = y(0)$

$y + 27 - 12y = 0$

$y^2 - 12y + 27 = 0$

$y^2 - 9y - 3y + 27 = 0$

$y(y-9) - 3(y-9) = 0$

$(y-9)(y-3)=0$

$y-9=0 \quad , y-3=0$

$y = 9 \quad , \quad y=3$

put $3^x = y$

$3^x = 9$

$3^x = 3^2$

$x = 2$

$3^x = 3$

$3^x = 3^1$

$x = 1$

S.S $= \{2, 1\}$

(vii) $5^{1+3x} + 5^{2-3x} - 126 = 0$

Sol: $5^{1+3x} + 5^{2-3x} - 126 = 0$

$5^1.5^{3x} + 5^2.5^{-3x}\ 126 = 0$

$5(5^{3x}) + \frac{25}{5^{3x}} - 126 = 0$

Let $5^{3x} = y$

So, that

$5y + \frac{25}{y} - 126 = 0$

$y(5y) + y\left(\frac{25}{y}\right) - y(126) = y(0)$

$5y^2 + 25 - 126y = 0$

$5y^2 - 126y + 25 = 0$

$5y^2 - 125y - y + 25 = 0$

$5y(y-25) - 1(y-25) = 0$

$(y-25)(5y-1) = 0$

$y - 25 = 0 , 5y - 1 = 0$

$y = 25, \ 5y = 1$

$, y = \frac{1}{5}$

Put $5^{3x} = y$

$5^{3x} = 25$

$5^{3x} = 5^2$

$3x = 2$

$x = \frac{2}{3}$

$5^{3x} = \frac{1}{5}$

$5^{3x} = 5^{-1}$

$3x = -1$

$x = -\frac{1}{3}$

S.S $= \left\{\frac{2}{3}, -\frac{1}{3}\right\}$

(viii) $\left(x^2 + \frac{1}{x^2}\right) + 2\left(x + \frac{1}{x}\right) - 33 = 0$

Sol: Let $x + \frac{1}{x} = y$

$\left(x + \frac{1}{x}\right)^2 = y^2$

$x^2 + \frac{1}{x^2} + 2 = y^2$

$x^2 + \frac{1}{x^2} = y^2 - 2$

So, that

$y^2 - 2 + 2y - 33 = 0$

$y^2 + 2y - 35 = 0$

$y^2 + 7y - 5y - 35 = 0$

$y(y+7) - 5(y+7) = 0$

$(y+7)(y-5) = 0$

$y + 7 = 0 , y - 5 = 0$

$y = -7 , y = 5$

Put $x + \frac{1}{x} = y$

$x + \frac{1}{x} = -7$

$x + \frac{1}{x} + 7 = 0$

$x(x) + x\left(\frac{1}{x}\right) + x(7) = x(0)$

$x^2 + 1 + 7x = 0$

$x^2 + 7x + 1 = 0$

$a = 1, b = 7, c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{\sqrt{(7)^2 - 4(1)(1)}}{2(1)}$

$$x = \frac{-7 \pm \sqrt{49-4}}{2}$$

$$x = \frac{-7 \pm \sqrt{45}}{2}$$

$$x = \frac{-7 \pm \sqrt{3^2 \times 5}}{2}$$

$$x = \frac{-7 \pm 3\sqrt{5}}{2}$$

$$x + \frac{1}{x} = 5$$

$$x + \frac{1}{x} - 5 = 0$$

$$x(x) + x\left(\frac{1}{x}\right) - 5(x) = 0$$

$$x^2 + 1 - 5x = 0$$

$$x^2 - 5x + 1 = 0$$

$a = 1, b = -5, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25-4}}{2}$$

$$x = \frac{5 \pm \sqrt{21}}{2}$$

$$S.S = \left\{\frac{-7 \pm 3\sqrt{5}}{2}, \frac{5 \pm \sqrt{21}}{2}\right\}$$

(ix) $\mathbf{2\left(x^2 + \frac{1}{x^2}\right) - 25\left(x + \frac{1}{x}\right) - 17 = 0}$

Sol. $2\left(x^2 + \frac{1}{x^2}\right) - 25\left(x + \frac{1}{x}\right) - 17 = 0$

Let $x - \frac{1}{x} = y$

$$\left(x - \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} - 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 + 2$$

So, that

$$2(y^2 + 2) - 25y - 17 = 0$$

$$2y^2 + 4 - 25y - 17 = 0$$

$$2y^2 - 25y - 13 = 0$$

$$2y^2 - 25y + y - 13 = 0$$

$$2y(y - 13) + 1(y - 13) = 0$$

$$(y-13)(2y+1) = 0$$

$y - 13 = 0$

$y = 13$

$$x - \frac{1}{x} = 13$$

$$x(x) - x\left(\frac{1}{x}\right) = 13(x)$$

$$x^2 - 1 = 13x$$

$$x^2 - 13x - 1 = 0$$

$a = 1, b = -13, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{13 \pm \sqrt{169+4}}{2}$$

$$x = \frac{13 \pm \sqrt{173}}{2}$$

$2y + 1 = 0$

$2y = -1$

$$y = -\frac{1}{2}$$

$$x - \frac{1}{x} = -\frac{1}{2}$$

$$x(x) - x\left(\frac{1}{x}\right) = -\frac{1}{2}(x)$$

$$x^2 - 1 = -\frac{x}{2}$$

$$x^2 + \frac{x}{2} - 1 = 0$$

$$2(x^2) + 2\left(\frac{x}{2}\right) - 2(1) = 2(0)$$
$$2x^2 + x - 2 = 0$$
$$a = 2,\ b = 1,\ c = -2$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-2)}}{2(2)}$$
$$x = \frac{-1 \pm \sqrt{1+16}}{4}$$
$$x = \frac{-1 \pm \sqrt{17}}{4}$$
$$S.S = \left\{\frac{13 \pm \sqrt{173}}{2}, \frac{-1 \pm \sqrt{17}}{4}\right\}$$

(x) $2x^4 - 5x^3 - 14x^2 - 5x + 2 = 0$

Sol: $2x^4 - 5x^3 - 14x^2 - 5x + 2 = 0$

Divide by x^2

$$\frac{2x^4}{x^2} - \frac{5x^3}{x^2} - \frac{14x^2}{x^2} - \frac{5x}{x^2} + \frac{2}{x^2} = \frac{0}{x^2}$$
$$2x^2 - 5x - 14 - \frac{5}{x} + \frac{2}{x^2} = 0$$
$$2x^2 + \frac{2}{x^2} - 5x - \frac{5}{x} - 14 = 0$$
$$2\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) - 14 = 0$$

Let $x + \frac{1}{x} = y$

$$\left(x + \frac{1}{x}\right)^2 = y^2$$
$$x^2 + \frac{1}{x^2} + 2 = y^2$$
$$x^2 + \frac{1}{x^2} = y^2 - 2$$

So, that

$$2(y^2 - 2) - 5y - 14 = 0$$
$$2y^2 - 4 - 5y - 14 = 0$$
$$2y^2 - 5y - 18 = 0$$
$$2y^2 - 9y + 4y - 18 = 0$$
$$y(2y - 9) + 2(2y - 9) = 0$$
$$(2y - 9)(y + 2) = 0$$
$$2y - 9 = 0,\ y + 2 = 0$$
$$2y = 9,\ y = -2$$
$$y = \frac{9}{2}$$

Put $x + \frac{1}{x} = y$

$$x + \frac{1}{x} = \frac{9}{2}$$
$$2x(x) + 2x\left(\frac{1}{x}\right) = 2x\left(\frac{9}{2}\right)$$
$$2x^2 + 2 = 9x$$
$$2x^2 - 9x + 2 = 0$$
$$a = 2,\ b = -9,\ c = 2$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(2)}}{2(2)}$$
$$x = \frac{9 \pm \sqrt{81 - 16}}{4}$$
$$x = \frac{9 \pm \sqrt{65}}{4}$$

$$x + \frac{1}{x} = -2$$
$$x(x) + x\left(\frac{1}{x}\right) = -2(x)$$
$$x^2 + 1 = -2x$$
$$x^2 + 2x + 1 = 0$$
$$(x)^2 + 2(x)(1) + (1)^2 = 0$$
$$(x + 1) = 0$$
$$\sqrt{(x+1)^2} = \sqrt{0}$$
$$x + 1 = 0$$
$$x = -1$$
$$S.S = \left\{\frac{9 \pm \sqrt{65}}{4}, -1\right\}$$

(xi) (x + 1) (x +2) (x – 4) (x – 5) + 8 = 0

Sol: $(x+1)(x+2)(x-4)(x-5)+8 = 0$

Re-arrange the factors, we get

$$[(x+1)(x-4)][(x+2)(x-5)]+8 = 0$$
$$(x^2-4x+x-4)(x^2-5x+2x-10)+8 = 0$$
$$(x^2-3x-4)(x^2-3x-10)+8 = 0$$
$$\text{Let } x^2-3x = y$$

So, that

$$(y-4)(y-10)+8 = 0$$
$$y^2-10y-4y+40+8 = 0$$
$$y^2-14y+48 = 0$$
$$y^2-8y-6y+48 = 0$$
$$y(y-8)-6(y-8) = 0$$
$$(y-8)(y-6) = 0$$
$$y-8=0\ ,\ y-6 = 0$$
$$y=8\ ,\ y = 6$$
$$\text{put } x^2-3x = y$$

$$x^2-3x = 8$$
$$x^2-3x-8 = 0$$
$$a=1,\ b=-3,\ c=-8$$
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2-4(1)(-8)}}{2(1)}$$
$$x = \frac{3 \pm \sqrt{9+32}}{2}$$
$$x = \frac{3 \pm \sqrt{41}}{2}$$
$$x^2-3x = 6$$
$$x^2-3x-6 = 0$$
$$a=1,\ b=-3,\ c=-6$$
$$x = \frac{-\pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2-4(1)(-6)}}{2(1)}$$
$$x = \frac{3 \pm \sqrt{9+24}}{2}$$
$$x = \frac{3 \pm \sqrt{33}}{2}$$

$$S.S = \left\{\frac{3 \pm \sqrt{41}}{2}, \frac{3 \pm \sqrt{33}}{2}\right\}$$

(xii) (x+3) (x+4) (x+5) (x+6) = –1

Sol: $(x+3)(x+4)(x+5)(x+6) = -1$

Re-arrange the factors, we get

$$[(x+3)(x+6)][(x+4)(x+5)] = -1$$
$$(x^2+6x+3x+18)(x^2+5x+4x+20) = -1$$
$$(x^2+9x+18)(x^2+9x+20) = -1$$
$$\text{Let } x^2+9x = y$$
$$(y+18)(y+20) = -1$$
$$y^2+20y+18y+360 = -1$$
$$y^2+38y+360+1 = 0$$
$$y^2+38y+361 = 0$$
$$(y)^2+2(y)(19)+(19)^2 = 0$$
$$(y+19)^2 = 0$$
$$\sqrt{(y+19)^2} = \sqrt{0}$$
$$y+19 = 0$$
$$y = -19$$
$$\text{Put } x^2+9x = y$$
$$x^2+9x = -19$$
$$x^2+9x+19 = 0$$
$$a=1,\ b=9,\ c = 19$$
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-9 \pm \sqrt{(9)^2-4(1)(19)}}{2(1)}$$
$$x = \frac{-9 \pm \sqrt{81-76}}{2}$$
$$x = \frac{-9 \pm \sqrt{5}}{2}$$
$$S.S = \left\{\frac{-9 \pm \sqrt{5}}{2}\right\}$$

5.4 Real World Problems

Example 14: A tradesman buys a number of articles for Rs. 300 four of them are broken in transit but by selling the remaining at a profit of Rs. 1.25 each, he gains Rs. 350 altogether. How many articles did he buy?

Solution: Suppose he buys x articles for Rs. 300

$\because$ Each cost $= \frac{300}{x}$ Rs.

He sold (x–4) articles for Rs.350

Each sold for $\frac{350}{x-4}$ Rs.

Now, $\frac{350}{x-4} - \frac{300}{x} = 1.25$

$\frac{350}{x-4} - \frac{300}{x} = \frac{125}{100} = \frac{5}{4}$

Multiplying both sides by $4(x-4)$, we get

$350(4x) - 300(x-4)(4) = 5x(x-4)$

$1400x - 1200x + 4800 = 5x^2 - 20x$

$5x^2 - 20x + 1200x - 1400x - 4800 = 0$

$5x^2 - 220x - 4800 = 0$

(Dividing by 5)

$x^2 - 44x - 960 = 0$

$x^2 - 60x + 16x - 960 = 0$

$x(x - 60) + 16(x - 60) = 0$

$(x - 60)(x + 16) = 0$

$x - 60 = 0, x + 16 = 0$

$\because \quad x = 60, -16$

Negative value is rejected

Number of articles purchased = 60

Challenge!

Amna bought 50 kg of fruits consisting of mangoes and grapes. She paid twice as much per kg for the mango as she did for the grapes. If Amna bought Rs. 18000 worth of mangoes and Rs. 6000 worth grapes, then how many kg's of each fruit did she buy?

Sol. Let m be the weight of mangoes (in kg)

Let g be the weight of grapes (in kg)

Let x be the price per kg of grapes

Total weight m + g = 50

Price of mangoes = 2x

Total cost

m.(2x) = 18000

$mx = \frac{18000}{2}$

$mx = 9000$

For grapes $g.x = 6000$

$\frac{m.x}{g.x} = \frac{9000}{6000}$

$\frac{m}{g} = \frac{3}{2}$

$m = 1.5\,g$

put $m = 1.5\,g$

$m + g = 50$

$1.5g + g = 50$

$2.5g = 50$

$g = \frac{50}{2.5} = 20\text{kg}$

Find the weight of mangoes

$m = 50 - 20$

$m = 30$

So, Amna bought 30kg of mangoes and 20kg of grapes

Example 15: A train is scheduled to cover a distance of 120 Km at a certain average speed, owing to a service checkup this average speed is reduced by 5 km/h, and in consequence the journey takes 20 minutes more than the scheduled time. What was the scheduled speed?

Solution: Let x km/h be scheduled speed;

then scheduled time for 120 km $= \frac{120}{x}$ hrs

20 minute $= \frac{20}{60}$ hrs $= \frac{1}{3}$ hrs

But $(x - 5)$ km/h is actual speed: therefore,

actual time for 120 km $= \frac{120}{x-5}$

$\because \quad \frac{120}{x-5} - \frac{120}{x} = \frac{1}{3}$

Multiply each term by $3x(x-5)$

$360x - 360(x-5) = x(x-5)$

$x^2 - 5x - 1800 = 0$

$(x-45)(x+40)=0$

$x-45=0,\ x+40=0$

$\because\ x=45,\ x=-40$

Negative value is rejected

The scheduled speed is 45 km/hr

Solved Exercise 5.5

1. A train travels a distance of 240 km at a uniform rate, if it had finished 4 km an hour slower, it would have taken 2 hours more over the journey; find its rate of travelling.

Sol: Suppose original speed be x Km/h

Time taken at original speed $\frac{240}{x}$ h

New speed = $(x-4)$ Km/h

Time taken at new speed= $\frac{240}{x-4}$ Km/h

Difference in two times = 2 h

According to given condition

$$\frac{240}{x-4}-\frac{240}{x}=2$$

Multiply in both sides by $x(x-4)$, we get

$$x(x-4)\left(\frac{240}{x-4}\right)-x(x-4)\left(\frac{240}{x}\right)=2(x)(x-4)$$

$$x(240)-(x-4)(240)=2x(x-4)$$

$$240x-(240x-960)=2x^2-8x$$

$$240x-240x+960=2x^2-8x$$

$$-2x^2+240x-240x+8x+960=0$$

$$-2x^2+8x+960=0$$

$$-2(x^2-4x-480)=0$$

$$x^2-4x-480=0$$

$$x^2-24x+20x-480=0$$

$$x(x-24)+20(x-24)=0$$

$$(x-24)(x+20)=0$$

$$x-24=0,\ x+20=0$$

$$x=24,\ x=-20$$

$$x=24,\ -20$$

Negative value is rejected

The rate of travelling = 24 km/h

2. Arshia and Ibraheem complete a job together in 4 hours. If Arshia takes 6 hours, the find how much time will Ibraheem take?

Sol: Let Ibraheem's time be x hours

Rate of work together: $\frac{1}{4}$ of the job per hour

Rate of Arshia's Time= $\frac{1}{6}$ of the job per hour

Rate of Ibraheem's time= $\frac{1}{x}$ of the job per hour

According to the given condition

$$\frac{1}{6}+\frac{1}{x}=\frac{1}{4}$$

$$\frac{1}{x}=\frac{1}{4}-\frac{1}{6}$$

$$\frac{1}{x}=\frac{3-2}{12}$$

$$\frac{1}{x}=\frac{1}{12}$$

$$12=x$$

So, time taken by Ibraheem to complete the job = 12 hours

3. One pipe fills in 5 hours, another in 8 hours. Second pipe closed after 2 hours. Find the total time.

Sol: Rate of pipe A = $\frac{1}{5}$ tank/hr

Rate of pipe B = $\frac{1}{8}$ tank/hr

Work done by both in first 2 hours

$$=2\left(\frac{1}{5}+\frac{1}{8}\right)$$

$$=2\left(\frac{8+5}{40}\right)$$

$$=2\left(\frac{13}{40}\right)$$

$= \frac{13}{20}$

Remaining work $= 1 - \frac{13}{20}$

$= \frac{20-13}{20} = \frac{7}{20}$

Time taken by pipe A for remaining work

$= \frac{\frac{7}{20}}{\frac{1}{5}}$

$= \frac{7}{20} \times \frac{5}{1} = \frac{7}{4}$ hour = 1 hr 45 minutes

Total time = 2hr+ 1 hr 45 min

= 3 hours 45 minutes

4. Huria can complete a project in 12 hours by working alone. If Abdul Hadi join her and they finish it together in 5 hours, how long would it take Abdul Hadi to do the project alone?

Sol: let Hadi's time = x hours

Combined time Rate $= \frac{1}{5}$

Huria's time Rate $= \frac{1}{12}$

Hadi's time Rate $= \frac{1}{x}$

Equation by given condition

$$\frac{1}{12} + \frac{1}{x} = \frac{1}{5}$$

$$\frac{1}{x} = \frac{1}{5} - \frac{1}{12}$$

$$\frac{1}{x} = \frac{12-5}{60}$$

$$\frac{1}{x} = \frac{7}{60}$$

$$60 = 7x$$

$$\frac{60}{7} = x$$

Hadi will take time $= \frac{60}{7}$ hours = 8 hours, 34 minutes

$$7\overline{)60} = 8,\quad 56,\quad 4$$

$$7\overline{)240} = 34,\quad 21,\quad 30,\quad 28,\quad 2$$

$4 \times 60 = 240$ min

5. Two cars start from opposite towns and head towards each other. The distance between them is 240 km. One travels at x km/h and the other at $(x + 10)$ km/h. They meet after 2 hours. Find the speed of each car.

Sol: Speed of car A = x km/hr

Speed of car B = $(x+10)$ km/hr

Total distence = 240 km

Total time = 2 hr

According to given condition

$$[x+(x+10)] \times 2 = 240$$

$$(x+x+10) = \frac{240}{2}$$

$$2x+10 = 120$$

$$2x = 120 - 10$$

$$2x = 110$$

$$x = \frac{110}{2} = 55$$

So, speed of the first car (x) = 55 km/hr

Speed of the second car $(x+10)$

= 55+10 = 65 km/hr

6. Rashid can paint a house in 6 days, but if he gets a helper he can do it in 4 days. How long would it take

the helper to paint the house alone?

Sol: Let the time taken by helper = x day

$$\text{Rate of helper} = \frac{1}{x} \text{ house/day}$$

$$\text{Rate of Rashid} = \frac{1}{6} \text{ house/day}$$

$$\text{Combined Rate} = \frac{1}{4} \text{ house/day}$$

According to the given condition

$$\frac{1}{x}+\frac{1}{6}=\frac{1}{4}$$

$$\frac{1}{x}=\frac{1}{4}-\frac{1}{6}$$

$$\frac{1}{x}=\frac{3-2}{12}$$

$$\frac{1}{x}=\frac{1}{12}$$

$$x=12$$

Time taken by helper to point the house alone = 12 days

7. Ishmal runs 10 Km in $\frac{2x}{x+4}$ hours. If her average speed is 8 km/h, find the value of x.

Sol: Speed = 8 Km/h

Distance = 10km

$$\text{Time} = \frac{2x}{x+4} \text{ hours}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$8 = \frac{10}{\frac{2x}{x+4}}$$

$$8 = 10 \times \frac{x+4}{2x}$$

$$8(2x) = 10(x+4)$$

$$16x = 10x+40$$

$$16x - 10x = 40$$

$$6x = 40$$

$$x = \frac{40}{6} = \frac{20}{3} \text{ or } 6.67$$

8. Fahad runs 600m at a certain pace, and then doubling his pace, does another 600 m. If he took $2\frac{1}{2}$ to cover the distance 1200 m, find the pace he started at, in metres per seconds.

Sol: Let the speed Fahad started = x m/s

$$\text{Pace (I) Time} = \frac{\text{Distance}}{\text{speed}} = \frac{600}{x}$$

$$\text{Pace (II) Time} = \frac{\text{Distance}}{\text{speed}} = \frac{600}{x} = \frac{300}{x}$$

Total time = $2\frac{1}{2}$ minutes = 150 seconds

Equation according to given condition

$$\frac{600}{x}+\frac{300}{x}=150$$

$$\frac{900}{x}=150$$

$$\frac{900}{150}=x$$

$$6=x$$

$$x=6$$

So, the pace started by Fahad = 6m/s

9. A cyclist travels 30 km at a certain speed. If the speed had been 5 km/h faster, the journey would have taken 1 hour less. Find the speed.

Sol: Let the speed = x km/h

$$\text{Time for original speed} = \frac{30}{x}$$

$$\text{Time for new speed} = \frac{30}{x+5}$$

Equation from given condition

$$\frac{30}{x}-\frac{30}{x+5}=1$$

$$x(x+5)\left(\frac{30}{x}\right)-x(x+5)\left(\frac{30}{x+5}\right)=x(x+5)(1)$$

$$(x+5)(30)-x(30) = x^2+5x$$

$$\cancel{30x}+150-\cancel{30x} = x^2+5x$$

$$-x^2-5x+150=0$$

$$x^2+5x-150=0$$

$$x^2+15x-10-150=0$$

$$x(x+15)-10(x+15)=0$$

$$(x+15)(x-10)=0$$

$x+15=0$, $x=-15$ | $x-10=0$, $x=10$

Negative ignored

so the speed = 10 km/h

Solved Review Exercise 5

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) What type of expression $\frac{2x-1}{x^2+4}$ is:

(a) a polynomial (b) an algebraic fraction
(c) a numerical fraction (d) an equation

(ii) Lowest form of $\frac{18a^2bc^2}{12ac}$ is:

(a) $\frac{9abc}{6}$ (b) $\frac{3abc}{2}$ (c) $\frac{18b}{12}$ (d) $\frac{3}{2}$

(iii) Lowest form of $\frac{15xyz}{12}$ is:

(a) $\frac{3xyz}{2}$ (b) $\frac{3}{2}$ (c) xyz (d) $\frac{xyz}{2}$

(iv) Simplified form of $\frac{3x+1}{5}+\frac{2x+3}{5}$ is:

(a) $\frac{5x+4}{10}$ (b) $\frac{5x+4}{5}$ (c) $\frac{5x+4}{25}$ (d) $5x+4$

(v) Simplified form of $\left(\frac{x^2-y^3}{x+y}\right)(x-y)$ is:

(a) $(x+y)^2$ (b) x^2+y^2 (c) $x-y$ (d) $(x-y)^2$

(vi) $(x^3-y^3)\div(x-y)$ in simplified form is:

(a) x^2-xy+y^2 (b) x^2+xy+y^2 (c) x^2-y^2 (d) x^2+y^2

(vii) An equation of the form $\frac{5}{x}+\frac{1-x}{3}=\frac{1}{6x}$ is called:

(a) Radical equation (b) Reciprocal equation
(c) Fractional equation (d) Exponential equation

(viii) An equation of the form $5^x + 64.5^{-x} - 20 = 0$ is called:

(a) Radical equation (b) Exponential equation

(c) Reciprocal equation (d) Fractional equation

(ix) Roots of $y^2 - 24y + 128 = 0$ are:

(a) 8, –16 (b) 8,16 (c) –8,16 (d) –8, –16

(x) Linear factors of $3x^2 + 10x + 3 = 0$ are:

(a) $(x+3), (3x+1)$ (b) $(x+3), (3x-1)$ (c) $(x-3), (3x+1)$ (d) $(x-3), (3x-1)$

ANSWERS:

(i) an algebraic fraction **(ii)** $\frac{3abc}{2}$ **(iii)** $\frac{3xyz}{2}$ **(iv)** $\frac{5x+4}{5}$

(v) $(x-y)^2$ **(vi)** x^2+xy+y^2 **(vii)** Fractional equation **(viii)** Exponential equation

(ix) 8, 16 **(x)** $(x+3), (3x+1)$

2. Reduce the following to the lowest form:

(i) $\frac{x^2-7x+12}{x^2-6x+9}$

Sol. $\frac{x^2-7x+12}{x^2-6x+9}$

$= \frac{x^2-4x-3x+12}{x^2-3x-3x+9}$

$= \frac{x(x-4)-3(x-4)}{x(x-3)-3(x-3)}$

$= \frac{(x-4)(x-3)}{(x-3)(x-3)}$

$= \frac{x-4}{x-3}$

(ii) $\frac{x^2+3x-18}{7x-21}$

Sol. $\frac{x^2+3x-18}{7x-21}$

$= \frac{x^2+6x-3x-18}{7(x-3)}$

$= \frac{x(x+6)-3(x+6)}{7(x-3)}$

$= \frac{(x+6)(x-3)}{7(x-3)}$

$= \frac{x+6}{7}$

3. Simplify:

(i) $\frac{5}{x^2+x-6} - \frac{1}{2x^2-7x+6}$

Sol. $\frac{5}{x^2+x-6} - \frac{1}{2x^2-7x+6}$

$= \frac{5}{x^2+3x-2x-6} - \frac{1}{2x^2-4x-3x+6}$

$= \frac{5}{x(x+3)-2(x+3)} - \frac{1}{2x(x-2)-3(x-2)}$

$$= \frac{5}{(x+3)(x-2)} - \frac{1}{(x-2)(2x-3)}$$

$$= \frac{5(2x-3)-(x+3)}{(x-2)(x+3)(2x-3)} = \frac{10x-15-x-3}{(x-2)(x+3)(2x-3)}$$

$$= \frac{9x-18}{(x-2)(x+3)(2x-3)} = \frac{9(x-2)}{(x-2)(x+3)(2x-3)}$$

$$= \frac{9}{(x+3)(2x-3)}$$

(ii) $\dfrac{x^3-27}{x^2-9} \div \dfrac{x^2+3x+9}{x^2+6x+9}$

Sol. $\dfrac{x^3-27}{x^2-9} \div \dfrac{x^2+3x+9}{x^2+6x+9}$

$$= \frac{(x)^3-(3)^3}{(x)^2-(3)^2} \div \frac{x^2+3x+9}{x^2+3x+3x+9}$$

$$= \frac{(x-3)\left[(x)^2+(x)(3)+(3)^2\right]}{(x-3)(x+3)} \div \frac{x^2+3x+9}{x(x+3)+3(x+3)}$$

$$= \frac{(x-3)(x^2+3x+9)}{(x-3)(x+3)} \div \frac{x^2+3x+9}{(x+3)(x+3)}$$

$$= \frac{x^2+3x+9}{(x+3)} \times \frac{(x+3)(x+3)}{x^2+3x+9} = \frac{(x+3)(x+3)(x^2+3x+9)}{(x+3)(x^2+3x+9)}$$

$$= x+3$$

(iii) $\dfrac{a^2-(b-c)^2}{(a+b)^2-c^2} \times \dfrac{a^2-(b+c)^2}{(a-b)^2-c^2}$

Sol. $\dfrac{a^2-(b-c)^2}{(a+b)^2-c^2} \times \dfrac{a^2-(b+c)^2}{(a-b)^2-c^2}$

$$= \frac{(a-(b-c))(a+(b-c))}{((a+b)-c)((a+b)+c)} \times \frac{(a+(b+c))(a-(b+c))}{((a-b)-c)((a-b)+c)}$$

$$= \frac{\overset{1}{\cancel{(a+b-c)}}\,\overset{1}{\cancel{(a-b+c)}}}{\underset{1}{\cancel{(a+b-c)}}\,\underset{1}{\cancel{(a+b+c)}}} \times \frac{\overset{1}{\cancel{(a+b+c)}}\,\overset{1}{\cancel{(a-b-c)}}}{\underset{1}{\cancel{(a-b-c)}}\,\underset{1}{\cancel{(a-b+c)}}}$$

$$= 1$$

4. Solve the following equations:

(i) $x^4 - 16x^2 + 63 = 0$

Sol. $x^4 - 16x^2 + 63 = 0$

Let $x^2 = y$

$(x^2)^2 = y^2$

$x^4 = y^2$

So, that

$y^2 - 16y + 63 = 0$

$y^2 - 9y - 7y + 63 = 0$

$y(y-9) - 7(y-9) = 0$

$(y-9)(y-7) = 0$

$y - 9 = 0 \quad , \quad y - 7 = 0$

$y = 9 \quad , \quad y = 7$

Put $x^2 = y$

$x^2 = 9$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

$x^2 = 7$

$\sqrt{x^2} = \sqrt{7}$

$x = \pm\sqrt{7}$

S.S = $\{\pm 3, \pm\sqrt{7}\}$

(ii) $4x^4 - 16x^2 + 15 = 0$

Sol. $4x^4 - 16x^2 + 15 = 0$

Let $x^2 = y$

$(x^2)^2 = y^2$

$x^4 = y^2$

So, that

$4y^2 - 16y + 15 = 0$

$4y^2 - 10y - 6y + 15 = 0$

$2y(2y-5) - 3(2y-5) = 0$

$(2y-5)(2y-3) = 0$

$2y - 5 = 0 \quad , \quad 2y - 3 = 0$

$2y = 5 \quad , \quad 2y = 3$

$y = \frac{5}{2} \quad , \quad y = \frac{3}{2}$

Put $x^2 = y$

$x^2 = \frac{5}{2}$

$\sqrt{x^2} = \sqrt{\frac{5}{2}}$

$x = \pm\sqrt{\frac{5}{2}} = \pm\frac{\sqrt{10}}{2}$

$x^2 = \frac{3}{2}$

$\sqrt{x^2} = \sqrt{\frac{3}{2}}$

$x = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{6}}{2}$

S.S = $\{\pm\frac{\sqrt{10}}{2}, \pm\frac{\sqrt{6}}{2}\}$

(iii) $3^{2x} - 12\cdot3^x + 27 = 0$

Sol. $3^{2x} - 12\cdot3^x + 27 = 0$

Let $3^x = y$

$(3^x)^2 = y^2$

$3^{2x} = y^2$

So,

$y^2 - 12y + 27 = 0$

$y^2 - 9y - 3y + 27 = 0$

$y(y-9) - 3(y-9) = 0$

$(y-9)(y-3) = 0$

$y - 9 = 0 \quad , \quad y - 3 = 0$

$y = 9 \quad , \quad y = 3$

Put $3^x = y$

$3^x = 9$

$3^x = 3^2$

$x = 2$

$3^x = 3$

$3^x = 3^1$

$x = 1$

S.S = $\{1, 2\}$

(iv) $\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) - 2 = 0$

Sol. $\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) - 2 = 0$

Let $x + \frac{1}{x} = y$

$$\left(x+\frac{1}{x}\right)^2 = y^2$$

$$x^2+\frac{1}{x^2}+2 = y^2$$

$$x^2+\frac{1}{x^2} = y^2-2$$

So, that

$$y^2-2-3y-2=0$$
$$y^2-3y-4=0$$
$$y^2-4y+y-4=0$$
$$y(y-4)+1(y-4)=0$$
$$(y-4)(y+1)=0$$
$$y-4=0 \quad , \quad y+1=0$$
$$y=4 \quad , \quad y=-1$$

Put $x+\frac{1}{x}=y$

$$x+\frac{1}{x}=4$$

$$x(x)+x\left(\frac{1}{x}\right)=x(4)$$

$$x^2+1=4x$$
$$x^2-4x+1=0$$
$$a=1,\ b=-4,\ c=1$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-(4)(1)(1)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{16-4}}{2}$$

$$x=\frac{4\pm\sqrt{12}}{2}$$

$$x=\frac{4\pm2\sqrt{3}}{2}$$

$$x=\frac{2\left(2\pm\sqrt{3}\right)}{2}$$

$$x=2\pm\sqrt{3}$$

$$x+\frac{1}{x}=-1$$

$$x(x)+x\left(\frac{1}{x}\right)=x(-1)$$

$$x^2+1 = -x$$

$$x^2+x+1=0$$

$$a=1,\ b=1,\ c=1$$

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(1)\pm\sqrt{(+1^2)-4(1)(1)}}{2(1)}$$

$$x=\frac{-1\pm\sqrt{1-4}}{2}$$

$$x=\frac{-1\pm\sqrt{-3}}{2}$$

$$\text{S.S}=\{2\pm\sqrt{3},\ \frac{-1\pm\sqrt{-3}}{2}\}$$

(v) $\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)=4$

Sol. $\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)=4$

Let $x+\frac{1}{x}=y$

$$\left(x+\frac{1}{x}\right)^2=y^2$$

$$=x^2+\frac{1}{x^2}+2=y^2$$

$$x^2+\frac{1}{x^2}=y^2-2$$

So,

$$y^2-2+y=4$$
$$y^2+y-2-4=0$$

$y^2 + y - 6 = 0$
$y^2 + 3y - 2y - 6 = 0$
$y(y + 3) - 2(y + 3) = 0$
$(y + 3)(y - 2) = 0$
$y + 3 = 0 \quad , \quad y - 2 = 0$
$y = -3 \quad , \quad y = 2$

Put $x + \dfrac{1}{x} = y$

$$x + \frac{1}{x} = -3$$

$$x(x) + x\left(\frac{1}{x}\right) = -3(x)$$

$x^2 + 1 = -3x$
$x^2 + 3x + 1 = 0$
$a = 1, b = 3, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - (4)(1)(1)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 - 4}}{2}$$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

$$x + \frac{1}{x} = 2$$

$$x(x) + x\left(\frac{1}{x}\right) = 2(x)$$

$x^2 + 1 = 2x$

$x^2 - 2x + 1 = 0$

$(x - 1)^2 = 0$

$\sqrt{(x-1)^2} = \sqrt{0}$

$x - 1 = 0$

$x = 1$

$$S.S = \{1, \frac{-3 \pm \sqrt{5}}{2}\}$$

(vi) $(x + 9)(x - 3)(x - 7)(x + 5) = 385$

Sol. Re-arranging the factors, we get

$[(x + 9)(x - 7)][(x + 5)(x - 3)] = 385$
$(x^2 - 7x + 9x - 63)(x^2 - 3x + 5x - 15) = 385$
$(x^2 + 2x - 63)(x^2 + 2x - 15) = 385$
Let $x^2 + 2x = y$
$(y - 63)(y - 15) = 385$
$y^2 - 15y - 63y + 945 = 385$
$y^2 - 78y + 945 - 385 = 0$
$y^2 - 78y + 560 = 0$
$y^2 - 70y - 8y + 560 = 0$
$y(y - 70) - 8(y - 70) = 0$
$(y - 70)(y - 8) = 0$
$y - 70 = 0 \quad , \quad y - 8 = 0$
$y = 70 \quad , \quad y = 8$
Put $x^2 + 2x = y$
$x^2 + 2x = 70$
$x^2 + 2x - 70 = 0$
$a = 1, b = 2, c = -70$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-70)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 + 280}}{2}$$

$$x = \frac{-2 \pm \sqrt{284}}{2}$$

$$x = \frac{-2 \pm \sqrt{4 \times 71}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{71}}{2}$$

$$x = \frac{2(-1 \pm \sqrt{71})}{2}$$

$$x = -1 \pm \sqrt{71}$$

$x^2 + 2x = 8$

$x^2 + 2x - 8 = 0$

$x^2 + 4x - 2x - 8 = 0$

$x(x + 4) - 2(x + 4) = 0$

$(x + 4)(x - 2) = 0$

$x + 4 = 0, x - 2 = 0$

$x = -4, x = 2$

$S.S = \{2, -4, -1 \pm \sqrt{71}\}$

(vii) $(x - 1)(x - 2)(x + 5)(x - 8) = -360$

Sol. $(x - 1)(x - 2)(x + 5)(x - 8) = -360$

$[(x - 1)(x - 2)][(x + 5)(x - 8)] = -360$

$(x^2 - 2x - x + 2)(x^2 - 8x + 5x - 40) = -360$

$(x^2 - 3x + 2)(x^2 - 3x - 40) = -360$

Let $x^2 - 3x = y$

$(y + 2)(y - 40) = -360$

$y^2 - 40y + 2y - 80 = -360$

$y^2 - 38y - 80 + 360 = 0$

$y^2 - 38y + 280 = 0$

$y^2 - 28y - 10y + 280 = 0$

$y(y - 28) - 10(y - 28) = 0$

$(y - 28)(y - 10) = 0$

$y - 28 = 0 \quad , \quad y - 10 = 0$

$y = 28 \quad , \quad y = 10$

Put $x^2 - 3x = y$

$x^2 - 3x = 28$

$x^2 - 3x - 28 = 0$

$x^2 - 7x + 4x - 28 = 0$

$x(x - 7) + 4(x - 7) = 0$

$(x - 7)(x + 4) = 0$

$x - 7 = 0 \quad , \quad x + 4 = 0$

$x = 7 \quad\quad x = -4$

$x^2 - 3x = 10$

$x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$

$x(x - 5) + 2(x - 5) = 0$

$(x - 5)(x + 2) = 0$

$x - 5 = 0 \quad , \quad x + 2 = 0$

$x = 5 \quad , \quad x = -2$

$S.S = \{-4, -2, 5, 7\}$

5. Ahmad takes 2 hours to paint 50 glasses. Faiza takes 2 hours to paint 45 glasses. Working together, how long should it take them to paint 150 glasses?

Sol. Ahmad rate per hour $= \dfrac{50\,\text{glasses}}{2\,\text{hours}} = 25$ glasses/hour

Ahmad rate per hour $= \dfrac{45\,\text{glasses}}{2\,\text{hours}} = 22.5$ glasses/hour

Combined rate $= 22.5 + 25 = 47.5$ glasses/hour

Time for point 150 glasses $= \dfrac{150}{47.5} = 3.16$ hours

6. A tap can fill a tank in 6 hours. Another tap can empty it in 9 hours. If both taps are opened together, how long will it take to fill the empty tank?

Sol. Filling rate $= +\dfrac{1}{6}$ tank / hour

Emptying rate $= -\dfrac{1}{9}$ tank / hour

$$\text{Net rate} = \frac{1}{6} - \frac{1}{9} = \frac{3-2}{18}$$

$$= \frac{1}{18} \text{ tank/hour}$$

so, it would tanks 18 hours to fill the tank

7. Maham bought a certain number of toys for Rs. 300. If each toy had cost Rs. 5 less, she could have bought two more for the same amount. How many did she buy?

Sol. Let the number of toys bought by Maham = x

Original price per toy = $\frac{300}{x}$

New price per toy = $\frac{300}{x+2}$

According to given condition = $\frac{300}{x} - \frac{300}{x+2} = 5$

Divide by 5 on both sides = $\frac{60}{x} - \frac{60}{x+2} = 1$

Multiply by LCM $x(x+2)$ on both sides

$x(x+2)\ \frac{(60)}{x} - x(x+2)\left(\frac{60}{x+2}\right) = x(x+2)$

$(x+2)\ 60 - x\ (60) = x^2 + 2x$

$60x + 120 - 60x = x^2 + 2x$

$-x^2 - 2x + 60x - 60x + 120 = 0$

$-x^2 - 2x + 120 = 0$

$x^2 + 2x - 120 = 0$

$x^2 + 12x - 10x - 120 = 0$

$x(x+12) - 10(x+12) = 0$

$(x+12)(x-10) = 0$

$x + 12 = 0$	$x - 10 = 0$
$x = -12$	$x = 10$

Negative Ignored

$x = 10$

So, number of toys Maham boug = 10 toys

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

5.1 Algebraic Fraction

Encircle the correct option.

1. Which of these is an algebraic fraction?

(a) $ax + b = 0$ (b) $\frac{3x}{x - x}$ (c) $\frac{3y}{x}$ (d) $ax^2 - bx + c = 0$

2. $\dfrac{27x^2y}{2x+4y}$ **is a type of expression.**

(a) an algebraic fraction (b) polynomial (c) equation (d) inequality

3. $\dfrac{x+5}{x+2}$ **is a:**

(a) polynomial (b) equation
(c) surd (d) rational expression

4. **Which is not an rational expression:**

(a) $\dfrac{3x^{-2}}{y}$ (b) $\dfrac{2x}{3y}$ (c) $\dfrac{x+y}{x^2-y^2}$ (d) $\dfrac{5ab}{10b}$

5. **Which is the example of an algebric fraction?**

(a) $\dfrac{5}{y}$ (b) $\dfrac{3x+4}{5x}$ (c) $\dfrac{a+b}{a-b}$ (d) All of them

5.1.2 Simplification of Rational Expressions

6. **Lowest form of** $\dfrac{3a^2b}{24b}$ **is:**

(a) $\dfrac{8a^2b}{a}$ (b) $\dfrac{a^2}{8}$ (c) $\dfrac{15}{16}b$ (d) $\dfrac{24a}{8b}$

7. **Lowest form of** $\dfrac{m^2-n^2}{m+n}$ **is:**

(a) m^3-n^3 (b) $(m+n)(m-n)$ (c) $m-n$ (d) $\dfrac{(m-n)(m-n)}{m+n}$

8. **What is the lowest form of** $\dfrac{6x^2y+8xy}{2xy}$

(a) $3x+4$ (b) $\dfrac{3xy+4xy}{2}$ (c) $\dfrac{2xy}{2xy}$ (d) $6x+8y$

9. **What is the lowest form of** $\dfrac{c^3-d^3}{c-d}$

(a) $\dfrac{(c+d)(c-d)}{c-d}$ (b) $(c-d)^2$ (c) $c-d$ (d) c^2+cd+d^2

10. **The lowest form of** $\dfrac{xy}{x^3y-xy^3}$ **is:**

(a) $\dfrac{1}{x^2-y^2}$ (b) $\dfrac{x^3y}{xy}$ (c) $\dfrac{x}{x^2-y^2}$ (d) $\dfrac{(x-y)^2}{x+y}$

11. **Simplify** $\dfrac{5x+10}{x+2}$ **provided.**

(a) 5 (b) $x+5$ (c) 10 (d) $5x$

12. **The lowest form $\dfrac{x^2-16}{x^2+8x+16}$ is:**

(a) $\dfrac{1}{8x}$ (b) -1 (c) $\dfrac{x-4}{x+4}$ (d) $\dfrac{x+4}{x-4}$

13. **What is the lowest form of $\dfrac{a+11}{a^2-121}$**

(a) $(a+1)(a-1)$ (b) $\dfrac{1}{a+11}$ (c) $\dfrac{1}{a-11}$ (d) $\dfrac{a+1}{a-11}$

14. **Lowest form of $\dfrac{x^2+7x+10}{x^2-4}$ is:**

(a) $\dfrac{x+5}{x+2}$ (b) $\dfrac{x-5}{x-2}$ (c) $\dfrac{7x+10}{x+2}$ (d) $\dfrac{x+5}{x-2}$

15. **Lowest form of $\dfrac{4a}{a^3-a}$ is**

(a) $\dfrac{4}{a^2+1}$ (b) $\dfrac{4}{a^2-1}$ (c) $\dfrac{4a}{a+1}$ (d) $\dfrac{1}{(a+1)(a-1)}$

5.2 Manipulation of Rational Expressions

16. $\dfrac{a}{b}+\dfrac{c}{d}=$ ____________.

(a) $\dfrac{ac}{bd}$ (b) $\dfrac{a+c}{b+d}$ (c) $\dfrac{ad+bc}{bd}$ (d) $\dfrac{ac}{b-d}$

17. **What is the LCM for $\dfrac{1}{x}+\dfrac{1}{x+2}$?**

(a) $x(x+2)$ (b) x^2+2 (c) $x(x-2)$ (d) x^2-2

18. $\dfrac{1}{a}-\dfrac{1}{b}=$ ____________.

(a) $\dfrac{a+b}{ab}$ (b) $\dfrac{b-a}{ab}$ (c) $\dfrac{1}{ab}$ (d) $\dfrac{a}{b}\times\dfrac{b}{c}$

19. $\dfrac{2}{c}+\dfrac{3}{f}=$ ____________.

(a) $\dfrac{5}{c+f}$ (b) $\dfrac{2c+3f}{cf}$ (c) $\dfrac{2f+3c}{cf}$ (d) $\dfrac{6}{c+f}$

20. $\dfrac{4}{x-3}-\dfrac{2}{x+1}=$ ____________.

(a) $\dfrac{2x+10}{(x-3)(x+1)}$ (b) $\dfrac{2x-2}{(x-3)(x+1)}$ (c) $\dfrac{2}{(x-3)(x+1)}$ (d) $\dfrac{2x+2}{x^2-2x-3}$

21. **Which value of x makes $\frac{x-5}{x^2-5}$ undefined?**

(a) 5 only (b) −5 only (c) 5 and −5 (d) 0

22. $\dfrac{\frac{1}{x}}{\frac{1}{y}} =$ ______________.

(a) $\frac{y}{x}$ (b) $\frac{1}{xy}$ (c) $\frac{x}{y}$ (d) $\frac{xy}{1}$

23. **Simplify $\frac{24x^2y}{5c} \times \frac{10c}{48y}$ is:**

(a) x^2 (b) $240xy$ (c) y^2 (d) $\frac{34}{5xy}$

24. **What is the simplified form $\frac{3x}{4y} \div \frac{3x}{8y}$?**

(a) $\frac{9x^2}{32y^2}$ (b) $\frac{3x}{8y}$ (c) $\frac{x}{y}$ (d) 2

25. **What is the simplified form of $(a^3 + b^3) \div (a + b)$?**

(a) $a^3 - b^3$ (b) $(a^2 - ab + b^2)$ (c) $(a^2 + ab + b^2)$ (d) $(a^2 - ab - b^2)$

26. **What is the simplified form of $a^2 - y^2 \times \frac{4x}{a-y}$**

(a) $4x\,(a + y)$ (b) $4x + (a - b)$ (c) $\frac{(a+y)}{a-y}$ (d) $\frac{4}{a-y}$

27. $a^4 - b^4 \times \frac{1}{a+b} =$ ______________.

(a) $\frac{a^2+b^2}{ab}$ (b) $\frac{a^3-b^2}{a+b}$ (c) $\frac{a^2-b^2}{a+b}$ (d) $(a^2 + b^2)(a - b)$

28. **What is the simplified form of $\frac{5x+1}{y} - \frac{4}{y}$?**

(a) $\frac{5x-3}{y}$ (b) $\frac{ax+1}{y}$ (c) 0 (d) $\frac{20x+4}{y^2}$

29. **What is the simplified form of $\frac{3x^2-3xy}{z} \div \frac{3x}{m}$?**

(a) $\frac{3x-4y}{zm}$ (b) $\frac{12x^2y}{mz}$ (c) $\frac{m(x-y)}{z}$ (d) $\frac{9xy-x}{m}$

30. **What is the simplified form of $\frac{1}{15a^2c^2} \times \frac{3ac}{4\ell}$**

(a) $20\,ac\ell$ (b) $\frac{1}{20ac\ell}$ (c) $\frac{3ac}{9ac\ell}$ (d) $\frac{15a^2c^2}{12ac}$

31. **Reduced quadratic equation form of $3x^4 - 11x^2 + 5 = 0$ is:**

(a) $\frac{3x^4}{x} - \frac{11}{x} - 5 = 0$ (b) $3y^2 + 11y - 5 = 0$
(c) $3y^2 + 11y = 5$ (d) $3y^2 - 11y + 5 = 0$

32. **An equation which remains unchanged when variable is replaced by its reciprocal is called:**

(a) Reciprocal equation (b) Exponential equation
(c) Radical equation (d) Simple equation

33. **$2^x - 5 \cdot 2^{2x} + 3 = 0$ is called:**

(a) Radical equation (b) Linear equation
(c) Exponential equation (d) Simple equation

34. **If $x^2 = \frac{3}{2}$ then $x =$ _______?**

(a) $\frac{3}{2}$ (b) $+\sqrt{\frac{3}{2}}$ (c) $-\sqrt{\frac{3}{2}}$ (d) $\pm\frac{\sqrt{6}}{2}$

35. **To reduce $ax^{2n} + bx^n + c = 0$ in to quadratic equation we put**

(a) $x^{2n} = y$ (b) $x^n = y$ (c) $x = y^{2n}$ (d) $y = n^x$

36. **The real roots of $x^6 - 9x^3 + 8 = 0$ are**

(a) 1, 3 (b) 0, 1 (c) 1, 2 (d) 3, 5

37. **If $4^x = 2$ then x equals**

(a) $-\frac{1}{2}$ (b) 2 (c) –2 (d) $\frac{1}{2}$

38. **If $3^x = 1$ then x equals**

(a) –3 (b) 2 (c) 0 (d) –1

39. **A reciprocal equation remains unchanged when x is replaced by:**

(a) $\frac{1}{x}$ (b) $-\frac{1}{x}$ (c) $\frac{1}{x^2}$ (d) $-x^2$

40. **The linear factors of $2y^2 - 11y + 12 = 0$ are**

(a) $(y + 11)(y + 2)$ (b) $(y - 3)(y + 4)$ (c) $(2y - 3)(y - 4)$ (d) $(2y + 12)(y + 3)$

41. **The linear factors of $y^2 - 24y + 128 = 0$ are**

(a) $(y - 16)(y - 8)$ (b) $(y + 8)(y + 16)$ (c) $(y - 16)(y + 16)$ (d) $(y + 9)(y + 6)$

ANSWERS:

1.	$\frac{3y}{x}$	2.	an algebraic fraction	3.	rational expression		
4.	$\frac{3x^{-2}}{y}$	5.	All of them	6.	$\frac{a^2}{8}$	7.	$m - n$
8.	$3x + 4$	9.	$c^2 + cd + d^2$	10.	$\frac{1}{x^2 - y^2}$	11.	5
12.	$\frac{x-4}{x+4}$	13.	$\frac{1}{a-11}$	14.	$\frac{x+5}{x-2}$	15.	$\frac{4}{a^2-1}$
16.	$\frac{ad+bc}{bd}$	17.	$x(x + 2)$	18.	$\frac{b-a}{ab}$		

19. $\dfrac{2f+3c}{cf}$ 20. $\dfrac{2x+10}{(x-3)(x+1)}$ 21. 5 and −5 22. $\dfrac{y}{x}$ 23. x^2

24. 2 25. (a^2-ab+b^2) 26. $4x(a+y)$ 27. $(a^2+b^2)(a-b)$

28. $\dfrac{5x-3}{y}$ 29. $\dfrac{m(x-y)}{z}$ 30. $\dfrac{1}{20ac\ell}$ 31. $3y^2-11y+5=0$

32. Reciprocal equation 33. Exponential equation 34. $\pm\dfrac{\sqrt{6}}{2}$

35. $x^n=y$ 36. 1, 2 37. $\dfrac{1}{2}$ 38. 0 39. $\dfrac{1}{x}$

40. $(2y-3)(y-4)$ 41. $(y-16)(y-8)$

Additional Short Question

Short Answer Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

5.1 Algebraic Fraction

○ Give short answers.

1. Define an algebraic fraction:

Ans. An algebraic fraction is a fraction where the numerator, denominator or both contain algebraic expressions.

2. What value of x makes the fraction $\dfrac{7}{x-5}$ undefined?

Ans. x cannot be 5 because thus denominator becomes zero.

3. Define a rational expression:

Ans. A rational expression is the goutiest $\dfrac{P(x)}{Q(x)}$ where $Q(x)\neq 0$ of two polynomials P(x) and $Q(x)\in Z$.

4. Is $\dfrac{19}{x}$ an algebric fraction.

Ans. Yes, $\dfrac{19}{x}$ is an algebraic fraction because the denominator contains a variable (x).

Simplification of rational expressions

5. Reduce to lowest form $\dfrac{18a^2c^2y^4}{6a^2y^2-12ac}$

Sol. $\dfrac{18a^2c^3y^4}{6a^2y^2-12ac}$

$$=\frac{18a^2c^3y^4}{6a\left(ay^2-2c\right)}=\frac{3a^{2-1}c^3y^4}{ay^2-2c}=\frac{3ac^3y^4}{ay^2-2c}$$

6. Reduce to the lowest form $\frac{5x^2-15xy}{9xy-27y^2}$

Sol. $\frac{5x^2-15xy}{9xy-27y^2}$

$$=\frac{5x(x-3y)}{9y(x-3y)}=\frac{5x}{9y}$$

7. Reduce to the lowest form $\frac{3x+4}{6x^2-x-12}$

Sol. $\frac{3x+4}{6x^2-x-12}$

$$=\frac{3x+4}{6x^2-9x+8x-12}$$

$$=\frac{3x+4}{3x(2x-3)+4(2x-3)}$$

$$=\frac{3x+4}{(3x+4)(2x-3)}$$

$$=\frac{1}{(2x-3)}$$

8. Reduce to the lowest form $\frac{4x+8}{4}$

Sol. $\frac{4x+8}{4}$

$$=\frac{4(x+2)}{4}=x+2$$

9. Reduce to the lowest form $\frac{x^2+5x+6}{x^2-4}$

Sol. $\frac{x^2+5x+6}{x^2-4}$

$$=\frac{x^2+3x+2x+6}{(x)^2-(2)^2}$$

$$=\frac{x(x+3)+2(x+3)}{(x+2)(x-2)}$$

$$=\frac{(x+2)(x+3)}{(x+2)(x-2)}$$

$$=\frac{x+3}{x-2}$$

10. Reduce to the lowest form $\frac{x^6+y^6}{x^2+y^2}$

Sol. $\frac{x^6+y^6}{x^2+y^2}$

$$=\frac{(x^2)^3+(y^2)^3}{x^2+y^2}$$

$$=\frac{(x^2+y^2)\left((x^2)^2-(x^2)(y^2)+(y^2)^2\right)}{(x^2+y^2)}$$

$$=\frac{(x^2+y^2)(x^4-x^2y^2+y^4)}{(x^2+y^2)}$$

$$=x^4-x^2y^2+y^4$$

5.2 Manipulation of Rational Expressions

11. Simplify $\frac{x}{x-y}+\frac{2x}{x+y}$

Sol. $\frac{x}{x-y}+\frac{2x}{x+y}$

$$=\frac{x(x+y)+2x(x-y)}{(x-y)(x+y)}$$

$$=\frac{x^2+xy+2x^2-2xy}{(x-y)(x+y)}$$

$$=\frac{3x^2-xy}{(x-y)(x+y)}=\frac{x(3x-y)}{(x-y)(x+y)}$$

12. Simplify $\frac{x}{5}+\frac{5}{x}$

Sol. $\frac{x}{5}+\frac{5}{x}$

$$=\frac{x^2+25}{5x}$$

13. Simplify $\frac{2}{x+3}-\frac{1}{x}$

Sol. $\frac{2}{x+3}-\frac{1}{x}$

$$= \frac{2x-(x+3)}{x(x+3)}$$

$$= \frac{2x-x-3}{x(x+3)}$$

$$= \frac{x-3}{x(x+3)}$$

14. Simplify $\frac{3}{x^2-4}-\frac{1}{x-2}$

Sol. $\frac{3}{x^2-4}-\frac{1}{x-2}$

$$= \frac{3}{(x+2)(x-2)}-\frac{1}{x-2}$$

$$= \frac{3-(x+2)}{(x+2)(x-2)}$$

$$= \frac{3-x-2}{(x+2)(x-2)}$$

$$= \frac{1-x}{x^2-4}$$

15. Simplify $\frac{1}{6x}+\frac{1}{4x^2}$

Sol. $\frac{1}{6x}+\frac{1}{4x^2}$

$$= \frac{2x+3}{12x^2}$$

16. Simplify $\frac{x^2-9}{4}\times\frac{8}{x-3}$

Sol. $\frac{x^2-9}{4}\times\frac{8}{x-3}$

$$= \frac{(x+3)(x-3)}{4}\times\frac{8}{x-3}$$

$$= \frac{8(x+3)(x-3)}{4(x-3)}$$

$$= 2(x+3)$$

17. Simplify $\frac{2x^2+5x-3}{x^2-9}\times\frac{x+3}{2x-1}$

Sol. $\frac{2x^2+5x-3}{x^2-9}\times\frac{x+3}{2x-1}$

$$= \frac{2x^2+6x-x-3}{(x)^2-(3)^2}\times\frac{x+3}{2x-1}$$

$$= \frac{2x(x+3)-1(x+3)}{(x+3)(x-3)}\times\frac{x+3}{2x-1}$$

$$= \frac{(2x-1)(x+3)(x+3)}{(2x-1)(x+3)(x-3)}$$

$$= \frac{x+3}{x-3}$$

18. $\frac{x^3-8}{x^2-4}\times\frac{x+2}{x^2+2x+4}$

Sol. $= \frac{(x^3)-(2)^3}{(x)^2-(2)^2}\times\frac{x+2}{x^2+2x+4}$

$$= \frac{(x-2)\left(x^2+(2)(x)+(2)^2\right)}{(x+2)(x-2)}\times\frac{x+2}{x^2+2x+4}$$

$$= \frac{(x-2)(x+2)\left(x^2+2x+4\right)}{(x-2)(x+2)\left(x^2+2x+4\right)}=1$$

19. Simplify $\frac{x^2-9}{4x^2-9}\div\frac{x+3}{2x-3}$

Sol. $\frac{x^2-9}{4x^2-9}\div\frac{x+3}{2x-3}$

$$\frac{(x)^2-(3)^2}{(2x)^2-(3)^2}\div\frac{x+3}{2x-3}$$

$$\frac{(x+3)(x-3)}{(2x+3)(2x-3)}\times\frac{2x-3}{x+3}$$

$$= \frac{(x+3)(x-3)(2x-3)}{(x+3)(2x+3)(2x-3)}$$

$$= \frac{x-3}{2x+3}$$

20. Simplify $\dfrac{x^2-x-12}{x^2-16}\div\dfrac{x^2+6x+9}{x+4}$

Sol. $\dfrac{x^2-x-12}{x^2-16}\div\dfrac{x^2+6x+9}{x+4}$

$$=\frac{x^2-4x+3x-12}{(x)^2-(4)^2}\div\frac{(x)^2+2(x)(3)+(3)^2}{x+4}$$

$$=\frac{x(x-4)+3(x-4)}{(x+4)(x-4)}\div\frac{(x+3)^2}{x+4}$$

$$=\frac{(x+3)(x-4)}{(x+4)(x-4)}\times\frac{x+4}{(x+3)^2}$$

$$=\frac{(x-4)(x+4)(x+3)}{(x-4)(x+4)(x+3)(x+3)}$$

$$=\frac{1}{x+3}$$

21. Simplify $\dfrac{10x^3}{x^3-y^3}\div\dfrac{5x}{x^2+xy+y^2}$

Sol. $\dfrac{10x^3}{x^3-y^3}\div\dfrac{5x}{x^2+xy+y^2}$

$$=\frac{10x^3}{(x-y)(x^2+xy+y^2)}\times\frac{x^2+xy+y^2}{5x}$$

$$=\frac{10x^3\times(x^2+xy+y^2)}{5x(x-y)(x^2+xy+y^2)}=\frac{2x^{3-1}}{x-y}=\frac{2x^2}{x-y}$$

22. Simplify $\dfrac{y^5}{y-3}\times\dfrac{y^2-9}{y^4}$

Sol. $\dfrac{y^5}{y-3}\times\dfrac{y^2-9}{y^4}$

$$=\frac{y^5}{y-3}\times\frac{(y)^2-(3)^2}{y^4}$$

$$=\frac{y^5}{y-3}\times\frac{(y-3)(y+3)}{y^4}=\frac{y^5(y-3)(y+3)}{y^4(y-3)}$$

$$=y^{5-4}(y+3)$$

$$=y(y+3)$$

5.3 Solution of Equations Reducible to Quadratic Equations

23. For $x^6 - 9x^3 + 7 = 0$, what substitution would you use to reduce it to a quadratic equation.

Sol. Let $x^3 = y$

$(x^3)^2 = y^2$

$x^6 = y^2$

The equation becomes $y^2 - 9y + 7 = 0$

24. In the equation $5^{2x} - 6 \cdot 5x + 5 = 0$, if we let $5x = y$, what is the resulting quadratic equation?

Sol. If $y = 5^x$, then

$y^2 = (5^x)^2$

$y^2 = 5^{2x}$

The equation becomes $y^2 - 6y + 5 = 0$

25. Define exponential equation:

Ans. Equations in which variable occur in exponents are called exponential equations for example $2^x - 3.2^{2x} = -56$

26. Define "Reciprocal" equations.

Ans. An equation which remains unchanged when variable is replaced by its reciprocal.

27. The sum of a natural number and its reciprocal is $\frac{10}{3}$. Find the number.

Sol. Let the number be x, and its reciprocal is $\frac{1}{x}$ to set up equation.

$$x + \frac{1}{x} = \frac{10}{3}$$

$$3x(x) + 3x\left(\frac{1}{x}\right) = 3x\left(\frac{10}{3}\right)$$

$$3x^2 + 3 = 10x$$

$$3x^2 - 10x + 3 = 0$$

$$3x^2 - 9x - x + 3 = 0$$

$$3x(x - 3) - 1(x - 3) = 0$$

$$(x - 3)(3x - 1) = 0$$

$$x - 3 = 0$$

$$x = 3$$

$$3x - 1 = 0$$

$$3x = 1$$

$$x = \frac{1}{3}$$

So, $x = 3, \frac{1}{3}$

Since it is a natural number is 3.