UNIT 03 Matrices and Determinants

Students Learning Outcomes

After completing this unit, the students will be able to:

- Display information in the form of matrix of order 2.
- Solve situations involving sum, difference and product of two matrices.
- Calculate the product of the scalar quantity and a matrix.
- Evaluate the determinant and inverse of a matrix of order 2-by-2.
- Solve the simultaneous linear equations in two variables using matrix inversion method and Cramer's rule.
- Explain, with examples, how mathematics plays a key role in the development of new scientific theories and technologies, [e.g., Mathematical models and simulations are used to design and optimize new materials and drugs, and to understand the behaviours of complex systems such as the human brain.
- Apply concepts of matrices to real world problems (such as engineering, economics, computer graphics and physics).

3.1 Matrices

A matrix is a rectangular array in shape, whose elements (entries) are written within square brackets in a specific order, in rows and columns. The arrays obey certain algebraic operations.

Generally, the matrices are denoted by the capital letters of the English alphabets while their elements are denoted by the small letters.

e.g $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B=\begin{bmatrix} 5 & 7 \end{bmatrix}$, $C=\begin{bmatrix} 3 \\ 2 \end{bmatrix}$,

$D=\begin{bmatrix} -2 & 0 \\ 0 & 5 \end{bmatrix}$, $E=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $F=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Rows and Columns of a Matrix

Row: Entry or entries written in the horizontal line or lines form the row or rows of a matrix.

Column: Entry or entries written in vertical line or lines form the column or columns of a matrix.

For example,

$$\begin{array}{cc} & \begin{array}{cccc} \text{Column 1} & \text{Column 2} & \text{Column 3} & \text{Column 4} \end{array} \\ A = & \begin{bmatrix} 2 & 3 & 7 & 1 \\ 1 & 5 & 3 & 2 \\ 9 & 1 & 8 & 4 \end{bmatrix} \begin{array}{l} \leftarrow \text{Row 1} \\ \leftarrow \text{Row 2} \\ \leftarrow \text{Row 3} \end{array} \end{array}$$

Matrix A has 3 rows and 4 columns.

$$\begin{array}{cc} & \begin{array}{ccc}\text{Column 1} & \text{Column 2} & \text{Column 3}\end{array} \\ B = & \begin{bmatrix} 7 & 4 & 8 \\ 3 & 0 & 10 \\ 2 & 1 & 11 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow \text{Row 1} \\ \leftarrow \text{Row 2} \\ \leftarrow \text{Row 3} \end{array}$$

Matrix B has 3 rows and 3 columns.

$$C = [5]$$

Matrix C has 1 row and 1 column.

Elements or Entries

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, then 1,2, 3,4, 5, 6 are called elements or entries of matrix A.

Order or Size of a Matrix:

If a matrix A has 'm' number of rows and 'n' number of columns, then the order or size of the matrix A is "m×n" (read as m-by-n). Order of matrix $B = \begin{bmatrix} 5 & -1 & 0 \\ 9 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ is 3-by-3. If order of a matrix is 3-by-4, it means it has 3 rows and 4 columns. It does not mean 3-by-4 = 12, however 12 gives the number of elements in the matrix.

Equal Matrices:

Two matrices A and B are said to be equal if and only if they are of same order and their corresponding elements are same.

For example: $A = \begin{bmatrix} 2 & 3 \\ 4 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4-2 & 3 \\ 4 & 4-6 \end{bmatrix}$ are equal matrices.

Similarly, $C = [a \quad b]$, $D = [3 \quad 4]$ will be equal matrices if and only if a = 3 and b = 4.

Solved Exercise 3.1

1. Write the number of rows and number of columns in each matrix.

$$\mathbf{A} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}, \mathbf{C} = [8 \ -10 \ 11], \mathbf{D} = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 0 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 5 & 9 & -2 \\ -3 & 4 & 5 \end{bmatrix}$$

Sol. Number of rows of the matrix A = 2
Number of columns in matrix A = 2
Number of rows of the matrix B = 3
Number of columns of the matrix B = 1
Number of rows of the matrix C = 1
Number of columns of the matrix C = 3
Number of rows of the matrix D = 3
Number of columns of the matrix D = 3
Number of rows of the matrix E = 2
Number of columns of the matrix E = 3

2. Write the order of each matrix.

$$A=\begin{bmatrix}1\\0\end{bmatrix},\ B=[3\ \ 4],\ C=\begin{bmatrix}3&5\\5&6\end{bmatrix},\ D=\begin{bmatrix}0&3&4\\5&0&9\end{bmatrix},\ E=\begin{bmatrix}0&1\\3&4\\5&6\end{bmatrix},\ F=[5]$$

Sol. $A=\begin{bmatrix}1\\0\end{bmatrix}$

Order of the matrix A is 2-by-1

$C=\begin{bmatrix}3&5\\5&6\end{bmatrix}$

Order of the matrix C is 2-by-2

$E=\begin{bmatrix}0&1\\3&4\\5&6\end{bmatrix}$

Order of the matrix E is 3-by-2

$B=[3\ \ 4]$

Order of the matrix B is 1-by-2

$D=\begin{bmatrix}0&3&4\\5&0&9\end{bmatrix}$

Order of the matrix D is 2-by-3

$F=[5]$

Order of the matrix F is 1-by-1

3. Which of the following matrices are equal?

$$A=\begin{bmatrix}0&3\\4&1\end{bmatrix},\quad B=\begin{bmatrix}9\\-7\end{bmatrix},\quad C=\begin{bmatrix}2\times3&2-1\\2\times2&4-2\\4+4&3+0\end{bmatrix}$$

$$D=\begin{bmatrix}5+4\\-8+1\end{bmatrix},\quad E=\begin{bmatrix}6&1\\4&2\\8&3\end{bmatrix},\quad F=\begin{bmatrix}3-3&3\\3+1&1\end{bmatrix}$$

Sol. $A=\begin{bmatrix}0&3\\4&1\end{bmatrix}$ and $F=\begin{bmatrix}3-3&3\\3+1&1\end{bmatrix}\Rightarrow F=\begin{bmatrix}0&3\\4&1\end{bmatrix}$ are equal matrices because they are of same order and the corresponding elements are same i.e., A = F

$B=\begin{bmatrix}9\\-7\end{bmatrix}$ and $D=\begin{bmatrix}5+4\\-8+1\end{bmatrix}\Rightarrow D=\begin{bmatrix}9\\-7\end{bmatrix}$ are equal matrices because they are of same order and the corresponding elements are same i.e., B = D

$C=\begin{bmatrix}2\times3&2-1\\2\times2&4-2\\4+4&3+0\end{bmatrix}\Rightarrow C=\begin{bmatrix}6&1\\4&2\\8&3\end{bmatrix}$ and $E=\begin{bmatrix}6&1\\4&2\\8&3\end{bmatrix}$ are equal matrices because they are of same order and their corresponding elements are same i.e. C = E

4. If $\begin{bmatrix}a+2&c-3\\b-1&d+4\end{bmatrix}=\begin{bmatrix}5&8\\6&4\end{bmatrix}$, then find the values of a, b, c and d.

Sol. Given that $\begin{bmatrix}a+2&c-3\\b-1&d+4\end{bmatrix}=\begin{bmatrix}5&8\\6&4\end{bmatrix}$

As both matrices are equal, so corresponding elements are equal.

$a+2=5$	$c-3=8$	$b-1=6$	$d+4=4$
$a=5-2$	$c=8+3$	$b=6+1$	$d=4-4$
$a=3$	$c=11$	$b=7$	$d=0$

5. If $\begin{bmatrix} 2x+1 & 4 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 7 & y \end{bmatrix}$, then find the values of x and y.

Sol. Given that $\begin{bmatrix} 2x+1 & 4 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 7 & y \end{bmatrix}$

As both matrices are equal, so their corresponding elements are same.

$$2x + 1 = 9 \qquad y = 5$$
$$2x = 9 - 1$$
$$2x = 8$$
$$x = \frac{8}{2}$$
$$x = 4$$
$$x = 4,\ y = 5$$

6. If $\begin{bmatrix} a+b & 2d-1 \\ 3b+2 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 11 & c \end{bmatrix}$, then find the values of a, b, c and d.

Sol. Given that $\begin{bmatrix} a+b & 2d-1 \\ 3b+2 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 11 & c \end{bmatrix}$

As both matrices are equal, so corresponding elements are same.

$$a + b = 10 \quad \_\_\_ \text{(i)}$$
$$3b + 2 = 11 \quad \_\_\_ \text{(ii)}$$
$$4 = c \quad \_\_\_ \text{(iii)}$$
$$2d - 1 = 5 \quad \_\_\_ \text{(iv)}$$

From (iii)

$$c = 4$$

From (ii)

$$3b + 2 = 11$$
$$3b = 11 - 2$$
$$3b = 9$$
$$b = \frac{9}{3} = 3 \Rightarrow b = 3$$

Putting b = 3 in (i)

$$a + b = 10$$
$$a + 3 = 10$$
$$a = 10 - 3 = 7 \Rightarrow a = 7$$

From (iv)

$$2d - 1 = 5$$
$$2d = 5 + 1$$
$$2d = 6$$
$$d = \frac{6}{2} = 3 \Rightarrow d = 3$$

$$a = 7,\ b = 3,\ c = 4,\ d = 3$$

7. If $\begin{bmatrix} p+q & 5 \\ 11 & p-2q \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 11 & 0 \end{bmatrix}$, then find the values of p and q.

Sol. Given that $\begin{bmatrix} p+q & 5 \\ 11 & p-2q \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 11 & 0 \end{bmatrix}$

As both matrices are equal, so their corresponding elements are same.

$$p + q = 6 \quad \text{\_\_\_ (i)}$$
$$p - 2q = 0 \quad \text{\_\_\_ (ii)}$$

From (ii)

$$p - 2q = 0$$
$$p = 2q$$

Putt p = 2q in (i)

$$p + q = 6$$
$$2q + q = 6$$
$$3q = 6$$
$$q = \frac{6}{3} = 2 \Rightarrow q = 2$$

Putting q = 2 into (i)

$$p + q = 6$$
$$p + 2 = 6$$
$$p = 6 - 2 = 4 \Rightarrow p = 4$$

Hence, $p = 4, q = 2$

3.2 Types of Matrices

(i) Row Matrix: A matrix having only one row is called a row matrix.

For example: A = [0 1 4], B = [a b], C = [a] are row matrices of order 1-by-3, 1-by-2 and 1-by-1 respectively.

(ii) Column Matrix: A matrix having only one column is called a column matrix.

For example: $A = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}, B = \begin{bmatrix} a \\ b \end{bmatrix}$, C = [2] are column matrices of order 3-by-1, 2-by-1 and 1-by-1 respectively.

(iii) Zero matrix or Null matrix or Void Matrix

If all the entries in a matrix are zero, it is called zero matrix, null matrix or a void matrix. It is represented by O.

For example: $[0], \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ are zero matrices of order 1-by-1, 2-by-2 and 2-by-3 respectively.

(iv) Square Matrix: A matrix in which number of rows is equal to the number of columns is called a square matrix. A matrix of order *n*-by-*n* is often referred to as a square matrix of order *n*.

For example: $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & l \end{bmatrix}$ is a square matrix of order 3.

In a square matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

a_{11}, a_{22}, a_{33} are called entries of principal diagonal or main diagonal or leading diagonal.

(v) Rectangular Matrix: If the number of rows and number of columns in a matrix are not equal, then it is called a rectangular matrix.

For example: $A = [a\ b], B = \begin{bmatrix} a \\ b \end{bmatrix}, C = \begin{bmatrix} 4 & 0 & 7 \\ 8 & 3 & 1 \end{bmatrix}$ are rectangular matrices.

(vi) Diagonal Matrix: A square matrix is called a diagonal matrix if atleast any one of the elements of its diagonal is non-zero and non-diagonal elements are zero.

For example: $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ are diagonal matrices of order 3-by-3 and 2-by-2 respectively.

(vii) Scalar Matrix: A diagonal matrix is called a scalar matrix if all the entries in the diagonal are the same and non-zero.

For example: $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, C = \begin{bmatrix} \sqrt{3} \end{bmatrix}$ are scalar matrices of order 3-by-3 2-by-2 and 1-by-1 respectively.

(viii) Identity Matrix or Unit Matrix: A scalar matrix is called an identity or unit matrix if all of its main diagonal entries are 1 and the rest are zero.

For example: $P = [1], Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are identity matrices of order 1-by-1, 2-by-2 and 3-by-3 respectively. We represent these matrices as:

$$I_1 = [1], I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

> **Remember!**
> A scalar matrix and identity matrix are also diagonal matrices but all diagonal matrices are not scalar or identity matrices.

(ix) Transpose of a Matrix: A matrix obtained by interchanging the rows and columns is called the transpose of the given matrix. If A is a matrix, then its transpose is written as A^t.

For example: $A = \begin{bmatrix} 1 & 2 & 4 \\ 8 & 0 & 5 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & 8 \\ 2 & 0 \\ 4 & 5 \end{bmatrix}$

(x) Symmetric Matrix: A square matrix A is symmetric if it is equal to its transpose, i.e., $A^t = A$.

For example: $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$, then $A^t = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = A$

Thus, A is a symmetric matrix.

(xi) Skew-Symmetric Matrix: A square matrix A is called skew-symmetric if $A^t = -A$.

For example: $B = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$, then $B^t = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} = -B$

Thus, B is a skew-symmetric matrix.

(xii) Negative of a Matrix: The negative of a matrix A is obtained by multiplying each of its elements by -1. It is denoted by $-A$.

For example: If $A = [3 \quad -6]$, then $-A = [-3 \quad 6]$

Solved Exercise 3.2

1. From the following matrices identify unit matrices, row matrices, column matrices and null matrices.

$A = [5 \quad 7 \quad 8]$, $B = [0]$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $E = \begin{bmatrix} 6 \\ 0 \\ 8 \end{bmatrix}$, $F = \begin{bmatrix} 7 \\ 1 \\ 9 \end{bmatrix}$

Sol. Matrix A is a row matrix of order 1-by-3

Matrix B is a null matrix of order 1-by-1

It is also a row matrix and a column matrix.

Matrix C is a unit matrix of order 2-by-2

Matrix D is a null matrix of order 2-by-2

Matrix E is a column matrix of order 3-by-1

Matrix F is a column matrix of order 3-by-1

2. Identify type of the given matrices as row, column, square and rectangular matrices.

$A = \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 6 \\ 4 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 5 & 8 \\ 0 & 4 & -2 \end{bmatrix}$, $D = \begin{bmatrix} 5 & -5 \\ 2 & 7 \end{bmatrix}$

$E = \begin{bmatrix} 3 & 2 \\ 4 & 1 \\ 5 & 0 \end{bmatrix}$, $F = [5 \quad -3 \quad 7]$, $G = \begin{bmatrix} 3 & 0 & 1 \\ 1 & 3 & 4 \\ 5 & 2 & -3 \end{bmatrix}$, $H = \begin{bmatrix} 3 & 5 \\ 4 & 4 \\ 5 & 2 \end{bmatrix}$

Sol. Matrix F is a row matrix.

Matrix A is a column matrix

Matrix B, D and G are square matrix

Matrix A, C, E, F and H are rectangular matrix.

3. Identify diagonal, scalar and unit matrices.

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 7 & 0 \\ 0 & 6 \end{bmatrix}$, $E = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

Sol. Matrices C and D are diagonal matrices of order 2-by-2

Matrices B and E are scalar matrices of order 2-by-2

Matrix A is unit matrix of order 2-by-2

4. Find transpose of each of the following matrices:

$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 7 \\ 6 \end{bmatrix}$, $C = [5 \quad -2 \quad 4]$, $D = \begin{bmatrix} 2 & 5 \\ 3 & 6 \\ 4 & 7 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 7 \\ 6 \end{bmatrix}$, $C = [5 \ -2 \ 4]$, $D = \begin{bmatrix} 2 & 5 \\ 3 & 6 \\ 4 & 7 \end{bmatrix}$

$A^t = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$, $B^t = [3 \ 7 \ 6]$, $C^t = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$ $D^t = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$

5. Find negative of the following matrices:

$A = \begin{bmatrix} -3 & 0 \\ 5 & 6 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 3 \\ -2 & 2 \end{bmatrix}$, $C = \begin{bmatrix} -9 & 1 \\ 1 & -7 \end{bmatrix}$

Sol. $-A = \begin{bmatrix} 3 & 0 \\ -5 & -6 \end{bmatrix}$ $-B = \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix}$ $-C = \begin{bmatrix} 9 & -1 \\ -1 & 7 \end{bmatrix}$

6. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 6 \\ 5 & 8 \end{bmatrix}$, then verify that

(i) $(A^t)^t = A$ (ii) $(B^t)^t = B$

Sol. (i) $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

$A^t = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$

$(A^t)^t = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = A$

So, verified that $(A^t)^t = A$

(ii) $B = \begin{bmatrix} 7 & 6 \\ 5 & 8 \end{bmatrix}$

$B^t = \begin{bmatrix} 7 & 5 \\ 6 & 8 \end{bmatrix}$

$(B^t)^t = \begin{bmatrix} 7 & 6 \\ 5 & 8 \end{bmatrix} = B$

So, verified that $(B^t)^t = B$

7. Show that $L = \begin{bmatrix} 2 & 3 & 4 \\ 3 & -2 & 5 \\ 4 & 5 & 0 \end{bmatrix}$ is a symmetric matrix.

Sol. $L = \begin{bmatrix} 2 & 3 & 4 \\ 3 & -2 & 5 \\ 4 & 5 & 0 \end{bmatrix}$

$L^t = \begin{bmatrix} 2 & 3 & 4 \\ 3 & -2 & 5 \\ 4 & 5 & 0 \end{bmatrix} = L$

$L^t = L$ so, L is a symmetric matrix

3.3 Algebraic Operations on Matrices

Basic operations on matrices are:

(i) Multiplication of a matrix by a scalar. (ii) Addition / subtraction of two matrices.

(iii) Multiplication of two matrices.

There is no concept of dividing a matrix by another matrix and thus, the operation $\frac{A}{B}$, where A and B are matrices, is not defined.

Multiplication of a Matrix by a Scalar

For a given matrix $A=\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ and a scalar k, we define a new matrix as $kA=\begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$,

that is each element has been multiplied by a scalar k. In particular if $k= -1$ -1, then

$-A=\begin{bmatrix} -a & -b & -c \\ -d & -e & -f \end{bmatrix}$.

Addition of Matrices

If A and B are two matrices of the **same order**, these can be added, then their sum denoted by $A + B$, is again a matrix of the **same order**. They are said to be **conformable for addition**. $A + B$ is obtained by adding the **corresponding entries** of A and B.

Example 1: If $A=\begin{bmatrix} 3 & 8 & 2 \\ 5 & 3 & 1 \\ 6 & 0 & 7 \end{bmatrix}$, $B=\begin{bmatrix} 4 & 1 & -2 \\ 3 & 0 & 6 \\ 2 & 3 & 4 \end{bmatrix}$, then find $A + B$ and $B + A$.

Solution : Given that $A=\begin{bmatrix} 3 & 8 & 2 \\ 5 & 3 & 1 \\ 6 & 0 & 7 \end{bmatrix}$, $B=\begin{bmatrix} 4 & 1 & -2 \\ 3 & 0 & 6 \\ 2 & 3 & 4 \end{bmatrix}$

Order of A = 3-by-3 and order of B = 3-by-3. A and B are conformable for addition

Now, $A+B=\begin{bmatrix} 3+4 & 8+1 & 2-2 \\ 5+3 & 3+0 & 1+6 \\ 6+2 & 0+3 & 7+4 \end{bmatrix}=\begin{bmatrix} 7 & 9 & 0 \\ 8 & 3 & 7 \\ 8 & 3 & 11 \end{bmatrix}$

and $B+A=\begin{bmatrix} 4+3 & 1+8 & -2+2 \\ 3+5 & 0+3 & 6+1 \\ 2+6 & 3+0 & 4+7 \end{bmatrix}=\begin{bmatrix} 7 & 9 & 0 \\ 8 & 3 & 7 \\ 8 & 3 & 11 \end{bmatrix}$

Subtraction of Matrices

If A and B are two matrices of the **same order**, they can be subtracted and their difference is denoted as $A - B$ or $B - A$, it is again a matrix of the same order. They are said to be **conformable for subtraction**. $A-B$ obtained by subtracting the corresponding entries of B from A.

Example 2: If $A=\begin{bmatrix} 5 & 9 \\ 4 & 8 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 6 \\ 2 & 1 \end{bmatrix}$, then find $A-B$ and $B-A$.

Solution: $A=\begin{bmatrix} 5 & 9 \\ 4 & 8 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 6 \\ 2 & 1 \end{bmatrix}$

Now, $A-B=\begin{bmatrix} 5-3 & 9-6 \\ 4-2 & 8-1 \end{bmatrix}=\begin{bmatrix} 2 & 3 \\ 2 & 7 \end{bmatrix}$

and $B-A=\begin{bmatrix} 3-5 & 6-9 \\ 2-4 & 1-8 \end{bmatrix}=\begin{bmatrix} -2 & -3 \\ -2 & -7 \end{bmatrix}$

we see $A - B \neq B - A$

Example 3: If $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 & 1 \\ 5 & 2 & 7 \end{bmatrix}$, then find $A + B$ and $B - A$, if possible.

Solution: $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 & 1 \\ 5 & 2 & 7 \end{bmatrix}$

Order of A is 2-by-2 and order of B is 2-by-3. Therefore, A and B are not conformable for addition and subtraction.

Commutative and Associative Laws of Addition of Matrices

a. Commutative Law of Addition of Matrices

For any two matrices A and B of same orders $A + B = B + A$. This law is called commutative law of matrices with respect to addition.

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 & 3 \\ 4 & 2 & 2 \\ 0 & -1 & 0 \end{bmatrix}$

Then $A + B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 3 \\ 4 & 2 & 2 \\ 0 & -1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1+3 & 2+1 & 3+3 \\ 4+4 & 5+2 & 6+2 \\ 7+0 & 8-1 & 9+0 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 6 \\ 8 & 7 & 8 \\ 7 & 7 & 9 \end{bmatrix} \quad \text{...(i)}$$

and $B + A = \begin{bmatrix} 3 & 1 & 3 \\ 4 & 2 & 2 \\ 0 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$$= \begin{bmatrix} 3+1 & 1+2 & 3+3 \\ 4+4 & 2+5 & 2+6 \\ 0+7 & -1+8 & 0+9 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 6 \\ 8 & 7 & 8 \\ 7 & 7 & 9 \end{bmatrix} \quad \text{...(ii)}$$

Hence, from (i) and (ii) $A + B = B + A$

Commutative law of addition of matrices holds.

b. Associative Law of Addition of Matrices

For any three matrices A, B and C of same order $(A + B) + C = A + (B + C)$. This law is called associative law of matrices with respect to addition.

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 & 3 \\ 4 & 2 & 2 \\ 0 & -1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

then $(A + B) + C = \left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 3 \\ 4 & 2 & 2 \\ 0 & -1 & 0 \end{bmatrix} \right) + \begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+3 & 2+1 & 3+3 \\ 4+4 & 5+2 & 6+2 \\ 7+0 & 8-1 & 9+0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & 6 \\ 8 & 7 & 8 \\ 7 & 7 & 9 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 11 & 7 & 11 \\ 8 & 8 & 10 \end{bmatrix} \quad \text{...(i)}$$

and $A + (B + C) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \left(\begin{bmatrix} 3 & 1 & 3 \\ 4 & 2 & 2 \\ 0 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix} \right)$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 3+0 & 1+2 & 3+0 \\ 4+3 & 2+0 & 2+3 \\ 0+1 & -1+1 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 3 \\ 7 & 2 & 5 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 11 & 7 & 11 \\ 8 & 8 & 10 \end{bmatrix} \quad \text{...(ii)}$$

Hence, from (i) and (ii) $(A + B) + C = A + (B + C)$

Associative law of addition of matrices holds.

Additive Identity of a Matrix

The additive identity of a matrix is a matrix that, when added to any matrix A of the same order, leaves it unchanged.

Mathematically, $A + O = A = O + A$, where O is the zero matrix (also called the **additive identity matrix**).

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Then $A + O = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = A$

and $O + A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = A$

Additive Inverse of a Matrix

The additive inverse of a matrix A is another matrix $-A$ such that

$$A + (-A) = O = (-A) + A.$$

Let $A = \begin{bmatrix} -2 & 1 & 3 \\ 4 & 2 & 0 \\ -3 & 0 & 5 \end{bmatrix}$, then $-A = \begin{bmatrix} 2 & -1 & -3 \\ -4 & -2 & 0 \\ 3 & 0 & -5 \end{bmatrix}$

New $A+(-A)=\begin{bmatrix}-2 & 1 & 3\\ 4 & 2 & 0\\ -3 & 0 & 5\end{bmatrix}+\begin{bmatrix}2 & -1 & -3\\ -4 & -2 & 0\\ 3 & 0 & -5\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=O$

$$(-A)+A=\begin{bmatrix}2 & -1 & -3\\ -4 & -2 & 0\\ 3 & 0 & -5\end{bmatrix}+\begin{bmatrix}-2 & 1 & 3\\ 4 & 2 & 0\\ -3 & 0 & 5\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=O$$

Thus, A and $-A$ are additive inverse of each other.

Solved Exercise 3.3

1. Which of the following matrices are conformable for addition and subtraction?

$$A=\begin{bmatrix}3 & 4\\ -2 & 1\end{bmatrix},\ B=\begin{bmatrix}3\\ 4\end{bmatrix},\ C=[5\ \ 2],\ D=\begin{bmatrix}1 & 7\\ 2 & 5\end{bmatrix},\ E=[2],$$

$$F=[7\ \ 11],\ G=\begin{bmatrix}a\\ b\end{bmatrix},\ H=[3],\ M=\begin{bmatrix}\ell\\ m\end{bmatrix}$$

Ans. Matrix A and matrix D are conformable for addition and subtraction because those are matrices of same order.

Matrix B and matrix G and M are conformable for addition and subtraction because those are matrices of same order.

Matrix C and matrix F are conformable for addition and subtraction because those are matrices of same order.

Matrix E and Matrix H are conformable for addition and subtraction because those matrices are of same order.

2. If $X=\begin{bmatrix}1 & -1\\ -2 & 2\end{bmatrix}$, $Y=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ and $Z=\begin{bmatrix}3 & 0\\ 0 & -2\end{bmatrix}$, then find the following:

(i) $X+Y$ (ii) $Y+7Z$ (iii) $4X-Z$
(iv) $X+2Y+3Z$ (v) $X-4Y+Z$ (vi) $Z-Z$

Sol. Given that

$$X=\begin{bmatrix}1 & -1\\ -2 & 2\end{bmatrix},\ Y=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\text{ and }Z=\begin{bmatrix}3 & 0\\ 0 & -2\end{bmatrix}$$

(i) X + Y

$$X+Y=\begin{bmatrix}1 & -1\\ -2 & 2\end{bmatrix}+\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & -1+2\\ -2+3 & 2+4\end{bmatrix}=\begin{bmatrix}2 & 1\\ 1 & 6\end{bmatrix}$$

(ii) Y + 7Z

$$Y+7Z=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}+7\begin{bmatrix}3 & 0\\ 0 & -2\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}+\begin{bmatrix}21 & 0\\ 0 & -14\end{bmatrix}=\begin{bmatrix}1+21 & 2+0\\ 3+0 & 4-14\end{bmatrix}=\begin{bmatrix}22 & 2\\ 3 & -10\end{bmatrix}$$

(iii) 4X − Z

$$4X-Z = 4\begin{bmatrix}1 & -1\\-2 & 2\end{bmatrix}-\begin{bmatrix}3 & 0\\0 & -2\end{bmatrix}$$

$$=\begin{bmatrix}4 & -4\\-8 & 8\end{bmatrix}-\begin{bmatrix}3 & 0\\0 & -2\end{bmatrix}=\begin{bmatrix}4-3 & -4-0\\-8-0 & 8+2\end{bmatrix}=\begin{bmatrix}1 & -4\\-8 & 10\end{bmatrix}$$

(iv) X + 2Y + 3Z

$$X+2Y+3Z=\begin{bmatrix}1 & -1\\-2 & 2\end{bmatrix}+2\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}+3\begin{bmatrix}3 & 0\\0 & -2\end{bmatrix}$$

$$=\begin{bmatrix}1 & -1\\-2 & 2\end{bmatrix}+\begin{bmatrix}2 & 4\\6 & 8\end{bmatrix}+\begin{bmatrix}9 & 0\\0 & -6\end{bmatrix}$$

$$=\begin{bmatrix}1+2+9 & -1+4+0\\-2+6+0 & 2+8-6\end{bmatrix}=\begin{bmatrix}12 & 3\\4 & 4\end{bmatrix}$$

(v) X−4Y + Z

$$X-4Y+Z=\begin{bmatrix}1 & -1\\-2 & 2\end{bmatrix}-4\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}+\begin{bmatrix}3 & 0\\0 & -2\end{bmatrix}$$

$$=\begin{bmatrix}1 & -1\\-2 & 2\end{bmatrix}-\begin{bmatrix}4 & 8\\12 & 16\end{bmatrix}+\begin{bmatrix}3 & 0\\0 & -2\end{bmatrix}$$

$$=\begin{bmatrix}1-4+3 & -1-8+0\\-2-12+0 & 2-16-2\end{bmatrix}=\begin{bmatrix}0 & -9\\-14 & -16\end{bmatrix}$$

(vi) Z−Z

$$Z-Z=\begin{bmatrix}3 & 0\\0 & -2\end{bmatrix}-\begin{bmatrix}3 & 0\\0 & -2\end{bmatrix}$$

$$=\begin{bmatrix}3-3 & 0-0\\0-0 & -2+2\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

3. Find the additive inverse of the following matrices:

(i) $P=\begin{bmatrix}5\\-7\end{bmatrix}$

Sol. $-P=\begin{bmatrix}-5\\7\end{bmatrix}$

(ii) $Q=[9\ \ -3]$

Sol. $-Q\,[-9\quad 3]$

(iii) $R=\begin{bmatrix}2 & -1\\-2 & 3\end{bmatrix}$

Sol. $-R\begin{bmatrix}-2 & +1\\2 & -3\end{bmatrix}$

(iv) $S=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Sol. Given that $S=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$S=-\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$ is the additive inverse of S.

4. If $A = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$ then verify the following:

(i) $A + B = B + A$ **(ii) $(A + B) + C = A + (B + C)$**

(iii) $(2A + B) + C = 2A + (B + C)$ **(iv) $3(A + B) = 3A + 3B$**

(i) $A + B = B + A$

Sol. $A + B = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

$$= \begin{bmatrix} 2+3 & 3+4 \\ -3+5 & 2+6 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix} \text{— (i)}$$

$$B+A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3+2 & 4+3 \\ 5-3 & 6+2 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix} \text{— (ii)}$$

Hence, from (i) and (ii) $A + B = B + A$

(ii) $(A + B) + C = A + (B + C)$

Sol. $(A+B) + C = \left[\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}\right] + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 2+3 & 3+4 \\ -3+5 & 2+6 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 5+1 & 7-2 \\ 2+0 & 8+5 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 13 \end{bmatrix} \text{— (i)}$$

$$A + (B + C) = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + \left[\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}\right]$$

$$= \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + \begin{bmatrix} 3+1 & 4-2 \\ 5+0 & 6+5 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 5 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4 & 3+2 \\ -3+5 & 2+11 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 13 \end{bmatrix} \text{—(ii)}$$

Hence, from (i) an (ii) $(A + B) + C = A + (B + C)$

(iii) $(2A + B) + C = 2A + (B + C)$

Sol. $(2A + B) + C = \left[2\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}\right] + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$

$$= \left[\begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}\right] + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 4+3 & 6+4 \\ -6+5 & 4+6 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 10 \\ -1 & 10 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 7+1 & 10-2 \\ -1+0 & 10+5 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ -1 & 15 \end{bmatrix} \text{— (i)}$$

$$2A + (B+C) = 2\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + \left[\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}\right]$$

$$= \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} 3+1 & 4-2 \\ 5+0 & 6+5 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 5 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & 6+2 \\ -6+5 & 4+11 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ -1 & 15 \end{bmatrix} \text{— (ii)}$$

Hence, From (i) and (ii) $(2A+B) + C = 2A + (B+C)$

(iv) $3(A + B) = 3A + 3B$

Sol. $$3(A + B) = 3\left[\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}\right]$$

$$= 3\begin{bmatrix} 2+3 & 3+4 \\ -3+5 & 2+6 \end{bmatrix} = 3\begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 15 & 21 \\ 6 & 24 \end{bmatrix} \text{______ (i)}$$

$$3A + 3B = 3\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + 3\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 9 \\ -9 & 6 \end{bmatrix} + \begin{bmatrix} 9 & 12 \\ 15 & 18 \end{bmatrix} = \begin{bmatrix} 6+9 & 9+12 \\ -9+15 & 6+18 \end{bmatrix} = \begin{bmatrix} 15 & 21 \\ 6 & 24 \end{bmatrix} \text{______ (ii)}$$

Hence, from (i) and (ii) $3(A + B) = 3A + 3B$

5. If $A = \begin{bmatrix} 5 & 6 \\ 7 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & -6 \\ -7 & 2 \end{bmatrix}$, then show that B is additive inverse of A and A is additive inverse of B.

Sol. Given that $A = \begin{bmatrix} 5 & 6 \\ 7 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & -6 \\ -7 & 2 \end{bmatrix}$

As we know that if A and B are two matrices of same order such that $A+B=O=B+A$ then A and B are called additive inverse of each other.

$$\text{Now } A + B = \begin{bmatrix} 5 & 6 \\ 7 & -2 \end{bmatrix} + \begin{bmatrix} -5 & -6 \\ -7 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5-5 & 6-6 \\ 7-7 & -2+2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \text{______ (i)}$$

$$B + A = \begin{bmatrix} -5 & -6 \\ -7 & +2 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & -2 \end{bmatrix} = \begin{bmatrix} -5+5 & -6+6 \\ -7+7 & +2-2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \text{______ (ii)}$$

Hence, From (i) and (ii) $A + b = 0 = B + A$

6. If $A = \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & -1 \\ 3 & 0 \end{bmatrix}$, then verify that

(i) $(A+B)^t = A^t + B^t$ (ii) $(A-B)^t = A^t - B^t$

(i) $(A+B)^t = A^t + B^t$

Sol. Given that $A = \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & -1 \\ 3 & 0 \end{bmatrix}$

$$A + B = \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 8 & -1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 6+8 & -2-1 \\ 0+3 & 3+0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 14 & -3 \\ 3 & 3 \end{bmatrix}$$

$$(A+B)^t = \begin{bmatrix} 14 & 3 \\ -3 & 3 \end{bmatrix} \text{— (i)}$$

$$A^t + B^t = \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix}^t + \begin{bmatrix} 8 & -1 \\ 3 & 0 \end{bmatrix}^t = \begin{bmatrix} 6 & 0 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 8 & 3 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6+8 & 0+3 \\ -2-1 & 3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 3 \\ -3 & 3 \end{bmatrix} \text{— (ii)}$$

Hence, from (i) and (ii) $(A+B)^t = A^t + B^t$

(ii) $(A-B)^t = A^t - B^t$

Sol. Given that $A = \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & -1 \\ 3 & 0 \end{bmatrix}$

$$A - B = \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 8 & -1 \\ 3 & 0 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 6-8 & -2+1 \\ 0-3 & 3-0 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -3 & 3 \end{bmatrix}$$

$$(A-B)^t = \begin{bmatrix} -2 & -3 \\ -1 & 3 \end{bmatrix} \text{— (i)}$$

$$A^t - B^t = \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix}^t - \begin{bmatrix} 8 & -1 \\ 3 & 0 \end{bmatrix}^t = \begin{bmatrix} 6 & 0 \\ -2 & 3 \end{bmatrix} - \begin{bmatrix} 8 & 3 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 6-8 & 0-3 \\ -2+1 & 3-0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -3 \\ -1 & 3 \end{bmatrix}$$

Hence, From (i) and (ii) $(A-B)^t = A^t - B^t$

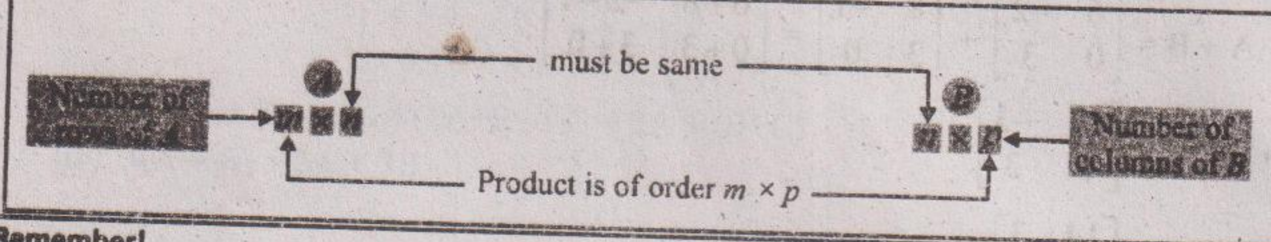
3.4 Multiplication of Matrices

A matrix A is said to be conformable for multiplication with a matrix B if the number of columns of A is equal to the number rows of B.

Thus, if A and B are given two matrices, then the product of matrices A and B is denoted by AB and its order will be m–by–p

The order of AB is m–by–p = number of rows of A by number of columns of B

Remember!



Remember!

For Multiplication

If A and B are two matrices two and we multiply A with B when:

(i) Number of columns in A = Number of rows in B.

(ii) Multiplication of two matrices A and B is written as AB.

(iii) To find AB, we start with the first row of A and multiply its each element with the corresponding elements of the first column of B and add the products. This gives the first element of the first row of the product matrix AB.

Next, we multiply each element of the first row of A with the correponding element of second column of B and add the products. This gives the second element of first row of the product matrix AB.

Similarly, we multiply all the rows of A with each columns of B and add products to get remaining elements of the matrix AB.

Example 4: If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}, B=\begin{bmatrix} \ell & m \\ n & p \end{bmatrix}$**, then find:**

(i) AB, if possible. **(ii) BA, if possible.**

Sol. (i) Number of columns in matrix A = 2

Number of rows in matrix B = 2

Thus, AB is possible

$$AB=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} \ell & m \\ n & p \end{bmatrix}$$

$$AB=\begin{bmatrix} a\ell+bn & am+bp \\ c\ell+dn & cm+dp \end{bmatrix}$$

(ii) Number of columns in matrix B = 2

Number of rows in matrix A = 2

Thus, BA is also possible

$$BA=\begin{bmatrix} \ell & m \\ n & p \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$BA = \begin{bmatrix} \ell a + mc & \ell b + md \\ na + pc & nb + pd \end{bmatrix}$$ We note that $AB \neq BA$

Example 5: If $A = \begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix}, B = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$**, then find**

(i) AB, if possible. **(ii) BA, if possible.**

Sol. (i) Number of columns in matrix A = 2
Number of rows in matrix B = 2

Thus, AB is possible

Now $AB = \begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 2\times(-2) + 0\times 1 \\ 5\times(-2) + (-3)\times 1 \end{bmatrix} = \begin{bmatrix} -4 + 0 \\ -10 - 3 \end{bmatrix} = \begin{bmatrix} -4 \\ -13 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 \\ 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix}$$

Number of columns in matrix B = 1
Number of rows in matrix A = 2
Thus, BA is not possible

Skilled Practice!

If $A = [5 \quad 3]$ and $B = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$, then find AB and its order.

Sol. $AB = [5 \quad 3]\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$= [5\,(-2) + 3\,(1)]$
$= [-10 + 3] = [-7]$

So, $AB = [-7]$ and its order is 1-by-1.

Commutative Law of Multiplication of Matrices

Consider $A = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 \\ 0 & -2 \end{bmatrix}$

$$AB = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 5\times1 + 2\times0 & 5\times5 + 2(-2) \\ 3\times1 + 4\times0 & 3\times5 + 4\times(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 5+0 & 25-4 \\ 3+0 & 15-8 \end{bmatrix} = \begin{bmatrix} 5 & 21 \\ 3 & 7 \end{bmatrix} \quad ...(i)$$

Now $BA = \begin{bmatrix} 1 & 5 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 1\times5+5\times3 & 1\times2+5\times4 \\ 0\times5+(-2)\times3 & 0\times2+(-2)\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 5+15 & 2+20 \\ 0-6 & 0-8 \end{bmatrix} = \begin{bmatrix} 20 & 22 \\ -6 & -8 \end{bmatrix} \quad ...(ii)$$

From (i) and (ii), it is verified that $AB \neq BA$

Thus, commutative law under multiplication does not hold in general in the multiplication of matrices.

Associative Law of Multiplication of Matrices

If A, B and C are any three matrices, then (AB) C = A (BC) is called associative law of multiplication of matrices.

Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ 4 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1(2)+0(0) & 1(1)+0(3) \\ 2(2)+1(0) & 2(1)+1(3) \end{bmatrix} = \begin{bmatrix} 2+0 & 1+0 \\ 4+0 & 2+3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$$

Now, $(AB)C = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 2(1)+1(4) & 2(2)+1(0) \\ 4(1)+5(4) & 4(2)+5(0) \end{bmatrix} = \begin{bmatrix} 2+4 & 4+0 \\ 4+20 & 8+0 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 24 & 8 \end{bmatrix} \quad ...(i)$$

$$BC = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2(1)+1(4) & 2(2)+1(0) \\ 0(1)+3(4) & 0(2)+3(0) \end{bmatrix} = \begin{bmatrix} 2+4 & 4+0 \\ 0+12 & 0+0 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 12 & 0 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 6 & 4 \\ 12 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(6)+0(12) & 1(4)+0(0) \\ 2(6)+1(12) & 2(4)+1(0) \end{bmatrix}$$

$$= \begin{bmatrix} 6+0 & 4+0 \\ 12+12 & 8+0 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 24 & 8 \end{bmatrix} \quad ...(ii)$$

Hence, from (i) and (ii)

(AB) C = A (BC)

Associative law of multiplication of matrices holds.

Distributive Laws of Multiplication Over Addition and Subtraction

(a) Let A, B and C be any three matrices, then distributive laws of multiplication over addition are as follows:

(i) $A(B+C) = AB + AC$ (Left distributive law over addition)

(ii) $(A+B)C = AC + BC$ (Right distributive law over addition)

Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix}$

$$A(B+C) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 7 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1(4)+2(7) & 1(1)+2(6) \\ 0(4)+1(7) & 0(1)+1(6) \end{bmatrix}$$

$$= \begin{bmatrix} 4+14 & 1+12 \\ 0+7 & 0+6 \end{bmatrix} = \begin{bmatrix} 18 & 13 \\ 7 & 6 \end{bmatrix} \quad ...(i)$$

$$AB = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 3+4 & 1+8 \\ 0+2 & 0+4 \end{bmatrix} = \begin{bmatrix} 7 & 9 \\ 2 & 4 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 1+10 & 0+4 \\ 0+5 & 0+2 \end{bmatrix} = \begin{bmatrix} 11 & 4 \\ 5 & 2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 7 & 9 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 11 & 4 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 7+11 & 9+4 \\ 2+5 & 4+2 \end{bmatrix} = \begin{bmatrix} 18 & 13 \\ 7 & 6 \end{bmatrix} \quad ...(i)$$

Hence from (i) and (ii)

$$A(B+C) = AB + AC$$

Similarly, we can verify right distributive law.

(b) If A, B and C be any three matrices, then distributive laws of multiplication over subtraction are as follow:

(i) $A(B-C) = AB - AC$ (Left distributive law over subtraction)

(ii) $(A-B)C = AC - BC$ (Right distributive law over subtraction)

Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}, C = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

$$A(B-C) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4-2 & 3-1 \\ 1-0 & 2-1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(2)+2(1) & 1(2)+2(1) \\ 0(2)+1(1) & 0(2)+1(1) \end{bmatrix} = \begin{bmatrix} 2+2 & 2+2 \\ 0+1 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \quad ...(i)$$

$$AB=\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}4&3\\1&2\end{bmatrix}=\begin{bmatrix}1(4)+2(1)&1(3)+2(2)\\0(4)+1(1)&0(3)+1(2)\end{bmatrix}$$

$$=\begin{bmatrix}4+2&3+4\\0+1&0+2\end{bmatrix}=\begin{bmatrix}6&7\\1&2\end{bmatrix}$$

$$AC=\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}2&1\\0&1\end{bmatrix}=\begin{bmatrix}1(2)+2(0)&1(1)+2(1)\\0(2)+1(0)&0(1)+1(1)\end{bmatrix}$$

$$=\begin{bmatrix}2+0&1+2\\0+0&0+1\end{bmatrix}=\begin{bmatrix}2&3\\0&1\end{bmatrix}$$

$$AB-AC=\begin{bmatrix}6&7\\1&2\end{bmatrix}-\begin{bmatrix}2&3\\0&1\end{bmatrix}=\begin{bmatrix}6-2&7-3\\1-0&2-1\end{bmatrix}=\begin{bmatrix}4&4\\1&1\end{bmatrix}\quad ...(ii)$$

Hence from (i) and (ii)

$A(B-C)=AB-AC$

Similarly, we can verify right distributive law over subtraction.

Multiplicative Identity of a Matrix

The multiplicative Identity of a matrix is a matrix that, when multiplied with any matrix A, does not change it. i.e., $AI = IA = A$

Let $A=\begin{bmatrix}2&3\\4&1\end{bmatrix}$

$$AI=\begin{bmatrix}2&3\\4&1\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}2(1)+3(0)&2(0)+3(1)\\4(1)+1(0)&4(0)+1(1)\end{bmatrix}=\begin{bmatrix}2+0&0+3\\4+0&0+1\end{bmatrix}=\begin{bmatrix}2&3\\4&1\end{bmatrix}=A$$

$$IA=\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}2&3\\4&1\end{bmatrix}=\begin{bmatrix}1(2)+0(4)&1(3)+0(1)\\0(2)+1(4)&0(3)+1(1)\end{bmatrix}$$

$$=\begin{bmatrix}2+0&3+0\\0+4&0+1\end{bmatrix}=\begin{bmatrix}2&3\\4&1\end{bmatrix}=A$$

Hence, I is multiplicative identity of a matrix A.

Solved Exercise 3.4

1. Find AB and BA, if possible.

(i) $A=\begin{bmatrix}1&2\\-1&0\end{bmatrix}, B=\begin{bmatrix}3&2\\1&-1\end{bmatrix}$

Sol. Given that $A=\begin{bmatrix}1&2\\-1&0\end{bmatrix}$ and $B=\begin{bmatrix}3&2\\1&-1\end{bmatrix}$

Number of columns in matrix $A=2$

Number of rows in matrix B = 2
Thus, AB is possible.

$$AB = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(3)+2(1) & 1(2)+2(-1) \\ -1(3)+0(1) & -1(2)+0(-1) \end{bmatrix} = \begin{bmatrix} 3+2 & 2-2 \\ -3+0 & -2-0 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -3 & -2 \end{bmatrix}$$

Given that $B\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$

Numbers of columns in matrix B = 2
Numbers of rows in matrix A = 2
Thus BA is possible

$$BA = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 3(1)+2(-1) & 3(2)+2(0) \\ 1(1)+(-1)(-1) & 1(2)+1(0) \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & 6+0 \\ 1+1 & 2+0 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 2 & 2 \end{bmatrix}$$

(ii) $A = [1 \quad -2], B = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

Sol. Given that $A = [1 \quad -2]$ and $B = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

Number of columns in matrix A = 2
Number of rows in matrix B = 2
Thus, AB is possible.

$$AB = [1 \quad -2]\begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$= [1(3) + (-2)(-4)]$$

$$= [3 + 8] = [11]$$

And number of columns in matrix B = 1
Number of rows in matrix A = 1
Thus, BA is possible.

$$BA = \begin{bmatrix} 3 \\ -4 \end{bmatrix}[1 \quad -2]$$

$$= \begin{bmatrix} 3\times 1 & 3\times(-2) \\ -4\times 1 & -5\times(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -6 \\ -4 & 8 \end{bmatrix}$$

(iii) $A = \begin{bmatrix} 4 \\ 4 \end{bmatrix}, B = [2 \quad 5]$

Sol. Given that $A = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$ and $B = [2 \quad 5]$

Number of columns in matrix A = 1
Number of rows in matrix B = 1
Thus, AB is possible.

$$AB = \begin{bmatrix} 4 \\ 4 \end{bmatrix}[2 \quad 5]$$

$$= \begin{bmatrix} 4(2) & 4(5) \\ 4(2) & 4(5) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 20 \\ 8 & 20 \end{bmatrix}$$

And number of columns in matrix B = 2
Number of rows in matrix A = 2
Thus, BA is possible.

$$BA = [2 \quad 5]\begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$= [2(4) + 5(4)]$$

$$= [8 + 20]$$

$$= [28]$$

(iv) $\mathbf{A}=\begin{bmatrix}1 & 2\\-1 & 1\\3 & 0\end{bmatrix}, \mathbf{B}=\begin{bmatrix}-1 & 4 & 1\\2 & 3 & 1\end{bmatrix}$

Sol. Given that $A=\begin{bmatrix}1 & 2\\-1 & 1\\3 & 0\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 4 & 1\\2 & 3 & 1\end{bmatrix}$

Number of columns in matrix A = 2
Number of rows in matrix B = 2
Thus, AB is possible.

$$AB=\begin{bmatrix}1 & 2\\-1 & 1\\3 & 0\end{bmatrix}\begin{bmatrix}-1 & 4 & 1\\2 & 3 & 1\end{bmatrix}=\begin{bmatrix}1(-1)+2(2) & 1(4)+2(3) & 1(1)+2(1)\\-1(-1)+1(2) & -1(4)+1(3) & -1(1)+1(1)\\3(-1)+0(2) & 3(4)+0(3) & 3(1)+0(1)\end{bmatrix}$$

$$=\begin{bmatrix}-1+4 & 4+6 & 1+2\\1+2 & -4+3 & -1+1\\-3+0 & 12+0 & 3+0\end{bmatrix}=\begin{bmatrix}3 & 10 & 3\\3 & -1 & 0\\-3 & 12 & 3\end{bmatrix}$$

And number of columns in matrix B = 3
Number of rows in matrix A = 3
Thus, BA is possible.

$$BA=\begin{bmatrix}-1 & 4 & 1\\2 & 3 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\-1 & 1\\3 & 0\end{bmatrix}=\begin{bmatrix}-(1)+4(-1)+1(3) & -1(2)+4(1)+1(0)\\2(1)+3(-1)+1(3) & 2(2)+3(1)+1(0)\end{bmatrix}$$

$$=\begin{bmatrix}-1-4+3 & -2+4+0\\2-3+3 & 4+3+0\end{bmatrix}=\begin{bmatrix}-2 & 2\\2 & 7\end{bmatrix}$$

(v) $\mathbf{A}=\begin{bmatrix}1 & 2 & 2\\3 & 1 & 1\end{bmatrix}, \mathbf{B}=\begin{bmatrix}1 & 5\\2 & 4\\1 & 6\end{bmatrix}$

Sol. Given that $A=\begin{bmatrix}1 & 2 & 2\\3 & 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 5\\2 & 4\\1 & 6\end{bmatrix}$

Number of columns in matrix A = 3
Number of rows in matrix B = 3
Thus, AB is possible.

$$A=\begin{bmatrix}1&2&2\\3&1&1\end{bmatrix}\begin{bmatrix}1&5\\2&4\\1&6\end{bmatrix}=\begin{bmatrix}1(1)+2(2)+2(1)&1(5)+2(4)+2(6)\\3(1)+1(2)+1(1)&3(5)+1(4)+1(6)\end{bmatrix}$$

$$=\begin{bmatrix}1+4+2&5+8+12\\3+2+1&15+4+6\end{bmatrix}=\begin{bmatrix}7&25\\6&25\end{bmatrix}$$

And number of columns in matrix B = 2
Number of rows in matrix A = 2
Thus, BA is possible.

$$BA=\begin{bmatrix}1&5\\2&4\\1&6\end{bmatrix}\begin{bmatrix}1&2&2\\3&1&1\end{bmatrix}=\begin{bmatrix}1(1)+5(3)&1(2)+5(1)&1(2)+5(1)\\2(1)+4(3)&2(2)+4(1)&2(2)+(4)(1)\\1(1)+6(3)&1(2)+6(1)&1(2)+6(1)\end{bmatrix}$$

$$=\begin{bmatrix}16&7&7\\14&8&8\\19&8&8\end{bmatrix}$$

2. Verify each statement, using $A=\begin{bmatrix}5&1\\-1&4\end{bmatrix}$, $B=\begin{bmatrix}2&-1\\0&3\end{bmatrix}$ and $C=\begin{bmatrix}5&1\\1&4\end{bmatrix}$.

(i) $AB \neq BA$

Sol. $$AB=\begin{bmatrix}5&1\\-1&4\end{bmatrix}\begin{bmatrix}2&-1\\0&3\end{bmatrix}=\begin{bmatrix}5(2)+1(0)&5(-1)+1(3)\\-1(2)+4(0)&-1(-1)+4(3)\end{bmatrix}$$

$$=\begin{bmatrix}10+0&-5+3\\-2+0&1+12\end{bmatrix}=\begin{bmatrix}10&-2\\-2&13\end{bmatrix} \quad \text{...(i)}$$

$$BA=\begin{bmatrix}2&-1\\0&3\end{bmatrix}\begin{bmatrix}5&1\\-1&4\end{bmatrix}=\begin{bmatrix}2(5)+(-1)(-1)&2(1)+(-1)(4)\\0(5)+3(-1)&0(1)+3(4)\end{bmatrix}$$

$$=\begin{bmatrix}10+1&2-4\\0-3&0+12\end{bmatrix}=\begin{bmatrix}11&-2\\-3&12\end{bmatrix} \quad \text{...(ii)}$$

Hence, verified from (i) and (ii) that $AB \neq BA$

(ii) $A(B-C) = AB - AC$

Sol. $$B-C=\begin{bmatrix}2&-1\\0&3\end{bmatrix}-\begin{bmatrix}5&1\\1&4\end{bmatrix}=\begin{bmatrix}2-5&-1-1\\0-1&3-4\end{bmatrix}$$

$$=\begin{bmatrix}-3&-2\\-1&-1\end{bmatrix}$$

$$A(B-C)=\begin{bmatrix}5&1\\-1&4\end{bmatrix}\begin{bmatrix}-3&-2\\-1&-1\end{bmatrix}$$

$$=\begin{bmatrix} 5(-3)+1(-1) & 5(-2)+1(-1) \\ -1(-3)+4(-1) & -1(-2)+4(-1) \end{bmatrix}=\begin{bmatrix} -15-1 & -10-1 \\ 3-4 & +2-4 \end{bmatrix}$$

$$=\begin{bmatrix} -16 & -11 \\ -1 & -2 \end{bmatrix} \quad ...(i)$$

$$AB=\begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}=\begin{bmatrix} 5(2)+1(0) & 5(-1)+1(3) \\ -1(2)+4(0) & -1(-1)+4(3) \end{bmatrix}$$

$$=\begin{bmatrix} 10+0 & -5+3 \\ -2+0 & 1+12 \end{bmatrix}=\begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix}$$

$$AC=\begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix}=\begin{bmatrix} 5(5)+1(1) & 5(1)+1(4) \\ -1(5)+4(1) & -1(1)+4(4) \end{bmatrix}$$

$$=\begin{bmatrix} 25+1 & 5+4 \\ -5+4 & -1+16 \end{bmatrix}=\begin{bmatrix} 26 & 9 \\ -1 & 15 \end{bmatrix}$$

$$AB-AC=\begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix}-\begin{bmatrix} 26 & 9 \\ -1 & 15 \end{bmatrix}=\begin{bmatrix} 10-26 & -2-9 \\ -2-(-1) & 13-15 \end{bmatrix}$$

$$=\begin{bmatrix} -16 & -11 \\ -2+1 & -2 \end{bmatrix}=\begin{bmatrix} -16 & -11 \\ -1 & -2 \end{bmatrix} \quad .... (ii)$$

Hence, verified from (i) and (ii) that A(B–C) = AB – AC

(iii) A(BC) = (AB)C

Sol. $$BC=\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix}=\begin{bmatrix} 2(5)+(-1)(1) & 2(1)+(-1)(4) \\ 0(5)+3(1) & 0(1)+3(4) \end{bmatrix}$$

$$=\begin{bmatrix} 10-1 & 2-4 \\ 0+3 & 0+12 \end{bmatrix}=\begin{bmatrix} 9 & -2 \\ 3 & 12 \end{bmatrix}$$

$$A(BC)=\begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 9 & -2 \\ 3 & 12 \end{bmatrix}=\begin{bmatrix} 5(9)+1(3) & 5(-2)+1(12) \\ -1(9)+4(3) & -1(-2)+4(12) \end{bmatrix}$$

$$=\begin{bmatrix} 45+3 & -10+12 \\ -9+12 & 2+48 \end{bmatrix}=\begin{bmatrix} 48 & 2 \\ 3 & 50 \end{bmatrix} \quad .... (i)$$

$$AB=\begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}=\begin{bmatrix} 5(2)+1(0) & 5(-1)+1(3) \\ -1(2)+4(0) & -1(-1)+4(3) \end{bmatrix}$$

$$=\begin{bmatrix} 10+0 & -5+3 \\ -2+0 & 1+12 \end{bmatrix}=\begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix}$$

$$(AB)C=\begin{bmatrix}10 & -2\\ -2 & 13\end{bmatrix}\begin{bmatrix}5 & 1\\ 1 & 4\end{bmatrix}=\begin{bmatrix}10(5)+(-2)(1) & 10(1)+(-2)(4)\\ (-2)(5)+13(1) & -2(1)+13(4)\end{bmatrix}$$

$$=\begin{bmatrix}50-2 & 10-8\\ -10+13 & -2+52\end{bmatrix}=\begin{bmatrix}48 & 2\\ 3 & 50\end{bmatrix}\text{ .... (ii)}$$

Hence, verified from (i) and (ii) that A(BC) = (AB)C

(iv) $(BC)^t = C^tB^t$

Sol. $$BC=\begin{bmatrix}2 & -1\\ 0 & 3\end{bmatrix}\begin{bmatrix}5 & 1\\ 1 & 4\end{bmatrix}=\begin{bmatrix}2(5)+(-1)(1) & 2(1)+(-1)(4)\\ 0(5)+3(1) & 0(1)+3(4)\end{bmatrix}$$

$$BC=\begin{bmatrix}10-1 & 2-4\\ 0+3 & 0+12\end{bmatrix}=\begin{bmatrix}9 & -2\\ 3 & 12\end{bmatrix}$$

$$(BC)^t=\begin{bmatrix}9 & 3\\ -2 & 12\end{bmatrix}\quad\text{...(i)}$$

$$B=\begin{bmatrix}2 & -1\\ 0 & 3\end{bmatrix}, C=\begin{bmatrix}5 & 1\\ 1 & 4\end{bmatrix}$$

$$B^t=\begin{bmatrix}2 & 0\\ -1 & 3\end{bmatrix}, C^t=\begin{bmatrix}5 & 1\\ 1 & 4\end{bmatrix}$$

$$C^tB^t\begin{bmatrix}5 & 1\\ 1 & 4\end{bmatrix}\begin{bmatrix}2 & 0\\ -1 & 3\end{bmatrix}=\begin{bmatrix}5(2)+1(-1) & 5(0)+1(3)\\ 1(2)+4(-1) & 1(0)+4(3)\end{bmatrix}$$

$$=\begin{bmatrix}10-1 & 0+3\\ 2-4 & 0+12\end{bmatrix}=\begin{bmatrix}9 & 3\\ -2 & 12\end{bmatrix}\quad\text{...(ii)}$$

Hence, verified from (i) and (ii) that $(BC)^t = C^t B^t$

(v) (B + C) A = BA + CA

Sol. $$B+C=\begin{bmatrix}2 & -1\\ 0 & 3\end{bmatrix}+\begin{bmatrix}5 & 1\\ 1 & 4\end{bmatrix}=\begin{bmatrix}2+5 & -1+1\\ 0+1 & 3+4\end{bmatrix}$$

$$=\begin{bmatrix}7 & 0\\ 1 & 7\end{bmatrix}$$

$$(B+C)A=\begin{bmatrix}7 & 0\\ 1 & 7\end{bmatrix}\begin{bmatrix}5 & 1\\ -1 & 4\end{bmatrix}=\begin{bmatrix}7(5)+0(-1) & 7(1)+0(4)\\ 1(5)+7(-1) & 1(1)+7(4)\end{bmatrix}$$

$$=\begin{bmatrix}35-0 & 7+0\\ 5-7 & 1+28\end{bmatrix}=\begin{bmatrix}35 & 7\\ -2 & 29\end{bmatrix}\quad\text{...(i)}$$

$$BA=\begin{bmatrix}2 & -1\\ 0 & 3\end{bmatrix}\begin{bmatrix}5 & 1\\ -1 & 4\end{bmatrix}=\begin{bmatrix}2(5)+(-1)(-1) & 2(1)+(-1)(4)\\ 0(5)+3(-1) & 0(1)+3(4)\end{bmatrix}$$

$$= \begin{bmatrix} 10+1 & 2-4 \\ 0-3 & 0+12 \end{bmatrix} = \begin{bmatrix} 11 & -2 \\ -3 & 12 \end{bmatrix}$$

$$CA = \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 5(5)+1(-1) & 5(1)+1(4) \\ 1(5)+4(-1) & 1(1)+4(4) \end{bmatrix}$$

$$= \begin{bmatrix} 25-1 & 5+4 \\ 5-4 & 1+16 \end{bmatrix} = \begin{bmatrix} 24 & 9 \\ 1 & 17 \end{bmatrix}$$

$$BA + CA = \begin{bmatrix} 11 & -2 \\ -3 & 12 \end{bmatrix} + \begin{bmatrix} 24 & 9 \\ 1 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} 11+24 & -2+9 \\ -3+1 & 12+17 \end{bmatrix} = \begin{bmatrix} 35 & 7 \\ -2 & 29 \end{bmatrix} \quad ...(ii)$$

Hence, verified from (i) and (ii) that $(B + C) A = BA + CA$

3. If $\begin{bmatrix} 4 & a \\ b & 3 \end{bmatrix}\begin{bmatrix} 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$, then find the values of a and b.

Sol. $\begin{bmatrix} 4 & a \\ b & 3 \end{bmatrix}\begin{bmatrix} 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 4(6)+a(6) \\ b(6)+3(6) \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 24+6a \\ 6b+18 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

Corresponding elements are same

$24 + 6a = 6$

$6a = 6 - 24$

$6a = - 18$

$a = -\frac{18}{6}$

$a = - 3$

$6b + 18 = 3$

$6b = 3 - 18$

$6b = -15$

$b = -\frac{15}{6}$

$b = -\frac{5}{2}$

So, $a = -3, b = -\frac{5}{2}$

4. If $\begin{bmatrix} x & 1 \\ y & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 4 & -2 \end{bmatrix}$, then find the value of x and y.

Sol. $\begin{bmatrix} x & 1 \\ y & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 4 & -2 \end{bmatrix}$

$\begin{bmatrix} x(1)+1(3) & x(0)+1(-1) \\ y(1)+2(3) & y(0)+2(-1) \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 4 & -2 \end{bmatrix}$

$\begin{bmatrix} x+3 & 0-1 \\ y+6 & 0-2 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 4 & -2 \end{bmatrix}$

$\begin{bmatrix} x+3 & -1 \\ y+6 & -2 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 4 & -2 \end{bmatrix}$

$x + 3 = 7$

$x = 7 - 3$

$x = 4$

$y + 6 = 4$

$y = 4 - 6$

$y = -2$

So, $x = 4$ and $y = -2$

Determinant of a Matrix of Order 2

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a matrix of order 2. Then the determinant of A is defined as

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

For example, Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix. Then $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ is called the determinant of matrix A and ad − bc is called value of determinant. We can note that the elements of A and the element of |A| are the same. The bracket [ ] of the matrix is replaced by two vertical lines "| |" in the determinant.

Example 6: Find the value of the determinant of $B = \begin{bmatrix} 8 & 2 \\ 4 & 3 \end{bmatrix}$.

Solution: $|B| = \begin{vmatrix} 8 & 2 \\ 4 & 3 \end{vmatrix}$

$= 8 \times 3 - 4 \times 2$
$= 24 - 8 = 16$

> **Remember!**
> The value of the determinant is not altered by changing the rows into columns and columns into rows.

Singular and Non-Singular Matrices

A square matrix A is said to be **singular** if |A| = 0. A square matrix A is said to be **non-singular** if |A| ≠ 0.

For example, Let $A = \begin{bmatrix} 4 & 8 \\ 2 & 4 \end{bmatrix}$

Now $|A| = \begin{vmatrix} 4 & 8 \\ 2 & 4 \end{vmatrix} = 4 \times 4 - 2 \times 8 = 16 - 16 = 0$

Thus, matrix A is a singular matrix.

For example, Let $M = \begin{bmatrix} 7 & 3 \\ 4 & 9 \end{bmatrix}$

Now $|M| = \begin{vmatrix} 7 & 3 \\ 4 & 9 \end{vmatrix} = 7 \times 9 - 4 \times 3$

$= 63 - 12 = 51 \neq 0$

Thus, matrix M is a non-singular matrix.

Adjoint of a Matrix

The adjoint of a square matrix is an important concept in matrix algebra, especially when finding the inverse of a matrix or solving system of equations.

The adjoint of a square matrix A of order 2 is obtained by interchanging the diagonal elements and changing the signs of other elements. It is denoted as Adj A.

If $A = \begin{bmatrix} 4 & 2 \\ -6 & 1 \end{bmatrix}$, then Adj $A = \begin{bmatrix} 1 & -2 \\ 6 & 4 \end{bmatrix}$

Multiplicative Inverse of a Non-Singular Matrix

We know $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverse (or reciprocal) as the product $\left(\frac{3}{4}\times\frac{4}{3}\right)$ is 1.

Similarly, if we have two square matrices of same order and their product is a unit matrix, then these matrices are called multiplicative inverse of each other.

> **Remember!**
> The inverse of a singular matrix does not exist.

Let A be any square matrix and there exists a square matrix B having the same order as A such that AB = BA = I (a unit matrix). Then A and B are multiplicative inverse of each other. Hence $AA^{-1} = A^{-1}A = I$.

We denote multiplicative inverse of A by A^{-1}.

Adjoint Method to Find Inverse of a square Matrix

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a non-singular matrix, then inverse of A is:

i.e., $A^{-1} = \dfrac{\text{Adj}\,A}{|A|}$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example 7: If $A = \begin{bmatrix} 8 & 5 \\ 4 & 3 \end{bmatrix}$, then find multiplicative inverse of A and verify that $AA^{-1} = A^{-1}A = I$

Sol. $A = \begin{bmatrix} 8 & 5 \\ 4 & 3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 8 & 5 \\ 4 & 3 \end{vmatrix}$$
$$= 8 \times 3 - 5 \times 4$$
$$= 24 - 20 = 4 \neq 0$$

A is non-singular, so A^{-1} is possible.

$$A^{-1} = \frac{1}{|A|}\text{Adj}\,A$$

$$= \frac{1}{4}\begin{bmatrix} 3 & -5 \\ -4 & 8 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{-5}{4} \\ -1 & 2 \end{bmatrix}$$

Let us verify that A and A^{-1} are multiplicative inverse of each other.

and $AA^{-1} = \begin{bmatrix} 8 & 5 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} \frac{3}{4} & \frac{-5}{4} \\ -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 8\times\frac{3}{4}+5\times(-1) & 8\times\left(\frac{-5}{4}\right)+\times 2 \\ 4\times\frac{3}{4}+3\times(-1) & 4\times\left(\frac{-5}{4}\right)+3\times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-5 & 10+10 \\ 3-3 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Now, $A^{-1}A = \frac{1}{4}\begin{bmatrix} 3 & -5 \\ -4 & 8 \end{bmatrix}\begin{bmatrix} 8 & 5 \\ 4 & 3 \end{bmatrix}$

$$= \frac{1}{4}\begin{bmatrix} 3\times 8+(-5)\times 4 & 3\times 5+(-5)\times 3 \\ (-4)\times 8+8\times 4 & (-4)\times 5+8(3) \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 24-20 & 15-15 \\ -32+32 & -20+24 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Thus $AA^{-1} = A^{-1}A = I$

Hence $\begin{bmatrix} 8 & 5 \\ 4 & 3 \end{bmatrix}$ and $\begin{bmatrix} \frac{3}{4} & \frac{-5}{4} \\ -1 & 2 \end{bmatrix}$ are multiplicative inverse of each other.

Challenge!

If $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}$, then verify

$(AB)^{-1} = B^{-1} A^{-1}$

Can we say it Reversal Law of Inverses?

L.H.S

$$AB = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2(3)+1(2) & 2(4)+1(4) \\ 3(3)+2(2) & 3(4)+2(4) \end{bmatrix}$$

$$= \begin{bmatrix} 6+2 & 8+4 \\ 9+4 & 12+8 \end{bmatrix}$$

$$AB = \begin{bmatrix} 8 & 12 \\ 13 & 20 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 8 & 12 \\ 13 & 20 \end{vmatrix}$$

$$= (8 \times 20) - (13 \times 12)$$

$$= 160 - 156$$

$$= 4 \neq 0$$

$$\text{Adj}(AB) = \begin{bmatrix} 20 & -12 \\ -13 & 8 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|} \times \text{Adj}(AB)$$

$$= \frac{1}{4}\begin{bmatrix} 20 & -12 \\ -13 & 8 \end{bmatrix}$$

Now $$= \begin{bmatrix} \frac{20}{4} & \frac{-12}{4} \\ \frac{-13}{4} & 2 \end{bmatrix}$$

R.H.S

$$B = \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}$$

$$|B| = (3 \times 4) - 2(4)$$

$$= 12 - 8 = 4 \neq 0$$

$$\text{Adj}B = \begin{bmatrix} 4 & -4 \\ -2 & 3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{4} \times \text{Adj}B$$

$$= \frac{1}{4}\begin{bmatrix} 4 & -4 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{4} & -\frac{4}{4} \\ -\frac{2}{4} & \frac{3}{4} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ \frac{-1}{2} & \frac{3}{4} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$|A| = (2 \times 2) - (3 \times 1)$$

$$= 4 - 3 = 1$$

$$\text{Adj } A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} 1 & -1 \\ \frac{-1}{2} & \frac{3}{4} \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1(2)+(-1)(-3) & 1(-1)+(-1)(2) \\ -\frac{1}{2}(2)+\frac{3}{4}(-3) & -\frac{1}{2}(-1)+\frac{3}{4}(2) \end{bmatrix}$$

$$= \begin{bmatrix} 2+3 & -1-2 \\ -1-\frac{9}{4} & +\frac{1}{2}+\frac{3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -3 \\ \frac{-4-9}{4} & \frac{4}{2} \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ \frac{-13}{4} & 2 \end{bmatrix}$$

So, proved that $(AB)^{-1} = B^{-1}A^{-1}$

Yes, we can say it Reversal Law of Inverses.

Example 8: Prove that $A = \begin{bmatrix} 4 & -2 \\ -6 & 4 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{3}{2} & 1 \end{bmatrix}$ **are multiplicative inverse**

of each other.

Sol. Given that

$$A = \begin{bmatrix} 4 & -2 \\ -6 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{3}{2} & 1 \end{bmatrix}$$

Now $AB = \begin{bmatrix} 4 & -2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{3}{2} & 1 \end{bmatrix}$

$$= \begin{bmatrix} 4\times1+(-2)\times\frac{3}{2} & 4\times\frac{1}{2}+(-2)\times1 \\ (-6)\times1+4\times\frac{3}{2} & (-6)\times\frac{1}{2}+4\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 2-2 \\ -6+6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

and $BA = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{3}{2} & 1 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ -6 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 1\times4+\frac{1}{2}\times(-6) & 1\times(-2)+\frac{1}{2}\times4 \\ \frac{3}{2}\times4+1\times(-6) & \frac{3}{2}\times(-2)+1\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Thus $AB = BA = I$

Therefore $A = \begin{bmatrix} 4 & -2 \\ -6 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{3}{4} & 1 \end{bmatrix}$

are multiplicative inverse of each other.

$$BA = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{3}{2} & 1 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ -6 & 4 \end{bmatrix}$$

Remember!
If A is a singular matrix i.e; $|A| = 0$, so in such a case, there is no inverse of matrix A.

Example 9: If $A = \begin{bmatrix} x & 8 \\ 5 & 10 \end{bmatrix}$ is a singular matrix, then find the value of x.

Sol. If $A = \begin{bmatrix} x & 8 \\ 5 & 10 \end{bmatrix}$ is a singular matrix, then $|A| = 0$

Now $|A| = \begin{vmatrix} x & 8 \\ 5 & 10 \end{vmatrix}$

Since A is singular, therefore,

$$|A| = 10x - 40 = 0$$

i.e. $10x = 40$

$\Rightarrow \quad x = 4$

Challenge!

Take any 2 by 2 matrix and check whether it is singular or non-singular. Also find its adjoint.

Sol. Let

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$$

$$= 3(4) - 1(2)$$

$$= 12 - 2 = 10 \neq 0$$

So, it is a non singular matrix

$$\text{Adj}\,A = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

Solved Exercise 3.5

1. Find the values of each of the determinant.

(i) $\begin{vmatrix} 10 & 5 \\ 4 & 6 \end{vmatrix}$

Sol. $\begin{vmatrix} 10 & 5 \\ 4 & 6 \end{vmatrix}$

$$= 10(6) - 4(5) = 60 - 20 = 40$$

(ii) $\begin{vmatrix} -5 & 8 \\ -3 & -7 \end{vmatrix}$

Sol. $\begin{vmatrix} -5 & 8 \\ -3 & -7 \end{vmatrix}$

$= -5(-7) - (-3)(8) = 35 + 24 = 59$

(iii) $\begin{vmatrix} 3 & 8 \\ 0 & 2 \end{vmatrix}$

Sol. $\begin{vmatrix} 3 & 8 \\ 0 & 2 \end{vmatrix}$

$= 3(2) - 0(8) = 6 - 0 = 6$

2. Find whether the following matrices are singular or non-singular.

$A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 7 & 21 \\ 2 & 6 \end{bmatrix},$

$C = \begin{bmatrix} 13 & 5 \\ 7 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix}$

$= 5(2) - (3)(3) = 10 - 9 = 1 \neq 0$

Thus, matrix A is a non-singular matrix.

$B = \begin{bmatrix} 7 & 21 \\ 2 & 6 \end{bmatrix}$

$|B| = \begin{vmatrix} 7 & 21 \\ 2 & 6 \end{vmatrix}$

$= 7(6) - 2(21) = 42 - 42 = 0$

Thus, matrix B is a singular matrix.

$C = \begin{bmatrix} 13 & 5 \\ 7 & 3 \end{bmatrix}$

$|C| = \begin{vmatrix} 13 & 5 \\ 7 & 3 \end{vmatrix}$

$= 13(3) - 7(5) = 39 - 35 = 4 \neq 0$

Thus, matrix C is a non-singular matrix.

$D = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

$D = \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix}$

$= 2(0) - 0(0) = 0 - 0 = 0$

Thus, matrix D is a singular matrix.

3. Find the value of x when $A = \begin{bmatrix} x & 6 \\ 5 & 15 \end{bmatrix}$ is a singular matrix.

Sol. $A = \begin{bmatrix} x & 6 \\ 5 & 15 \end{bmatrix}$

According to the given condition, matrix A is a singular matrix, so $|A| = 0$

$\begin{vmatrix} x & 6 \\ 5 & 15 \end{vmatrix} = 0$

$x(15) - 5(6) = 0$

$15x - 30 = 0$

$15x = 30$

$x = \frac{30}{15}$

$x = 2$

4. Find the adjoint of the following matrices.

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, B = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}, C = \begin{bmatrix} -3 & 5 \\ -3 & 2 \end{bmatrix}$

Sol. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, B = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}, C = \begin{bmatrix} -3 & 5 \\ -3 & 2 \end{bmatrix}$

$\text{Adj}\,A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \text{Adj}\,B = \begin{bmatrix} 7 & 2 \\ -3 & 5 \end{bmatrix},$

$\text{Adj}\,C = \begin{bmatrix} 2 & -5 \\ 3 & -3 \end{bmatrix}$

5. Find multiplicative inverse of the following matrices:

(i) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

Sol. Let

$A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$|A| = \begin{vmatrix} 5 & 0 \\ 0 & 5 \end{vmatrix} = 5(5) - 0(0) = 25 - 0 = 25 \neq 0$

A is a non-singular, so A^{-1} is possible

$\text{Adj}\,A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj}A$$

$$= \frac{1}{25}\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{5}{25} & \frac{0}{25} \\ \frac{0}{25} & \frac{5}{25} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$$

(ii) $\begin{bmatrix} -4 & 8 \\ 7 & 2 \end{bmatrix}$

Sol. Let

$$A = \begin{bmatrix} -4 & 8 \\ 7 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -4 & 8 \\ 7 & 2 \end{vmatrix}$$

$$= -(4) - 7(8) = -8 - 56 = -64 \neq 0$$

A is non-singular, so A^{-1} is possible

$$\text{Adj}A = \begin{bmatrix} 2 & -8 \\ -7 & -4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj}A$$

$$= \frac{1}{-64}\begin{bmatrix} 2 & -8 \\ -7 & -4 \end{bmatrix} = \begin{bmatrix} \frac{2}{-64} & \frac{-8}{-64} \\ \frac{-7}{-64} & \frac{-4}{-64} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{32} & \frac{1}{8} \\ \frac{7}{64} & \frac{1}{16} \end{bmatrix}$$

(iii) $\begin{bmatrix} 40 & 8 \\ 5 & 2 \end{bmatrix}$

Sol. Let

$$A = \begin{bmatrix} 40 & 8 \\ 5 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 40 & 8 \\ 5 & 2 \end{vmatrix}$$

$$= 40(2) - 5(8) = 80 - 40 = 40 \neq 0$$

$$\text{Adj}A = \begin{bmatrix} 2 & -8 \\ -5 & 40 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj}A$$

$$= \frac{1}{40} \times \begin{bmatrix} 2 & -8 \\ -5 & 40 \end{bmatrix} = \begin{bmatrix} \frac{2}{40} & \frac{-8}{40} \\ \frac{-5}{40} & \frac{40}{40} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{20} & -\frac{1}{5} \\ -\frac{1}{8} & 1 \end{bmatrix}$$

(iv) $\begin{bmatrix} 3 & 5 \\ 5 & -3 \end{bmatrix}$

Sol. Let

$$A = \begin{bmatrix} 3 & 5 \\ 5 & -3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 5 \\ 5 & -3 \end{vmatrix}$$

$$= 3(-3) - 5(5) = -9 - 25 = -34 \neq 0$$

A is non-singular, So A^{-1} is possible

$$\text{Adj}A = \begin{bmatrix} -3 & -5 \\ -5 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj}A$$

$$= \frac{1}{-34}\begin{bmatrix} -3 & -5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} \frac{-3}{-34} & \frac{-5}{-34} \\ \frac{-5}{34} & \frac{3}{-34} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{34} & \frac{5}{34} \\ \frac{5}{-34} & \frac{-3}{34} \end{bmatrix}$$

(v) $\begin{bmatrix} 10 & 8 \\ 3 & 3 \end{bmatrix}$

Sol. Let

$$A = \begin{bmatrix} 10 & 8 \\ 3 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 10 & 8 \\ 3 & 3 \end{vmatrix}$$

$$= 10(3) - 3(8) = 30 - 24 = 6 \neq 0$$

A is a non-singular, So A^{-1} is possible

$$\text{Adj}\,A = \begin{bmatrix} 3 & -8 \\ -3 & 10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj}\,A$$

$$= \frac{1}{6}\begin{bmatrix} 3 & -8 \\ -3 & 10 \end{bmatrix} = \begin{bmatrix} \frac{3}{6} & -\frac{8}{6} \\ -\frac{3}{6} & \frac{10}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{4}{3} \\ -\frac{1}{2} & \frac{5}{3} \end{bmatrix}$$

(vi) $\begin{bmatrix} -2 & -3 \\ 4 & 5 \end{bmatrix}$

Sol. Let

$$A = \begin{bmatrix} -2 & -3 \\ 4 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -2 & -3 \\ 4 & 5 \end{vmatrix}$$

$$= -2(5) - 4(-3) = -10 + 12 = 2 \neq 0$$

A is non-singular, so A^{-1} is possible.

$$\text{Adj}\,A = \begin{bmatrix} 5 & 3 \\ -4 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj}\,A$$

$$= \frac{1}{2}\begin{bmatrix} 5 & 3 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & \frac{3}{2} \\ \frac{-4}{2} & -\frac{2}{2} \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & \frac{3}{2} \\ -2 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix}$$

6. If $|A| = \begin{vmatrix} -2 & -3 \\ 4 & 5 \end{vmatrix}$ then find A^{-1} and prove that $AA^{-1} = A^{-1}A = I$.

Sol. $A = \begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 5 & -3 \\ 2 & -1 \end{vmatrix}$$

$$= 5(-1) - 2(-3) = -5 + 6 = 1 \neq 0$$

A is a non-singular, so A^{-1} is possible.

$$\text{Adj}\,A = \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj}\,A$$

$$= \frac{1}{1}\begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 5(-1)+(-3)(-2) & 5(3)+(-3)(5) \\ 2(-1)+(-1)(-2) & 2(3)+(-1)(5) \end{bmatrix}$$

$$= \begin{bmatrix} -5+6 & 15-15 \\ -2+2 & 6-5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \quad \text{...(i)}$$

$$A^{-1}A = \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1(5)+3(2) & -1(-3)+3(-1) \\ -2(5)+5(2) & -2(-3)+5(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -5+6 & 3-3 \\ -10+10 & 6-5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \quad \text{...(ii)}$$

Hence, verified from(i) and (ii) that $AA^{-1} = A^{-1}A = I$

7. Show that the following matrices are multiplicative inverse of each other.

(i) $\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

Sol. Multiply both matrices

$$\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$=\begin{bmatrix}2(2)+(-1)(3) & 2(1)+(-1)(2)\\ -3(2)+2(3) & -3(1)+2(2)\end{bmatrix}$$

$$=\begin{bmatrix}4-3 & 2-2\\ -6+6 & -3+4\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Since $\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}=I$

So, these are the multiplicative inverses of each other.

(ii) $\begin{bmatrix}4 & 15\\ 2 & 8\end{bmatrix},\begin{bmatrix}4 & -\frac{15}{2}\\ -1 & 2\end{bmatrix}$

Sol. Multiply both matrices

$$\begin{bmatrix}4 & 15\\ 2 & 8\end{bmatrix}\begin{bmatrix}4 & -\frac{15}{2}\\ -1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}4(4)+15(-1) & 4\left(-\frac{15}{2}\right)+15(2)\\ 2(4)+8(-1) & 2\left(-\frac{15}{2}\right)+8(2)\end{bmatrix}$$

$$=\begin{bmatrix}16-15 & -30+30\\ 8-8 & -15+16\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

$$\begin{bmatrix}4 & 15\\ 2 & 8\end{bmatrix}\begin{bmatrix}4 & -\frac{15}{2}\\ -1 & 2\end{bmatrix}=I$$

So these are the multiplicative inverses of each other.

8. Prove that $(AB)^{-1} = B^{-1}A^{-1}$, if

(i) $A=\begin{bmatrix}-3 & -2\\ 5 & 6\end{bmatrix}, B=\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}$

Sol. $AB=\begin{bmatrix}-3 & -2\\ 5 & 6\end{bmatrix}\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}$

$$=\begin{bmatrix}-3(2)+(-2)(-3) & -3(-1)+(-2)(2)\\ 5(2)+6(-3) & 5(-1)+6(2)\end{bmatrix}$$

$$=\begin{bmatrix}-6+6 & 3-4\\ 10-18 & -5+12\end{bmatrix}$$

$$AB=\begin{bmatrix}0 & -1\\ -8 & 7\end{bmatrix}$$

$$|AB|=\begin{vmatrix}0 & -1\\ -8 & 7\end{vmatrix}$$

$$=0(7)-(-8)(-1)=0-8=-8\neq 0$$

AB is a non-singular, so $(AB)^{-1}$ is possible.

$$Adj(AB)=\begin{bmatrix}7 & 1\\ 8 & 0\end{bmatrix}$$

$$(AB)^{-1}=\frac{1}{|AB|}\times Adj(AB)$$

$$=\frac{1}{-8}\begin{bmatrix}7 & 1\\ 8 & 0\end{bmatrix}$$

$$=\begin{bmatrix}\frac{7}{-8} & \frac{1}{-8}\\ \frac{8}{-8} & \frac{0}{-8}\end{bmatrix}=\begin{bmatrix}-\frac{7}{8} & -\frac{1}{8}\\ -1 & 0\end{bmatrix} \quad ...(i)$$

$$B=\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}$$

$$|B|=\begin{vmatrix}2 & -1\\ -3 & 2\end{vmatrix}$$

$$=2(2)-(-3)(-1)=4-3=1\neq 0$$

B is non singular, so B^{-1} is possible.

$$AdjB=\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}$$

$$B^{-1}=\frac{1}{|B|}\times AdjB$$

$$=\frac{1}{1}\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}=\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}$$

$$A=\begin{bmatrix}-3 & -2\\ 5 & 6\end{bmatrix}$$

$$|A|=\begin{vmatrix}-3 & -2\\ 5 & 6\end{vmatrix}$$

$$=(-3\times 6)-5(-2)=-18+10=-8\neq 0$$

A is a non-singular, So A^{-1} is possible

$$AdjA=\begin{bmatrix}6 & 2\\ -5 & -3\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times AdjA$$

$$=\frac{1}{-8}\begin{bmatrix}6 & 2\\ -5 & -3\end{bmatrix}$$

$$= \begin{bmatrix} \frac{6}{-8} & \frac{2}{-8} \\ \frac{-5}{-8} & \frac{-3}{-8} \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} & -\frac{1}{4} \\ \frac{5}{8} & \frac{3}{8} \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -\frac{3}{4} & -\frac{1}{4} \\ \frac{5}{8} & \frac{3}{8} \end{bmatrix}$$

$$= \begin{bmatrix} 2\left(-\frac{3}{4}\right)+1\left(\frac{5}{8}\right) & 2\left(-\frac{1}{4}\right)+1\left(\frac{3}{8}\right) \\ 3\left(-\frac{3}{4}\right)+2\left(\frac{5}{8}\right) & 3\left(-\frac{1}{4}\right)+2\left(\frac{3}{8}\right) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{3}{2}+\frac{5}{8} & -\frac{1}{2}+\frac{3}{8} \\ -\frac{9}{4}+\frac{10}{8} & -\frac{3}{4}+\frac{3}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-12+5}{8} & \frac{-4+3}{8} \\ \frac{-18+10}{8} & 0 \end{bmatrix} = \begin{bmatrix} \frac{-7}{8} & \frac{-1}{8} \\ \frac{-8}{8} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{7}{8} & -\frac{1}{8} \\ -1 & 0 \end{bmatrix} \quad \text{...(ii)}$$

Hence, verify from (i) and (ii) that $(AB)^{-1} = B^{-1}A^{-1}$

(ii) $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 8 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & -1 \\ 5 & 2 \end{bmatrix}$

Sol. $AB = \begin{bmatrix} 1 & 2 \\ 8 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 5 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 1(0)+2(5) & 1(-1)+2(2) \\ 8(0)+0(5) & 8(-1)+0(2) \end{bmatrix}$$

$$= \begin{bmatrix} 0+10 & -1+4 \\ 0+0 & -8+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 10 & 3 \\ 0 & -8 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 10 & 3 \\ 0 & -8 \end{vmatrix}$$

$$= 10(-8) - 0(3) = -80 - 0 = -80 \neq 0$$

AB is a non-singular matrix

$$\text{Adj}(AB) = \begin{bmatrix} -8 & -3 \\ 0 & 10 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|} \times \text{Adj}(AB)$$

$$= \frac{1}{-80} \begin{bmatrix} -8 & -3 \\ 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-8}{-80} & \frac{-3}{-80} \\ \frac{0}{-80} & \frac{10}{-80} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{10} & \frac{-3}{80} \\ 0 & -\frac{1}{8} \end{bmatrix} \quad \text{...(i)}$$

$$B = \begin{bmatrix} 0 & -1 \\ 5 & 2 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 0 & -1 \\ 5 & 2 \end{vmatrix}$$

$$= 0(2) - 5(-1) = 0 + 5 = 5 \neq 0$$

B is a non-singular matrix, So B^{-1} is possible.

$$\text{Adj}B = \begin{bmatrix} 2 & 1 \\ -5 & 0 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times \text{Adj}B$$

$$= \frac{1}{5} \begin{bmatrix} 2 & 1 \\ -5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{5} & \frac{1}{5} \\ \frac{-5}{5} & \frac{0}{5} \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} \\ -1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 8 & 0 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 8 & 0 \end{vmatrix}$$

$$= 1(0) - 8(2) = 0 - 16 = -16 \neq 0$$

A is a non-singular matrix, so A^{-1} is possible.

$$\text{Adj}\,A = \begin{bmatrix} 0 & -2 \\ -8 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj}\,A$$

$$= \frac{1}{-16}\begin{bmatrix} 0 & -2 \\ -8 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0}{-16} & \frac{-2}{-16} \\ -\frac{8}{-16} & \frac{1}{-16} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{8} \\ \frac{1}{2} & -\frac{1}{16} \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & \frac{1}{8} \\ \frac{1}{2} & \frac{-1}{16} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{5}(0)+\left(\frac{1}{5}\right)\left(\frac{1}{2}\right) & \frac{2}{5}\left(\frac{1}{8}\right)+\frac{1}{5}\left(-\frac{1}{16}\right) \\ -1(0)+0\left(\frac{1}{2}\right) & -1\left(\frac{1}{8}\right)+0\left(-\frac{1}{16}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 0+\frac{1}{10} & \frac{2}{40}-\frac{1}{80} \\ 0+0 & -\frac{1}{8}+0 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & \frac{4-1}{80} \\ 0 & -\frac{1}{8} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{10} & \frac{3}{80} \\ 0 & -\frac{1}{8} \end{bmatrix} \quad ...(ii)$$

Hence, verified from (i) and (ii) that $(AB)^{-1} = B^{-1}A^{-1}$

3.6 Solution of Simultaneous Linear Equations

A **linear equation** is an algebraic equation in which the highest power of the variable is 1. A general form of a linear equation in two variables is:

$ax + by = c$, where a, b, c are real numbers, x and y are variables and $a, b \neq 0$.

Simultaneous linear equations are a set of two or more linear equations with two or more variables that are solved together (simultaneously) to find a common solution. The general form of system of two linear equations in two variables is given below:

$$ax + by = m$$
$$cx + dy = n$$

where a, b, c, d, m and n are real numbers.

Here we will find the solution of two simultaneous equations in two variables by the following methods:

(i) Matrix Inversion Method

(ii) Cramer's Rule

Matrix Inversion Method

Look at the solution of the following pair of equation.

Let $\quad ax + by = m$

and $\quad cx = dy = n$

These equations can be written in the matrix form as: $\begin{bmatrix} ax+by \\ cd+dy \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix}$

or $\quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix} \quad$ (i)

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} m \\ n \end{bmatrix}$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Now if $|A| \neq 0$, then (i) can be written as

$Ax = B$

and $A^{-1}AX = A^{-1}B \quad$ [pre-multiply by A^{-1}]

$IX = A^{-1}B \quad$ [as $A^{-1}A = 1$]

Thus $\quad X = A^{-1}B \quad$ [as $IX = X$]

$$X = \frac{\text{Adj}\,A}{|A|}.B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix}}{ad-bc}$$

$$= \frac{\begin{bmatrix} dm-bn \\ ab-cm \end{bmatrix}}{ad-bc} = \begin{bmatrix} \dfrac{dm-bn}{ad-bc} \\ \dfrac{an-cm}{ad-bc} \end{bmatrix}$$

$$x = \frac{dm-bn}{ad-bc},\ y = \frac{an-cm}{ad-bc}$$

Example 10: Solve the following by matrix inversion method:

$$2x + 3y = 13$$
$$4x - 5y = -7$$

Solution: Given that $2x + 3y = 13$

$$4x - 5y = -7$$

Writing the equations in matrix form:

$$\begin{bmatrix} 2x+3y \\ 4x-5y \end{bmatrix} = \begin{bmatrix} 13 \\ -7 \end{bmatrix}$$

or $$\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 7 \end{bmatrix} \quad \text{(i)}$$

Let $A = \begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 13 \\ -7 \end{bmatrix}$

Then (i) can be written as

$AX = B$

or $X = A^{-1}B$

Now $A = \begin{vmatrix} 2 & 3 \\ 4 & -5 \end{vmatrix}$

$$|A| = \begin{vmatrix} 2 & 3 \\ 4 & -5 \end{vmatrix}$$
$$= (2)(-5) - (3)(4)$$
$$= -10 - 12 = -22 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -5 & -3 \\ -4 & 2 \end{bmatrix}$$

Therefore $A^{-1} = \dfrac{1}{|A|} \, adj\ A$

$$= \frac{1}{-22}\begin{bmatrix} -5 & -3 \\ -4 & 2 \end{bmatrix}$$

Now $X = A^{-1}B$ can be written as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-22}\begin{bmatrix} -5 & -3 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 13 \\ -7 \end{bmatrix}$$

$$= \frac{1}{-22}\begin{bmatrix} (-5)\times 13 + (-3)\times(-7) \\ (-4)\times 13 + \quad 2\times(-7) \end{bmatrix}$$

$$= \frac{1}{-22}\begin{bmatrix} -65 + 21 \\ -52 - 14 \end{bmatrix} = \frac{1}{-22}\begin{bmatrix} -44 \\ -66 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Thus, $x = 2, y = 3$

Solution set is $\{(2, 3)\}$

Challenge

Take any two linear equations in two variables and solve them by matrix inversion method.

Sol. $3x + 4y = 25$

$6x - 2y = 10$

Uriting these equations in natrix form

$$\begin{bmatrix} 3x+4y \\ 6x-2y \end{bmatrix} = \begin{bmatrix} 25 \\ 10 \end{bmatrix}$$

or $$\begin{bmatrix} 3 & 4 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 25 \\ 10 \end{bmatrix} \text{ — (i)}$$

Let $A = \begin{bmatrix} 3 & 4 \\ 6 & -2 \end{bmatrix}, X \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 25 \\ 10 \end{bmatrix}$

Than (i) can be written as

$AX = B$

or $X = A^{-1} B$

No $A = \begin{bmatrix} 3 & 4 \\ 6 & -2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & 4 \\ 6 & -2 \end{vmatrix}$$

$$= 3(-2) - 6(4) = -6 - 24 = -30$$

$$\text{Adj } A = \begin{bmatrix} -2 & -4 \\ -0 & 3 \end{bmatrix}$$

$$= \frac{1}{-30}\begin{bmatrix} -2 & -4 \\ -6 & +3 \end{bmatrix}$$

Now $x = A^{-1} B$ can be written as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-30}\begin{bmatrix} -2 & -4 \\ -6 & +3 \end{bmatrix}\begin{bmatrix} 25 \\ 10 \end{bmatrix}$$

$$= \frac{1}{-30}\begin{bmatrix} -2(25)+(-4)(10) \\ -6(25)+3(10) \end{bmatrix} = \frac{1}{-30}\begin{bmatrix} -50-40 \\ -150+30 \end{bmatrix}$$

$$= \frac{1}{-30}\begin{bmatrix} -90 \\ -120 \end{bmatrix} = \begin{bmatrix} \frac{-90}{-30} \\ \frac{-120}{-30} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

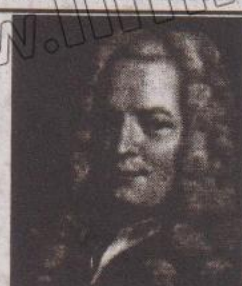
$x = 3, y = 4$

SS = {(3, 4)}

History!

Gabriel Cramer was a Genevan mathematician. Cramer showed promised in Mathematics from an early age. At 18 he received his doctorate in Mathematics.

In 1750 he published Cramer's rule, giving a general formula for the solution for any unknown in linear equation system having a unique solution, in terms of determinants applied by the system. This rule is still standard.



Gabriel Cramer
(July 31, 1704 - January 4, 1752)

Cramer's Rule

Consider the following system of linear equations.

$$ax + by = m$$
$$cx + dy = n$$

These equations can be written in the matrix form as

Where $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \end{bmatrix}$ and $B=\begin{bmatrix} m \\ n \end{bmatrix}$

$$AX = B \quad (i)$$

If $|A| \neq 0$, then from equation (i)

$A^{-1}AX = A^{-1}B$ [pre multiplication by A^{-1}]

$IX = A^{-1}B$

or $X = A^{-1}B$ [as $IX = X$]

$$X = \frac{\text{Adj}A}{|A|} \times B$$

$$\left[A^{-1} = \frac{\text{Adj } A}{|A|}\right]$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix}}{|A|}$$

$$= \frac{\begin{bmatrix} dm-bn \\ -cm+an \end{bmatrix}}{|A|}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{dm-bn}{|A|} \\ \frac{-cm+an}{|A|} \end{bmatrix}$$

or $x = \frac{dm-bn}{|A|}, y \frac{an-cm}{|A|}$

or $x = \frac{\begin{vmatrix} m & b \\ n & d \end{vmatrix}}{|A|} = \frac{|A_x|}{|A|}$

$$y = \frac{\begin{vmatrix} a & m \\ c & n \end{vmatrix}}{|A|} = \frac{|A_y|}{|A|}$$

Challenge!

Take any two linear equations in two variables and solve them by using cramer's rule.

Sol: $2x + 4y = 2$

$5x - 2y = -19$

writing these equation in matrix form.

Sol.

$$\begin{bmatrix} 2x+4y \\ 5x-2y \end{bmatrix} = \begin{bmatrix} 2 \\ -19 \end{bmatrix}$$

or $\begin{bmatrix} 2 & 4 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -19 \end{bmatrix}$

Let $A = \begin{bmatrix} 2 & 4 \\ 5 & -2 \end{bmatrix}$, $Ax = \begin{bmatrix} 2 & 4 \\ -19 & -2 \end{bmatrix}$,

$$Ay = \begin{bmatrix} 2 & 2 \\ 5 & -19 \end{bmatrix}$$

$$\text{Now } A = \begin{bmatrix} 2 & 2 \\ 5 & -19 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 4 \\ 5 & -2 \end{vmatrix}$$

$$|A| = \begin{vmatrix} 2 & 4 \\ 5 & -2 \end{vmatrix}$$

$$= 2(-2) - (5)(4)$$
$$= -4 - 20 = -24 \neq 0$$

$$|Ax| = \begin{vmatrix} 2 & 4 \\ -19 & -2 \end{vmatrix}$$

$$= 2(-2) - (-19)(4)$$
$$= -4 + 76 = 72$$

$$x = \frac{|Ax|}{|A|}$$

$$x = \frac{72}{-24} = -3$$

$$|Ay| = \begin{vmatrix} 2 & 2 \\ 5 & 19 \end{vmatrix}$$

$$= (2)(-19) - (5)(2)$$
$$= -38 - 10 = -48$$

$$|Ay| = \begin{bmatrix} 2 & 2 \\ 5 & 19 \end{bmatrix} = \frac{-48}{-24} = 2$$

So $x = -3$, $y = 2$

S.S = $\{(-3, 2)\}$

Example 11: Use Cramer's rule to solve the system of equations:

$$2x + 3y = 13$$
$$4x - 5y = -7$$

Solution: $2x + 3y = 13$

$$4x - 5y = -7$$

Matrix form of the equations is:

$$\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ -7 \end{bmatrix}$$

Here, we have

$$A = \begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}, A_x = \begin{bmatrix} 13 & 3 \\ -7 & -5 \end{bmatrix}, A_y = \begin{bmatrix} 2 & 13 \\ 4 & -7 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 3 \\ 4 & -5 \end{vmatrix}$$

$$= (2)(-5) - (3)(4)$$
$$= -10 - 12$$
$$= -22 \neq 0$$

$$|Ax| = \begin{vmatrix} 13 & 3 \\ -7 & -5 \end{vmatrix}$$

$$= (13)(-5) - (-7)(3)$$
$$= -65 + 21 = -44$$

$$x = \frac{|Ax|}{|A|}$$

$$= \frac{-44}{-22} = 2$$

$$|A_y| = \begin{vmatrix} 2 & 13 \\ 4 & -7 \end{vmatrix}$$

$$= (2)(-7) - (13)(4)$$
$$= -14 - 52 = -66$$

$$y = \frac{|A_y|}{|A|} = \frac{-66}{-22} = 3$$

Solution set = $\{(2, 3)\}$

3.7 Applications of Matrices in Real World Problems

Example 12: Three forces act on a particle which must be in equilibrium i.e.

$F_1 + F_2 + F_3 = 0$, where $F_1 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $F_2 = \begin{bmatrix} -3 \\ x \end{bmatrix}$,

$F_3 = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$ Find the value of x.

Solution: Since $F_1 + F_2 + F_3 = 0$

$$\begin{bmatrix} 5 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ x \end{bmatrix} + \begin{bmatrix} -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5-3-2 \\ 2+x-4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ x-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x - 2 = 0$$
$$x = 2$$

Example 13: Eight years ago Huria's was $\frac{3}{4}$ of Jannat's age. After four years Huria's age will be $\frac{6}{7}$ of Jannat's age. Find their present ages by using matrices.

Solution: Suppose Huria's present age is x years and Jannat's present age is y years. Eight years ago their ages were $x - 8$ and y–8. According to first condition:

$$x - 8 = \frac{3}{4}(y - 8)$$

$$\text{or} \quad 4x - 32 = 3y - 24$$

$$\text{or} \quad 4x - 3y = - 24 + 32$$

$$4x - 3y = 8 \quad \text{(i)}$$

After 4 years their ages will be $x + 4$ and $y + 4$.

Applying the second condition:

$$x + 4 = \frac{6}{7}(y + 4)$$

$$\text{or } 7x + 28 = 6y + 24$$

$$7x - 6y = -4 \quad \text{(ii)}$$

Matrix form of (i) and (ii) is

$$\begin{bmatrix} 4 & -3 \\ 7 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

Here, $A = \begin{bmatrix} 4 & -3 \\ 7 & -6 \end{bmatrix}$, $A_x = \begin{bmatrix} 8 & -3 \\ -4 & -6 \end{bmatrix}$,

$$A_y = \begin{bmatrix} 4 & 8 \\ 7 & -4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & -3 \\ 7 & -6 \end{vmatrix}$$

$$= (4)(-6) - (7)(-3)$$

$$= -24 + 21$$

$$= -3 \neq 0$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 8 & -3 \\ -4 & -6 \end{vmatrix}}{-3}$$

$$= \frac{-48 - 12}{-3} = \frac{-60}{-3} = 20$$

$$= = \frac{-60}{-3} = 20$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 4 & 8 \\ 7 & -4 \end{vmatrix}}{-3}$$

$$= \frac{-16 - 56}{-3} = \frac{-72}{-3} = 24$$

$\therefore$ Present ages of Huria and Jannat are 20 years and 24 years respectively.

3.8 The Role of Mathematics in Scientific Theories and Technological Advancement

Mathematics is often described as the language of science. It plays a fundamental role in developing scientific theories and advancing modern technologies. From understanding the laws of nature to designing life-saving drugs, Mathematics provides the tools needed to describe, predict and optimize real-world phenomena,

Drug Discovery

Mathematical algorithms help simulate how a drug will interact with cells or proteins in the body, speeding up the discovery of effective treatments.

Neuroscience

Mathematical models are used to simulate brain activity, helping researchers understand learning, memory and diseases like epilepsy or Alzheimer's.

Climate Science

Complex equations describe the interaction of air, water and energy, helping scientists to predict weather and long-term climate change.

Exercise 3.6

1. Solve by matrix inversion method, if possible.

(i) $2x + 5y = 19$
$4x - 3y = -1$

Sol. Given that $2x + 5y = 19$
$4x - 3y = -1$

Writing the equations in matrix form

$$\begin{bmatrix} 2x+5x \\ 4x-3y \end{bmatrix} = \begin{bmatrix} 19 \\ -1 \end{bmatrix}$$

or
$$\begin{bmatrix} 2 & 5 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ -1 \end{bmatrix} \text{—(i)}$$

Let $A = \begin{bmatrix} 2 & 5 \\ 4 & -3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 19 \\ -1 \end{bmatrix}$

Then (i) can be writen as:

$AX = B$

or $X = A^{-1}B$

Now $A = \begin{bmatrix} 2 & 5 \\ 4 & -3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 5 \\ 4 & -3 \end{vmatrix}$$
$$= (2\times-3)-(4\times5)$$
$$= -6-20$$
$$= -26 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\times\text{Adj } A$$

$$= \frac{1}{-26}\begin{bmatrix} -3 & -5 \\ -4 & 2 \end{bmatrix}$$

Now $X = A^{-1}B$ Can be writen as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-26}\begin{bmatrix} -3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 19 \\ -1 \end{bmatrix}$$

$$= \frac{1}{-26}\begin{bmatrix} -3(19)+(-5)(-1) \\ -4(19)+2(-1) \end{bmatrix}$$

$$= \frac{1}{-26}\begin{bmatrix} -57+5 \\ -76-2 \end{bmatrix}$$

$$= \frac{1}{-26}\begin{bmatrix} -52 \\ -78 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-52}{-26} \\ \frac{-78}{-26} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Thus $x = 2, y = 3$

Solution set $\{(2, 3)\}$

(ii) $3x + 2y = 7$
$5x - y = 16$

Sol. Given that $3x + 2y = 7$
$5x - y = 16$

Writing the equations in matrix form

$$\begin{bmatrix} 3x+2y \\ 5x-y \end{bmatrix} = \begin{bmatrix} 7 \\ 16 \end{bmatrix}$$

or
$$\begin{bmatrix} 3 & 2 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 16 \end{bmatrix} \text{—(i)}$$

Let $A = \begin{bmatrix} 3 & 2 \\ 5 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 7 \\ 16 \end{bmatrix}$

Then (i) can be writen as:

$AX = B$

or $X = A^{-1}B$

Now $A = \begin{bmatrix} 3 & 2 \\ 5 & -1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & 2 \\ 5 & -1 \end{vmatrix}$$
$$= 3(-1)-5(2)$$
$$= -3-10$$
$$= -13 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -1 & -2 \\ -5 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj}A$$

$$= \frac{1}{-13}\begin{bmatrix} -1 & -2 \\ -5 & 3 \end{bmatrix}$$

Now $X = A^{-1}B$ Can be writen as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-13}\begin{bmatrix} -1 & -2 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 16 \end{bmatrix}$$

$$= \frac{1}{-13}\begin{bmatrix} -1(7)+(-2)(16) \\ -5(7)+\ 3(16) \end{bmatrix}$$

$$= \frac{1}{-13}\begin{bmatrix} -7-32 \\ -35+48 \end{bmatrix}$$

$$= \frac{1}{-13}\begin{bmatrix} -39 \\ 13 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-39}{-13} \\ \frac{13}{-13} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Thus x = 3, y = –1

Solution set y{(3, –1)}

(iii) $\boldsymbol{x - 2y = 9}$

$\boldsymbol{2x + 7y = -4}$

Sol. Given that $x - 2y = 9$

$2x + 7y = -4$

Writing the equations in matrix form

$$\begin{bmatrix} x\ -2y \\ 2x+7y \end{bmatrix} = \begin{bmatrix} 9 \\ -4 \end{bmatrix}$$

or $$\begin{bmatrix} 1 & -2 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -4 \end{bmatrix} \text{—(i)}$$

Let $A = \begin{bmatrix} 1 & -2 \\ 2 & 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 9 \\ -4 \end{bmatrix}$

Then (i) can be writen as:

$$A\,X = B$$

or $$X = A^{-1}B$$

Now $$A = \begin{bmatrix} 1 & -2 \\ 2 & 7 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -2 \\ 2 & 7 \end{vmatrix}$$

$$= 1(7)-2(-2)$$

$$= 7+4$$

$$= 11 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} 7 & 2 \\ -2 & 1 \end{bmatrix}$$

Therefore $A^{-1} = \frac{1}{|A|} \times \text{Adj } A$

$$= \frac{1}{11}\begin{bmatrix} 7 & 2 \\ -2 & 1 \end{bmatrix}$$

Now $X = A^{-1}B$ Can be writen as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 7 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 9 \\ -4 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 7(9)+2(-4) \\ -2(9)+1(-4) \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 63-8 \\ -18-4 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 55 \\ -22 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{55}{11} \\ \frac{-22}{11} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

Thus x = 5, y = –2

Solution set is {(5, –2)}

(iv) $\boldsymbol{3x + 2y = 2}$

$\boldsymbol{x - 2y = -2}$

Sol. Given that $3x + 2y = 2$

$x - 2y = -2$

Writing the equations in matrix form

$$\begin{bmatrix} 3x+2y \\ x-2y \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

or $$\begin{bmatrix} 3 & 2 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \text{—(i)}$$

Let $A = \begin{bmatrix} 3 & 2 \\ 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

Then (i) can be writen as

$$AX = B$$

or $$X = A^{-1}B$$

Now $$A = \begin{bmatrix} 3 & 2 \\ 1 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 1 & -2 \end{vmatrix}$$
$$= 3(-2)-1(-2)$$
$$= -6-2$$
$$= -8 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -2 & -2 \\ -1 & 3 \end{bmatrix}$$

Therefore $A^{-1} = \frac{1}{|A|} \times \text{Adj}A$

$$= \frac{1}{-8}\begin{bmatrix} -2 & -2 \\ -1 & 3 \end{bmatrix}$$

Now $X = A^{-1}B$ Can be writen as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} -2 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} -2(2)+(-2)(-2) \\ -1(2)+3(-2) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} -4+4 \\ -2-6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} 0 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{0}{-8} \\ \frac{-8}{-8} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Thus $x = 0, y = -1$

Solution set is $\{(0, -1)\}$

2. Use Cramer's rule to solve the following pair of linear equations, if possible.

(i) $x + 4y = 4$
$2x - y = 5$

Sol. Given that $x + 4y = 4$
$2x - y = 5$

Writing the equations in matrix form

$$\begin{bmatrix} x+4y \\ 2x-y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

or $$\begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

Here, we have

$A = \begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}, A_x = \begin{bmatrix} 4 & 4 \\ 5 & -1 \end{bmatrix}, A_y = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 4 \\ 2 & -1 \end{vmatrix}$$
$$= 1(-1)-2(4)$$
$$= -1-8$$
$$= -9 \neq 0$$

$$|A_x| = \begin{vmatrix} 4 & 4 \\ 5 & -1 \end{vmatrix}$$
$$= 4(-1) - 5(4)$$
$$= -4 - 20$$
$$= -24$$

$$x = \frac{|A_x|}{|A|} = \frac{-24}{-9} = \frac{8}{3}$$

$$|A_y| = \begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix}$$
$$= 1(5) - 2(4)$$
$$= 5 - 8 = -3$$
$$y = \frac{|A_y|}{|A|}$$
$$= \frac{-3}{-9} = \frac{1}{3}$$

Solution set = $\left\{\left(\frac{8}{3}, \frac{1}{3}\right)\right\}$

(ii) $x + 2y = 7$
$3x - 2y = -3$

Sol. Given that $x + 2y = 7$
$3x - 2y = -3$

Writing the equations in matrix form

$$\begin{bmatrix} x+2y \\ 3-2y \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$$

or $$\begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$$

Here, we have

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}, A_x = \begin{bmatrix} 7 & 2 \\ -3 & -2 \end{bmatrix}, A_y = \begin{bmatrix} 1 & 7 \\ 3 & -3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix}$$
$$= 1(-2) - 3(2)$$
$$= -2 - 6 = -8 \neq 0$$

$$|A_x| = \begin{vmatrix} 7 & 2 \\ -3 & -2 \end{vmatrix}$$
$$= 7(-2) - (-3)(2)$$
$$= -14 + 6$$
$$= -8$$
$$x = \frac{|A_x|}{|A|}$$
$$= \frac{-8}{-8}$$
$$= 1$$

$$|A_y| = \begin{vmatrix} 1 & 7 \\ 3 & -3 \end{vmatrix}$$
$$= 1(-3) - 3(7)$$
$$= -3 - 21 = -24$$
$$y = \frac{|A_y|}{|A|}$$
$$= \frac{-24}{-8}$$
$$= 3$$

Solution set = $\{(1, 3)\}$

(iii) $2x - 5y = -6$
$4x - 3y = -12$

Sol. Given that $2x - 5y = -6$
$4x - 3y = -12$

Writing the equations in matrix form

$$\begin{bmatrix} 2x-5y \\ 4x-3y \end{bmatrix} = \begin{bmatrix} -6 \\ -12 \end{bmatrix}$$

or $$\begin{bmatrix} 2 & -5 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -12 \end{bmatrix}$$

Here, we have

$$A = \begin{bmatrix} 2 & -5 \\ 4 & -3 \end{bmatrix}, A_x = \begin{bmatrix} -6 & -5 \\ -12 & -3 \end{bmatrix},$$

$$A_y = \begin{bmatrix} 2 & -6 \\ 4 & -12 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -5 \\ 4 & -3 \end{vmatrix}$$
$$= 2(-3) - 4(-5)$$
$$= -6 + 20 = 14 \neq 0$$

$$|A_x| = \begin{vmatrix} -6 & -5 \\ -12 & -3 \end{vmatrix}$$
$$= (-6)(-3) - (-12)(-5)$$
$$= 18 - 60 = -42$$
$$x = \frac{|A_x|}{|A|}$$
$$= \frac{-42}{14}$$
$$= -3$$

$$|A_y| = \begin{vmatrix} 2 & -6 \\ 4 & -12 \end{vmatrix}$$
$$= 2(-12) - 4(-6)$$
$$= -24 + 24 = 0$$
$$y = \frac{|A_y|}{|A|}$$
$$= \frac{0}{14} = 0$$

Solution set = $\{(-3, 0)\}$

(iv) $3x + 2y = -1$

$5x + 6y = 5$

Sol. Given that $3x + 2y = -1$

$5x + 6y = 5$

Writing the equations in matrix form

$$\begin{bmatrix} 3x+2y \\ 5x+6y \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

or

$$\begin{bmatrix} 3 & 2 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

Here, we have

$$A = \begin{bmatrix} 3 & 2 \\ 5 & 6 \end{bmatrix}, A_x = \begin{bmatrix} -1 & 2 \\ 5 & 6 \end{bmatrix}, A_y = \begin{bmatrix} 3 & -1 \\ 5 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 5 & 6 \end{vmatrix}$$

$$= 3(6)-5(2)$$

$$= 18-10 = 8$$

$$|A_x| = \begin{vmatrix} -1 & 2 \\ 5 & 6 \end{vmatrix}$$

$$= -1(6)-5(2)$$

$$= -6-10$$

$$= -16$$

$$x = \frac{|A_x|}{|A|}$$

$$= \frac{-16}{8}$$

$$= -2$$

$$|A_y| = \begin{vmatrix} 3 & -1 \\ 5 & 5 \end{vmatrix}$$

$$= 3(5)-5(-1)$$

$$= 15 + 5 = 20$$

$$y = \frac{|A_y|}{|A|}$$

$$= \frac{20}{8}$$

$$= \frac{5}{2}$$

3. An electrical engineer wants to determine the current in two branches A and B of a simple electrical circuit. The system of the equations is:

$x + y = 7$

$2x - y = 2$

where x is the current in branch A and y is the current in branch B. Find x and y by using matrices.

Sol. System of equation

$x + y = 7$

$2x - y = 2$

Writing the equations in matrix form

$$\begin{bmatrix} x+y \\ 2x-y \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

or

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix} \text{—(i)}$$

Let $A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$

Then (i) can be writen as

$$AX = B$$

or

$$X = A^{-1}B$$

Now $A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$$

$$= 1(-1)-2(1)$$

$$= -1-2 = -3 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj} A$$

$$= \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$$

Now $X = A^{-1}B$ can be writen as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{-3}\begin{bmatrix} -1(7)+(-1)(2) \\ -2(7)+1(2) \end{bmatrix}$$

$$= \frac{1}{-3}\begin{bmatrix} -7 - 2 \\ -14 + 2 \end{bmatrix}$$

$$= \frac{1}{-3}\begin{bmatrix} -9 \\ -12 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-9}{-3} \\ \frac{-12}{-3} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

Thus $x = 3, y = 4$

current in branch A-(x) = 3

current in branch B (y) = 4

4. Three forces act on a particle and must be in equilibrium i.e.

$$F_1 + F_2 + F_3 = 0,$$

where $F_1 = \begin{bmatrix} 8 \\ x \end{bmatrix}, F_2 = \begin{bmatrix} -2 \\ -7 \end{bmatrix}, F_3 = \begin{bmatrix} y \\ -1 \end{bmatrix}$.

Find the value of *x* and *y*.

Sol. Given

$$F_1 + F_2 + F_3 = 0$$

$$\begin{bmatrix} 8 \\ x \end{bmatrix} + \begin{bmatrix} -2 \\ -7 \end{bmatrix} + \begin{bmatrix} y \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 8-2+y \\ x-7-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6+y \\ x-8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x - 8 = 0$	$6 + y = 0$
$x = 8$ _(i)	$y = -6$

So, $x = 8, y = -6$

5. Two support beams, A and B are holding up a combined load of 100 kN. Twice the load on beam A and three times the load on beam B equals 240 kN. Find the load of beam A and beam B by using matrices.

Sol. Make equations from the given data

$$A + B = 100$$
$$2A + 3B = 240$$

Writing the equations in matrix form

$$\begin{bmatrix} A+B \\ 2A+3B \end{bmatrix} = \begin{bmatrix} 100 \\ 240 \end{bmatrix}$$

or $$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 100 \\ 240 \end{bmatrix} \text{—(i)}$$

Let $C = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} A \\ B \end{bmatrix}, D = \begin{bmatrix} 100 \\ 240 \end{bmatrix}$

Then (i) can be writen as:

$$CX = D$$

or $$X = C^{-1}D$$

Now $$C = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$$

$$|C| = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$
$$= 1(3)-(2)(1)$$
$$= 3 - 2 = 1 \neq 0$$

$$\text{Adj } C = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$$

$$C^{-1} = \frac{1}{|C|} \times \text{Adj}C$$

$$= \frac{1}{1}\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$$

Now $X = C^{-1}D$ can be writen as

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 100 \\ 240 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 3(100)+(-1)(240) \\ -2(100)+1(240) \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 300-240 \\ -200+240 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 60 \\ 40 \end{bmatrix}$$

So, Load on beam A = 60 kN
Load on beam B = 40 kN

6. In a 2D game world, two characters are moving along straight paths. One character moves along a line where the total of twice their horizontal position and vertical position is 5, while the other moves along a line where their horizontal position is one more than their vertical position. Find their point of intersection by using matrices.

Sol. Let the horizontal position = x
the vertical position = y
First condition
$2x + y = 5$ ___(i)
Second condition
$x = y + 1$
$x - y = 1$ ___(ii)
$2x + y = 5$
$x - y = 1$
writing the equations in matrix form

$$\begin{bmatrix} 2x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

or $$\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix} \quad \text{—(i)}$$

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

Then (i) can be written as
$A \times = B$
or $X = A^{-1} B$

Now $A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$

$$|A| = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$$
$$= 2(-1) - (1)(1)$$
$$= -2 - 1$$
$$= -3 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$
$$= \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$$

Now $X = A^{-1} B$ can be written as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 1 \end{bmatrix}$$
$$= \frac{1}{-3}\begin{bmatrix} -1(5)+(-1)(1) \\ -1(5)+2(1) \end{bmatrix}$$
$$= \frac{1}{-3}\begin{bmatrix} -5-1 \\ -5+2 \end{bmatrix}$$
$$= \frac{1}{-3}\begin{bmatrix} -6 \\ -3 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{-6}{-3} \\ \frac{-3}{-3} \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

So, $x = 2, y = 1$
the intersecting point is (2, 1)

7. Two years ago a man was 5 times as old as his son was. After 6 years he will be 3 times as old as his son. Find their present ages by using matrices.

Sol. Let man's present age is x years and present age of son is y years.
Two years ago age of man = x–2 years
Two years ago age of son = y–2 years

According to first condition:

$x-2=5(y-2)$

$x-2=5y-10$

$x-5y=-10+2$

$x-5y=-8$ (i)

After 6 years the age of father will be $=x+6$ years

After 6 years the age of son will be$=y+6$ years

According to second condition:

$x+6=3(y+6)$

$x+6=3y+18$

$x-3y=18-6$

$x-3y=12$ — (ii)

Matrix form of (i) and (ii)

$$\begin{bmatrix} x-5y \\ x-3y \end{bmatrix}=\begin{bmatrix} -8 \\ 12 \end{bmatrix}$$

or $$\begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -8 \\ 12 \end{bmatrix} \text{ — (iii)}$$

Let $A=\begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix}, X=\begin{bmatrix} x \\ y \end{bmatrix}, B=\begin{bmatrix} -8 \\ 12 \end{bmatrix}$

Then (iii) can be written as

$A \times = B$

or $\times = A^{-1} B$

Now $A=\begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix}$

$$|A|=\begin{vmatrix} 1 & -5 \\ 1 & -3 \end{vmatrix}$$

$=1(-3)-1(-5)$

$=-3+5=2\neq 0$

$$\text{Adj } A=\begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times \text{Adj } A$$

$$=\frac{1}{2}\begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix}$$

Now $X=A^{-1}B$ can be written as

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{2}\begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -8 \\ 12 \end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix} -3(-8+5(12) \\ -1(-8)+1(12) \end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix} 24+60 \\ 8+12 \end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix} 84 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} \frac{84}{2} \\ \frac{20}{2} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 42 \\ 10 \end{bmatrix}$$

$x=42,\ y=10$

So, Man,s present age = 42 years

Son's present age = 10 years

8. Two cyclists are 44 km apart and start out at the same time. If they go towards one another they meet in 2 hours, but if they go in the same direction the faster overtakes the slower in $7\frac{1}{2}$ hours. Find their speeds by using matrices.

Sol. Let x = the speed of the faster cyclist (in km/h)

y = the speed of the slower cyclist (in km/h)

condition I (moving to wards each other.)

$2(x+y)=44$

$$x+y=\frac{44}{2}$$

$x+y=22$ — (i)

condition II (moving in the same direction)

$$\frac{15}{2}(x-y)=44$$

$$15(x-y)=88$$
$$15x-15y=88 \text{ — (ii)}$$

writing the equations in matrix form

$$\begin{bmatrix} x+y \\ 15-15y \end{bmatrix}=\begin{bmatrix} 22 \\ 88 \end{bmatrix}$$

or $$\begin{bmatrix} 1 & 1 \\ 15 & -15 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 22 \\ 88 \end{bmatrix} \text{ — (ii)}$$

Let $A=\begin{bmatrix} 1 & 1 \\ 15 & -15 \end{bmatrix}, X=\begin{bmatrix} x \\ y \end{bmatrix}, B=\begin{bmatrix} 22 \\ 88 \end{bmatrix}$

Then (iii) can be written as

$$AX=B$$

or $$X=A^{-1}B$$

Now $$A=\begin{bmatrix} 1 & 1 \\ 15 & -15 \end{bmatrix}$$

$$|A|=\begin{bmatrix} 1 & 1 \\ 15 & -15 \end{bmatrix}$$
$$=1(-5)-15(1)$$
$$=-15-15=-30\neq 0$$

$$\text{Adj } A=\begin{bmatrix} -15 & -1 \\ -15 & 1 \end{bmatrix}$$

$$A-1=\frac{1}{|A|}\times \text{Adj } A$$
$$=\frac{1}{-30}\begin{bmatrix} -15 & -1 \\ -15 & 1 \end{bmatrix}$$

Now $n=A^{-1}B$ can be written as

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{-30}\begin{bmatrix} -15 & -1 \\ -15 & 1 \end{bmatrix}\begin{bmatrix} 22 \\ 88 \end{bmatrix}$$
$$=\frac{1}{-30}\begin{bmatrix} -15(22)+(-1)(88) \\ -15(22)+1(88) \end{bmatrix}$$
$$=\frac{1}{-30}\begin{bmatrix} -330-88 \\ -330+88 \end{bmatrix}$$
$$=\frac{1}{-30}\begin{bmatrix} -418 \\ -242 \end{bmatrix}$$
$$=\begin{bmatrix} \frac{-418}{-30} \\ \frac{-242}{-30} \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 13.93 \\ 8.07 \end{bmatrix}$$
$$x=13.93,\ y=8.07$$

So, speed of the faster cyclist = 13.93 km/h
speed of the slower cyclist = 8.07 km/h

9. The numerator of a fraction is 7 less than the denominator. If the numerator is increased by 3, the new fraction can be cancelled down to $\frac{3}{4}$. Find the original fraction by using matrices.

Sol: Let

x = the numerator of the fraction
y = the denominator of the fraction

Condition I

$$x=y-7$$
$$x-y=-7 \text{ — (i)}$$

Condition II $$\frac{x+3}{y}=\frac{3}{4}$$

$$4(x+3)=3y$$
$$4x+12=3y$$
$$4x-3y=-12 \text{ — (ii)}$$

writing the two equations in matrix form

$$\begin{bmatrix} x-y \\ 4x-3y \end{bmatrix}=\begin{bmatrix} -7 \\ -12 \end{bmatrix}$$

or $$\begin{bmatrix} 1 & -1 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -7 \\ -12 \end{bmatrix} \text{ — (iii)}$$

Let $A = \begin{bmatrix} 1 & -1 \\ 4 & -3 \end{bmatrix}$, $X \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} -7 \\ -12 \end{bmatrix}$

Then (iii) can be written as

$$AX = B$$

or $$X = A^{-1} B$$

Now $$A = \begin{bmatrix} 1 & -1 \\ 4 & -3 \end{bmatrix}$$

$$|A| = \begin{bmatrix} 1 & -1 \\ 4 & -3 \end{bmatrix}$$

$$= 1(-3) - 4(-1)$$

$$= -3 + 4 = 1 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -3 & 1 \\ -4 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$

$$= \frac{1}{1}\begin{bmatrix} -3 & 1 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 1 \\ -4 & 1 \end{bmatrix}$$

No $X = A^{-1} B$ can be written as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} -7 \\ -12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3(-7)+1(-12) \\ -4(-7)+1(-12) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} +21-12 \\ +28-12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 16 \end{bmatrix}$$

So, $x = 9$, $y = 16$

$$\text{fraction} = \frac{9}{16}$$

Solved Review Exercise 3

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) If $\begin{bmatrix} a+2 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$, then what is the value of a?

(a) 3 (b) 5
(c) 6 (d) 7

(ii) $A = [3\ \ 5\ \ 0]$ is a ________ matrix.

(a) row (b) square
(c) column (d) null

(iii) $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is a ________ matrix.

(a) identity (b) square
(c) row (d) column

(iv) $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a/an _____ matrix.

(a) rectangular (b) identity
(c) column (d) row

(v) If $A^t = -A$, then A is ______ matrix.

(a) symmetric (b) row
(c) rectangular
(d) skew-symmetric

(vi) If $A=[3\ \ \ 4]$, $B=[7\ \ \ 8]$, then $A+B =$

(a) $[21\ \ \ 32]$ (b) $[24\ \ \ 28]$
(c) $[10\ \ \ 12]$ (d) $[11\ \ \ 11]$

(vii) If $A= [3\ \ \ 4]$, $B=[7\ \ \ 8]$, then $B - A =$

(a) $[10\ \ \ 12]$ (b) $[11\ \ \ 11]$
(c) $[4\ \ \ 4]$ (d) $[-4\ \ \ -4]$

(viii) What is the additive inverse of $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$?

(a) $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} -3 \\ -4 \end{bmatrix}$

(c) $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ (d) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(ix) If $A=\begin{bmatrix} 3 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix}$, then order of A^t is:

(a) 3-by-2 (b) 2-by-3
(c) 3-by-3 (d) 2-by-2

(x) $\begin{vmatrix} 3 & 1 \\ 0 & 4 \end{vmatrix} =$

(a) 11 (b) 12
(c) 13 (d) −11

ANSWERS:

(i) 5 (ii) row (iii) column
(iv) identity (v) skew-symmetric
(vi) [10 12] (vii) [4 4] (viii) $\begin{bmatrix} -3 \\ -4 \end{bmatrix}$
(ix) 2-by-3 (x) 12

2. If $A=\begin{bmatrix} 4 & 2 \\ 7 & 6 \end{bmatrix}, B=\begin{bmatrix} 5 & 5 \\ 8 & 8 \end{bmatrix}$, then find

(i) $(A-B)^t$

Sol. $A-B=\begin{bmatrix} 4 & 2 \\ 7 & 6 \end{bmatrix}-\begin{bmatrix} 5 & 5 \\ 8 & 8 \end{bmatrix}$

$A-B=\begin{bmatrix} 4-5 & 2-5 \\ 7-8 & 6-8 \end{bmatrix}$

$A-B=\begin{bmatrix} -1 & -3 \\ -1 & -2 \end{bmatrix}$

$(A-B)^t=\begin{bmatrix} -1 & -3 \\ -1 & -2 \end{bmatrix}^t$

$(A-B)^t=\begin{bmatrix} -1 & -1 \\ -3 & -2 \end{bmatrix}$

(ii) $B^t - A^t$

Sol. $B=\begin{bmatrix} 5 & 5 \\ 8 & 8 \end{bmatrix}, A=\begin{bmatrix} 4 & 2 \\ 7 & 6 \end{bmatrix}$

$B^t=\begin{bmatrix} 5 & 8 \\ 5 & 8 \end{bmatrix}, A^t=\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}$

$B^t-A^t=\begin{bmatrix} 5 & 8 \\ 5 & 8 \end{bmatrix}-\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}$

$=\begin{bmatrix} 5-4 & 8-7 \\ 5-2 & 8-6 \end{bmatrix}$

$=\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

(iii) $2A+3B$

Sol. $2A+3B=2\begin{bmatrix} 4 & 2 \\ 7 & 6 \end{bmatrix}+3\begin{bmatrix} 5 & 5 \\ 8 & 8 \end{bmatrix}$

$=\begin{bmatrix} 8 & 4 \\ 14 & 12 \end{bmatrix}+\begin{bmatrix} 15 & 15 \\ 24 & 24 \end{bmatrix}$

$=\begin{bmatrix} 8+15 & 4+15 \\ 14+24 & 12+24 \end{bmatrix}$

$=\begin{bmatrix} 23 & 19 \\ 38 & 36 \end{bmatrix}$

3. If $A=\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, B=\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}, C=\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}$, then verify that

(i) $2(A+B)=2A+2B$

Sol. $2(A+B)=2\left[\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}+\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}\right]$

$=2\begin{bmatrix} 2+5 & 3+6 \\ 4+2 & 5+1 \end{bmatrix}$

$=2\begin{bmatrix} 7 & 9 \\ 6 & 6 \end{bmatrix}$

$=\begin{bmatrix} 14 & 18 \\ 12 & 12 \end{bmatrix}$ ——— (i)

$2A=2\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}=\begin{bmatrix} 4 & 6 \\ 8 & 10 \end{bmatrix}$

$2B=2\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}=\begin{bmatrix} 10 & 12 \\ 4 & 2 \end{bmatrix}$

$2A+2B=\begin{bmatrix} 4 & 6 \\ 8 & 10 \end{bmatrix}+\begin{bmatrix} 10 & 12 \\ 4 & 2 \end{bmatrix}$

$=\begin{bmatrix} 4+10 & 6+12 \\ 8+4 & 10+2 \end{bmatrix}$

$=\begin{bmatrix} 14 & 18 \\ 12 & 12 \end{bmatrix}$ ——— (i)

Hence, verified from (i) and (ii) that

$2(A+B)=2A+2B$

(ii) (A + B) + C = A + (B + C)

Sol. $A+B=\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}+\begin{bmatrix}5 & 6\\2 & 1\end{bmatrix}$

$=\begin{bmatrix}2+5 & 3+6\\4+2 & 5+1\end{bmatrix}=\begin{bmatrix}7 & 9\\6 & 6\end{bmatrix}$

$(A+B)+C=\begin{bmatrix}7 & 9\\6 & 6\end{bmatrix}+\begin{bmatrix}3 & 2\\1 & 7\end{bmatrix}$

$=\begin{bmatrix}7+3 & 9+2\\6+1 & 6+7\end{bmatrix}$

$=\begin{bmatrix}10 & 11\\7 & 13\end{bmatrix}$ ——— (i)

$B+C=\begin{bmatrix}5 & 6\\2 & 1\end{bmatrix}+\begin{bmatrix}3 & 2\\1 & 7\end{bmatrix}$

$=\begin{bmatrix}5+3 & 6+2\\2+1 & 1+7\end{bmatrix}=\begin{bmatrix}8 & 8\\3 & 8\end{bmatrix}$

$A+(B+C)=\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}+\begin{bmatrix}3 & 8\\3 & 8\end{bmatrix}$

$=\begin{bmatrix}2+8 & 3+8\\4+3 & 5+8\end{bmatrix}$

$=\begin{bmatrix}10 & 11\\7 & 13\end{bmatrix}$ ——— (i)

Hence, verified from (i) and (ii) that (A+B)+C= A + (B + C)

(iii) (A + B) C = AC + BC

Sol. $A+B=\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}+\begin{bmatrix}5 & 6\\2 & 1\end{bmatrix}$

$=\begin{bmatrix}2+5 & 3+6\\4+2 & 5+1\end{bmatrix}=\begin{bmatrix}7 & 9\\6 & 6\end{bmatrix}$

$(A+B)C=\begin{bmatrix}7 & 9\\6 & 6\end{bmatrix}\begin{bmatrix}3 & 2\\1 & 7\end{bmatrix}$

$=\begin{bmatrix}7(3)+9(1) & 7(2)+9(7)\\6(3)+6(1) & 6(2)+6(7)\end{bmatrix}$

$=\begin{bmatrix}21+9 & 14+63\\18+6 & 12+42\end{bmatrix}$

$=\begin{bmatrix}30 & 77\\24 & 54\end{bmatrix}$ ——— (i)

$AC=\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}\begin{bmatrix}3 & 2\\1 & 7\end{bmatrix}$

$=\begin{bmatrix}2(3)+3(1) & 2(2)+3(7)\\4(3)+5(1) & 4(2)+5(7)\end{bmatrix}$

$=\begin{bmatrix}6+3 & 4+21\\12+5 & 8+35\end{bmatrix}$

$=\begin{bmatrix}9 & 25\\17 & 43\end{bmatrix}$

$BC=\begin{bmatrix}5 & 6\\2 & 1\end{bmatrix}\begin{bmatrix}3 & 2\\1 & 7\end{bmatrix}$

$=\begin{bmatrix}5(3)+6(1) & 5(2)+6(7)\\2(3)+1(1) & 2(2)+1(7)\end{bmatrix}$

$=\begin{bmatrix}15+6 & 10+42\\6+1 & 4+7\end{bmatrix}=\begin{bmatrix}21 & 52\\7 & 11\end{bmatrix}$

$AC+BC=\begin{bmatrix}9 & 25\\17 & 43\end{bmatrix}+\begin{bmatrix}21 & 52\\7 & 11\end{bmatrix}$

$=\begin{bmatrix}9+21 & 25+52\\17+7 & 43+11\end{bmatrix}$

$=\begin{bmatrix}30 & 77\\24 & 54\end{bmatrix}$ ——— (ii)

Hence, verified from (i) and (ii) that (A+B) C = AC + BC

(iv) C(A − B) = CA − CB

Sol. $A-B=\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}-\begin{bmatrix}5 & 6\\2 & 1\end{bmatrix}$

$=\begin{bmatrix}-3 & -3\\2 & 4\end{bmatrix}$

$C(A-B)=\begin{bmatrix}3 & 2\\1 & 7\end{bmatrix}\begin{bmatrix}-3 & -3\\2 & 4\end{bmatrix}$

$$= \begin{bmatrix} 3(-3)+2(2) & 3(-3)+2(4) \\ 1(-3)+7(2) & 1(-3)+7(4) \end{bmatrix}$$

$$= \begin{bmatrix} 6+8 & 9+10 \\ 2+28 & 3+28 \end{bmatrix}$$

$$= \begin{bmatrix} -9+4 & -9+8 \\ -3+14 & -3+28 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 \\ 11 & 25 \end{bmatrix} \text{ ——— (i)}$$

$$CA = \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2(3)+2(4) & 3(3)+2(5) \\ 1(2)+7(4) & 1(3)+7(5) \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 19 \\ 30 & 38 \end{bmatrix}$$

$$CB = \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3(5)+2(2) & 3(6)+2(1) \\ 1(5)+7(2) & 1(6)+7(1) \end{bmatrix}$$

$$= \begin{bmatrix} 15+4 & 18+2 \\ 5+14 & 6+7 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 20 \\ 19 & 13 \end{bmatrix}$$

$$CA - CB = \begin{bmatrix} 14 & 19 \\ 30 & 38 \end{bmatrix} - \begin{bmatrix} 19 & 20 \\ 19 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 14-19 & 19-20 \\ 30-19 & 38-13 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 \\ 11 & 25 \end{bmatrix} \text{ ——— ii}$$

Hence, verified from (i) and (ii) that $C(A-B) = CA - CB$

(v) $(AB)^{-1} = B^{-1}A^{-1}$

Sol. $$AB = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2(5)+3(2) & 2(6)+3(1) \\ 4(5)+5(2) & 4(6)+5(1) \end{bmatrix}$$

$$AB = \begin{bmatrix} 16 & 15 \\ 30 & 29 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 16 & 15 \\ 30 & 29 \end{vmatrix}$$

$$= (16 \times 29) - (30 \times 15)$$

$$= 464 - 450 = 14 \neq 0$$

$$Adj\,(AB) = \begin{bmatrix} 29 & -15 \\ -30 & 16 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|} \times Adj\,(AB)$$

$$= \frac{1}{14}\begin{bmatrix} 29 & -15 \\ -30 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{29}{14} & \frac{-15}{14} \\ \frac{-30}{14} & \frac{16}{14} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{29}{14} & \frac{-15}{14} \\ \frac{-15}{7} & \frac{8}{7} \end{bmatrix} \text{ ——— (i)}$$

$$B = \begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 5 & 6 \\ 2 & 1 \end{vmatrix}$$

$$= 5(1) - 2(6)$$

$$= 5 - 12 = -7 \neq 0$$

$$Adj\,B = \begin{bmatrix} 1 & -6 \\ -2 & 5 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times Adj B$$

$$B^{-1} = \frac{1}{-7}\begin{bmatrix} 1 & -6 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{7} & \frac{-6}{-7} \\ \frac{-2}{-7} & \frac{5}{-7} \end{bmatrix} = \begin{bmatrix} -\frac{1}{7} & \frac{6}{7} \\ \frac{2}{7} & \frac{-5}{7} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$$

$$= 2(5) - 4(3)$$

$$= 10 - 12 = -2 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj} A$$

$$= \frac{1}{-2} \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{-2} & \frac{-3}{-2} \\ \frac{-4}{-2} & \frac{-2}{-2} \end{bmatrix} = \begin{bmatrix} \frac{-5}{2} & \frac{3}{2} \\ 2 & -1 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} -\frac{1}{7} & \frac{6}{7} \\ \frac{2}{7} & \frac{-5}{7} \end{bmatrix} \begin{bmatrix} \frac{-5}{2} & \frac{3}{2} \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \left(-\frac{1}{7}\right)\left(-\frac{5}{2}\right) + \left(\frac{6}{7}\right)(2) & \left(-\frac{1}{7}\right)\left(\frac{3}{2}\right) + \frac{6}{7}(-1) \\ \frac{2}{7}\left(-\frac{5}{2}\right) + \left(\frac{-5}{7}\right)(2) & \frac{2}{7}\left(\frac{3}{2}\right) + \left(\frac{-5}{7}\right)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{14} + \frac{12}{7} & -\frac{3}{14} - \frac{6}{7} \\ -\frac{5}{7} - \frac{10}{7} & \frac{3}{7} + \frac{5}{7} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5+24}{14} & \frac{-3-12}{14} \\ \frac{-15}{7} & \frac{8}{7} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{29}{14} & \frac{-15}{14} \\ \frac{-15}{7} & \frac{8}{7} \end{bmatrix} \text{ \_\_\_\_ (ii)}$$

Hence, prooved that $(AB)^{-1} = B^{-1}A^{-1}$ from (i) and (ii).

(vi) $AA^{-1} = A^{-1}A = I$

Sol. $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$$

$$= 2(5) - 4(3)$$

$$= 10 - 12 = -2 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj} A$$

$$= \frac{1}{-2} \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{-2} & \frac{-3}{-2} \\ \frac{-4}{-2} & \frac{2}{-2} \end{bmatrix} = \begin{bmatrix} \frac{-5}{2} & \frac{3}{2} \\ 2 & -1 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} \frac{-5}{2} & \frac{3}{2} \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2\left(-\frac{5}{2}\right) + 3(2) & 2\left(\frac{3}{2}\right) + (3)(-1) \\ 4\left(\frac{-5}{2}\right) + 5(2) & 4\left(\frac{3}{2}\right) + 5(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -5+6 & 3-3 \\ -10+10 & 6-5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \text{ \_\_\_\_ (i)}$$

$$A^{-1}A=\begin{bmatrix}-\frac{5}{2} & \frac{3}{2}\\ 2 & -1\end{bmatrix}\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$$

$$=\begin{bmatrix}-\frac{5}{2}(2)+\frac{3}{2}(4) & -\frac{5}{2}(3)+\frac{3}{2}(5)\\ 2(2)+(-1)(4) & 2(3)+(-1)(5)\end{bmatrix}$$

$$=\begin{bmatrix}-5+6 & -\frac{15}{2}+\frac{15}{2}\\ 4-4 & 6-5\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I \quad \text{\_\_\_\_\_\_ (ii)}$$

Hence, Proved from (i) and (ii) that $AA^{-1}=A^{-1}A=I$

(vii) $(AB)^t = B^tA^t$

Sol. $$AB=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}\begin{bmatrix}5 & 6\\ 2 & 1\end{bmatrix}$$

$$AB=\begin{bmatrix}2(5)+3(2) & 2(6)+3(1)\\ 4(5)+5(2) & 4(6)+5(1)\end{bmatrix}$$

$$AB=\begin{bmatrix}10+6 & 12+3\\ 20+10 & 24+5\end{bmatrix}$$

$$AB=\begin{bmatrix}16 & 15\\ 30 & 29\end{bmatrix}$$

$$(AB)^t=\begin{bmatrix}16 & 30\\ 15 & 29\end{bmatrix} \quad \text{\_\_\_\_\_\_ (i)}$$

$$B=\begin{bmatrix}5 & 6\\ 2 & 1\end{bmatrix},\ A=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$$

$$B^t=\begin{bmatrix}5 & 2\\ 6 & 1\end{bmatrix},\ A^t=\begin{bmatrix}2 & 4\\ 3 & 5\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}5 & 2\\ 6 & 1\end{bmatrix}\begin{bmatrix}2 & 4\\ 3 & 5\end{bmatrix}$$

$$=\begin{bmatrix}5(2)+2(3) & 5(4)+2(5)\\ 6(2)+1(3) & 6(4)+1(5)\end{bmatrix}$$

$$=\begin{bmatrix}10+6 & 20+10\\ 12+3 & 24+5\end{bmatrix}$$

$$=\begin{bmatrix}16 & 30\\ 15 & 29\end{bmatrix} \quad \text{\_\_\_\_\_\_ (ii)}$$

Hence, proved from (i) and (ii) that $(AB)^t = B^tA^t$

(viii) (AB) C = A (BC)

Sol. $$AB=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}\begin{bmatrix}5 & 6\\ 2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}2(5)+3(2) & 2(6)+3(1)\\ 4(5)+5(2) & 4(6)+5(1)\end{bmatrix}$$

$$=\begin{bmatrix}10+6 & 12+3\\ 20+10 & 24+5\end{bmatrix}$$

$$=\begin{bmatrix}16 & 15\\ 30 & 29\end{bmatrix}$$

$$(AB)\,C=\begin{bmatrix}16 & 15\\ 30 & 29\end{bmatrix}\begin{bmatrix}3 & 2\\ 1 & 7\end{bmatrix}$$

$$=\begin{bmatrix}16(3)+15(1) & 16(2)+15(7)\\ 30(3)+29(1) & 30(2)+29(7)\end{bmatrix}$$

$$=\begin{bmatrix}48+15 & 32+105\\ 90+29 & 60+203\end{bmatrix}$$

$$=\begin{bmatrix}63 & 137\\ 119 & 263\end{bmatrix} \quad \text{\_\_\_\_\_\_ (i)}$$

$$BC=\begin{bmatrix}5 & 6\\ 2 & 1\end{bmatrix}\begin{bmatrix}3 & 2\\ 1 & 7\end{bmatrix}$$

$$=\begin{bmatrix}5(3)+6(1) & 5(2)+6(7)\\ 2(3)+(1)(1) & 2(2)+1(7)\end{bmatrix}$$

$$=\begin{bmatrix}15+6 & 10+42\\ 6+1 & 4+7\end{bmatrix}$$

$$=\begin{bmatrix}21 & 52\\ 7 & 11\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}\begin{bmatrix}21 & 52\\ 7 & 11\end{bmatrix}$$

$$=\begin{bmatrix}2(21)+3(7) & 2(52)+3(11)\\ 4(21)+5(7) & 4(52)+5(11)\end{bmatrix}$$

$$=\begin{bmatrix}42+21 & 104+33\\ 84+35 & 208+55\end{bmatrix}$$

$$= \begin{bmatrix} 63 & 137 \\ 119 & 263 \end{bmatrix} \text{\_\_\_\_\_\_\_\_(ii)}$$

Hence, Proved from (i) and (ii) that
(AB) C = A(BC)

4. If $A=\begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}, B=\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$, then find

(i) |B|

Sol. $B=\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

$$|B|=\begin{vmatrix} 5 & 3 \\ 2 & 4 \end{vmatrix}$$
$$= 5(4) - 2(3)$$
$$= 20 - 6 = 14$$

(ii) AdjB

Sol. $B=\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

$$AdjB=\begin{bmatrix} 4 & -3 \\ -2 & 5 \end{bmatrix}$$

(iii) A^{-1}

Sol. $A=\begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$

$$|A|=\begin{vmatrix} 7 & 3 \\ 2 & 1 \end{vmatrix}$$
$$=7(1) - 2(3)$$
$$= 7-6 = 1 \neq 0$$

So, A^{-1} is possible

$$Adj\,A = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj A$$

$$= \frac{1}{1}\begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$$

(iv) $A^{-1}A$

Sol. $A=\begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$

$$|A|=\begin{vmatrix} 7 & 3 \\ 2 & 1 \end{vmatrix}$$
$$= 7(1) - 2(3)$$
$$= 7 - 6 = 1 \pm 0$$

So, A^{-1} is possible

$$Adj\,A = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj A$$

$$= \frac{1}{1}\begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}\begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7(1)-3(2) & 1(3)+(-3)(1) \\ -2(7)+7(2) & -2(3)+7(1) \end{bmatrix}$$

$$= \begin{bmatrix} 7-6 & 3-3 \\ -14+14 & -6+7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

(v) $(AB)^t$

Sol. $AB = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

$$AB = \begin{bmatrix} 7(5)+3(2) & 7(3)+3(4) \\ 2(5)+1(2) & 2(3)+1(4) \end{bmatrix}$$

$$AB = \begin{bmatrix} 35+6 & 21+12 \\ 10+2 & 6+4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 41 & 33 \\ 12 & 10 \end{bmatrix}$$

$$(AB)^t = \begin{bmatrix} 41 & 12 \\ 33 & 10 \end{bmatrix}$$

(vi) $(B^t)^t$

Sol. $B=\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

$$B^t = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$$

$$(B^t)^t = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$$

5. Use matrix inversion method and Cramer's rule to solve the following pair of linear equations, if possible:

(i) $3x + 4y = 7$
$5x - y = 2$

Sol. By inversion method

Given that $3x + 4y = 7$

$5x - y = 2$

writing the equations in matrix form

$$\begin{bmatrix} 3x+4y \\ 5x-y \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

or $$\begin{bmatrix} 3 & 4 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix} \text{---(i)}$$

let $A = \begin{bmatrix} 3 & 4 \\ 5 & -1 \end{bmatrix}, \times = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$

Then (i) can be written as.

$A \times = B$

or $\times = A^{-1} B$

Now $A = \begin{bmatrix} 3 & 4 \\ 5 & -1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & 4 \\ 5 & -1 \end{vmatrix}$$

$= 3(-1) - 5(4)$

$= -3 - 20 = -23 \neq 0$

$$\text{Adj } A = \begin{bmatrix} -1 & -4 \\ -5 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$

$$= \frac{1}{-23}\begin{bmatrix} -1 & -4 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{-23}\begin{bmatrix} -1(7)+(-4)(2) \\ -5(7)+3(2) \end{bmatrix}$$

$$= \frac{1}{-23}\begin{bmatrix} -7-8 \\ -35+6 \end{bmatrix} = \frac{1}{-23}\begin{bmatrix} -15 \\ -29 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-15}{-23} \\ \frac{-29}{-23} \end{bmatrix}$$

So, $x = \frac{15}{23}, y = \frac{29}{23}$

Solution Set: $\left\{\left(\frac{15}{23}, \frac{29}{23}\right)\right\}$

By cramers rule

Given that $3x + 4y = 7$

$5x - y = 2$

Writing the equations in matrix form

$$\begin{bmatrix} 3x+4y \\ 5x-y \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

or $$\begin{bmatrix} 3 & 4 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

Here, we have

$$A = \begin{bmatrix} 3 & 4 \\ 5 & -1 \end{bmatrix}, A_x = \begin{bmatrix} 7 & 4 \\ 2 & -1 \end{bmatrix}, A_y = \begin{bmatrix} 3 & 7 \\ 5 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 4 \\ 5 & -1 \end{vmatrix}$$

$= 3(-1) - 5(4)$

$= -3 - 20 = -23 \neq 0$

$$|A_x| = \begin{vmatrix} 7 & 4 \\ 2 & -1 \end{vmatrix}$$

$= 7(-1) - (4)$

$= -7 - 8 = -15$

$$x = \frac{|A_x|}{|A|}$$

$$= \frac{-15}{-23} = \frac{15}{23}$$

$$|A_y|=\begin{vmatrix}3 & 7\\ 5 & 2\end{vmatrix}$$
$$= 3(2) - 5(7)$$
$$= 6 - 35 = -29$$
$$y = \frac{|Ay|}{|A|}$$
$$= \frac{-29}{-23} = \frac{29}{23}$$

(ii) $\boldsymbol{x - 6y = -15}$
$\boldsymbol{2x + 6y = -3}$

Sol. By inversion method

Given that $x - 6y = -15$
$2x + 6y = -3$

Writing the equations in matrix form

$$\begin{bmatrix}x-6y\\ 2x+6y\end{bmatrix}=\begin{bmatrix}-15\\ -3\end{bmatrix}$$

or $\begin{bmatrix}1 & -6\\ 2 & 6\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-15\\ -3\end{bmatrix}$ ——— (i)

Let $A = \begin{bmatrix}1 & -6\\ 2 & 6\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}-15\\ -3\end{bmatrix}$

Then (i) can be written as

$\therefore$ $A\times = B$

or $\times = A^{-1}B$

Now $A = \begin{bmatrix}1 & -6\\ 2 & 6\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & -6\\ 2 & 6\end{vmatrix}$$
$$= 1(6) - 2(-6)$$
$$= 6 + 12$$
$$= 18 \neq 0$$

$$\text{Adj } A = \begin{bmatrix}6 & 6\\ -2 & 1\end{bmatrix}$$

There for $A^{-1}=\frac{1}{|A|}\times \text{Adj}A$

$$= \frac{1}{18}\begin{bmatrix}6 & 6\\ -2 & 1\end{bmatrix}$$

Now $\times = A^{-1}B$ can be written as.

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{18}\begin{bmatrix}6 & 6\\ -2 & 1\end{bmatrix}\begin{bmatrix}-15\\ -3\end{bmatrix}$$
$$= \frac{1}{18}\begin{bmatrix}6(-15)+6(-3)\\ -2(-15)+1(-3)\end{bmatrix}$$
$$= \frac{1}{18}\begin{bmatrix}-90-18\\ +30-3\end{bmatrix}$$
$$= \frac{1}{18}\begin{bmatrix}-108\\ 27\end{bmatrix} = \begin{bmatrix}\frac{-108}{18}\\ \frac{27}{18}\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-6\\ \frac{3}{2}\end{bmatrix}$$

So, $x = -6$, $y = \frac{3}{2}$

Solution set: $\left\{\left(-6, \frac{3}{2}\right)\right\}$

By cramer's method

Given that $x - 6y = -15$
$2x + 6y = -3$

Writing the equations in matrix form

$$\begin{bmatrix}x-6y\\ 2x+6y\end{bmatrix}=\begin{bmatrix}-15\\ -3\end{bmatrix}$$

or $\begin{bmatrix}1 & -6\\ 2 & 6\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-15\\ -3\end{bmatrix}$

here, we have

$$A=\begin{bmatrix}1 & -6\\ 2 & 6\end{bmatrix}, A_x=\begin{bmatrix}-15 & -6\\ -3 & 6\end{bmatrix}, A_y=\begin{bmatrix}1 & -15\\ 2 & -3\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & -6\\ 2 & 6\end{vmatrix}$$
$$= 1(6) - 2(-6)$$
$$= 6 + 12 = 18 \neq 0$$

$$|A_x|=\begin{vmatrix}-15 & -6\\ -3 & 6\end{vmatrix}$$

$= (-15)(6) - (-3)(-6)$
$= -90 - 18 = -108$

$$x = \frac{|A_x|}{|A|}$$

$$= \frac{-108}{18} = -6$$

$$|A_y| = \begin{vmatrix} 1 & -15 \\ 2 & -3 \end{vmatrix}$$

$= 1(-3) - (2)(-15)$
$= -3 + 30 = 27$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{27}{18} = \frac{3}{2}$$

So, $x = -6, y = \frac{3}{2}$

Solution set: $\left\{\left(-6, \frac{3}{2}\right)\right\}$

(iii) $\mathbf{2x + y = 5}$
$\mathbf{x + 3y = 3}$

Sol. By inversion method

Given that $2x + y = 5$
$x + 3y = 3$

Writing the equations in matrix form

$$\begin{bmatrix} 2x+y \\ x+3y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

or $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$ ______ (i)

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

Then (i) can be written as
$A \times = B$
or $\times = A^{-1} B$

Now $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}$$

$= 2(3) - 1(1)$
$= 6 - 1 = 5 \neq 0$

$$\text{Adj } A = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj}A$$

$$= \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$$

Now $X = A^{-1}B$ can be written as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \cdot \frac{5}{3}$$

$$= \frac{1}{5}\begin{bmatrix} 3(5)+(-1)(3) \\ -1(5)+(2)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 15-3 \\ -5+6 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 12 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{12}{5} \\ \frac{1}{5} \end{bmatrix}$$

So, $x = \frac{12}{5}, y = \frac{1}{5}$

Solution Set $= \left\{\left(\frac{12}{5}, \frac{1}{5}\right)\right\}$

By cramer's rule

Given that $2x + y = 5$
$x + 3y = 3$

Writing the equations in matrix form

$$\begin{bmatrix} 2x+y \\ x+3y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

or $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

Here, we have

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}, A_x = \begin{bmatrix} 5 & 1 \\ 3 & 3 \end{bmatrix}, A_y = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & 1\\1 & 3\end{vmatrix}$$

$$= 2\times(3)-1(1)=6-1=5\neq 0$$

$$|A_x|=\begin{vmatrix}5 & 1\\3 & 3\end{vmatrix}$$

$$=5(3)-3(1)=15-3=12$$

$$x=\frac{|A_x|}{|A|}$$

$$=\frac{12}{5}$$

$$|A_y|=\begin{vmatrix}2 & 5\\1 & 3\end{vmatrix}$$

$$=2(3)-1(5)=6-5=1$$

$$y=\frac{|A_y|}{|A|}$$

$$=\frac{1}{5}$$

So, $x=\frac{12}{5}, y=\frac{1}{5}$

Solution Set $\left\{\left(\frac{12}{5},\frac{1}{5}\right)\right\}$

6. Find two numbers by using matrices such that twice the first added to the second makes 21 and twice the second added to the first makes 27.

Sol. Let the first number x and second number y.

According to the first condition

$2x+y=21$ ______(i)

According to the second condition

$x+2y=27$ ______(ii)

Writing these equations in matrix form

$$\begin{bmatrix}2x+y\\x+2y\end{bmatrix}=\begin{bmatrix}21\\27\end{bmatrix}$$

or $\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}21\\27\end{bmatrix}$ ______(i)

Let $A=\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}$ $B=\begin{bmatrix}21\\27\end{bmatrix}$

Then (i) can be written as

$$AX=B$$

or $X=A^{-1}B$

Now $A=\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$

$$|A|=\begin{vmatrix}2 & 1\\1 & 2\end{vmatrix}$$

$$=2(2)-1(1)$$

$$=4-1=3\neq 0$$

$$\text{Adj } A=\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times \text{Adj } A$$

$$=\frac{1}{3}\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$$

Now $x = A^{-1}B$ can be written as

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{3}\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}\begin{bmatrix}21\\27\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{3}\begin{bmatrix}2(21)+(-1)(27)\\-1(21)+2(27)\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{3}\begin{bmatrix}42-27\\-21+54\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{3}\begin{bmatrix}15\\33\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{15}{3}\\\frac{33}{3}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}5\\11\end{bmatrix}$$

So, $x=5, y=11$

Hence, these two numbers are 5 and 11.

7. 4 knives and 6 forks cost Rs. 136, whereas 6 knives and 5 forks cost

Rs. 164. Find the cost of a knife and a fork by using matrices.

Sol. Let the cost of a knife be x Rupees and cost of a frok is y Rupees.

According to the first condition

$4x + 6y = 136$ _____ (i)

According to the second condition

$6x + 5y = 164$ _____ (ii)

Writing these equations in matrix form

$$\begin{bmatrix} 4x+6y \\ 6x+5y \end{bmatrix} = \begin{bmatrix} 136 \\ 164 \end{bmatrix}$$

or $$\begin{bmatrix} 4 & 6 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 136 \\ 164 \end{bmatrix}$$ _____ (i)

Let $A\begin{bmatrix} 4 & 6 \\ 6 & 5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 136 \\ 164 \end{bmatrix}$

Then (i) can be written as.

$$A\times = B$$

or $$X = A^{-1}B$$

Now $$A = \begin{bmatrix} 4 & 6 \\ 6 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 6 \\ 6 & 5 \end{vmatrix}$$

$$= 4(5) - 6(6)$$

$$= 20 - 36 = -16 \neq 0$$

$$\text{Adj}A = \begin{bmatrix} 5 & -6 \\ -6 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj}A$$

$$= \frac{1}{-16}\begin{bmatrix} 5 & -6 \\ -6 & 4 \end{bmatrix}$$

Now $X = A^{-1}B$ can be written as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-16}\begin{bmatrix} 5 & -6 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} 136 \\ 164 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-16}\begin{bmatrix} 5(136)+(-6)(164) \\ -6(136)+4(164) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-16}\begin{bmatrix} 680-984 \\ -816+656 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-16}\begin{bmatrix} -304 \\ -160 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-304}{-16} \\ \frac{-160}{-16} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 10 \end{bmatrix}$$

So, x = 19, y = 10

cost of a knife = Rs. 19

cost of a frok = Rs. 10

8. A shop employs, 5 men and 3 women, pays total daily wages Rs. 3500. If the number of men is reduced to 2 and 3 extra women are taken on, the daily wages amount to Rs. 5000. Find daily wages of a man and a woman by using matrices.

Sol. Let the wages of a man Rs. x and wages of a woman is Rs. y.

According to the first condition

$5x + 3y = 3500$ _____ (i)

According to the second condetion

$2x + 6y = 5000$ _____ (i)

Writing these equations in matrix form.

$$\begin{bmatrix} 5x+3y \\ 2x+6y \end{bmatrix} = \begin{bmatrix} 3500 \\ 5000 \end{bmatrix}$$

or $$\begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3500 \\ 5000 \end{bmatrix}$$

Let $A = \begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 3500 \\ 5000 \end{bmatrix}$

Then (i) can be written as.

$$A\times = B$$

or $$\times = A^{-1}B$$

Now $$A = \begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}$$

$$|A| = (5)(6) - (2)(3) = 30 - 6 = 24 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} 6 & -3 \\ -2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$

$$= \frac{1}{24}\begin{bmatrix} 6 & -3 \\ -2 & 5 \end{bmatrix}$$

Now $X = A^{-1}B$ can be written as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{24}\begin{bmatrix} 6 & -3 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 3500 \\ 5000 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{24}\begin{bmatrix} 6(3500)+(-3)(5000) \\ -2(3500)+5(5000) \end{bmatrix}$$

$$= \frac{1}{24}\begin{bmatrix} 21000-15000 \\ -7000+25000 \end{bmatrix}$$

$$= \frac{1}{24}\begin{bmatrix} 6000 \\ 18000 \end{bmatrix} = \begin{bmatrix} \frac{6000}{24} \\ \frac{18000}{24} \end{bmatrix}$$

$$= \begin{bmatrix} 250 \\ 750 \end{bmatrix}$$

So, $x = 250$, $y = 750$

Hence, Daily wages of a man = Rs.250

Daily wages of a women = Rs. 750

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

3.1 Matrices

Encircle the correct option.

1. **In the matrix $\begin{bmatrix} 12 \\ 34 \\ 56 \end{bmatrix}$, how many rows are there?**

   (a) 6 (b) 3 (c) 2 (d) 4

2. **In the matrix A = [1 2 3], how many columns are there.**

   (a) 1 (b) 2 (c) 3 (d) 4

3. **What are the entries of the second row of matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$**

   (a) 1,3 (b) 2,4 (c) 1,2 (d) 3,4

4. **What is the entry of the first row end second column of matrix $\begin{bmatrix} 1 & 7 \\ 4 & 3 \end{bmatrix}$**

   (a) 7 (b) 1 (c) 4 (d) 3

3.1.3 Order or Size of a Matrix

5. **Which of these is a 2 × 2 Matrix?**

   (a) $\begin{bmatrix} 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 4 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

6. **Identify the order of the matrix** $B = \begin{bmatrix} 7 & 8 & 9 \end{bmatrix}$

(a) 3-by-3 (b) 3-by-1 (c) 1-by-3 (d) 1-by-1

7. **A Matrix has order 3 by 2. How many elements are in each column?**

(a) 3 (b) 2 (c) 1 (d) 4

8. **How many columns are in a 2 by 3 matrix?**

(a) 2 (b) 3 (c) 1 (d) 4

9. **What is the order of the matrix** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$**?**

(a) 3 by 2 (b) 3 by 3 (c) 2×2 (d) 6

3.1.4 Equal Matrices

10. **If** $\begin{bmatrix} x & 5 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 0 & y \end{bmatrix}$**, what is the value of x**

(a) 0 (b) 5 (c) 1 (d) 3

11. **Find the value of a if** $\begin{bmatrix} a+b \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$ **and b = 3**

(a) 2 (b) 4 (c) 7 (d) 9

12. **If** $A = \begin{bmatrix} 2x & 1 \end{bmatrix} = \begin{bmatrix} 10 & 1 \end{bmatrix}$ **then what is x?**

(a) 20 (b) 1 (c) 5 (d) 10

3.2 Types of Matrices

13. **What is the type of the Matrix** $A = [-7 \;\; 0 \;\; 4]$**?**

(a) row (b) square (c) null (d) column

14. **What is the type of the matrix** $B = \begin{bmatrix} 4 \\ 9 \\ 9 \end{bmatrix}$**?**

(a) Identity (b) Square (c) column (d) row

15. **Which is a null matrix?**

(a) $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 7 \\ 0 \end{bmatrix}$

16. **What is the type of the matrix** $B = \begin{bmatrix} 4 & -7 \\ 2 & 3 \end{bmatrix}$

(a) square (b) row (c) null (d) column

17. **Which is the order of a square matrix?**

(a) 1×3 (b) 3×2 (c) 3×3 (d) 3×1

18. **What is the type of the matrix** $C = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

(a) row (b) square (c) null (d) rectangular

19. **Which is the order of a rectangular matrix?**

(a) 1×3 (b) 3×3 (c) 2×2 (d) 1×1

20. **Which is not a diagonal Matrix?**

(a) $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 4 \\ 0 & 2 \end{bmatrix}$

21. **What is the type of the matrix $A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$?**

(a) row (b) scalar (c) null (d) rectangular

22. **Identify the unit matrix.**

(a) $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

23. **Transpose of the matrix $A = \begin{bmatrix} -7 \\ 3 \\ 2 \end{bmatrix}$ is:**

(a) $\begin{bmatrix} -7 & 3 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -7 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 2 \\ -7 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 3 & -7 \end{bmatrix}$

24. **A square matrix A is symmetric if:**

(a) $A = A^{-1}$ (b) $A = -A$ (c) $A^t = A$ (d) $A^t = -A$

25. **Which is the example of skew symmetric matrix?**

(a) $\begin{bmatrix} 0 & -7 \\ 7 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 5 \\ 5 & 7 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

26. **A Matrix where every element is Zero is called.**

(a) Identity (b) skew (c) row (d) null

3.3 Algebraic operations on Matrices

27. **If matrix A is of size 3 × 2 and matrix B is of size 3 × 2, what is the size of resulting matrix A + B?**

(a) 3 × 2 (b) 6 × 4 (c) 2 × 3 (d) 3 × 3

28. **If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$, the A + B will be**

(a) $\begin{bmatrix} 8 & 2 \\ 4 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 4 \\ 6 & 4 \end{bmatrix}$

29. **If $2A = \begin{bmatrix} 10 & 4 \\ 6 & 8 \end{bmatrix}$, then matrix A is?**

(a) $\begin{bmatrix} 8 & 2 \\ 4 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 20 & 8 \\ 12 & 16 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 4 \\ 6 & 4 \end{bmatrix}$

30. **If $A = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 7 \\ 2 & 0 \end{bmatrix}$, then A − B is:**

(a) $\begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$

31. **If $A = \begin{bmatrix} 2 & 3 & -7 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 & -3 \end{bmatrix}$, then $B - A$ is.**

(a) $\begin{bmatrix} 2 & 1 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 2 & 10 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 2 & 4 \end{bmatrix}$

32. **Commutative law of Addition of Matrices is.**

(a) $A + B = A - B$ (b) $A + B = B + A$ (c) $A + B = AB$ (d) $A + B = 0$

33. **The additive identity of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ is:**

(a) $\begin{bmatrix} -1 & -2 \\ -3 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

34. **The additive inverse of matrix $A = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$ is:**

(a) $\begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ +3 & 2 \end{bmatrix}$

35. **If $A = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ the $3A - A$ will be:**

(a) $\begin{bmatrix} 4 \\ 10 \end{bmatrix}$ (b) $\begin{bmatrix} 6 \\ 15 \end{bmatrix}$ (c) $\begin{bmatrix} 4 \\ 10 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$

36. **Which are conformable for addition.**

(a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

37. **For matrices A and B which is always true?**

(a) $A - B = B - A$ (b) $(A - B)^t = A^t - B^t$ (c) $(A-B)^t = A^t B^t$ (d) $AB = I$

38. **If $Q = \begin{bmatrix} 2 & -3 \\ 0 & -4 \end{bmatrix}$ then $-4Q$ is:**

(a) $\begin{bmatrix} -8 & 12 \\ 0 & 16 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & -12 \\ 0 & 16 \end{bmatrix}$ (c) $\begin{bmatrix} 8 & 12 \\ 16 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 9 \\ 4 & -12 \end{bmatrix}$

3.3.1 Multiplication of Matrices

39. **Which pair of matrices is conformable for multiplication.**

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

40. **What is product of $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \end{bmatrix}$?**

(a) $\begin{bmatrix} 0 \end{bmatrix}$ (b) $\begin{bmatrix} 7 \end{bmatrix}$ (c) $\begin{bmatrix} 5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 0 \end{bmatrix}$

41. **If $A = \begin{bmatrix} 5 & 7 \\ 0 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then AB is?**

(a) $\begin{bmatrix} 7 & 5 \\ 4 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -4 \\ 5 & 7 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 7 \\ 0 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -7 \\ 0 & 4 \end{bmatrix}$

42. **If $A = [-3 \quad 7]$ and B $\begin{bmatrix} -4 \\ -11 \end{bmatrix}$ the order of AB will be:**

(a) 1 by 2 (b) 2 by 2 (c) 1 by 1 (d) 2 by 1

43. **The associative law of multiplication of matrices is**

(a) AB = BA (b) AB = I (c) AB+C=AB+AC (d) (AB)C = A (BC)

44. **Multiplicative identity of a matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ is**

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & -3 \\ -4 & -5 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$

3.5 Multiplicative Inverse of a matrix

45. **If $A = \begin{bmatrix} 5 & 3 \\ 3 & 4 \end{bmatrix}$, then $|A|$ will be.**

(a) 29 (b) 0 (c) 11 (d) 16

46. **What is the value of $\begin{vmatrix} 0 & -3 \\ -2 & 4 \end{vmatrix}$?**

(a) 4 (b) 10 (c) 6 (d) –6

47. **If $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $|B|$ will be.**

(a) ab – ed (b) ad – bc (c) cb – all (d) ac – bd

3.5.2 Singular and Non - Singular Matrices

48. **Which of the following is a singular matrix?**

(a) $\begin{bmatrix} 4 & 10 \\ 2 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 3 \\ 10 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

49. **Which of the following is a non - singular matrix?**

(a) $\begin{bmatrix} 3 & 2 \\ 9 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & 12 \\ 2 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 7 \\ 4 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 20 \\ 2 & 8 \end{bmatrix}$

50. **If $\begin{vmatrix} 9 & 6 \\ 3 & k \end{vmatrix} = 0$, what is the value of k?**

(a) 4 (b) 2 (c) 5 (d) 0

51. **Which of these is not a singular matrix?**

(a) $\begin{bmatrix} 4 & 7 \\ 2 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} 21 & 9 \\ 7 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 12 & 8 \\ 6 & 4 \end{bmatrix}$

3.5.4 Multiplicative Inverse of a Non-singular matrix

52. For which multiplicative inversse is not possible.

(a) $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ (b) $\begin{bmatrix}5 & 6\\7 & 8\end{bmatrix}$ (c) $\begin{bmatrix}7 & 14\\2 & 4\end{bmatrix}$ (d) $\begin{bmatrix}a & d\\c & d\end{bmatrix}$

53. The Adjoint of matrix D = $\begin{bmatrix}a & b\\c & d\end{bmatrix}$ is:

(a) $\begin{bmatrix}d & -b\\-c & a\end{bmatrix}$ (b) $\begin{bmatrix}a & c\\b & d\end{bmatrix}$ (c) $\begin{bmatrix}-a & c\\b & -d\end{bmatrix}$ (d) $\begin{bmatrix}a & d\\b & c\end{bmatrix}$

54. For which multiplicative inverse is not possible.

(a) $\begin{bmatrix}1 & 3\\3 & 4\end{bmatrix}$ (b) $\begin{bmatrix}5 & 6\\7 & 9\end{bmatrix}$ (c) $\begin{bmatrix}7 & 14\\3 & 6\end{bmatrix}$ (d) $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$

55. $(AB)^{-1}$ = ________

(a) $(BA)^{-1}$ (b) $B^{-1}A^{-1}$ (c) AB (d) $(A+B)^{m}$

56. AA^{-1} = ________ **if A = $\begin{bmatrix}4 & 9\\2 & 5\end{bmatrix}$**

(a) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}-4 & -9\\-2 & -5\end{bmatrix}$ (c) $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}5 & -9\\-2 & 4\end{bmatrix}$

57. There is no invers of B if.

(a) $|B| = -1$ (b) $|B| = 4$ (c) $|B| = 0$ (d) $|B| = 10$

ANSWERS:

1. 3	**2.** 3	**3.** 3,4	**4.** 7	**5.** $\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}$
6. 1 by 3	**7.** 3	**8.** 3	**9.** 3 by 2	**10.** 3
11. 4	**12.** 5	**13.** row	**14.** column	**15.** $\begin{bmatrix}0\\0\end{bmatrix}$
16. Square	**17.** 3×3	**18.** rectangular	**19.** 1×3	**20.** $\begin{bmatrix}-3 & 4\\0 & 2\end{bmatrix}$
21. scalar	**22.** $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	**23.** $[-7\ 3\ 2]$	**24.** $A^t = A$	**25.** $\begin{bmatrix}0 & -7\\7 & 0\end{bmatrix}$
26. null	**27.** 3×2	**28.** $\begin{bmatrix}6 & 8\\10 & 12\end{bmatrix}$	**29.** $\begin{bmatrix}5 & 2\\3 & 4\end{bmatrix}$	**30.** $\begin{bmatrix}3 & -2\\1 & 2\end{bmatrix}$
31. $[2\ 2\ 4]$	**32.** $A + B = B + A$		**33.** $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$	**34.** $\begin{bmatrix}-1 & 0\\3 & -2\end{bmatrix}$
35. $\begin{bmatrix}4\\10\end{bmatrix}$	**36.** $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}, \begin{bmatrix}2 & 1\\0 & 3\end{bmatrix}$		**37.** $(A - B)^t = A^t - B^t$	

38. $\begin{bmatrix} -8 & 12 \\ 0 & 16 \end{bmatrix}$ 39. $\begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ 40. $[5]$ 41. $\begin{bmatrix} 5 & 7 \\ 0 & -4 \end{bmatrix}$ 42. 1 by 1

43. (AB) C = A (BC) 44. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 45. 11 46. –6

47. ad – bc 48. $\begin{bmatrix} 4 & 10 \\ 2 & 5 \end{bmatrix}$ 49. $\begin{bmatrix} 3 & 7 \\ 4 & 2 \end{bmatrix}$ 50. 2 51. $\begin{bmatrix} 4 & 7 \\ 2 & 9 \end{bmatrix}$

52. $\begin{bmatrix} 7 & 14 \\ 2 & 4 \end{bmatrix}$ 53. $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ 54. $\begin{bmatrix} 7 & 14 \\ 3 & 6 \end{bmatrix}$ 55. $B^{-1} A^{-1}$ 56. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

57. $|B| = 0$

Additional Short Question

Short Answer Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

3.1.1 Rows and Columns of a Matrix

1. **Define a matrix.**

Ans. A matrix is a rectangular array in shape, whose elements (entries) are written within square brackets in a specific order.

2. **Define a row in the content of a matrix.**

Ans.A row consists of entries written in a horizontal line.

3. **Define a column in context of a matrix.**

Ans.A column consists of entries in a vertical line.

4. **How many rows and columns are there in this matrix B = $\begin{bmatrix} 7 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix}$**

Ans. Matrix B has two rows and three columns. Order or size of a Matrix.

5. **Write the order of the matrix A = $[1 \quad 3 \quad 7]$**

Ans. The order of the matrix A = 1–by–3

6. **Write the order of the matrix B = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 0 & 7 \end{bmatrix}$**

Ans. The order of the matrix B = 3-by-2

7. **Are the orders of the matrices A = $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and B $[1 \quad 2]$ the same?**

Ans. No, the orders of both matrices are not same. The order of A is 2 by 1 and order of B is 1 by 2.

8. **Define equal matrices.**

Ans. Two matrices A and B are said to be equal if and only if they are of same order and their corresponding elements are same.

9. **If $\begin{bmatrix} 5x-1 & 4 \\ 11 & y \end{bmatrix} = \begin{bmatrix} 14 & 4 \\ 11 & 9 \end{bmatrix}$, then find the values of x and y**

Sol. Corresponding entries of the equal matrices are same. So,

$5x - 1 = 14 \qquad y = 9$

$5x = 14 + 1$

$5x = 15$

$x = \frac{15}{5}$

$x = 3$

So, $x = 3$, $y = 9$

10. **Are matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and Matrix $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ are same or not? Give reason.**

Ans. No, matrix A and Matrix B are not equal matrices. Both have same order but corresponding entries of the second row are not same.

11. **Find a,b if $\begin{bmatrix} 7 & -4 \\ b & 0 \end{bmatrix} \begin{bmatrix} a-1 & -4 \\ 9 & 0 \end{bmatrix}$**

Sol. corresponding entries of equal matrices are same.

$a - 1 = 7 \qquad , \; b = 9$

$a = 7 + 1$

$a = 8$

So, $a = 8$, $b = 9$

12. **Define a "Row matrix".**

Ans. A row matrix is a matrix that has only one row. For example $A = [1 \; 0 \; -3]$

13. **Define a "Column matrix".**

Ans. A column matrix is a matrix that has only one column. For example $B = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$

14. **What is a "square matrix".**

Ans. A matrix in which number of rows is equal to the number of columns is called a square matrix. For example, $A = \begin{bmatrix} -3 & 2 \\ 0 & 7 \end{bmatrix}$

15. **Define "Rectangular matrix".**

Ans. If the number of rows and number of columns in a matrix are not equal, then it is called a rectangular matrix.

For example: $A = [3 \; 4 \; 5]$

16. **Define "Diagonal matrix".**

Ans. A square matrix is called a diagonal matrix if at least any one of elements of its diagonal is non-zero and non-diagonal elements are zero.

For example $A = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}$

17. Define "Scalar matrix".

Ans. A diagonal matrix is called a scalar matrix if all the entries in the diagonal are the same.

18. Define "Identity matrix".

Ans. A scalar matrix is called an identity or unit matrix if all main diagonal entries are 1 and the rest are zero.

For example $Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

19. Find the transpose of matrix $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

20. Find the transpose of matrix $C=[-1 \ \ 0 \ \ -19]$

sol. $C = [-1 \ \ 0 \ \ -19]$

$C^t = \begin{bmatrix} -1 \\ 0 \\ -19 \end{bmatrix}$

21. If $C=\begin{bmatrix} 1 & 2 \\ 11 & 7 \end{bmatrix}$, then verify that $(C^t)^t=C$

Ans. $C = \begin{bmatrix} 1 & 2 \\ 11 & 7 \end{bmatrix}$

$C^t = \begin{bmatrix} 1 & 11 \\ 2 & 7 \end{bmatrix}$

$(C^t)^t = \begin{bmatrix} 1 & 2 \\ 11 & 7 \end{bmatrix} = C$

Hence, verified that $(C^t)^t = C$

22. Find the negative of matrix $B=\begin{bmatrix} -1 & 0 \\ 4 & 7 \end{bmatrix}$

Sol. $B=\begin{bmatrix} -1 & 0 \\ 4 & 7 \end{bmatrix}$

$-B = \begin{bmatrix} 1 & 0 \\ -4 & -7 \end{bmatrix}$

23. If $A=\begin{bmatrix} 9 & 8 \\ 8 & 4 \end{bmatrix}$, then show it is a symmetric matrix.

Sol. $A = \begin{bmatrix} 9 & 8 \\ 8 & 4 \end{bmatrix}$

$A^t = \begin{bmatrix} 9 & 8 \\ 8 & 4 \end{bmatrix} = A$

Hence, verified that A is a symmetric matrix because $A^t = A$

24. Show that $M=\begin{bmatrix} 0 & 13 \\ -13 & 0 \end{bmatrix}$ is a skew-symmetric matrix.

Sol. $M = \begin{bmatrix} 0 & 13 \\ -13 & 0 \end{bmatrix}$

$M^t = \begin{bmatrix} 0 & -13 \\ 13 & 0 \end{bmatrix} = -M$

Hence, verified that M is a skew symmetric matrix.

3.3 Algebraic Operations on Matrices

25. If $Z = [-3 \ \ 0 \ \ 15]$, then find $-7Z$

Ans. $Z = [-3 \ \ 0 \ \ 15]$

$-7Z = -7[-3 \ \ 0 \ \ 15]$

$-7Z = [21 \ \ 0 \ \ -105]$

26. If $Y=\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ and $Z = \begin{bmatrix} 7 & 6 \\ 4 & -3 \end{bmatrix}$, then find $Y-5Z$

Ans. $Y-5Z = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} - 5\begin{bmatrix} 7 & 6 \\ 4 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 35 & 30 \\ 20 & -15 \end{bmatrix}$

$= \begin{bmatrix} 1-35 & 0-30 \\ 1-20 & 2+15 \end{bmatrix}$

$= \begin{bmatrix} 1-35 & 0-30 \\ 1-20 & 2+15 \end{bmatrix}$

$$= \begin{bmatrix} -34 & -30 \\ -19 & 17 \end{bmatrix}$$

27. If $Z = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$, then find $Z - 3Z$.

Sol. $Z = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$

$$Z - 3Z = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} - 3\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ -9 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2-6 & 3-9 \\ -3+9 & 2-6 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -6 \\ 6 & -4 \end{bmatrix}$$

28. If $A = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$, then find $A^t + B^t$

Ans. $A = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$

$A^t = [1 \quad 7]$ $B^t = [0 \quad 4]$

$A^t + B^t = [1 \quad 7] + [0 \quad 4]$

$= [1 + 0 \quad 7 + 4]$

$= [1 \quad 11]$

Multiplication of Matrices

29. If $A = \begin{bmatrix} 3 & 0 \\ -3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$, then find AB

Sol. $AB = \begin{bmatrix} 3 & 0 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} -5 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 3(-5)+0(1) \\ -3(-5)+5(1) \end{bmatrix}$$

$$= \begin{bmatrix} -15+0 \\ 15+5 \end{bmatrix} = \begin{bmatrix} -15 \\ 20 \end{bmatrix}$$

30. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find BA

Sol. $BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 1(1)+0(3) & 1(2)+0(4) \\ 0(1)+1(3) & 0(2)+1(4) \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 2+0 \\ 0+3 & 0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

31. If $B = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 7 \\ 4 & 2 \end{bmatrix}$, then Find $(BC)^t$

Sol. $BC = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 7 \\ 4 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 1(1)+0(4) & 1(7)+0(2) \\ 2(1)+3(4) & 2(7)+3(2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 7+0 \\ 2+12 & 14+6 \end{bmatrix}$$

$$BC = \begin{bmatrix} 1 & 7 \\ 14 & 20 \end{bmatrix}$$

$$(BC)^t = \begin{bmatrix} 1 & 14 \\ 7 & 20 \end{bmatrix}$$

Multiplication Inverse of a Matrix

Determinant of a Matrix of order.

32. Find the value of $\begin{vmatrix} -3 & 2 \\ -7 & 4 \end{vmatrix}$.

Ans. $\begin{vmatrix} -3 & 2 \\ -7 & 4 \end{vmatrix}$

$= -3(4) - (-7)(2)$

$= -12 + 14 = 2$

33. Find the determinant of

$$A = \begin{bmatrix} -7 & 35 \\ -1 & 5 \end{bmatrix}$$

Ans. $A = \begin{bmatrix} -7 & 35 \\ -1 & 5 \end{bmatrix}$

$$|A| = \begin{bmatrix} -7 & 35 \\ -1 & 5 \end{bmatrix}$$

$= -7(5) - (-1)(35)$

$= -35 + 35$

$= 0$

34. **Find the determinat of** $C=\begin{bmatrix}-7 & -5\\ -3 & 0\end{bmatrix}$

Sol. $C=\begin{bmatrix}-7 & -5\\ -3 & 0\end{bmatrix}$

$|C|=\begin{vmatrix}-7 & -5\\ -3 & 0\end{vmatrix}$

$=-7(0)-(-3)(-5)$

$=0-15$

$=-15$

Singular and Non-Singular Matrices

35. **Find whether the matrix** $A=\begin{bmatrix}4 & 7\\ 12 & 21\end{bmatrix}$ **is singular or non singular**

Sol. $A=\begin{bmatrix}4 & 7\\ 12 & 21\end{bmatrix}$

$|A|=\begin{bmatrix}4 & 7\\ 12 & 21\end{bmatrix}$

$=4(21)-12(7)$

$=84-84=0$

So, A is a singular matrix

36. **Show that** $A=\begin{bmatrix}19 & 3\\ 17 & 2\end{bmatrix}$ **is a non-singular matrix.**

Sol. $A=\begin{bmatrix}19 & 3\\ 17 & 2\end{bmatrix}$

$|A|=\begin{vmatrix}19 & 3\\ 17 & 2\end{vmatrix}$

$=19(2)-17(3)$

$=38-51$

$=-13\neq 0$

So, A is a non - singular because $|A|\neq 0$

37. **Find the value of K is** $B=\begin{bmatrix}K & 6\\ 7 & 14\end{bmatrix}$ **is a singular matrix.**

Sol. $B=\begin{bmatrix}K & 6\\ 7 & 14\end{bmatrix}$

$|B|=\begin{vmatrix}K & 6\\ 7 & 14\end{vmatrix}=0$

$K(14)-7(6)=0$

$14K-42=0$

$14K=42$

$K=\frac{42}{14}$

$K=3$

Adjoint of a Matrix

38. **Find the adjoint of matrix** $A=\begin{bmatrix}7 & 4\\ 0 & 3\end{bmatrix}$

Sol. $A=\begin{bmatrix}7 & 4\\ 0 & 3\end{bmatrix}$

$\text{Adj } A=\begin{bmatrix}3 & -4\\ 0 & 7\end{bmatrix}$

39. **Find the adjoint of** $M=\begin{bmatrix}s & t\\ u & v\end{bmatrix}$

Sol. $M=\begin{bmatrix}s & t\\ u & v\end{bmatrix}$

$\text{Adj } M=\begin{bmatrix}v & -t\\ -u & s\end{bmatrix}$

Multiplicative inverse of a Non-singular matrix

40. **When does the inverse of a matrix not exist?**

Ans. The inverse of a singular matrix does not exist.

41. **Find** A^{-1} **of** $A=\begin{bmatrix}2 & 1\\ 3 & 0\end{bmatrix}$

Sol. $A=\begin{bmatrix}2 & 1\\ 3 & 0\end{bmatrix}$

$|A|=\begin{bmatrix}2 & 1\\ 3 & 0\end{bmatrix}$

$=2(0)-3(1)$

$=0-3$

$=-3\neq 0$

A is non singular, so, inverse is possible

$$\text{Adj } A = \begin{bmatrix} 0 & -1 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj} A$$

$$= \frac{1}{-3}\begin{bmatrix} 0 & -1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0}{-3} & \frac{-1}{-3} \\ \frac{-3}{-3} & \frac{2}{-3} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{3} \\ 1 & -\frac{2}{3} \end{bmatrix}$$

42. Show that $\begin{bmatrix} 8 & 5 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} \frac{3}{4} & -\frac{5}{4} \\ -1 & 2 \end{bmatrix}$ **are multiplicative inverse of each other.**

Ans. $\begin{bmatrix} 8 & 5 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} \frac{3}{4} & \frac{-5}{4} \\ -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 8\left(\frac{3}{4}\right)+5(-1) & 8\left(-\frac{5}{4}\right)+5(2) \\ 4\left(\frac{3}{4}\right)+3(-1) & 4\left(-\frac{5}{4}\right)+3(2) \end{bmatrix}$$

$$= \begin{bmatrix} 6-5 & -10+10 \\ 3-3 & -5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

So, $\begin{bmatrix} 8 & 5 \\ 4 & 3 \end{bmatrix}$ and $\begin{bmatrix} \frac{3}{4} & \frac{-5}{4} \\ -1 & 2 \end{bmatrix}$ are multiplicatiove inverse of each other because their product a unit matrix.

43. Find the inverse of B = $\begin{bmatrix} -3 & -7 \\ 9 & 21 \end{bmatrix}$

Sol. $B = \begin{bmatrix} -3 & -7 \\ 9 & 21 \end{bmatrix}$

$$|B| = \begin{vmatrix} -3 & -7 \\ 9 & 21 \end{vmatrix}$$

$$= -3(21) - 9(-7)$$

$$= -63 + 63$$

$$= 0$$

So, inverse of matrix B is not possible because it is a singular matrix.

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