UNIT 02 Quadratic Equations and Inequalities

Student's Learning Outcomes

After completing this unit, the students will be able to:

- Solve quadratic equations of the form $ax^2 + bx + c = 0$, $a\ (\neq 0)$, b, c are real numbers by factorization, quadratic formula, completing square and graphs.
- Find the intersecting point graphically when intersection occurs between
  - a linear function and coordinate axes.
  - two linear functions a linear and a quadratic function.
- Sketch and interpret the graphs of the quadratic function $y = ax^2+bx+c$, $a(\neq 0)$, b and c are real numbers.
- Establish relationship between roots and coefficients of quadratic equations.
- Form a quadratic equation when roots are given.
- Find discriminant of a given quadratic equation.
- Identify the nature of roots of a quadratic equation through discriminant.
- Solve quadratic inequalities in one unknown.
- Solve problems of "changing the subject of formula".
- Solve fractional equations that can be reduced to quadratic equations.
- Apply the concept of quadratic equations and quadratic inequalities, to real world problems (such as in physics, engineering and finance, i.e. calculating max and min heights in projectile motion, determining the maximum price on a company's budget, stability of population, growth of business, the relationship between hours worked and amount earned etc.)

2.1 Quadratic Equation

A quadratic equation is a second degree polynomial equation in one variable.
For example, the following equations are quadratic equations:

(i) $2x^2 + 3x + 7 = 0$ **(ii)** $x^2 - 5x - 4 = 0$ **(iii)** $3y^2 - 6y + 3 = 0$

> **Note**
> If a, b, c ∈ R, a ≠ 0 and b ≠ 0, then equation $ax^2 + bx + c = 0$ is called the standard form of the quadratic equation in x.
> If a = 0, the equation $ax^2 + bx + c = 0$ becomes $bx + c = 0$ which is a linear equation.
> If b = 0, the equation $ax^2 + bx + c = 0$ becomes $ax^2 + c = 0$ which is pure quadratic equation.

Solution of Quadratic Equation of the Form

$ax^2 + bx + c = 0$, **where** $a, b, c \in R$ **and** $a \neq 0$

(a) Factorization Method:

The solution of a quadratic equation by factorization is based on the following result: If a, b ∈ R and ab = 0, then a = 0 or b = 0. This is called zero product property.

Example 1: Solve by factorization method: $x^2 + \frac{7}{3}x = 2$

Sol. $x^2 + \frac{7}{3}x = 2$

$$3x^2 + 7x = 6$$ [multiplying both sides by 3]
$$3x^2 + 7x - 6 = 0$$
$$3x^2 + 9x - 2x - 6 = 0$$
$$3x(x+3) - 2(x+3) = 0$$
$$(x+3)(3x-2) = 0$$

> **Do you know?**
> Every quadratic equation has at most two roots.

Now if either of the factor $x + 3$ or $3x - 2$ is zero, then their product is zero. So,

$$x + 3 = 0 \text{ or } 3x - 2 = 0$$
$$x = -3 \text{ or } x = \frac{2}{3}$$

Hence, solution set is $\left\{-3, \frac{2}{3}\right\}$

> **Note**
> An equation in which variable appears in the denominator of one or more terms is called a fractional equation.
> (i) $\frac{2x}{5} - \frac{6}{x} = \frac{3}{7}, x \neq 0$
> (ii) $\frac{x+3}{x-1} = \frac{2x-1}{x+5}, x \neq 1, -5$ are fractional equations.

Example 2: Solve by factorization method: $\frac{2x-3}{2} = \frac{4x-6}{x}, x \neq 0$

Sol. $\frac{2x-3}{2} = \frac{4x-6}{x}$

> Multiplying both sides by LCM of the denominators.
> i.e., 2x to eliminate denominator

$$\frac{2x(2x-3)}{2} = \frac{(4x-6)(2x)}{x}$$
$$2x^2 - 3x = 8x - 12$$

> Transposing all terms to left hand side of '=' sign.

$$2x^2 - 3x - 8x + 12 = 0$$
$$2x^2 - 11x + 12 = 0$$
$$2x^2 - 8x - 3x + 12 = 0$$
$$2x(x-4) - 3(x-4) = 0$$
$$(2x-3)(x-4) = 0$$
$$2x - 3 = 0 \text{ or } x - 4 = 0$$
$$x = \frac{3}{2} \text{ or } x = 4$$

Hence, solution set is $\left\{\frac{3}{2}, 4\right\}$

(b) Solution of a Quadratic Equation by Completing Square Method

Example 3: Solve by completing square method: $3x^2 - 32 = -10x$

Sol: $3x^2 - 32 = -10x$

Step I: Simplifying equation so that the terms in x^2 and x are on one side of the equation and the term without x on the other side.

$$3x^2 + 10x = 32 \text{ [Transposing]}$$

Step II: Make the coefficient of x^2 unity and positive by dividing throughout by the coefficient of x^2.

$$x^2+\frac{10}{3}x=\frac{32}{3}$$

Step III: Add to each side of the equation the square of half the coefficient of x.

$$x^2+\frac{10}{3}x+\left(\frac{5}{3}\right)^2=\frac{32}{3}+\left(\frac{5}{3}\right)^2$$

$$\left(x+\frac{5}{3}\right)^2=\frac{32}{3}+\frac{25}{9}=\frac{96+25}{9}=\frac{121}{9}$$

$$\left(x+\frac{5}{3}\right)^2=\left(\frac{11}{3}\right)^2$$

Step IV: Taking square root of both the side.

$$\therefore \quad \pm\left(x+\frac{5}{3}\right)=\frac{11}{3}$$

$$x+\frac{5}{3}=\pm\frac{11}{3}$$

$$x+\frac{5}{3}=\frac{11}{3} \quad \text{or} \quad x+\frac{5}{3}=-\frac{11}{3}$$

$$x=\frac{11}{3}-\frac{5}{3} \quad \text{or} \quad x=-\frac{11}{3}-\frac{5}{3}$$

$$x=\frac{11-5}{3} \quad \text{or} \quad x=\frac{-11-5}{3}$$

$$x=\frac{6}{3} \quad \text{or} \quad x=\frac{-16}{3}=-5\frac{1}{3}$$

$$x=2 \quad \text{or} \quad x=-5\frac{1}{3}$$

Hence, solution set is $\left\{2,-5\frac{1}{3}\right\}$

Skilled Practice!

Solve by completing square method.

(i) $5x^2-18=2x$ **(ii)** $2x^2-7x=-20$

Sol. (i) $5x^2-18=2x$

Dividing by 5 (the coefficient of x^2) on both sides of the equation.

$$\frac{5x^2}{5}-\frac{18}{5}=\frac{2}{5}x$$

$$x^2-\frac{18}{5}=\frac{2}{5}x$$

$$x^2-\frac{2}{5}x=\frac{18}{5}$$

We add square of half of the coefficient of x. *i.e*, $\left(\frac{1}{5}\right)^2$ on both sides of the equation.

$$x^2-\frac{2}{5}x+\left(\frac{1}{5}\right)^2 = \frac{18}{5}+\left(\frac{1}{5}\right)^2$$

$$\left(x-\frac{1}{5}\right)^2 = \frac{18}{5}+\frac{1}{25}$$

$$\left(x-\frac{1}{5}\right)^2 = \frac{90+1}{25}=\frac{91}{25}$$

$$\sqrt{\left(x-\frac{1}{5}\right)^2} = \sqrt{\frac{91}{25}}$$

$$\left(x-\frac{1}{5}\right) = \pm\frac{\sqrt{91}}{5}$$

$$x = \frac{1}{5}\pm\frac{\sqrt{91}}{5}=\frac{1\pm\sqrt{91}}{5}$$

$$S.S = \left\{\frac{1\pm\sqrt{91}}{5}\right\}$$

(ii) $2x^2-7x=-20$

Dividing by a (the coefficient of x^2), on both sides of the equation.

$$\frac{2}{2}x^2-\frac{7}{2}x = \frac{-20}{2}$$

$$x^2-\frac{7}{2}x = -10$$

We add square of half of the coefficient of x. *i.e.*, $\left(\frac{7}{4}\right)^2$ on both sides of the equation.

$$x^2-\frac{7}{2}x+\left(\frac{7}{4}\right)^2 = -10+\left(\frac{7}{4}\right)^2$$

$$\left(x-\frac{7}{4}\right)^2 = -10+\frac{49}{16}$$

$$\left(x-\frac{7}{4}\right)^2 = \frac{-160+49}{16}=\frac{-111}{16}$$

$$\sqrt{\left(x-\frac{7}{4}\right)^2} = \sqrt{\frac{-111}{16}}$$

$$x-\frac{7}{4} = \pm\frac{\sqrt{-111}}{4}$$

$$x = +\frac{7}{4} \pm \frac{\sqrt{-111}}{4} = \frac{7 \pm \sqrt{-111}}{4}$$

$$S.S = \left\{ \frac{7 \pm \sqrt{-111}}{4} \right\}$$

Hence, solution set is $\left\{ 2, -5\frac{1}{3} \right\}$

Challenge!

The square of every real quantity is non-negative and therefore greater than zero. Thus, $(a - b)^2$ is non-negative i.e., $a^2 - 2ab + b^2 \geq 0$. Do you agree:

Sol. $\frac{x + y}{2} > \sqrt{xy}$

$$\left(\sqrt{x} - \sqrt{y}\right)^2 \geq (0)^2 \quad \text{(assume)}$$

$$\left(\sqrt{x}\right)^2 - 2\sqrt{x}\sqrt{y} + \left(\sqrt{y}\right)^2 \geq 0$$

$$x + y \geq 2\sqrt{xy}$$

Dividing both sides by 2, we get

$$\frac{x + y}{2} \geq \frac{2\sqrt{xy}}{2}$$

$$\frac{x + y}{2} \geq \sqrt{xy}$$

yes, we agree.

(c) Solution of a Quadratic Equation by Quadratic Formula

Derivation of Quadratic Formula by Completing Square Method

After suitable reduction and transposition every quadratic equation can be written in the form: $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

Now, $ax^2 + bx + c = 0$ We can rewrite the equation as:

Step I: $ax^2 + bx = -c$ (Transposing)

Step II: Dividing by a (the co-efficient of x^2), on both sides of the equation.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Step III: We add square of half of the co-efficient of x i.e; $\left(\frac{b}{2a}\right)^2$ on both side of the equation.

> **Remember!**
>
> $ax^2 + bx + c = 0$, can equivalently be expressed as
>
> $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ since, $a \neq 0$

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

Step IV: Taking square root of both the sides.

$$\pm\left(x+\frac{b}{2a}\right)=\frac{\sqrt{b^2-4ac}}{2a}$$

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

or $$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a} \quad \text{(Transposing)}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

This is the quadratic formula.

Remember!

Moving a term from one side of an equation to the other by changing its sign is called transposition

Example 4: Solve by using quadratic formula: $3x^2 + 7x - 6 = 0$

Sol. $3x^2 + 7x - 6 = 0$ (given)

$ax^2 + bx + c = 0$ (standard form)

Comparing the given equation and the standard form.

$$a = 3, b = 7, c = -6$$

Substituting the values of a, b, c in the quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-7\pm\sqrt{(7)^2-4(3)(-6)}}{2(3)}$$

$$=\frac{-7\pm\sqrt{49+72}}{6}=\frac{-7\pm\sqrt{121}}{6}=\frac{-7\pm 11}{6}$$

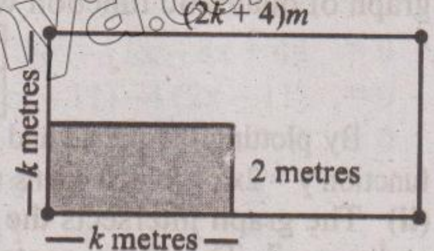
$$=\frac{-7+11}{6},\frac{-7-11}{6}=\frac{4}{6},\frac{-18}{6}$$

$$x=\frac{2}{3},-3$$

Hence, solution set is $\left\{\frac{2}{3},-3\right\}$

Activity

A rectangular garden is in front of Kiran's house, whose dimensions are (2k + 4) metres and k metres. A smaller rectangular portion of the garden of dimensions k metres and 2 metres is leveled. Find the area of unlevelled portion of the garden.



Sol: Length of garden (ℓ) $= 2k + 4$ m

Width of garden (w) $=$ k m

Area of garden $= \ell \times w$

$= (2k + 4)(k)$

$= (2k^2 + 4k)\ m^2$

Length of levelled area $=$ km

width of levelled area $=$ 2m

Area of levelled area $=$ km × 2m

$= 2km^2$

Unlevelled Area $=$ Total area – Leavelled Area

$= 2k^2 + 4k - 2k$

$= (2k^2 + 2k)\ m^2$

$= 2k(k + 1)\ m^2$

(d) Solution of Quadratic Equation by Graphs

> **Do you know!**
> The graph of a quadratic function is called parabola.

To solve the quadratic equation graphically means to find the points where the graph of quadratic equation crosses the x-axis. The x-values of these points are the solution of the quadratic equation. If the coefficient of x^2 in the quadratic equation is positive, then the graph will be $\cup$ shaped and if the coefficient of x^2 in quadratic equation is negative, then the graph will be $\cap$ shaped. To sketch the graph of quadratic function, we find its vertex by completing square method. $a(x - h)^2 + k$ is called the vertex form of quadratic function having vertex (h, k) and a is the coefficient of x^2.

Example 5: (i) Sketch and interpret the graph of quadratic function

$$y = 2x^2 - x - 10.$$

(ii) Solve the quadratic equation $2x^2 - x - 10 = 0$ graphically.

Sol. (i) Let us sketch the graph of $y = 2x^2 - x - 10$.

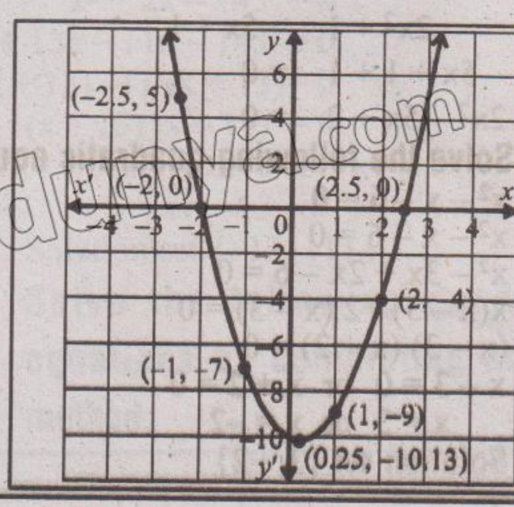
$$y = 2\left(x^2 - \frac{1}{2}x - 5\right)$$

$$= 2\left[x^2 - \frac{1}{2}x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 5\right]$$

$$= 2\left[\left(x - \frac{1}{4}\right)^2 - \frac{1}{16} - 5\right]$$

$$= 2\left[\left(x - \frac{1}{4}\right)^2 - \frac{81}{16}\right] = 2\left(x - \frac{1}{4}\right)^2 - \frac{2(81)}{16}$$

$$= 2\left(x - \frac{1}{4}\right)^2 - \frac{81}{8} = 2(x - 0.25)^2 - 10.13$$



Compare it with $y = a(x-h)^2 + k$. So, the vertex is $(0.25, -10.13)$. To sketch the graph of quadratic function we take some more points.

x	-2.5	-1	1	2
y	5	-7	-9	-4

By plotting the points and vertex, graph is sketched above. The graph of the given quadratic function $y = 2x^2 - x - 10$ opens upwards and it intersects the x-axis at $(-2, 0)$ and $(2.5, 0)$.

(ii) The graph intersects the x-axis at $x = 2.5$ and $x = -2$. Therefore, roots of the quadratic equation $2x^2 - x - 10 = 0$ are 2.5 and -2. Hence, solution set is $\{2.5, -2\}$.

Remember!
For finding the solution of quadratic equation $ax^2 + bx + c = 0$ graphically, first of all we draw the graph of quadratic function $y = ax^2 + bx + c$.

Solved Exercise 2.1

1. Write the following quadratic equations in standard form:

(i) $3x - 1 = 2x^2$

Sol. $3x - 1 = 2x^2$

$-2x^2 + 3x - 1 = 0$

$2x^2 - 3x + 1 = 0$

(ii) $2x(x+1) = 4(2x+3)$

Sol. $2x(x+1) = 4(2x+3)$

$2x^2 + 2x = 8x + 12$

$2x^2 + 2x - 8x - 12 = 0$

$2x^2 - 6x - 12 = 0$

$2(x^2 - 3x - 6) = 0$

$x^2 - 3x - 6 = 0$

(iii) $2x^2 - 4x = 4x + 7$

Sol. $2x^2 - 4x = 4x + 7$

$2x^2 - 4x - 4x - 7 = 0$

$2x^2 - 8x - 7 = 0$

(iv) $4(3x - 2) = 9x^2$

Sol. $4(3x - 2) = 9x^2$

$12x - 8 = 9x^2$

$-9x^2 + 12x - 8 = 0$

$9x^2 - 12x + 8 = 0$

(v) $2x + \frac{1}{x} = 5 - \frac{1}{x}, x \neq 0$

Sol. $2x + \frac{1}{x} = 5 - \frac{1}{x}$

Multiplying both sides by x

$x(2x) + x\left(\frac{1}{x}\right) = x(5) - x\left(\frac{1}{x}\right)$

$2x^2 + 1 = 5x - 1$

$2x^2 - 5x + 1 + 1 = 0$

$2x^2 - 5x + 2 = 0$

(vi) $\frac{6x+6}{20-x} = \frac{1}{x}, x \neq 0, 20$

Sol. $\frac{6x+6}{20-x} = \frac{1}{x}$

Multiplying both sides by $x(20-x)$

$x(20-x)\frac{(6x+6)}{20-x} = x(20-x)\left(\frac{1}{x}\right)$

$x(6x+6) = 20 - x$

$6x^2 + 6x = 20 - x$

$6x^2 + 6x + x - 20 = 0$

$6x^2 + 7x - 20 = 0$

2. Solve the following quadratic equations by factorization method:

(i) $x^2 - x - 6 = 0$

Sol. $x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$x(x-3) + 2(x-3) = 0$

$(x-3)(x+2) = 0$

$x - 3 = 0$ or $x + 2 = 0$

$x = 3$ or $x = -2$

Solution set $\{3, -2\}$

(ii) $x^2 + 3x - 28 = 0$

Sol. $x^2 + 3x - 28 = 0$

$x^2 + 7x - 4x - 28 = 0$

$x(x+7) - 4(x+7) = 0$

$(x+7)(x-4) = 0$

$x + 7 = 0$ or $x - 4 = 0$

$x = -7$ or $x = 4$

Solution set $\{-7, 4\}$

(iii) $6x^2 + 13x - 5 = 0$

Sol.
$$6x^2 + 13x - 5 = 0$$
$$6x^2 + 15x - 2x - 5 = 0$$
$$3x(2x + 5) - 1(2x + 5) = 0$$
$$(2x + 5)(3x - 1) = 0$$
$$2x + 5 = 0 \quad , \quad 3x - 1 = 0$$
$$2x = -5 \quad , \quad 3x = 1$$
$$x = \frac{-5}{2} \quad , \quad x = \frac{1}{3}$$

Solution set $\left\{\frac{-5}{2}, \frac{1}{3}\right\}$

(iv) $x^2 - \frac{3}{2}x = \frac{9}{2}$

Sol. $x^2 - \frac{3}{2}x = \frac{9}{2}$

Multiplying both sides by 2, we get
$$2(x^2) - 2\left(\frac{3}{2}x\right) = 2\left(\frac{9}{2}\right)$$
$$2x^2 - 3x = 9$$
$$2x^2 - 3x - 9 = 0$$
$$2x^2 - 6x + 3x - 9 = 0$$
$$2x(x - 3) + 3(x - 3) = 0$$
$$(x - 3)(2x + 3) = 0$$
$$x - 3 = 0 \quad \text{or} \quad 2x + 3 = 0$$
$$x = 3 \quad \text{or} \quad 2x = -3$$
$$x = \frac{-3}{2}$$

Solution set $\left\{3, \frac{-3}{2}\right\}$

(v) $\frac{3x - 8}{x - 2} = \frac{5x - 2}{x + 5}, x \neq 2, -5$

Sol. $\frac{3x - 8}{x - 2} = \frac{5x - 2}{x + 5}$

By cross multiplication
$$(x + 5)(3x - 8) = (x - 2)(5x - 2)$$
$$x(3x - 8) + 5(3x - 8) = x(5x - 2) - 2(5x - 2)$$
$$3x^2 - 8x + 15x - 40 = 5x^2 - 2x - 10x + 4$$
$$3x^2 + 7x - 40 = 5x^2 - 12x + 4$$
$$3x^2 - 5x^2 + 7x + 12x - 40 - 4 = 0$$
$$-2x^2 + 19x - 44 = 0$$
$$2x^2 - 19x + 44 = 0$$
$$2x^2 - 11x - 8x + 44 = 0$$
$$x(2x - 11) - 4(2x - 11) = 0$$
$$(2x - 11)(x - 4) = 0$$
$$2x - 11 = 0 \quad \text{or} \quad x - 4 = 0$$
$$2x = 11 \quad \text{or} \quad x = 4$$
$$x = \frac{11}{2}$$

Solution set $\left\{\frac{11}{2}, 4\right\}$

(vi) $\frac{1}{x - 1} - \frac{1}{x + 3} = \frac{1}{35}, x \neq 1, -3$

Sol. $\frac{1}{x - 1} - \frac{1}{x + 3} = \frac{1}{35}$
$$\frac{x + 3 - (x - 1)}{(x-1)(x+3)} = \frac{1}{35}$$
$$\frac{x + 3 - x + 1}{(x-1)(x+3)} = \frac{1}{35}$$
$$\frac{4}{(x-1)(x+3)} = \frac{1}{35}$$

By cross multiplication
$$35 \times 4 = (x - 1)(x + 3)$$
$$140 = x(x + 3) - 1(x + 3)$$
$$140 = x^2 + 3x - x - 3$$
$$140 = x^2 + 2x - 3$$
$$-x^2 - 2x + 140 + 3 = 0$$
$$-x^2 - 2x + 143 = 0$$
$$x^2 + 2x - 143 = 0$$
$$x^2 + 13x - 11x - 143 = 0$$
$$x(x + 13) - 11(x + 13) = 0$$
$$(x + 13)(x - 11) = 0$$
$$x + 13 = 0 \quad \text{or} \quad x - 11 = 0$$
$$x = -13 \quad \text{or} \quad x = 11$$

Solution set $\{-13, 11\}$

3. Solve the following quadratic equations by completing square method:

(i) $2x^2 + 5x + 2 = 0$

Sol. $2x^2 + 5x + 2 = 0$

Dividing by 2 (the coefficient of x^2) on both sides of the equation.

$$\frac{2x^2}{2} + \frac{5}{2}x + \frac{2}{2} = \frac{0}{2}$$

$$x^2 + \frac{5}{2}x + 1 = 0$$

$$x^2 + \frac{5}{2}x = -1$$

We add square of half of the coefficient of x.i.e, $\left(\frac{5}{4}\right)^2$ on both sides of the equation.

$$x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = -1 + \left(\frac{5}{4}\right)^2$$

$$\left(x + \frac{5}{4}\right)^2 = -1 + \frac{25}{16}$$

$$\left(x + \frac{5}{4}\right)^2 = \frac{-16+25}{16}$$

$$\left(x + \frac{5}{4}\right)^2 = \frac{9}{16}$$

Taking square root of both the sides.

$$\sqrt{\left(x + \frac{5}{4}\right)^2} = \sqrt{\frac{9}{16}}$$

$$x + \frac{5}{4} = \pm\frac{3}{4}$$

$$x + \frac{5}{4} = \frac{3}{4} \quad \text{or} \quad x + \frac{5}{4} = -\frac{3}{4}$$

$$x = \frac{3}{4} - \frac{5}{4} \quad \text{or} \quad x = -\frac{3}{4} - \frac{5}{4}$$

$$x = \frac{3-5}{4} \quad \text{or} \quad x = \frac{-3-5}{4}$$

$$x = -\frac{2}{4} \quad \text{or} \quad x = -\frac{8}{4}$$

$$x = -\frac{1}{2} \quad \text{or} \quad x = -2$$

Solution Set $= \left\{-\frac{1}{2}, -2\right\}$

(ii) $x^2 + x = 42$

Sol. $x^2 + x = 42$

We add square of half of the co-efficient of x.i.e., $\left(\frac{1}{2}\right)^2$ on both sides of the equation.

$$x^2 + x + \left(\frac{1}{2}\right)^2 = 42 + \left(\frac{1}{2}\right)^2$$

$$\left(x + \frac{1}{2}\right)^2 = 42 + \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{168+1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{169}{4}$$

Taking square root of both the sides.

$$\sqrt{\left(x + \frac{1}{2}\right)^2} = \sqrt{\frac{169}{4}}$$

$$x + \frac{1}{2} = \pm\frac{13}{2}$$

$$x + \frac{1}{2} = \frac{13}{2} \quad \text{or} \quad x + \frac{1}{2} = -\frac{13}{2}$$

$$x = \frac{13}{2} - \frac{1}{2} \quad \text{or} \quad x = -\frac{13}{2} - \frac{1}{2}$$

$$x = \frac{13-1}{2} \quad \text{or} \quad x = \frac{-13-1}{2}$$

$$x = \frac{12}{2} \quad \text{or} \quad x = \frac{-14}{2}$$

$$x = 6 \quad \text{or} \quad x = -7$$

Solution Set = {6, −7}

(iii) $12x^2 + 7x = 12$

Sol. $12x^2 + 7x = 12$

Dividing by 12 (the coefficient of x^2) on both sides of the equation.

$$\frac{12}{12}x^2 + \frac{7}{12}x = \frac{12}{12}$$

$$x^2+\frac{7}{12}x=1$$

We add square of half of the co-efficient of x.i.e, $\left(\frac{7}{24}\right)^2$ on both sides of the equation.

$$x^2+\frac{7}{12}x+\left(\frac{7}{24}\right)^2=1+\left(\frac{7}{24}\right)^2$$

$$\left(x+\frac{7}{24}\right)^2=1+\frac{49}{576}$$

$$\left(x+\frac{7}{24}\right)^2=\frac{576+49}{576}$$

$$\left(x+\frac{7}{24}\right)^2=\frac{625}{576}$$

Taking square root of both the sides.

$$\sqrt{\left(x+\frac{7}{24}\right)^2}=\sqrt{\frac{625}{576}}$$

$$x+\frac{7}{24}=\pm\frac{25}{24}$$

$$x=\frac{-7}{24}+\frac{25}{24} \text{ or } x=\frac{-7}{24}-\frac{25}{24}$$

$$x=\frac{-7+25}{24} \text{ or } x=\frac{-7-25}{24}$$

$$x=\frac{18}{24} \text{ or } x=\frac{-32}{24}$$

$$x=\frac{3}{4} \text{ or } x=\frac{-4}{3}$$

Solution Set is $\left\{\frac{3}{4},\frac{-4}{3}\right\}$

(iv) $\frac{x+3}{2x-7}=\frac{2x-1}{x-3}$, $x\neq\frac{7}{2},3$

Sol. $\frac{x+3}{2x-7}=\frac{2x-1}{x-3}$

$(x+3)(x-3)=(2x-1)(2x-7)$

by cross multiplication

$x^2-3x+3x-9=4x^2-14x-2x+7$

$x^2-9=4x^2-16x+7$

$x^2-4x^2+16x-9-7=0$

$-3x^2+16x-16=0$

$3x^2-16x+16=0$

Dividing by 3 (the co-efficient of x^2) on both sides of the equation.

$$\frac{3x^2}{3}-\frac{16}{3}x+\frac{16}{3}=\frac{0}{3}$$

$$x^2-\frac{16}{3}x+\frac{16}{3}=0$$

$$x^2-\frac{16}{3}x=\frac{-16}{3}$$

We add square of half of the co-efficient of *x.i.e*, $\left(\frac{8}{3}\right)^2$ on both sides of the equation.

$$x^2-\frac{16}{3}x+\left(\frac{8}{3}\right)^2=\frac{-16}{3}+\left(\frac{8}{3}\right)^2$$

$$\left(x-\frac{8}{3}\right)^2=\frac{-16}{3}+\frac{64}{9}$$

$$\left(x-\frac{8}{3}\right)^2=\frac{-48+64}{9}$$

$$\left(x-\frac{8}{3}\right)^2=\frac{16}{9}$$

$$\sqrt{\left(x-\frac{8}{3}\right)^2}=\sqrt{\frac{16}{9}}$$

Taking square root of both the sides.

$$x-\frac{8}{3}=\pm\frac{4}{3}$$

$$x-\frac{8}{3}=\frac{4}{3} \quad \text{or} \quad x-\frac{8}{3}=-\frac{4}{3}$$

$$x=\frac{4}{3}+\frac{8}{3} \quad \text{or} \quad x=-\frac{4}{3}+\frac{8}{3}$$

$$x=\frac{12}{3} \quad \text{or} \quad x=\frac{4}{3}$$

$x=4$ or

Solution set is $\left\{4, \frac{4}{3}\right\}$

(v) $\dfrac{1}{1+x} - \dfrac{1}{3-x} = \dfrac{6}{35}, x \neq -1, 3$

Sol. $\dfrac{1}{1+x} - \dfrac{1}{3-x} = \dfrac{6}{35}$

$$\frac{(3-x)-(1+x)}{(1+x)(3-x)} = \frac{6}{35}$$

$$\frac{3-x-1-x}{3-x+3x-x^2} = \frac{6}{35}$$

$$\frac{2-2x}{-x^2+2x+3} = \frac{6}{35}$$

$35(2-2x) = 6(-x^2+2x+3)$

(By cross multiplication)

$70 - 70x = -6x^2 + 12x + 18$

$6x^2 - 70x - 12x + 70 - 18 = 0$

$6x^2 - 82x + 52 = 0$

$2(3x^2 - 41x + 26) = 0$

(Taking 2 as a common)

$3x^2 - 41x + 26 = 0$

Dividing by 3 (the coefficient of x^2) on both sides of the equation)

$$\frac{3}{3}x^2 - \frac{41}{3}x + \frac{26}{3} = 0$$

$$x^2 - \frac{41}{3}x + \frac{26}{3} = 0$$

$$x^2 - \frac{41}{3}x = -\frac{26}{3}$$

We add square of half of the co-efficient of x.i.e., $\left(\frac{41}{6}\right)^2$ on both sides of the equation.

$$x^2 - \frac{41}{3}x + \left(\frac{41}{6}\right)^2 = -\frac{26}{3} + \left(\frac{41}{6}\right)^2$$

$$\left(x - \frac{41}{6}\right)^2 = -\frac{26}{3} + \frac{1681}{36}$$

$$\left(x - \frac{41}{6}\right)^2 = \frac{-312+1681}{36}$$

$$\left(x - \frac{41}{6}\right)^2 = \frac{1369}{36}$$

Taking square root of both the sides.

$$\sqrt{\left(x - \frac{41}{6}\right)^2} = \sqrt{\frac{1369}{36}}$$

$$x - \frac{41}{6} = \pm\frac{37}{6}$$

$x - \dfrac{41}{6} = \dfrac{37}{6}$ or $x - \dfrac{41}{6} = -\dfrac{37}{6}$

$x = \dfrac{37}{6} + \dfrac{41}{6}$ or $x = -\dfrac{37}{6} + \dfrac{41}{6}$

$x = \dfrac{78}{6}$ or $x = \dfrac{4}{6}$

$x = 13$ or $x = \dfrac{2}{3}$

Solution Set = $\left\{13, \frac{2}{3}\right\}$

(vi) $\dfrac{3x-1}{4x+7} = 1 - \dfrac{6}{x+7}, x \neq \dfrac{-7}{4}, -7$

Sol. $\dfrac{3x-1}{4x+7} = 1 - \dfrac{6}{x+7}$

$$\frac{3x-1}{4x+7} = \frac{x+7-6}{x+7}$$

$$\frac{3x-1}{4x+7} = \frac{x+1}{x+7}$$

$(3x-1)(x+7) = (x+1)(4x+7)$

by cross multiplication

$3x^2 + 21x - x - 7 = 4x^2 + 7x + 4x + 7$

$3x^2 + 20x - 7 = 4x^2 + 11x + 7$

$3x^2 - 4x^2 + 20x - 11x = 7 + 7$

$-x^2 + 9x = 14$

$x^2 - 9x = -14$

$x^2 - 9x = -14$

We add square of half of the co-efficient of x.i.e., $\left(\frac{9}{2}\right)^2$ on the both sides of the equation.

$$x^2-9x+\left(\frac{9}{2}\right)^2=-14+\left(\frac{9}{2}\right)^2$$

$$\left(x-\frac{9}{2}\right)^2=-14+\frac{81}{4}$$

$$\left(x-\frac{9}{2}\right)^2=\frac{-56+81}{4}$$

$$\left(x-\frac{9}{2}\right)^2=\frac{25}{4}$$

Taking square root of both sides.

$$\sqrt{\left(x-\frac{9}{2}\right)^2}=\sqrt{\frac{25}{4}}$$

$$x-\frac{9}{2}=\pm\frac{5}{2}$$

$$x-\frac{9}{2}=\frac{5}{2} \quad \text{or} \quad x-\frac{9}{2}=-\frac{5}{2}$$

$$x=\frac{5}{2}+\frac{9}{2} \quad \text{or} \quad x=-\frac{5}{2}+\frac{9}{2}$$

$$x=\frac{14}{2} \quad \text{or} \quad x=\frac{4}{2}$$

$$x=7 \quad \text{or} \quad x=2$$

Solution Set = {7, 2}

4. Use quadratic formula to solve the following equations:

(i) $2x^2-5x+3=0$

Sol. $2x^2-5x+3=0$ (given)

$ax^2+bx+c=0$ (standard form)

comparing the given equation with the standard form.

$a=2,\ b=-5,\ c=3$

Substituting the values of a,b,c in the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(3)}}{2(3)}$$

$$x=\frac{5\pm\sqrt{25-24}}{4}$$

$$x=\frac{5\pm\sqrt{1}}{4}$$

$$x=\frac{5\pm1}{4}$$

$$x=\frac{5+1}{4} \quad , \quad x=\frac{5-1}{4}$$

$$x=\frac{6}{4}=\frac{3}{2} \quad , \quad x=\frac{4}{4}=1$$

Solution set: $\left\{\frac{3}{2},1\right\}$

(ii) $2x^2-7x-15=0$

Sol. $2x^2-7x-15=0$ (given)

$ax^2+bx+c=0$ (Standard form)

Comparing the given equation with the standard form.

$a=2,\ b=-7,\ c=-15$

Substituting the values of a,b,c in the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(2)(-15)}}{2(2)}$$

$$x=\frac{7\pm\sqrt{49+120}}{4}$$

$$x=\frac{7\pm\sqrt{169}}{4}$$

$$x=\frac{7\pm13}{4}$$

$$x=\frac{7+13}{4} \quad , \quad x=\frac{7-13}{4}$$

$$x=\frac{20}{4}=5 \quad , \quad x=\frac{-6}{4}=\frac{-3}{2}$$

Solution set: $\left\{5,\frac{-3}{2}\right\}$

(iii) $2x^2 + 7x = 15$

Sol. $2x^2 + 7x = 15$ (Given)

$2x^2 + 7x - 15 = 0$

$ax^2 + bx + c = 0$ (Standard form)

Comparing the given equation with the standard form.

$a = 2, b = 7, c = -15$

Substituting the values of a,b,c in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(2)(-15)}}{2(2)}$$

$$x = \frac{-7 \pm \sqrt{49+120}}{4}$$

$$x = \frac{-7 \pm \sqrt{169}}{4}$$

$$x = \frac{-7 \pm 13}{4}$$

$$x = \frac{-7+13}{4}, \quad x = \frac{-7-13}{4}$$

$$x = \frac{6}{4} = \frac{3}{2}, \quad x = \frac{-20}{4} = -5$$

Solution set: $\left\{\frac{3}{2}, -5\right\}$

(iv) $x^2 + 11 = 7x$

Sol. $x^2 + 11 = 7x$

$x^2 - 7x + 11 = 0$ (Given)

$ax^2 + bx + c = 0$ (Standard form)

Comparing the given equation with the standard form.

$a = 1, b = -7, c = 11$

Substituting the values of a,b,c in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(11)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49-44}}{2}$$

$$x = \frac{7 \pm \sqrt{5}}{2}$$

Solution set: $\left\{\frac{7 \pm \sqrt{5}}{2}\right\}$

(v) $\frac{x+4}{x-4} + \frac{x-2}{x-3} = 6\frac{1}{3}, x \neq 4, 3$

Sol. $\frac{x+4}{x-4} + \frac{x-2}{x-3} = 6\frac{1}{3}$

$$\frac{(x-3)(x+4) + (x-4)(x-2)}{(x-4)(x-3)} = \frac{19}{3}$$

$$\frac{x^2 + 4x - 3x - 12 + x^2 - 2x - 4x + 8}{x^2 - 3x - 4x + 12} = \frac{19}{3}$$

$$\frac{x^2 + x^2 + 4x - 3x - 2x - 4x - 12 + 8}{x^2 - 7x + 12} = \frac{19}{3}$$

$$\frac{2x^2 - 5x - 4}{x^2 - 7x + 12} = \frac{19}{3}$$

$3(2x^2 - 5x - 4) = 19(x^2 - 7x + 12)$

$6x^2 - 15x - 12 = 19x^2 - 133x + 228$

$6x^2 - 19x^2 - 15x + 133x - 12 - 228 = 0$

$-13x^2 + 118x - 240 = 0$

$13x^2 - 118x + 240 = 0$ (Given)

$ax^2 + bx + c = 0$ (Standard form)

Comparing the given equation with the standard form.

$a = 13, b = -118, c = 240$

Substituting the values of a,b,c in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-118) \pm \sqrt{(-118)^2 - 4(13)(240)}}{2(13)}$$

$$x = \frac{118 \pm \sqrt{13924 - 12480}}{26}$$

$$x = \frac{118 \pm \sqrt{1444}}{26}$$

$$x = \frac{118 \pm 38}{26}$$

$$x = \frac{118+38}{26} \quad , \quad x = \frac{118-38}{26}$$

$$x = \frac{156}{26} = 6 \quad , \quad x = \frac{80}{26} = \frac{40}{13}$$

Solution set: $\left\{6, \frac{40}{13}\right\}$

(vi) $\mathbf{\frac{3x-3}{x+1} = \frac{2x-1}{x-1}, x \neq -1, 1}$

Sol. $\frac{3x-3}{x+1} = \frac{2x-1}{x-1}$

$(x-1)(3x-3) = (x+1)(2x-1)$

$3x^2 - 3x - 3x + 3 = 2x^2 - x + 2x - 1$

$3x^2 - 6x + 3 = 2x^2 + x - 1$

$3x^2 - 2x^2 - 6x - x + 3 + 1 = 0$

$x^2 - 7x + 4 = 0$ (Given)

$ax^2 + bx + c = 0$ (Standard form)

Comparing the given equation with the standard form.

$a = 1, b = -7, c = 4$

Substituting the values of a,b,c in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49-16}}{2}$$

$$x = \frac{7 \pm \sqrt{33}}{2}$$

Solution set: $\left\{\frac{7 \pm \sqrt{33}}{2}\right\}$

5. Solve the following quadratic equations graphically:

(i) $\mathbf{x^2 - 3x - 18 = 0}$

Sol. $x^2 - 3x - 18 = 0$

Let

$y = x^2 - 3x - 18$

$$y = x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 18$$

$$y = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - 18$$

$y = (x - 1.5)^2 - 2.25 - 18$

$y = (x - 1.5)^2 - 20.25$

$y = a(x - h)^2 + k$

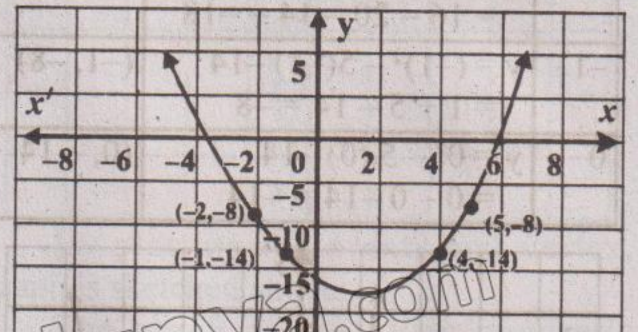
[Vertex form of quadratic function]

By compairing vertex is (1.5, –20.25)

To sketch the graph of quadratic function, we take some more points.

Table of values for $y = x^2 - 3x - 18$

x	$y = x^2 - 3x - 18$	(x, y)
5	$y = (5)^2 - 3(5) - 18$ $= 25 - 15 - 18 = -8$	(5, –8)
4	$= (4)^2 - 3(4) - 18$ $= 16 - 12 - 18 = -14$	(4, –14)
–2	$= (-2)^2 - 3(-2) - 18$ $= 4 + 6 - 18 = -8$	(–2, –8)
–1	$= (-1)^2 - 3(-1) - 18$ $= 1 + 3 - 18 = -14$	(–1, –14)



By Plotting the point and vertex, graph is sketched above. The graph of the given quadratic function $y = x^2 - 3x - 18$ opens upward and it intersects the x-axis at (–3, 0) and (6,0).

The graph intersects the x-axis at x = 6 and x=–3. Therefore, roots of the quadratic equation $x^2 - 3x - 18 = 0$ are –3 and 6.

Hence, Solution set is {–3, 6}

(ii) $x^2 - 5x - 14 = 0$

Sol. $x^2 - 5x - 14 = 0$

Let

$$y = x^2 - 5x - 14$$

$$y = x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 14$$

$$y = \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} - 14$$

$$y = \left(x - \frac{5}{2}\right)^2 - 6.25 - 14$$

$$y = (x - 2.5)^2 - 20.25$$

$$y = (x - h)^2 + k$$

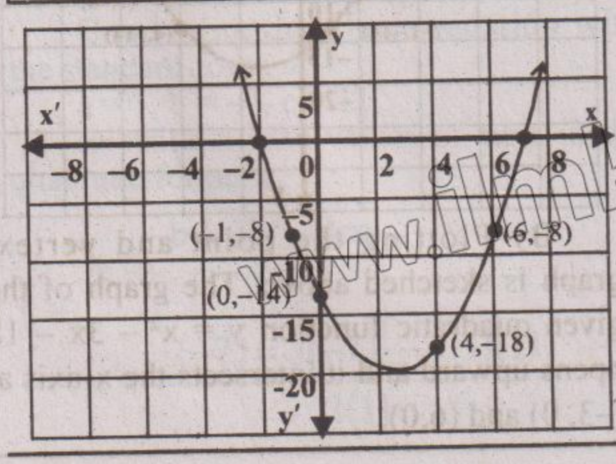
[the vertex form of quadratic function]

By compairing the vertex is (2.5, –20.25)

To sketch the graph of quadratic function we take some more points.

Table of values for function $y=x^2-5x-14$

x	$y = x^2 - 5x - 14$	(x, y)
6	$y = (6)^2 - 5(6) - 14$ $= 36 - 30 - 14 = -8$	(6, –8)
4	$y = (4)^2 - 5(4) - 14$ $= 16 - 20 - 14 = -18$	(4, –18)
–1	$y = (-1)^2 - 5(-1) - 14$ $= 1 + 5 - 14 = -8$	(–1, –8)
0	$y = 0^2 - 5(0) - 14$ $= 0 - 0 - 14 = -14$	(0, – 14



By ploting the points and vetex, graph is sketched above. The graph of the given quadratic function $y = x^2 - 5x - 14 = 0$ opens upwards and it intersects the x-axis at (–2,0) and (7,0).

The graph intersects the x – axis at x = 7 and x = –2. Therefore, roots of the quadratic equation $x^2 - 5x - 14 = 0$ are 7 and –2.

Hence, solution set is {–2, 7}

(iii) $2x^2 + 13x + 6 = 0$

Sol. $2x^2 + 13x + 6 = 0$

Let $\quad y = 2x^2 + 13x + 6$

$$y = 2\left[x^2 + \frac{13}{2}x + \frac{6}{2}\right]$$

$$y = 2\left[x^2 + \frac{13}{2}x + 3\right]$$

$$y = 2\left[x^2 + \frac{13}{2}x + \left(\frac{13}{4}\right)^2 - \left(\frac{13}{4}\right)^2 + 3\right]$$

$$y = 2\left[\left(x + \frac{13}{4}\right)^2 - \frac{169}{16} + 3\right]$$

$$y = 2\,[(x + 3.25)^2 - 10.57 + 3]$$

$$y = 2\,[(x + 3.25)^2 - 7.57]$$

$$y = 2\,[(x + 3.25)^2 - 15.14]$$

$$y = a\,(x - h)^2 + k$$

[the vertex form of quadratic function]

By compairing the vertex is

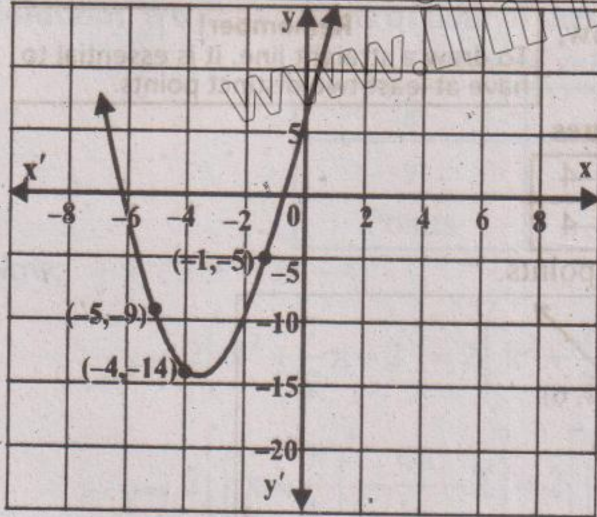
(–3.25, –15.14)

To sketch the graph of quadratic function we take some more points.

Table of the values for function $y = 2x^2 + 13x + 6$

x	$y = 2x^2 + 13x + 6$	(x, y)
–1	$y = 2(-1)^2 + 13(-1) + 6$ $= 2 - 13 + 6 = -5$	(–1, –5)
–4	$y = 2(-4)^2 + 13(-4) + 6$ $= 32 - 52 + 6 = -14$	(–4, –14)
–5	$y = 2(-5)^2 + 13(-5) + 6$ $= 50 - 65 + 6 = -9$	(–5, –9)

0	$y = 2(0)^2 + 13(0) + 6$ $= 2(0) + 0 + 6$ $= 0 + 0 + 6 = 6$	(0,6)



By plotting the points and vertex, graph is sketched above. The graph of the given quadratic function $2x^2 + 13x + 6 = 0$ opens upwards and it intersects the x-axis at (–6,0) and (–0.5,0).

The graph intersects the x – axis at x = –6 and x = –0.5. Therefore, roots of the quadratic equation are –6 and –0.5.

Hence, the solution set is {–6, –0.5}.

(iv) $4x^2 + 12x - 27 = 0$

Sol: $4x^2 + 12x - 27 = 0$

Let $y = 4x^2 + 12x - 27$

$$y = 4\left[x^2 + \frac{12}{4}x - \frac{27}{4}\right]$$

$$y = 4\left[x^2 + 3x - \frac{27}{4}\right]$$

$$y = 4\left[x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - \frac{27}{4}\right]$$

$$y = 4\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{27}{4}\right]$$

$$y = 4\left[\left(x + \frac{3}{2}\right)^2 - \frac{36}{4}\right]$$

$$y = 4\left(x + \frac{3}{2}\right)^2 - 4 \times \frac{36}{4}$$

$y = 4(x + 1.5)^2 - 36$

$y = a(x - h)^2 - 36$

[the vertex form of the quadratic function]

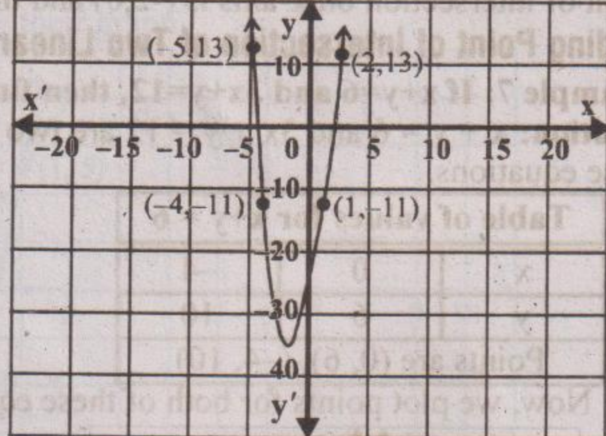
By compairing the vertex is (–1.5, –36)

The sketch the graph of quadratic function we take some more points.

Table of the values for function

$y = 4x^2 + 12x - 27$

x	$y = 4x^2 + 12x - 27$	(x,y)
1	$= 4(1)^2 + 12(1) - 27$ $= 4 + 12 - 27 = -11$	(1, –11)
2	$= 4(2)^2 + 12(2) - 27$ $= 16 + 24 - 27 = 13$	(2, 13)
–4	$= 4(-4)^2 + 12(-4) - 27$ $= 64 - 48 - 27 = -11$	(–4, –11)
–5	$= 4(-5)^2 + 12(-5) - 27$ $= 100 - 60 - 27 = 13$	(–5, +13)



By plotting the points and vertex, graph is sketched above.

The graph of the given quadratic function $y = 4x^2 + 12x - 27$ opens upwards and it intersects the x-axis at (–4.5,0) and (1.5, 0).

The graph intersects the x-axis at x = –4.5 and x = 1.5. Therefore, roots of the quadratic equation $4x^2 + 12x - 27 = 0$ are –4.5 and 1.5.

Hence, the solution set is {–4.5, 1.5}

2.2 Finding Intersection Point(s) Graphically

Example 6: Find the points of intersection of $y = 2x + 4$ with coordinate axes graphically.

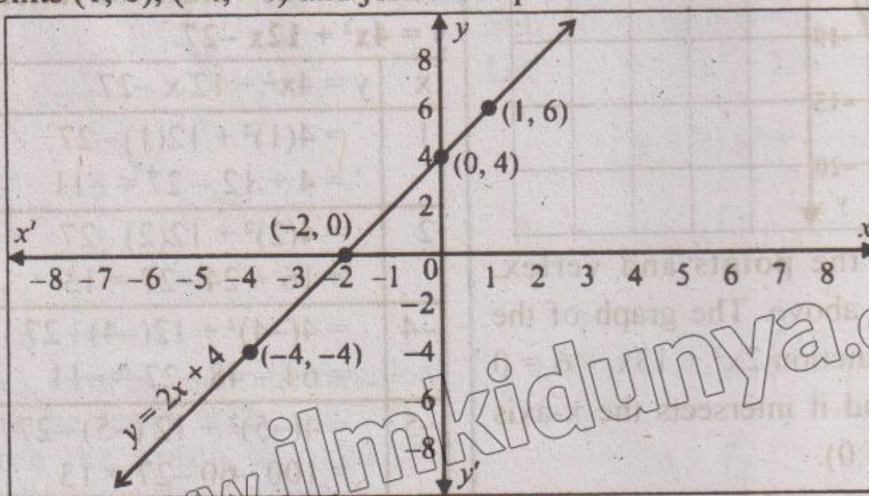
Solution: $y = 2x + 4$ is an equation of a line. Now, we draw graph of $y = 2x + 4$.

> **Remember!**
> To draw a straight line, it is essential to have at least two distinct points.

Table of values

x	1	–4
y	6	–4

We plot points (1, 6), (–4, –4) and join these points.



Point of intersection on x-axis is (-2,0) and the point of intersection on y-axis is (0,4).

Finding Point of Intersection of Two Linear Functions Graphically

Example 7: If x+y=6 and 3x+y=12, then find their point of intersection graphically.

Solution: $x + y = 6$ and $3x + y = 12$ are two linear equations. We draw graphs of both of these equations.

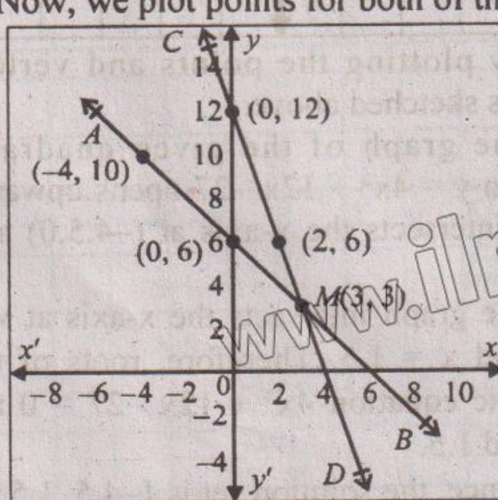
Table of values for x+y = 6

x	0	–4
y	6	10
Points are (0, 6), (–4, 10)		

Table of values for 3x+y= 12

x	0	2
y	12	6
Points are: (0, 12), (2, 6)		

Now, we plot points for both of these equations.



$\overleftrightarrow{AB}$ is the graph of $x + y = 6$

$\overleftrightarrow{CD}$ is the graph of $3x + y = 12$

Hence, the point of intersection is M(3, 3)

Finding Points of Intersection of a Linear Function and a Quadratic Function Graphically

Example 8: If $y = 2 + 3x$ and $y = 2x^2 + 7x - 4$, then find their points of intersection graphically.

Solution: We draw graphs of both of these equations.

Table of values for $y = 2 + 3x$

x	0	–2
y	2	–4
Points	(0, 2)	(–2, –4)

Now, $y = 2x^2 + 7x - 4$

$$y = 2\left(x^2 + \frac{7}{2}x - 2\right) = 2\left(x^2 + \frac{7}{2}x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 - 2\right)$$

$$y = 2\left[\left(x + \frac{7}{4}\right)^2 - \frac{49}{16} - 2\right] = 2\left[\left(x + \frac{7}{4}\right)^2 - \frac{81}{16}\right] = 2\left(x + \frac{7}{4}\right)^2 - 2\left(\frac{81}{16}\right)$$

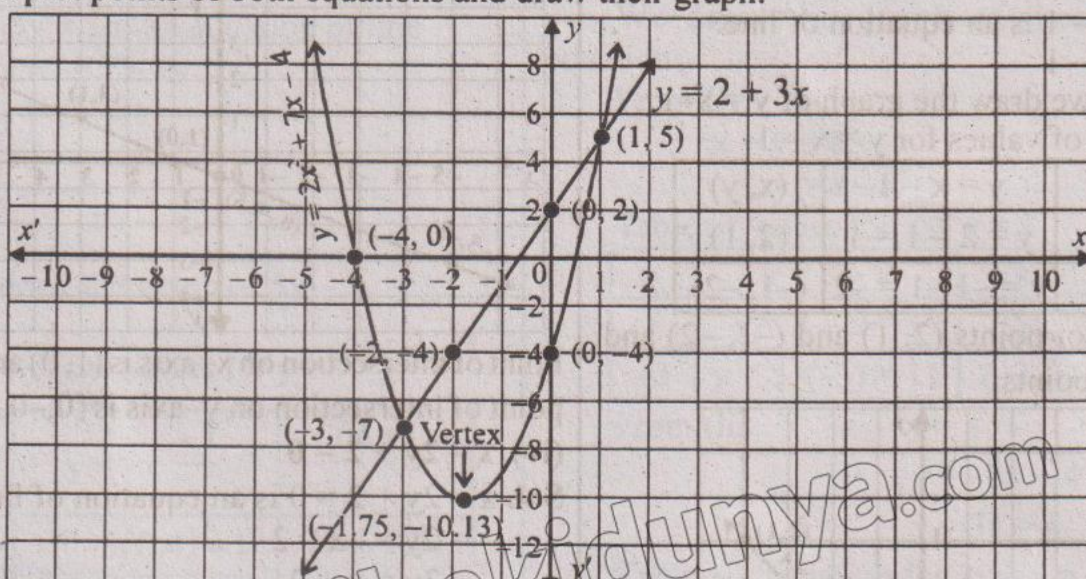
$$y = 2(x + 1.75)^2 - 10.13$$

So, the vertex is (–1.75, –10.13)

Table of values for $y = 2x^2 + 7x - 4$

x	–4	–3	0	1
y	0	–7	–4	5

We plot points of both equations and draw their graph.



Points of intersection are: (1, 5) and (–3, –7)

Solved Exercise 2.2

1. Find the points of intersection of the following linear equations with coordinate axes graphically.

(i) $x + y = 8$

Sol. $x + y = 8$ is an equation of a line.

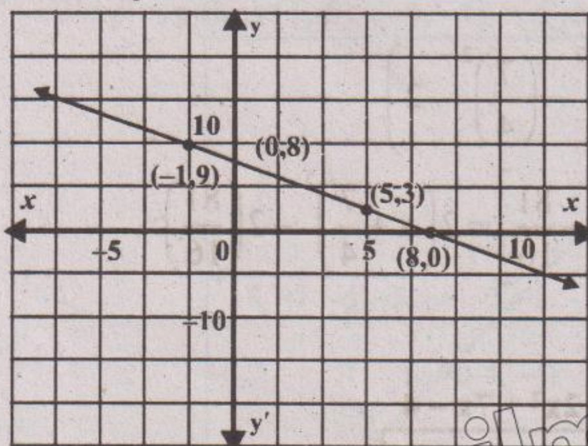
$y = 8 - x$

Now we draw the graph of $x + y = 8$.

Table of values for $y = 8 - x$

x	$y = 8 - x$	(x, y)
5	$y = 8 - 5 = 3$	(5, 3)
–1	$y = 8 - (-1) = 9$	(–1, 9)

We plot points (5, 3) and (–1, 9) and join these points.



Point of intersection on x-axis is (8, 0) and the point of intersection on y-axis is (0,8).

(ii) $x - y = 1$

Sol. $x - y = 1$ is an equation of line.

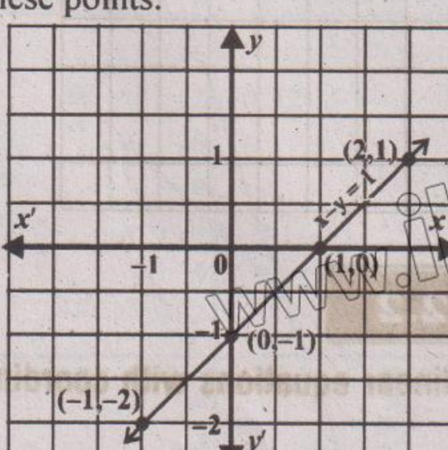
or $y = x - 1$

Now we draw the graph of $y = x - 1$.

Table of values for $y = x - 1$

x	$y = x - 1$	(x, y)
2	$y = 2 - 1 = 1$	(2, 1)
–1	$y = -1 - 1 = -2$	(–1, –2)

We plot points (2, 1) and (–1, –2) and join these points.



Point of intersection on x–axis is (1, 0) and the point of intersection on y–axis is (0,–1)

(iii) $x - 2y = 1$

Sol. $x - 2y = 1$ is an equation of line.

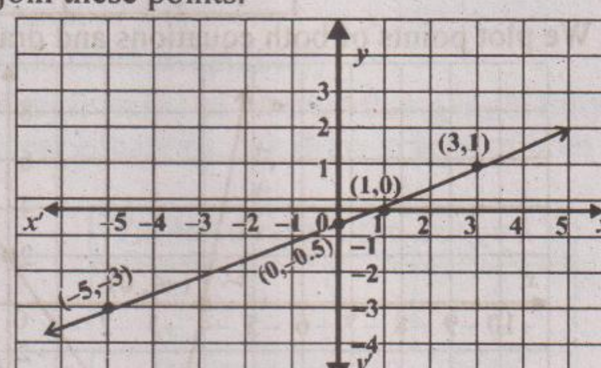
$$-2y = -x + 1$$
$$2y = x - 1$$
$$y = \frac{x-1}{2}$$

Now we draw the graph of $x - 2y = 1$.

Table of the values for $y = \frac{x-1}{2}$

x	$y = \frac{x-1}{2}$	(x, y)
3	$y = \frac{3-1}{2} = 1$	(3, 1)
–5	$y = \frac{-5-1}{2} -3$	(–5, –3)

We plot points (3, 1) and (–5, –3) and join these points.



Point of intersection on x–axis is (1, 0) and the point of intersection on y–axis is (0,–0.5).

(iv) $x - 2y + 2 = 0$

Sol. $x - 2y + 2 = 0$ is an equation of line.

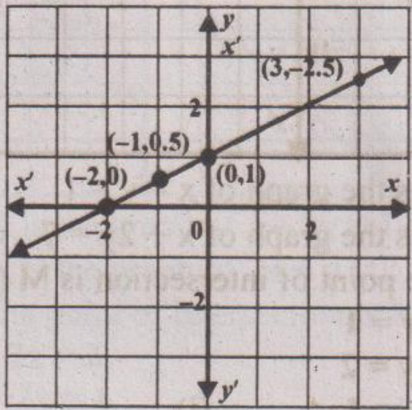
$$-2y = -x - 2$$
$$2y = x + 2$$
$$y = \frac{x+2}{2}$$

Now we draw the graph of $x - 2y + 2 = 0$.

Table of the values for $y = \frac{x+2}{2}$

x	$y=\dfrac{x+2}{2}$	(x,y)
3	$y=\dfrac{3+2}{2}=2.5$	(3, 2.5)
–1	$y=\dfrac{-1+2}{2}=0.5$	(–1, 0.5)

We plot points (3, 2.5) and (–1, 0.5) and join these points.



Point of intersection on x–axis is (–2, 0) and the point of intersection on y–axis is (0,1).

(v) 5x – 5y = 1

Sol. 5x – 5y = 1 is an equation of line

$-5y = -5x + 1$

$5y = 5x - 1$

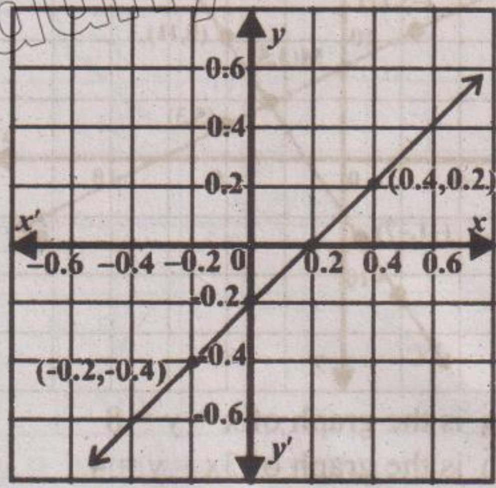
$y = \dfrac{5x-1}{5}$

Now we draw the graph of 5x – 5y = 1.

Table of the values for $y = \dfrac{5x-1}{5}$

x	$y=\dfrac{5x-1}{5}$	(x, y)
0.4	$y=\dfrac{5(0.4)-1}{5}$ $=\dfrac{2-1}{5}=0.2$	(0.4, 0.2)
–0.2	$y=\dfrac{-1-1}{5}=-0.4$	(–0.2, –0.4)

We plot points (0.4, 0.2) and (–0.2, –0.4) and join these points.



The point of intersection on x–axis is (0.2,0) and the point of intersection on y–axis is (0,–0.2)

2. Solve the following system of linear equations graphically:

(i) x + y = 8

3x – y = 4

Sol. x + y = 8 ________ (i)

3x – y = 4 ________ (ii)

We draw the graph of these two equations.

From (i)

$x + y = 8$

$y = 8 - x$

Table of values for x + y = 8

x	y = 8 – x	(x, y)
5	y = 8 – 5 = 3	(5, 3)
–3	y = 8–(–3) = 11	(–3, 11)

From (ii) $3x - y = 4$

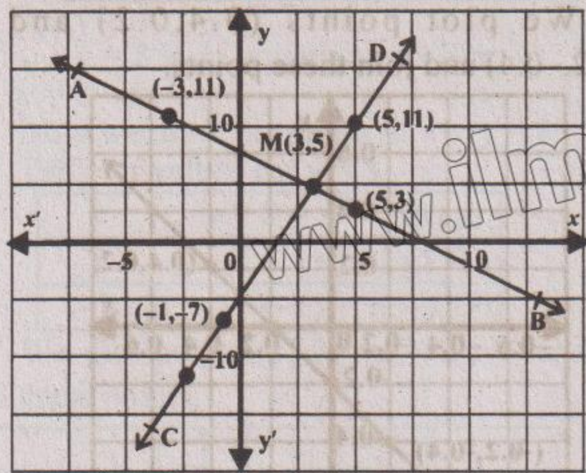
$-y = -3x + 4$

$y = 3x - 4$

Table of the values for 3x – y = 4

x	y = 3x – 4	(x, y)
5	y = 3(5) – 4 = 11	(5, 11)
–1	y = 3(–1) – 4 = –7	(–1, –7)

Now we plot points for both of these equations.



$\overline{AB}$ is the graph of $x + y = 8$
$\overline{CD}$ is the graph of $3x - y = 4$

Hence, the point of intersection is M (3, 5)

(ii) $\mathbf{x - y = 1}$
$\mathbf{x + 2y = 7}$

Sol. $x - y = 1$ ________ (i)
$x + 2y = 7$ ________ (ii)

We draw the graph of these two equations.

From (i)

$$x - y = 1$$
$$-y = -x + 1$$
$$y = x - 1$$

Table of values for $x - y = 1$

x	$y = x - 1$	(x, y)
7	$y = 7 - 1 = 6$	(7, 6)
–5	$y = -5 - 1 = -6$	(–5, –6)

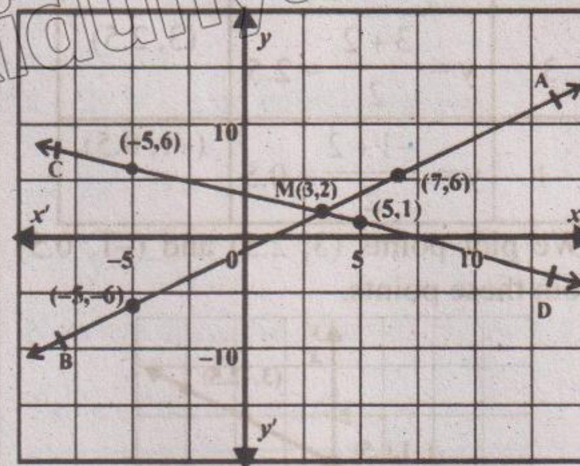
From (ii) $\quad x + 2y = 7$

$$2y = 7 - x$$
$$y = \frac{7 - x}{2}$$

Table of the values for $x + 2y = 7$

x	$y = \frac{7-x}{2}$	(x, y)
5	$y = \frac{7-5}{2} = 1$	(5, 1)
–5	$y = \frac{7-(-5)}{2} = 6$	(–5, 6)

Now we plot points for both of these equations.



$\overline{AB}$ is the graph of $x - y = 1$
$\overline{CD}$ is the graph of $x + 2y = 7$

Hence, the point of intersection is M (3, 2).

(iii) $\mathbf{x - 2y = 1}$
$\mathbf{2x + y = 2}$

Sol. $x - 2y = 1$ ________ (i)
$2x + y = 2$ ________ (ii)

We draw the graph of these two equations.

From (i)

$$x - 2y = 1$$
$$-2y = -x + 1$$
$$2y = x - 1$$
$$y = \frac{x - 1}{2}$$

Table of values for $x - 2y = 1$

x	$y = \frac{x-1}{2}$	(x, y)
5	$y = \frac{5-1}{2} = 2$	(5, 2)
–5	$y = \frac{-5-1}{2} = -3$	(–5, –3)

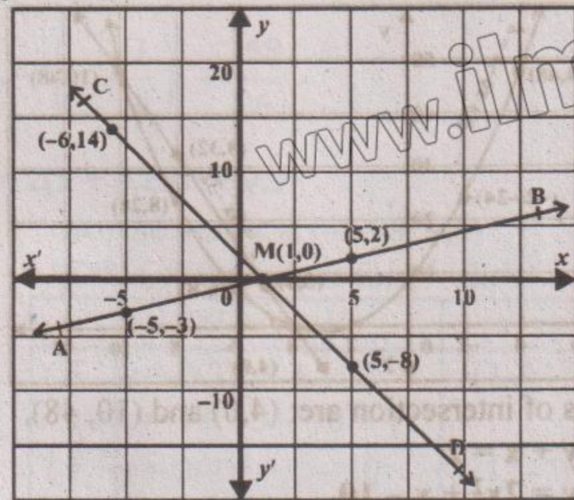
From (ii) $\quad 2x + y = 2$

$$y = 2 - 2x$$

Table of the values for $-2x + y = 2$

x	$y = 2 - 2x$	(x, y)
5	$y = 2 - 2(5) = -8$	(5, –8)
–6	$y = 2 - 2(-6) = 14$	(–6, 14)

Now plot points for both of these equations.



$\overline{AB}$ is the graph of $x - 2y = 1$

$\overline{CD}$ is the graph of $2x + y = 2$

Hence, the point of intersection is M (1, 0).

(iv) $y = 2x + 2$

$3x + 2y = 4$

Sol. $y = 2x + 2$ ________ (i)

$3x + 2y = 4$ ________ (ii)

We draw the graph of these two equations.

From (i)

$y = 2x + 2$

Table of the values for $y = 2x + 2$

x	y = (2x + 2)	(x, y)
5	$y = 2(5) + 2 = 12$	(5, 12)
–5	$y = 2(-5) + 2 = -8$	(–5, –8)

From (ii) $\quad 3x + 2y = 4$

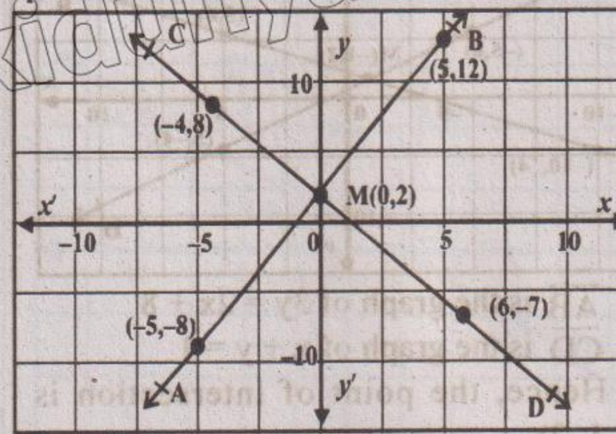
$$2y = -3x + 4$$

$$y = \frac{-3x+4}{2}$$

Table of the values for $y = \dfrac{-3x+4}{2}$

x	$y = \dfrac{-3x+4}{2}$	(x, y)
6	$y = \dfrac{-3(6)+4}{2} = -7$	(6, –7)
–4	$y = \dfrac{-3(-4)+4}{2} = 8$	(–4, 8)

Now we plot points for both of these equations.



$\overline{AB}$ is the graph of $y = 2x + 2$

$\overline{CD}$ is the graph of $3x + 2y = 4$

Hence, the point of intersection is M (0, 2).

(v) $3y = 2x + 8$

$x + y = 1$

Sol. $3y = 2x + 8$ ________ (i)

$x + y = 1$ ________ (ii)

We draw the graph of these two equations.

From (i)

$3y = 2x + 8$

$$y = \frac{2x+8}{3}$$

Table of the values for $3y = 2x + 8$

x	$y = \dfrac{2x+8}{3}$	(x, y)
5	$y = \dfrac{2(5)+8}{3} = 6$	(5, 6)
–10	$y = \dfrac{2(-10)+8}{3} = -4$	(–10, –4)

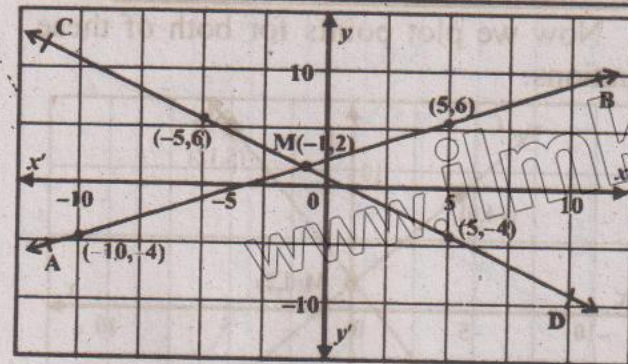
From (ii) $\quad x + y = 1$

$$y = 1 - x$$

Table of the values for $x + y = 1$

x	y = 1 – x	(x, y)
5	$y = 1 - 5 = -4$	(5, –4)
–5	$y = 1 - (-5) = 6$	(–5, 6)

Now we plot points for both of these equations.



$\overleftrightarrow{AB}$ is the graph of $3y = 2x + 8$

$\overleftrightarrow{CD}$ is the graph of $x + y = 1$

Hence, the point of intersection is M $(-1, 2)$

3. Solve the following equations graphically.

(i) $y = 8x - 32$

$y = x^2 - 6x + 8$

Sol. $y = 8x - 32$ ______ (i)

$y = x^2 - 6x + 8$ ______ (ii)

We draw the graph of both of these two equations.

From (i)

$$y = 8x - 32$$

Table of the values for $y = 8x - 32$

x	$y = 8x - 32$	(x, y)
8	$y = 8(8) - 32 = 32$	(8, 32)
5	$y = 8(5) - 32 = 8$	(5, 8)

Now $\quad y = x^2 - 6x + 8$

$= x^2\ 6x + (3)^2 - (3)^2 + 8$

$= (x-3)^2 - 9 + 8$

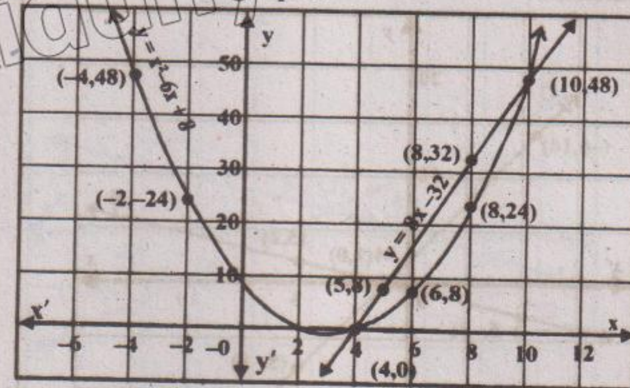
$= (x - 3)^2 - 1$

So the vertex is $(3, -1)$

Table of the values for $y = x^2 - 6x + 8$

x	$y = x^2 - 6x + 8$	(x, y)
8	$y = (8)^2 - 6(8) + 8$ $= 64 - 48 + 8 = 24$	(8, 24)
–4	$y = (-4)^2 - 6(-4) + 8$ $= 16 + 24 + 8 = 48$	(–4, 48)
–2	$y = (-2)^2 - 6(-2) + 8$ $= 4 + 12 + 8 = 24$	(–2, 24)
6	$y = (6)^2 - 6(6) + 8$ $= 36 - 36 + 8 = 8$	(6, 8)

Now we plot points of both equations and draw their graph.



Points of intersection are: (4,0) and (10, 48).

(ii) $y + x = 2$

$y = 2x^2 + x - 10$

Sol. $y + x = 2$ ______ (i)

$y = 2x^2 + x - 10$ ______ (ii)

We draw the graph of both of these two equations.

From (i)

$$y + x = 2$$

$$y = 2 - x$$

Table of the values for $y = 2-x$

x	$y = 2 - x$	(x, y)
10	$y = 2 - 10 = -8$	(10, –8)
0	$y = 2 - 0 = 2$	(0, 2)

Now $\quad y = 2x^2 + x - 10$

$$= 2\left(x^2 + \frac{x}{2} - 5\right)$$

$$= 2\left[x^2 + \frac{x}{2} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 5\right]$$

$$= 2\left[\left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - 5\right]$$

$$= 2\left[\left(x + \frac{1}{4}\right)^2 - \frac{1-80}{16}\right]$$

$$= 2\left[\left(x + \frac{1}{4}\right)^2 - \left(\frac{-79}{16}\right)\right]$$

$$= 2\left[\left(x+\frac{1}{4}\right)^2+\frac{79}{16}\right]$$

$$= 2\left(x+\frac{1}{4}\right)^2+\frac{158}{16}$$

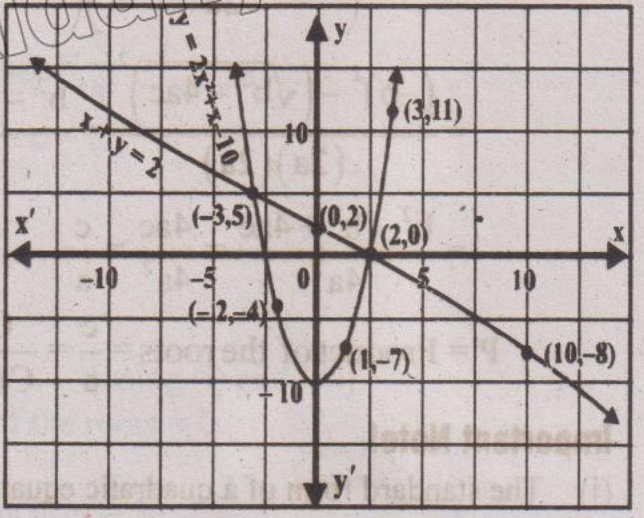
$$= 2(x+0.25)^2+9.9$$

So, the vertex is $(-0.25, 9.9)$

Table of the values for $y = 2x^2 + x - 10$

x	$y = 2x^2 + x - 10$	(x, y)
3	$y = 2(3)^2 + 3 - 10$ $= 18 + 3 - 10 = 11$	(3, 11)
1	$y = 2(1)^2 + 1 - 10$ $= 2 + 1 - 10 = -7$	(1, –7)
–2	$y = 2(-2)^2 - 2 - 10$ $= 8 - 12 = -4$	(–2, –4)
–3	$y = 2(-3)^2 - 3 - 10$ $= 18 - 13 = 5$	(–3, 5)

Now, we plot points of both equations and draw their graphs.



Points of intersection are:

(2, 0) and (–3, 5)

2.3 Relationship Between the Roots and Coefficients of a Quadratic Equation

The standard form of a quadratic equation is $ax^2 + bx + c = 0$, $a \neq 0$ and $a, b, c \in R$.
The roots of this equation are:

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Let these roots be α, β

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}, \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Now S = Sum of the roots

$$= \alpha+\beta = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-2b}{2a} = \frac{-b}{a}$$

$$S = \text{Sum of the roots} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

P = Product of the roots

$$= \alpha\beta = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$= \frac{(-b)^2 - \left(\sqrt{b^2-4ac}\right)^2}{(2a)(2a)} = \frac{b^2-(b^2-4ac)}{4a^2}$$

$$= \frac{b^2-b^2+4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

$$P = \text{Product of the roots} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Important Note!

(i) The standard form of a quadratic equation is $ax^2 + bx + c = 0$ where $a \neq 0$ and $a, b, c \in R$

(ii) Sum of the roots $= -\frac{b}{a}$, Product of the roots $= \frac{c}{a}$

(iii) When the coefficient of x^2 in a quadratic equation is unity (one).

- Sum of roots is equal to the coefficient of x with sign changed.
- Product of the roots is equal to the constant term.

(iv) If the constant term of $ax^2 + bx + c = 0$ is zero, then sum of its roots $= \frac{-b}{a}$ and products of the roots = 0

Example 9: Find the sum and the product of the roots of the equation $3x^2 + 5x - 12 = 0$ without solving.

Sol. Standard form of the quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$ and a, b, c $\in$ R. The given equation is $3x^2 + 5x - 12 = 0$.

Here $a = 3, b = 5, c = -12$

$$S = \text{Sum of the roots} = -\frac{b}{a} = -\frac{5}{3}$$

$$P = \text{Product of the roots} = \frac{c}{a} = \frac{-12}{3} = -4$$

Example 10: Find the sum and the product of the roots of the quadratic equation $x^2 + 6x + 8 = 0$

Sol. Coefficient of x^2 is unity

Sum of the roots = -6 [Coefficient of x with sign changed]

Product of the roots = 8 [Constant term]

Form a Quadratic Equation When Roots are Given

The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$ and a, b, c $\in$ R.

Let α, β be its roots. We have established a relation between the roots and coefficients.

$$S = \text{Sum of the roots} = \alpha + \beta = -\frac{b}{a} \quad \text{\_\_\_\_\_ (i)}$$

$$P = \text{Product of the roots} = \alpha\beta = \frac{c}{a} \quad \text{\_\_\_\_\_ (ii)}$$

The given quadratic equation is $ax^2 + bx + c = 0$.
Dividing this equation by a, we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Writing this equation in terms of α, β

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0 \qquad \text{[using (i) and (ii)]}$$

or $x^2 - (\text{Sum of the roots})\,x + \text{Product of the roots} = 0$

or $x^2 - Sx + P = 0$

Example 11: Form the equation whose roots are $\frac{2}{3}$ and $\frac{3}{4}$.

Sol. Method-I

Here $x = \frac{2}{3}, x = \frac{3}{4}$

$\therefore \quad x - \frac{2}{3} = 0$ or $x - \frac{3}{4} = 0$

Both of these statements are included in

$$\left(x - \frac{2}{3}\right)\left(x - \frac{3}{4}\right) = 0$$

$$x^2 - \frac{3}{4}x - \frac{2}{3}x + \frac{1}{2} = 0$$

$$x^2 - \frac{17}{12}x + \frac{1}{2} = 0$$

$12x^2 - 17x + 6 = 0$ is the required equation.

Method-II

Roots of the required equation are $\frac{2}{3}, \frac{3}{4}$

$$S = \text{Sum of the roots} = \frac{2}{3} + \frac{3}{4}$$

$$= \frac{8+9}{12} = \frac{17}{12} \quad \text{\_\_\_\_\_ (i)}$$

$$P = \text{Product of the roots} = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2} \quad \text{\_\_\_\_\_ (ii)}$$

Required equation is

$$x^2 - Sx + P = 0$$

Putting values of S and P in above equation

$$x^2 - \frac{17}{12}x + \frac{1}{2} = 0 \qquad \text{[From (i) and (ii)]}$$

So, $\quad 12x^2 - 17x + 6 = 0$

Example 12: If α, β are roots of the equation $x^2 - 7x + 10 = 0$, then form an equation whose roots are $2\alpha + 1$ and $2\beta + 1$.

Sol. As α, β are roots of $x^2 - 7x + 10 = 0$, therefore

$$\alpha + \beta = -\frac{-7}{1} = 7 \quad \text{(i)}, \qquad \alpha\beta = \frac{10}{1} = 10 \quad \text{(ii)}$$

S = Sum of roots of the required equation $2\alpha + 1 + 2\beta + 1$

$$= 2(\alpha + \beta) + 2$$
$$= 2(7) + 2 = 16$$

P = Product of roots of the required equation $= (2\alpha + 1)(2\beta + 1)$

$$= 4\alpha\beta + 2(\alpha + \beta) + 1$$
$$= 4(10) + 2(7) + 1 = 55$$

The required equation will be

$$x^2 - Sx + P = 0$$
$$x^2 - 16x + 55 = 0 \qquad \text{[Putting values of S and P]}$$

Example 13: Find the condition that the roots of $ax^2 + bx + c = 0$ may be equal in magnitude but opposite in sign, where $a \neq 0$.

Sol. $ax^2 + bx + c = 0$ (given equation)

Let $\alpha, -\alpha$ be the roots of this equation

$$S = \text{Sum of the roots} = \alpha + (-\alpha) = -\frac{b}{a}$$

$$0 = -\frac{b}{a} \qquad \text{where } a \neq 0$$

$\therefore \quad b = 0$ is the required condition.

Challenge

If one root of the quadratic equation is twice the other and their product is 8, find the equation and both roots.

Sol: Let the roots α, β

Condition I

$$\beta = 2\alpha$$

Condition II

$$\alpha\beta = 8$$
$$\alpha(2\alpha) = 8 \qquad \because \beta = 2\alpha$$
$$2\alpha^2 = 8$$
$$\alpha^2 = \frac{8}{2}$$
$$\alpha^2 = 4$$

Taking square root of both sides, we get

$$\sqrt{\alpha^2} = \sqrt{4}$$

$$\alpha = \pm 2$$

If $\alpha = 2$ then $\beta = 2(2) = 4$

So, roots are 2 and 4.

S = Sum of the roots = 2 + 4 = 6

P = Product of the roots = (2) (4) = 8

Required equation

$$x^2 - Sx + P = 0$$

$$x^2 - 6x + 8 = 0$$

or

If $\alpha = -2$ then $B = 2(-2) = -4$

roots = −2 are −4

S = Sum of the roots = −2 + (−4) = −6

P = Product of the roots = −2 (−4) = + 8

Required equation is

$$x^2 - Sx + P = 0$$

$$x^2 - (-6)x + 8 = 0$$

$$x^2 + 6x + 8 = 0$$

Solved Exercise 2.3

1. Form a quadratic equation whose roots are given below:

(i) −4, 9

Sol. Roots of the required equation are −4, 9

S = Sum of the roots = −4 + 9 = 5 ______ (i)

P = Product of the roots = (−4) (9) = −36 ______ (ii)

Required equation

$$x^2 - Sx + P = 0$$

Putting values of S and P in above equation

$$x^2 - 5x + (-36) = 0 \text{ (from (i) and (ii))}$$

$$x^2 - 5x - 36 = 0$$

(ii) 5, −7

Sol. Roots of the required equation are 5, −7

S = Sum of the roots = 5 + (−7) = −2 ______ (i)

P = Product of the roots = (5) (−7) = −35 ______ (ii)

Required equation is

$$x^2 - Sx + P = 0$$

Putting values of S and P in the above equation

$$x^2 - (-2)x + (-35) = 0 \text{ (from (i) and (ii))}$$

$$x^2 + 2x - 35 = 0$$

(iii) $\frac{-7}{5}, \frac{-6}{5}$

Sol. Roots of the required equation are $\frac{-7}{5}, \frac{-6}{5}$

S = Sum of the roots $= \frac{-7}{5} + \left(\frac{-6}{5}\right)$

$$= \frac{-7-6}{5} = \frac{-13}{5} \text{ ______ (i)}$$

$$P = \text{Product of the roots} = \left(\frac{-7}{5}\right)\left(\frac{-6}{5}\right) = \frac{42}{25} \text{ ______ (ii)}$$

Required equation is $x^2 - Sx + P = 0$

Putting values of S and P in the above equation

$$x^2 - \left(\frac{-13}{5}x\right) + \frac{42}{25} = 0 \text{ (from (i), and (ii))}$$

$$x^2 + \frac{13}{5}x + \frac{42}{25} = 0$$

Multiplying throughout by 25, we get

$$25\left(x^2\right) + 25\left(\frac{13}{5}\right)x + 25\left(\frac{42}{25}\right) = 25(0)$$

$$25x^2 + 65x + 42 = 0$$

(iv) $\frac{-3}{2}, \frac{7}{2}$

Sol. Roots of the required equation are $\frac{-3}{2}, \frac{7}{2}$

$$S = \text{Sum of the roots} = \frac{-3}{2} + \frac{7}{2}$$

$$= \frac{-3+7}{2} = \frac{4}{2} = 2 \text{ ______ (i)}$$

$$P = \text{Product of the roots} = = \left(\frac{-3}{2}\right)\left(\frac{7}{2}\right) = \frac{-21}{4} \text{ ______ (ii)}$$

Required equation is $x^2 - Sx + P = 0$

Putting values of S and P in above equation

$$x^2 - 2x + \left(\frac{-21}{4}\right) = 0 \text{ (from (i) and (ii))}$$

$$x^2 - 2x - \frac{21}{4} = 0$$

Multiplying throughout by 4, we get

$$4(x^2 - 4(2x)) - 4\left(\frac{21}{4}\right) = 4(0)$$

$$4x^2 - 8x - 21 = 0$$

(v) $3+\sqrt{5}, 3-\sqrt{5}$

Sol. Roots of the required equation are $3+\sqrt{5}, 3-\sqrt{5}$

$$S = \text{Sum of the roots} = \left(3+\sqrt{5}\right)+\left(3-\sqrt{5}\right)$$
$$= 3+\sqrt{5}+3-\sqrt{5} = 6 \quad \text{\_\_\_\_\_\_ (i)}$$
$$P = \text{Product of the roots} = \left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right)$$
$$= (3)^2-\left(\sqrt{5}\right)^2$$
$$= 9-5 = 4 \quad \text{\_\_\_\_\_\_ (ii)}$$

Required equation is $x^2 - Sx + Px = 0$

Putting values of S and P in above equation

$$x^2 - 6x + 4 = 0 \text{ (from (i) and (ii))}$$

(vi) $-2+\sqrt{3}, -2-\sqrt{3}$

Sol. Roots of the required equation are $-2+\sqrt{3}, -2-\sqrt{3}$

$$S = \text{Sum of the roots} = \left(-2+\sqrt{3}\right)+\left(-2-\sqrt{3}\right)$$
$$= -2+\sqrt{3}-2-\sqrt{3}$$
$$= -4 \quad \text{(i)}$$
$$P = \text{Product of the roots} = \left(-2+\sqrt{3}\right)\left(-2-\sqrt{3}\right)$$
$$= (-2)^2-\left(\sqrt{3}\right)^2$$
$$= 4-3 = 1 \quad \text{\_\_\_\_\_\_ (ii)}$$

Required equation is $x^2 - Sx + P = 0$

Putting values of S and P in the above equation

$$x^2 - (-4)x + 1 = 0 \text{ (from (i) and (ii))}$$
$$x^2 + 4x + 1 = 0$$

2. Find the quadratic equation with roots exceeding by 2 than those of roots of $x^2 + 9x + 20 = 0$.

Sol. $x^2 + 9x + 20 = 0$ (given)

$a = 1, b = 9, c = 20$

Let roots of the given equation α, β.

$$S = \alpha+\beta = \frac{-b}{a} = \frac{-9}{1} = -9 \quad \text{\_\_\_\_\_\_ (i)}$$

$$P = \alpha\beta = \frac{c}{a} = \frac{20}{1} = 20 \quad \text{\_\_\_\_\_\_ (ii)}$$

New roots $a + 2, \beta + 2$

$$S` = \text{Sum of new roots} = (\alpha+2)+(\beta+2)$$
$$= \alpha+2+\beta+2$$
$$= \alpha+\beta+4 \quad \text{\_\_\_\_\_\_ (iii)}$$
$$P` = \text{Product of new roots} = (\alpha+2)(\beta+2)$$
$$= \alpha\beta + 2\alpha + 2\beta + 4$$
$$= \alpha\beta + 2(\alpha+\beta) + 4 \quad \text{\_\_\_\_\_\_ (iv)}$$

Required equation $x^2 - Sx + P = 0$

Putting the values of S and P in the above equation

$$x^2 - (\alpha + \beta + 4)x + \alpha\beta + 2(\alpha + \beta) + 4 = 0 \quad \text{\_\_\_\_\_\_ (v)}$$

Putting the values of $\alpha + \beta$ and $\alpha\beta$ from (i) and (ii) in equation (v), we get

$x^2 - (-9 + 4)x + 20 + 2(-9) + 4 = 0$

$x^2 - (-5)x + 20 - 18 + 4 = 0$

$x^2 + 5x + 6 = 0$

3. Find the equation whose roots are double the roots of $x^2 - px + q = 0$.

Sol. $x^2 - px + q = 0$ (Given)

$a = 1, b = -p, c = q$

Let the roots of the given equation are α and β

$$S = \alpha + \beta = \frac{-b}{a} = \frac{-(-p)}{1} = p \quad \text{\_\_\_\_\_\_ (i)}$$

$$P = \alpha\beta = \frac{c}{a} = \frac{q}{1} = q \quad \text{\_\_\_\_\_\_ (ii)}$$

New roots $2\alpha, 2\beta$

$S` = \text{Sum of new roots} = 2\alpha + 2\beta = 2(\alpha + \beta)$ ______ (iii)

$P` = \text{Product of new roots} = (2\alpha)(2\beta) = 4\alpha\beta$ ______ (iv)

Required equation is $x^2 - Sx + P = 0$

Putting the values of S and P from (iii) and (iv)

$$x^2 - 2(\alpha + \beta)x + 4\alpha\beta = 0 \quad \text{\_\_\_\_\_\_ (v)}$$

Putting the values of $\alpha+\beta$ and $\alpha\beta$ from (i) and (ii) in equation (v), we get

$$x^2 - 2(p)x + 4(q) = 0$$

$$x^2 - 2px + 4q = 0$$

4. If α, β are the roots of the equation $x^2 + 2x + 4 = 0$, then find the equation whose roots are:

(i) $\frac{1}{\alpha}, \frac{1}{\beta}$ **(ii)** $\frac{\alpha}{B}, \frac{B}{\alpha}$ **(iii)** $2\alpha - \frac{1}{\beta}, 2\beta - \frac{1}{\alpha}$

(iv) α^2, β^2 **(v)** $2\alpha - 1, 2\beta - 1$

Sol. $x^2 + 2x + 4 = 0$ (Given)

$a = 1, b = 2, c = 4$

Let the roots of the given equation are α and β.

$$S = \alpha + \beta = \frac{-b}{a} = \frac{-2}{1} = -2 \quad \text{\_\_\_\_\_\_ (i)}$$

$$P = \alpha\beta = \frac{c}{a} = \frac{4}{1} = 4 \quad \text{\_\_\_\_\_\_ (ii)}$$

(i) $\frac{1}{\alpha},\frac{1}{\beta}$

Given roots are $\frac{1}{\alpha},\frac{1}{\beta}$

S = Sum of the given roots $=\frac{1}{\alpha}+\frac{1}{\beta}$

$=\frac{\beta+\alpha}{\alpha\beta}$

$=\frac{\alpha+\beta}{\alpha\beta}$

Putting the values of $\alpha+\beta$ and $\alpha\beta$ from (i) and (ii)

$=\frac{-2}{4}=\frac{-1}{2}$ (A)

P = Product of the given roots $=\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)=\frac{1}{\alpha\beta}$

Putting the value of $\alpha\beta$ from (ii)

$=\frac{1}{4}$ ______ (B)

Required equation is

$x^2-Sx+P=0$

Putting the values of S and P from (A) and (B)

$$x^2-\left(-\frac{1}{2}\right)x+\frac{1}{4}=0$$

$$x^2+\frac{1}{2}x+\frac{1}{4}=0$$

$$4x^2+4\left(\frac{1}{2}\right)x+4\left(\frac{1}{4}\right)=4(0)$$

$$4x^2+2x+1=0$$

(ii) $\frac{\alpha}{\beta},\frac{\beta}{\alpha}$

Given root are $\frac{\alpha}{\beta},\frac{\beta}{\alpha}$

S = Sum of the given roots $=\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

$=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

Putting the values of $\alpha+\beta$ and $\alpha\beta$ from (i) and (ii)

$$=\frac{(-2)^2-2(4)}{4}$$

$$=\frac{4-8}{4}$$

$=\frac{-4}{4}=-1$ ______ (A)

P = Product of the given roots $=\left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right)=1$ ______ (B)

Required equation is $x^2-Sx+P=0$

Putting the values of S and P from (A) and (B)

$$x^2-(-1)x+1=0$$

$$x^2+x+1=0$$

(iii) $2\alpha-\frac{1}{\beta},2\beta-\frac{1}{\alpha}$

Given roots are $2\alpha-\frac{1}{\beta},2\beta-\frac{1}{\alpha}$

S = Sum of the given roots

$$=\left(2\alpha-\frac{1}{\beta}\right)+\left(2\beta-\frac{1}{\alpha}\right)$$

$$=2\alpha-\frac{1}{\beta}+2\beta-\frac{1}{\alpha}$$

$$=2\alpha+2\beta-\frac{1}{\alpha}-\frac{1}{\beta}$$

$$=2(\alpha+\beta)-\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$$

$$=2(\alpha+\beta)-\left(\frac{\beta+\alpha}{\alpha\beta}\right)$$

$= 2(\alpha+\beta) - \left(\frac{\alpha+\beta}{\alpha\beta}\right)$

Putting the values of $\alpha + \beta$ and $\alpha\beta$ from (i) and (ii)

$= 2(-2) - \left(-\frac{2}{4}\right)$

$= -4 + \frac{1}{2}$

$= \frac{-8+1}{2} = \frac{-7}{2}$ ______ (A)

P = Product of the given roots

$= \left(2\alpha - \frac{1}{\beta}\right)\left(2\beta - \frac{1}{\alpha}\right)$

$= 4\alpha\beta - \frac{2\alpha}{\alpha} - \frac{2\beta}{\beta} + \frac{1}{\alpha\beta}$

$= 4\alpha\beta - 2 - 2 + \frac{1}{\alpha\beta}$

$= 4\alpha\beta - 4 + \frac{1}{\alpha\beta}$

Putting the values of $\alpha\beta$ from (ii)

$= 4(4) - 4 + \frac{1}{4}$

$= 16 - 4 + \frac{1}{4}$

$= 12 + \frac{1}{4}$

$= \frac{48+1}{4}$

$= \frac{49}{4}$ ______ (B)

The required equation is $x^2 - Sx + P = 0$

Putting the values of S and P from A and B

$x^2 - \left(-\frac{7}{2}\right)x + \frac{49}{4} = 0$

$4x^2 + 4\left(\frac{7}{2}\right)x + 4\left(\frac{49}{4}\right) = 0$

$4x^2 + 14x + 49 = 0$

(iv) α^2, β^2

Given roots are α^2 and β^2

S = Sum of the given roots

$= \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

Putting the values of $\alpha + \beta$ and $\alpha\beta$ from (i) and (ii)

$= (-2)^2 - 2(4)$

$= 4 - 8 = -4$ ______ (A)

P = Product of the given roots $= \alpha^2\beta^2$

$= (\alpha\beta)^2$

Putting the value of $\alpha\beta$ from (ii)

$= (4)^2 = 16$ ______ (B)

The required equation is $x^2 - Sx + P = 0$

Putting the values of S and P from A and B

$x^2 - (-4)x + 16 = 0$

$x^2 + 4x + 16 = 0$

(v) $2\alpha - 1, 2\beta - 1$

Given roots are $2\alpha - 1, 2\beta - 1$

S = Sum of the given roots

$= (2\alpha - 1) + (2\beta - 1)$

$= 2\alpha - 1 + 2\beta - 1$

$= 2\alpha + 2\beta - 1 - 1$

$= 2(\alpha + \beta) - 2$

Putting the value of $\alpha + \beta$ from (i)

$= 2(-2) - 2$

$= -4 - 2 = -6$ ______ (A)

P = Product of the given roots

$= (2\alpha - 1)(2\beta - 1)$

$= 4\alpha\beta - 2\alpha - 2\beta + 1$

$= 4\alpha\beta - 2(\alpha + \beta) + 1$

Putting the values of $\alpha + \beta$ and $\alpha\beta$ from (i) and (ii)

$= 4(4) - 2(-2) + 1$

$= 16 + 4 + 1 = 21$ ______ (B)

Required equation is $x^2 - Sx + P = 0$

Putting the values of S and P from (A) and (B)

$\therefore \quad x^2 - (-6)x + 21 = 0$

$\therefore \quad x^2 + 6x + 21 = 0$

5. Find the condition that roots of $ax^2 + bx + c = 0$ should be reciprocals of each other.

Sol. $ax^2 + bx + c = 0$

$S = \text{Sum of the roots} = -\dfrac{b}{a}$ ______ (i)

$P = \text{Product of the roots} = \dfrac{c}{a}$ ______ (ii)

If roots are a and $\dfrac{1}{a}$, then product

$P = a \times \dfrac{1}{a} = 1$ ______ (iii)

$P = \dfrac{c}{a}$ from (ii)

Putting P = 1 from (iii)

$1 = \dfrac{c}{a}$

$a = c$

The roots should be reciprocal of each other if a = c

6. Find the value of k given that one root of $x^2 - (2k + 4)x + (7k + 1) = 0$ is 3.

Sol. $x^2 - (2k + 4)x + (7k + 1) = 0$ ______ (i) (Given)

one root of the given equation = 3

Putting 3 in (i)

$(3)^2 - (2k + 4)3 + (7k + 1) = 0$

$9 - 6k - 12 + 7k + 1 = 0$

$-6k + 7k + 9 - 12 + 1 = 0$

$k - 2 = 0$

$k = 2$

7. Find the value of m in the equation $2x^2 + 3x + m = 0$ when sum of its roots is equal to double the product of its roots.

Sol. $2x^2 + 3x + m = 0$

$a = 2,\ b = 3,\ c = m$

$S = -\dfrac{b}{a} = -\dfrac{3}{2}$ ______ (i)

$P = \dfrac{c}{a} = \dfrac{m}{2}$ ______ (ii)

According to the given condition

$S = 2P$

Putting the values of S and P from (i) and (ii)

$-\dfrac{3}{2} = 2\left(\dfrac{m}{2}\right)$

$-\dfrac{3}{2} = m$

$m = -\dfrac{3}{2}$

8. If α, β are the roots of $x^2 + ax + b = 0$ and α^2, β^2 are the roots of $x^2 + Ax + B = 0$, prove that $A = 2b - a^2$, $B = b^2$.

Sol. From $x^2 + ax + b = 0$

Roots are α and β

$\alpha + \beta = -\dfrac{b}{a} = -\dfrac{a}{1} = -a$ ______ (i)

$\alpha\beta = \dfrac{c}{a} = \dfrac{b}{1} = b$ ______ (ii)

For $x^2 + Ax + B = 0$

roots are α^2, β^2

$\alpha^2 + \beta^2 = -\dfrac{b}{a}$

$\alpha^2 + \beta^2 = -\dfrac{A}{1}$

$\alpha^2 + \beta^2 = -A$

$(\alpha + \beta)^2 - 2\alpha\beta = -A$

Putting the values of $\alpha + \beta$ and $\alpha\beta$ from (i) and (ii)

$(-a)^2 - 2(b) = -A$

$a^2 - 2b = -A$

$A = 2b - a^2$ (Proved)

$$\alpha^2\beta^2 = \frac{c}{a}$$

$$\alpha^2\beta^2 = \frac{B}{1}$$

$\alpha^2\beta^2 = B$

$(\alpha\beta)^2 = B$

Putting value of $\alpha\beta$ from (ii)

$b^2 = B$

$B = b^2$ (Proved)

9. If α, β are the roots of $x^2 + px + q = 0$, then find the condition that

(i) $\alpha = \beta$

Sol. $x^2 + px + q = 0$ (given equation)

Root of equation are α, β

$$\alpha+\beta = -\frac{b}{a} = \frac{-p}{1} = -p$$

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$

If $\alpha = \beta$, then

$\alpha + \alpha = 2\alpha = -p$

$$\alpha = \frac{-p}{2}$$

$$\alpha^2 = \left(-\frac{p}{2}\right)^2$$

$$\alpha^2 = \frac{p^2}{4} \text{ ——— (i)}$$

$\alpha\alpha = \alpha^2 = q$

$\alpha^2 = q$ ——— (ii)

Equation the values of α^2, we get

$$\frac{p^2}{4} = q$$

$p^2 = 4q$

So the required condition is $p^2 = 4q$

(ii) $\alpha = \dfrac{1}{\beta}$

Sol. $x^2 + px + q = 0$ (Given)

Roots of equation are α, β

$$\alpha+\beta = -\frac{b}{a} = \frac{-p}{1} = -p \text{ ——— (i)}$$

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q \text{ ——— (ii)}$$

$$\alpha = \frac{1}{\beta} \quad \text{(condition)}$$

If one root is reciprocal of other then their product will be

$\alpha\beta = q$ from (i)

$$\frac{1}{\beta}(\beta) = q$$

$1 = q$

$q = 1$

So, the required equation is $q = 1$

2.4 Discriminant of a Quadratic Equation

The quadratic equation $ax^2+bx+c=0$, where $a \neq 0$ and $a,b,c \in R$ has two roots.

Let these roots be α and β.

Now, $\alpha = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$

Discriminant

The expression $b^2 - 4ac$, under the radical sign in quadratic formula is called the Discriminant (Disc.) of the equation, $ax^2 + bx + c = 0$. It determines the nature of character of the roots of the equation.

Nature of the Roots of a Quadratic Equation

Before proceeding to discuss the nature or character of the roots of a quadratic equation, it is important that the students should clearly distinguish among the terms rational, irrational, real and imaginary.

Now, $\sqrt{25}$ or 5, $4\frac{1}{2}, \frac{5}{6}$ are rational and real. $\sqrt{7}, \sqrt{5}, \sqrt{3}$ are irrational and real.

$\sqrt{-3}, \sqrt{-7}, \sqrt{-4}$ **are imaginary or unreal.**

As we know that the two roots of $ax^2 + bx + c = 0$, where $a \neq 0$ and $a, b, c \in R$ are $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$.

Then we have the following results:

(i) If $b^2 - 4ac > 0$ and is a perfect square, then the roots are rational (real) and unequal.

(ii) If $b^2 - 4ac > 0$ and is not a perfect square, then the roots are irrational (real) and unequal.

(iii) If $b^2 - 4ac = 0$, then the roots are real and equal.

(iv) If $b^2 - 4ac < 0$, then the roots are imaginary (complex conjugates).

By applying these tests, the nature of the roots of any quadratic equation may be determined without solving the equation.

Example:14

Examine the nature of roots of the following quadratic equations:

(i) $3x^2 - 8x - 2 = 0$

(ii) $4x^2 + 4x + 1 = 0$

(iii) $2x^2 + 3x + 4 = 0$

(iv) $x^2 + 4x + 3 = 0$

(i) $3x^2 - 8x - 2 = 0$

Sol. (i) Here, $a = 3, b = -8, c = -2$

$$\begin{aligned} \text{Disc} &= b^2 - 4ac \\ &= (-8)^2 - 4(3)(-2) \\ &= 64 + 24 = 88 > 0 \end{aligned}$$

As the discriminant is positive and not a perfect square. Therefore, the roots are irrational and unequal.

(ii) $4x^2 + 4x + 1 = 0$

Sol. Here, $a = 4, b = 4, c = 1$

$$\begin{aligned} \text{Disc.} &= b^2 - 4ac \\ &= (4)^2 - 4(4)(1) \\ &= (4)^2 - 4(4)(1) \\ &= 16 - 16 = 0 \end{aligned}$$

As the discriminant is zero. Therefore, the roots are real and equal.

(iii) $2x^2 + 3x + 4 = 0$

Sol. Given that $2x^2 + 3x + 4 = 0$

Here, $a = 2, b = 3, c = 4$

$$\begin{aligned} \text{Disc} &= b^2 - 4ac \\ &= (3)^2 - 4(2)(4) \\ &= 9 - 32 \\ &= -23 < 0 \end{aligned}$$

Which is negative. Therefore, the roots are imaginary.

(iv) $x^2 + 4x + 3 = 0$

Sol. Given that $x^2 + 4x + 3 = 0$

Here, $a = 1, b = 4, c = 3$

$$\begin{aligned} \text{Disc.} &= b^2 - 4ac \\ &= (4)^2 - 4(1)(3) \\ &= 16 - 12 \\ &= 4 = 2^2 > 0 \end{aligned}$$

As the discriminant is positive and perfect square. Therefore, the roots are rational and unequal.

Note: If the roots of $ax^2 + bx + c = 0$ are rational, $b^2 - 4ac$ must be a perfect square.

Example 15 Find the value of k if the equation $(k + 1)x^2 + (2k - 1)x + (k - 1) = 0$ has equal roots.

Sol. Here $a = k + 1, b = 2k - 1, c = k - 1$

Disc $= b^2 - 4ac = 0$ (roots are equal)

$$\begin{aligned} (2k - 1)^2 - 4(k + 1)(k - 1) &= 0 \\ (4k^2 - 4k + 1) - 4(k^2 - 1) &= 0 \\ 4k^2 - 4k + 1 - 4k^2 + 4 &= 0 \\ -4k + 5 &= 0 \\ 4k &= 5 \\ k &= \frac{5}{4} \end{aligned}$$

Solved Exercise 2.4

1. Examine the nature of roots of the following quadratic equations:

(i) $3x^2 - 9x - 2 = 0$

Sol. $3x^2 - 9x - 2 = 0$

Here, $a = 3, b = -9, c = -2$

Disc. $= b^2 - 4ac$

$= (-9)^2 - 4(3)(-2)$

$= 81 + 24$

$= 105 > 0$

As the discriminant is positive and not a perfect square. So the roots are irrational and unequal.

(ii) $x^2 + 6x + 9 = 0$

Sol. $x^2 + 6x + 9 = 0$

Here, $a = 1, b = 6, c = 9$

Disc $= b^2 - 4ac$

$= (6)^2 - 4(1)(9)$

$= 36 - 36$

$= 0$

As the discriminant is zero so, the roots are real and equal.

(iii) $2x^2 + 4x + 5 = 0$

Sol. $2x^2 + 4x + 5 = 0$

Here, $a = 2, b = 4, c = 5$

Disc. $= b^2 - 4ac$

$= (4)^2 - 4(2)(5)$

$= 16 - 40$

$= -24 < 0$

As the discriminant is negative. So, the root are imaginary.

(iv) $7x^2 - 6x - 1 = 0$

Sol. $7x^2 - 6x - 1 = 0$

Here, $a = 7, b = -6, c = -1$

Disc $= b^2 - 4ac$

$= (-6)^2 - 4(7)(-1)$

$= 36 + 28$

$= 64 = 8^2$

As the discriminant is positive and perfect square. So, the roots are rational and unequal.

(v) $5x^2 - 2x + 10 = 0$

Sol. Here, $a = 5, b = -2, c = 10$

Disc. $= b^2 - 4ac$

$= (-2)^2 - 4(5)(10)$

$= 4 - 200$

$= -196 < 0$

As the discriminant is negative. So, the roots are imaginary.

(vi) $x^2 - 8x + 16 = 0$

Sol. $x^2 - 8x + 16 = 0$

Here, $a = 1, b = -8, c = 16$

Disc $= b^2 - 4ac$

$= (-8)^2 - 4(1)(16)$

$= 64 - 64$

$= 0$

As the discriminant is zero. So, the roats are real and equal.

2. For what values of t, the roots of $3x^2 + x + 9t = 0$ are real and unequal?

Sol. $3x^2 + x + 9t = 0$

Here, $a = 3, b = 1, c = 9t$

Disc. $= b^2 - 4ac > 0$ (roots are real and unequal)

$(1)^2 - 4(3)(9t) > 0$

$1 - 108t > 0$

$-108t > -1$

$108t < 1$ (Multiplying both sides by -1 ; also change $>$ into $<$.)

$t < \frac{1}{108}$ (Dividing both sides by 108)

3. If the quadratic equation $16x^2 + 7px + 49 = 0$ has equal roots, then find the values of p.

Sol. $16x^2 + 7px + 49 = 0$

Here, $a = 16, b = 7p, c = 49$

Disc $= b^2 - 4ac = 0$ (roots are equal)

$(7p)^2 - 4(16)(49) = 0$

$49p^2 - 3136 = 0$

$49p^2 = 3136$

$p^2 = \frac{3136}{49}$

$p^2 = 64$

$\sqrt{p^2} = \sqrt{64}$

$P = \pm 8$

4. **If the quadratic equation $4u^2 + 8u + q = 0$ has unequal and real roots, find the possible values for q.**

Sol. $4u^2 + 8u + q = 0$

Here, $a = 4$, $b = 8$, $c = q$

Disc $= b^2 - 4ac > 0$ (roots are unequal *and real)*

$$(8)^2 - 4(4)(q) > 0$$
$$64 - 16q > 0$$
$$-16q > -64$$

$16q < 64$ (Multiplying both sides by -1; also change $>$ into $<$.)

$q < \frac{64}{16}$ (Dividing both sides by 16)

$q < 4$

5. **Find the value of m, if the quadratic equation $mx^2 - 8x + 1 = 0$ has real and equal roots.**

Sol. $mx^2 - 8x + 1 = 0$

Here, $a = m$, $b = -8$, $c = 1$

Disc $= b^2 - 4ac = 0$ (roots are real and equal)

$$(-8)^2 - 4(m)(1) = 0$$
$$64 - 4m = 0$$
$$-4m = -64$$
$$4m = 64$$
$$m = \frac{64}{4}$$
$$m = 16$$

2.5 Solution of Quadratic Inequalities in One Unknown

Inequality In Mathematics an inequality is a relation which makes a un-equal comparison between two numbers or other mathematical expressions. We have learned how to use the inequality symbols $<$ (less than), $>$ (greater than), $\leq$ (less than or equal to) and (greater than or equal to).

Here we learn how to solve quadratic inequalities in one unknown.

There are following four steps to solve quadratic inequality:

(i) Solve the corresponding equation.
(ii) Determine intervals.
(iii) Test points in each interval.
(iv) Conclusion.

Example 16: Solve the inequality $x^2 - 5x + 6 > 0$.

Solution: $x^2 - 5x + 6 > 0$

Step I: Solve the corresponding or associated equation.

Set the quadratic expression equal to zero (0) to find the roots.

$$x^2 - 5x + 6 = 0$$
$$x^2 - 3x - 2x + 6 = 0$$
$$x(x-3) - 2(x-3) = 0$$
$$(x-2)(x-3) = 0 \quad \text{[Factorize]}$$

Now, $x - 2 = 0$ or $x - 3 = 0$

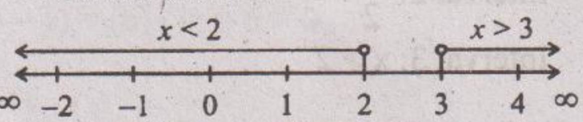
So, the solutions are $x = 2$ and $x = 3$.

Here, critical points (2,0) and (3,0).

Step II: Determine intervals

The critical points divide the number line into three intervals.

- **Interval 1:** $x < 2$.
- **Interval 2:** $2 < x < 3$
- **Interval 3:** $x > 3$



Step III: Test points in each interval.

Choose a test point in each interval and substitute it into the inequality.

- For $x = 1$: $(1-2)(1-3) = 2 > 0$.

  So, this interval satisfies the inequality.

- For $x = 2.5$: $(2.5-2)(2.5-3) = -0.25 < 0$.

  So, this interval does not satisfy the inequality.

- For $x = 4$ $(4-2)(4-3) = 2 > 0$.

  So, this interval satisfies the inequality.

Step IV: Conclusion:

The solution of this inequality $x^2 - 5x + 6 \geq 0$ is $x \in (-\infty, 2) \cup (3, \infty)$

Example:17 Solve the inequality $2x^2 - 5x + 2 \leq 0$.

Solution: $2x^2 - 5x + 2 \leq 0$.

Step I: Solve the corresponding or associated equation.

$$2x^2 - 5x + 2 = 0$$

Factorizing the quadratic equation

$2x^2 - 4x - x + 2 = 0$

$2x(x-2) - 1\,(x-2) = 0$

$(x-2)(2x-1) = 0$

$$(2x-1)(x-2) = 0$$

Now, $2x - 1 = 0$ or $x - 2 = 0$

So, the solutions are $x = \dfrac{1}{2}$ and $x = 2$

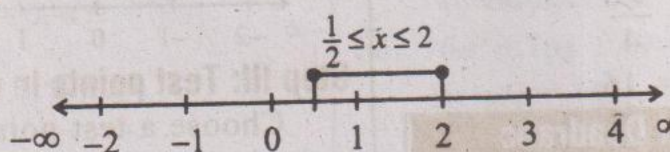
> **Note**
> For solution, we use open interval (a,b) when equation has <or> sign. We use closed interval [a, b] when equation has ≤ or ≥ sign.

Here, critical points are $\left(\dfrac{1}{2}, 0\right)$ and $(2, 0)$.

Step II: Determine Intervals

The critical points $\left(\dfrac{1}{2}, 0\right)$, $(2,0)$ divides the number line into three intervals.

- Interval 1: $x < \dfrac{1}{2}$
- Interval 2: $\dfrac{1}{2} < x < 2$
- Interval 3: $x > 2$



Step III: Test points in each Intervals.

- For $x = 0$: $(2(0) - 1)(0 - 2) = (-1)(-2) = 2 > 0$,
  So, this interval does not satisfy the inequality.
- For $x = 1$: $(2(1) - 1)(1 - 2) = (1)(-1) = -1 < 0$,
  So, this interval satisfies the inequality.
- For $x = 3$: $(2(3) - 1)(3 - 2) = (6 - 1)(1) = 5 > 0$,
  So, this interval does not satisfy the inequality.

Step IV Conclusion:

The solution of the inequality is: $x \in \left[\dfrac{1}{2}, 2\right]$.

Solved Exercise 2.5

1. Solve the following in equalities:

(i) $x^2 + 3x - 4 > 0$

Sol. $x^2 + 3x - 4 > 0$

The corresponding equation is

$x^2 + 3x - 4 = 0$

$x^2 + 4x - x - 4 = 0$

$x(x + 4) - (x + 4) = 0$

$(x + 4)(x - 1) = 0$

$x + 4 = 0$ or $x - 1 = 0$

$x = -4$ or $x = 1$

So, the solutions are $x = -4$, $x = 1$: Hence, the critical points are $(-4, 0)$ and $(1, 0)$.

These critical points divides the number line into three intervals

Interval 1: $x < -4 \Rightarrow x \in (-\infty, -4)$

Interval 2: $4 < x < 1 \Rightarrow x \in (-4, 1)$

Interval 3: $x > 1 \Rightarrow x \in (1, \infty)$

Test points in each interval

For $x = -5$ [in the interval $(-\infty, -4)$]

For $x = -5$ [in the interval $(-\infty, -4)$] : $(-5 + 4)(-5 - 1) = (-1)(-6) = 6 > 0$

So, this interval satisfies the inequality.

For $x = 0$ [interval $(-4, 1)$] : $(0 + 4)(0 - 1) = (4)(-1) = -4 < 0$

So, this interval does not satisfy the inequality.

For $x = 2$ [in the interval $(1, \infty)$] : $(2 + 4)(2 - 1) = (6)(1) = 6 > 0$

So, this interval satisfies the inequality.

Conclusion:

The solution of this inequality is $(-\infty, -4) \cup (1, \infty)$

(ii) $2x^2 - 8x + 6 > 0$

Sol. $2x^2 - 8x + 6 > 0$

The corresponding equation is

$2x^2 - 8x + 6 = 0$

$2x^2 - 6x - 2x + 6 = 0$

$2x(x - 3) - 2(x - 3) = 0$

$(x - 3)(2x - 2) = 0$

$x - 3 = 0$ or $2x - 2 = 0$

$x = 3$ or

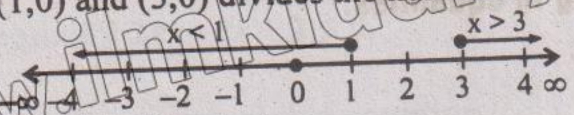
$$x = \frac{2}{2}$$

$x = 1$

So, the solutions are $x = 1, x = 3$.

Hence, the critical points are $(1, 0)$ and $(3, 0)$

These critical points $(1,0)$ and $(3,0)$ divides the number line into three intervals.



Interval 1: $x < 1 \Rightarrow x \in (-\infty, 1)$

Interval 2: $1 < x < 3 \Rightarrow x \in (1, 3)$

Interval 3: $x > 3 \Rightarrow x \in (3, \infty)$

Test points in each interval.

For $x = 0$ [in the interval $(-\infty, 1)$] : $(0 - 3)(2(0) - 2)$

$= (-3)(-2) = 6 > 0$

So, this interval satisfies the inequality.

For $x = 2$ [in the interval $(1, 3)$] : $(2 - 3)(2(2) - 2) = (-1)(4 - 2) = (-1)(2) = -2 < 0$

So, this interval does not satisfy the inequality.

For $x = 4$[interval $(3, \infty)$] : $(4 - 3)(2(4) - 2) = (1)(8 - 2) = (1)(6) = 6 > 0$

So, this interval satisfies the inequality.

Conclusion:

The solution of this inequality is $(-\infty, 1) \cup (3, \infty)$

(iii) $x^2 + x - 6 < 0$

Sol. $x^2 + x - 6 < 0$

The corresponding equation

$x^2 + x - 6 = 0$

$x^2 + 3x - 2x - 6 = 0$

$x(x + 3) - 2(x + 3) = 0$

$(x + 3)(x - 2) = 0$

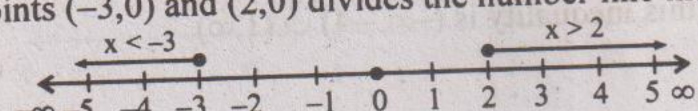
$x + 3 = 0$ or $x - 2 = 0$

$x = -3$ or $x = 2$

So, the solutions are $x = -3$, and $x = 2$

Hence, the critical points are $(-3, 0)$ and $(2, 0)$

These critical points $(-3,0)$ and $(2,0)$ divides the number line into three intervals.



Interval 1: $x < -3 \Rightarrow x \in (-\infty, -3)$

Interval 2: $-3 < x < 2 \Rightarrow x \in (-3, 2)$

Interval 3: $x > 2 \Rightarrow x \in (2, \infty)$

Test points in each interval.

For $x = -4$ [in the interval $(-\infty, -3)$] : $(-4 + 3)(-4 - 2) = (-1)(-6) = 6 > 0$

So, this interval does not satisfy the inequality.

For $x = 0$ [in the interval $(-3, 2)$] : $(0 + 3)(0 - 2) = (3)(-2) = -6 < 0$

So, this interval satisfies the inequality.

For $x = 3$ [in the interval $(2, \infty)$] : $(3 + 3)(3 - 2) = (6)(1) = 6 > 0$

So, this interval does not satisfy the inequality.

Conclusion:

The solution of this inequality is $(-3, 2)$

(iv) $\mathbf{x^2 - 6x + 9 < 0}$

Sol. $x^2 - 6x + 9 < 0$

The corresponding equation is

$x^2 - 6x + 9 = 0$

$x^2 - 3x - 3x + 9 = 0$

$x(x - 3) - 3(x - 3) = 0$

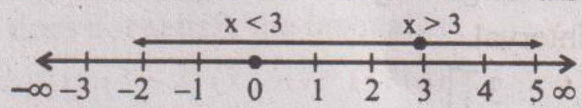
$(x - 3)(x - 3) = 0$

$x - 3 = 0$ or $x - 3 = 0$

$x = 3$ or $x = 3$

So, the solutions is only $x = 3$ (only one critical point)

this critical point divides the number line into two intervals.



Interval 1: $x < 3 \Rightarrow x \in (-\infty, 3)$

Interval 2: $x > 3 \Rightarrow x \in (3, \infty)$

Test points in each interval.

For $x = 0$ [in the interval $(-\infty, 3)$] : $(0 - 3)(0 - 3) = (-3)(-3) = 9 > 0$

So, this interval does not satisfy the inequality.

For $x = 4$ [in the interval $(3, \infty)$] $= (4 - 3)(4 - 3) = (1)(1) = 1 > 0$

So, this interval does not satisfy the inequality.

For $x = 3$ [in the interval $(3, \infty)$] $= (3 - 3)(3 - 3) = 0 < 0$

So, this interval does not satisfy the inequality

There is no real number that satisfies the inequality.

No Solution (Empty or ϕ)

(v) $\mathbf{4x^2 - 16x + 15 \leq 0}$

Sol. $4x^2 - 16x + 15 \leq 0$

The corresponding equation is

$4x^2 - 16x + 15 = 0$ 60

$4x^2 - 10x - 6x + 15 = 0$

$2x(2x - 5) - 3(2x - 5) = 0$

$(2x - 5)(2x - 3) = 0$

$2x - 5 = 0$ or $2x - 3 = 0$

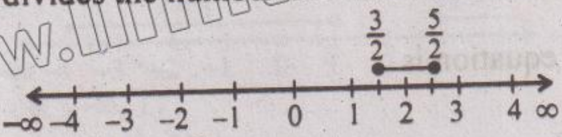
$2x = 5$ or $2x = 3$

$x = \frac{5}{2}$ or $x = \frac{3}{2}$

So, the solutions are $x = \frac{3}{2}$ and $\frac{5}{2}$

The critical points are $\left(\frac{3}{2}, 0\right)$ and $\left(\frac{5}{2}, 0\right)$

These critical points divides the number line into three intervals.



Interval 1: $x < \frac{3}{2} \Rightarrow x \in \left(-\infty, \frac{3}{2}\right]$,

Interval 2: $\frac{3}{2} < x < \frac{5}{2} \Rightarrow x \in \left[\frac{3}{2}, \frac{5}{2}\right]$

Interval 3: $x > \frac{5}{2} \Rightarrow x \in \left[\frac{5}{2}, \infty\right)$

Test points in each interval.

For $x = 0$ [interval $\left(-\infty, \frac{3}{2}\right]$] : $(2(0) - 5)\ (2\ (0) - 3] = (-5)\ (-3) = 15 > 0$

So, this interval does not satisfy the inequality.

For $x = 2$[interval $\left[\frac{3}{2}, \frac{5}{2}\right]$] : $(2\ (2) - 5)\ (2(2) - 3) = (4 - 5)(4 - 3) = (-1)(1) = -1 < 0$

So, this interval satisfies the inequality.

For $x = 3$ [interval $\left[\frac{5}{2}, \infty)\right]$: $(2\ (3) - 5)\ (2(3) - 3) = (6 - 5)\ (6 - 3) = (+1)\ (3) = +3 > 0$

So, this interval does not satisfy the inequality.

Conclusion:

The solution of the inequality is $\left[\frac{3}{2}, \frac{5}{2}\right]$.

(vi) $-x^2 + 3x - 2 \geq 0$

Sol. $-x^2 + 3x - 2 \geq 0$

The corresponding equation is

$-x^2 + 3x - 2 = 0$

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

$x(x - 2) - 1\ (x - 2) = 0$

$(x - 2)\ (x - 1) = 0$

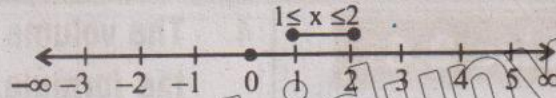
$x - 2 = 0$ or $x - 1 = 0$

$x = 2$ or $x = 1$

So, the solutions are $x = 2$ and $x = 1$

The critical points are $(1,0)$ and $(2, 0)$

These critical points divides the number line into three intervals.



Interval 1: $x < 1 \Rightarrow x \in (-\infty, 1]$
Interval 2: $1 < x < 2 \Rightarrow x \in [1, 2]$
Interval 3: $x > 2 \Rightarrow x \in [2, \infty]$
Test points in each interval.

For $x = 0$: $\left[\text{in the interval}\left(-\infty,-1\right]\right]$: $(0-2)(0-1) = (-2)(-1) = 2 > 0$

So, this interval satisfies the inequality.

For $x = \frac{3}{2}$ [interval $\left[1,2\right]$]: $\left(\frac{3}{2}-2\right)\left(\frac{3}{2}-1\right) = \left(\frac{3-4}{2}\right)\left(\frac{3-2}{2}\right)$

$$= \left(\frac{-1}{2}\right)\left(\frac{1}{2}\right) = -\frac{1}{4} < 0$$

So, this interval does not satisfy the inequality.

For $x = 3$ [interval $\left[2,\infty\right]$: $(3-2)(3-1) = (1)(2) = 2 > 0$

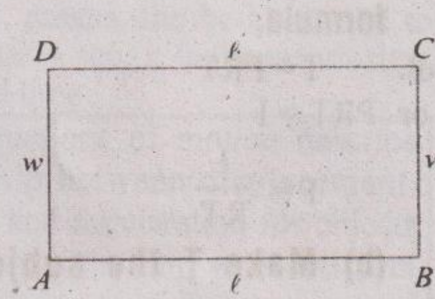
So, this interval satisfies the inequality.

Conclusion: The solution of the inequality is $[1, 2]$

2.6 Changing the subject of Formula

A formula expresses the precise relationship between various measurable quantities in a problem or situation in an equation form.

The figure on the right is of a rectangle. Its length is ℓ units and width is w units. Let P be its perimeter.



Now, $P = \ell + w + \ell + w$
$= 2\ell + 2w$
$P = 2(\ell + w)$ units

We have constructed a formula for perimeter (P) of a rectangle. In the above formula $P = 2(\ell + w)$, the letter P on the left-hand-side is defined as the **subject** of the formula, while the letters on the right-hand-side are called terms of the formula.

Perimeter (P) of square of length (ℓ) is $P = 4\ell$

We change the subject P and rewrite this formula as $\ell = \frac{1}{4}P$, ℓ has become subject of the formula. If value of P is given, then we can directly find the value of ℓ.

Example 18: The area of a circle is $A = \pi r^2$, Make r the subject of the formula.

Solution: $A = \pi r^2$

or $\pi r^2 = A$

$$r^2 = \frac{A}{\pi}$$

$$\therefore\ r = \sqrt{\frac{A}{\pi}}\quad \text{[r is always positive]}$$

Solved Exercise 2.6

1. Make F the subject of the formula,

$C° = \frac{5}{9}(F° - 32)$.

Sol. $C = \frac{5}{9}(F° - 32)$

or $\frac{5}{9}(F° - 32) = C$

$\frac{9}{5} \times \frac{5}{9}(F° - 32) = \frac{9}{5}C$

$F° - 32 = \frac{9}{5}C$

$F° = \frac{9}{5}C + 32$

2. The formula for finding simple interest is $I = PRT$.

(a) Make P the subject of the formula.

Sol. $I = PRT$

or $PRT = I$

$P = \frac{I}{RT}$

(b) Make T the subject of the formula.

Sol. $I = PRT$

or $PRT = I$

$P = \frac{I}{PR}$

3. Make 'a' the subject of the formula $S = 2a + (n-1)d$.

Sol. $S = 2a + (n-1)d$

or $2a + (n-1)d = S$

$2a = S - (n-1)d$

$a = \frac{S-(n-1)d}{2}$

4. The volume of a cylinder is given by the formula, $V = \pi r^2 h$. Make 'h' the subject of the formula.

Sol. $V = \pi r^2 h$

or $\pi r^2 h = V$

$h = \frac{V}{\pi r^2}$

5. The area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, make h the subject of the formula.

Sol. $A = \frac{1}{2}h(b_1 + b_2)$

or $\frac{1}{2}h(b_1 + b_2) = A$

$h(b_1 + b_2) = 2A$

$h = \frac{2A}{b_1 + b_2}$

6. If $y = mx + c$, then make 'x' the subject of this equation.

Sol. $y = mx + c$

or $mx + c = y$

$mx = y - c$

$x = \frac{y-c}{m}$

7. Perimeter (P) of a rectangle is $P = 2(\ell + w)$, make ℓ as the subject of this formula.

Sol. $P = 2(\ell + w)$

or $2(\ell + w) = P$

$\ell + w = \frac{P}{2}$

$\ell = \frac{P}{2} - w$

$\ell = \frac{P-2w}{2}$

8. The equation of a parabola is $y^2 = 4ax$, make 'x' as a subject of this equation.

Sol. $y^2 = 4ax$

or $4ax = y^2$

$$x = \frac{y^2}{4a}$$

9. If $P = S - C$, where S is selling price and C is cost price. Make S as subject of the equation.

Sol. $P = S - C$

or $S - C = P$

$S = P + C$

10. Volume of the cone is $V = \frac{1}{3}\pi r^2 h$, make 'h' as subject of this formula.

Sol. $V = \frac{1}{3}\pi r^2 h$

or $\frac{1}{3}\pi r^2 h = V$

$\pi r^2 h = 3V$

$$h = \frac{3V}{\pi r^2}$$

2.7 Applications of Quadratic Equation in Real World

Example19: A ball is thrown upward with an initial velocity of 20 ms^{-1}. Calculate the maximum height it reaches above ground level. Calculate time of return to the ground.

Solution: Maximum height

At its highest point, the ball's velocity is zero. So, we have the following quantities.

Initial velocity = v_i = 20 ms^{-1}

Final velocity = v_f = 0 ms^{-1}

Gravitational acceleration = g = −10 m s^{-2}

Distance (displacement) = S = ?

The relevant equation of motion is $v_f^2 = v_i^2 + 2gs$

Substituting values of v_i, v_f and g, we get

$0^2 = (20)^2 + 2(-10)s$

$0 = 400 - 20s$

so, $s = \frac{400}{20} = 20\text{m}$

Maximum Time of Return: When the ball returns to the point from which it was thrown, its displacement (s) is zero. We have the following information:

Displacement = s = 0

Initial velocity = v_i = 20ms^{-1}

Gravitational acceleration = g = −10 ms^{-2}

Time = t = ?

The relevant equation of motion:

$$s = v_i t + \frac{1}{2} g t^2$$

Putting the values of s, v_i and g

$0 = 20t + \frac{1}{2}(-10)t^2$

$0 = 20t - 5t^2$

$0 = t[20 - 5t]$

Now $t = 0$ sec or $20 - 5t = 0 \Rightarrow t = \frac{20}{5} = 4$ sec

(i) $t = 0$ sec. means, the ball had zero displacement at the time it was thrown.

(ii) $t = 4$ sec. means, the ball returned to the ground after 4 sec, it is the return time or the required time.

> **Note:** The equations of motion describe the relationship between displacement, velocity, time and acceleration for objects moving in a straight line with uniform acceleration.
>
> First equation : $v_f = v_i + at$
>
> Second equation : $s = v_i t + \frac{1}{2}at^2$
>
> Third equation : $v_f^2 = v_i^2 + 2as$

Example 20: A company models its profit P(rupees in thousands) by the equation $P(x) = -2x^2 + 80x - 500$, where x is the price per item (in rupees). Find the price that gives the maximum profit and calculate the maximum profit.

Solution: $P(x) = -2x^2 + 80x - 500$

Since $-2 < 0$, the parabola opens downward and vertex gives the maximum point.

$$x=\frac{-b}{2a}=\frac{-80}{2(-2)}=\frac{-80}{-4}=20$$

Hence, the price per item that gives maximum profit is Rs. 20.

$$P(20)=-2(20)^2+80(20)-500$$
$$=-2(400)+1600-500$$
$$=300 \text{ (in thousands)}$$

Hence, maximum profit is Rs. 300,000.

> **Note:**
> **We can also find vertex by using**
> $$x=\frac{-b}{2a},\ y=f\left(\frac{-b}{2a}\right)$$

Example 21: A ball is thrown upwards from the ground. Its height after t seconds is modeled by:

$$h(t)=-5t^2+20t \text{ (in metres)}$$

For what time interval is the ball at least 15 metres above the ground?

Solution: Here, $-5t^2+20t\geq 15$

$$-5t^2+20t-15\geq 0$$
$$5t^2-20t+15\leq 0$$

By solving the quadratic equation $5t^2-20t+15=0$.

$$t=\frac{-(-20)\pm\sqrt{(-20)^2-4(5)(15)}}{2(5)}$$
$$=\frac{20\pm\sqrt{400-300}}{10}$$
$$=\frac{20\pm\sqrt{100}}{10}$$
$$=\frac{20\pm 10}{10}$$
$$=\frac{20-10}{10},\frac{20+10}{10}$$
$$=1,3$$

So, the ball is at least 15 metres high when $1\leq t\leq 3$ seconds.

Solved Exercise 2.7

1. A town's population is modeled by $P(t)=-2t^2+40t+800$, where t is years since 2020. Find the year when the population will be at least 1000.

Sol. We need to find t when $P(t)\geq 1000$.

Here,

$$-2t^2+40t+800\geq 1000.$$
$$-2t^2+40t+800-1000\geq 0.$$
$$-2t^2+40t-200\geq 0.$$

Dividing both sides by -2, we get

$$t^2-20t+100\leq 0$$
$$(t)^2-2(t)(10)+(10)^2\leq 0.$$
$$(t-10)^2\leq 0.$$

By taking square root

$t-10=0$ (A squared number can never negative)

$$t=10$$

Where, t represents years since 2020.

Year = 2020 + 10 = 2030

2. A company models its profit P in thousands of rupees by the equation:
$P(x)=-5x^2+150x-1000$, where x is the price per item in rupees. Find the price that gives maximum profit.

Sol. $P(x)=-5x^2+150x-1000$

This is downward – opening parabola (because of $-5x$). The maximum profit occurs at the vertex.

$a=-5$, $b=150$, $c=-1000$

Use the vertex formula for the x-coordinate

$$x=\frac{-b}{2a}=\frac{-150}{2(-5)}=\frac{-150}{-10}=15$$

So, the price that gives maximum profit is 15 Rupees.

3. A toy car rolls down an incline and covers a distance given by the equation $d = t^2 - 0.5t$ metres, where t is the time in seconds. Find the time when the car has travelled a distance 12.5 metres.

Sol. We have to find t when $d = 12.5$

$$d = t^2 - 0.5t$$
$$12.5 = t^2 - 0.5t$$
$$0 = t^2 - 0.5t - 12.5$$
or $\quad t^2 - 0.5t - 12.5 = 0$

Multiply by 2 to remove decimal

$$2t^2 - t - 25 = 0$$
$$a = 2, b = -1, c = -25$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-25)}}{2(2)}$$
$$t = \frac{1 \pm \sqrt{1 + 200}}{4}$$
$$t = \frac{1 \pm \sqrt{201}}{4}$$
$$t = \frac{1 \pm 14.18}{4}$$
$$t = \frac{1 + 14.18}{4}, t = \frac{1 - 14.18}{4}$$
$$t = \frac{15.18}{4}, t = \frac{-13.18}{4} \Rightarrow t = -3.295$$

$t = 3.795$ (As time can't be negative because it starts from 0 (present) and *moves in a positive direction*).

So, the car has travelled 12.5 meters in 3.8 seconds (approximately).

4. A ball's height (in metres) after t seconds is $h(t) = -4t^2 + 24t$. For what time interval is the ball at least 20m above the ground?

Sol. We need to find interval when $h(t) \geq 20$.

Set up the inequality

$$-4t^2 + 24t \geq 20$$
$$-4t^2 + 24t - 20 \geq 0$$

Divide both sides by (-4)

$$t^2 - 6t + 5 \leq 0$$

Associated quadratic equation

$$t^2 - 6t + 5 = 0$$
$$t^2 - 5t - t + 5 = 0$$
$$t(t-5) - 1(t-5) = 0$$
$$(t-5)(t-1) = 0$$
$$t - 5 = 0 \quad \text{or} \quad t - 1 = 0$$
$$t = 5 \quad \text{or} \quad t = 1$$

Since the inequality is ≤ 0 (and the simplified Parabola opens upward), the solution lies between the roots.

Check a test point $t = 2$

$$(2-5)(2-1) = (-3)(+1) = -3 < 0$$

So, the interval is correct.

So, the ball is at least 20m above the ground for the time interval $1 \leq t \leq 5$.

5. A ball is thrown upward with an initial velocity of $40ms^{-1}$. Calculate the maximum height it reaches above ground level.

Sol. At its highest point, the ball's velocity is zero. So we have the following quantities.

Initial velocity $= v_i = 40ms^{-1}$

Final velocity $= v_f = 0ms^{-1}$

Gravitational acceleration $= g = -10ms^{-2}$

Distance (displacement) $= s = ?$

The relevant equation of motion is

$$v_f^2 = v_i^2 + 2gs$$

Substituting values of v_i, v_f and g, we get

$$0^2 - (40)^2 = 2(-10)s$$
$$0 - 1600 = -20s$$
$$-1600 = -20s$$
$$\Rightarrow 20s = 1600$$

$$s = \frac{1600}{20}$$

$$s = 80 \text{ m}$$

So, the maximum height that the ball reached above the ground level is 80m.

6. A freelancer's earnings follow the model $E(h) = -2h^2 + 40h$, where E is earning in rupees and h is hours worked per week. What is the maximum number of hours he should work to maximize earnings?

Sol. $E(h) = -2h^2 + 40h$

We need to maximize the function $E(h)$. This is finding the vertex of the parabola. Identify coefficients:

$a = -2, b = 40$

Use vertex Formula for h:

$$h = \frac{-b}{2a}$$

$$h = \frac{-40}{2(-2)} = \frac{-40}{-4} = 10$$

So, he should work 10 hours per week to maximize the earning.

Solved Review Exercise 2

1. For possible answers are given for the following questions. Choose the correct answer.

(i) The type of the equation $2x^2 - x + 1 = 0$ is:

(a) quadratic (b) linear (c) third degree (d) pure quadratic

(ii) What is the discriminant of $x^2 + 5x - 5 = 0$?

(a) -20 (b) 20 (c) 25 (d) 45

(iii) The solution set of $3x^2 - 9 = 0$ is:

(a) $\{3\}$ (b) $\{\pm 3\}$ (c) $\{\pm\sqrt{3}\}$ (d) $\{\sqrt{3}\}$

(iv) Sum of the roots of $3x^2 + 5x - 12 = 0$ is:

(a) $\frac{5}{3}$ (b) $-\frac{5}{3}$ (c) $\frac{12}{3}$ (d) $\frac{3}{5}$

(v) Product of the roots of $3x^2 + 5x - 12 = 0$ is:

(a) -4 (b) 3 (c) 4 (d) 5

(vi) What are the roots of $(x - 3)(x + 3) = 0$?

(a) $3, -3$ (b) $3, 3$ (c) $-3, -3$ (d) $9, 0$

(vii) 3 and 2 are the roots of:

(a) $x^2 + 5x + 6 = 0$ (b) $x^2 + 6x + 5 = 0$ (c) $x^2 - 5x + 6 = 0$ (d) $x^2 + 6x - 5 = 0$

(viii) If $b^2 - 4ac > 0$ and is a perfect square, then the roots of $ax^2 + bx + c = 0$ are:

(a) equal (b) unequal (c) imaginary (d) irrational

(ix) If $b^2 - 4ac = 0$, then the roots of $ax^2 + bx + c = 0$ are:

(a) unequal (b) irrational (c) imaginary (d) equal

(x) Subject "c" of $x - 2c = b$ is:

(a) $x + b$ (b) $b - x$ (c) $\frac{x-b}{2}$ (d) $\frac{b-x}{2}$

ANSWERS:

(i) quadratic (ii) 45 (iii) $\{\pm\sqrt{3}\}$ (iv) $-\frac{5}{3}$ (v) -4

(vi) 3, −3 (vii) $x^2 + 5x + 6 = 0$ (viii) unequal (ix) equal (x) $\frac{x-b}{2}$

2. Solve the following quadratic equations by factorization method, by completing square method and by quadratic formula:

(i) $8x^2 = x + 7$

Sol. By factorization

$$8x^2 = x + 7$$
$$8x^2 - x - 7 = 0$$
$$8x^2 - 8x + 7x - 7 = 0$$
$$8x(x-1) + 7(x-1) = 0$$
$$(x-1)(8x+7) = 0$$
$$x - 1 = 0 \quad \text{or} \quad 8x + 7 = 0$$
$$x = 1 \quad \text{or} \quad 8x = -7$$
$$x = -\frac{7}{8}$$

So, solution set: $\left\{1, \frac{-7}{8}\right\}$

By completing square

$$8x^2 = x + 7$$

Dividing both sides by 8, we get

$$\frac{8x^2}{8} = \frac{x}{8} + \frac{7}{8}$$
$$x^2 = \frac{x}{8} + \frac{7}{8}$$
$$x^2 - \frac{x}{8} = \frac{7}{8}$$

We add square of half of the co-efficient of x i.e, $\left(\frac{1}{16}\right)^2$ on both sides of the equation.

$$x^2 - \frac{x}{8} + \left(\frac{1}{16}\right)^2 = \frac{7}{8} + \left(\frac{1}{16}\right)^2$$

$$\left(x - \frac{1}{16}\right)^2 = \frac{7}{8} + \frac{1}{256}$$
$$\left(x - \frac{1}{16}\right)^2 = \frac{224+1}{256}$$
$$\left(x - \frac{1}{16}\right)^2 = \frac{225}{256}$$

Taking square root of both the sides.

$$\sqrt{\left(x - \frac{1}{16}\right)^2} = \sqrt{\frac{225}{256}}$$
$$x - \frac{1}{16} = \pm\frac{15}{16}$$
$$x - \frac{1}{16} = \frac{15}{16} \quad \text{or} \quad x - \frac{1}{16} = -\frac{15}{16}$$
$$x = \frac{15}{16} + \frac{1}{16} \quad \text{or} \quad x = -\frac{15}{16} + \frac{1}{16}$$
$$x = \frac{16}{16} = 1 \quad \text{or} \quad x = \frac{-14}{16} = \frac{-7}{8}$$
$$S,S = \left\{1, -\frac{7}{8}\right\}$$

By quadratic formula

$$8x^2 = x + 7$$
$$8x^2 - x - 7 = 0 \text{ (given)}$$

$ax^2+bx+c=0$ (standard form)

Compairing the given equation with the standard form.

$a = 8, b = -1, c = -7$

Substituting the values of a,b,c in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(8)(-7)}}{2(8)}$$

$$x = \frac{1 \pm \sqrt{1+224}}{16}$$

$$x = \frac{1 \pm \sqrt{225}}{16} = \frac{1 \pm 15}{16}$$

$$x = \frac{1+15}{16}, \quad x = \frac{1-15}{16}$$

$$x = \frac{16}{16} = 1, \quad x = \frac{-14}{16} = \frac{-7}{8}$$

$$\text{S.S} = \left\{1, -\frac{7}{8}\right\}$$

(ii) $2x^2 - x - 10 = 0$

Sol. By factorization

$$2x^2 - x - 10 = 0$$

$$2x^2 - 5x + 4x - 10 = 0$$

$$x(2x-5) + 2(2x-5) = 0$$

$$(2x-5)(x+2) = 0$$

$$2x - 5 = 0 \text{ or } x + 2 = 0$$

$$2x = 5 \text{ or } x = -2$$

$$x = \frac{5}{2}$$

$$\text{So, S,S} = \left\{\frac{5}{2}, -2\right\}$$

By completing square

$$2x^2 - x - 10 = 0$$

Dividing both sides by 2, we get

$$\frac{2x^2}{2} - \frac{x}{2} - \frac{10}{2} = \frac{0}{2}$$

$$x^2 - \frac{x}{2} - 5 = 0$$

$$x^2 - \frac{x}{2} = 5$$

We add square of half of the co-efficient of x.i.e., $\left(\frac{1}{4}\right)^2$ on both sides of the equation.

$$x^2 - \frac{x}{2} + \left(\frac{1}{4}\right)^2 = 5 + \left(\frac{1}{4}\right)^2$$

$$\left(x - \frac{1}{4}\right)^2 = 5 + \frac{1}{16}$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{80+1}{16}$$

$$\sqrt{\left(x - \frac{1}{4}\right)^2} = \sqrt{\frac{81}{16}}$$

$$x - \frac{1}{4} = \pm\frac{9}{4}$$

$$x - \frac{1}{4} = \frac{9}{4} \text{ or } x - \frac{1}{4} = -\frac{9}{4}$$

$$x = \frac{9}{4} + \frac{1}{4} \text{ or } x = -\frac{9}{4} + \frac{1}{4}$$

$$x = \frac{10}{4} = \frac{5}{2} \text{ or } x = -\frac{8}{4} = -2$$

$$\text{So, S.S} = \left\{\frac{5}{2}, -2\right\}$$

By quadratic formula

$$2x^2 - x - 10 = 0 \text{ (given)}$$

$ax^2+bx+c=0$ (standard form)

Compairing the given equation with the standard form.

$a = 2$, $b = -1$, $c = -10$

Substituting the values of a,b,c in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-10)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{1+80}}{4} \Rightarrow x = \frac{1 \pm \sqrt{81}}{4}$$

$$x = \frac{1 \pm 9}{4}$$

$$x = \frac{1+9}{4}, \quad x = \frac{1-9}{4}$$

$$x = \frac{10}{4} = \frac{5}{2}, \quad x = -\frac{8}{4} = -2$$

So, S.S $= \left\{\frac{5}{2}, -2\right\}$

3. Form a quadratic equation whose roots are; 6, $\frac{3}{2}$.

Sol. Given roots are 6, $\frac{3}{2}$

S = Sum of the given roots = $6+\frac{3}{2}=\frac{12+3}{2}=\frac{15}{2}$ ____(i)

P = Product of the given roots = $(6)\left(\frac{3}{2}\right)=\frac{18}{2}=9$ ____(ii)

Required quadratic equation is

$x^2 - Sx + P = 0$

Putting values of S and P from (i) and (ii)

$$x^2-\frac{15}{2}x+9=0$$

Multiplying throughout by 2, we get

$$2(x^2)-2\left(\frac{15}{2}x\right)+2(9) = 2(0)$$

$$2x^2 - 15x + 18 = 0$$

4. Examine the nature of the roots of the following equations:

(i) $15x^2 + 11x + 2 = 0$

Sol.

$15x^2 + 11x + 2 = 0$

$a = 15, b = 11, c = 2$

Disc $= b^2 - 4ac$

$= (11)^2 - 4(15)(2)$

$= 121 - 120$

$x = 1 = (1)^2 > 0$

As the discriminant is positive and perfect square.

Therefore, the roots are real, rational and unequal.

(ii) $x^2 - x - 1 = 0$

Sol. $x^2 - x - 1 = 0$

$a = 1, b = -1, c = -1$

Disc $= b^2 - 4ac$

$= (-1)^2 - 4(1)(-1)$

$= 1 + 4$

$= 5 > 0$ and not a perfect square.

So, the roots are irrational (real) and unequal.

5. If a ball is thrown upward with a velocity v, the maximum height it reaches can be determined by using a formula $h = \frac{v^2}{2g}$. Rearrange the formula to make v the subject.

Sol.

$$h = \frac{v^2}{2g} \text{ or } \frac{v^2}{2g} = h$$

$$v^2 = 2gh$$

Taking square root of both sides, we get

$$\sqrt{v^2} = \sqrt{2gh}$$

$$v = \sqrt{2gh}$$

6. If the equation $x^2 + 2(1+k)x + k^2 = 0$ has equal roots, then find the value of k.

Sol. $x^2 + 2(1 + k)x + k^2 = 0$

$a = 1, b = 2(1 + k), c = k^2$

Discriminant = 0 (as roots are equal)

$$b^2 - 4ac = 0$$

$$\{2(1 + k)\}^2 - 4(1)(k^2) = 0$$

$$4(1 + 2k + k^2) - 4k^2 = 0$$

$$4 + 8k + 4k^2 - 4k^2 = 0$$

$$4 + 8k = 0$$

$$8k = -4$$

$$k = -\frac{4}{8}$$

$$k = -\frac{1}{2}$$

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

2.1 Quadratic Equation

Encircle the correct option.

1. **What is the degree of polynomial $-x + 5x^2 - 7$?**
(a) 1 (b) 2 (c) 5 (d) -7

2. **A quadratic, equation is a polynomial equation of degree.**
(a) one (b) two (c) three (d) zero

3. **What is the standard form of a quadratic equation in one variable x?**
(a) $ax + b = 0$ (b) $ax^3 + bx^2 + c = 0$ (c) $ax^2 + bx + c = 0$ (d) $a^2 + x = c$

4. **If $a = 0$ in the equation $ax^2 + bx + c = 0$, the equation becomes:**
(a) a linear equation (b) a quadratic equation
(c) a pure quadratic equation (d) a cubic equation

5. **An equation of the form $ax^2+bx+c = 0$ becomes pure quadratic equation when:**
(a) $a = 0$ (b) $x = 0$ (c) $c = 0$ (d) $b = 0$

2.1.1 Solution of Quadratic Equation of the Form $ax^2 + bx + c = 0$, where $a,b,c \in R$ and $a \neq 0$

6. **The "Zero Product Property " states that if $a,b \in R$ and $ab = 0$ then:**
(a) Both a and b must be zero (b) Either $a = 0$ or $b = 0$
(c) a must be equal to b (d) $a + b$ must be zero

7. **The maximum number of real roots every quadratic equation has?**
(a) two (b) three (c) zero (d) one

8. **Which is a fractional equation?**
(a) $x^2 + \frac{3}{2}x + 4 = 0$ (b) $ax^2 + bx + c = 0$
(c) $\frac{2x}{5} + \frac{6}{x} + 4 = 0$ (d) $x + 3 = 0$

9. **The solution set of equation $2x^2 - 11x + 12 = 0$ is:**
(a) $\left\{\frac{3}{2}, 4\right\}$ (b) $\{0, 4\}$ (c) $\left\{-3, \frac{1}{3}\right\}$ (d) $\{1, -5\}$

10. **To make the coefficient of x^2 unity in $3x^2 - 32 = -10x$, you must:**
(a) subtract 3 from both sides (b) divide every term by 3
(c) multiply every term by 10 (d) square the number 3

11. **By taking square root of both sides of $\left(x+\frac{5}{3}\right)^2 = \frac{121}{9}$. What is the result?**

(a) $x+\frac{5}{3}=\frac{11}{3}$ (b) $x+\frac{5}{3}=\pm\frac{11}{3}$ (c) $x^2=\frac{121}{9}$ (d) $x+\frac{25}{9}=\frac{11}{3}$

12. **$ax^2 + bx + c = 0$ can equivalently be expressed as:**

(a) $a^2 + bx = -c$ (b) $2x^2 + \frac{b}{a}x + c$ (c) $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ (d) $\frac{a}{b}x^2 + bx + c = 0$

13. **For equation $3x^2 + 7x - 6 = 0$, which is correct?**

(a) $a=7, b=3, c=0$ (b) $a=3, b=6, c=7$ (c) $a=-6, b=7, c=3$ (d) $a=3, b=7, c=-6$

14. **The vertex form of a quadratic function is:**

(a) $ax^2 + bx + c = 0$ (b) $x = -\frac{b}{a}$ (c) $a(x-h)^2 + k$ (d) $(x-h)(x-k)$

15. **The graph of a quadratic equation is called:**

(a) parabola (b) hyperbola (c) ellipse (d) circle

2.2 Finding Intersection Point (s) Graphically

16. **To point intersection of $y = 2x+4$ with x-axis graphically is:**

(a) (–1,0) (b) (–2,0) (c) (–3,0) (d) (–4,0)

17. **The point of intersection for $x+y = 6$ and $3x+y = 12$ graphically is:**

(a) M(2, 6) (b) M(0, 6) (c) M(3, 3) (d) M(0, 12)

18. **To draw a straight line, it is essential to have at least two distinct:**

(a) lines (b) origins (c) equations (d) points

2.3 Relationship Between the Roots and Coefficients of a Quadratic Equation

19. **Which formula represents sum of root of equation $ax^2 + bx + c = 0$?**

(a) $\frac{b}{a}$ (b) $-\frac{b}{a}$ (c) $\frac{a}{b}$ (d) $\frac{c}{d}$

20. **The product of the roots for $2x^2 + 5x - 7 = 0$ is:**

(a) $\frac{5}{2}$ (b) $-\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $-\frac{7}{2}$

21. **In equation $5x^2 - 6x + 1 = 0$, the sum of the roots is equal to:**

(a) $\frac{6}{5}$ (b) $-\frac{6}{5}$ (c) $\frac{1}{5}$ (d) $-\frac{1}{5}$

22. **If α and B are root of $7x^2 - 4x - 8 = 0$ then α+B is:**

(a) $\frac{8}{7}$ (b) $-\frac{4}{7}$ (c) $\frac{4}{7}$ (d) $-\frac{8}{7}$

23. **The formula to form a quadratic equation is:**

(a) $x^2 + Sx + P = 0$ (b) $x^2 - Sx - P = 0$ (c) $x^2 - Sx + P = 0$ (d) $x^2 + Sx - P = 0$

24. **Which equation has roots 4 and 5?**
(a) $x^2-9x+20=0$ (b) $x^2+9x-20=0$ (c) $x^2-9x-20=0$ (d) $x^2+20x+9=0$

25. **What are the roots of $(x-5)(x+7)=0$?**
(a) $-5, -7$ (b) $-5, 7$ (c) $1, 7$ (d) $5, -7$

2.4 Discriminant of a Quadratic Equation

26. **The expression b^2-4ac in a quadratic equation is known as:**
(a) variable (b) constant (c) coefficient (d) discriminant

27. **if $b^2-4ac>0$ and is a perfect square, the roots are:**
(a) rational, real and unequal (b) irrational and equal
(c) imaginary and unequal (d) rational and equal

28. **When $b^2-4ac=0$, the roots are:**
(a) imaginary (b) irrational and unequal
(c) real and equal (d) real and unequal

29. **Which of the following is irrational and real root according to discriminant?**
(a) $\sqrt{25}$ (b) 5 (c) $\sqrt{7}$ (d) $4/3$

30. **If the roots are "complex conjugates" the discriminant must be:**
(a) positive (b) zero (c) negative (d) a perfect square

31. **The discriminant of $4x^2+4x+1=0$ is:**
(a) 0 (b) 4 (c) 17 (d) 16

32. **The discriminant of $2x^2+3x+4=0$ is:**
(a) 23 (b) 24 (c) -23 (d) 12

33. **If the roots of $ax^2+bx+c=0$ are rational, b^2-4ac must be a:**
(a) zero (b) perfect square (c) negative (d) cube

2.5 Solution of Quadratic Inequalities in One Unknown

34. **Which of the following is not the symbol of inequality?**
(a) $\leq$ (b) $>$ (c) $=$ (d) $<$

35. **To solve the $x^2-5x+6>0$, set the quadratic expression equal to zero to find the:**
(a) square (b) cube (c) roots (d) y-intercept

36. **The critical points of equation $x^2-5x+6=0$ are:**
(a) (1,0) and (2,0) (b) (2,0) and (3,0) (c) (3,0) and (4,0) (d) (4,0) and (5,0)

37. **For solution we use open internal (a, b) when the equation has sign:**
(a) $<$ and $<$ (b) $<$ or $>$ (c) [a, b] (d) $\leq$ or $\leq$

38. **For solution we use closed interval [a,b], when equation has sign:**
(a) $\leq$ or $\geq$ (b) $<$ or $>$ (c) $=$ (d) $>$

39. **The corresponding equation to the inequality: $3x^2-5x+7\leq 0$ is:**
(a) $3x^2+5x=-7$ (b) $3x-5x^2-7=0$ (c) $3x^2-5x+7=0$ (d) $3x^2=-5x+7$

2.6 Changing the Subject of Formula

40. If I = PRT, then T is:

(a) IPR (b) $\frac{IR}{P}$ (c) PRT (d) $\frac{I}{PR}$

41. If $P = 2(l + w)$, then w is?

(a) $\frac{P-2l}{2}$ (b) $l - W$ (c) $\frac{Pl-w}{2}$ (d) PlW

42. If $S = 2a + (n-1)d$, then d is?

(a) $\frac{S+2l}{n}$ (b) $\frac{3-2d}{-1}$ (c) $\frac{S-2a}{n-1}$ (d) $\frac{S+2a}{n-1}$

43. If $y^2 = 4ax$ then x is:

(a) $y^2 = 4a$ (b) $\frac{y^2}{4a}$ (c) $\frac{y^2}{a} = 4$ (d) $4ay^2 = 0$

44. If $P = \frac{1}{2}m(v_1^2 - v_2^2) + mgt$ then t = ____?

(a) $\frac{2P - m(v_1^2 - v_2^2)}{2mg}$ (b) $\frac{P}{mg} - \frac{1}{2}(v_1^2 - v_2^2)$ (c) $\frac{2P + m(v_1^2 - v_2^2)}{2mg}$ (d) $\frac{P - m(v_1^2 - v_2^2)}{mg}$

45. w = ____? if $V = u + aw$:

(a) $V - u - a$ (b) $\frac{a}{V-u}$ (c) $\frac{V+u}{a}$ (d) $\frac{V-u}{a}$

46. If $A = \frac{1}{2}hb$ then h = ____?

(a) $\frac{A}{2b}$ (b) $\frac{2A}{b}$ (c) 2Ab (d) $\frac{b}{2A}$

47. If $y = 3x - 5$, then x = ____?

(a) $3y + 5$ (b) $\frac{y-5}{3}$ (c) $\frac{y+5}{3}$ (d) $\frac{y}{3} + 5$

2.7 Applications of Quadratic Equation in Real World

48. For object moving in a straight line with uniform acceleration, V_f = ____?

(a) $\frac{1}{2}at^2$ (b) $v_i^2 + 2as$ (c) $v_i + at$ (d) $v_i - at$

49. For object moving in a straight line S = ____?

(a) $v_i t + \frac{1}{2}at^2$ (b) $v_i + at$ (c) $v_i - 2as$ (d) $v_i t + at^2$

50. Using formula $v_f = v_i + at$, if $v_i = 30$, $v_f = 0$, $a = -2$ then t = ____?

(a) 15 (b) 10 (c) 0 (d) 30

51. We can also find vertex by:

(a) $x = \frac{b}{a}, y = \frac{c}{a}$ (b) $x = 2b, y = f$ (c) $x = \frac{h}{c}, y = hb$ (d) $x = \frac{-b}{2a}, y = f\left(\frac{-b}{2a}\right)$

ANSWERS:

1. 2 2. two 3. $ax^2+bx+c=0$ 4. a linear equation 5. $b=0$ 6. Either $a=0$ or $b=0$

7. two 8. $\frac{2}{5}x+\frac{6}{x}+4=0$ 9. $\left\{\frac{3}{2},4\right\}$ 10. divide every term by 3 11. $x+\frac{5}{3}=\pm\frac{11}{3}$

12. $x^2+\frac{b}{a}x+\frac{c}{a}=0$ 13. $a=3, b=7, c=-6$ 14. $a(x-h)^2+k$ 15. parabola 16. $(-2,0)$ 17. $M(3, 3)$

18. points 19. $-\frac{b}{a}$ 20. $-\frac{7}{2}$ 21. $\frac{6}{5}$ 22. $\frac{4}{7}$ 23. $x^2-Sx+P=0$ 24. $x^2-9x+20=0$

25. 5, −7 26. Discriminant 27. rational, real and unequal 28. real and equal 29. $\sqrt{7}$ 30. negative

31. 0 32. −23 33. perfect square 34. = 35. roots 36. (2,0) and (3,0)

37. < or > 38. ≤ or ≥ 39. $3x^2-5x+7=0$ 40. $\frac{I}{PR}$ 41. $\frac{P-2\ell}{2}$ 42. $\frac{S-2a}{n-1}$

43. $\frac{y^2}{4a}$ 44. $\frac{2P-m(v_1^2-v_2^2)}{2mg}$ 45. $\frac{V-u}{a}$ 46. $\frac{2A}{b}$ 47. $\frac{y+5}{3}$ 48. v_i+at

49. $v_it+\frac{1}{2}at^2$ 50. 15 51. $x=\frac{-b}{2a}, y=f\left(\frac{-b}{2a}\right)$

Additional Short Question

Short Answer Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

2.1 Quadratic Equation

1. What is a quadratic equation.

Ans. A quadratic equation is a second-degree polynomial equation in one variable.

For example: $2x^2+3x+7=0$

2. What is the standard form of a quadratic equation?

Ans. The standard form of a quadratic equation is $ax^2+bx+c=0$ in x where $a, b, c \in R$ and $a \neq 0, b \neq 0$.

3. What is a "Pure Quadratic Equation"?

Ans. A pure quadratic equation occurs when $b=0$.

For example: $ax^2+c=0$

4. What is the "Zero Product property"?

Ans. The zero product property states that if a and b are numbers and $ab=0$, then either $a=0$ or $b=0$.

5. Write the quadratic equation $5(2x+3)=-9x^2$ in standard form.

Sol.
$$5(2x+3)=-9x^2$$
$$10x+15=-9x^2$$
$$9x^2+10x+15=0$$

2.1.1 Solution of Quadratic Equation of the Form $ax^2 + bx + c = 0$, where $a, b, c \in R$ and $a \neq 0$

6. Solve the equation by factorization method $9x^2 - 12x - 5 = 0$.

Sol.
$$9x^2 - 12x - 5 = 0$$
$$9x^2 - 15x + 3x - 5 = 0$$
$$3x(3x-5) + 1(3x-5) = 0$$
$$(3x-5)(3x+1) = 0$$
$$3x - 5 = 0 \text{ or } 3x + 1 = 0$$
$$3x = 5 \text{ or } 3x = -1$$
$$x = \frac{5}{3} \text{ or } x = -\frac{1}{3}$$
$$S.S = \left\{\frac{5}{3}, -\frac{1}{3}\right\}$$

7. Solve the equation $x^2 + 2x - 35 = 0$ by factorization method.

Sol.
$$x^2 + 2x - 35 = 0$$
$$x^2 + 7x - 5x - 35 = 0$$
$$x(x+7) - 5(x+7) = 0$$
$$(x+7)(x-5) = 0$$
$$x + 7 = 0 \text{ or } x - 5 = 0$$
$$x = -7 \text{ or } x = +5$$
$$S.S = \{-7, +5\}$$

8. Solved the equation by completing square method: $x^2 - 2x - 889 = 0$

Sol. $x^2 - 2x - 889 = 0$
$$x^2 - 2x = +889$$
We add square of half of the co-efficient of x *i.e.*, $(1)^2$ on both sides of the equation.
$$x^2 - 2x + (1)^2 = 889 + (1)^2$$
$$(x-1)^2 = 889 + 1$$
Taking square root of both sides, we get
$$\sqrt{(x-1)^2} = \sqrt{890}$$
$$x - 1 = \pm\sqrt{890}$$
$$x = 1 \pm \sqrt{890}$$
So, $S.S = \left\{1 \pm \sqrt{890}\right\}$

9. Solve $x^2 - 4x - 5 = 0$ by completing square method.

Sol.
$$x^2 - 4x - 5 = 0$$
$$x^2 - 4x = 5$$
We add square of half of the co-efficient of x i.e, $(2)^2$ on both sides of the equation.
$$x^2 - 4x + (2)^2 = 5 + (2)^2$$
$$(x-2)^2 = 5 + 4$$

$$(x-2)^2 = 9$$

Taking square root of both sides, we get

$$\sqrt{(x-2)^2} = \sqrt{9}$$

$$x-2 = \pm 3$$

$$x-2 = -3$$

$$x-2 = 3 \text{ or } x = -3+2$$

$$x = 3+2 \text{ or } x = -1$$

$$x = 5$$

$S.S = \{5, -1\}$

10. Solve $x^2 - 5x + 2 = 0$ by using the quadratic formula.

Sol. $x^2 - 5x + 2 = 0$ (given)

$ax^2+bx+c=0$ (standard form)

Compairing the given equation with the standard form.

$$a = 1, b = -5, c = 2$$

Substituting the values of a,b,c in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25-8}}{2}$$

$$x = \frac{5 \pm \sqrt{17}}{2}$$

$$S.S = \left\{\frac{5+\sqrt{17}}{2}, \frac{5-\sqrt{17}}{2}\right\}$$

11. Solve the equation $3x^2 - 32 = -10x$ by using quadratic formula.

Sol. $3x^2 - 32 = -10x$

$3x^2 + 10x - 32 = 0$ (given)

$ax^2 + bx + c = 0$ (standard form)

Compairing the given equation with the standard form

$$a = 3, b = 10, c = -32$$

Substituting the values of a,b,c in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(3)(-32)}}{2(3)}$$

$$x = \frac{-10 \pm \sqrt{100+384}}{6}$$

$$x = \frac{-10 \pm \sqrt{484}}{6}$$

$$x = \frac{-10 \pm 22}{6}$$

$$x = \frac{-10+22}{6} \text{ or } x = \frac{-10-22}{6}$$

$$x = \frac{12}{6} = 2 \text{ or } x = -\frac{32}{6} = -\frac{16}{3} = -5\frac{1}{3}$$

So, $S.S = \{2, -5\frac{1}{3}\}$

2.2 Finding Intersection Point (s) Graphically

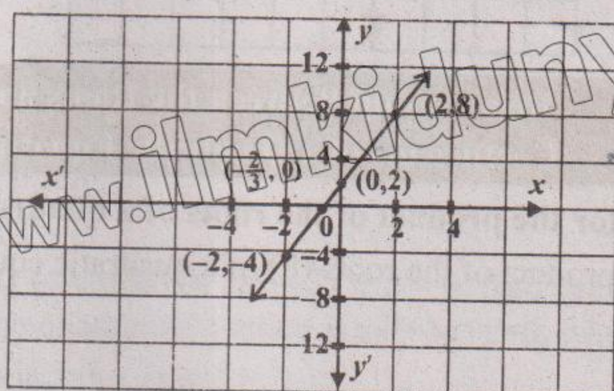
12. Find the point of intersecting of the linear equation $y = 2 + 3x$ with coordinate axes graphically.

Sol. $y = 2+3x$ is an equation of line. Now we draw the graph of $y = 2 + 3x$

Table of the values for $y = 2 + 3x$

x	y = 2 + 3x	Points (x,y)
-2	$y = 2 + 3(-2)$ $= 2 - 6 = -4$	$(-2, -4)$
$\frac{-2}{3}$	$y = 2 + 3\left(\frac{-2}{3}\right)$ $= 2 - 2 = 0$	$\left(\frac{-2}{3}, 0\right)$
0	$y = 2 + 3(0)$ $= 2 + 0 = 2$	$(0, 2)$
2	$y = 2 + 3(2)$ $= 2 + 6 = 8$	$(2, 8)$

We plot points $(-2,-4)$, $\left(\frac{-2}{3}, 0\right)$, $(0,2)$ and $(2,8)$ and join these points.



Point of intersection on x-axis is $(-\frac{2}{3}, 0)$ and the point of intersection on y-axis is (0, 2).

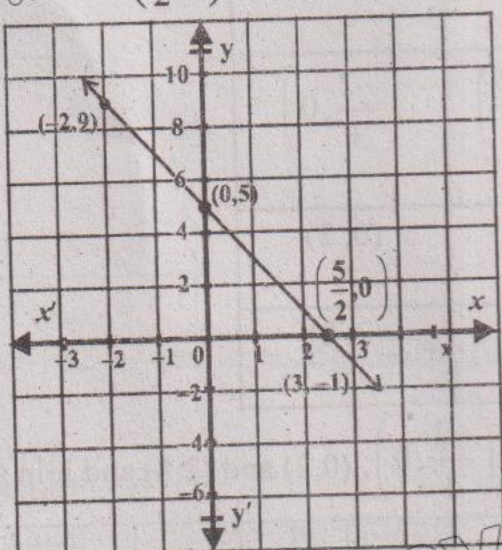
13. Find the point of intersecting of the linear equation 2x+y=5 with coordinate axes.

Sol. $2x + y = 5$ is an equation of line. Now we draw the graph of $2x + y = 5$

Table of the values for $y = 5 - 2x$

x	y = 5 – 2x	Points (x,y)
–2	$y = 5 - 2(-2)$ $= 5 + 4 = 9$	(–2, 9)
0	$y = 5 - 2(0)$ $= 5 - 0 = 5$	(0,5)
$\frac{5}{2}$	$y = 5 - 2\left(\frac{5}{2}\right)$ $= 5 - 5 = 0$	$\left(\frac{5}{2}, 0\right)$
3	$y = 5 - 2(3)$ $= 5 - 6 = -1$	(3, –1)

We plot the points (–2,9), (0,5), $\left(\frac{5}{2}, 0\right)$ and (3,–1) and join these points.



2.3 Relationship Between the Roots and Coefficients of a Quadratic Equation

14. Write the formula for the product of the roots of a quadratic equation.

Ans. The formula for the product of the roots (P) of a quadratic equation is given by:

$$P = \frac{c}{a}$$

15. **Write the formula for the sum of the roots of a quadratic equation.**

Ans. The formula for the sum of the roots (S) of a quadratic equation is given by:

$$S = \frac{-b}{a}$$

16. **Form the equation whose roots are $\frac{3}{4}$ and $\frac{4}{5}$.**

Sol. Roots of the required equation are $\frac{3}{4}$ and $\frac{4}{5}$

$$S = \text{Sum of the roots} = \frac{3}{4} + \frac{4}{5}$$

$$= \frac{15+16}{20} = \frac{31}{20}$$

$$P = \text{Product of the roots} = \frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}$$

Required equation is

$$x^2 - Sx + P = 0$$

Putting the values of S and P

$$x^2 - \frac{31}{20}x + \frac{3}{5} = 0$$

Multiplying throughtout by 20, we get

$$20(x^2) - 20\left(\frac{31}{20}x\right) + 20\left(\frac{3}{5}\right) = 20(0)$$

$$20x^2 - 31x + 12 = 0$$

17. **Form the equation whose roots are $5 + \sqrt{7}$ and $5 - \sqrt{7}$**

Sol. Roots of the required equation are $5 + \sqrt{7}$ and $5 - \sqrt{7}$

$$S = \text{Sum of the roots} = 5 + \sqrt{7} + 5 - \sqrt{7}$$

$$= 10$$

$$P = \text{Product of the roots} = (5 + \sqrt{7})(5 - \sqrt{7})$$

$$= (5)^2 - (\sqrt{7})^2$$

$$= 25 - 7 = 18$$

Required equation is

$$x^2 - Sx + P = 0$$

$$x^2 - 10x + 18 = 0$$

18. **If α, β are the roots of equation $3x^2 - 3x + 5 = 0$, find the values of $\alpha^2 + \beta^2$**

Sol. Standard form of the quadratic equation is $ax^2+bx+c=0$, where $a \neq 0$ and $a, b, c \in R$.

The given equation is $3x^2 - 3x + 5 = 0$

Here, $a = 3, b = -3, c = 5$

As α,β are the roots of the given equation $3x^2 - 3x + 5 = 0$,

therefore $S = \alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{3} = 1$

$P = \alpha\beta = \frac{c}{a} = \frac{5}{3}$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (1)^2 - 2\left(\frac{5}{3}\right)$

$= 1 - \frac{10}{3}$

$= \frac{3-10}{3} = \frac{-7}{3}$

19. If α,β are the roots of equation $x^2 - 3x - 11 = 0$, then find $\frac{1}{\alpha} + \frac{1}{\beta}$

Sol. Standard form of the quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$ and $a,b,c \in R$.

The given equation is

$x^2 - 3x - 11 = 0$

Here $a = 1, b = -3, c = -11$

As α,β are the roots of the given equation $x^2 - 3x - 11 = 0$, therefore

$S = \alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3$

$P = \alpha\beta = \frac{c}{a} = \frac{-11}{1} = -11$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{\alpha + \beta}{\alpha\beta}$

$= \frac{3}{-11} = -\frac{3}{11}$

20. Find the sum and product of the root for $3x^2 - 11x + 7 = 0$

Sol. Standard form of the quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$ and $a,b,c \in R$.

The given equation is

$3x^2 - 11x + 7 = 0$

Here, $a = 3, b = -11, c = 7$

$S =$ Sum of the roots $\frac{-b}{a} = \frac{-(-11)}{3} = \frac{11}{3}$

$$P = \text{Product of the roots } \frac{c}{a} = \frac{7}{3}$$

21. What happens to the product of the roots if constant term c is zero.

Sol. If $c = 0$, then the product of the roots is 0.

$$P = \frac{c}{a} = \frac{0}{a} = 0$$

22. In equation $5x^2 - 10 = 0$. What is the sum of the roots.

Sol. Standard form of the quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$ and $a, b, c \in R$.

The given equation is $5x^2 - 10 = 5x^2 + 0x - 10 = 0$

$a = 5$, $b = 0$ and $c = -10$

$$S = \frac{-b}{a} = -\frac{0}{5} = 0$$

23. Calculate the sum and product of the roots for $3x^2 - x - 17 = 0$

Sol. Standard form of the quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$ and $a, b, c \in R$.

The given equation is.

$$3x^2 - x - 17 = 0$$

Here, $a = 3, b = -1, c = -17$

$$S = \frac{-b}{a} = -\frac{-(-1)}{3} = \frac{1}{3}$$

$$P = \frac{c}{a} = -\frac{17}{3}$$

2.4 Discriminant of a Quadratic Equation.

24. What is the discriminant of the equation $ax^2 + bx + c = 0$?

Ans. The discriminant of the equation $ax^2 + bx + c = 0$ is $b^2 - 4ac$.

25. If $b^2 - 4ac = 0$, what is the nature of the roots.

Ans. The roots are real and equal.

26. Describe the nature of roots if $b^2 - 4ac > 0$ and is a perfect square.

Ans. The roots are rational (real) and unequal.

27. Examine the nature of the roots of $3x^2 - 8x + 16 = 0$

Sol. $3x^2 - 8x + 16 = 0$

$a = 3, b = 8, c = 16$

$$\begin{aligned} \text{Disc} &= b^2 - 4ac \\ &= (-8)^2 - 4(3)(16) \\ &= 64 - 192 \\ &= -128 < 0. \end{aligned}$$

Which is negative, therefore

the roots are imaginary and unequal.

28. Examine the nature of the roots $25x^2 - 30x + 9 = 0$

Sol. $25x^2 - 30x + 9 = 0$

$a = 25, b = -30, c = 9$

$\text{Disc} = b^2 - 4ac$

$= (-30)^2 - 4(25)(9)$

$= 900 - 900 = 0$

As the discriminant is zero. Therefore the roots are real and equal.

29. Examine the nature of the roots of $5x^2 - x - 2 = 0$

Sol. $5x^2 - x - 2 = 0$

$a = 5, b = -1, c = -2$

$\text{Disc} = b^2 - 4ac$

$= (-1)^2 - 4(5)(-2)$

$= 1 + 40 = 41 > 0$ and is not a perfect square.

So, the roots are irrational and unequal.

30. Examine the nature of the roots of $x^2 - 7x + 12 = 0$.

Sol. $x^2 - 7x + 12 = 0$

$a = 1, b = -7, c = 12$

$\text{Dise} = b^2 - 4ac$

$= (-7)^2 - 4(1)(12)$

$= 49 - 48$

$= 1 = (1)^2 > 0$ and is a perfect square.

So, the roots are rational and unequal.

2.5 Solution of Quadratic Inequalities in One Unknown

31. What are the four steps to solve quadratic inequality?

Ans. There are following four steps to solve quadratic inequality.

(i) Solve the corresponding equation.

(ii) Determine intervals

(iii) Test points in each interval.

(iv) Conclusion

32. If a quadratic equation has roots x = 2 and x = 3 and the critical points are (2,0) and (3,0). What are the resulting intervals on number line?

Ans. As a quadratic equation has roots x = 2 and x = 3. Therefore, the critical points (2,0) and (3,0) divide the number line into three intervals.

$(-\infty, 2)\ (2, 3),\ (3, \infty)$

33. What are the inequality symbols?

Ans. These are the inequalities symbols. $<$ (less than), $>$ (greater than) $\leq$ (less then or equal to) and $\geq$ (greater than or equal to)

2.6 Changing the Subject of Formula

34. If $y^2 = 4ax$, make 'a' as the subject of the equation.

Sol. $y^2 = 4ax$

$\Rightarrow 4ax = y^2$

$$a = \frac{y^2}{4x}$$

35. If $P = 2(\ell + w)$, make w as the subject of the formula.

Sol. $P = 2(\ell + w)$

or $2(\ell + w) = P$

$$(\ell + w) = \frac{P}{2}$$

$$w = \frac{P}{2} - \ell$$

$$w = \frac{P - 2\ell}{2}$$

36. If $V = \pi r^2 h$, make 'r' the subject of this formula.

Sol. $V = \pi r^2 h$

or $\pi r^2 h = V$

$$r^2 = \frac{V}{\pi h}$$

$$r = \sqrt{\frac{V}{\pi h}}$$

37. Make 'd' as the subject of the formula $S = 2a + (n-1)d$

Sol. $S = 2a + (n-1)d$

or $2a + (n-1)d = S$

$$(n-1)d = S - 2a$$

$$d = \frac{S-2a}{n-1}$$

38. If I = PRT then make 'R' the subject of this formula.

Sol. I = PRT

or PRT = I

$$R = \frac{I}{PT}$$

2.7 Applications of Quadratic Equation in Real World

39. A ball rolling down a hill accelerates at 4m/s². After 3 seconds, it has travelled 30 meters. What was its initial velocity.

Sol. Given that $a = 4\text{m/s}^2$, $t = 3s$, $S = 30\text{m}$

We use the equation of motion $S = v_i t + \frac{1}{2} at^2$ because it relates the displacement(s) of the object to the initial velocity (v_i), acceleration (a) and time (t).

$$S = v_i t + \frac{1}{2} a t^2$$

$$30 = v_i(3) + \frac{1}{2}(4)(3)^2$$

$$30 = 3v_i + 18$$

$$30 - 18 = 3v_i$$

$$12 = 3v_i$$

$$\frac{12}{3} = v_i$$

$$v_i = 4 \text{ m/s}$$

40. Write three equations for objects moving in a straight line with uniform acceleration.

Sol. First equation: $V_f = v_i + at$

Second equation: $S = v_i t + \frac{1}{2} at^2$

Third equation: $v_f^2 = vi^2 + 2as$

41. Find acceleration if $v_i = 20\text{m/s}$, $v_f = 0$ and $S = 40$ m.

Sol. Given that $v_i = 20\text{m/s}$, $v_f = 0$ and $S = 40\text{m}$

$$v_{f^2} - = v_i^2 \times 2as$$

The relevant equation of motion is.

$$0^2 = (20)^2 + 2(a)(40)$$

$$0 = 400 + 80a$$

$$-80a = 400$$

$$a = \frac{400}{-80}$$

$$a = -5 \text{ m/s}^2$$

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