UNIT 04 Functions and Graphs

Students' Learning Outiones.

After completing this unit, the students will be able to:

- Explain operations on functions and compositions of functions.
- Define inverse functions and demonstrate their domain and range with examples.
- Formulate composite functions as defined by $(fog)(x) = f(g(x))$.
- Apply concepts from functions to real-world problems (such as finance, transportation and sales).
- Plot graphs of absolute valued functions.
- Solve absolute value linear equations and inequalities in one variable and express the solution as a range of values on a number line.
- Apply concepts of absolute valued functions to real-world problems (such as to calculate energy wave, magnitude and distance).

4.1 Function

If X and Y are two non-empty sets, then a function from a set X to a set Y is a rule that assigns each element of set X exactly one element in set Y. It is written as $f: X \to Y$, where the set X is called the domain and the set Y is called the co-domain of function f. While the range is the subset of the co-domain that contains only the actual output values produced by the function f.

History!
Gottfried Wilhelm Leibniz is credited with first introducing the word "function" in a mathematical context. He used the term in 1673 to describe a quantity related to a curve, such as its coordinates or slope.

Example 1: If $f(x) = x^2 + 3x + 1$, then evaluate

(i) $f(0)$ **(ii)** $f(-2)$ **(iii)** $f(a)$ **(iv)** $f(x+1)$

Solution: Given that $f(x) = x^2 + 3x + 1$

(i) $f(0) = (0)^2 + 3(0) + 1 = 1$

(ii) $f(-2) = (-2)^2 + 3(-2) + 1$
$= 4 - 6 + 1 = -1$

(iii) $f(a) = a^2 + 3a + 1$

(iv) $f(x+1) = (x+1)^2 + 3(x+1) + 1$
$= x^2 + 2x + 1 + 3x + 3 + 1$
$= x^2 + 5x + 5$

Remember!
Range is always a subset of the co-domain.

Skilled Practice!

If $f(x) = 2x^2 + 5x$, then find $f(x+2)$.

Sol. $f(x) = 2x^2 + 5x$

$f(x+2)^2 = 2(x+2)^2 + 5(x+2)$
$= 2[x^2 + 4x + 4] + 5x + 10$
$= 2x^2 + 8x + 8 + 5x + 10$
$= 2x^2 + 13x + 18$

Example 2: Find the domain and range of $f(x) = (x + 1)^2$.

Solution: As $f(0) = (0 + 1)^2 = 1$; $f(4) = (4 + 1)^2 = 25$; $f(-10) = (-10 + 1)^2 = 81$;

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}+1\right)^2 = \frac{9}{4}; f\left(\sqrt{3}\right) = \left(\sqrt{3}+1\right)^2 = 7.46; f(-1) = 0; \text{ so on}$$

For every real number x, $f(x) = (x + 1)^2$ is a non-negative real number. So,
Domain f = set of all real numbers ; Range f = set of all non-negative real numbers.

Operations on Functions

Operations on functions involves performing algebraic operations (addition, subtraction, multiplication and division)

a. Addition of Functions

If $f(x)$ and $g(x)$ are two functions, then their sum is written as:

$$(f + g)(x) = f(x) + g(x)$$

Example 3: Let $f(x) = 2x + 3$ and $g(x) = x^2 + x + 1$. Add $f(x)$ and $g(x)$.

Solution:

$$\begin{aligned} f(x) &= 2x + 3 \\ g(x) &= x^2 + x + 1 \\ (f+g)(x) &= f(x) + g(x) \\ &= (2x + 3) + (x^2 + x + 1) = x^2 + 3x + 4 \end{aligned}$$

b. Subtraction of Functions

If $f(x)$ and $g(x)$ are two functions, then their difference is written as:

$$(f - g)(x) = f(x) - g(x)$$

Example 4: Let $f(x) = 5x + 2$, $g(x) = x^2 - 3$. Find $f(x) - g(x)$

Solution:

$$\begin{aligned} f(x) &= 5x + 2 \\ g(x) &= x^2 - 3 \\ (f-g)(x) &= f(x) - g(x) \\ &= (5x + 2) - (x^2 - 3) \\ &= 5x + 2 - x^2 + 3 = -x^2 + 5x + 5 \end{aligned}$$

c. Multiplication of Functions

If $f(x)$ and $g(x)$ are two functions, then their product is written as:

$(f \cdot g)(x) = f(x) \cdot g(x)$

Example 5: Let $f(x) = x + 1$, $g(x) = 2x - 3$, find $f(x) \cdot g(x)$.

Solution:

$$\begin{aligned} f(x) &= x + 1 \\ g(x) &= 2x - 3 \\ (f \cdot g)(x) &= f(x) \cdot g(x) \\ &= (x + 1)(2x - 3) = 2x^2 - 3x + 2x - 3 \\ &= 2x^2 - x - 3 \end{aligned}$$

d. Division of Functions

If $f(x)$ and $g(x)$ are two functions, then their quotient is written as:

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \text{ for } g(x) \neq 0$$

Example 6: Let $f(x) = x^2 - 4$, $g(x) = x + 2$. Find $\left(\frac{f}{g}\right)(x)$, where $x \neq -2$.

Solution: $f(x) = x^2 - 4$

$g(x) = x + 2$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$\text{So,} = \frac{x^2-4}{x+2} = \frac{(x+2)(x-2)}{(x+2)} = x - 2$$

Composition of Functions

Composition of function is a process of combining two or more functions to produce a new single function. The composition operator "o" takes two functions f and g and results a new function.

Example 7: If $f(x) = 2x + 3$, $g(x) = x^2$, then find

(i) $(fog)(x)$ **(ii)** $(gof)(x)$ **(iii)** $(fof)(x)$ **(iv)** $(gog)(x)$

Solution: Given $f(x) = 2x + 3$, $g(x) = x^2$

(i) $(fog)(x) = f(g(x))$
$= f(x^2)$
$= 2x^2 + 3$

(ii) $(gof)(x) = g(f(x))$
$= g(2x + 3)$
$(gof)(x) = (2x + 3)^2$
$= 4x^2 + 12x + 9$

(iii) $(fof)(x) = f(f(x))$
$= f(2x + 3)$
$= 2(2x + 3) + 3$
$= 4x + 9$

(iv) $(gog)(x) = g(g(x))$
$= g(x^2)$
$(gog)(x) = (x^2)^2 = x^4$

Remember!
In general,
$(fog)(x) \neq (gof)(x)$

One-to-One Function
A function f is said to be one-to-one (or injective) if each input maps to a unique output.

Onto Function
A function is called onto (or surjective if every element in the codomain has at least one preimage in the domain.

Bijective Function
A function is called bijective if it is both one-to-one and onto.

Inverse Function

A function f takes an input x from a set X and maps it to an output y in a set Y. i.e.,

$f: X \to Y, f(x) = y$

Then the inverse function of f denoted by f^{-1}, reverses this process. it takes y as input and gives x as output:

Domain of f = Range of f^{-1},

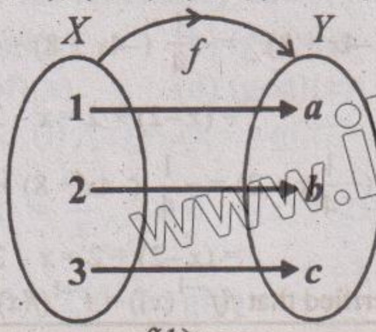
Range of f = Domain of f^{-1},

Let us understand it with the help of an example:

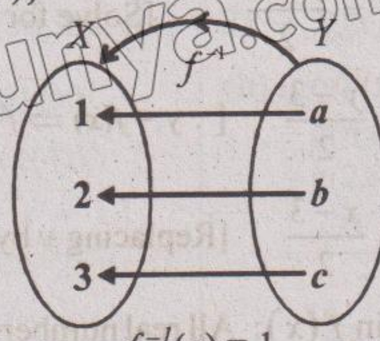
Consider $X = \{1, 2, 3\}$, $Y = \{a, b, c\}$

Note
If $y = f(x)$ then
$f^{-1}(y) = x$ is called inverse of $f(x)$

and $f: \{(1, a), (2, b), (3, c)\}$, then $f^{-1}: \{(a, 1), (b, 2), (c, 3)\}$



$f(1) = a$
$f(2) = b$
$f(3) = c$

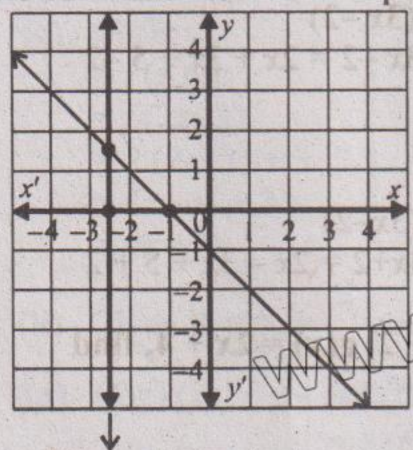


$f^{-1}(a) = 1$
$f^{-1}(b) = 2$
$f^{-1}(c) = 3$

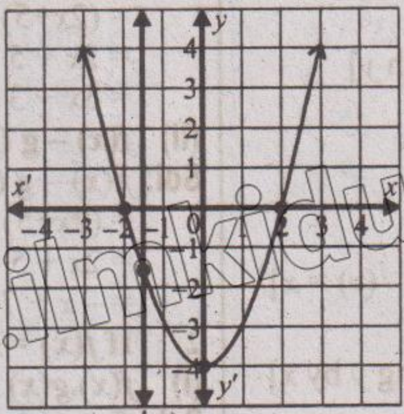
It is important to note that a function has an inverse if and only if it is one-to-one and onto (bijective).

Vertical Line Test

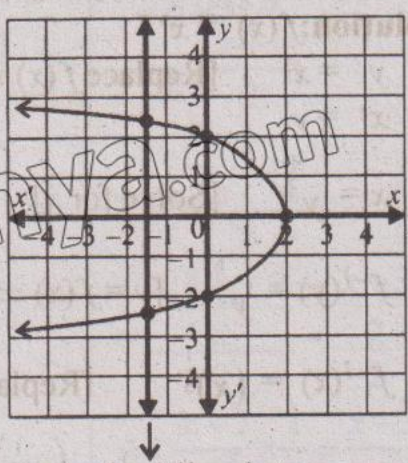
A curve/line drawn in a graph represents a function, if every vertical line intersects the curve/line at most one point.



Vertical line intersects at one point



Vertical line intersects at one point



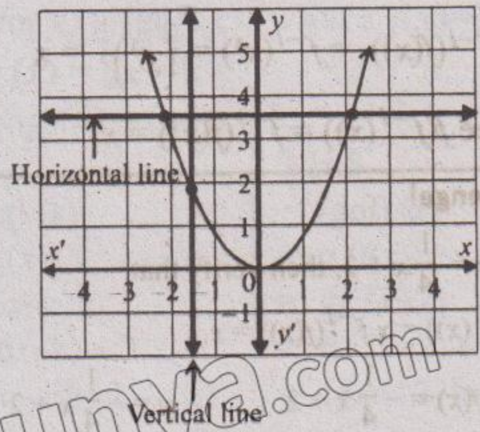
Vertical line intersects at two points

Graphs of Figure 4.8 and Figure 4.9 represent graphs of functions and graph of Figure 4.10 does not represent the graph of a function.

Horizontal Line Test

If every horizontal line intersects the curve at most one point, then the graph is a one-to-one function.

The vertical line intersects the graph of $f(x) = x^2$ at only one point, so it is the graph of a function. It fails in horizontal test, so it is not a one to one function.



Example 8: Find the inverse of $f(x) = 2x + 3$. Also find domain and range of $f^{-1}(x)$.

Solution: $f(x) = 2x + 3$

$y = 2x + 3$

$y = 2x + 3$ [Replace $f(x)$ with y]

$y - 3 = 2x$

$\therefore x = \dfrac{y-3}{2}$ [Solve for x]

$f^{-1}(y) = \dfrac{y-3}{2}$ [$\because y = f(x) \Rightarrow f^{-1}(y) = x$]

$f^{-1}(x) = \dfrac{x-3}{2}$ [Replacing y by x]

$\begin{cases} \text{Domain } f(x)\text{: All real numbers} \\ \text{Range } f(x)\text{: All real numbers} \end{cases}$

$\begin{cases} \text{Domain } f^{-1}(x)\text{: All real numbers} \\ \text{Range } f^{-1}(x)\text{: All real numbers} \end{cases}$

Example 9: If $f(x) = x^3$, find $f^{-1}(x)$. Also verify that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

Solution: $f(x) = x^3$

$y = x^3$ [Replace $f(x)$ with y]

$x^3 = y$

$x = y^{\frac{1}{3}}$ [Solve for x]

$f^{-1}(y) = y^{\frac{1}{3}}$ [$y = f(x) \Rightarrow f^{-1}(y) = x$]

$f^{-1}(x) = (x)^{\frac{1}{3}}$ [Replacing y by x]

$f(f^{-1}(x)) = f(x)^{\frac{1}{3}} = \left(x^{\frac{1}{3}}\right)^3 = x$

$f^{-1}(f(x)) = f^{-1}(x^3) = \left(x^3\right)^{\frac{1}{3}} = x$

Hence $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Challenge!

If $f(x) = \frac{1}{4}x + 2$, then verify that $f(f^{-1}(x)) = x\, f^{-1}(f(x)) = x$.

Sol: $f(x) = -\frac{1}{4}x + 2$; $y = -\frac{1}{4}x + 2$

or $-\frac{1}{4}x + 2 = y$; $-\frac{1}{4}x = y - 2$

$\frac{1}{4}x = -y + 2$; $x = 4(-y+2)$

$x = -4y + 8$; $f^{-1}(y) = -4y + 8$

$f^{-1}(x) = -4y + 8$

$f(f^{-1}(x))\, f(-4x+8) = -\frac{1}{4}(-4x+8) + 2$

$= (x-2) + 2 = x - 2 + 2 = x$

$f^{-1}(f(x))\, f^{-1}(-\frac{1}{4}x + 2) = -\frac{1}{4}(-4x+8) + 2$

$= (x-2) + 2 = x - 2 + 2 = x$

So, verified that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Solved Exercise 4.1

1. If $f(x) = 2x + 5$, $g(x) = 3x - 2$, then find

(i) $f(x) + g(x)$

Sol: $f(x) + g(x)$

$= (2x+5) + (3x-2)$

$= 2x + 5 + 3x - 2 = 2x + 3x + 5 - 2$

$= 5x + 3$

(ii) $f(x) - g(x)$

Sol: $f(x) - g(x)$

$= (2x+5) - (3x-2)$

$= 2x + 5 - 3x + 2 = 2x - 3x + 5 + 2$

$= -x + 7$

2. If $f(x) = x + 2$, $g(x) = 2x + 4$, find

(i) $f(x).g(x)$

Sol: $f(x).g(x)$

$= (x+2)(2x+4)$

$= 2x^2 + 4x + 4x + 8 = 2x^2 + 8x + 8$

$= 2(x^2 + 4x + 4)$

(ii) $g(x).f(x)$

Sol: $g(x).f(x)$

$= (2x+4)(x+2)$

$= 2x^2 + 4x + 4x + 8 = 2x^2 + 8x + 8$

$= 2(x^2 + 4x + 4)$

(iii) $\dfrac{f(x)}{g(x)}$

Sol: $\dfrac{f(x)}{g(x)} = \dfrac{x+2}{2x+4} = \dfrac{x+2}{2(x+2)} = \dfrac{1}{2}$

(iv) $\dfrac{g(x)}{f(x)}$

Sol: $\dfrac{g(x)}{f(x)} = \dfrac{2x+4}{x+2} = \dfrac{2(x+2)}{(x+2)} = 2$

3. For the functions f and g, find

(a) $(fog)(x)$ (b) $(gof)(x)$

(c) $(fof)(x)$ (d) $(gog)(x)$

Where, (i) $f(x) = 2x + ;\ g(x) = x^3$

(ii) $f(x) = \dfrac{2}{x},\ x \neq 0;\ g(x) = 2x^2 - 1$

(iii) $f(x) = 2x - 1;\ g(x) = \dfrac{x+1}{2}$

(i) Given that $f(x) = 2x + 3$, $g(x) = x^3$

(a) $(fog)(x)$
$= (f(g(x))$
$= f(x^3)$
$= 2(x^3) + 3$
$= 2x^3 + 3$

(b) $(gof)(x)$
$= g(f(x))$
$= g(2x + 3)$
$= (2x + 3)^3$

(c) $(fof)x$
$= f(f(x))$
$= f(2x+3)$
$= 2(2x+3) + 3$
$= 4x + 6 + 3$
$= 4x + 9$

(d) $(gog)x$
$= g(g(x))$
$= g(x^3)$
$= (x^3)^3$
$= x^9$

(ii) Given that $f(x) = \dfrac{2}{x},\ x \neq 0;\ g(x) = 2x^2 - 1$

(a) $(fog)(x)$
$= f(g(x))$
$= f(2x^2 - 1)$
$= \dfrac{2}{2x^2 - 1}$

(b) $(fog)(x)$
$= g(f(x))$
$= g\left(\dfrac{2}{x}\right) = 2\left(\dfrac{2}{x}\right)^2 - 1$
$= 2\left(\dfrac{4}{x^2}\right) - 1$
$= \dfrac{8}{x^2} - 1 = \dfrac{8 - x^2}{x^2}$

(c) $(fof)(x)$
$= f(f(x))$
$= f\left(\dfrac{2}{x}\right)$
$= \dfrac{2}{\frac{2}{x}} = 2 \times \dfrac{x}{2}$
$= x$

(d) $(fog)(x)$
$= g(g)(x))$
$= g(2x^2 - 1)$
$= 2(2x^2 - 1)^2 - 1$
$= 2(4x^4 - 4x^2 + 1) - 1$
$= 8x^4 - 8x^2 + 2 - 1$
$= 8x^4 - 8x^2 + 1$

(iii) Given that $f(x) = 2x - 1;\ g(x) = \dfrac{x+1}{2}$

(a) $(fog)(x)$
$= f(g(4))$
$= f\left(\dfrac{x+1}{2}\right)$
$= 2\left(\dfrac{x+1}{2}\right) - 1$
$= x + 1 - 1$
$= x$

(b) $(gof)(x)$
$= g(f(x))$
$= g(2x - 1)$
$= \dfrac{(2x - 1) + 1}{2}$
$= \dfrac{2x - 1 + 1}{2}$
$= \dfrac{2x}{2} = x$

(c) $(fof)(x)$
$= f(f(x))$
$= f(2x - 1)$
$= 2(2x - 1) - 1$
$= 4x - 2 - 1$
$= 4x - 3$

(d) $(gog)(x)$
$= g(g(x))$
$= g\left(\dfrac{x+1}{2}\right)$
$= \dfrac{\frac{x+1}{2} + 1}{2}$
$= \dfrac{\frac{x+1+2}{2}}{2} = \dfrac{\frac{x+3}{2}}{2}$
$= \dfrac{x+3}{2} \times \dfrac{1}{2} = \dfrac{x+3}{4}$

4. Find the value of k, such that $(fog)(x) = (gof)(x)$, where $f(x) = 3x + 2$, $g(x) = 6x - k$

Sol: $(fog)(x) = (gof)(x)$

L.H.S
$(fog)(x)$
$= f(g(x))$
$= f(6x - k)$
$= 3(6x - k) + 2$
$= 18x - 3k + 2$ __ (i)

R.H.S
$(gof)(x)$
$= g(f(x))$
$= g(3x + 2)$
$= 6(3x + 2) - k$
$= 18x + 12 - k$ _ (ii)

Equate (i) and (ii)

$18x - 3k + 2 = 18x + 12 - k$

$18x - 18x - 3k + k = 12 - 2$

$-2k = 10$

$2k = -10$

$k = -\frac{10}{2}$

$k = -5$

5. Given that $f(x) = 3x + 2$ and $g(x)=2x + 3$. Find

(i) $g(f(4))$ (ii) $f(f(3))$ (iii) $f(g(-2))$

Sol: Given $f(x) = 3x+2$ and $g(x) = 2x+3$

(i) $g(f(4))$

$f(4) = 3(4) + 2 = 12+2=14$

$g(14) = 2(14) +3 =28 +3 =31$

(ii) $f(f(3))$

$f(3) = 3(3) + 2 =11$

$f(11) = 3(11) +2 = 33+2 = 35$

(iii) $f(g(-2))$

$g(-2) = 2(-2)+3 = -4+3= -1$

$f(-1) = 3(-1) +2 = -3 + 2 = -1$

6. Find $f^{-1}(x)$ in each of the following.

(i) $f(x) = 2x - 3$

Sol: $f(x) = 2x - 3$

$y = 2x - 3$ [Replace $f(x)$ with y]

or $2x - 3 = y$

$2x = y+3$

$x = \frac{y+3}{2}$ [solve for x]

$f^{-1}(y) = \frac{y+3}{2} \quad \therefore y = f(x) \Rightarrow f^{-1}(y) = x$

$f^{-1}(x) = \frac{x+3}{2}$ [Replacing y by x]

(ii) $f(x) = 4x^3 - 1$

Sol: $f(x) = 4x^3 - 1$

$y = 4x^3 - 1$ [Replace $f(x)$ with y]

or $4x^3 - 1 = y$

$4x^3 = y + 1$

$x^3 = \frac{y+1}{4}$

$\sqrt[3]{x^3} = \sqrt[3]{\frac{y+1}{4}}$

$x = \sqrt[3]{\frac{y+1}{4}}$ [solve for x]

$f^{-1}(y) = \sqrt[3]{\frac{y+1}{4}} \quad \therefore y = f(x) \Rightarrow f^{-1}(y)=x$

$f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}}$ [Replacing y by x]

(iii) $f(x) = \sqrt{x-1}$

Sol: $f(x) = \sqrt{x-1}$

$y = \sqrt{x-1}$ [Replacing $f(x)$ with y]

or $\sqrt{x-1} = y$

$(\sqrt{x-1})^2 = y^2$

$x-1 = y^2$

$x = y^2+1$

$f^{-1}(y) = y^2+1 \quad \therefore y = f(x) \Rightarrow f^{-1}(y) = x$

$f^{-1}(x) = x^2+1$ [Replacing y by x]

(iv) $f(x) = \frac{x+1}{3x-2}$

Sol: $f(x) = \frac{x+1}{3x-2}$

$y = \frac{x+1}{3x-2}$ [Replace $f(x)$ with y]

$y(3x - 2) = x + 1$

$3xy - 2y = x + 1$

$3xy - x = 2y + 1$

$x(3y - 1) = 2y + 1$

$x = \frac{2y+1}{3y-1}$ [Solve for x]

$f^{-1}(y) = \frac{2y+1}{3y-1} \quad \therefore y = f(x) \Rightarrow f^{-1}(y) = x$

$f^{-1}(x) = \frac{2x+1}{3x-1}$ [Replacing y by x]

7. The function f and g are defined such that $f(x) = 4x + 2$ and $g(x) = 6x - 18$.

(i) Find $f^{-1}(x)$ and $g^{-1}(x)$.

Sol: $f(x) = 4x+2$

$y = 4x+2$ [Replace $f(x)$ with y]

or $4x+2 = y$

$4x = y-2$

$x = \dfrac{y-2}{4}$ (Solve for x)

$f^{-1}(y) = \dfrac{y-2}{4}$ [$\because y = f(x) \Rightarrow f^{-1}(y) = x$]

$f^{-1}(x) = \dfrac{x-2}{4}$ [Replacing y by x]

Now $g(x) = 6x-18$

$y = 6x - 18$ [Replace g (x) with y]

or $6x-18 = y$

$6x = y+18$

$x = \dfrac{y+18}{6}$ (solve for x)

$g^{-1}(y) = \dfrac{y+18}{6}$ [$\because y = g(x) \Rightarrow g^{-1}(y) = x$]

$g^{-1}(y) = x$

$g^{-1}(x) = \dfrac{x+18}{6}$ [Replace y by x]

(ii) Find x if $f^{-1}(x) = g^{-1}(x)$.

Sol: $f(x) = 4x+2$

$y = 4x+2$ (Replace (x) with y)

or $4x + 2 = y$

$4x = y-2$

$x = \dfrac{y-2}{4}$ (Solve for x)

$f^{-1}(y) = \dfrac{y-2}{4}$ [$\because y = f(x) \Rightarrow f^{-1}(y) = x$]

$f^{-1}(x) = \dfrac{x-2}{4}$ ___(i) [Replacing y by x]

$g(x) = 6x - 18$

$y = 6x - 18$

or $6x - 18 = y$

$6x = y + 18$

$x = \dfrac{y+18}{6}$

$g^{-1}(y) = \dfrac{y+18}{6}$ [$\because y = g(x) \Rightarrow g^{-1}(y) = x$]

$g^{-1}(x) = \dfrac{x+18}{6}$ ___(i)[Replacing y by x]

Given condition

$f^{-1}(x) = g^{-1}(x)$

$\dfrac{x-2}{4} = \dfrac{x+18}{6}$ from (i) and (ii)

$6(x-2) = 4(x+18)$

$6x - 12 = 4x + 72$

$6x - 4x = 72 + 12$

$2x = 84$

$x = \dfrac{84}{2}$

$x = 42$

8. Verify that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

(i) $f(x) = x - 6$

Sol. $f(x) = x - 6$

$y = x - 6$ (Replace $f(x)$ with y)

or $x - 6 = y$

$x = y + 6$ (Solve for x)

$f^{-1}(y) = y+6$ [$\because y = f(x) \Rightarrow f^{-1}(y) = x$]

$f^{-1}(x) = x + 6$ (Replacing y by x)

$f(f^{-1}(x)) = f(x + 6) = (x + 6) - 6$

$= x + 6 - 6 = x$

$f^{-1}(f(x)) = f^{-1}(x - 6) = (x + 6) - 6$

$= x + 6 - 6 = x$

Hence, verified that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

(ii) $f(x) = 7x - 4$

Sol. $f(x) = 7x - 4$

$y = 7x - 4$ (Replace $f(x)$ with y)

or $7x - 4 = y$

$7x = y + 4$

$x = \dfrac{y+4}{7}$ (Solve for x)

$f^{-1}(y) = \dfrac{y+4}{7}$ $[\because y = f(x) \Rightarrow f^{-1}(y) = x]$

$f(x) = \dfrac{x+4}{7}$ (Replacing y by x)

$f(f^{-1}(x)) = f\left(\dfrac{x+4}{7}\right) = 7\left(\dfrac{x+4}{7}\right) - 4$

$= x + 4 - 4 = x$

$f^{-1}(f(x)) = f^{-1}\left(\dfrac{x+4}{7}\right) = 7\left(\dfrac{x+4}{7}\right) - 4$

$= x + 4 - 4 = x$

Hence, verified that

$f(f^{-1}(x)) = f^{-1}(f(x)) = x$

(iii) $f(x) = \dfrac{x-3}{4}$

Sol. $f(x) = \dfrac{x-3}{4}$

$y = \dfrac{x-3}{4}$ (Replace $f(x)$ with y)

or $\dfrac{x-3}{4} = y$

$x - 3 = 4y$

$x = 4y + 3$ (Solve for x)

$f^{-1}(y) = 4y + 3$ $[\because y = f(x) \Rightarrow f^{-1}(y) = x]$

$f^{-1}(x) = 4x + 3$ (Replacing y by x)

$f(f^{-1}(x)) = f(4x + 3)$

$= \dfrac{4x+3-3}{4} = \dfrac{4x}{4} = x$

$f^{-1}(f(x)) = f^{-1}(4x + 3)$

$= \dfrac{4x+3-3}{4} = \dfrac{4x}{4} = x$

Hence, verified that

$f(f^{-1}(x)) = f^{-1}(f(x)) = x$

(iv) $f(x) = \dfrac{x-4}{x+2}$

Sol. $f(x) = \dfrac{x-4}{x+2}$

$y = \dfrac{x-4}{x+2}$ (Replace $f(x)$ with y)

$y(x + 2) = x - 4$

$xy + 2y = x - 4$

$xy - x = -2y - 4$

$x(y - 1) = -2y - 4$

$x = \dfrac{-2y-4}{y-1}$ (Solve for x)

$f^{-1}(y) = \dfrac{-2y-4}{y-1}$ $[\because y = f(x) \Rightarrow f^{-1}(y) = x]$

$f^{-1}(x) = \dfrac{-2x-4}{x-1}$ (Replacing y by x)

$$f(f^{-1}(x)) = f\left(\frac{-2x-4}{x-1}\right) = \frac{\dfrac{-2x-4}{x-1} - 4}{\dfrac{-2x-4}{x-1} + 2}$$

$$= \frac{\dfrac{-2x-4-4(x-1)}{x-1}}{\dfrac{-2x-4+2(x-1)}{x-1}}$$

$$= \frac{\dfrac{-2x-4-4x+4}{x-1}}{\dfrac{-2x-4+2x-2}{x-1}} = \frac{\dfrac{-6x}{x-1}}{\dfrac{-6}{x-1}}$$

$$= \frac{-6x}{x-1} \times \frac{x-1}{-6} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{-2x-4}{x-1}\right) = \frac{\dfrac{-2x-4}{x-1} - 4}{\dfrac{-2x-4}{x-1} + 2}$$

$$= \frac{\dfrac{-2x-4-4(x-1)}{x-1}}{\dfrac{-2x-4+2(x-1)}{x-1}}$$

$$= \frac{\frac{-2x-4-4x+4}{x-1}}{\frac{-2x-4+2x-2}{x-1}} = \frac{\frac{-6x}{x-1}}{\frac{-6}{x-1}}$$

$$= \frac{-6x}{x-1} \times \frac{x-1}{-6} = x$$

Hence, verified that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

9. Without finding $f^{-1}(x)$, find domain and range of $f^{-1}(x)$:

(i) $f(x) = 12x - 3$

Sol. $f(x) = 12x - 3$

Domain of $f(x)$ = All real numbers $= (-\infty, \infty)$

Range of $f(x)$ = All real numbers $= (-\infty, \infty)$

Domain of $f^{-1}(x)$ = All real numbers $= (-\infty, \infty)$

Range of $f^{-1}(x)$ = All real numbers $= (-\infty, \infty)$

(ii) $f(x) = \frac{1}{2}x + 8$

Sol. $f(x) = \frac{1}{2}x + 8$

Domain of $f(x)$ = All real numbers $= (-\infty, \infty)$

Range of $f(x)$ = All real number $= (-\infty, \infty)$

Domain of $f^{-1}(x)$ = All real numbers $= (-\infty, \infty)$

Range of $f^{-1}(x)$ = All real number $= (-\infty, \infty)$

(iii) $f(x) = \frac{x}{1+x}$

Sol. $f(x) = \frac{x}{1+x}$

Domain of $f(x) = x$ cannot be zero $= (-\infty, 1) \cup (1, \infty)$

Range of $f(x) = x$ cannot be zero $= (-\infty, -1) \cup (-1, \infty)$

Domain of $f^{-1}(x) = (-\infty, 1) \cup (1, \infty)$

Range of $f^{-1}(x) = (-\infty, -1) \cup (-1, \infty)$

(iv) $f(x) = \sqrt{x-2}$

Sol. $f(x) = \sqrt{x-2}$

Domain of $f(x) = x \geq 2 = [2, \infty)$

Range of $f(x)$ = square roots always non negative $= [0, \infty)$

Domain $f^{-1}(x) = [0, \infty)$

Range $f^{-1}(x) = [2, \infty]$

10. Given that $f(x) = x^2 + 9$ and $g(x)=x+21$. Find the values of a such that: $f(a) = g(a)$.

Sol. $f(x) = x^2 + 9$, $g(x) = x + 21$

$f(a) = a^2 + 9$, $g(a) = a + 21$

as $f(a) = g(a)$ (given)

$a^2 + 9 = a + 21$

$a^2 - a + 9 - 21 = 0$

$a^2 - a - 12 = 0$

$a^2 - 4a + 3a - 12 = 0$

$a(a-4) + 3(a-4) = 0$

$(a-4)(a+3) = 0$

$a - 4 = 0$	$a + 3 = 0$
$a = 4$	$a = -3$

Hence, $a = 4, -3$

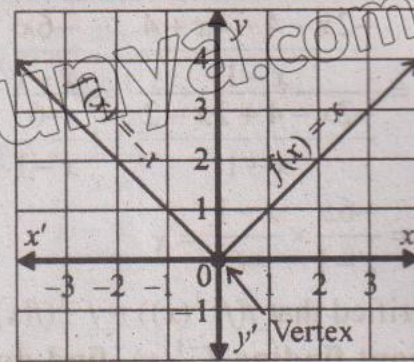
4.2 Graphs of Absolute Valued Functions

An absolute valued function is a type of function that includes an algebraic expression enclosed within absolute value bars. Its general form is $f(x) = |x|$. The absolute value of a number represents its distance from zero on the number line and is always non-negative.

Remember!
The graphs of all other absolute value functions are variations of the parent function $f(x) = |x|$ obtained through transformations.

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

The graph of $f(x) = |x|$ V-shape and is symmetric with respect to the y-axis. The point where the graph changes direction is called the vertex (0, 0).



The vertex form of an absolute value function is $g(x) = a|x - h| + k$, where $a \neq 0$ and (h, k) is the vertex. Its graph is symmetric about the line $x = h$.

Example 10: Plot the graph of the following functions:

(i) $f(x) = |x| + 2$

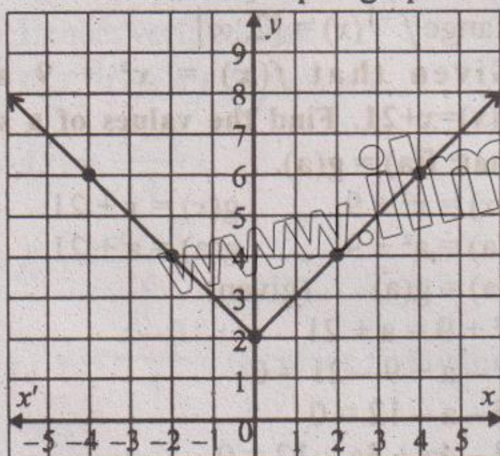
Sol.

Step I Make a table of values.

x	−4	−2	0	2	4...
y	6	4	2	4	6...

Step II Plot the ordered pairs.

Step III Draw the V-shaped graph.



(ii) $g(x) = |x - 3|$

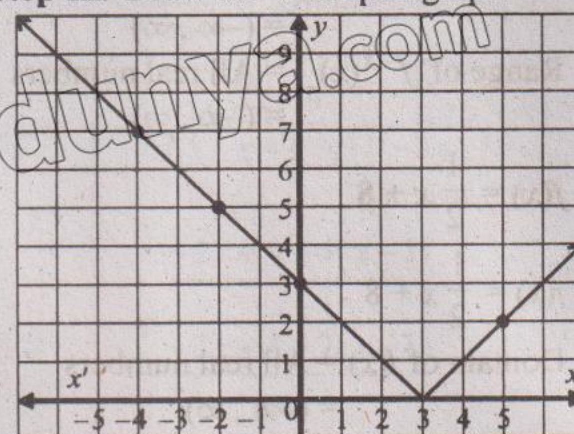
Sol.

Step I Make a table of values.

x	−4	−2	0	3	5...
y	7	5	3	0	2...

Step II Plot the ordered pairs.

Step III Draw the V-shaped graph.



Example 11: Plot the graph of $f(x) = 2|x + 1| - 3$.

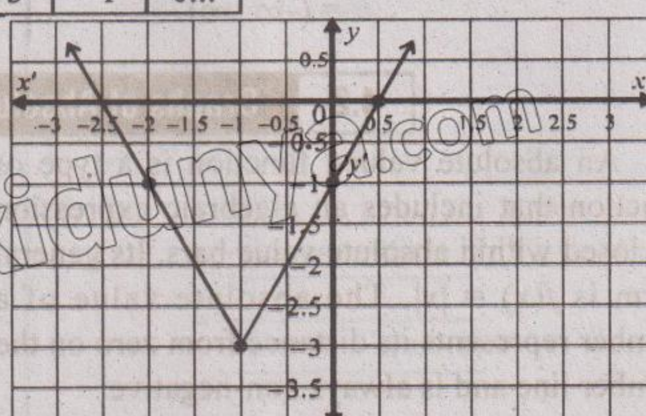
Sol. $f(x) = 2|x + 1| - 3$

Step I Make a table of values.

x	−2	−1	0	0.5
$f(x)$	−1	−3	−1	0...

Step II Plot the ordered pairs.

Step III Draw the V-shaped graph of each function. Notice that the vertex of the graph of f is (−1, −3) and the graph is symmetric about $x = -1$



4.3 Solution of Absolute Value Linear Equation in One Variable

An absolute value linear equation is an equation in which the absolute value is applied to a linear expression.

Example 12: Solve $2|x-3| = 16$ and express the solution on number line.

Sol. $2|x-3| = 16$

$|x-3| = 8$

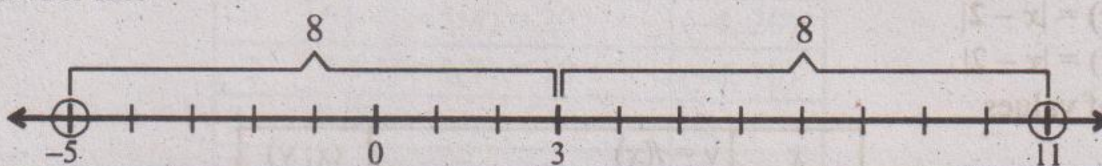
$\pm(x-3) = 8$ or $x-3 = \pm 8$

Now $x-3 = 8$ or $x-3 = -8$

$x = 11$ or $x = -5$

Solution set $\{11, -5\}$

Number line



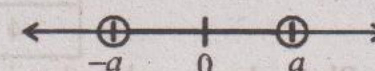
4.4 Solution of Absolute Value Linear Inequality in One Variable

An absolute value linear inequality in one variable is an inequality that involves the absolute value of a linear expression with only one variable. It has forms such as:

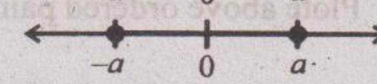
$|ax + b| < 0$, $|ax + b| > 0$, $|ax + b| \geq 0$, $|ax + b| \leq 0$

Note:

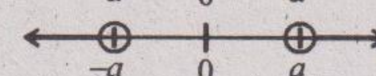
(i) $-a < x < a$ is equivalent to $(-a, a)$



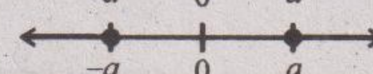
(ii) $-a \leq x \leq a$ is equivalent to $[-a, a]$



(iii) $a < -a$ or $x > a$ is equivalent to $(-\infty, -a) \cup (a, \infty)$



(iv) $x \leq -a$ or $x \geq a$ is equivalent to $(-\infty, -a) \cup [a, \infty)$



Example 13: Solve the inequality $|x-1| \geq 5$ and express the solution on number line.

Sol. $|x-1| \geq 5$

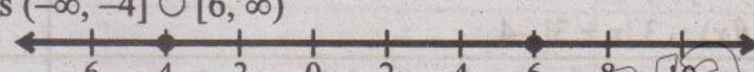
$\pm(x-1) \geq 5$

$-(x-1) \geq 5$ or $x-1 \geq 5$

$x-1 \leq -5$ or $x-1 \geq 5$

$x \leq -4$ or $x \geq 6$

The solution is $(-\infty, -4] \cup [6, \infty)$



Example 14: Express the solution of $40|2x-5| + 1 > 201$ on number line.

Sol. First isolate the absolute value expression on one side of the inequality.

$40|2x-5| + 1 > 201$

$40|2x-5| > 200$

$\frac{40|2x-5|}{40} > \frac{200}{40}$ (Divide each side by 40)

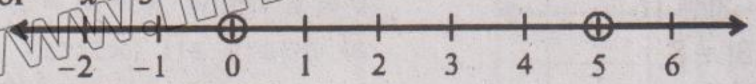
$|2x-5| > 5$

$\pm(2x-5) > 5$

$-(2x-5) > 5$ or $2x-5 > 5$

$2x-5 < -5$ or $2x > 10$

$x < 0$ or $x > 5$



The solution is $(-\infty, 0) \cup (5, \infty)$

Solved Exercise 4.2

1. Plot graph for the following absolute valued functions:

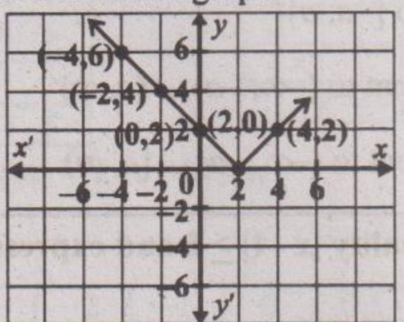
(i) $f(x) = |x-2|$

Sol. $f(x) = |x-2|$

Table of values

x	$y = f(x)$ $f(x) = \|x-2\|$	(x, y)
-4	$\|-4-2\| = \|-6\| = 6$	$(-4, 6)$
-2	$\|-2-2\| = \|-4\| = 4$	$(-2, 4)$
0	$\|0-2\| = \|-2\| = 2$	$(0, 2)$
2	$\|2-2\| = \|0\| = 0$	$(2, 0)$
4	$\|4-2\| = \|2\| = 2$	$(4, 2)$

Plote above ordered pairs and draw the graph

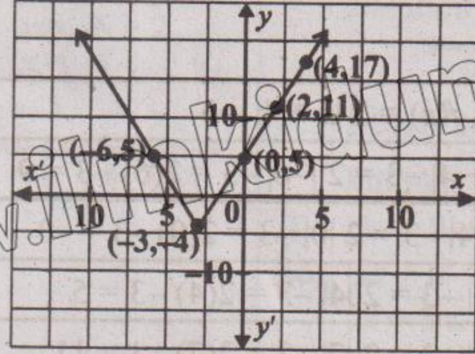


(ii) $f(x) = 3|x+3| - 4$

Sol. Table of the values

x	$y = f(x)$ $f(x) = 3\|x+3\| - 4$	(x,y)
-6	$3\|-6+3\| - 4 = 3\|-3\| - 4 = 3(3) - 4 = 5$	$(-6, 5)$
-3	$3\|-3+3\| - 4 = 3\|0\| - 4 = 3(0) - 4 = -4$	$(-3, -4)$
0	$3\|0+3\| - 4 = 3(3) - 4 = 9 - 4 = 5$	$(0, 5)$
2	$3\|2+3\| - 4 = 3\|5\| - 4 = 3(5) - 4 = 11$	$(2, 11)$
4	$3\|4+3\| - 4 = 3\|7\| - 4 = 3(7) - 4 = 17$	$(4, 17)$

Plot the above ordered pairs and draw the graph

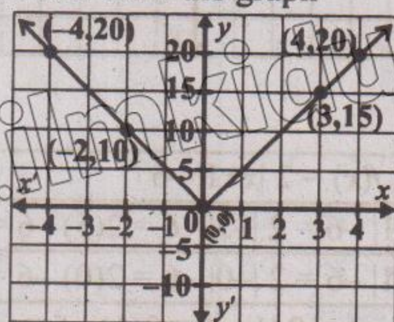


(iii) $f(x) = 5|x|$

Sol. Table of the values

x	$y = f(x), f(x) = 5\,|x|$	(x, y)
-4	$5\,|-4| = 5(4) = 20$	$(-4, 20)$
-2	$5\,|-2| = 5(2) = 10$	$(-2, 10)$
0	$5\,|0| = 5(0) = 0$	$(0, 0)$
3	$5\,|3| = 5(3) = 15$	$(3, 15)$
4	$5\,|4| = 5(4) = 20$	$(4, 20)$

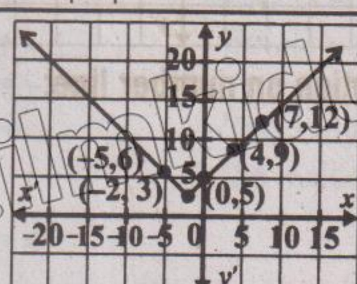
Plot the above ordered pairs and draw the graph



(iv) $f(x) = |x + 2| + 3$

Sol. Table of the values

x	$y = f(x), f(x) = |x + 2| + 3$	(x, y)
7	$|7 + 2| + 3 = |9| + 3 = 9 + 3 = 12$	$(7, 12)$
4	$|4 + 2| + 3 = |6| + 3 = 6 + 3 = 9$	$(4, 9)$
0	$|0 + 2| + 3 = |2| + 3 = 2 + 3 = 5$	$(0, 5)$
-2	$|-2 + 2| + 3 = |0| + 3 = 0 + 3 = 3$	$(-2, 3)$
-5	$|-5 + 2| + 3 = |-3| + 3 = 3 + 3 = 6$	$(-5, 6)$

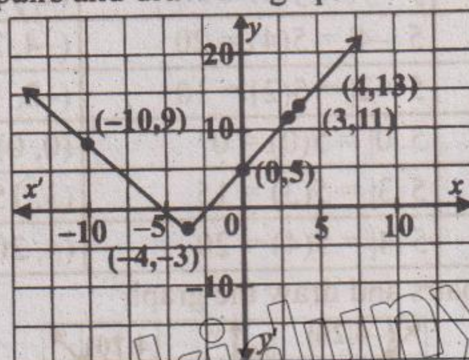


(v) $f(x) = 2|x + 4| - 3$

Sol. Table of the values

x	$y = f(x)$, $f(x) = 2\|x + 4\| - 3$	(x, y)
–10	$2\|-10 + 4\| - 3 = 2\|-6\| - 3 = 2(6) - 3 = 9$	(–10, 9)
–4	$2\|-4 + 4\| - 3 = 2\|0\| - 3 = 2(0) - 3 = -3$	(–4, –3)
0	$2\|0 + 4\| - 3 = 2\|4\| - 3 = 2(4) - 3 = 5$	(0, 5)
3	$2\|2 + 4\| - 3 = 2\|7\| - 3 = 2(7) - 3 = 11$	(3, 11)
4	$2\|4 + 4\| - 3 = 2\|8\| - 3 = 2(8) - 3 = 13$	(4, 13)

Plot the above ordered pairs and draw the graph

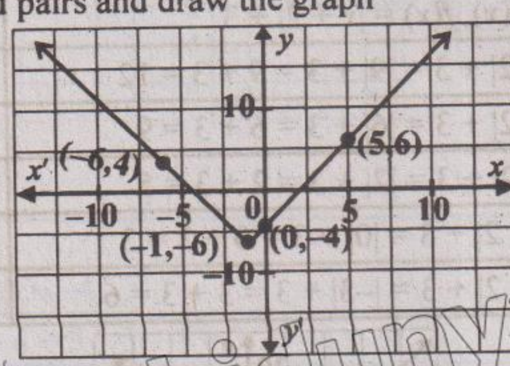


(vi) $f(x) = 2|x + 1| - 6$

Sol. Table of the values

x	$y = f(x)$, $f(x) = 2\|x+1\| - 6$	(x, y)
–6	$2\|-6 + 1\| - 6 = 2\|-5\| - 6 = 2(5) - 6 = 4$	(–6, 4)
–1	$2\|-1 + 1\| - 6 = 2\|-0\| - 6 = 2(0) - 6 = -6$	(–1, – 6)
0	$2\|0 + 1\| - 6 = 2\|1\| - 6 = 2(1) - 6 = -4$	(0, –4)
5	$2\|5 + 1\| - 6 = 2\|6\| - 6 = 6$	(5, 6)

Plot the above ordered pairs and draw the graph



2. Solve and express the solution on number line:

(i) $|x - 2| = 6$

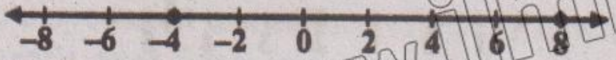
Sol. $|x - 2| = 6$

$x - 2 = \pm 6$

$x - 2 = 6, x - 2 = -6$

$x-2=6$ | $x-2=-6$
$x=6+2$ | $x=-6+2$
$x=8$ | $x=-4$

Solution set = $\{8, -4\}$



(ii) $|2x+1|=7$

Sol. $|2x+1|=7$

$2x+1=\pm 7$

$2x+1=7,\ 2x+1=-7$

$2x+1=7$	$2x+1=-7$
$2x=7-1$	$2x=-7-1$
$2x=6$	$2x=-8$
$x=\dfrac{6}{2}$	$x=\dfrac{-8}{2}$
$x=3$	$x=-4$

Solution set = $\{3, -4\}$

(iii) $|4x-9|=3$

Sol. $|4x-9|=3$

$4x-9=\pm 3$

$4x-9=3,\ 4x-9=-3$

$4x-9=3$	$4x-9=-3$
$4x=3+9$	$4x=-3+9$
$4x=12$	$4x=6$
$x=\dfrac{12}{4}$	$x=\dfrac{6}{4}=\dfrac{3}{2}$
$x=3$	

Solution set = $\left\{3, \dfrac{3}{2}\right\}$

(iv) $|7-2x|=1$

Sol. $|7-2x|=1$

$7-2x=\pm 1$

$7-2x=1,\ 7-2x=-1$

$7-2x=1$	$7-2x=-1$
$-2x=1-7$	$7-2x=-1-7$
$-2x=-6$	$-2x=-8$
$2x=6$	$2x=8$
$x=\dfrac{6}{2}=3$	$x=\dfrac{8}{2}=4$

Solution set = $\{3, 4\}$

3. Solve and express the solution on number line:

(i) $|5x+8|\le 3$

Sol. $|5x+8|\le 3$

$5x+8\le\pm 3$

$5x+8\le 3$ or $5x+8\ge -3$

$5x\le 3-8$ or $5x\ge -3-8$

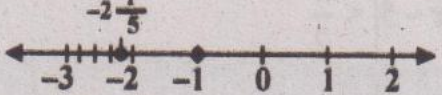
$5x\le -5$ or $5x\ge -11$

$x\le -\dfrac{5}{5}$ or $x\ge -\dfrac{11}{5}$

$x\le -1$ or $x\ge -2\dfrac{1}{5}$

$-2\dfrac{1}{5}\le x\le -1$

$\left[-2\dfrac{1}{5}, -1\right]$



(ii) $|4x-12|\le 0$

Sol. $|4x-12|\le 0$

Since an absolute value can not be less than zero, the only possibility is:

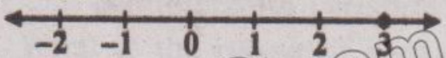
$|4x-12|=0$

$4x-12=0$

$4x=12$

$x=\dfrac{12}{4}$

$x=3$



(iii) $|3-4x|>0$

Sol. $|3-4x|>0$

An absolute is always positive or zero. The only value that does not work when the inside equal zero.

$3-4x\ne 0$

$3-4x\ne -3$

$$4x \neq 3$$

$$x \neq \frac{3}{4}$$

$$S.S = \left(-\infty, \frac{3}{4}\right) \cup \left(\frac{3}{4}, \infty\right)$$

(iv) $1-2\left|\frac{3}{2}x-5\right| > -3$

Sol. $1-2\left|\frac{3}{2}x-5\right| > -3$

$$-2\left|\frac{3}{2}x-5\right| > -3-1$$

$$-2\left|\frac{3}{2}x-5\right| > -4$$

$$2\left|\frac{3}{2}x-5\right| < 4$$

$$\left|\frac{2}{x}x-5\right| < \frac{4}{2}$$

$$\left|\frac{3}{2}x-5\right| < 2$$

$$\frac{3}{2}x-5 < \pm 2$$

$$\frac{3}{2}x-5 < 2 \quad \text{or} \quad \frac{3}{2}x-5 > -2$$

$$\frac{3}{2}x < 2+5 \quad \text{or} \quad \frac{3}{2}x > -2+5$$

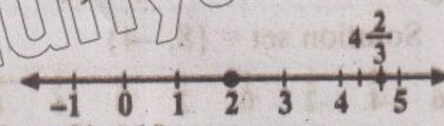
$$\frac{3}{2}x < 7 \quad \text{or} \quad \frac{3}{2}x > 3$$

$$x < 7\times\frac{2}{3} \quad \text{or} \quad x > 3\times\frac{2}{3}$$

$$x < \frac{14}{3} \quad \text{or} \quad x > 2$$

$$x < 4\frac{2}{3}$$

$$S.S \left\{2, 4\frac{2}{3}\right\} \quad 2 < x < \frac{14}{3}$$



(v) $|3x-2| \leq 12$

Sol. $|3x-2| \leq 12$

$$3x-2 \leq \pm 12$$

$$3x-2 \leq 12 \quad \text{or} \quad 3x-2 \geq -12$$

$$3x \leq 12+2 \quad \text{or} \quad 3 \geq -12+2$$

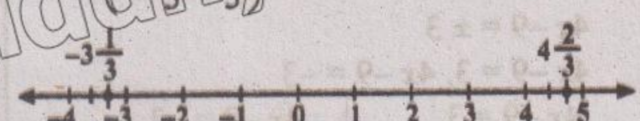
$$3x \leq 14 \quad \text{or} \quad 3x \geq -10$$

$$x \leq \frac{14}{3} \quad \text{or} \quad x \geq -\frac{10}{3}$$

$$x \leq 4\frac{2}{3} \quad \text{or} \quad x \geq -3\frac{1}{3}$$

$$-3\frac{1}{3} \leq x \leq 4\frac{2}{3}$$

$$S.S = \left(-3\frac{1}{3}, 4\frac{2}{3}\right)$$



(vi) $|1-2x| > 5$

Sol. $|1-2x| > 5$

$$1-2x > \pm 5$$

$$1-2x > 5 \quad \text{or} \quad 1-2x < -5$$

$$-2x > 5-1 \quad \text{or} \quad -2x < -5-1$$

$$-2x > 4 \quad \text{or} \quad -2x < -6$$

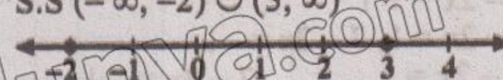
$$2x > 4 \quad \text{or} \quad 2x < 6$$

$$x < \frac{-4}{2} \quad \text{or} \quad x > \frac{6}{2}$$

$$x < -2 \quad \text{or} \quad x > 3$$

$x < -2$ or $x > 3$

$S.S\ (-\infty, -2) \cup (3, \infty)$



4.5 Solving Real-Life Problems Involving Function

Functions are widely used to model and solve real-life problems in areas such as finance, transportation and sales. In finance, functions can represent income, expenses, interest or profit over time. In transportation, functions can describe the relationship between distance, speed and time, helping to calculate travel times or fuel consumption. In sales, functions are used to model revenue, cost and profit based on the number of items sold. Solving these problems involves identifying the relevant variables, expressing their relationship as a function or equation and using algebraic methods to find unknown values, make decisions or predict outcomes.

Example 15: $E(n) = 1,500n$ is the total earning function, where n is the number of days. If labourer works 10 days, find his earning.

Solution: Given that:

$$E(n) = 1,500n$$

If the labourer works for 10 days, his earning will be $E(10) = 1,500 \times 10$

$$= \text{Rs. } 15,000$$

Example 16: $f(x) = 200 + 15x$ is a function of charging taxi fare, where x represents the number of kilometres. Find proposed fare for a distance of 50 Km.

Solution: $f(x) = 200 + 15x$

Required fare $= f(50) = 200 + 15 \times 50$

$= 200 + 750 = \text{Rs. } 950$

4.6 Solving Real-Life Problems Involving Absolute Valued Functions

Absolute value functions are essential for solving real-life problems where distance, magnitude or fluctuations are involved, regardless of direction. In energy wave analysis, such as sound or seismic waves, absolute value functions model the amplitude or strength of a wave. In physics and engineering, magnitude is often expressed using absolute values to describe the size of a force, charge or vector, independent of direction. In distance-related problems, such as the location of an object from a reference points, absolute value functions measure how far one quantity is from another, ensuring the result is always non-negative. These functions help describe situations with symmetry or two-sided variation.

Example 17: A sensitive device operates best at $220V$ but can tolerate a variation of up to $5V$. The condition is modeled as: $|V - 220| \leq 5$. What is the range of acceptable voltages?

Solution $|V - 220| \leq 5$

or $-5 \leq V - 220 \leq 5$

or $-5 \leq V - 220 \leq V \leq 5 + 220$ or $215 \leq V \leq 225$

So, acceptable voltage is between $215\ V$ and $225\ V$.

Example 18:

An earthquake sensor issues a warning if the recorded magnitude differs from a safe level of 4.5 by more than 1.2 units. The condition is modeled by $|M - 4.5| > 1.2$.

For which values of magnitude M will the sensor issue a warning?

Solution: $|M - 4.5| > 1.2$

$\pm(M - 4.5) > 1.2$

$-(M - 4.5) > 1.2$ or $M - 4.5 > 1.2$

$M - 4.5 < -1.2$ or $M - 4.5 > 1.2$

$M < 3.3$ or $M > 5.7$

Hence, the sensor issues a warning when $M < 3.3$ or $M > 5.7$.

Solved Exercise 4.3

1. A function $B(t) = 5{,}000 + 200t$ represents the total balance (in rupees) after t months. What will be the balance after 6 months?

Sol: B (t) = 5,000 + 200t (Given function)

Put t = 6 in function

B (6) = 5,000 + 200 (6)

B (6) = 5,000 + 1200

B (6) = 6,200

Balance = Rs.6,200

2. A function $f(k) = 150 + 20k$ represents the total fare (in rupees) for k kilometers. How much will the fare be for a 12 kilometres ride?

Sol:

$f(k) = 150 + 20k$ (Given function)

Put k = 12 in the function

$f(12) = 150 + 20(12)$

$f(12) = 150 + 240$

$f(12) = 390$

So, Fare = Rs.390

3. The cost of manufacturing fancy sofa set would be fixed charges Rs. 5500 which is modeled as $f(n) = 5500$ n, where n is the number of sofa sets. Find the cost of 50 sofa sets.

Sol:

$f(x) = 5500x$ (Given function)

Put $x = 50$ in the function

$f(50) = 5500(50)$

$f(50) = 275{,}000$

So, total cost = Rs.275,000

4. A function $T(d) = \dfrac{d}{60}$ represents the time T in hours to travel a distance d kilometers. How long will it take to travel 180km?

Sol: $T(d) = \dfrac{d}{60}$ (Given function)

Put d = 180 in the function

$T(180) = \frac{180}{60}$

$T(180) = 3$

So, Time = 3 hours

5. A company charges Rs. 100 for an encoding work. In addition, the company charges Rs. 5 per page of printed output. The model of function $f(x) = 100 + 5x$, where x represents the number of pages printed out. How much will company charge for 55 page encoding and printing work?

Sol:

$f(x) = 100+5x$ (Given function)

put $x = 55$ in the function

$f(55) = 100 + 5(55)$

$f(55) = 100 + 275$

$f(55) = 375$

Total charge = Rs. 375

6. A chemical reaction is stable at 37°C. The process must be stopped if the temperature deviates by more than 2.5°C. The condition is modeled as: $|T - 37°| > 2.5°$, T be the temperature. For what temperature values must the process be stopped?

Sol: $|-37°| > 2.5°$

$T-37 > \pm 2.5$

$T-37 > 2.5$ or $T-37 < -2.5$

$T > 2.5 + 37$ or $T < -2.5 + 37$

$T > 39.5$ or $T < 34.5$

The process must be stopped if the temperature is obove 39.5°c or below 34.5°c

$T < 34.5°C$ or $T > 39.5°C$

7. A factory produces metal rods that must be 2.5 metres long, with a tolerance of ±0.04 metres. An absolute value inequality models this: $|x - 2.5| \leq 0.04$. What is the range of acceptable lengths?

Sol: $|x - 2.5| \leq 0.04$

$x-2.5 \leq \pm 0.04$

$x-2.5 \leq 0.04$ or $x-2.5 \geq -0.04$

$x \leq 0.04 + 2.5$ or $x \geq -0.04 + 2.5$

$x \leq 2.54$ or $x \leq 2.46$

$2.46\text{ m} \leq x \leq 2.54$

8. A machine part must be aligned so that its centre is exactly at 0. If it shifts more than 0.1 mm, the part is rejected. The model is given by |JC| > 0.1. What positions of the centre cause rejection?

Sol: $|x| > 0.1$

$x > \pm 0.1$

$x > 0.1$ or $x < -0.1$

So, Rejection occurs for any positon greater Than 0.1mm or less than -0.1 mm

$x < -0.1$mm or $x > 0.1$mm

Solved Review Exercise 4

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) If $f(x) = \dfrac{5x-6}{3}$, then what is the value of $f(3)$?

(a) -1 (b) 3 (c) 9 (d) 15

(ii) A function f from X to Y is represented by:

(a) f : XY (b) $f: Y \to X$ (c) $f: X \to Y$ (d) $f: \dfrac{X}{Y}$

(iii) $(f o g)(x) =$

(a) $(f+g)(x)$ (b) $(f-g(x)$ (c) $f(g(x))$ (d) $f(x) \div g(x)$

(iv) If $f(x) = 2x + 3$, $g(x) = x + 1$, then $f(x) + g(x) =$

(a) $3x$ (b) $3x + 4$ (c) 4 (d) $2x^2 + 3$

(v) If $f(x) = 5x + 2$, $h(x) = 2x - 2$, then what is $f(x) - h(x) =$

(a) $3x$ (b) $5x^2 - 4$ (c) $3x + 4$ (d) $-3x - 4$

(vi) If $f(x) = 3x + 1$, $g(x) = 2x$, then what is $g(x) \times f(x) =$

(a) $6x + 2x$ (b) $5x^2 + 1$ (c) $x + 1$ (d) $6x^2 + 2x$

(vii) If $f(x) = x^2 - 4$, $g(x) = x + 2$, $x \neq -2$ then $\dfrac{f(x)}{g(x)} =$

(a) $\dfrac{1}{x-2}$ (b) $\dfrac{1}{x+2}$ (c) $x + 2$ (d) $x - 2$

(viii) What is the shape of the graph of an absolute value function?

(a) U-shaped (b) V-shaped (c) L-shaped (d) M-shaped

(ix) If a graph represents a function, then every vertical line must intersect it at:

(a) 4 points (b) 3 points (c) 2 points (d) 1 point

(x) If $f(x) = x^3$, then $f(-2) =$

(a) -8 (b) 8 (c) 4 (d) -6

ANSWERS:

(i)	3	(ii)	$f: X \to Y$	(iii)	$f(g(x))$	(iv)	$3x + 4$	(v)	$3x + 4$
(vi)	$6x^2 + 2x$	(vii)	$x - 2$	(viii)	V-shaped	(ix)	1 point	(x)	-8

2. If $f(x) = 25 - x^2$ and $g(x) = 5 + x$, find

(i) $f(x) + g(x)$

Sol. $f(x) + g(x)$ (given)

$= (25 - x^2) + (5 + x)$

$= 25 - x^2 + 5 + x$

$= -x^2 + x + 25 + 5$
$= -x^2 + x + 30$

(ii) $f(x) - g(x)$

Sol. $f(x) - g(x)$
$= (25 - x^2) - (5 + x)$
$= 25 - x^2 - 5 - x$
$= -x^2 - x + 25 - 5$
$= -x^2 - x + 20$

(iii) $f(x)\,g(x)$

Sol. $f(x)\,g(x)$
$= (25 - x^2)(5 + x)$
$= 25(5 + x) - x^2(5 + x)$
$= 125 + 25x - 5x^2 - x^3$
$= -x^3 - 5x^2 + 25x + 125$

(iv) $\dfrac{f(x)}{g(x)}$

Sol. $\dfrac{f(x)}{g(x)}$

$$= \frac{25 - x^2}{(5+x)} = \frac{(5)^2 - (x)^2}{(5+x)}$$

$$= \frac{(5+x)(5-x)}{(5+x)}$$

$= 5 - x$

(v) $f(7)$

Sol. $f(x) = 25 - x^2$
$f(7) = 25 - (7)^2$
$= 25 - 49$
$= -24$

(vi) $g(-8)$

Sol. $g(x) = 5 + x$
$g(-8) = 5 + (-8)$
$= 5 - 8 = -3$

3. If $f(x) = x^3$ and $g(x) = 14 + 2x$, find

(i) $(fog)(x)$

Sol. $= f(g(x))$
$= (14 + 2x)$
$= (14 + 2x)^3$

(ii) $(gof)(x)$

Sol. $= g(f(x))$
$= (f(x))$
$= 14 + 2x^3$
$= 2(7 + x^3)$

(iii) $(fof)(x)$

Sol. $= (f(x))$
$= f(x^3)$
$= (x^3)^3$
$= x^9$

(iv) $(gog)(x)$

Sol. $= g(g(x))$
$= g(14 + 2x)$
$= 14 + 2(14 + 2x)$
$= 14 + 28 + 4x$
$= 42 + 4x$
$= 2(21 + 2x) = 2(2x + 21)$

4. Find $f^{-1}(x)$ if

(i) $f(x) = 9x - 1$

Sol. $f(x) = 9x - 1$
$y = 9x - 1$ (Replace $f(x)$ with y)
or $9x - 1 = y$
$9x = y + 1$

$$x = \frac{y+1}{9} \quad \text{(Solve for } x\text{)}$$

$$f^{-1}(y) = \frac{y+1}{9} \quad \because y = f(x) \Rightarrow f^{-1}(y) = x$$

$$f^{-1}(x) = \frac{x+1}{9} \quad \text{(Replacing y by } x\text{)}$$

(ii) $f(x) = \dfrac{5}{x-1}, x \neq 1$

Sol. $f(x) = \dfrac{5}{x-1}$

$$y = \frac{5}{x-1} \quad \text{(Replace } f(x) \text{ with y)}$$

$y(x - 1) = 5$

$$(x - 1) = \frac{5}{y}$$

$$x = \frac{5}{y} + 1$$

$$x = \frac{5+y}{y} \quad \text{(Solve for x)}$$

$f^{-1}(y) = \dfrac{5+y}{y} \quad \therefore y = f(x) \Rightarrow f^{-1}(y) = x$

$f^{-1}(x) = \dfrac{5+x}{x}$ (Replace y by x)

(iii) $f(x) = \sqrt{x-5}$

Sol. $y = \sqrt{x-5}$ (Replace $f(x)$ with y)

$y = \sqrt{x-5}$

$\left(\sqrt{x-5}\right)^2 = y^2$

$x - 5 = y^2$

$x = y^2 + 5$ (Solve for x)

$f^{-1}(y) = y^2 + 5$ [$\because$ $y = f(x) \Rightarrow f^{-1}(y) = x$]

$f^{-1}(x) = x^2 + 5$ (Replacing y by x)

(iv) $f(x) = \dfrac{3-x}{2}$

Sol. $y = \dfrac{3-x}{2}$ (Replace $f(x)$ with y)

or $\dfrac{3-x}{2} = y$

$3 - x = 2y$

$-x = 2y - 3$

$x = -2y + 3$ (solve for x)

$f^{-1}(y) = -2y + 3$ [$\because$ $y = f(x) \Rightarrow f^{-1}(y) = x$]

$f^{-1}(x) = -2x + 3$ (Replacing y by x)

$f^{-1}(x) = 3 - 2x$

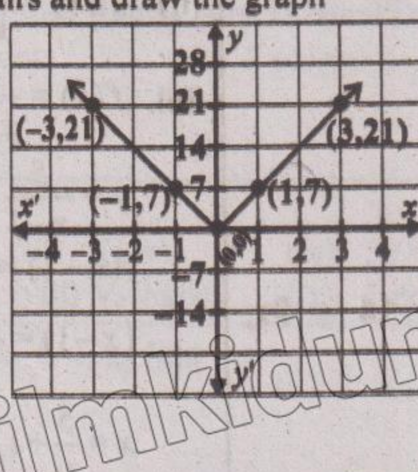
5. Plot graph for the following absolute valued functions:

(i) $f(x) = 7|x|$

Sol. Table of the values

x	$y = f(x), f(x) = 7\|x\|$	(x, y)
-3	$7\|-3\| = 7(3) = 21$	$(-3, 21)$
-1	$7\|-1\| = 7(1) = 7$	$(-1, 7)$
0	$7\|0\| = 7(0) = 0$	$(0, 0)$
1	$7\|1\| = 7(1) = 7$	$(1, 7)$
3	$7\|3\| = 7(3) = 21$	$(3, 21)$

Plot the above ordered pairs and draw the graph



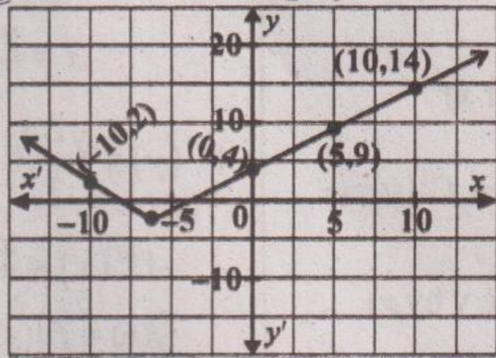
(ii) $f(x) = |x + 6| - 2$

Sol. Table of the values

x	$y = f(x), f(x) = \|x + 6\| - 2$	(x, y)
-10	$\|-10 + 6\| - 2 = \|-4\| + 2 = 4 - 2 = 2$	$(-10, 2)$

–6	$\|-6+6\|-2 = \|0\|-2 = 0-2 = -2$	(–6, –2)
0	$\|0+6\|-2 = \|6\|-2 = 6-2 = 4$	(0, 4)
5	$\|5+6\|-2 = \|11\|-2 = 11-2 = 9$	(5, 9)
10	$\|10+6\|-2 = \|16\|-2 = 16-2 = 14$	(10, 14)

Plot the above ordered pairs and draw the graph



6. Solve and express the solution on number line:

(i) $|3x-2| = 1$

Sol. $|3x-2| = 1$

$3x - 2 = \pm 1$

$3x - 2 = 1,\quad 3x - 2 = -1$

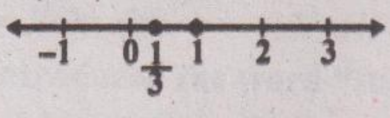
$3x = 1 + 2,\quad 3x = -1 + 2$

$3x = 3,\quad 3x = 1$

$x = \frac{3}{3},\quad x = \frac{1}{3}$

$x = 1,$

$S.S = \left\{\frac{1}{3}, 1\right\}$



(ii) $|6x+1| = 9$

Sol. $6x + 1 = \pm 9$

$6x + 1 = 9,\quad 6x + 1 = -9$

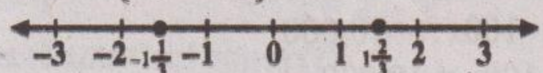
$6x = 9 - 1,\quad 6x = -9 - 1$

$6x = 8,\quad 6x = -10$

$x = \frac{8}{6},\quad x = \frac{-10}{6}$

$x = \frac{4}{3} = 1\frac{1}{3}\quad x = -\frac{5}{3} = -1\frac{2}{3}$

$S.S = \left\{1\frac{1}{3}, -1\frac{2}{3}\right\}$



7. Solve and express the solution on number line:

(i) $|7-2x| \le 1$

Sol. $\pm(7-2x) \le 1$

$7 - 2x \le 1$ or $7 - 2x \ge -1$

$-2x \le 1 - 7$ or $-2x \ge -1 - 7$

$-2x \le -6$ or $-2x \ge -8$

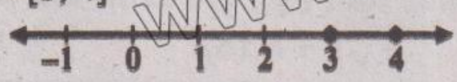
$2x \ge 6$ or $2x \le 8$

$x \ge \frac{6}{2}$ or $x \le \frac{8}{2}$

$x \ge 3$ or $x \le 4$

$3 \le x \le 4$

$S.S = [3, 4]$



(ii) $|6x+18| \le 24$

Sol. $\pm(6x+18) \le 24$

$6x + 18 \le 24$ or $6x + 18 \ge -24$

$6x \le 24 - 18$ or $6x \ge -24 - 18$

$6x \le 6$ or $6x \ge -42$

$x \le \frac{6}{6}$ or $x \ge \frac{-42}{6}$

$x \le 1$ or $x \ge -7$

$-7 \le x \le 1$

$S.S = [-7, 1]$

8. Given that $f(x) = 3x + 7$, find

(i) $f^{-1}(x)$

Sol. $f(x) = 3x + 7$

$y = 3x + 7$ (Replace $f(x)$ with y)

or $3x + 7 = y$

$3x = y - 7$

$x = \frac{y-7}{3}$ (Solve for x)

$f^{-1}(y) = \frac{y-7}{3}$ $\because$ $y = f(x) \Rightarrow f^{-1}(y) = x$

$f(x) = \frac{x-7}{3}$ (Replacing y by x)

(ii) the value of x for which $f(x) = f^{-1}(x)$

Sol. $f(x) = 3x + 7$

$y = 3x + 7$ (Replace $f(x)$ with y)

or $3x + 7 = y$

$3x = y - 7$

$x = \frac{y-7}{3}$ (Solve for x)

$f^{-1}(y) = \frac{y-7}{3}$ $\because$ $y = f(x) \Rightarrow f^{-1}(y) = x$

$f^{-1}(x) = \frac{x-7}{3}$ (Replacing y by x)

$f(x) = f^{-1}(x)$ (Given)

$3x + 7 = \frac{x-7}{3}$

$3(3x + 7) = x - 7$

$9x + 21 = x - 7$

$9x - x = -7 - 21$

$8x = -28$

$x = \frac{-28}{8} = \frac{-7}{2}$

9. A company earns a profit of Rs. 100 for each item sold after a fixed monthly expense of Rs. 10,000. A function $P(x) = 100x - 10,000$ represents the profit after selling x items. How many items must be sold to break even (profit = 0)?

Sol. $P(x) = 100x - 10,000$ (Given function)

$0 = 100x - 10,000$

$-100x = -10,000$

$100x = 10,000$

$x = \frac{10000}{100}$

$x = 100$

So, 100 items must be sold to break even.

10. A store offers a 15% discount on a product. A function $D(p) = 0.85p$ gives the sale price after the discount on an original price p. What is the sale price of an item that originally costs Rs. 2,000?

Sol. $D(p) = 0.85\ P$ (i) represents the function of solve price after 15% discount on original price

Given original price (p) = Rs. 2000

Put (p) = Rs. 2000 in (i)

$D(2000) = 0.85(2000) = 1700$

So, the sale price = Rs. 1700

11. A GPS system is considered accurate if the actual position differs from the reported position by no more than 6 meters. If the actual location is at point $x = 100$, the allowed range is defined as: $|x - r| \le 6$, where r is the reported location. What is the range of acceptable reported locations?

Sol. $|x - r| \le 6$

$x = 100$ (Given)

$|100 - r| \le 6$

$\pm(100 - r) \le 6$

$100 - r \le 6$ or $100 - r \ge -6$

$-r \le 6 - 100$ or $-r \ge -6 - 100$

$-r \le -94$ or $-r \ge -106$

$r \ge 94$ or $r \le 106$

$106 \ge r \ge 94$ or $94 \le r \le 106$

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques

(Knowledge, Understanding, Application, Analytical & Conceptual)

4.1 Functions

Encircle the correct option.

1. **How is a function from set X to set Y mathematically written?**
   (a) $f = X+Y$ (b) $f: Y \to X$ (c) $f: X \to Y$ (d) $f = X - Y$
2. **Who introduced the word "function" in a mathematical context?**
   (a) Isaac Newton (b) Rene Descartes
   (c) Gottfried Wilhelm Leibniz (d) Albert Einstein
3. **What is the name given to the set X in the function f: $X \to Y$.**
   (a) Range (b) Co-domain (c) Domain (d) Variable
4. **If $f(x) = x+3x^2+1$, what is the value of $f(-2)$?**
   (a) 1 (b) −1 (c) 11 (d) 0
5. **If $f(x) = \dfrac{x^2+x}{6}$, what is the value of $f(3)$?**
   (a) 2 (b) 3 (c) 9 (d) 6
6. **If $g(y) = 3x^2 + 5x$, what is the value of g (5)?**
   (a) 45 (b) 25 (c) 75 (d) 100
7. **What is the value of $f(\sqrt{3})$ for function $f(x) = (x+1)^2$**
   (a) $(\sqrt{3})^2$ (b) $4 + 2\sqrt{3}$ (c) $3+4\sqrt{2}$ (d) $3+\sqrt{3}\ \sqrt{2}$

8. **If $f(a) = 5$ the 5 is called ________ of the function**
(a) f^{-1} (b) domain (c) image (d) power

9. **The range of $f(x) = (x+1)$ is:**
(a) set of negative numbers (b) set of integers
(c) set of all non-negative real numbers (d) equal to domain

4.1.1 Operations on Functions

10. **If $f(x) = 5x + 1$ and $g(x) = -3x + 7$. Then value of $f(x)+g(x)$ is:**
(a) $2x + 8$ (b) $8x + 8$ (c) $2x - 8$ (d) $15x + 8$

11. **If $f(x) = x + 4$ and $g(x) = x - 4$, what is $(f.g)(x)$?**
(a) $x^2 - 8x - 16$ (b) $2x$ (c) $x^2 + 16$ (d) $x^2 - 16$

12. **If $f(x) = x^2$ and $g(x) = 2x - 1$, the value of $(f - g)(x)$ is:**
(a) $x^2 + 2x + 1$ (b) $-x^2 + x + 1$ (c) $x^2 - 2x - 1$ (d) $x^2 - 2x + 1$

13. **If $f(x) = 3x + 2$ and $g(x) = x - 5$, what is $(f + g)(x)$?**
(a) $3x^2 - 13x - 10$ (b) $2x + 7$ (c) $4x - 3$ (d) $4x + 3$

14. **If $f(x) = 10x^3$ and $g(x) = 5x$, what is $\frac{f(x)}{g(x)}$?**
(a) $2x^2$ (b) $10x^4$ (c) $50x^4$ (d) $10x^2$

15. **If $f(x) = x^2 - 1$ and $g(x) = x + 1$, what is value of $(\frac{f}{g})(x)$?**
(a) $x^2 - x$ (b) $x + 1$ (c) $x - 1$ (d) $x^3 - 1$

16. **If $f(x) = x^3 + y^3$ and $g(x) = x + y$, what is value of $\frac{f(x)}{g(x)}$?**
(a) $(x^2 + xy + y^2)$ (b) $(x^2 - xy + y^2)$ (c) $(x+y)^3$ (d) $(x-y)^3$

17. **If $f(x) = x + k$ and $g(x) = -3 - x$, what is value of k if $f(x) + g(x) = 4$.**
(a) 12 (b) 7 (c) 9 (d) -3

18. **If $f(x) = x - 3$ and $g(x) = x^2 - 9$, what is $\frac{f(x)}{g(x)}$?**
(a) $x + 3$ (b) $x - 3$ (c) $\frac{1}{x-3}$ (d) $\frac{1}{x+3}$

4.1.2 Compositon of Functions

19. **If $f(x) = 2x + 3$, $g(x) = x^2$, then value of $(fog)(x)$ is:**
(a) $2x^2 + 3$ (b) $2x + 3$ (c) $x^2 + 3$ (d) $2 - x + 3$

20. **If $f(x) = -3x - 7$, $g(x) = x^3$, then what is $(fog)(x)$?**
(a) $3x + 7$ (b) $x^3 - x - 7$ (c) $-3x^3 - 7$ (d) $3x^3 + 7$

21. **If a function is both one-to-one and onto, then it is called:**
(a) single function (b) bijective function
(c) negative function (d) into function

22. **One-to-one function is also called:**
(a) injective (b) bijective (c) negative (d) continuous

23. **It g $(x) = x^2$, then value of (gog) (x)?**
(a) x^3 (b) x^5 (c) x^4 (d) x^6

24. **If $f(x) =\ x+5$, then value of $(fof)\ (x)$ is:**
(a) $x + 10$ (b) $x - 10$ (c) $x - 25$ (d) $x + 25$

25. **If $f(x) = x^3$ and g $(x) = x^2 + 4$, the value of $(gof)\ (x)$ is:**
(a) $x + 3$ (b) $x^6 - 6$ (c) $x^2 - 4$ (d) $x^6 + 4$

4.1.3 Inverse Functions

26. **If $y = f(x)$, then $f^{-1}(y) = x$ is called:**
(a) negative of $f(x)$ (b) inverse of $f(x)$ (c) function of $f(x)$ (d) domain of $f(x)$

27. **Domain of $f =$ __________.**
(a) f^{-1} (b) inverse (fx) (c) Range of f^{-1} (d) $x = y$

28. **If $f\ \{(2,a), (5,b)\}$ the f^{-1} is:**
(a) $\{(-2,a), (-5.b\}$ (b) $\{(a,2),(5,b)\}$ (c) $\{(a,b), (b,5)\}$ (d) $\{(a,5), (b,2)\}$

29. **Domain of f^{-1} __________.**
(a) Domain of f (b) Range of f (c) Domain f^{-1} (d) Range of (y)

30. **A function has an inverse if and only if it is:**
(a) only onto (b) only into
(c) only one to one (d) one to one and onto

31. **The inverse of the function $f(x) = 2x-5$ is:**
(a) $f^{-1}(x) = \dfrac{1}{2x-5}$ (b) $f^{-1}(x) = \dfrac{x-5}{2}$
(c) $f^{-}(x) = 5X + 2$ (d) $f^{-1}(x) = \dfrac{x+5}{2}$

32. **$f^{-1}(x)$ of $f(x) = x^2 + 3$ is:**
(a) $\dfrac{x-3}{2}$ (b) $\sqrt{x-3}$ (c) $\sqrt{x+3}$ (d) $\sqrt{\dfrac{x-3}{2}}$

33. **If $f(x)= x^3+1$, then $f^{-1}(9)$ will be.**
(a) 3 (b) 730 (c) 8 (d) 2

34. **What is the domain of $f^{-1}(x)$ if the range of $f(x)$ is $[0,\infty)$?**
(a) $[0,\infty)$ (b) $(-\infty,0)$ (c) $(0, \infty)$ (d) $(-\infty,\infty)$

35. **If $f(x)=\dfrac{1}{x}$, what is $f^{-1}(x)$?**
(a) x^2 (b) $\dfrac{1}{x}$ (c) $-x$ (d) x

4.2 Graphs of Absolute Valued Functions

36. **The absolute value of a number is always:**
(a) negative (b) only prime number
(c) non–negative (d) only even number

37. **The point where the graph changes direction is called.**
(a) Domain (b) Range (c) vertex (d) function

38. **Which is the vertex form of an absolute value function?**
(a) $g(x) = -a|h - x|$ (b) $g(x) = -a|x| + k$
(c) $g(x) = a|x - h| + k$ (d) $g(x) = |x - h| \times k$

39. **Which is correct ordered pair for the graph of $f(x) = |x - 5|$**
(a) $(-4, 9)$ (b) $(4, -9)$ (c) $(3, 8)$ (d) $(-1, -6)$

40. **The correct ordered pair for the graph of $g(x) = |x| + 3$ is:**
(a) $(-6, -3)$ (b) $(0, 4)$ (c) $(5, 8)$ (d) $(3, -6)$

41. **Which ordered pair is not correct for the graph of function $f(x) = |x - 1|$**
(a) $(3, 2)$ (b) $(-5, 6)$ (c) $(-9, 10)$ (d) $(-6, -7)$

4.3 Solution of Absolute Valued Linear Equation in One Variable

42. **The solution of equation $|x| = 7$ is:**
(a) -7 only (b) 7 only (c) 7 and -7 (d) No solution

43. **$|x - 10| = 0$ its solution is:**
(a) ± 10 (b) -10 (c) 10 (d) No solution

44. **The solution of $|3x| = 15$ is:**
(a) $x = 5, x = -5$ (b) $x = 15$ (c) $x = 5$ (d) $x = 18, x = 12$

45. **Which of the following equations has exactly two solution.**
(a) $|x| + 5 = 5$ (b) $|3x| = -1$ (c) $|x - 8| = 4$ (d) $|x - 2| + 5 = 2$

4.4 Solution of Absolute Value Linear inequality in One Variable

46. **$-5 \leq x \leq 5$ is equivalent to:**
(a) $(-5, 5)$ (b) $[-5, 5]$ (c) $(-5, \infty)$ (d) $(\infty, 5)$

47. **$-a < x < a$ is equivalent to:**
(a) $(-a, a)$ (b) $[a, -a]$ (c) $(-\infty, a)$ (d) (a, ∞)

48. **Which is correct inequality for "The distance between x and 0 is at least 10 units."**
(a) $|x| < 10$ (b) $|x| \leq 10$ (c) $|x| > 10$ (d) $|x| \geq 10$

49. **The solution set of $|3x - 6| \geq 9$**
(a) $x < 1$ or $x < -5$ (b) $x = 1, 5$ (c) $x < 1$ or $x > -5$ (d) $x \leq -1$ or $x > 5$

50. **Solution for: $2|x| - 4 < 6$ is:**
(a) $x < 5$ (b) $-5 < x < 5$ (c) $x < 1$ or $x > 5$ (d) $-1 < x < 1$

ANSWERS:

1. $f: X \rightarrow Y$ 2. Gottfried Wilhelm Leibniz 3. Domain 4. -1 5. 2
6. 100 7. $4 + 2\sqrt{3}$ 8. image 9. set of all non-negative real numbers
10. $2x + 8$ 11. $x^2 - 16$ 12. $x^2 - 2x + 1$ 13. $4x - 3$ 14. $2x^2$ 15. $x - 1$
16. $(x^2 - xy + y^2)$ 17. 7 18. $\frac{1}{x+3}$ 19. $2x^2 + 3$ 20. $-3x^3 - 7$
21. bijective function 22. injective 23. x^4 24. $x + 10$ 25. $x^6 + 4$
26. inverse of $f(x)$ 27. Range of f^{-1} 28. $\{(a,2),(b,5)\}$
29. Range of f 30. one to one and onto 31. $f^{-1}(x) = \frac{x+5}{2}$ 32. $\sqrt{x-3}$

33. 2 **34.** $[0,\infty)$ **35.** $\frac{1}{x}$ **36.** non-negative **37.** vertex

38. $g(x) = a|x-h| + k$ **39.** $(-4, 9)$ **40.** $(5, 8)$ **41.** $(-6, -7)$ **42.** 7 and −7

43. 10 **44.** $x = 5, x = -5$ **45.** $|x-8| = 4$ **46.** $[-5, 5]$ **47.** $(-a, a)$

48. $|x| > 10$ **49.** $x < -1$ or $x > 5$ **50.** $-5 < x < 5$

Additional Short Question

Short Answer Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

4.1 Function

1. Define a "Function".

Ans. If X and Y are two non-empty sets, then a function from a set X to a set Y is a rule that assigns each element of set X exactly one element in set Y.

2. Describe the domain of a function.

Ans. The domain of a function is the set of all possible input values (x-values) for which the function is defined.

3. Describe the range of a function.

Ans. Range is the set of all possible output values (y-values) the function produces from those inputs.

4. If $f(x) = x^2 - 5x + 3$, then find f(3).

Sol.
$$\begin{aligned} f(x) &= x^2 - 5x + 3 \\ f(3) &= (3)^2 - 5(3) + 3 \\ &= 9 - 15 + 3 \\ &= 9 + 3 - 15 \\ &= -3 \end{aligned}$$

5. If $g(x) = (x-1)^3$, then find g(−4).

Sol.
$$\begin{aligned} g(x) &= (x-1)^3 \\ &= (-4-1)^3 \\ &= (-5)^3 \\ &= -125 \end{aligned}$$

6. If $f(x) = 2^x$, then find f(5).

Ans.
$$\begin{aligned} f(x) &= 2^x \\ &= 2^5 \\ &= 32 \end{aligned}$$

7. Find the domain of function of $f(x) = \dfrac{3}{x+2}$.

Ans. Since $x = -2$ makes the function undefined. So, the domain is all real number except −2.

Domain: $(-\infty, -2) \cup (-2, \infty)$

8. **Find the domain and range for functions $f(x)=\sqrt[3]{x}$**

Ans. We can take cube root of any number positive or negative. No restriction on what goes in or what comes out.

Domain: All real numbers $(-\infty, \infty)$

Range: All real numbers $(-\infty, \infty)$

4.1.1 Operations on Functions

9. **If $f(x) = 5x^2 - 3$ and $g(x) = x^2 - 5$. Add $f(x)$ and $g(x)$**

Sol. $f(x) = 5x^2 - 3$, $g(x) = x^2 - 5$

$$\begin{aligned} f(x) + g(x) &= (5x^2 - 3) + (x^2 - 5) \\ &= 5x^2 - 3 + x^2 - 5 \\ &= 6x^2 - 8 \end{aligned}$$

10. **If $f(x) = 3x - 7$ and $g(x) = -x - 3$, then find $f(x) - g(x)$**

Sol. $f(x) = 3x - 7$, $g(x) = -x - 3$

$$\begin{aligned} f(x) - g(x) &= (3x - 7) - (-x - 3) \\ &= 3x - 7 + x + 3 \\ &= 4x - 4 \end{aligned}$$

11. **If $f(x) = 7x - 3$, $g(x) = 2x^2 + 5$, the find $(f.g)(x)$.**

Sol. $f(x) = 7x - 3$, $g(x) = 2x^2 + 5$

$$\begin{aligned} f(x)\,g(x) &= (7x - 3)(2x^2 + 5) \\ &= 14x^3 + 35x - 6x^2 - 15 \\ &= 14x^3 - 6x^2 + 35x - 15 \end{aligned}$$

12. **Let $f(x) = x - 3$, $g(x) = x^2 + 4x - 21$, then find $\dfrac{g(x)}{f(x)}$.**

Sol. $f(x) = x - 3$, $g(x) = x^2 + 4x - 21$

$$\begin{aligned} = \frac{g(x)}{f(x)} &= \frac{x^2+4x-21}{x-3} \\ &= \frac{x^2+7x-3x-21}{x-3} \\ &= \frac{x(x+7)-3(x+7)}{(x-3)} \\ &= \frac{(x+7)(x-3)}{(x-3)} \\ &= x + 7 \end{aligned}$$

13. **If $f(x) = (a + b)$, $g(x) = (a^2 - ab + b^2)$, then find $f(x)\,g(x)$.**

Sol. $f(x) = a + b$, $g(x) = a^2 - ab + b^2$

$$\begin{aligned} f(x)\,g(x) &= (a + b)(a^2 - ab + b^2) \\ &= a^3 - a^2 b + ab^2 + a^2b - ab^2 + b^3 \\ &= a^3 + b^3 \end{aligned}$$

14. If $f(x)=\frac{3}{x}, g(x)=\frac{2}{x^2}+3$ then find $(f+g)(x)$

Sol. $f(x) = \frac{3}{x}, g(x) = \frac{2}{x^2}+3$

$(f+g)(x) = \frac{3}{x}+\frac{2}{x^2}+3$

$= \frac{3x+2+3x^2}{x^2}$

$= \frac{3x^2+3x+2}{x^2}$

Composition of functions

15. If $f(x) = 3x - 5$, $g(x) = x^2$, then find $(fog)(x)$

Sol. Given $f(x) = 3x - 5$, $g(x) = x^2$

$(fog)x = f(g(x))$

$= f(x^2)$

$= 3x^2 - 5$

16. If $f(x) = -4x - 3$ and $g(x) = x^2$, then find $(gof)\,x$

Sol. Given:

$f(x) = -4x - 3, g(x) = x^2$

$(gof)(x) = g(f(x))$

$= g(-4x-3)$

$= (-4x-3)^2$

$= (-4x-3)(-4x-3)$

$= 16x^2 + 12x + 12x + 9$

$= 16x^2 + 24x + 9$

17. If $f(x) = 3x - 4$, then find $(fof)(x)$

Sol. Given

$f(x) = 3x - 4$

$(fof)(x) = f(f(x))$

$= f(3x-4)$

$= 3(3x-4)-4$

$= 9x - 12 - 4$

$= 9x - 16$

18. If $g(x) = x^5$, then find $(gog)(x)$

Sol. Given

$g(x) = x^5$

$(gog)(x) = g(g(x))$

$= g(x^5)$

$= (x^5)^5$

$= x^{25}$

19. Define one-to-one function:

Ans. A function f is said to be one-to-one if each input maps to a unique output.

20. Define onto function

Ans. A function is called onto if every element in the codomain has at least one pre-image in the domain.

21. Define bijective function:

Ans. A function f is called bijective if it is both one-to-one and onto.

Inverse Function

22. Define inverse function.

Ans. The inverse function of f denoted by f^{-1}, reverses the process of a function. It takes y as input and gives x as output.

23. If f: {(3, a), (7, b), (1, c)} then find f^{-1}.

Sol. f^{-1}: {(a, 3), (b, 7), (c,1)}

24. Find the inverse of $f(x) = 5x - 3$

Sol.
$f(x) = 5x - 3$
$y = 5x - 3$ (Replace $f(x)$ with y)
$5x = y + 3$
$x = \frac{y+3}{5}$ (Solve for x)
$f^{-1}(y) = \frac{y+3}{5}$ $\because y = f(x) \Rightarrow f^{-1}(y) = x$
$f^{-1}(x) = \frac{x+3}{5}$ (Replacing y by x)

25. Find the inverse of $f(x) = 5x^2 + 1$.

Sol.
$f(x) = 5x^2 + 1$
$y = 5x^2 + 1$ (Replace $f(x)$ with y)
or $5x^2 + 1 = y$
$5x^2 = y - 1$
$x^2 = \frac{y-1}{5}$
$\sqrt{x^2} = \sqrt{\frac{y-1}{5}}$
$x = \sqrt{\frac{y-1}{5}}$ (Solve for x)
$f^{-1}(y) = \sqrt{\frac{y-1}{5}}$ $\because$ $y = f(x) \Rightarrow f^{-1}(y) = x$
$f^{-1}(x) = \sqrt{\frac{x-1}{5}}$ (Replacing y by x)

26. Find the inverse of $f(x)$ $\sqrt{2x+1}$

Sol.
$f(x) = \sqrt{2x+1}$
$y = \sqrt{2x+1}$ (Replace $f(x)$ by y)
or $\sqrt{2x+1} = y$
$\left(\sqrt{2x+1}\right)^2 = y^2$
$2x + 1 = y^2$
$2x = y^2 - 1$
$x = \frac{y^2-1}{2}$ (Solve for x)
$f^{-1}(y) = \frac{y^2-1}{2}$ $\because$ $y = f(x) \Rightarrow f^{-1}(y) = x$

$f^{-1}(x) \quad = \dfrac{x^2-1}{2}$ (Replacing y by x)

27. If $f(x) = x^2$ then find $f(f^{-1}(x))$

Sol. $f(x) = x^2$

$y = x^2$ (Replace $f(x)$ with y)

or $x^2 = y$

$\sqrt{x^2} = \sqrt{y}$

$x = y^{\frac{1}{2}}$ (Solve for x)

$f^{-1}(y) = y^{\frac{1}{2}} \quad \because \quad y = f(x) \Rightarrow f^{-1}(y) = x$

$f^{-1}(x) = x^{\frac{1}{2}}$ (Replacing y by x)

$$f(f^{-1}(x)) = f\left(x^{\frac{1}{2}}\right) = \left(x^{\frac{1}{2}}\right)^2 = x$$

4.2 Graph of Absolute valued functions

28. Define an absolute value function.

Ans. An absolute value function is a type of function that includes an algebraic expression enclosed within absolute value bars.

29. What is the range of the basic absolute value function $f(x) = |x|$?

Ans. Since the absolute value represents distance and can never be negative, the range is all non-negative real numbers.

30. What is the characteristic shape of the graph of $f(x) = |x|$?

Ans. The graph of $f(x) = |x|$ has a V-shape.

31. State the vertex form of an absolute value function.

Ans. The vertex form is $g(x) = a\,|x - h| + k$.

32. Around which line is the graph of $g(x) = a\,|x - h| + k$ symmetric?

Ans. The graph is symmetric about the vertical line $x = h$.

4.3 Solution of absolute Linear Equations in one variable

33. Solve $3\,|x-2| = 18$ and express on number line.

Sol. $3\,|x-2| = 18$

$|x-2| = 18$

$|x-2| = \dfrac{18}{3}$

$|x-2| = 6$

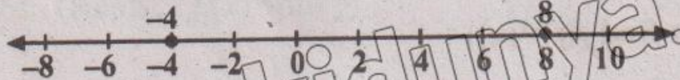
$(x-2) = \pm 6$

$x-2 = 6$

$x = 6+2$ or $x = -6+2$

$x = 8$ or $x = -4$

S.S $= \{8, -4\}$



34. Solve $|5 - 2x| = 3$, and express solution on number line.

Sol. $5 - 2x = \pm 3$

$5 - 2x = 3$ or $5 - 2x = -3$

$-2x = 3 - 5$ or $-2x = -3 - 5$

$-2x = -2$ or $-2x = -8$

$2x = 2$ or $2x = 8$

$x = \dfrac{2}{2}$ or $x = \dfrac{8}{2}$

$x = 1$ or $x = 4$

S.S $= \{1, 4\}$

4.4 Solution of Absolute Value Linear Inequality in One Variable

35. Solve the inequality $|x-2| \geq 7$ and express the solution on number line.

Sol. $|x - 2| \geq 7$

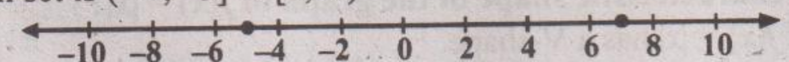
$\pm (x - 2) \geq 7$

$x - 2 \geq 7$ or $x - 2 \leq -7$

$x \geq 7 + 2$ or $x \leq -7 + 2$

$x \geq 9$ or $x \leq -5$

The solution set is $(-\infty, -5] \cup [9, \infty)$



36. What is an absolute value linear inequality in one variable?

Ans. It is an inequality that involves the absolute value of a linear expression containing only one variable.

37. What is the difference between the intervals $[-a, a]$ and $(-a, a)$?

Ans. $[-a, a]$ uses square brackets, meaning the end points are included. $(-a, a)$ uses parentheses, meaning end points are excluded.

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