Chapter 19

Nuclear and Particle Physics

Learning Objectives

After studying this chapter, the students will be able to:

- recognize the equivalence between energy and mass as represented by $E = mc^2$ and state use of this equation
- define and use the terms mass defect and binding energy
- sketch the variation of binding energy per nucleon with nucleon number
- recall what is meant by nuclear fusion and nuclear fission
- explain the relevance of binding energy per nucleon to nuclear reactions, including nuclear fusion and nuclear fission
- explain how the neutrons produced in fission create a chain reaction and that this is controlled in a nuclear reactor [including the action of coolant, moderators and control rods]
- calculate the energy released in nuclear reactions using $E=\Delta m c^2$
- explain that fluctuations in count rate provide evidence for the random nature of radioactive decay
- explain that radioactive decay is both spontaneous and random
- define activity and decay constant, and state the use of $A = -\lambda N$
- explain half-life with examples
- use to solve numerical problems $\lambda = 0.693/T_{1-2}$
- state the exponential nature of radioactive decay
- use the relationship $N = N_o e^{-\lambda t}$ [where N could represent activity, number of undecayed nuclei or received count rate) to solve problems analytically and graphically and N_o is the initial number of Nuclides]
- describe the function of the principle components of a water moderated power reactor [core, fuel, rods, moderator, control rods, heat exchange, safety rods and shielding]
- explain why uranium fuel needs to be enriched before use
- compare the amount of energy released in a fission reaction with the (given) energy released in a chemical reaction.
- explain what is a medical tracer [a substance containing radioactive nuclei that can be introduced into the body and is then absorbed by the tissue being studied]
- explain annihilation reactions [they occur when a particle interacts with its antiparticle and that mass-energy and momentum are conserved in the process]
- calculate the energy of the gammaray photons emitted during the annihilation of an electron-positron pair
- explain that the gamma-ray photons from an annihilation event travel outside the body and can be detected

Ernest Rutherford's experiments in 1911 indicated the existence of a dense, positively charged central part (the nucleus) of very small size surrounded by electrons. In 1920, Rutherford further suggested the positive charge due to protons inside the nucleus and also predicted the presence of another particle having no charge. The prediction came true when James Chadwick discovered neutron in 1932.

The nuclear physics is the study of various aspects of atomic nuclei and sub-atomic particles. We will particularly discuss here unstable nuclei giving off radiations, their decay process, nuclear reactions such as fission and fusion, benefiting mankind with the release of huge amount of energy. Radioactive isotopes of various elements are used as radioactive traces for medical diagnostics and treatment, agriculture, scientific research and industry.

An atomic nucleus is represented by the symbol $_{AZ}X$ often called a nuclide. 'X' represents the chemical symbol of the element, 'Z' the atomic or charge number which indicates the numbers of protons. 'A' stands for mass number which indicates the total number of nucleons (protons and neutrons). The number of neutrons 'N' in the nucleus is given by A-Z. Thus, we write the elements of hydrogen, carbon and uranium as $^{1}_{1}H$, $^{12}_{6}C$, $^{235}_{92}U$ respectively. The nucleus is very dense with a radius of the order 10^{-14} m, surrounded by a cloud of elections giving the atomic radius of the order 10^{-10} m.

Do you know?

From α-particles scattering experiments Lord Rutherford concluded that most of the part of an atom is empty and that mass is concentrated in a very small region called nucleus.

19.1 MASS DEFECT AND BINDING ENERGY

The results of experiments on the masses of different nuclei show that the mass of the nucleus is always less than the total mass of all the protons and neutrons making up the nucleus. The lost or missing mass is called mass defect Δm given by the equation:

$$\Delta m = [Zm_p + (A - Z)\, m_n] - m_{nucleus} \quad \ldots\ldots\ldots\ldots (19.1)$$

where m_p is the mass of a proton, m_n is the mass of a neutron and $m_{nucleus}$ is the mass of the entire nucleus. The missing mass is converted to energy, used in the formation of the nucleus. This energy can be calculated from Einstien's mass-energy relation:

$$\Delta E = (\Delta m)\, c^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19.2)$$

this energy is the potential energy called the Binding Energy (B.E) of the nucleus. It is defined as:

The work done on the nucleus to break it into constituents neutrons and protons.

Binding energy is considered to have negative value similar to absolute gravitational potential value taken as zero on the surface of the Earth. A better measure is the building energy per nucleon. It can be considered as the average energy needed to separate a nucleus into its individual nucleons (neutrons and protons).

We usually use the masses of sub-atomic particles in atomic mass unit (amu or u) which is 1/112th of the mass of an unbound natural atom of ${}^{2}_{6}C$.

$$1\,u = 1.66\times10^{-24}\,g$$

or

$$1\,u = 1.66\times10^{-27}\,kg$$

Example 19.1 Find the mass defect and binding energy of the helium nucleus given that;

Mass of proton $m_p = 1.672623\times10^{-27}\,kg$

Mass of neutron $m_n = 1.674929\times10^{-27}\,kg$

Mass of nucleus $= 6.646786\times10^{-27}\,kg$

Solution Using mass defect equation:

$$\Delta m = 2m_p + 2m_n - m_{nucleus}$$

$$= 2(1.672623)\times10^{-27}\,kg + 2(1.674929)\times10^{-27}\,kg - 6.646786\times10^{-27}\,kg$$

$$= 6.695104\times10^{-27}\,kg - 6.646786\times10^{-27}\,kg$$

$$\Delta m = 0.048318\times10^{-27}\,kg$$

Using Einstein's Equation:

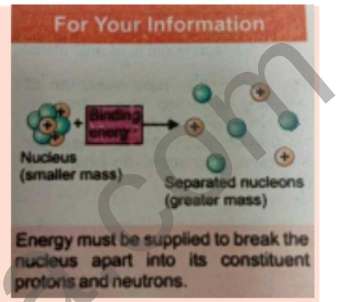
$$B.E = \Delta m c^2$$

$$= 0.048318\times10^{-27}\,kg\times(3.0\times10^{8}\,m\,s^{-1})^2$$

$$B.E = 4.34862\times10^{-12}\,J$$

Changing into eV

$$B.E = \frac{4.34862\times10^{-12}\,J}{1.6\times10^{-19}\,J\,(eV)^{-1}} = 27\,MeV$$


For Your Information

Energy must be supplied to break the nucleus apart into its constituent protons and neutrons.

For your information

Some Atomic Masses

Particle	Mass (u)
e	0.00055
n	1.008665
^{1}H	1.007276
^{2}H	2.014102
^{3}H	3.01605
^{3}He	3.01603
^{4}He	4.002603
^{7}Li	7.016004
^{10}Be	10.013534
^{14}N	14.0031
^{17}O	16.9991

Binding Energy per Nucleon

It is useful to draw a graphical curve of binding energy per nucleon against the nucleon number 'A'. The position of a nucleus on this curve gives information about the stability of the nucleus and whether or not we can get energy by their fission or by fusion. Figure 19.1 shows that initially the hydrogen nucleus with just one proton has a binding energy zero. The fall from hydrogen to helium is large, and suggests a large energy release when hydrogen is converted to helium.

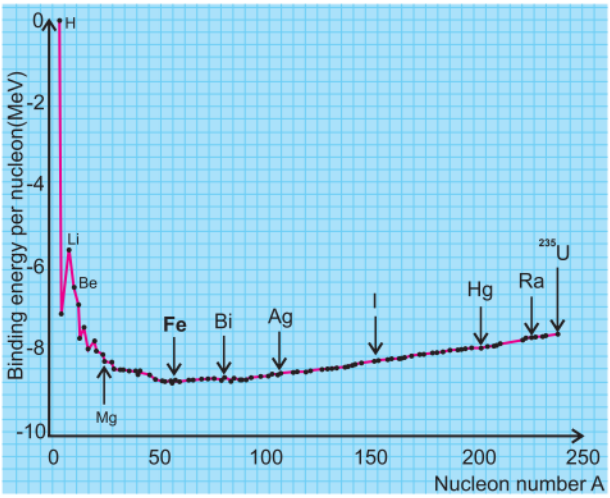


Fig. 19.1: Average binding energy per nucleon against nucleon number

The curve reaches a maximum value of about 8.7 MeV for that of iron ${}^{z6}_{26}\mathrm{Fe}$ which is the most stable nuclide. The region of most stable nuclei is between nucleon number 'A' equal to 50 and 80. Then, there is a steady decrease in binding energy per nucleon up to the end of the curve.

Thus binding energy per nucleon is less for very light and very heavy nuclei. This is very significant. If a heavy nucleus is split into two nuclei that lie near the lowest part of the curve would be more tightly bound (stable). Such a process we call nuclear fission. The mankind is already benefiting from this process getting huge amount of power production.

Similarly, the nucleons in any pair of neighboring nuclei on the lighter side of the curve would be more tightly bound (stable) if the pair combine to form a single nucleus. If a deuterium and tritium fuse to form a more stable helium nucleus, huge amount of energy is released. But such a process is not easy to initiate and needs extremely high temperature such as occurring naturally in our Sun and other stars where tremendous amount of energy is released by fusion processes. In fact, energy is obtained from any nuclear reaction in which the binding energy per nucleon of the products increases.

19.2 NUCLEAR FISSION

Otto Hahn and Fritz Strassmann of Germany while working upon the nuclear reactions made a startling discovery. They observed that when slow moving neutrons are bombarded on ${}^{235}_{92}\mathrm{U}$, then as a result of the nuclear reaction ${}^{141}_{56}\mathrm{Ba}$, ${}^{92}_{36}\mathrm{Kr}$ and an average of three neutrons are obtained. It may be remembered that the mass of both krypton and barium is less than that of the mass of uranium. This nuclear reaction was different from other nuclear reactions, in two ways:

Firstly as a result of the breakage of the uranium nucleus, two nuclei of almost equal size are obtained, whereas in the other nuclear reactions the difference between the masses of the reactants and the products was not large. Secondly, a very large amount of energy is given out in this reaction.

A reaction in which a heavy nucleus like that of uranium splits up into two nuclei of roughly equal size along with the emission of energy during the reaction is called fission reaction.

Fission reaction of ${}^{235}_{92}\mathrm{U}$ can be represented by the equation:

$${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3{}^{1}_{0}\mathrm{n} + Q \quad \text{..........(19.3)}$$

here Q is the energy given out in this reaction. By comparing the total energy on the left side of the equation with total energy on the right side, we find that in the fission of one uranium nucleus, about 200 MeV energy is given out. It may be kept in mind that there is no difference between the sum of the mass and the charge numbers on both sides of the equation. Fission reaction is shown in Fig.19.2. Fission reaction can be easily explained with the help of graph of Fig. 19.1. This graph shows that the binding energy per nucleon is greatest for the middle elements of the periodic table and this binding energy per nucleon is a little less for the light or very heavy elements i.e., the nucleons in the light or very heavy elements are not so rigidly bound.

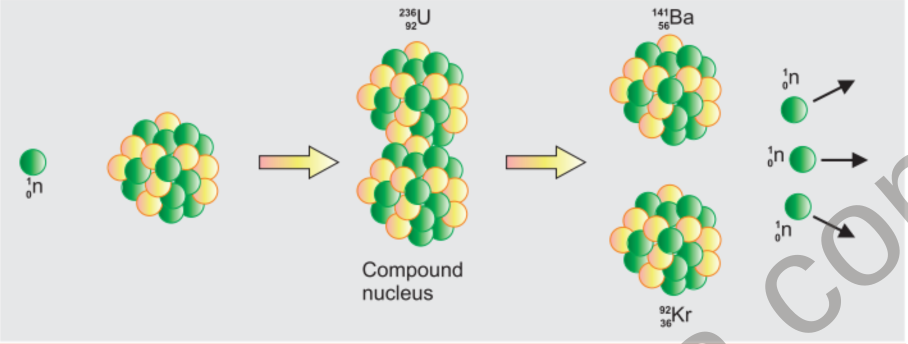


Fig.19.2: Process of fission reaction

For example, the binding energy per nucleon for uranium is about 7.6 MeV and the products of the fission reaction of uranium, namely barium and krypton, have binding energy of about 8.5 MeV per nucleon. Thus, when a uranium nucleus breaks up into barium and krypton, as a result of fission reaction, an energy at the rate of (8.5-7.6) = 0.9 MeV per nucleon is given out. This means that an energy $235 \times 0.9 = 211.5$ MeV is given out in the fission of one uranium nucleus.

The fission process of uranium does not always produce the same fragments (Ba, Kr). In fact, any of the two nuclei present in the upper horizontal part of binding energy could be produced. Two possible fission reactions of uranium are given below as an example:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{132}_{50}Sn + ^{101}_{42}Mo + 3\,^{1}_{0}n + Q \quad ....(19.4)$$

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^{1}_{0}n + Q \quad ....(19.5)$$

Hence, in the uranium fission reaction, several products may be produced. All of these products (fission fragments) are radioactive. Fission reaction is not confined to uranium alone; it is possible in many other heavy elements. However, it has been observed that fission takes place very easily with the slow neutrons in uranium-235 and plutonium-239, and mostly these two are used for fission purposes.

Fission Chain Reaction

We have observed that during fission reaction, a nucleus of uranium-235 absorbs a neutron and breaks into two nuclei of almost equal masses besides emitting two or three neutrons. By properly using these neutrons, fission reaction can be produced in more uranium atoms such that a fission reaction can continuously maintain itself.

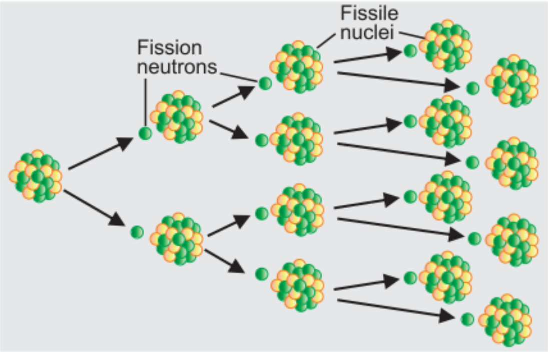


Fig. 19.3: Fission Chain Reaction

This process is called fission chain reaction. Suppose that we have a definite amount of $^{235}_{92}U$ and a slow neutron originating from any source produces fission reaction in one atom of uranium. Out of this reaction about three neutrons are emitted. If conditions are appropriate, these neutrons produce fission in some more atoms of uranium. In this way, this process rapidly proceeds and in an infinitesimal small time, a large amount of energy along with huge explosion is produced. Figure19.3 is the representation of fission chain reaction.

It is possible to produce such conditions in which only one neutron, out of all the neutrons created in one fission reaction, becomes the cause of further fission reaction. The other neutrons either escape out or are absorbed in any other medium except uranium. In this case the fission chain reaction proceeds with its initial speed. To understand these conditions carefully, look at Fig. 19.4. In Fig.19. 4 (a) a fission reaction in a thin sheet of fissile material (such as enriched uranium-235. The resulting neutrons scatter in the air and so they cannot produce any fission chain reaction. Figure 19.4 (b) shows some favourable conditions for chain reaction. Some of the neutrons produced in the first fission reaction produce only one more fission reaction but here also no chain reaction is produced. In Fig.19.4 (c) a spherical lump of highly fissile heavy material mostly uranium-235 is shown. If the lump is sufficiently big, then most of the neutrons produced by the fission reaction get absorbed in before they escape out of the sphere and produce chain reaction.

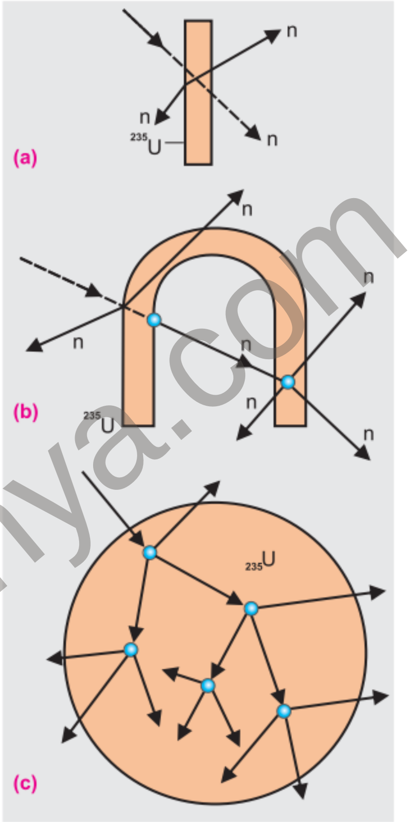
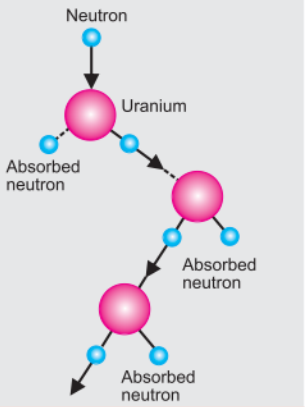


Fig. 19.4

Such a mass of uranium in which one neutron, out of all the neutrons produced in one fission reaction, produces further fission is called critical mass.

The volume of this mass of uranium is called critical volume or size. If the mass of uranium is much greater than the critical mass, then the chain reaction proceeds at a rapid speed and a huge explosion is produced. Atom bomb works at this principle. If the mass of uranium is less than the critical mass, the chain reaction does not proceed. If the mass of uranium is equal to the critical mass, the chain reaction proceeds at its initial speed and in this way we get a source of

Do you know?



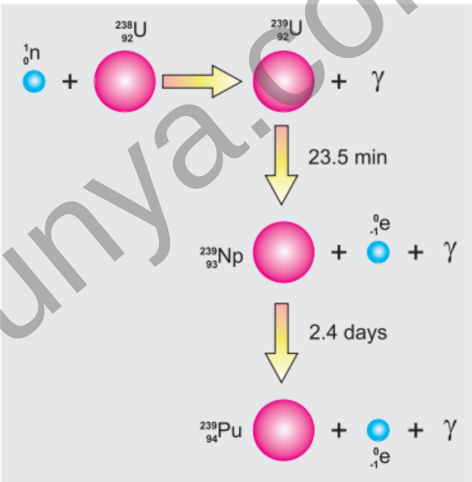
In a controlled chain reaction, only one neutron, on the average, from each fission event causes another nucleus to fission. As a result, energy is released at a steady or controoled rate.

energy. Energy, in an atomic reactor, is obtained according to this principle. The chain reaction is not allowed to run wild, as in an atomic bomb but is controlled by a series of rods, usually made of cadmium, that are inserted into the reactor. Cadmium is an element that is capable of absorbing a large number of neutrons without becoming unstable or radioactive. Hence, when the cadmium control rods are inserted into the reactor, they absorb neutrons to cut down on the number of neutrons that are available for the fission process. In this way, the fission reaction is controlled.

19.3 NUCLEAR REACTOR

In a nuclear power station, the reactor plays the same part as does furnace in a thermal power station. In a furnace, coal or oil is burnt to produce heat, while in a reactor fission reaction produces heat. When fission takes place in the atom of uranium or any other heavy atom, then an energy at the rate of about 200 MeV per nucleus is produced. This energy appears in the form of kinetic energy of the fission fragments. These fast moving fragments besides colliding with one another also collide with the uranium atoms. In this way, their kinetic energy gets transformed in heat energy. This heat is used to produce steam which in turn rotates the turbine. Turbine rotates the generator which produces electricity. A sketch of a nuclear power station is shown in Fig.19.5.

Do you know?



An induced nuclear reaction in which $^{238}_{92}U$ is transmitted into the transuranium element plutonium $^{239}_{94}Pu$.

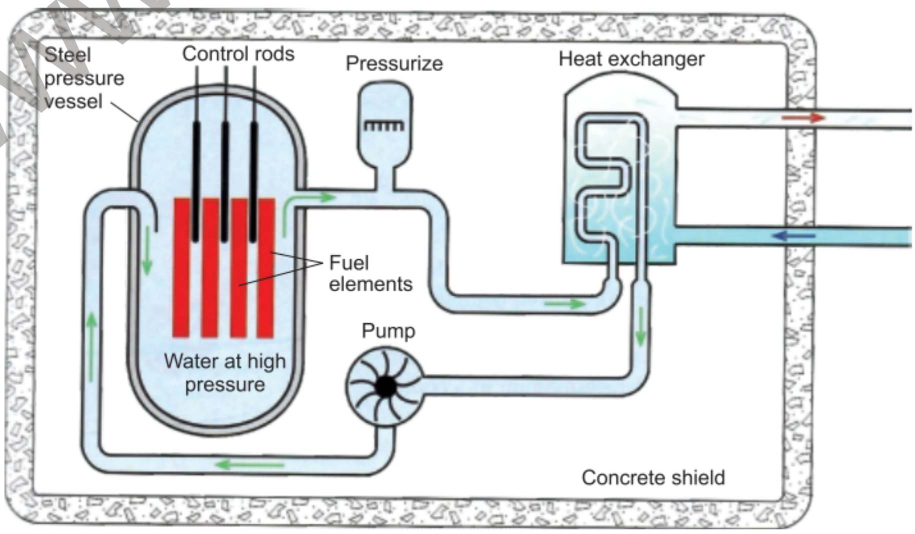


Fig. 19.5: Nuclear Power Station

A reactor usually has four important parts. These are:

1. The most important and vital part of a reactor is called core. Here the fuel is kept in the shape of cylindrical tubes. Reactor fuels are of various types. Uranium was used as fuel in the elementary reactors. In this fuel, the quantity of is enriched from 2.4 to 3.0 percent. It may be remembered that the quantity of in the naturally occurring uranium is only 0.7 percent. Nowadays plutonium-239 and uranium-233 are also being used as fuel.
2. The fuel rods are placed in a substance of small atomic weight, such as water or heavy water. They are called moderators. The function of these moderators is to slow down the speed of the neutrons produced during the fission process and to direct them towards the fuel. Heavy water, it may be remembered, is made of ${}^{2}_{1}H$, a heavy isotope of hydrogen instead of ${}^{1}_{1}H$. The neutrons produced in the fission reaction are very fast and energetic and are not suitable for producing fission in reactor fuel like or ${}^{239}_{94}Pu$ etc. For this purpose, slow neutrons are more useful. To achieve this moderators are used.
3. Besides moderator, there is an arrangement for the control of number of neutrons, so that of all the neutrons produced in fission, only one neutron produces further fission reaction. The purpose is achieved either by cadmium or by boron rods because they have the property of absorbing fast neutrons. The control rods made of cadmium or boron are moved in or out of the reactor core to control the neutrons that can initiate further fission reaction. In this way, the speed of the chain reaction is kept under control. In case of emergency or for repair purposes, control rods are allowed to fall back into the reactor and thus stop the chain reaction and shut down the reactor.
4. Heat is produced due to chain reaction taking place in the core of the reactor. The temperature of the core, therefore, rises to about 500 °C. To produce steam from this heat, it is transported to heat exchanger with the help of water, heavy water or any other liquid under high pressure. In the heat exchanger, this heat is used to produce high temperature steam from ordinary water. The steam is then used to run the turbine which in turn rotates the generator to produce electricity. The temperature of the steam coming out of the turbine is about 300 °C. This is further cooled to convert it into water again. To cool this steam, water from some river or sea is, generally, used.

   In Karachi nuclear power plant (KANUP), heavy water is being used as a moderator and for the transportation of heat also from the reactor core to heat exchanger, heavy water is used. However, to cool steam coming out of the turbine, sea water is being used.

Do you know?

This symbol is universally used to indicate an area where radioactivity is being handled or artificial radiations are being produced.

5. At present, three nuclear power plants are operational in Karachi whereas four at Chashma (Mianwali) in Pakistan with a total output capacity of more than 3000 MW. They are all pressurized water reactors (PWR). Their cooling source is the water from nearby Indus river.

The nuclear fuel once loaded into the reactor can operate it for a few months. There after the fissile material begins to decrease. Now the used fuel is removed and fresh fuel is fed instead. In the used up fuel, intense radioactive substances still remain present. The half-life of these radioactive remnant materials is many thousand years. The radiations and the particles emitted out of this nuclear waste are very injurious and harmful to the living things. Unfortunately, there is no proper arrangement of the disposal of the nuclear waste. This cannot be dumped into oceans or left in any place where they will contaminate the environment, such as through the soil or the air. They must not be allowed to get into the drinking water. The best place so far found to store these wastes is in the bottom of old salt mines, which are very dry and are thousands of metres below the Earth surface. Here, they can remain and decay without polluting the environment on the surface of the Earth.

Do you know?

Radioactive wastes are of three types i.e., high level, medium and low level. All these wastes are dangerous for ground water and land environment.

For your information

It is very difficult to dispose off radioactive waste safely due to their long half-lives for example, 'Pu" half life is 24,000 years, therefore, it remains dangerous for about 1,82,000 years.

19.4 NUCLEAR FUSION

The curve of the graph in Fig. 19.1 also shows that the binding energy per nucleon increases up to A = 50. Hence, when two light nuclei merge together to form a heavy nucleus whose mass numbers *A* is less than 50, then energy is given out. In the topic on mass defect and binding energy, we have observed that when two protons and two neutrons merge to form a helium nucleus, then about 27 MeV energy is given out.

A nuclear reaction in which two light nuclei merge to form a heavy nucleus is called fusion reaction.

During a fusion reaction some mass is lost and its equivalent energy is given out. In a fusion reaction, more energy per nucleon can be obtained as compared to the fission reaction. But unfortunately, it is more difficult to produce fusion. Two positively charged nuclei must be brought very close to one another. To do so, work has to be done against the electrostatic force of repulsion between the positively charged nuclei. Thus, a very large amount of energy is required to produce fusion reaction. It is true that a greater amount of energy can be obtained during a fusion reaction compared to that produced during a fission reaction, but in order to start this reaction, a very large amount of energy has to be spent.

The probability of occurring fusion of two lighter nuclei is great where one proton or one neutron is produced as given below:

$$^{2}_{1}H + ^{2}_{1}H \longrightarrow ^{3}_{1}H + ^{1}_{1}H + 4.0\,MeV \quad \text{..........(19.6)}$$

or $$^{2}_{1}H + ^{2}_{1}H \longrightarrow ^{3}_{2}He + ^{1}_{0}n + 3.3\,MeV \quad ....(19.7)$$

In both of these reactions, about 1.0 MeV energy per nucleon is produced which is equal to the energy produced during fission. If $^{2}_{1}H$ and $^{3}_{1}H$ are forced to fuse, then 17.6 MeV energy is obtained.

i.e., $$^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + ^{1}_{0}n + 17.6\,MeV \quad ....(19.8)$$

In above reactions $^{1}_{1}H$+ $^{3}_{1}H$ are termed as fusion fuels. We know that for fusion of two light nuclei, the work has to be done to overcome the repulsive force which exists between them. For this, the two nuclei are collided towards one another at a very high speed. One method to do so is to give these nuclei a very large velocity with the help of an accelerator as shown in Fig. 19.6.

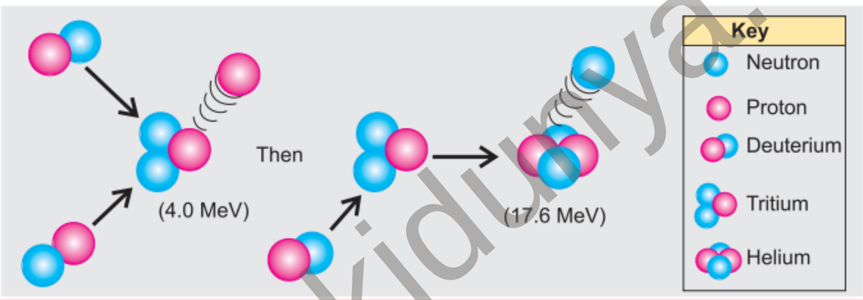
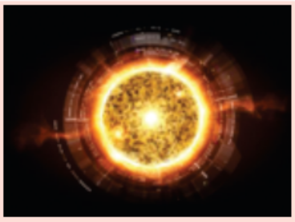


Fig. 19.6: Fusion reaction

This method has been used in the research study of nuclear fusion of $^{2}_{1}H$ and $^{3}_{1}H$. But this method of nuclear fusion for getting continuously energy cannot be used on a large scale. There is another method to produce fusion reaction .It is based upon the principle that the speed of atoms of a substance increases with the increase in the temperature of that substance. To start a fusion reaction, the temperature at which the required speed of the light nuclei can be obtained is about 10 million degrees Celsius. At such extraordinarily high temperature, the reaction that takes place is called thermonuclear reaction. Ordinarily such a high temperature cannot be achieved. However, during the explosion of an atom bomb this temperature can be had for a very short time.

For your information



An "artificial Sun" refers to experimental nuclear fusion reactors like China's EAST and HL-2M/HL-3. They heat hydrogen isotopes to over 100 million °C, hotter than the sun's core to mimic solar fusion for clear, limitless energy.

Until now the fusion reaction has been observed in the experimental testing of hydrogen bomb by trigging it by an atomic bomb. A very large amount of energy can be had from a fusion reaction, but till now this reaction has not been brought under control like a fission reaction and so is not being used to produce electricity. Efforts are in full swing in this field and it is hoped that in near future, some method would be found to control this reaction as well. However, the fusion process is taking place in the core of stars including our Sun.

19.5 ACTIVITY AND HALF-LIFE

We have seen that whenever an α or β-particle is emitted from a radioactive element, it is transformed into some other element. This radioactive decay process is quite random and is not subjected to any symmetry. Fluctuation in decay curves is also an evidence of its random nature. This means that we cannot foretell about any particular atom as to when will it decay. It could decay immediately or it may remain unchanged for thousands of years. Thus, we cannot say anything about the life of any particular atom of a radioactive element.

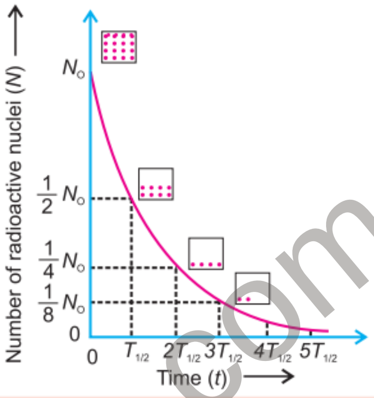


Fig. 19.7

While we cannot predict when a single nucleus will decay but in a very large sample the decay follows a pattern or consistency when we draw the number of radioactive nuclei against time (Fig. 19.7).

The activity of a radioactive substance is the number of particles it emits per second.

Each emission brings a change in the nucleus and is called decay rate. Its SI unit is Becquerel (Bq).

1 Bq = 1 decay per second

The count rate by a nuclear radiation detector is the number of counts recorded per second. It has been found directly proportional to the activity. As time goes, the activity of a source decreases in a consistent manner.

The time taken for the activity of a sample to decrease to half of any starting value is called half life.

Besides getting the definition of half-life, we can deduce two other conclusions from this example. These are, firstly no radioactive element can completely decay. It is due to the reason that in any half-life period, only half of the nuclei decay and in this way, an infinite time is required for all the atoms to decay.

Secondly, the number of atoms decaying in a particular period is proportional to the number of atoms present in the beginning of the period. If the number of atoms to start with is large then a large number of atoms will decay in this period and if the number of atoms present in the beginning is small then less atoms will decay.

We can represent these results with an equation. If at any particular time the number of radioactive atoms be N, then in an interval Δt, the number of decaying atom, ΔN is proportional to the time interval Δt and the initial number of atoms N, i.e..

$$\Delta N \propto -N\Delta t$$

or $$\Delta N = -\lambda N\Delta t \quad ....(19.9)$$

where "λ" is the constant of proportionality and is called decay constant. Eq. 19.9 shows that if the decay constant of any element is large, then in a particular interval more of its atoms will decay and if the constant "λ" is small, then in that very interval less number of atoms will decay. Thus, decay constant of any element is equal to the fraction of the decaying atoms per unit time. The unit of the decay constant is s^{-1}. The negative sign in Eq. 19.10 indicates the decrease in the number of atoms N. From Eq. 19.9 we can find activity:

$$\frac{\Delta N}{\Delta t} = -\lambda N \qquad \text{as} \quad \frac{\Delta N}{\Delta t} \text{ Activity (A),} \qquad \text{hence, } A = -\lambda N \quad \ldots\ldots\ldots\ldots.. (19.10)$$

The decay ability of any radioactive element is shown by the graphical curve (Fig.19.7).

We know that every radioactive element decays at a particular rate with time. If we draw a graph between number of atoms in the sample of the radioactive element present at different times and the time then a curve as shown in Fig. 19.7 will be obtained. This graph shows that in the beginning, the number of atoms present in the sample of the radioactive element were N_o, with the passage of time the number of these atoms decreased due to their decay. This graph is called decay curve.

After a period of one half-life $N_o / 2$ number of atoms of this radioactive element are left behind. If we wait further for another half-life period, then half of the remaining $N_o / 2$ atoms decay, and $1/2 \times N_o/2 = (1/2)^2 N_o$ atoms remain behind. After the expiry of further period of a half-life, half of the remaining $(1/2)^2 N_o$ atoms decay. The number of atoms that remain un-decayed is $1/2 \times (1/2)^2 N_o = (1/2)^3 N$. We can conclude from this example that if we have N_o number of any radioactive element, then after a period of *n* half-lives, the number of atoms left behind is $(1/2)^n N_o$.

In a similar way, we can use the general formula for decayed radioactive nuclides:

After first half-life $= N_o - \frac{N_o}{2} = \frac{N_o}{2}$

After second half-life $= N_o - \frac{N_o}{4} = \frac{3N_o}{4}$

After third half-life $= N_o - \frac{N_o}{8} = \frac{7N_o}{8}$

$$\vdots \qquad \vdots \qquad \vdots$$

For your information

Artificial Sun has been designed to mimic real Sun. It is an experimental advanced super conducting Tokamak (EAST) project in China. It was power magnetic field to confine super heated plasma reaching temperature over million °C in an attempt to fuse hydrogen isotopes into helium nucleus releasing clean energy, just like our Sun.

After n half-lives; $$N = N_o\left(1 - \frac{1}{2^n}\right) \quad \ldots\ldots\ldots\ldots.. (19.11)$$

It has been found that the estimate of decay of every radioactive element is according to the graph of Fig.19.7, but the half-life of every radioactive element is different. For example, the half-life of uranium-238 is 4.5×10^9 years while the half-life of radium-226 is 1620 years. The haif-life of some radioactive elements is very small, for example, the half-life of radon gas is 3.8 days and that of uranium-239 is 23.5 minutes.

From the above discussion, it is found that the estimate of any radioactive element can be made from its half-life or by determining its decay constant 'λ'. It can be proved with the help of calculus that the following relations exist between the decay constant 'λ' and the number of existing number of nucleons at any time t.

$$N = N_o e^{-\lambda t} \qquad \text{............(19.12)}$$

Where N_o is the initial number of nuclides and N is the undecayed nuclei after time 't'.

When N reduces to $N_o/2$, t is known as half-life $T_{1/2}$. The solution of Eq. 19.12 gives,

$$T_{1/2} = \frac{0.693}{\lambda} \qquad \text{............(19.13)}$$

Thus, if the decay constant 'λ' of any radioactive element is known, its half-life can be found.

Any stable element; besides the naturally occurring radioactive element, can be made radioactive. For this, very high energy particles are bombarded on the stable element. This bombardment excites the nuclei and the nuclei after becoming unstable become radioactive element. Such radioactive elements are called artificial radioactive elements.

Example 19.2 The half-life of polonium ($^{210}_{84}\text{Po}$) is 140 days. If there are 1000 atoms of polonium initially present in a sample, how many of them will decay in 280 days.

Solution

Original atoms of polonium $= 1000$

Half-life of polonium $= T_{1/2} = 140$ days

Number of atoms decaying in 140 days $= \dfrac{1000}{2}$

Number of polonium atoms left $= 1000 - 500 = 500$

Number of atoms decaying in next 140 days $= \dfrac{500}{2} = 250$

Total atoms of polonium decaying in 280 days $= 500 + 250 = 750$

Example 19.3 Amercium-241 is an artificially produced radioactive element that emit 5.9×10^{-6} g particles. Its sample of mass 5.1 μg is found to have an activity 5.9×10^{5} Bq. Determine, for this sample:

(i) the total sample of nuclei (ii) the decay constant (iii) half-life in years

Solution

(I) Using Avagadro's number $N_o = 6.02 \times 10^{23}$

Number of nucleons $= \dfrac{\text{mass} \times N_o}{\text{Number of nucleons}}$

$$= \frac{5.1\times 10^{-6}\,\text{g} \times 6.02\times 10^{-23}}{241}$$

$$= 1.27\times 10^{16}$$

(ii) Using Eq. 19.16

$$A = -\lambda N$$

Putting the values $5.9 \times 10^{5}\ \text{s}^{-1} = -\lambda \times 1.27 \times 10^{16}$

$$\lambda = 4.65 \times 10^{-11}\ \text{s}^{-1}$$

(iii) Half-life $T_{1/2} = \dfrac{0.693}{\lambda}$

$$= \frac{0.693}{60 \times 60 \times 24 \times 365} \times 1.49 \times 10^{10}\ \text{s}$$

$$= \frac{1.49 \times 10^{10}\ \text{s}}{60 \times 60 \times 24 \times 365}$$

$$= 472 \text{ years}$$

19.6 RADIOACTIVE TRACERS

A radioactive isotope behaves in just the same way as the normal isotope inside a living organism. But the location and concentration of a radioactive isotope can be determined easily by measuring the radiation it emits. Thus, a radioactive isotope acts as an indicator or tracer that makes it possible to follow the course of a chemical or biological process. The technique is to substitute radioactive atoms for stable atoms of the same kind in a substance and then to follow the 'tagged' atoms with the help of radiation detector in the process.

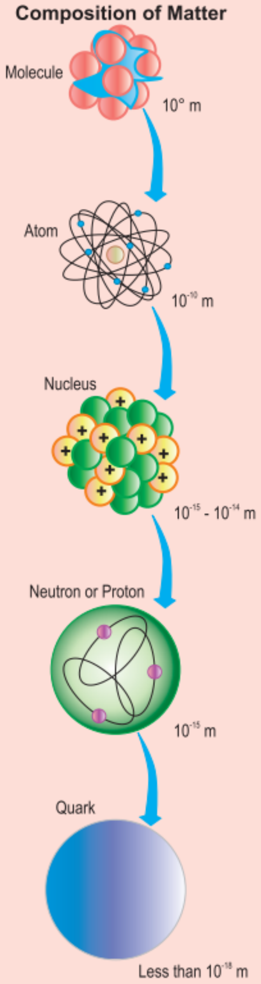
For example, some chemicals such as hydrogen and sodium present in water and food are distributed uniformly throughout the human body. Certain other chemicals are selectively absorbed by certain organs.

Radioisotopes of many elements can be made easily by bombardment with neutrons and other particles. As such isotopes have become available and are inexpensive, their use in medicine, agriculture, scientific research and industries has expanded tremendously.

Radioisotopes are used to find out what happens in many complex chemical reactions and how they proceed. Similarly, in biology, they have helped in investigating into chemical reactions that take place in plants and animals. By mixing a small amount of radioactive isotope with fertilizer, we can easily measure how much fertilizer is taken up by a plant using radiation detector. From such measurements, farmers can know the proper amount of fertilizer to use.

For your information

Composition of Matter



Through the use of radiation-induced mutations, improved varieties of certain crops such as rice, chickpea, wheat and cotton have been developed. They have improved plant structure. The plants have shown more resistance to diseases and pest, and give better yield and grain quality.

Use in Medical Diagnostic and Treatment

Tracers are widely used in medicine to study the process of digestion and the way chemical substances move about in the human body.

Radio-iodine, for example, is absorbed mostly by the thyroid gland, phosphorus by bones and cobalt by liver. They can serve as tracers. Small quantity of low activity radioisotope mixed with stable isotope is administered by injection or otherwise to a patient and its location in diseased tissue can be ascertained by means of radiation detectors. For example, radioactive iodine can be used to check that a person's thyroid gland is working properly. A diseased or hyperactive gland absorbs more than twice the amount of normal thyroid gland. For radioactive Iodine-131 is also used to combat cancer of the thyroid gland. A similar method can be used to study the circulation of blood using radioactive isotope sodium-24.

Experiments on cancerous cells have shown that those cells that multiply rapidly absorb more radiation and are more easily destroyed than normal cells by ionizing radiation.

Radioactive tracers in imaging devices have helped in the understanding and diagnosis and treatment of many diseases.

In some cases, radioisotopes in capsules known as 'seed' are implanted in the malignant tissue for local and short ranged treatment. For skin cancer, phosphorous-32 or strontiun-90 may be used but the dose has to be carefully controlled to avoid healthy tissues.

The patient undergoing radiation treatment often feel ill as the radiation also damage some healthy tissues.

Table 19.1

Some Radioisotopes and their uses

Isotope	Half-life	Example of use
Sodium ^{24}Na	15 hours	Plasma volume
Iron ^{59}Fe	45 days	Iron in plasma
Technetium ^{99}Te	6 hours	Thyroid uptake scans
Iodine ^{131}I	8 days	Kidney tests
Iodine ^{125}I	60 days	Plasma volume Vein flow

19.7 A ANNIHILATION REACTIONS

We have already discussed annihilation of positron-electron pair in the last chapter. Infact, annihilation is an event taking place whenever a particle and its antiparticle come close to each other. In elementary particles research, very high energies are required for investigations. Instead of collisions at their rest mass energies, they are accelerated to nearly the speed of light and then collided. At CERN's Large Hadron Collider (LHC), proton p^+-antiproton p^- and other heavy nuclei are collided at nearly with the speed of light to create conditions similar to early universe. p^+,-p^- and other heavy particles collision at energies of the order of TeV offer insight to early universe conditions. During such collisions electro-weak force mediation particles W^+, W^{-1} and Z bosons predicted by Prof. Abdus Salam have been discovered, along with top and bottom quarks and most significant Higgs bosons in July, 2012.

Example 19.4 Calculate the energy of the gamma ray photons emitted during the annihilation of an electron-positron pair .

Solution

Mass of positron = 9.1×10^{-31} kg

Mass of electron = 9.1×10^{-31} kg

Using Einstein mass-energy relation

$$E = 2mc^2$$

$$= 2 \times 9.1 \times 10^{-31}\ \text{kg} \times (3 \times 10^8\ \text{m s}^{-1})^2$$

$$E = 1.64 \times 10^{-13}\ \text{J}$$

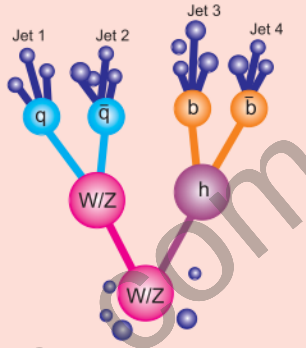
In eV,

$$E = \frac{1.64 \times 10^{-13}\ \text{J}}{1.6 \times 10^{-19}\ \text{J (eV)}^{-1}}$$

$$= 1.02\ \text{MeV}$$

Hence 2γ-ray photons will be emitted each with 0.51 MeV energy travelling in opposite directions to conserve momentum.

For your information



Schematic diagram of p^+ and p^- collision

For your information

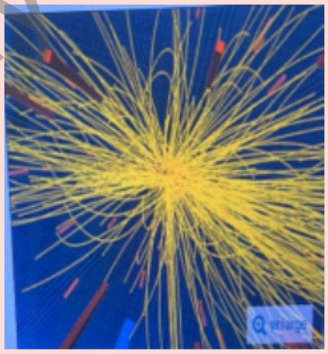


Image of a 7 TeV proton-proton collision in CMS producing more than 100 charged particles

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer:

19.1 What is the half-life of a radio nuclide if 1/16 of its mass is present after 2h?

(a) 15 min. (b) 30 min. (c) 45 min. (d) 60 min.

19.2 Gamma radiations are emitted due to:

(a) de-excitation of atoms (b) excitation of atoms
(c) de-excitation of nuclei (d) excitation of nuclei

19.3 Cadmium rods are used in nuclear reactor for:

(a) slowing down fast neutron (b) speeding up slow neutron
(c) absorbing neutrons (d) starting the nuclear reaction

19.4 The mass of fissionable material needed for self-sustaining chain reaction is called:

(a) supercritical mass (b) fermi mass
(c) critical mass (d) sub-critical mass

19.5 Graphite and heavy water are two common moderators used in a nuclear reactor. The function of the moderator is:

(a) to slow down the neutrons to thermal energies
(b) to absorb the neutrons and stop the chain reaction
(c) to cool the reactor
(d) to control the energy released in the reactor

19.6 The mass of a nucleus is always:

(a) an integral number of proton masses
(b) equal to the mass of all other nuclei of the same element
(c) equal to the sum of the masses of proton and Neutron
(d) less than the sum of masses of the constituent particles

19.7 Binding energy per nucleon is:

(a) greater for heavy nuclei (b) least for heavy nuclei
(c) greater for light nuclei d) greater for medium weight nuclei

19.8 What remains unchanged in a fusion event?

(a) Energy (b) Mass of nucleons
(c) The Number of nucleons (d) Temperature

19.9 Radioactive tracers are also employed to follow the path that various chemicals or food constituents take in:

(a) human bodies (b) animals (c) plants (d) all of these

19.9 The *K.E.* of the fission fragments is ultimately converted to:

(a) potential energy	(b) heat energy
(c) magnetic energy	(d) chemical energy

Short Answer Questions

19.1 Define binding energy and how is it calculated?

19.2 Comment on the statement. "In radioactive decay, if the probability of decay per unit time is doubled, its half-life will also be doubled."

19.3 After four half-lives, what percentage of a radioactive sample remains?

19.4 Why natural uranium is said to be low grade nuclear fuel? Describe briefly.

19.5 Give an application of a radioactive tracers in medicine.

19.6 How is energy generated in the Sun? Describe briefly.

19.7 What is meant by the half-life of a radioactive material?

19.8 What is meant by controlled and uncontrolled fission chain reaction? Describe briefly.

Constructed Response Questions

19.1 In what ways a nuclear power station is different to a fossil fuel burning power station called thermal power plant?

19.2 If fusion reactors are developed, what advantage are they likely to have over fission nuclear reactors?

19.3 The activity of a radioactive sample is found to decay by 10 percent in a year. What will be the activity after another year? Explain.

19.4 A source has an activity of 10×10^4 Bq and a half-life of 20 minutes. What will be the activity of the source after one hour?

19.5 What factors make a fusion reaction difficult to achieve?

19.6 In what way a nuclear reaction is different from chemical reaction? Explain.

19.7 Suggest a reason why neutron are absorbed in the boron rods and the rods become hot as a result of this reaction.

Comprehensive Questions

19.1 Define the terms mass defect and binding energy. What information is given by the graphical curve between binding energy per nucleon against nucleon number?

19.2 What is nuclear fission? Explain critical mass and nuclear fission chain reaction.

19.3 Describe a nuclear power reactor with its components and their functions.

19.4 Describe why the very lighter nuclei and very heavy nuclei are unstable. How are they useful as a source of huge amount of energy?

19.5 What are radioactive tracers? Give some applications in medical diagnostics and treatment.

19.6 What is half-life? Derive the relation between number of half-lives and time. How we can, calculate the remaining and decayed quantity after several half-lives?

Numerical Problems

19.1 Find the mass defect and binding energy of the deuterium. The mass of deuteron is 3.3435×10^{-27} kg.
(mass of proton $m_p = 1.6726 \times 10^{-27}$, mass of neutron $m_n = 1.6749 \times 10^{-27}$)
(Ans: 2.23 MeV)

19.2 The decay constant of strontium is $1.99 \times 10^{-5}\ s^{-1}$. Find its half-life. (Ans: 9.7 hours)

19.3 Half-life of krypton is 3.16 minutes. Out of 40 g of krypton, how much will be left after 12.64 minutes? (Ans: 2.5 g)

19.4 32 g sample of a radioactive material remain after 18 years. What is its half-life?
(Ans: 3 years)

19.5 An alpha particle has an energy of 5.56 MeV when released from the radiam atom. Determine its velocity. (Ans: $1.68 \times 10^7 ms^{-1}$)

19.6 Find the energy released during the nuclear reaction: ${}^7_3Li + {}^1_1H \rightarrow 2\,{}^4_2He$
Mass of ${}^4_2He = 4.00388u$, ${}^1_1H = 1.00814\,u$ and of ${}^7_3Li = 7.01823u$.
(Ans: 19.2 MeV)