Chapter 18

Quantum Physics

Students Learning Outcomes

After studying this chapter, the students will be able to:

- state that electromagnetic radiation has a particulate nature
- explain and apply the photonic model of light to solve problems
- [use $E = hf$ to solve problems, and use the electron volt (eV) as a unit of energy]
- explain that a photon has momentum [including that the momentum is given by $p = E/c$ (connect with the idea that light can exert a force)]
- describe that photoelectrons may be emitted from a metal surface when it is illuminated by electromagnetic radiation
- describe and use the terms threshold frequency and threshold wavelength
- explain photoelectric emission in terms of photon energy and work function energy
- state and apply; $hf = \phi + 1/2\, mv_{max}^2$
- explain why the maximum kinetic energy of photoelectrons is independent of intensity, whereas the photoelectric current is proportional to intensity
- evidence for light as a wave and as a particle
  [Explain that the photoelectric effect provides evidence for a particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for a wave nature]
- Discuss qualitatively the evidence provided by electron diffraction for the wave nature of particles
- explain and apply the de Broglie wavelength to solve problems [use to solve problems]; $\lambda = h/p$
- state that there are discrete electron energy levels in isolated atoms (e.g. atomic hydrogen)
- explain the appearance and formation of emission and absorption line spectra
- use of; $E = hf$ to solve problems.
- describe the Compton effect qualitatively.
- explain the phenomena of pair production and pair annihilation.
- explain how electron microscopes achieve very high resolution.
- state and explain Heisenberg's uncertainty principle qualitatively
- use the uncertainty principle to explain why empirical measurements must necessarily have uncertainty in them

Quantum Theory assumes the behaviour of electromagnetic radiations as discrete packets of energy and particles on a very small scale to behave as waves. It is probably the most successful theory in physics at providing explanations for the unresolved issues and accurate predictions.

However, the classical physics is still valid in ordinary processes of everyday life. We shall discuss various aspects of quantum theory in this chapter.

18.1 QUANTUM THEORY OF RADIATION

In 1901, Max Planck suggested that energy is radiated or absorbed in discrete packets of energy called quanta rather than as a continuous wave. Quanta is plural of quantum, a discrete packet of energy. Each quantum is associated with radiations of a single frequency. The energy E of each quantum is proportional to its frequency f related as

For your information

The one important constant h is a constant of quantum theory and another important constant c is the speed of light c which is a constant of special theory of relativity.

$$E = hf \qquad \text{............(18.1)}$$

where h is Planck's constant, its value is 6.63×10^{-34} J s.

Max Planck received Nobel Prize in physics in 1918 for his discovery of energy quanta.

The Photon

Max Planck suggested that as matter is not continuous but consists of a large number of tiny particles, so is the radiation energy from a source. He assumed that granular or particle nature of radiation from hot bodies was due to some property of the atoms producing it. Einstein extended his idea and postulated that packets or tiny bundles of energy are integral part of all electromagnetic radiations and that they could not be sub-divided. These indivisible tiny bundles of energy he called "photons". The beam of light with wavelength λ consists of a stream of photons travelling at speed c and carries energy; $E = hf$.

From the theory of relativity $E = mc^2$, the relativistic momentum of photon is; $p = mc$,

hence, $$E = pc \qquad \text{.................(18.2)}$$

Thus $pc = hf$ or $p = hf/c$ Since $c = f\lambda$, therefore,

Momentum of photon $$p = h/\lambda \qquad \text{................(18.3)}$$

The quantum theory may be extended to include any system such as a mass oscillating on a spring. However, energy steps are far too small to be detected, so any particle nature is invisible. Quantum effects are only important when observing atom sized objects, where h is a significant factor in any detectable energy change.

18.2 PHOTOELECTRIC EFFECT

The process of emission of electrons from a metal surface when exposed to light of suitable frequency or wavelength is called the photoelectric effect. The emitted electrons are known as photoelectrons.

The photoelectric effect is demonstrated by the apparatus shown in Fig. 18.1. An evacuated glass tube X contains two electrodes. The electrode B connected to the positive terminal of the battery acts as anode. The metal electrode C connected to negative terminal acts as cathode. When monochromatic light of suitable frequency is

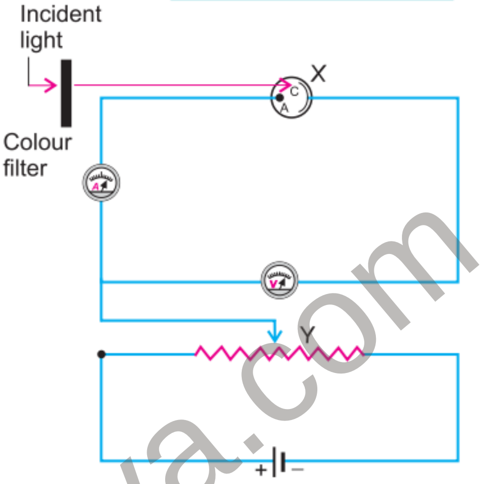


Fig. 18.1: Experimental arrangement to observe photoelectric effect

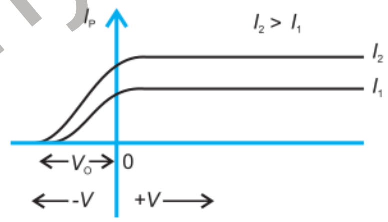


Fig. 18.2: Characteristic curves of photocurrent vs. applied voltage for two intensities of monochromatic light.

allowed to shine on cathode, it begins to emit electrons. These photoelectrons are attracted by the positive anode and the resulting current is measured by a micro-ammeter. The current stops when light is cut off, which proves that the current flows because of incident light. This current is, hence, called photoelectric current. The maximum energy of the photoelectrons can be determined by reversing the connections of the battery in the circuit i.e., now the anode A is negative and cathode C is at positive potential. In this condition, the photoelectrons are repelled by the anode and the photoelectric current decreases. If this potential is made more and more negative, at a certain value, called stopping potential V_o, the current becomes zero. Even the electrons of maximum energy are not able to reach collector plate. The maximum kinetic energy of photoelectrons is, thus

$$\frac{1}{2}mv^2_{max} = V_o e \quad \text{............}(18.4)$$

where m is mass, v is velocity and e is the charge on electron. If the experiment is repeated with light beam of higher intensity, the amount of current increases but the current stops for the same value of V_o. Figure 18.2 shows two curves of photoelectric current as a function of potential V where $I_2 > I_1$. However, if the intensity is kept constant and experiment is performed with different frequencies of incident light, we obtain the curves as shown in Fig. 18.3. The current is same but stopping potential is different for each frequency of incident light, which indicates the proportionality of maximum kinetic energy with frequency of light f.

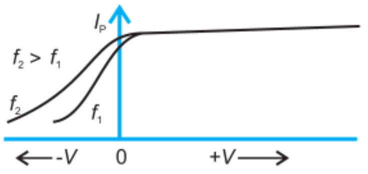


Fig. 18.3: Characteristic curves of photocurrent vs applied voltage for light of different frequencies.

The important results as indicated by the graph between $(K.E.)_{max}$ and frequency of incident light of the experiments are:

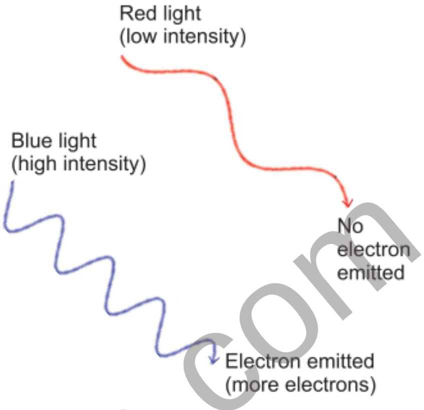
1. The electrons are emitted with different energies. The maximum energy of photoelectrons depends on the particular metal surface and the frequency of incident light.
2. There is a minimum frequency called threshold frequency below which no electrons are emitted, however, intense the light may be. For example, blue light produces

photoelectric emission in sodium metal but red light does not (Fig. 18.4).

This threshold frequency f_o varies from metal to metal. The corresponding wavelength of incident light is called threshold wavelength.

3. Electrons are emitted instantaneously; the intensity of light determines only their number.

These results could not be explained on the basis of electromagnetic wave theory of light. According to this theory, increasing the intensity of incident light should increase the *K.E.* of emitted electrons which contradicts the experimental result. The classical theory cannot also explain the threshold frequency of light.



18.4: Red and blue light incident on sodium metal

According to this theory, even the light of lesser energy should eventually transfer enough energy to liberate electrons. But this does not happen.

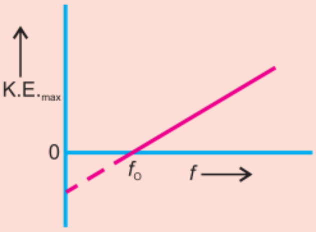
Explanation on the Basis of Quantum Theory

Einstein extended the idea of quantization of energy proposed by Max Planck that light is emitted or absorbed in quanta, a tiny packets of energy, known as photons. The energy of each photon of frequency *f* as given by quantum theory is:

$$E = hf$$

A photon could be absorbed by a single electron in the metal surface. The electron needs a certain minimum energy called the work function ϕ to escape from the metal surface. If the energy of incident photon is sufficient, the electron is ejected instantaneously from the metal surface. A part of the photon energy is used by the electron to break away from the metal and the rest appears as the kinetic energy of the electron. That is;

For your information



A graph of the maximum kinetic energy of photoelectrons vs. light frequency. Below a certain frequency *f*, no photoemission occurs.

$$\text{Incident photon energy} - \text{Work function} = (K.E.)_{max} \text{ of photoelectron}$$

or

$$hf - \phi = \frac{1}{2}mv_{max}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(18.5)$$

This is known as Einstein's photoelectric equation.

When $(K.E.)_{max}$ of the photoelectron is zero, the frequency *f* is equal to threshold frequency f_o, hence, Eq. 18.5 becomes:

$$hf - \phi = 0 \qquad \text{or} \qquad \phi = hf \ldots\ldots\ldots\ldots(18.6)$$

Hence, we can also write Einstein's photoelectric equation as

$$(K.E.)_{max} = hf - hf_o \quad \ldots\ldots\ldots\ldots\ldots(18.7)$$

It is to be noted that all the emitted electrons do not possess the maximum kinetic energy, some electrons come straight out of the metal surface and some lose energy in atomic collisions before coming out. Equation 18.7 holds good only for those electrons which come out with full surplus energy.

Albert Einstein was awarded Nobel Prize in physics in 1921 for his explanation of photoelectric effect.

The phenomenon of photoelectric effect cannot be explained if we assume that light consists of waves and energy is uniformly distributed over its wavefront. It can only be explained by assuming light consists of small packet of energy known as photons. Thus, it shows the particle nature of light.

Example 18.1 Find the energy of a photon in eV of the blue light of wavelength 450 nm.

Solution Wavelength $\lambda = 450\text{ nm} \quad = 450\times10^{-9}\text{ m}$

$h = 6.63\times10^{-34}\text{ J s}$

$$E = hf = \frac{hc}{\lambda}$$

$$= \frac{6.63\times10^{-34}\text{ J s}\times3\times10^{8}\text{ m s}^{-1}}{450\times10^{-9}\text{ m}}$$

$$= 4.42\times10^{-19}\text{ J}$$

As $1.6\times10^{-19}\text{ J}\times1\text{ eV}$ or $1\text{ J} = \frac{1}{1.6\times10^{-19}}\text{eV}$

$$E = \frac{4.42\times10^{-19}}{1.6\times10^{-19}}\text{ eV}$$

$$= 2.76\text{ eV}$$

Example 18.2 Yellow light of 577 nm wavelength is incident on a cesium metal surface. The stopping voltage is found to be 0.25 V. Find

(a) the maximum *K.E.* of photoelectrons

(b) the work function of cesium

Solution **(a)** Given data is

Wavelength $\lambda = 577\text{ nm} = 5.77\times10^{-9}\text{ m}$

Stopping voltage $V_o = 0.25\text{ V}$

As $(K.E.)_{max} = V_o e$

$= 0.25\text{ V}\times1.6\times10^{-19}\text{ C}$

$= 4\times10^{-20}\text{ J}$

Hence $(K.E.)_{max} = \frac{4\times10^{-20}}{1.6\times10^{-19}}\text{eV} \quad = 0.25\text{ eV}$

(b) Using photoelectric equation:

$$hf-\phi = \frac{1}{2}mv_{max}^2$$

or $$\phi = \frac{hc}{\lambda} - \frac{1}{2}mv_{max}^2$$

Putting the values

$$\phi = \frac{6.63\times10^{-34}\,\text{J s}\times3\times10^{8}\,\text{m s}^{-1}}{577\times10^{-9}\,\text{m}} - 4\times10^{-20}\,\text{J}$$

$$= 3.45\times10^{-19}\,\text{J} - 4\times10^{-20}\,\text{J}$$

As $1.6\times10^{-19}\,\text{J} = 1\,\text{eV}$, therefore,

Hence $$\phi = \frac{3.05\times10^{-19}\,\text{J}}{1.6\times10^{-19}\,\text{J}\,(\text{eV})^{-1}}$$

$$\phi = 1.9\,\text{eV}$$

For your information

Interaction of electromagnetic radiation with matter

(i) At low energies (less than 0.5 MeV), the dominant process is photoelectric effect.

(ii) At intermediate energies, the dominant process is Compton effect.

(iii) At higher energies (more than 1.2MeV), the dominant process is pair production.

18.3 COMPTON EFFECT

Arthur Holly Compton at Washington university in 1923 studied the scattering of X-rays by loosely bound electrons from a graphite target (Fig. 18.5). He measured the wavelength of X-rays scattered at an angel θ with the original direction. He found that wavelength λ_s, of the scattered X-rays is larger than the wavelength λ_i of the incident X-rays. This is known as Compton effect. The increase in wavelength of scattered X-rays could not be explained on the basis of classical wave theory. In order to explain this effect, Compton suggested that X-rays consist of photons with energy hc/λ and momentum h/λ.

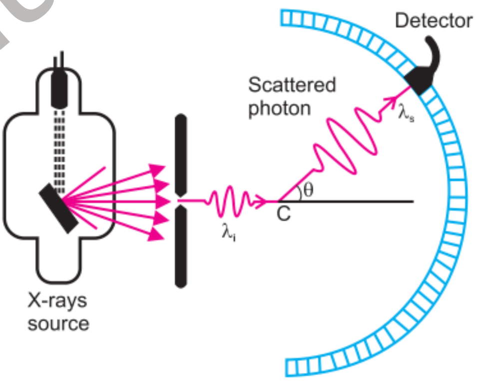


Fig. 18.5: Compton's scattering experiment

Compton Shift

Let us derive an expression for change in wavelength $\Delta\lambda$ known as Compton shift in wavelength.

Consider scattering of a photon by an electron in which the photon suffers collision with the electron like a billiard ball. Figure 18.6 shows the collision between an X-ray photon and an electron which is initially at rest.

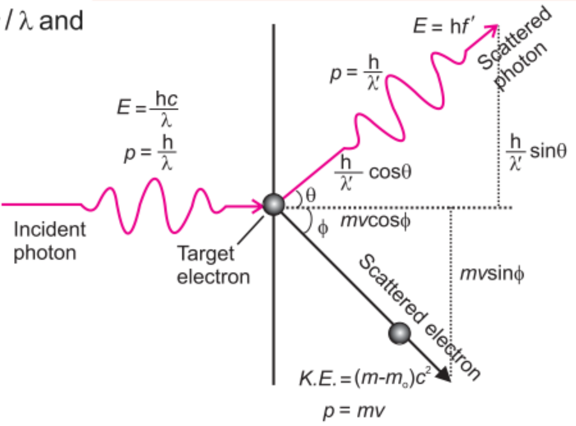


Fig. 18.6

The incident X-ray photon of energy ($hf = hc/\lambda$) strikes an electron, initially at rest. On collision, the photon loses some energy which is taken up by the electron. The photon is scattered at an angle θ with the original direction with a smaller energy ($hf' = hc/\lambda'$) and the electron recoils at an angle ϕ, with the original direction of the photon, with *K.E.* equal to $(m - m_o)c^2$, where m is the mass of electron when it is moving with velocity v and m_o is its rest mass. The difference of the total energy of the electron after collision (mc^2) and before collision, (m_oc^2) is equal to the *K.E.* of the electron.

Actually this is simple problem of the elastic collision of two balls. Both energy and momentum are conserved. So, by law of conservation of energy:

$$\text{Change in photon energy} = K.E. \text{ of electron}$$

$$hf - hf' = mc^2 - m_oc^2$$

$$\frac{hc}{\lambda} - \frac{hc}{\lambda'} = (m - m_o)c^2$$

Since momentum is a vector quantity, it must be conserved both for x and y-components. Hence, by law of conservation of momentum for x-component i.e., along original direction of photon:

$$(\text{Initial momentum}) = (\text{Final momentum})$$

or

$$\frac{h}{\lambda} + 0 = \frac{h}{\lambda'}\cos\theta + mv\cos\phi$$

For y-component i.e., in a direction perpendicular to the original direction of photon:

$$(\text{Initial momentum}) = (\text{Final momentum})$$

or

$$0 + 0 = \frac{h}{\lambda'}\sin\theta - mv\sin\phi$$

or

$$0 = \frac{h}{\lambda'}\sin\theta - mv\sin\phi$$

By solving the above equation, it is found that relation between original wavelength λ of photon, scattered wavelength λ' after collision and scattering angle θ is given by

$$\lambda' = \lambda + \frac{h}{m_oc}(1\text{-}\cos\theta)$$

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_oc}(1\text{-}\cos\theta)$$

the expression for Compton shift is, thus,

$$\Delta\lambda = \frac{h}{m_oc}(1\text{-}\cos\theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots (18.8)$$

The factor h/m_oc has dimensions of wavelength and is called Compton wavelength and has the numerical value

$$\frac{h}{m_oc} = \frac{6.63\times10^{-34}\,\text{J s}}{9.1\times10^{-31}\,\text{kg}\times3\times10^{8}\,\text{m s}^{-1}} = 2.43\times10^{-12}\,\text{m}$$

If the scattered X-ray photons are observed at $\theta = 90°$, the Compton shift $\Delta\lambda$ equals the Compton wavelength. The Eq. 18.6 was found to be in complete agreement with Compton's experimental result, which again is a striking confirmation of particle like interaction of electromagnetic waves with matter.

Arthur Holly Compton was awarded Nobel Prize in physics in 1927 for his discovery of the effect named after him.

We have already discussed in previous class, another kind of very high energy photon such that of a γ-rays with matter. It is pair production in which photon energy is changed into electron-positron pair given by Eienstein energy equation.

$$\text{Energy of photon} = \text{Energy need for pair-production} + K.E. \text{ of the particles}$$

$$hf = 2m_oc^2 + K.E.(e^-) + K.E.(e^+) \ldots\ldots\ldots\ldots\ldots (18.9)$$

the converse of pair-production is known as annihilation of matter in which a particle and its antiparticle interact and annihilate into high energy photons in the γ-rays range:

$$e^- + e^+ = \gamma + \gamma$$

18.4 WAVE NATURE OF PARTICLES

It has been observed that light displays a dual nature, it acts as a wave and it acts as a particle. Assuming symmetry in nature, the French physicist, Louis de Broglie proposed in 1924 that particles should also possess wave-like properties. As momentum *p* of photon is:

$$p = \frac{h}{\lambda} \quad \text{(Eq. 18.3)}$$

de Broglie suggested that momentum of a material particle of mass *m* moving with velocity *v* should be given by the same expression. Thus,

$$p = \frac{h}{\lambda} = mv$$

$$\text{or} \qquad \lambda = \frac{h}{\rho} = \frac{h}{mv} \ldots\ldots\ldots\ldots (18.10)$$

where λ is the wavelength associated with particle waves. Hence, an electron can be considered to be a particle and a wave. Equation 18.9 is called de Broglie relation.

An object of large mass and ordinary speed has such a small wavelength that its wave effects such as interference and diffraction are negligible. For example, a rifle bullet of mass 20 g flying with speed 330 m s^{-1} will have a wavelength λ given by

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\text{ J s}}{2\times10^{-2}\text{ kg}\times 330\text{ m s}^{-1}} = 1\times10^{-34}\text{ m}$$

This wavelength is so small that it is not measurable or detectable by any of its effects.

On the other hands, for an electron moving with a speed of 1×10^6 m s^{-1}:

$$\lambda = \frac{6.63\times10^{-34}\text{ J s}}{9.1\times10^{-31}\text{ kg}\times1\times10^{6}\text{ m s}^{-1}} = 7\times10^{-10}\text{ m}$$

This wavelength is in the X-rays range. Thus, diffraction effects for electrons are measurable whereas interference effects for bullets are not.

Example 18.3 Find the wavelength of two photons produced when a positron-electron pair annihilates. The rest mass energy of each photon is 0.51 MeV.

Solution

$$E = 0.51\ \text{MeV}$$

$$E = 0.51 \times 10^{6}\ \text{ev} \times 1.6 \times 10^{-19}\ \text{J/eV} = 8.16 \times 10^{-14}\ \text{J}$$

Wavelength $\lambda = ?$

As $E = hf = \dfrac{hc}{\lambda}$ or $\lambda = \dfrac{hc}{E}$

Putting the values

$$\lambda = \frac{6.63 \times 10^{-34}\ \text{J s} \times 3 \times 10^{8}\ \text{m s}^{-1}}{8.16 \times 10^{-14}\ \text{J}}$$

$$\lambda = 2.43 \times 10^{-12}\ \text{m} = 2.43\ \text{pm}$$

Davisson and Germer Experiment

A convincing evidence of the wave nature of electrons was provided by Clinton J. Davisson and Laster H. Germer. They showed that electrons are diffracted from metal crystals in exactly the same manner as X-rays or any other wave. The apparatus used by them is shown in Fig. 18.7, in which electrons from heated filament are accelerated by an adjustable applied voltage. The electron beam is then made incident on a nickel crystal. The scattered electrons from metal surface came off in regular peaks. They interpreted these peak pattern as a result of diffraction just like X-rays diffraction by NaCl crystal. The wavelength of the waves associated with electrons was found to be just that predicted by de Brogalie.

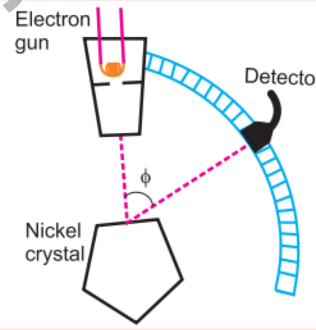


Fig.18.7: **Experimental arrangement of Davisson and Germer for electron diffraction**

The electron beam of energy *Ve* is made incident on a nickel crystal. The beam diffracted from crystal surface enters a detector and is recorded as a current I. The gain in *K.E.* of the electron as it is accelerated by a potential V in the electron gun is given by

$$\frac{1}{2}mv^2 = Ve$$

or $mv^2 = 2Ve$ or $m^2v^2 = 2mVe$

or $mv = \sqrt{2mVe}$

From de Broglie equation: $\lambda = \dfrac{h}{mv}$

Thus $$\lambda = \frac{h}{\sqrt{2mVe}} \qquad \text{............} \quad (18.11)$$

In one of the experiments, the accelerating voltage *V* was 54 volts, hence,

$$\lambda = \frac{h}{\sqrt{2mVe}} = \frac{6.63\times 10^{-34}\ \text{J s}}{\sqrt{2\times 9.1\times 10^{-31}\ \text{kg}\times 54\ \text{J C}^{-1}\times 1.6\times 10^{-19}\ \text{C}}}$$

$$\lambda = 1.66\times 10^{-10}\ \text{m}$$

This beam of electrons diffracted from crystal surface was obtained for a glancing angle of 65°. According to Bragg's equation:

$$2d\sin\theta = m\lambda$$

For 1st order diffraction $m = 1$

For nickel $d = 0.91\times 10^{-10}\text{m}$

Thus $2\times 0.91\times 10^{-10}\text{m}\times \sin 65^\circ = \lambda$, which gives

$$\lambda = 1.65\times 10^{-10}\text{m}$$

Thus, experimentally observed wavelength is in excellent agreement with theoretically predicted wavelength.

Diffraction patterns have also been observed with protons, neutrons, hydrogen atoms and helium atoms, thereby giving substantial evidence for the wave nature of particles.

For his work on the dual nature of particles, Prince Louis Victor de Broglie received the 1929 Nobel Prize in physics. Clinton Joseph Davisson and George Paget Thomson shared the Nobel Prize in 1937 for their experimental confirmation of the wave nature of particles.

18.5 WAVE-PARTICLE DUALITY

Interference and diffraction of light provide evidence for its wave nature, while photoelectric effect and Compton effect prove the particle nature of light. Similarly, the experiments of Davisson and Germer and G. P. Thomson reveal wave-like nature of electrons and in the experiment of J. J. Thomson to find *e/m*, we had to assume particle-like nature of the electron. In the same ways we are forced to assume both wave-like and particle-like properties for all matter: electrons, protons, neutrons, molecules, etc. and also light, X-rays, γ-rays, etc. have to be included in this. In other words, matter and radiation have a dual 'wave-particle' nature and this new concept is known as wave-particle duality. Niels Bohr pointed out in stating his principle of complementarity that both wave and particle aspects are required for the complete description of both radiation and matter. Both aspects are always present and either may be revealed by an experiment. However, both aspects cannot be revealed simultaneously in a single experiment. The two aspects of light are different 'faces' that light shows to experimenters. A particular aspect is determined by the nature of the experiment being done. If we put a diffraction grating in the path of a light beam, we reveal it as a wave. If we allow the light beam to hit a metal surface, we need to regard the beam as a stream of particles to explain our observations. There is no simple experiment that we can carry out with the beam that will require us to interpret it as a wave and as a particle at the same time. Light behaves as a stream of photons when it interacts with matter and behaves as a wave in traveling from a source to the place where it is detected. Thus, the duality of light had to be accepted as a fact of life. In effect, all micro-particles (electrons, protons, photons, atoms, etc.) propagate as if they were waves and exchange energies as if they were particles - that is referred to as the wave-particle duality.

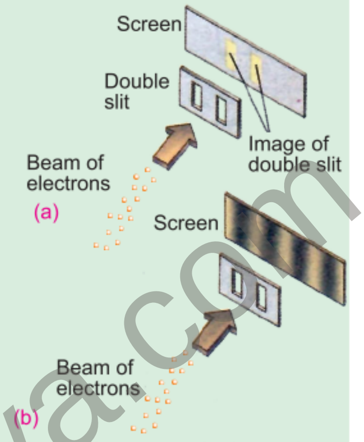
18.6 ELECTRON MICROSCOPE

Electron microscope uses the wave nature of electrons whose de Broglie wavelength is thousand times shorten than the wavelength of visible light. Which enables the electron microscope to see final details not visible to the optical microscope. The electrons from a source are passed through the electromagnetic lens called condenser which focuses as the beam on the specimen. The beam is accelerated on applying the voltages of several megavolts. The higher is the speed of electrons, the shorter is the wavelength and hence, higher is the resolution. An electromagnetic lens (objective lens) form the image of specimen. The image is further magnified by the electromagnetic projector lens to photograph the image on a film, called micrograph.

A three dimensional image of remarkable quality can be achieved by modern versions called scanning electron microscopes.

As stated above, electron microscope can see much smaller details down to nano-metric i.e., about the atomic levels of various materials. The uses include study of cell structure, viruses and bacteria in microbiology, investigation of metal fractures, materials and crystal structure.

Do you know?



(a) If electrons behaved as discrete particles with no wave properties, they would pass through one or the other of the two slits and strike the screen causing it to glow and produce exact images of the slits.

(b) In reality, the screen reveals a pattern of bright and dark fringes similar to light is used and interference occurs between the light waves coming from each slit.

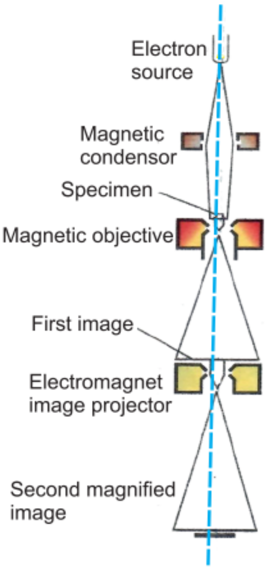
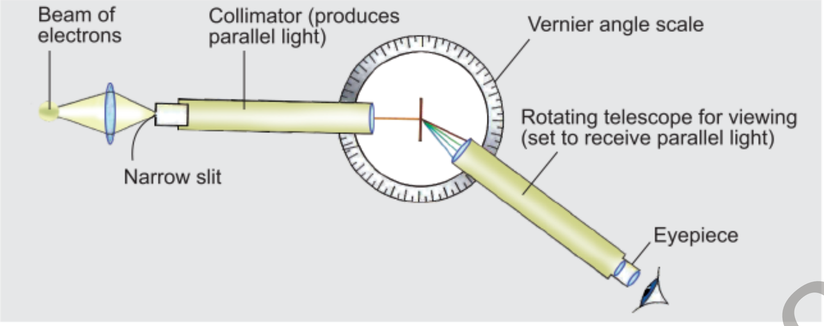


Fig. 18.8: A schematic diagram of the transmission electron microscope.

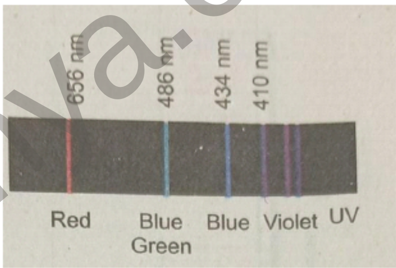
18.7 ATOMIC SPECTRA

The evidence of particle nature of quantized photons using spectrometer is the basis of atomic spectra. The experimental arrangement consists of a discharge tube, spectrometer and diffraction grating as shown in Fig. 18.9.



Fi.g 18.9: Spectrum produced with a diffracting grating

A series of lines are viewed in dark background through the eyepiece of the spectrometer. The vapours of different elements exhibit different patterns of such lines called spectral series which can be used to identify different elements. One such series was identified by J. J. Balmer in 1885 in the spectrum of atomic hydrogen shown in Fig. 18.10. It is in the visible region of the electromagnetic spectrum consisted of four spectral lines of wavelength. Its results were expressed by J.R. Rydberg in the following mathematical formula:



Fi.g 18.10

$$\frac{h}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where *n* is the discrete excited energy level of the electrons with values 3,4,5,... and R_H is the Rydberg constant. Its value is 1.0974 x 10^7 m^{-1}. Infact, spectral lines of atomic hydrogen extends in the invisible ultraviolet and infrared regions. In the ultraviolet region, the Lyman series contains the wavelengths given by the formula:

$$\frac{h}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \text{ .... } (18.12)$$

where *n* = 2, 3, 4,........

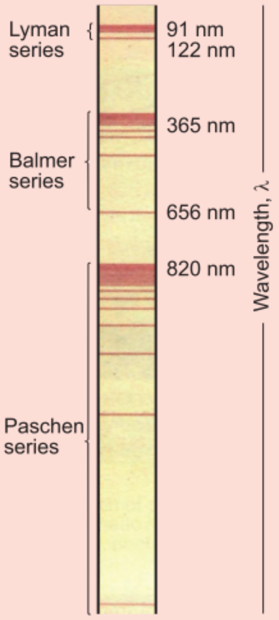
In the infrared region, the wavelengths of spectral lines are given by

$$\frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \text{ .... } (18.13)$$

where *n* = 4, 5, 6,.......

This series is known as Paschen series. Energy level diagram is given in Fig. 18.11 and different transitions are shown in Fig. 18.12.

For your information



Line spectrum of atomic hydrogen. Only the Balmer series lies in the visible region of the electromagnetic spectrum.

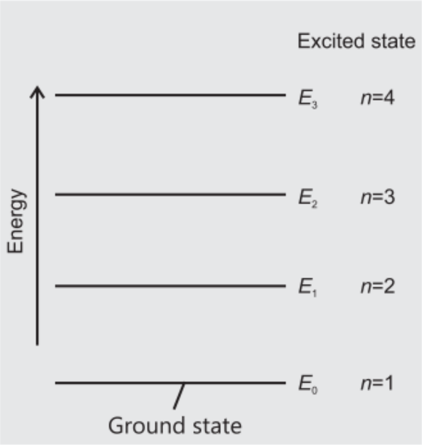


Fig. 18.11: Energy levels

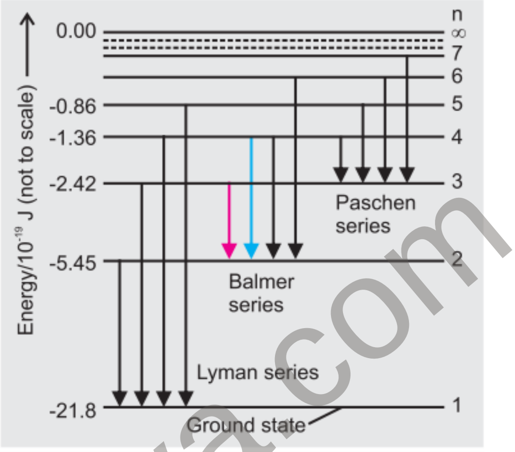


Fig. 18.12: Different transitions in the hydrogen atom

The above mentioned equations give the allowed energy value of the electron in the isolated atoms of hydrogen. The state for $n = 1$ is said to be the ground state whereas the state $n = 2, 3, 4, \ldots$ are called excited states. An atom absorbs energy in discrete amounts only and is raised to one of its excited states. The excited states have very short life. The electron soon returns to lower energy level by emitting photons observed as emission spectrum consisted of several spectral lines.

Absorption and Emission Spectra

When light with a continuous spectrum such as white light is passed through a gas at low pressure, and spectrum of light is then analysed, it is found that light of certain wavelengths is missing. In their places dark lines are seen. This type of spectrum is called absorption spectrum. As the light passes through the gas, electrons absorb discrete amount of energy and make transition to higher energy levels. Only photons of certain energy or frequency are absorbed. The wavelengths of light absorbed correspond exactly to energy needed to make such upward transitions. When these excited electrons return to lower levels, the photons are emitted of specific wavelengths given out in emission

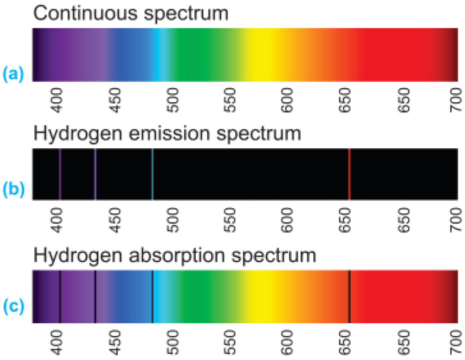


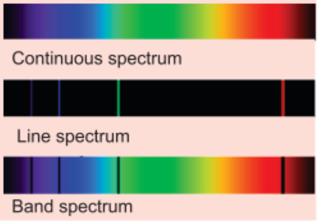
Fig. 18.13: Relation between an absorption spectrum and the emission spectrum of the same element: (a) spectrum of white light, (b) absorption spectrum of element (c) emission spectrum of the same element.

Do you know?

Photon must have energy exactly equal to the energy difference between the two shells for excitation of an atom but an electron with *K.E.* greater or equal can excite the gas atoms. This transition of energy is equl to; $hf = E_n - E_p$.

For your information

Different types of spectra



spectrum. It means the wavelengths missing from an absorption line spectrum are those present in the emission line spectrum as shown in Fig. 18.13.

18.8 UNCERTAINTY PRINCIPLE

Position and momentum of a particle cannot be measured simultaneously with perfect accuracy. There is always a fundamental uncertainty associated with any measurement. This uncertainty is not associated with the measuring instrument. It is a consequence of the wave-particle duality of matter and radiation. This was first proposed by Werner Heisenberg in 1927 and hence, is known as Heisenberg uncertainty principle. This fundamental uncertainty is completely negligible for measurements of position and momentum of macroscopic objects in daily life but is a predominant fact of life in the atomic domain. For example, a stream of light photons striking a flying tennis ball hardly affects its path, but one photon striking an electron drastically alters its motion. Since light has also wave properties, we would expect to be able to determine the position of the electron only to within one wavelength of the light being used. Hence, in order to observe the position of an electron with less uncertainty, we must use light of short wavelength. But it will alter the motion drastically making momentum measurement less precise. If light of wavelength λ is used to locate a micro-particle moving along x-axis, the uncertainty in its position measurement is: $\Delta x \approx \lambda$

At most, the photon of light can transfer all its momentum h/λ to the micro-particle whose own momentum will then be uncertain by an amount:

$$\Delta p \approx \frac{h}{\lambda}$$

Multiplying these two uncertainties gives:

Do you know?

In the sub-atomic world, few things can be predicted with 100% precision.

$$\Delta x.\Delta p \approx \lambda\left(\frac{h}{\lambda}\right) \approx h \text{...........} \quad (8.14)$$

Equation 18.14 is the mathematical form of position-momentum uncertainty principle. It states that:

> **The product of the uncertainty Δx in the position of a particle at some instant and the uncertainty Δp in the x-component of its momentum at the same instant approximately equals Planck's constant h.**

Using uncertainty principle, we can prove that electrons are not present inside atomic nucleus. As the diameter of a nucleus is of the order of 10^{-15}m for an electron to be in the nucleus, the uncertainty in its position can be 10^{-15} m. Using uncertainty principle, the uncertainty in the momentum will be:

$$\Delta p \approx \frac{h}{\Delta x} \approx \frac{6.63 \times 10^{-34}\ \text{J s}}{1 \times 10^{-15}} \approx 6.63 \times 10^{-19}\,\text{J s}$$

The corresponding energy uncertainty should be of the order of GeV. No such election in the atom has been found. Thus electrons do not exist inside the nucleus.

There is another form of uncertainty principle which relates the energy of a particle and the time at which it had that energy. i.e.,

$$\Delta E.\Delta t \approx h \text{ .... } (18.15)$$

$$\frac{x}{v}.\Delta pv \geq h$$

$$\Delta t.\Delta E \geq h \quad \text{or} \quad \Delta E.\Delta t \geq h$$

where $\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34}\text{ J s}$

$$\Delta E.\Delta t \approx h \text{ .... } (18.16)$$

Thus, more accurately we determined the energy of a particle, the more uncertain we will be of the time during which it has that energy.

According to Heisenberg's more careful calculations, he found that at the very best:

$\Delta x.\, \Delta p \geq \hbar$ where $\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34}\text{ J s}$

and $\Delta E.\Delta t \geq \hbar$

Warner Heisenberg was awarded Nobel Prize in 1932 for his contribution towards quantum physics.

Example 18.4 What can be the velocity of an electron enclosed in a box about the size of an atom of the order of 1.0×10^{-10} m?

Solution Size of atom $\Delta x = 1.0 \times 10^{-10}$ m

Mass of electron $m = 9.1 \times 10^{-31}$ kg

Velocity of electron $\Delta v = ?$

Using the formula $\Delta p.\Delta x \approx h$

As $\Delta p \approx m\Delta v$

Hence $m\Delta v.\Delta x = h \qquad \Delta v = \frac{h}{m \times \Delta x}$

Putting the values

$$\Delta v = \frac{6.63 \times 10^{-34}\text{ J s}}{9.1 \times 10^{-31}\text{ kg} \times 1.0 \times 10^{-10}\text{ m}}$$

$$\Delta v = 7.3 \times 10^{6}\text{ m s}^{-1}$$

Example 18.5 Find the uncertainty in the energy of a photon which is emitted from an atom radiating for about 10^{-8} second.

Solution $\Delta t = 10^{-8}$ s, $\quad h = 6.63 \times 10^{-34}$ J s $\quad$ and $\quad \Delta E = ?$

Using uncertainty principle:

$$\Delta E.\Delta t \approx h$$

or

$$\Delta E \approx \frac{h}{\Delta t}$$

$$\Delta E \approx \frac{6.63\times10^{-34}\ \text{J s}}{10^{-8}\ \text{s}} \approx 6.63\times10^{-26}\ \text{J}$$

As $1.6\times10^{-19}\ \text{J} = 1\text{eV}$, hence

$$\Delta E \approx \frac{6.63\times10^{-26}}{1.6\times10^{-19}}\ \text{eV}$$

$$\Delta E \approx 4\times10^{-7}\ \text{eV}$$

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

18.1 Which particle is emitted when UV light is made incident on a zinc surface?
(a) Photon (b) Positron (c) Electron (d) Alpha particle

18.2 The wave nature of electrons is supported by experiments on:
(a) line spectrum of atoms (b) the production of X-rays
(c) photoelectric effect (d) electron diffraction by crystalline material

18.3 The dimensions of Planck's constant are same as that of:
(a) momentum (b) torque
(c) gravitational constant (d) angular momentum

18.4 Which of the following has the most energetic photons?
(a) Light waves (b) Microwaves (c) X-rays (d) Gamma rays

18.5 de-Broglie waves are associated with:
(a) moving charged particles only (b) moving neutral particles only
(c) all moving particles (d) all particles whether in motion or at rest

18.6 An electron microscope employ the principle:
(a) electron have a wave nature
(b) electron can be focused by an electric field
(c) electron can be focused by a magnetic field
(d) all of the above

18.7 Electron, proton, neutron and alpha particle, all have the same speed, which particle will have the shortest wavelength?
(a) Electron (b) Proton (c) Neutron (d) Alpha particle

18.8 The Balmer series is obtained when all the transitions of electrons terminate at:
(a) $n=1$ (b) $n=2$ (c) $n=3$ (d) $n=4$

18.9 The spectrum in which different colours are not diffused into each other and are separated by dark spaces is usually known as:

(a) line spectrum	(b) continuous spectrum
(c) band spectrum	(d) absorption spectrum

18.10 Using uncertainty principle, it can be proved that:

(a) light has particle nature

(b) light has wave nature

(c) electron lies out of the nucleus

(d) there is always uncertainty in measuring accurately energy and momentum for atomic particles

18.11 According to uncertainty principle, in order to observe the position of an electron with greater accuracy, we must use light of:

(a) longer wavelength	(b) shorter wavelength
(c) single wavelength	(d) any wavelength

Short Answer Questions

18.1 How energy of a pocket is related to its frequency according to quantum theory? Describe briefly.

18.2 How does the photoelectric current vary with intensity of incident light on a metal surface?

18.3 Which has more energy, a photon of UV radiation or a photon of yellow light? Describe briefly.

18.4 Is work function of a metal related to threshold frequency in a photoelectric emission?

18.5 Explain briefly, why we can observe the wave-like properties of electrons but not of a flying tennis ball.

18.6 Radiation with a certain frequency causes electrons to be emitted from the surface of one metal and not from the surface of another metal. Why?

18.7 Will high frequency light has more number of electrons than a low frequency light? Describe briefly.

18.8 When does light behave as a wave and when does it behaves as consisted of particles?

18.9 Name two possible spectral line series in the spectrum of atomic hydrogen. In which region of electromagnetic spectrum each lies?

18.10 What is meant by shift, and on witch factors does it depend?

Constructed Response Questions

18.1 Why do solids give rise to a continuous spectrum while hot vapours emit line spectrum?

18.2 Light can emit electrons from a metal surface and light can also be diffracted. Comment on the statement.

18.3 Why X-rays have different properties form light even though both originate from the transition of elections between different energy levels in excited atoms? Describe briefly.

18.4 Is energy conserved when an atom emits a photon of light? Describe how.

18.5 How can spectrum of hydrogen atom contains many spectral lines when hydrogen atom contains one electron only?

18.6 What should be the speed of an electron if it is to be confined in a box of the size of a nucleus?

18.7 An incident X-ray on a metal surface is scattered. What is the wavelength, frequency, energy and speed of the scattered X-ray as compared to incident X-ray?

18.8 Why does energy and momentum conservation play a key role in driving Compton's effect?

18.9 Describe how a line emission spectrum leads to an understanding of the existence of discrete electron energy levels in atoms.

18.10 Mass of a photon is considered zero, then how does the photon possess momentum? Describe.

Comprehensive Questions

18.1 Explain photoelectric effect, its experimental arrangements and observations. Deduce photoelectric equation; $hf = \phi + 1/2\, mv^2_{max}$.

18.2 Describe de-Broglie waves associated with material particles and discuss wave particle duality.

18.3 Explain the appearance and formation of emission and absorption of line spectra from excited atoms. How can they be used to identify the presence of varies elements in a mixture of vapours?

18.4 Describe the transmission of electron microscope. What are the similarities and differences between electron microscope and an optical microscope?

18.5 State and explain uncertainty principle. What is its significance in particle physics?

Numerical Problems

18.1 A photon has wavelength 350 nm. What is its energy in joules and also in eV?

(Ans: 5.68×10^{-19} J, 3.55 eV)

18.2 A certain atom has a separation in energy of 3×10^{-18} J between two of its energy levels. What frequency photon is required to make the atom change from the lower state to higher state? (Ans: 4.5×10^{14} Hz)

18.3 What is the wavelength of photon with an energy of 10×10^{-19} J? (124 nm)

(Ans: 1.99×10^{-7} m)

18.4 Calculate the threshold frequency for sodium metal whose work function is 2.28 eV and the value of h is 6.63×10^{-34} J s. (Ans: 5.5×10^{14} Hz)

18.5 What can be the longest wavelength for photoelectric emission from tungsten if its work function is 4.54 eV. (Ans: 273 nm)

18.6 An electron is accelerated in an evacuated tube from rest through a potential difference of 550 V. What will be its final momentum? Calculate the wavelength associate with this electron. (Ans: 1.27×10^{-23} N s, 5.22×10^{-11} m)

18.7 The position of a free electron is determined with an uncertainty of 10^{-7} m. What is the uncertainty in its velocity? What would be the position of an electron after one second? (Ans: 6×10^{2} m s^{-1}, anywhere within a distance of 6×10^{2} m)

18.8 A stationary nickel nucleus of mass 9.95×10^{-26} kg emits a photon of energy 1.17 MeV. Find:

(i) the wavelength of the photon and its momentum.

(ii) the speed of the nucleus after the emission of photon.

(Ans: (I) 1.06×10^{-12} m, 6.25×10^{-22} N s, (ii) 6.29×10^{3} m s^{-1})

18.9 The work function of a metal is 3.5 eV. Light of wavelength 450 nm is incident on the surface. Find out whether electrons will be emitted by the photoelectric effect, from the surface. (Ans: Electrons will not be emitted)

18.10 A 90 keV X-ray photon is fired at a carbon target and Compton scattering occurs. Find the wavelength of the incident photon and the wavelength of the scattered photon for scattering angle of (a) 30° (b) 60°.

(Ans: 13.8 pm (a) 14.1 pm (b) 15 pm)