Chapter 17

Alternating Current

Students Learning Outcomes

After studying this chapter, the students will be able to:

- analyze equations of the form $x = x_o \sin(\omega t)$ representing a sinusoidally alternating current or voltage.
- understand and describe the terms period, frequency and the peak value as applied to an alternating current or voltage.
- differentiate between root-mean- square (rms) and peak values [using $I_{rms} = \frac{I_o}{\sqrt{2}}$ and $V_{rms} = \frac{V_o}{\sqrt{2}}$ for a sinusoidal alternating current and voltage.]
- know about the fact that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current.
- understand the phase of AC and how phase lags and leads in AC circuits.
- describe impedance – the unseen force as vector summation of resistance of resistors and reactance of capacitors and inductors.
- apply the knowledge to calculate the reactance of capacitors and inductors.
- identify inductors as important components of AC circuits termed as chokes [devices which present a high resistance to alternating current].
- distinguish graphically between half-wave and full-wave rectification.
- explain the use of a single diode for the half-wave rectification of an alternating current.
- explain the use of four diodes (Bridge Rectifier) for the full-wave rectification of an alternating current.
- describe the effect of a single capacitor in smoothing current flow [including the effect of the values of capacitance and the load resistance]
- state and prove expressions for mutual inductance (M) and self-inductance (L), their unit henry.

17.1 ALTERNATING CURRENT AND ITS CHARACTERISTICS

An alternating current AC means a sinusoidally varying current which can be represented by time dependent sine or cosine functions. The emf of an AC generator is given by the following equation, for voltage:

$$V = V_o \sin \omega t \quad \text{.............} (17.1)$$

where V represents magnitude of alternating current or voltage corresponding to time t, V_o represents maximum value of voltage and ω is the angular frequency of alternating voltage as shown in Fig 17.1.We can write Eq.17.1 for current as:

$$I = I_o \sin \omega t$$

where I is instantaneous value of alternating current at time t, I_o is its maximum or peak value ω is its angular frequency.

A periodically varying current or voltage is termed as alternating if every cycle corresponds to one time period (T) with two exactly symmetrical half cycles having one half is positive and remaining half is negative.

In case of current, the direction of current reverses after every half-cycle and in case of voltages, the polarity of potential difference reverses after every half cycle. The second condition to be an alternating one is that amplitude or the peak value of current or voltage i.e., the maximum value on both positive and negative sides remains constant in all cycles but changes occur most rapidly at the zero (crossover) points and most slowly at its peak (Fig. 17.1).

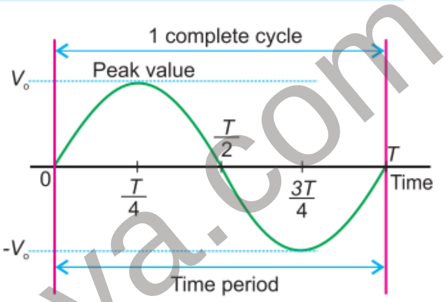


Fig.17.1: Waveform of alternating voltage

Some important terms related to alternating quantities are:

Waveform: The path traced by an alternating quantity, such as the voltage in Fig.17.1 plotted as a function of some variable such as time.

Cycle: One complete set of positive and negative values of an alternating quantity is called a cycle. Figure 17.1 shows one cycle of an alternating voltage.

Time period (T): The time taken to complete one cycle of an alternating quantity is called its time period and is measured in seconds.

Frequency (f): The number of cycles that occur in one second is called its frequency. The unit of frequency is hertz (Hz); where 1 Hz = 1 cycle per second and $f = 1 / T$.

Do you know?

Why is AC better than DC?
AC is easy to be transferred over longer distances even between two cities without much energy loss. DC cannot be transferred over a very long distance. It loses electric power.

Average or Mean Value of AC Waveform

The Average Value of an AC waveform is the arithmetic mean of all instantaneous values over a specific portion of the cycle.

If we calculate the average of a complete sine wave over a full cycle (0 to 2π), the result is **zero** (Fig. 17.2). This happens because the positive half-cycle and the negative half-cycle are mirror images of each other. They cancel out perfectly. That is not very useful. So, when we talk about the average value of an AC waveform, we usually mean the **average over**

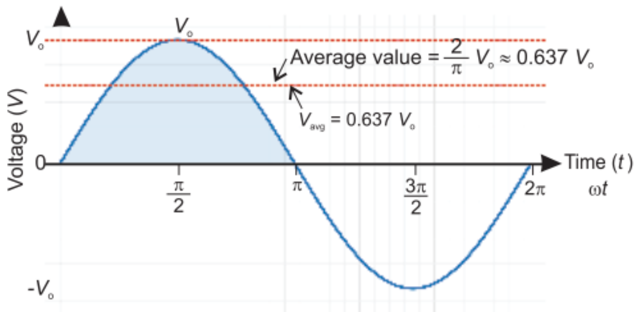


Fig.17.2: Average value of alternating voltage

one half-cycle (0 to π).

The average value is used in situations involving rectification. When an AC signal passes through a rectifier (such as in a power supply), the output is a pulsating DC signal. The average of this pulsating DC is what matters for many applications. For a full-wave rectified sine wave, the average values of current and voltage are given by

$I_{av} = 0.637\, I_o$ and $V_{av} = 0.637\, V_o$

where I_{av} and V_{av} represent average value of alternating current and alternating voltage respectively whereas I_o and V_o represents maximum or peak values of alternating current and voltage respectively (Fig.17.3).

Instantaneous Value: The value of an AC quantity at any instant of time is known as instantaneous value. The value of instantaneous voltage is given as:

$$V = V_o \sin(\omega t)$$

As $\omega = \frac{2\pi}{T}$

therefore, $V = V_o \sin \frac{2\pi}{T} t$

Also $f = \frac{1}{T}$

So $V = V_o \sin(2\pi f t)$

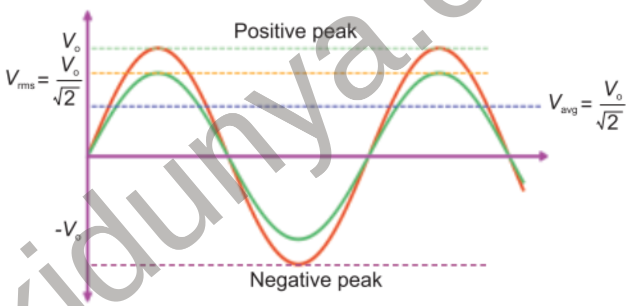


Fig.17.3: Graphically showing the rms value, average or mean value, peak value, and p-p value of alternating quantities-voltage.

The value of instantaneous current is:

$$I = I_o \sin(\omega t)$$

or $I = I_o \sin(2\pi f t)$ ....(17.2)

Peak or Maximum Value: It is the highest value that an AC waveform reaches during one cycle. It is the topmost point on the positive half-cycle or the bottommost point on the negative half-cycle (Figs.17.1 and 17.2). It is written by V_o and I_o for voltage and current respectively.

Peak to Peak Value: This is the total distance from the positive peak to the negative peak of the waveform. For a symmetrical sine wave:

$$V_{p\text{-}p} = 2 \times V_o$$

Brain teaser

How can we detect the presence of AC under a carpet?

we can use a non-contact voltage detector, known as a pen tester or a multi-meter with AC voltage setting. Non-contact detectors will sense the electromagnetic field generated by the current and indicate its presence with a light or sound. A multi-meter can provide a more precise measurement by directly detecting the voltage.

The p-p value of the voltage waveform shown in Fig.17.2 is 2 V_o.

Root-mean-square (rms) or Effective Value of Current and Voltage:

Root-mean- square (rms) values of current, or voltage, are a useful way of comparing alternating current, or voltage, to its equivalent direct current, or voltage. The rms values represent the direct current, or voltage, values that will produce the same heating effect, or power dissipation as the alternating current or voltage. The rms value of an alternating current is defined as:

The value of a constant current that produces the same power in a resistor as the alternating current.

$$I_{rms} = \frac{I_o}{\sqrt{2}} \quad \text{or} \quad I_{rms} = 0.707\, I_o \text{............ (17.3)}$$

The rms value of an alternating voltage is defined as:

The value of a constant voltage that produces the same power in a resistor as the alternating voltage.

$$V_{rms} = \frac{V_o}{\sqrt{2}} \quad \text{or} \quad V_{rms} = 0.707\, V_o \text{............ (17.4)}$$

where I_o = Peak or maximum current

V_o = Peak or maximum voltage

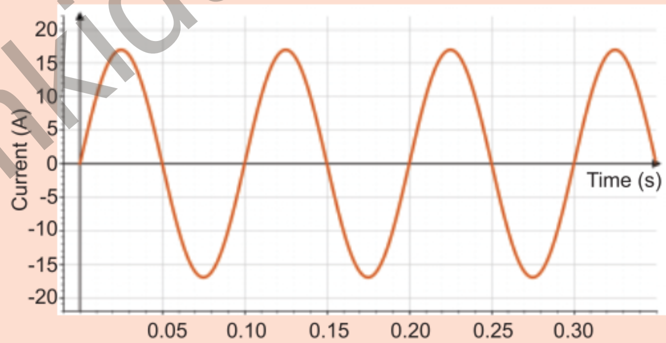
So, rms current, I_{rms} is equal to 0.707 I_o, which is about 70% of the peak current I_o.

Point to ponder!

Why do high voltage power lines crackle and hiss?

The hissing sound often heard near high voltage power lines is primarily caused by corona discharge, an electrical phenomenon where the air surrounding the conductor becomes ionized due to a strong electric field. This ionization creates a discharge that can produce a visible glow, radio noise, and audible hissing or crackling sounds, particularly when the voltage exceeds the breakdown strength of the air.

Example 17.1 The steady direct current, or voltage that delivers the same average power in a resistor as the alternating current, or voltage. Using the graph as shown in the figure and the equation for alternating current, calculate the value of the current at a time 0.67 s.



Solution

We know that for AC circuits;

$$I = I_o \sin(\omega t) \quad \text{and} \quad \omega = \frac{2\pi}{T}$$

The time period T is the time taken for one full cycle, and peak current I_o from the graph are:

$$T = 0.10\text{ s} \quad \text{and} \quad I_o = 17\text{ A}$$

Using the equation $I = I_o \sin(\omega t)$

As $\omega = \frac{2\pi}{T}$, therefore, $I = I_o \sin \frac{2\pi}{T} t$

$$\left(I = 17\text{A} \times \sin \frac{2 \times 3.14 \times 0.67\text{s}}{0.1\text{ s}} \right)$$

$$I = 17\text{A} \times \sin 42^\circ$$

$$I = 17\text{A} \times (0.917) = 15.58\text{ A}$$

Do you know?

How can you tell if a current is alternating?

Step 1: Determine the direction of the electric current.

Step 2: Determine whether the current's direction changes or stays the same.

Step 3: If the current periodically reverses direction, it is alternating current. If the current flows in a single direction, it is direct current.

17.2 RELATION BETWEEN MEAN POWER AND MAXIMUM POWER FOR AN ALTERNATING CURRENT

An alternating current is a current that periodically reverses direction, causing the voltage and current to fluctuate sinusoidally. A resistive load is one that does not store energy (like a capacitor or inductor) and simply dissipates energy as heat.

The instantaneous power P in a circuit is the power at any given moment in time t.

$$P = VI \text{ .... (17.5)}$$

where V is the instantaneous voltage and I is the instantaneous current. The maximum or peak P_{max} occurs when the current and voltage are at their peak values, I_o and V_o respectively. Also the mean power P_{mean} is the average power over one complete cycle of the AC waveform as shown in Fig.17.4.

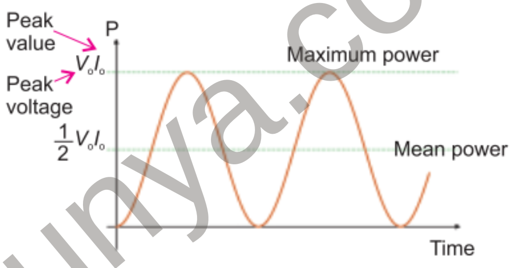


Fig.17.4: Mean power is exactly half the maximum power

For a sinusoidal AC, let I_o be the maximum or peak value of the current while voltage V_o is the maximum or peak value of the voltage, then the maximum or peak power P_{max} is given by

$$P_{max} = I_o^2 R \text{ ....(17.6)}$$

Also we have expression for the mean power P_{mean} represented by

$$P_{mean} = I_{rms}^2 R \text{ ....(17.7)}$$

As $I_o = \sqrt{2}\, I_{rms}$, so, using Eq. 17.6 becomes:

$$P_{max} = (\sqrt{2} I_{rms})^2 \; R = 2I_{rms}^2 R$$

From Eq. 17.7 ; $IR_{rms}^2 = P_{mean}$, thus

$$P_{max} = 2P_{mean}$$

or $$P_{mean} = \frac{P_{max}}{2} \text{ .... (17.8)}$$

Interesting Information

Digital Clamp Meter For Measuring AC and DC

Therefore, it can be concluded that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current or voltage because the instantaneous power in an AC circuit varies over time. While the instantaneous power reaches at maximum value at certain points in the cycle, it spends significant time at zero or lower values, resulting in an average power that is half the peak value.

Example 17.2 An alternating voltage supplied across a resistor of 50 Ω has a voltage of 220 V. Calculate the mean power in kilo watt of the system.

Solution $R = 50\,\Omega$ and $V = 220\,V$

We know that; $P_{max} = \dfrac{V^2}{R}$

Putting the values $P_{max} = \dfrac{(220\ V)^2}{50\ \Omega} = 968\ W$

As the mean power is half of the maximum (peak) power,

$$P_{mean} = \frac{P_{max}}{2} = \frac{968\ W}{2}$$

$$= 484\ W \quad \text{or} \quad 0.484\ kW$$

Fascinating Fact

Scientists have converted human blood sugar into electricity

17.3 PHASE OF A.C.

The angle θ which specifies the instantaneous value of the alternating voltage or current is called the phase. The instantaneous values of voltage and current are given by

$$V = V_o \sin\omega t$$

or $$V = V_o \sin\theta$$

$$I = I_o \sin\omega t$$

or $$I = I_o \sin\theta$$

The angle which gives the value of alternating quantity is known as phase.

The phase of points A, B, C, D and E are 0, π/2, π, 3π/2 and 2π respectively as shown in Fig.17.5(a). Thus each point of A.C cycle corresponds to a phase.

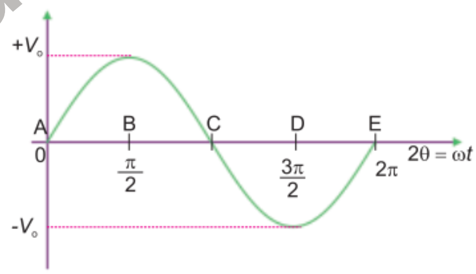


Fig.17.5: (a) AC cycle corresponds to a phase.

Phase Lag and Phase Lead

Referring to the Figs.17.5 (b) and 17.5(c), at $t = 0$, the angle θ is also 0°, the value of the phase angle θ at t equals to zero is called the **initial phase** of AC quantity. Thus, according to equations:

$$V = V_o \sin\theta \quad \text{and} \quad I = I_o \sin\theta \ ....(17.9)$$

which correspond to Figs.17.5(b) and 17.5(c) respectively, the initial phase of both voltage V and current I is zero. There are situations when current I and voltage V are not in phase i.e; they differ in phase. For example, the initial phase of current I may be positive or negative as compared to the initial phase of voltage V, which is zero. Consider a situation in which the initial phase of voltage V is zero and initial phase of current I is ϕ as shown in Fig.17.5 (b). This situation can be represented by the following equations:

$$V = V_o \sin\theta \quad \text{and} \quad I = I_o \sin(\theta + \phi)$$

Fig.17.5: (b) Gr
current leads vol

At $t = 0$; voltage $V = 0$, but the current is positive and $I = I_o \sin\phi$. It means that the current had its zero value earlier by an angle ϕ than voltage. Thus, we can say that the current is leading the voltage by an angle ϕ in this situation (Fig. 17.5-b). The angle ϕ is the phase difference between the voltage and the current.

Similarly, the initial phase of current I is negative as compared to the initial phase of voltage V, which is zero as shown in Fig.17.5(c).This situation can be represented by the following equations:

$$V = V_o \sin\theta \qquad \text{and} \qquad I = I_o \sin(\theta - \phi)$$

It means the value of V is zero, but the current I has some negative value (I – sin ϕ). It means that the current I will reach its zero value later on by an angle ϕ than the voltage V. Thus, we can say that current I is lagging behind the voltage V by an angle ϕ. The angle ϕ is called the phase diffference between V and I.

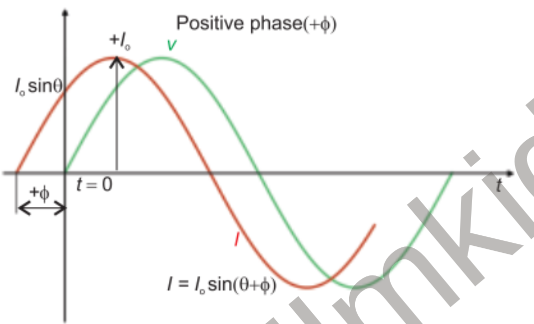


Fig.17.5: (b) Graphical representation showing current lead the voltage.

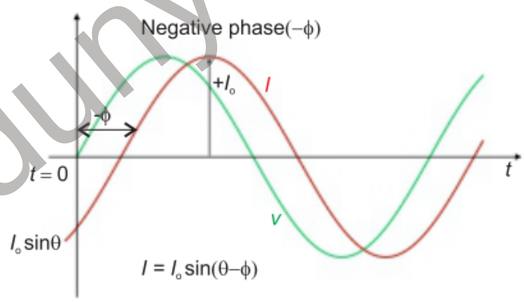


Fig.17.5(c): Graphical representation showing current lags behinds voltage.

Vector Representation of an Alternating Quantity

An alternating quantity can be represented by a anticlockwise rotating vector if it satisfies the conditions: **(i)** Its length on a certain scale represents the peak value or rms value of alternating quantity. **(ii)** It is in horizontal position at the instant. When the alternating quantity is zero and is increasing positively **(iii)** The angular frequency of the rotating vector is same as the angular frequency of alternating quantity. Fig.17.5 (d)

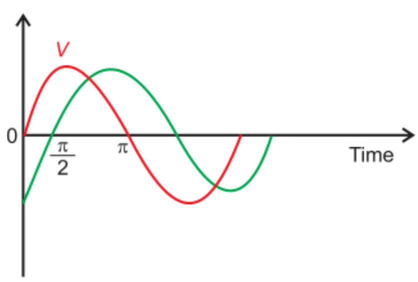


Fig.17.5 (d): Graphical representation showing voltage V leads current I by $\pi/2$

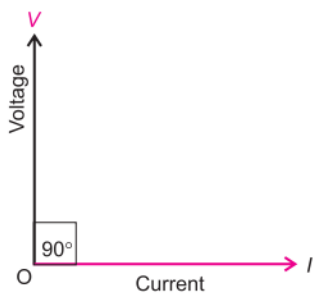


Fig. 17.5 (e): Vector diagram **OI** & **OV** showing V leads I by 90° or $\pi/2$ rad

shows an alternating voltage waveform leading the current wave form by 90° or $\pi/2$ rad. In Fig.17.5(e), vector **OI** represents peak or rms value of current which is taken as reference quantity. Similarly, **OV** represents peak or rms value of alternating voltage, which is leading the current by $\pi/2$ rad or 90°. Both vectors are rotating in anitclockwise direction with angular frequency.

17.4 AC CIRCUITS

The basic circuit element in a DC circuit is a resistor R which controls the current or voltage and the relation between them is given by Ohm's law ($V = IR$). But the basic circuit elements in AC circuit are resistor R, inductor L and capacitor C. These elements control the current and voltage through the circuit. The AC circuits with these components are discussed below:

17.5 AC THROUGH A RESISTOR

A resistor R connected with an AC source is shown in Fig.17.6 (a). The instantaneous voltage V is given as:

$$V = V_o \sin \omega t \quad \text{.................... 17.10}$$

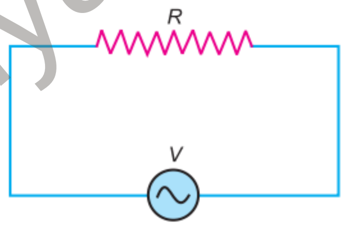


Fig.17.6(a): Showing a resistor R connected with an AC source

The instantaneous current I through the circuit is:

$$I = \frac{V}{R}$$

Using Eq.17.10; $I = \dfrac{V_o \sin \omega t}{R}$

$$I = I_o \sin \omega t \quad \text{.................17.11}$$

where $I_o = \dfrac{V_o}{R}$ is the peak or maximum value of current.

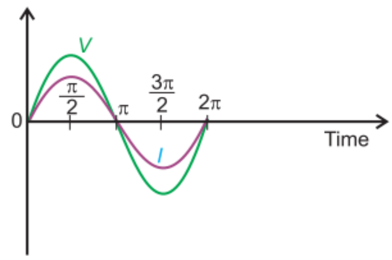


Fig. 17.6(b): Graphical representation for purely resistive circuit showing V and I in phase.

It follows from Eqs.(17.10) and (17.11) that the instantaneous values of both voltage and current are sine functions which vary with time. Figure 17.6(b) shows that both voltage and current pass their minimum and maximum values at the same time and thus their instantaneous values are said to be in phase with each other.

Also Fig.17.6(c) shows $\mathbf{V_R}$ and $\mathbf{I_R}$ vectors for resistance. V and I are drawn parallel because there is no phase difference between them. The opposition to AC which the circuit presents is the resistance given by

$$R = \frac{V}{I}$$

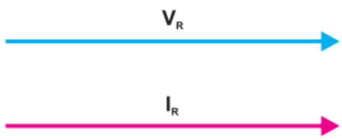


Fig.17.6(c): $\mathbf{V_R}$ and $\mathbf{I_R}$ vectors representation for resistance.

The power dissipation is proportional to the square of the current and makes no difference whether the current is direct or alternating i.e., whether the sign associated with the current is positive or negative. However, the power dissipation produced by AC having maximum value I_o is not the same as that produced by a direct current of maximum value I_o, because the alternating current is at this maximum value only for an instant during each half-cycle. Instantaneous power P dissipated across a resistor in AC circuit is:

$$P = VI \quad \text{or} \quad P = I^2R \quad \text{or} \quad P = \frac{V^2}{R} \quad \ldots\ldots\ldots\ldots\ldots (17.12)$$

and average power $<P> = <I^2R>$

As $<I^2> = \frac{I_o^2}{2} = I_{rms}^2$, so

Average power $<P> = \frac{I_o^2}{2}R = I_{rms}^2 R$

Also, it can be proved that;

Average power $<P> = I_{rms}V_{rms}$

It is to be noted that here P is measured in watts V in volts, I in amperes and R in ohms respectively, and the Eq.(17.12) for power holds good only when V and I are in phase .

For your information

12 V A.C DOORBELL

Low-voltage AC is safer to handle and reduces the risk of electrical shock.

Example 17.3 A 1 kW heating element is connected to a 250 V AC supply voltage. Calculate the amount of current taken from the supply and the resistance of the element when it is hot.

Solution Power P = 1 kW = 1000 W and

Applied voltage V = 250 V

We know that; $P = VI$ or $I = \frac{P}{V}$

$$I = \frac{1000\ \text{W}}{250\ \text{V}} = 4\ \text{A}$$

$$R = \frac{250\ \text{V}}{4\ \text{A}} = 62.5\ \Omega$$

Also $V = IR$ which suggests that, $R = \frac{V}{I}$

$$R = \frac{250\ \text{V}}{4\ \text{A}}$$

$$R = 62.5\ \Omega$$

17.6 AC THROUGH INDUCTOR

An inductor, also called a coil, or choke is a passive two terminal electrical component having a large value of self -inductance and negligible resistance that stores energy in a magnetic field when electric current flows through it. The inductor is used to slow down current surges or spikes by temporarily storing energy in an electromagnetic field and then releasing it back into the circuit.

Suppose an inductor of inductance L is connected with an AC source of frequency f with a negligible resistance as shown in Fig.17.7 (a). Suppose the current is:

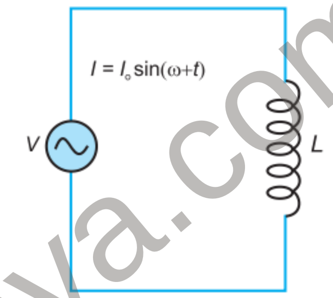


Fig.17.7 (a): Inductor L is connected with an AC source.

$$I = I_o \sin\omega t \quad ....(17.13)$$

If L is the inductance of the coil, then changing current sets up a back emf in the coil of magnitude:

$$V = L\frac{\Delta I}{\Delta t}$$

To maintain a constant current, the applied voltage must be equal to the back emf . The magnitude of applied voltage across the coil must have value given by

As $\quad I = I_o \sin\omega t$, so

$$V = L\frac{\Delta}{\Delta t}(I_o \sin\omega t)$$

$$V = LI_o\frac{\Delta}{\Delta t}\sin\omega t \quad ....(7.14)$$

For your information

$\Delta I/\Delta t$ is rate of change of current with time. This also represents slope of I-t curve.

As $\quad \frac{\Delta}{\Delta t}\sin(\omega t) = \omega\cos(\omega t)$, therefore,

$$V = \omega LI_o\cos(\omega t)$$

As $\quad \omega LI_o = V_o$, so

$$V = V_o\cos(\omega t)$$

As $\quad \cos\omega t = \sin(\omega t + \frac{\pi}{2})$, so

$$V = V_o\sin(\omega t + \frac{\pi}{2}) \quad .....(17.15)$$

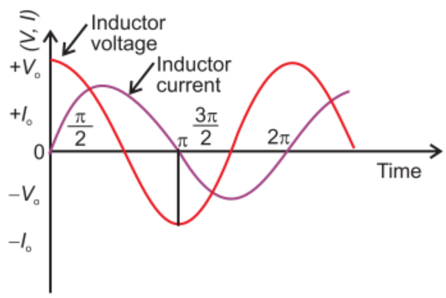


Fig.17.7(b): Graphically showing voltage is leading current in an inductor

It is seen from Eq.17.15, the voltage across the inductor L is leading the current, or in other words, current is lagging behind the voltage in AC circuit containing an inductor as shown graphically in Figs.17.7(b) and 17.7 (c). The current in an inductor always lags behind the voltage by 90° or

$\pi/2$ rad. The resistance offered by an inductor is called inductive reactance denoted by X_L and is given as

$$X_L = \frac{V_{rms}}{I_{rms}} \quad \text{.........(17.16)}$$

where V_{rms} is the rms value of alternating voltage in the inductor and I_{rms} is the rms current passing through it.

As $\quad V_{rms} = 0.707\,V_o$ and $I_{rms} = 0.707\,I_o$, so

$$X_L = \frac{0.707V_o}{0.707I_o} = \frac{V_o}{I_o}$$

As $V_o = \omega L I_o$,

so $$X_L = \frac{\omega L I_o}{I_o}$$

Hence $\quad X_L = \omega L = 2\pi f L$ .... (17.17)

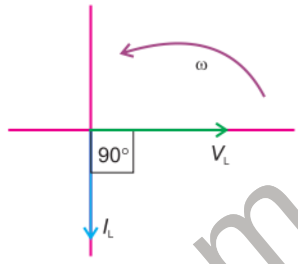


Fig.17.7(c): vector representation of V and I in an inductor.

It shows that inductive reactance X_L is directly proportional to both, frequency of current and the inductance L of the inductor. In case of DC, $f = 0$, so $X_L = 0$, while in case of large AC frequency, X_L is also large. Thus, we conclude that an inductor allows DC but blocks the AC. The unit of X_L is ohm.

Power dissipation in an inductor

The average power dissipated in a pure inductor is zero. This is because the inductor does not consume power in the traditional sense, as it stores energy in the magnetic field and returns it to the source during demagnetisation. The instantaneous power in the inductive circuit is zero and the average power dissipated per cycle is also zero. This behaviour is due to the phase difference between the voltage and current in an AC circuit, which is 90°. Therefore, the average power dissipated in a pure inductor is zero. Since inductor does not consume energy, it is used for controlling AC without consuming energy.

Example 17.4 A 400 mH coil of negligible resistance is connected to an AC circuit in which a current having its rms value of 6 mA is flowing. Find out the value of rms voltage across the coil if its frequency is 1 kHz.

Solution

Inductance of the coil $\quad L = 400\text{ mH} = 400 \times 10^{-3}\text{ H}$

and $\quad I_{rms} = 6\text{ mA} = 6 \times 10^{-3}\text{ A}$

Frequency $\quad f = 1\text{ kHz} = 1000\text{ Hz}$

Voltage across coil $\quad V_{rms} = ?$

We know that; $\quad V_{rms} = I_{rms} X_L$

As $\quad X_L = 2\pi f L \quad$ therefore,

For your information

Types of chokes

So $V_{rms} = I_{rms}(2\pi f L)$

$V_{rms} = 6 \times 10^{-3} A(2 \times 3.14 \times 1000\,Hz \times 400 \times 10^{-3} H) = 15\,V$

17.7 CHOKE

It is a coil of thick copper wire wound closely in a large number of turns over a soft iron laminated core.

A choke is often modeled as a series RL-circuit, consisting of a resistor R, having very small value of resistance, in series with an inductor *L*, of quite large inductance $X_L = 2\pi f L$, as shown in Fig.17.7(e).The inductance *L* of the choke coil is also very high due to the high permeability of iron core on which choke coil is wound. As the resistance *R* of a choke coil is negligibly small, therefore, the power factor (cosθ) of the choke coil is almost zero. Thus, the phase difference θ between the current and voltage for a choke coil is nearly equal to 90°. So practically, no power is dissipated as heat by a choke coil. Also choke is used to block high frequency AC while allowing DC and low frequency AC to pass. Choke coils are used to filter out high frequency AC noise from electronic circuits, ensuring a cleaner DC output. They are essential in switch-mode power supplies, helping to regulate voltage and filter out switching noise.

Fig. 17.7(e): A choke

Choke coils prevent unwanted RF signals from leaking out of circuits, protecting other sensitive components. Chokes in fluorescent lights generate transient voltages across the tube, making it conducive to the breakdown voltage of the gas inside. Choke coils can limit the rate of current changes in circuits, preventing damage to insulation from sudden surges.

Do you know?

Why capacitor and inductor behaves differently for AC and DC?

For DC supply, when *f* = 0, then $X_L = 2\pi fL = 0$, thus, inductor acts as conductor for DC

For AC supply when f = 0

Then $X_C = 1/2\pi f C = 1/0 = \infty$

Thus, capacitor blocks DC but allows AC to pass through it.

17.8 AC THROUGH A CAPACITOR

A capacitor does not allow direct current to pass through it because of the presence of an insulating medium between its plates. But alternating current can pass through a capacitor. In electric circuits, a capacitor is a reactive component. Unlike a resistor, a capacitor behaves differently in AC and DC circuits. It is because a capacitor can store energy in the form of electric field, whereas a resistor cannot store electrical energy in any form. Consider an alternating voltage source *V* is applied to a

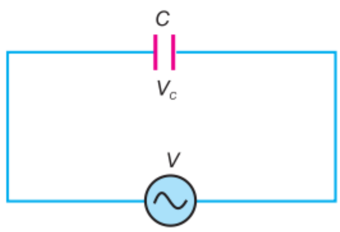


Fig.17.8(a): Showing capacitor connected with ac voltage source

capacitor of capacitance *C* as shown in Fig.17.8 (a).

When an alternating voltage is applied across the plates of a capacitor, it is charged in one direction and then in the other as the voltage reverses. The result is that electrons move to and fro around the circuit, connecting the plates, thus, constituting alternating current. The basic relationship between charge *q* and the voltage *V* across the plates: $q = CV$, holds good at every instant. Let V be the applied alternating voltage given by

$$V = V_o \sin\omega t \text{ .... (17.18)}$$

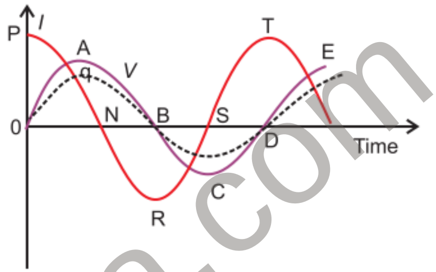


Fig.17.8 (b): Graphical representation of current and voltage for capacitor.

The change on the capacitor at any instant is give by

$$q = CV = CV_o \sin\omega$$

Thus
$$I = \frac{\Delta q}{\Delta t} = \frac{\Delta}{\Delta t}(CV_o \sin\omega t)$$

$$I = CV_o \frac{\Delta}{\Delta t}(\sin\omega t)$$

As $\frac{\Delta}{\Delta t}\sin\omega t = \omega\cos\omega t$, thus

$$I = CV_o\omega\cos\omega t$$

As $\cos(\omega t) = \sin\left(\omega t + \frac{\pi}{2}\right)$, so

$$I = \omega CV_o \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$I = \omega CV_o \sin\left(\omega t + \frac{\pi}{2}\right)$$

Here $\omega CV_o = I_o$,thus

$$I = I_o \sin\left(\omega t + \frac{\pi}{2}\right) \text{............(17.19)}$$

Do you know?

Father of ALTERNATING CURRENTS?
Nikola Tesla was born in 1856 in Austria-Hungary and emigrated to the U.S.A. in 1884 as a physicist. He pioneered the generation, transmission, and use of alternating current which can be transmitted over much greater distances than direct current.

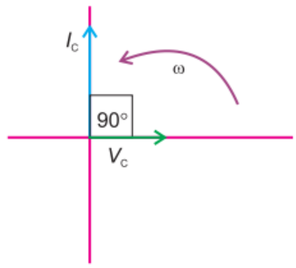


Fig. 17.8©: Vector representation of current and voltage for a capacitor.

Equations 17.18 and 17.19 show that inductor current *I* leads the voltage by 90° or $\pi/2$ rad or voltage lags behind the current *I* by 90° or $\pi/2$ rad as shown in Figs.17.8(b) and .17.8(c).

Resistance offered by capacitor is known as capacitive reactance denoted by *Xc* and is given by

$$X_C = \frac{V_{rms}}{I_{rms}}$$

where V_{rms} is the rms value of alternating voltage in the inductor and I_{rms} is the rms current passes through it.

As $V_{rms} = 0.707V_o$ and $I_{rms} = 0.707I_o$, so

$$X_C = \frac{0.707V_o}{0.707I_o} = \frac{V_o}{I_o}$$

As $I_o = \omega C V_o$

Hence $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ where $\omega = 2\pi f$

The unit of capacitive reactance X_C is ohm and is given as:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Which shows that for low frequencies, capacitor will have a large reactance X_C and current I will be small whereas at high frequencies reactance X_C will be small and current I through the same capacitor will be large.

17.9 IMPENDANCE

In AC circuits, the opposition to current flow is called impedance denoted by Z, which includes both resistance R and reactance X. The resistance R is the opposition to current flow due to resistors. Reactance is the opposition to current flow due to capacitors (capacitive reactance X_C) and inductors (inductive reactance X_L). A pure resistive circuit has only resistance R and no reactance.

The combined effect of resistance and reactances in such circuits is known as impedance.

It is the ratio of the rms value of the voltage to the rms value of the current. Thus,

$$Z = \frac{V_{rms}}{I_{rms}} \quad \text{..........(17.20)}$$

This SI unit of impedance is ohm.

Example 17.5 An AC circuit operates by a peak voltage of 200 V and 10 A as peak input current. Find the impedance of the circuit.

Solution

Peak voltage $V_o = 200\,V$, Peak current $I_o = 10\,A$

We know that; $V_{rms} = V_o \times 0.707 = 200\,V \times 0.707 = 141.4\,V$

and $I_{rms} = I_o \times 0.707 = 10\,A \times 0.707 = 7.07\,A$

As impedance $Z = \frac{V_{rms}}{I_{rms}}$, therefore,

$$Z = \frac{141.4V}{7.07A} = 20\,\Omega$$

17.10 AC THROUGH RC-SERIES CIRCUIT

Consider a network of resistance *R* and a capacitor *C* connected in series by an alternating voltage source *V* as shown in Fig.17.9 (a). As *R* and *C* are in series, so same current would flow through each of them. The potential difference *V* across the resistance *R* would be $I_{rms}R$ and it would be in phase with the current. The vector diagram of the voltage and current is shown in Fig.17.9 (b).Taking the current as reference, potential difference $V_R = I_{rms}R$ across the resistance is represented by a line along the current line because potential drop $I_{rms}R$ is in phase with current. The potential difference across the capacitor will be:

$$V_c = I_{rms}X_c = \frac{I_{rms}}{\omega C}$$

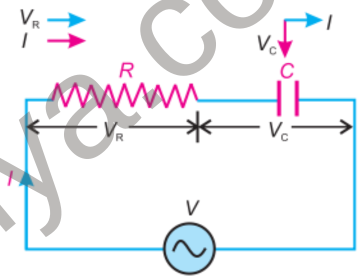


Fig. 17.9(a)

As this, voltage lags behinds the current by $\pi/2$ rad or 90°, so the line representing the vector $I_{rms}X_C$ is drawn at right angles to the current line Fig.17.9 (b).

The applied voltage V_{rms} that will send the current *I* in the circuit is obtained by resultant of the vectors $I_{rms}R$ and $\frac{I_{rms}}{\omega C}$ i.e;

$$V_{rms} = \sqrt{(V_R)^2 + (V_c)^2}$$

$$V_{rms} = \sqrt{(I_{rms}R)^2 + \left(\frac{I_{rms}}{\omega C}\right)^2}$$

$$V_{rms} = I_{rms}\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$V_{rms} = I_{rms}\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

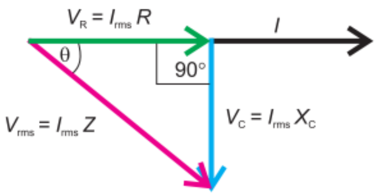


Fig. 17.9(b)

Impedance $Z = \frac{V_{rms}}{I_{rms}} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$ ....(17.21)

Equation 17.21 suggests that we can find the impedance of a series AC circuit by vector addition. The resistance *R* is represented by a horizontal line in the direction of current which is taken as reference. The reactance $X_c = 1/\omega C$ is shown by a line lagging the R-Q line by 90°as shown in Fig.17.9(c) .The impedance *Z* of the circuit is obtained by the vector summation of resistance and reactance. Figure 17.9(c) is known as impedance diagram of the circuit. The angle which the line representing the impedance *Z* makes with *R* line gives the phase difference between the voltage and current *I* in Fig. 17.9(c), the current is leading the voltage applied by the angle θ as shown in Fig.17.9(d) and is given by

$$\theta = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega CR}\right) = \tan^{-1}\left(\frac{1}{2\pi fC}\right)$$

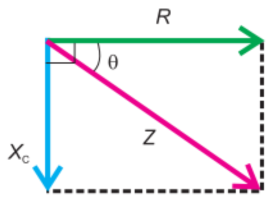


Fig. 17.9(c): Impedance diagram of RC series circuit.

The power consumed in RC-series circuit is primarily due to the resistor, as the capacitor only stores energy and does not dissipate it. The power consumed by the resistor can be calculated by using the formula:

$$P = I^2R$$

or $$P = VI\cos\theta \quad \text{.............. (17.22)}$$

where I is the current, R is the resistance, V is the voltage, and θ is the phase angle between voltage and current.

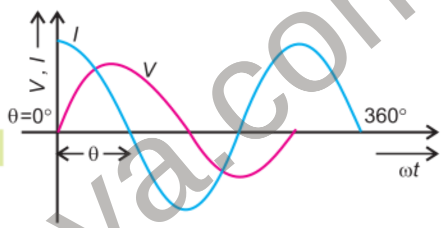


Fig. 17.9(d): Voltage and current waveform in RC-series circuit.

Example 17.6 A resistor of resistance 5 kΩ is connected in series with capacitor of capacitance 5 F across the AC source of 220 V that has frequency 50 Hz. Calculate the phase angle.

Solution Resistance $R = 5\text{ k}\Omega = 5000\,\Omega$,

Capacitance $C = 5\,\mu\text{F} = 5 \times 10^{-6}$ F

and Frequency f = 50 Hz

We know that;

Phase angle $\theta = \tan^{-1}\left(\frac{1}{\omega CR}\right)$

As $\omega = 2\pi f$, so

$$\theta = \tan^{-1}\left(\frac{1}{2\pi f\, CR}\right)$$

Putting the values

$$\theta = \tan^{-1}\left(\frac{1}{2 \times 3.14 \times 50\text{ Hz} \times 0.000005\text{ F} \times 5000\text{ W}}\right)$$

$$\theta = \tan^{-1}(0.1273) = 7.25^\circ$$

17.11 AC THROUGH RL-SERIES CIRCUIT

Consider an AC circuit containing a resistor R and inductor L connected in series with an AC source V as shown in Fig.17.10(a). As R and L are in series, therefore, same current I flows through both R and L.

$$I = I_o \sin\theta$$

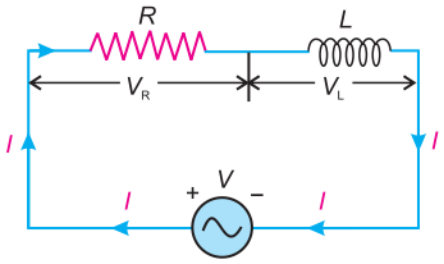


Fig.17.10 (a): *RL* series circuit connected with A.C. supply

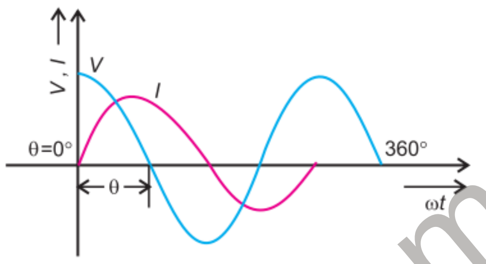


Fig. 17.10 (b): Voltage and Current waveform in of *RL* series circuit.

Let rms value of current through R is I. Then rms value of voltage V_R across R will be:

$$V_R = I_{rms} R$$

V will be in phase with the current I in the circuit, as R is shown vectorially by line **OA** in Fig.17.10(c). The value of rms voltage V_L across L will be:

$$V_L = I_{rms} X_L$$

Voltage V across L is ahead of current I through L by 90° as shown in Fig.17.10 (b) and vectorially by line **OB** in Fig.17.10(c).The resultant voltage V will be the vector sum of V_R and V_L shown vectorially by line **OP,** thus in Fig.17.10(c):

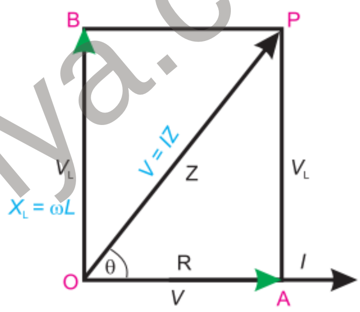


Fig. 17.10 (c): Vector diagram of V_R and V_L in of *RL* series circuit.

$$V_{rms} = \sqrt{V_R^2 + V_L^2}$$

$$V_{rms} = \sqrt{(I_{rms} R^2) + (I_{rms} X_L^2)}$$

As $X_L = \omega L$, so

$$V_{rms} = I_{rms}\sqrt{R^2 + (\omega L)^2}$$

$$\text{Impedance } Z = \frac{V_{rms}}{I_{rms}} = \sqrt{R^2 + (\omega L)^2} \quad .... (17.23)$$

$$\text{Angle} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

For your information

Electric Eels create powerful shocks by its 3 main organs consisting of 80% of Eels body length through specialized cells called electrolytes.

The angle θ which Z makes with R line, gives the phase difference between the applied voltage and current.

Power dissipation in RL-Series circuit

Power consumed in RL- series circuit is primarily due to the resistor, as the inductor stores energy but ideally does not dissipate it. The power consumed by the resistor is given by the formula:

$$P = I^2 R = VI\cos\theta$$

where I, R and V represents current, resistance, and voltage respectively whereas θ is the phase angle between voltage and current. The factor cos θ is known as power factor. It is to be noted that when we convert DC power P_{dc}, into AC power P_{ac}, we have to take an account of a quantity known as inverter efficiency. It usually may be 85 % to 90%. So, we can find AC power from DC power by using the following conversion as:

$$P_{ac} = (\text{Inverter efficiency}) \times P_{dc}$$

Do you know?

The electricity consumption of a 1.5 ton A.C is approximately 1.2 to 1.5 units per hour.

Brain teaser

How much AC power will be received from a DC solar panel rating 575 DC watt-75% inverting efficiency?

17.12 NAVIGATING AC THROUGH RLC-SERIES CIRCUIT

Consider a RLC-series circuit having an alternating voltage source with variable frequency as shown in Fig.17.11(a).The impedance diagram of RLC-series circuit is shown in Fig.17.11(b) in which inductive reactance $X_L = \omega L$ and capacitive reactance $X_C = 1/\omega C$ are directed opposite to each other.

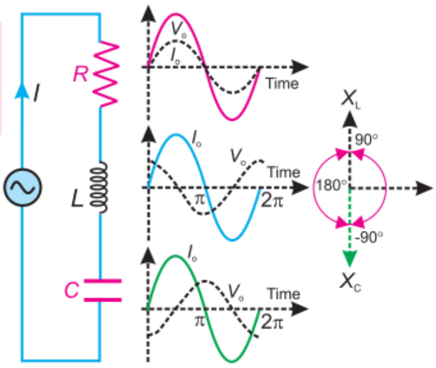


Fig. 17.11(a): RLC-series circuit with its AC source phasor diagram

The impedance of the RLC-series circuit is:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{.....................(17.24)}$$

or

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad \text{...................(17.25)}$$

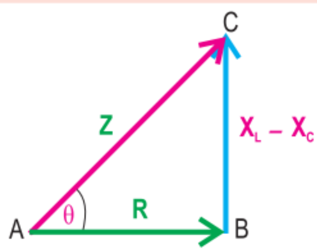


Fig.17.11 (b): The impedance vector diagram of RLC-series circuit

When frequency is high, then $X_L = \omega L$ is much greater than $X_C = 1/\omega C$, and hence the inductance X_L dominates at high frequency, so RLC-series circuit behaves as RL-series circuit.

When frequency is low, then $X_C = 1 / \omega C$ is much greater than $X_L = \omega L$, and hence the capacitance X_C dominates at low frequency, hence RLC series circuit behaves as RC series circuit as shown in Fig.17.11(b).

In between these frequencies, there will be a frequency ω_r at which $X_L = X_C$. At this condition, RLC-series circuit behaves as pure resistive R-circuit as shown in Fig.17.11(c).

This condition is called resonance. Thus, at resonance, the inductive reactance X_L being equal and opposite to capacitive reactance X_C, cancel each other. The value of resonance frequency f_r can be obtained by applying the condition:

$$X_L = X_C$$

$$\omega_r L = \frac{1}{\omega_r C}$$

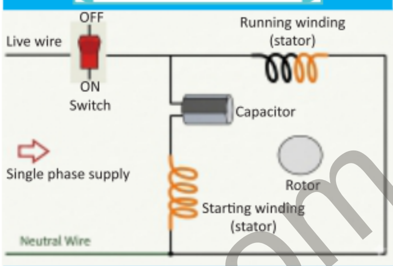
$$\omega_r^2 = \frac{1}{LC}$$

$$\omega_r = \frac{1}{\sqrt{LC}} \quad \text{As } \omega_r = 2\pi f_r \text{, so}$$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad \text{..............(17.26)}$$

Brain teaser



Guess the name of the AC circuit used in the ceiling fan?

Fig.17.11(c): Vector diagram of RLC-series circuit showing that it behaves as pure resistive *R*-circuit at $X_L = X_C$

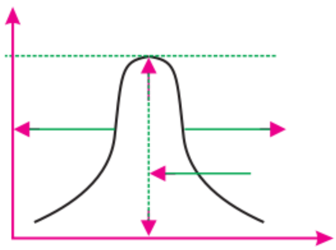


Fig.17.11 (d): Phasor diagram of RLC-series circuit resonance frequency at $X_L = X_C$

Key Properties of RLC Series Circuit

In essence, the series RLC-circuit is a versatile circuit element that can be designed to exhibit specific behaviour at different frequencies, making it important in various electronic applications.

1. Impedance and Resonance: At resonance impedance, is minimum ($Z = R$) and at resonance, the resonant frequency can be calculated by using the formula:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

2. Current: At resonance, the current in the circuit reaches its maximum value due to the minimum impedance (Fig. 17.11-d).

3. Phase Angle: At resonance, the phase angle is zero, indicating that the voltage and current are in phase and have power factor equal to 1.

4. Damping: The resistor in the circuit introduces damping, which affects the oscillations of the current. A higher resistance leads to more damping, causing the oscillations to decay faster. The forced but damped motion of the wheel on the car spring is analogous to an RLC-series AC circuit. The shock absorber damps the motion and dissipates energy, analogous to the resistance in an RLC-series circuit. The mass and spring determine the resonant frequency as shown in Fig.17.11(e).

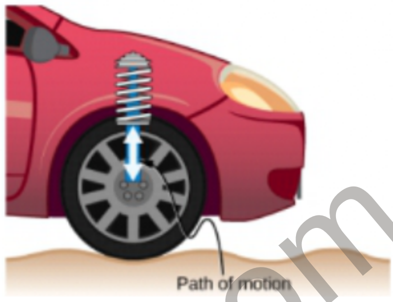


Fig.17.11 (e): On a car, the shock absorber damps motion and dissipates energy. This is much like the resistance in an RLC-circuit. The mass and spring determine the resonant frequency.

5. Tuning and Filtering: RLC-series circuits, especially at resonance, are widely used for tuning in radio receivers and other communication devices. They can be used as band-pass filters, allowing signals within a specific frequency range to pass while attenuating others.

6. Voltage Amplification: At resonance, the voltage across the inductor and capacitor can be significantly higher than the applied voltage due to the circulating current. This phenomenon is known as voltage amplification.

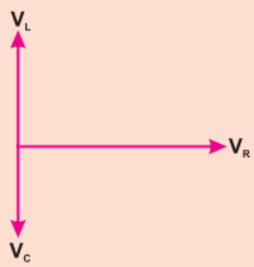
7. Power Dissipation: The power dissipation in RLC-series circuit is given by

$$P = VI\cos\theta \quad \ldots\ldots\ldots\ldots\ldots (17.27)$$

which is known as a true power in RLC-series circuit and it shows that a maximum power will be dissipated when θ = 0, which is only possible in resistor. However, the power dissipation in impedance is called apparent power and is equal to $VI\cos\theta$.

Example 17.7 In a LCR-circuit, an inductor having inductive voltage 20 V, capacitor with capacitive voltage11 V and a resistance with voltage 12 V are connected in series. Find phase difference between resultant voltage and current in the circuit. Also calculate the V_{rms} value of voltage of AC source.

Solution $V_L = 20\text{ V}$, $V_C = 11\text{ V}$ and $V_R = 12\text{ V}$



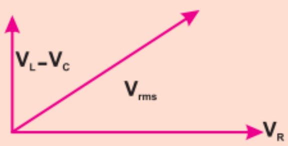
We know that; Phase difference for RLC-series circuit is given by

$$\tan\theta = \frac{(V_L - V_C)}{V_R}$$

$$\tan\theta = \frac{20\text{ V} - 11\text{ V}}{12\text{ V}} = 0.75$$

or $$\theta = \tan^{-1}(0.75) = 36.8^\circ$$

Also we know that;



$$V_{rms} = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

$$V_{rms} = \sqrt{(12\text{ V})^2 + (20\text{ V} - 11\text{ V})^2}$$

$$V_{rms} = \sqrt{144\text{ V}^2 + 81\text{ V}^2}$$

$$V_{rms} = \sqrt{225\text{ V}^2} = 15\text{ V}$$

17.13 AC THROUGH RLC-PARALLEL CIRCUIT

A RLC-parallel circuit is one where the resistor, inductor and capacitor are connected in parallel to each other with an AC source as shown in Fig.17.12 (a). The parallel arrangement affects the overall impedance and current distribution in the circuit, making it distinct from its series counterpart.

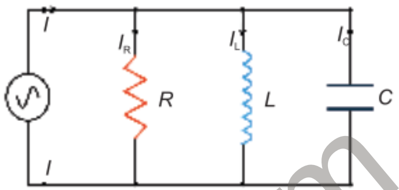


Fig.17.12(a): RLC-parallel circuit connected with an AC source

Key Properties of RLC-Parallel Circuit

1. Resonance and impedance: A parallel RLC-circuit resonates at $X_L = X_C$ and the impedance is at its maximum value in a way that parallel RLC-circuit behaves like a purely resistive circuit. The impedance of RLC-parallel circuit is given by

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} \quad \text{...............(17.27)}$$

2. Voltage and Current: In a RLC-parallel circuit, the voltage is same across all components, so for convenience, the voltage may be taken as a reference phasor. At the resonance, current is minimum but it can be significantly magnified, while the source current remains relatively low. So, it may be used for current magnification. The variation of current with frequency of the source is shown in Fig.17.12 (b). At resonance, the branch currents *IL* and *IC* may each be larger than the resonance current I_r.

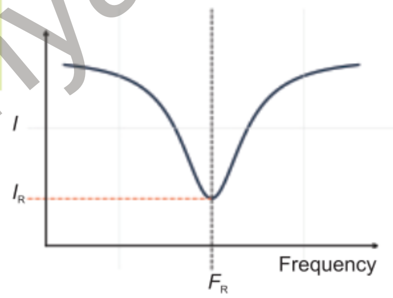


Fig.17.12(b): A graph showing resonance frequency with its resonance current I_r.

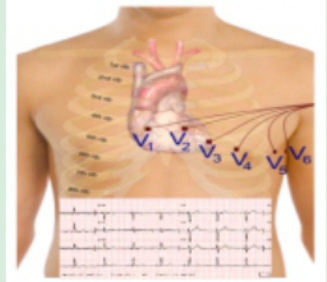
3. Admittance: Admittance (*Y*) is the reciprocal of impedance and is used to analyze parallel circuits. The total admittance of a parallel RLC-circuit is the sum of the admittances of the individual components.

4. Frequency and Bandwidth: The impedance, admittance, and currents in a parallel RLC-circuit are all frequency-dependent. The bandwidth of a parallel RLC-circuit is the range of frequencies over which the circuit operates effectively. The resonance frequency of RLC-parallel circuit is:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

The parallel RLC-circuit can also be used for tuning circuits, signal processing, and power systems. RLC-parallel circuits are indeed used in induction cooktops to match the desired heating frequency, specifically, the resonant behaviour of a parallel RLC-circuit is

Do you know?



ECTR OCARDIO GRAM (ECG) is used for diagnosing heart conditions by recording of small electric waves being generated during heart activity seen by CRO.

important for the operation of induction heating, which is the core principle behind induction cooktops i.e; the RLC-parallel circuit generates a high frequency current in the coil, which induces currents in the metal cookware, causing it to heat up as shown in Fig. 17.12(c).

Figure 17.12(c): RLC-parallel circuits-a core principle behind induction cooktops.

5. **Quality Factor (Q):** The quality factor is a measure of the sharpness of the resonance peak and indicates how much energy is stored in the circuit compared to the energy dissipated. In a parallel RLC-circuit, the Q-factor is inversely proportional to the resistance.

6. **Power Factor:** The power factor in a parallel RLC-circuit can vary depending on the frequency and the relative magnitudes of the inductive and capacitive reactances. At resonance, the power factor is unity.

17.14 RECTIFICATION

Rectification is the process of converting alternating current into direct current. Diodes allow current to flow in one direction (forward bias) but block it in the opposite direction (reverse bias). Rectification is of two types, half wave rectification and full wave rectification. Rectification is commonly used in electronic devices that require a stable DC power source, such as computers and smartphones, as most household and commercial power is AC.

Diode as a Rectifier

A diode allows large current to flow when forward biased. However, the current through a reverse biased diode is practically zero. It is due to this important property of the diode that it can be used for rectification i.e., to convert alternating current into unidirectional current or direct current.

> **Brain teaser**
>
> Cell phones store AC or DC!
> Cell phone batteries store DC in them, since it is easier to store DC than AC. DC is also safer compared to AC. The electric grid provides AC only. Therefore, the AC is converted to DC using a rectifier before charging the cell phones or any other portable devices such as laptops, flashlights, etc.

Half-Wave Rectifier

In a half-wave rectification, an AC signal is converted into pulsating direct current DC by

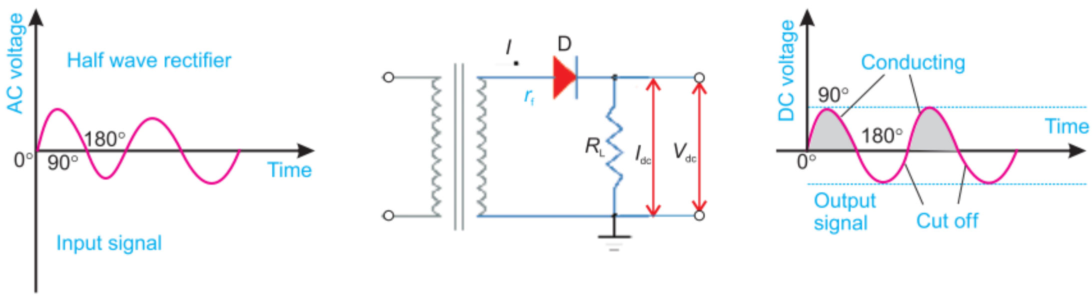


Figure 17.13: Working of a half-wave rectifier, showing graphically its input and output wave forms.

passing one half-cycle of waveform and blocking the other half. In Fig.17.13, alternating voltage of period T called input voltage is supplied to a diode D which is connected in series with a load resistance R_L. The working of half-wave rectifier is based on the fact that the diode allows the current flow only in one direction. Thus, it converts the AC signal into DC signal. Figure 17.13 shows the circuit of half-wave rectification and it is working showing graphically its input and output wave forms.During positive half-cycle of the input alternating voltage i.e., during the interval $0 \rightarrow T/2$, the diode D is forward bias, so it offers a very low resistance and current flows through R_L. The flow of current through R_L causes a potential drop across it which varies in accordance with the alternating input (Fig.17.13).

For your information

A circuit board view of half-wave rectifier with its different components

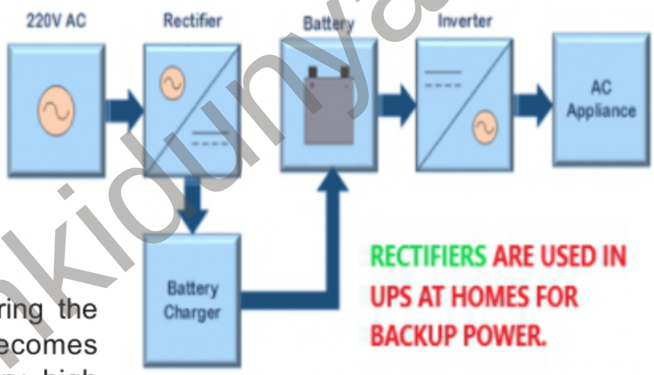


Fig.17.13(a): A circuit view of a UPS system used in our homes.

During negative half cycle, during the interval $T/2 \rightarrow T$, the diode becomes reverse bias and offers a very high résistance, so practically no current flows through R_L and potential drop across it is almost zero (Fig.17.13).The same events repeat during the next cycle and so on. The current across R_L flows in only one direction which means it is direct current. However, this current flows in pulses. The voltage which appears across load resistance R_L is known as output voltage.

Full0-Wave Rectification

Rectification during which both halves of alternating input voltage are converted into unidirectional current through a resistance is called full-wave rectification. Full-wave rectification is achieved by a bridge rectifier in which four individual rectifying diodes are connected in a closed loop "bridge" configuration to produce the desired output. The single secondary winding is connected to one side of the diode bridge network and the load to the

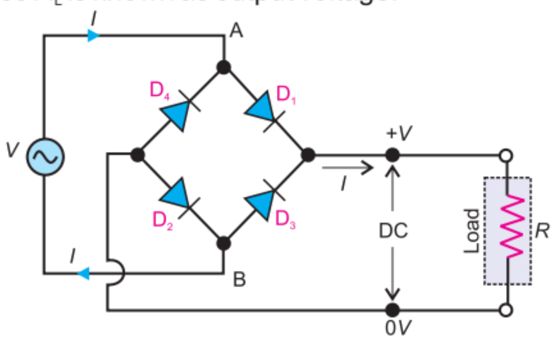


Figure 17.15(a): Full-wave bridge rectifier circuit showing 4 diode network with load resistance.

other side. The four diodes labelled diodes D_1 to D_4 are arranged in "series pairs" with only two diodes conducting current during each half-cycle as shown in Fig.17.15(a)

The Positive Half-cycle: We know that diode conducts only when it is forward bias. During the positive half-cycle of the supply i.e., during the time 0 → *T*/2, the terminal A of the bridge is positive w.r.t. its other terminal B. Now the diodes D_1 and D_2 become forward biased and conduct in series while diodes D_3 and D_4 are reverse biased and the current flows through the load resistance *R* as shown in Fig.17.15 (b)

Do you know?

Uses of the PN junction diode:
1. Rectification
2. Signal clipping
3. Regulation
4. Clamping voltage
5. Signal detection
6. Photovoltaic cells and
7. Switching light emission

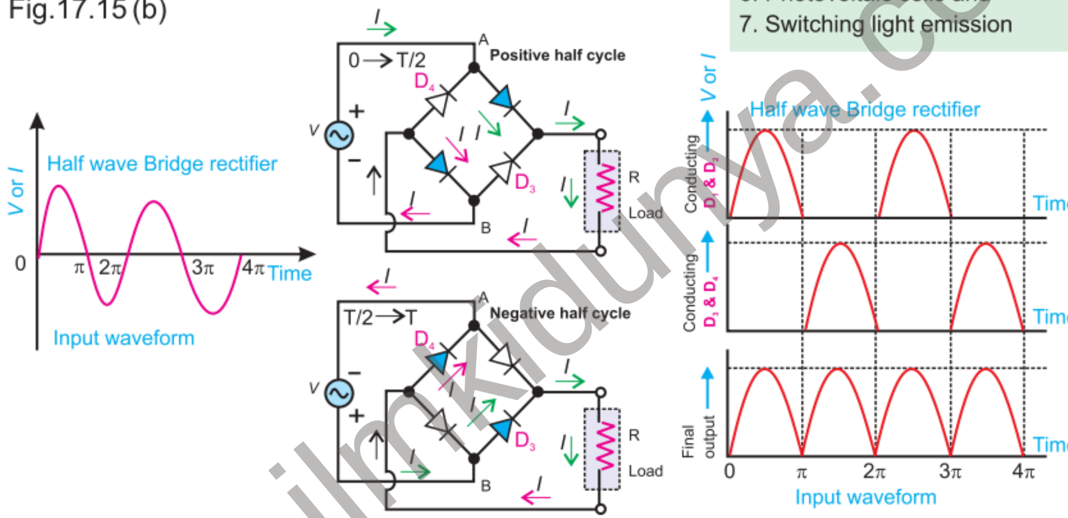


Figure 17.15(b) ,Figure 17.15 (c): Full-wave bridge rectifier circuit showing its working during its positive and negative half cycles and also representing graphically its input and output waveforms.

The Negative Half-cycle: During the negative half cycle of the supply, during the time *T*/2 → *T*, terminal A is negative and B is positive. Now the diodes D_3 and D_4 conduct in series, but diodes D_1 and D_2 switch "OFF" as they are now reverse biased. The current flowing through the load is the same direction as before as shown in Fig.17.15 (c).If we take a comparison of Figs.17.15(b) and 17.15(c), it can be seen that direction of current flow through the load resistance is same in both half of the cycle. Thus, both halves of the alternating input voltage send a unidirectional current through load resistance *R*. The output voltage is not smooth but pulsating. It can

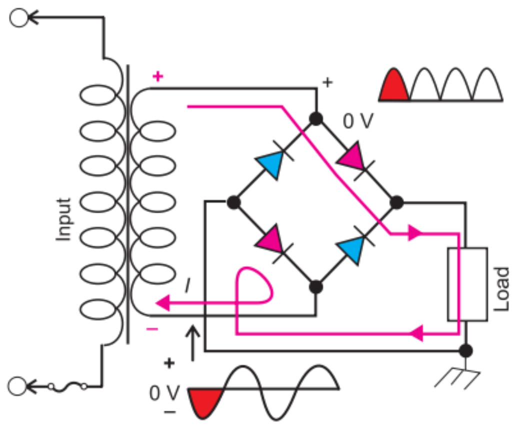


Figure 17.16(a): Circuit diagram showing the use of Full wave Bridge Rectifier in mobile charger circuit.

be made smooth using a circuit called filter which may consist of capacitors. Mobile phone chargers typically use full-wave bridge rectifier as part of their circuitry. The rectifier converts the alternating current from the wall outlet into direct current, which is necessary to charge the phone's battery.

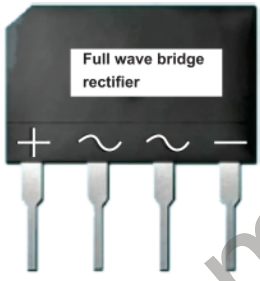


Figure 17.16(b): A Compact Full wave Bridge Rectifier used in electronics.

Full-Wave Bridge Rectifier with Single Smoothing Capacitor

In AC to DC conversion, smoothing filters are used to reduce the ripple voltage after rectification. When AC is rectified, the output is pulsating DC, which is not suitable for most electronic devices. Smoothing filters help to produce a more stable DC voltage by filleting the AC components (ripples).They are of different types. In capacitor filter, a capacitor is connected in parallel with the load. It charges when the rectified voltage increases and discharges when the voltage decreases, thereby smoothing the output.

Smoothing or reservoir capacitors connected in parallel with the load across the output of the full-wave bridge rectifier circuit, increase the average DC output level even higher as the capacitor acts like a storage device as shown in Fig.17.17. The smoothing capacitor converts the full-wave rippled output of the rectifier into a more smooth DC output voltage. We can see the effect it has on the rectified output waveform with different values of smoothing capacitor installed.

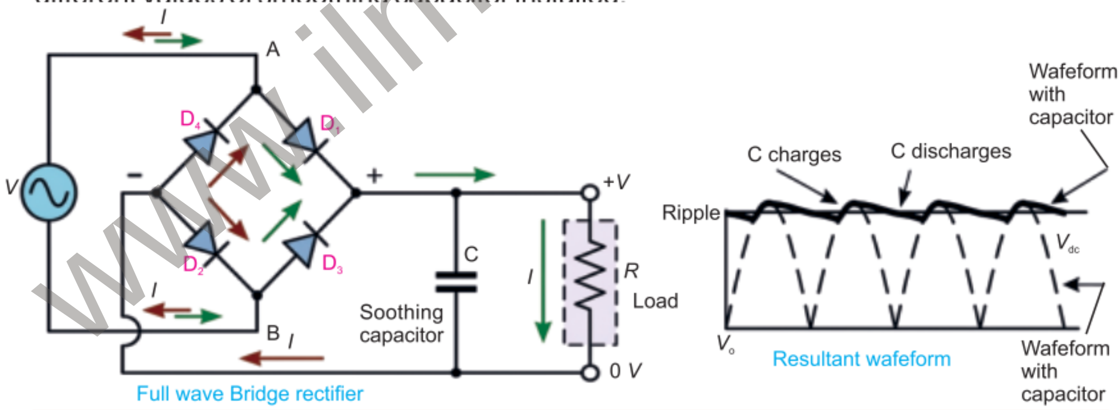


Fig.17.17: The smoothing capacitor converts the full-wave rippled output of the rectifier into a more smooth DC output voltage.

The maximum ripple voltage present in a full-wave rectifier bridge circuit is not determined by the value of the smoothing capacitor but by the frequency and load current, and is calculated as:

$$V_{ripple} = \frac{I_{DC}}{f_{ripple}C} \quad \text{or} \quad V_{ripple} = \frac{I_{DC}}{2fC}$$

Brain teaser

Our heart is driven by electric pulses; the high electric frequency of AC current can affect the frequency of the heart and can lead to a heart attack.

where I_{DC} is the load current in ampere, f is input frequency and C is capacitance in farad (F). It is to be noted that ripple frequency f_{ripple} is twice that of the input frequency,

i.e., $f_{ripple} = 2f$

17.16 SELF INDUCTION

Consider a circuit in which a coil is connected in series with a battery, a switch and a rheostat as shown in Fig.17.18. If we move the rheostat quickly, the primary current through the coil will change. The magnetic flux through the coil will also change, which finally induces an emf in the coil itself.

The phenomenon, in which changing current induces an emf inside the coil itself, is called self-induction.

Let ϕ represents flux passing through one loop of the coil. The flux passing through the coil of N turns would be $N\phi$. As this flux ϕ is proportional to the magnetic field produced which is in turn proportional to the current I, therefore,

$$N\phi \propto I$$

$$N\phi = LI \quad \ldots\ldots\ldots\ldots (17.29)$$

where L is proportionality constant and is called self-inductance of the coil.

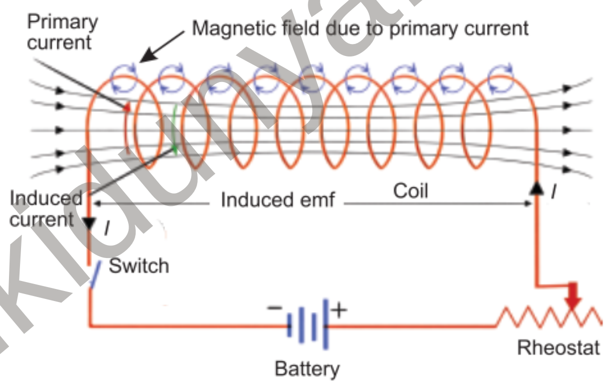


Fig.17.18: A coil showing production of induced current and induced emf in itself.

The self-inductance L depends upon the following factors:

1. Number of turns of the coil.
2. Area of cross-section of the coil.
3. Core material, by winding the coil around ferromagnetic iron core, the magnetic flux and hence inductance can be increased significantly relative to that for an air core.

By Faraday's law, the emf induced in the coil is given by

$$\varepsilon_L = -N\frac{\Delta\phi}{\Delta t} \quad \ldots\ldots\ldots\ldots (17.30)$$

$$\varepsilon_L = -\frac{\Delta(N\phi)}{\Delta t}$$

Using Eq. (17.28); $$\varepsilon_L = -\frac{\Delta(LI)}{\Delta t}$$

$$\varepsilon_L = -L\frac{\Delta I}{\Delta t} \quad \ldots\ldots\ldots\ldots (17.31)$$

or $$L = \frac{-\varepsilon_L}{\frac{\Delta I}{\Delta t}} \quad ....(17.32)$$

Here L is self-inductance of the coil and minus sign indicates that the induced emf opposes the applied voltage. In the above expression, If we take ε_L = 1 volt and $\Delta I/\Delta t = 1\,\text{A s}^{-1}$, then L = 1 henry (H), defined as:

If the emf induced in the coil is 1 V when the current flowing through it changes at the rate of 1 A s^{-1}, the coil has self-induce in a coil of 1 henry (H).

The self-inductance of the coil can also be defined as ratio of the emf to the rate of change of current in the same coil. The SI unit of self-inductance L is Vs A^{-1} having dimensions [ML^{2}T^{-2}A^{-2}].

By equating Eqs.17.30 and 17.31, we have

$$-N\frac{\Delta\phi}{\Delta t} = -L\frac{\Delta I}{\Delta t}$$

$$L = N\frac{\Delta\phi}{\Delta I} \quad ....(17.33)$$

which gives another form of equation for self-inductance of the coil.

Example 17.8 A coil has an inductance reactance of 160 Ω at a frequency of 50 Hz. Calculate self-inductance of the coil.

Solution Induced reaction $X_L = 160\,\Omega$, Frequency $f = 59$ Hz

We know that; $X_L = 2\pi f L$

or $L = X_L / 2\pi f$

$L = 160\,\Omega / (2 \times 3.14 \times 50\,\text{Hz}) = 0.509$ H

Energy Stored in an Inductor

Consider a coil connected to a battery and a switch in series as shown in Fig.17.19. When the switch is turned ON, voltage is applied across the ends of coil and current increases from zero to maximum value. Due to change of current, an emf is induced, which is opposite to that of battery. Work is done by battery to move charges against the induced emf.

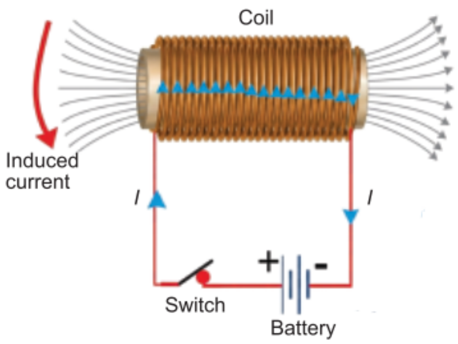


Figure 17.19: Circuit designed to calculate energy stored in an inductor.

Work done by battery in moving a small charge Δq is given by

$$W = \Delta q\,\varepsilon_L \quad ....(17.34)$$

where ε_L is the magnitude of self-induced emf and is given by

$$\varepsilon_L = L\frac{\Delta I}{\Delta t}$$

So, putting values in Eq.17.34, we have

$$W = \Delta q L\frac{\Delta I}{\Delta t} = \frac{\Delta q}{\Delta t}L\Delta I \text{......(17.35)}$$

where $\frac{\Delta q}{\Delta t}$ represents average current and is given by

$$\frac{\Delta q}{\Delta t} = \left(\frac{0+1}{2}\right) = \frac{1}{2}I$$

And $\Delta I = I - 0 = I$

Putting the values in Eq.17.35, we have

$$W = \left(\frac{1}{2}I\right)LI = \frac{1}{2}LI^2 \text{.....(17.36)}$$

This work is stored as potential energy in the inductor. Hence,

$$U_m = \frac{1}{2}LI^2 \text{...............} \quad (17.37)$$

It is to be noted that in an inductor, energy is stored in the magnetic field and can never be negative because it is proportional to square of current. The inductor stores energy while absorbing power, returns the previous stored energy when delivering power, so the net energy transfer can never be negative. Using the Eq.17.37 expressed in terms of magnetic field **B** of solenoid can be found having:

n = number of turns per unit length

A = Area of cross section of coil

B = Magnetic field strength = $\mu_o n I$ (Negative field due to solenoid) energy stored in an

I = current in solenoid $= \frac{B}{\mu_o n I}$

L = Self Inductance $= \mu_o n^2 A l$ where l is the length of solenoid

Putting the values in Eq.17.37 we have

$$U_m = \frac{1}{2}\mu_o n^2 A l\left(\frac{B}{\mu_o n}\right)^2$$

$$U_m = \frac{B^2}{2\mu_o}(Al)$$

As Al = volume, so $U_m = B^2\frac{1}{2\mu_o}\text{(volume)}$

or $\frac{U_m}{\text{volume}} = \frac{1}{\mu_o}B^2$

The energy stored U_m, per unit volume inside the solenoid is called energy density, and for an inductor it is given by

$$\text{Energy density} = B^2 \frac{1}{2\mu_o} \text{...............(17.38)}$$

17.17 MUTUAL INDUCTION

Consider two coils close to each other as shown in Fig. (17.20).One coil connected with a battery through a switch S and a rheostat, is called Primary coil and the other one connected to the galvanometer is called Secondary coil. If the current in the primary is changed by varying the resistance of the rheostat, the magnetic flux in the surrounding region changes. Since the secondary coil is in magnetic field of the primary, the changing flux also links with the secondary. This causes an induced emf in the secondary.

The phenomenon in which a changing current in one coil induces an emf in another coil is called mutual induction.

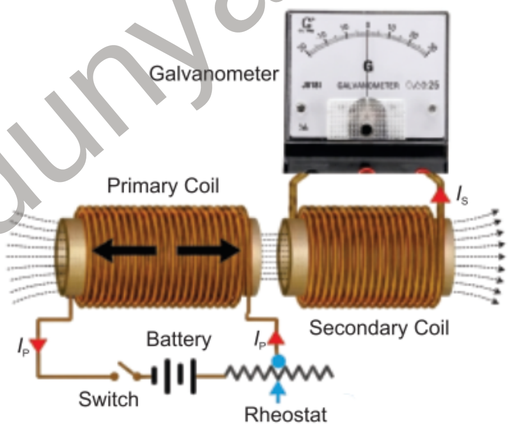


Figure (17.20): Circuit diagram to find Mutual induction between two coils.

According to Faraday's law, the emf induced in the secondary coil ε_s is proportional to the rate of change of flux $\Delta\phi_s/\Delta t$ passing through it and is given by

$$\varepsilon_s = -N_s \frac{\Delta\phi_s}{\Delta t} \text{.........(17.39)}$$

where N_s is the number of turns in the secondary coil.

Let ϕ_s represents flux passing through the secondary coil. The net flux passing through the secondary coil of N_s loops is $N_s\,\phi_s$. As this net flux is proportional to the magnetic field produced by the current I_p in the primary and the magnetic field itself is proportional to I_p, therefore

$$N_s\phi_s \propto I_p$$

$$N_s\phi_s = M I_p \text{............ (17.40)}$$

where M is proportionality constant and is called Mutual inductance of the two coils, given by

$$M = N_s \frac{\phi_s}{I_p}$$

The mutual inductance M depends upon the following factors,

1. Number of turns of both the primary and secondary coils.
2. Area of cross-section of the two coils.

3. Magnetic permeability of medium between the coils.
4. Nature of material on which two coils are wound.
5. Distance between two coils.
6. Orientation between the primary and secondary coils.

By Faraday's law, the emf in the secondary coil is given by the rate of change of flux through the secondary coil *r* as:

$$\varepsilon_s = -N\frac{\Delta\phi_s}{\Delta t}$$

Using Eq. (17.40), we have:

$$\varepsilon_s = -\frac{\Delta N_s\phi_s}{\Delta t}$$

$$\varepsilon_s = -\frac{\Delta M I_p}{\Delta t}$$

$$\varepsilon_s = -M\frac{\Delta I_p}{\Delta t} \quad \text{.............(17.41)}$$

Which shows that emf induced in the secondary coil is proportional to time rate of change of current in the primary. The negative sign indicates the fact that the induced emf is in such a direction that it opposes the change of current in the primary coil. The magnitude of mutual induction M is

$$M = \frac{E_s}{\frac{\Delta I_p}{\Delta t}} \quad \text{................(17.42)}$$

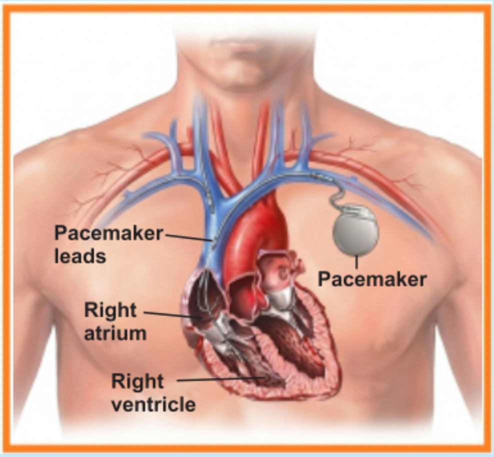
Mutual induction can be defined as ratio of average emf induced in the secondary to the time rate of change of current in the primary.

The SI unit of mutual inductance M is V s A^{-1}, which is called henry after Joseph Henry. One henry is the mutual inductance of the pair of coils in which the rate of change of current of one ampere per second in the primary causes an induced emf of one volt in the secondary.

Mutual inductance finds many applications in various devices such as transformers, electric motors, and generators etc. It also plays a key role in digital signal processing and is utilized in devices like pacemakers and metal detectors.

Brain teaser

Do you have an idea how a pacemaker works?



QUESTIONS

Multiple Choice Questions

Choose the correct answer.

17.1 Name of device that is used as a part of charging circuit for mobile phones is:
(a) full-wave bridge rectifier (b) half-wave rectifier
(c) AC generator (d) battery

17.2 A 10 μF capacitor is plugged into a 110 V_{rms}, 60 Hz voltage source, with an ammeter in series. The value of I_{rms} that passes through the capacitor is:
(a) 0.0415 A (b) 0.415 A (c) 415 A (d) 0.0415 Ma

17.3 In an RC-circuit a 10 kΩ resistance is connected in series to a capacitor of 0.05μF. The applied voltage for charging is 36 V, the RC-time constant to charge capacitor is:
(a) 0.0578 ms (b) 0.009 ms (c) 0.549 ms (d) 5.49 ms

17.4 In a purely inductive or capacitive circuit, the average power consumed is:
(a) equal to apparent power (b) minimum
(c) maximum (d) zero

17.5 Through which of the AC-circuit elements both emf and current are in phase?
(a) resistance (b) inductor (c) capacitor (d) LED

17.6 An inductor with a reactance of 120 Ω, a capacitor with a reactance of 150 Ω and a 24 Ω resistance are connected in series across a 220 V source. When the circuit is at resonance, then voltage across the inductor is:
(a) 1030 V (b) 1.1 KV (c) 1200 V (d) 11 kV

17.7 If magnetic field is doubled in an inductor, then, magnetic energy density becomes:
(a) 6 times (b) half (c) 4 times (d) constant

17.8 RC-time constant is measured in:
(a) ohm (b) henry (c) volts (d) second

17.9 Resonance frequency of RLC-series circuit is f_r. If the capacitance is made 4 time the initial value, then the resonance frequency becomes:
(a) half (b) one third (c) twice (d) four times

17.10 The magnitude of mutual inductance M between two coils when current changes at 20 A s^{-1} in one coil induces an emf of 50 mV in the other is:
(a) 5 mH (b) 2.5 mH (c) 0.0025 μH (d) 25 mH

Short Answer Questions

17.1 What impacts does resistance have on capacitance?

17.2 What happens when an AC line touches a DC line?

17.3 What is the difference between peak to peak and amplitude in case of sine wave?

17.4 Why are RC circuits used in timing circuits?

17.5 Why are choke coils preferred over resistors for limiting current in AC circuits?

17.6 Define rms values of voltage and current with their phasor diagrams.

17.7 Resonance in LRC-circuits has various applications, give any four of them.

Constructed Response Questions

17.1 What do you mean by rating of electrical appliances? Explain briefly.

17.2 The energy stored in an inductor is analogous to the kinetic energy of a moving mass? Justify.

17.3 What are safety measures to take when working with AC or DC?

17.4 When the frequency is at "off resonance" in LCR-series circuit, the voltages across the inductor and capacitor can be significantly larger than the source voltage? Justify.

17.5 How does self-induction manifest when a current is switched on or off in a circuit?

17.6 Is it possible to achieve mutual inductance using a combination of four coils? If yes, justify your answer.

17.7 The currents of the order of 0.1 A through human body is fatal what causes death; heating due to current or something else?

Comprehensive Questions

17.1 Explain the behaviour of a capacitor is an AC circuit. Derive the expression for capacitive reactance and discuss why current leads the voltage by 90° or $\pi/2$ rad.

17.2 Explain the behaviour of an inductor is an AC circuit. Derive the expression for inductive reactance and explain why current lags behind the voltage by 90° or $\pi/2$ rad.

17.3 Define RLC-parallel circuit. Also write its key properties.

17.4 Find expression for impedance in case of RC and RL-series circuits.

17.5 Define full-wave rectification? Draw circuit, input and output signal phasor diagrams to explain how full-wave rectification is achieved by use of full wave bridge rectifier.

17.6 State and explain in detail RLC-series circuit?

Numerical Problems

17.1 AC circuit configure by a resistance of 20 Ω , capacitor of capacitance 40 μF is

connected to an AC supply of 110 V having frequency 50 Hz. Calculate:
(a) capacitive reactance. (b) current in the circuits (c) suggest phase between the voltage and current in a capacitive circuit.

Ans: (a) 79.58 Ω (b) 1.34A (c) Current leads voltage by 90° or π / 2 rad.

17.2 An alternating current I is represented by the equation; $I = 420 \sin(100\,\pi t)$.Find **(a)** the rms current **(b)** the frequency

Ans: (a) 296.94 A (b) = 50 Hz

17.3 A 50 Hz AC of peak value 1 A flows through primary coil of a transformer. If the mutual inductance between the primary and secondary be 1.5 H, then find the mean value of induced voltage. **Ans:** 3000 V

17.4 A 220 V AC source of frequency 2 kHz is connected across a capacitor of 10 μF. Find out: (a) the reactance X_L of capacitor (b) the current in the circuit.

Ans: (a) 7.96 Ω (b) 27.63 A

17.5 In a coil of inductance 150 mH, a current changes steadily from 50 mA to 30 mA in 164 ms, find: (a) the magnitude (b) direction of induced emf.

Ans: (a) 18.29 mV (b) same direction as the original current.

17.6 A choke coil is needed to operate a lamp at 160 V_{rms} and 50 Hz. The lamp has an resistance of 5 Ω when running at 10 A_{rms}. Calculate the inductance of the choke coil. **Ans:** 48.4 mH