بِسۡمِ اللّٰہِ الرَّحۡمٰنِ الرَّحِیۡمِ

(In the Name of Allah, the Most Merciful, the Most Compassionate.)

PHYSICS 12

PECTAA
Transformation, Innovation and Excellence

PUNJAB EDUCATION, CURRICULUM, TRAINING AND ASSESSMENT AUTHORITY

This textbook is based on Updated / Revised National Curriculum of Pakistan-2023 and has been approved by the Punjab Education, Curriculum, Training and Assessment Authority (PECTAA).

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Experimental Edition

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Chapter 13

Thermal Physics

Students Learning Outcomes

After studying this chapter, the students will be able to:

- Explain how molecular movement causes the pressure exerted by a gas.
- Derive and use the relationship $PV = \frac{1}{3}Nm<v^2>$
  [where $<v^2>$ is the mean-square speed (a simple model considering one-dimensional collisions and then extending to three dimensions using $\frac{1}{3}<v^2> = <v_x^2>$ is sufficient)
- Calculate the root-mean-square speed of an ideal gas.
- Derive and use the formula for the average translational kinetic energy of a gas
- Illustrate that the model of ideal gasses is used a base from which the field of statistical mechanics emerged [and has helped explain the behavior of 'non-ideal' gasses through modifications to the model e.g. the behavior of stars]

Thermal physics is an important area of study which deals with the relationship between heat, work, temperature, and other forms of energy. The laws of thermodynamics describe the mechanism of the change of energy in a system and the system can perform useful work on its surroundings. A large system containing many atoms or molecules is called a macroscopic system, and a system consisting of a single atom or molecule is called a microscopic system. Macroscopic systems have properties such as temperature and pressure, these are thermal properties of the whole system. They can be observed and studied without reference to the molecular nature of matter. Microscopic systems have properties such as kinetic energy and momentum. In thermal physics, we deal with large number of particles of the order of Avogadro's number. These particles may be atoms or molecules in gases, liquids and solids. We can extend it to the electron motion in metals and neutrons in neutron stars.

There are number of applications of the kinetic theory of gases in daily life such as in automobile engines, turbines, pumps, air conditioners, production of food, and environment etc. These diverse applications make thermal physics an important area.

Thermal physics is an area which includes the knowledge of statistical mechanics, and kinetic theory of gases. In order to study thermal physics, the knowledge of statistical mechanics, and kinetic theory of gases is needed.

13.1 BROWNIAN MOTION

Brownian motion is the random and irregular motion of molecules in a gas. In 1927, Robert Brown, a botanist observed under microscope that tiny particles of pollens plant were moving around in the liquid randomly. These particles were identified as dust particles. Later, it was proved to be one of the effects of molecular motion.

A molecule in a gas changes its path after collision with another molecule. When it keeps on colliding with other molecule, the colliding molecule follows a random or zig-zag motion. In fact, collision transfers or exchanges the momentum and energy between the molecules (Fig 13.1).

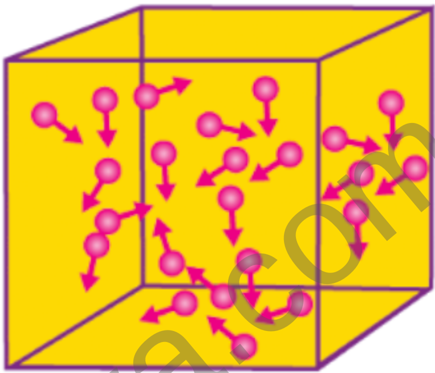

Fig. 13.1: **Particles are in random motion**

Brownian motion describes randomness and chaos, therefore, it represents one of the simple models of randomness. There are various reasons and causes of this motion which are given as under:

1. The size of the molecules is inversely proportional to the speed of the motion. We know that the transfer of momentum is inversely proportional to the mass of the particles. Lighter particles obtain greater speeds from collisions.
2. The speed of the particles is inversely proportional to the viscosity of the fluid. Low viscosity of the fluid results in the faster Brownian movement.

> **For your information**
>
> Albert Einstein explained the pollen movement in a liquid assisted by the molecules in 1905. In 1908, a French physicist J Perrin experimentally verified Einstein's explanation which earned him the 1926 Nobel Prize in physics.

3. Viscosity describes the magnitude of the internal friction in a fluid. It represents the resistance to flow of the fluid.
4. Brownian movement causes the molecules/particles in a fluid to be in constant motion.

13.2 KINETIC THEORY OF GASES

The kinetic theory of gases describes the molecular composition of the gas and their motion. In this theory, the gas pressure arises due to particles colliding with each other and the walls of the container. It also defines properties such as temperature, volume and pressure, as well as transport properties such as viscosity and thermal conductivity and diffusivity.

The kinetic molecular theory applies to the ideal gases and it has been defined in the previous class. However, the basic assumptions are:

1. Gases consist of very large number of tiny spherical particles that are far apart from one another compared to their sizes.
2. Gas particles are in constant rapid motion in random directions.

> **Do you know?**
>
> A small 'cube' of air can have as many as 10^{20} molecules. Volume of the gas particles is negligible compared to the volume of the empty spaces.

3. Collisions between gas particles and the container walls are perfectly elastic collisions.
4. There are no forces of attraction or repulsion between gas particles until the particles perform collisions with each other.

For your information

The mean free path is average distance a moving particle covered before it undergoes a collision that significantly alters its direction or energy.

5. The average kinetic energy of gas particles is dependent upon the temperature of the gas.

13.3 PRESSURE IN A GAS AND DERIVATION OF $PV = \frac{1}{3}Nm\langle v^2\rangle$

We can use the kinetic theory of the gas to derive an equation which relates the macroscopic properties of a gas (pressure and volume) to the microscopic properties of its molecules (mass and speed). It is assumed that a single molecule is moving in a cube-like box of each side L (Fig.13.2). This molecule has mass m, and is moving with speed "v". It moves back and forth, colliding at regular intervals with the walls and therefore, it contributes to the pressure of the gas. We are going to work out the pressure exerted by this one molecule on the wall of the box and then deduce the total pressure produced by all the molecules.

According to the kinetic theory of gases, a gas consists of a large number of tiny particles (atoms or molecules) that are in constant random motion. These particles move freely in all directions and frequently collide with one another and with the walls of the container. When gas molecules strike the walls, they exert a force on them due to the change in momentum during each collision. The pressure of a gas is defined as the force exerted per unit area of the container walls as a result of these collisions. Since the collisions are elastic, no energy is lost, and the average kinetic energy of the molecules is directly proportional to the absolute temperature of the gas i.e., $\langle K.E.\rangle \propto T$. Thus, an increase in temperature increases the speed of the molecules, leading to more collisions, and consequently, a higher pressure.

Brain teaser

What is the difference between mole (gram-molecule) and Avogadro's number? Does Avogadro's number depend on state of the substance?

The ideal gas obeys the gas law which would relate the gas temperature with other parameters and the gas law is:

$$PV = nRT \quad \ldots\ldots\ldots\ldots\ldots (13.1)$$

where P is the pressure, n is number of moles, R is general gas constant equals to $N_A k_B$, $k_B = 1.38 \times 10^{-23}$ J K^{-1} is the Boltzmann constant, ($N_A = 6.02 \times 10^{23}$ molecules) is the Avogadro number and T is the temperature of the gas. Therefore,

$$PV = nN_A k_B T$$

The number of moles in terms of number of particle N is defined as; $n = \frac{N}{N_A}$, then

$$PV = Nk_B T \quad \ldots\ldots\ldots\ldots\ldots (13.2)$$

As pressure is force per unit area, exerted on walls of the container, therefore, each time a gas particle (molecule or atom) collides with walls of the container, and exerts a force on the walls of the container. This force spreads over whole wall area of the container.

Consider a collision in which the molecule with velocity v strikes one face of the cube along x-axis. It rebounds elastically in the opposite direction having velocity $-v$, its momentum changes from mv to $-mv$.

Change in momentum along x-axis;

$$\Delta p = -mv_x - (mv_x) = -mv_x - mv_x = -2mv_x$$

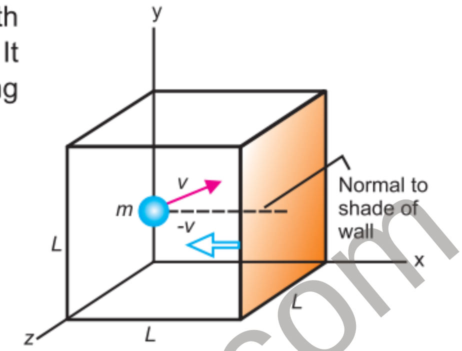

Fig. 13.2: Molecule moving with velocity v

The molecule travels a distance $2L$ between the consecutive collisions with the same wall of the cube. Hence, time between two consecutive collisions with one side of the cube is:

$$\frac{2L}{v_x} \quad \text{.........(13.3)}$$

The average force exerted by the wall of cube on the molecule can be found using Newton's second law of motion. We know that force is equal to the rate of change of momentum. The area of the wall is L^2. Thus,

$$F = \frac{\text{Change in momentum}}{\text{Time}} = \frac{|\Delta p|}{\Delta t} = \frac{+2mv_x}{\frac{2L}{v_x}} = \frac{mv_x^2}{L} \quad \text{..........(13.4)}$$

This expression represents the force exerted on the container wall by one molecule in the x-direction. The positive sign in the change of momentum is due to the force of the molecule by the wall which is in opposite direction to the change in momentum as a result of the Newton's third law of motion.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{F}{L^2}$$

Using equation, $$P = \frac{\frac{mv_x^2}{L}}{L^2}$$

Brain teaser

How does the Kinetic theory of gases relate to the Boyle's law and Charles' law?

$$P = \frac{mv_x^2}{L^3} = \frac{mv_x^2}{V} \quad \text{..........(13.5)}$$

This is the pressure exerted on container wall by one molecule.

The total force, we must add up the contributions of all the molecules that strike the wall, allowing for the possibility that they all have different speeds. Dividing the magnitude of the total force F_x along x-axis by the area of the wall (L^2), then gives the pressure P on that wall:

$$P = \frac{m}{V}(v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \text{........}) \quad \text{..........(13.6)}$$

where v_{1x}^2 is the velocity of particle 1 in the x-direction, and v_{2x}^2 the velocity of particle 2 in the x-direction and so on. If there are N number of particles in the container, then total mass is the number of particles times mass of the particle (i.e. $M = mN$).

As we know that velocity striking the wall is v_x and after striking the wall it is $-v_x$ which would result in average velocity zero. This means simple average does not work here. In order to avoid this zero, we need to do the square of the velocities before averaging.

$$< v_x^2 >= \frac{v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \dots\dots\dots + v_{Nx}^2}{N} = \frac{\sum_{i=1}^{N} v_{ix}^2}{N}$$

If we take the square root of the average square of the velocities, it is known as the root mean square (rms) velocity and is written as:

$$(v_{rms})_x = <\sqrt{v_x^2}> \quad \dots\dots\dots(13.8)$$

Since the molecules are moving randomly in all directions, only one-third of their total velocity will be directed along any one Cartesian axis. So,

$$<v^2>=<v_x^2>+<v_y^2>+<v_z^2> \quad \dots\dots\dots(13.9)$$

Since gas molecules are moving randomly and have the same average speed in all three directions. This means that;

$$<v^2>=<v_x^2>+<v_y^2>+<v_z^2>=3<v_x^2> \quad \dots\dots\dots(13.10)$$

Total average velocity is:

$$<v_x^2>=\frac{<v^2>}{3} \quad \dots\dots\dots(13.11)$$

The pressure is written as:

$$P=\frac{N}{3V}m<v^2> \quad \dots\dots\dots(13.12)$$

Then $$PV=\frac{1}{3}Nm<v^2> \quad \dots\dots\dots(13.13)$$

This is the pressure of the gas molecules on the wall. The pressure in terms of density can be written as:

$$\rho = \frac{\text{Total mass}}{\text{Volume}} = \frac{M}{V}$$

Rearranging the pressure, Eq. (13.13) for P, and substituting the density

$$PV=\frac{1}{3}\rho<v^2> \quad \dots\dots\dots(13.14)$$

This is the pressure of the gas which depends on the density and root mean square of velocity.

Since $$\rho=\frac{mN}{V}$$

$$P=\frac{mN}{3V}<v^2> \quad \text{or} \quad PV=\frac{1}{3}Nm<v^2>$$

$$P = \frac{2}{3}\frac{N}{V} < \frac{1}{2}mv^2 >$$

$$P = \frac{2}{3}N_o < \frac{1}{2}mv^2 > \text{ (Here } \frac{N}{V} = N_o \text{)}$$

$$P = \frac{2}{3}N_o < K.E._T >$$

Example 13.1 An ideal gas has a density of 4.5 kg m^{-3} at a pressure of 9.3×10^5 Pa and a temperature of 504 K. Determine the root-mean-square speed of the gas atoms.

Solution $\rho = 4.5$ kg m^{-3} , $P = 9.3 \times 10^5$ Pa , $v_{rms} = ?$

Pressure is defined as; $P = \frac{1}{3}\rho < v^2 >$

or $$< v^2 > = \frac{3P}{\rho}$$

$$< v^2 > = \frac{3 \times (9.3 \times 10^5 \text{ Pa})}{4.5 \text{ kg m}^{-3}} = 6.2 \times 10^5 \text{ m}^2 \text{ s}^{-2}$$

To find the rms value, take the square root of the mean-square-speed, thus

$$v_{rms} = \sqrt{< v^2 >} = \sqrt{6.2 \times 10^5 \text{ m}^2 \text{ s}^{-2}} = 787 \text{ m s}^{-1} \approx 7.9 \times 10^2 \text{ m s}^{-1}$$

Example 13.2 What is the rms speed of air molecules (O_2 and N_2) at room temperature e.g. 25 °C?

Solution Mass of oxygen O_2 = 32 u and Mass of nitrogen N_2 = 28 u

1u = 1.66×10^{-27} kg; $k_B = 1.38 \times 10^{-23}$ J K^{-1} ; T = 25 °C = 298 K

Mass of oxygen O_2 = $32 \times 1.66 \times 10^{-27}$ kg = 5.3×10^{-26} kg

Mass of Nitrogen N_2 = $28 \times 1.66 \times 10^{-27}$ kg = 4.6×10^{-26} kg

The rms speed of oxygen is:

$$v_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3 \times (1.38 \times 10^{-23} \text{ J K}^{-1}) \times 298 \text{ K}}{5.3 \times 10^{-26} \text{kg}}} = 481 \text{ m s}^{-1} \approx 4.81 \times 10^2 \text{ m s}^{-1}$$

Similarly, the rms speed of nitrogen is $v_{rms} = 515$ m $s^{-1} \approx 5.15 \times 10^2$ m s^{-1}

The speed of air molecules is much greater than the speed of sound which is about 340 m s^{-1}.

13.4 AVERAGE TRANSLATIONAL KINETIC ENERGY OF A GAS

We have calculated the pressure of the gas molecule with n (=N/V) molecules by using the gas law $PV = (nRT)$ and kinetic theory.

$$PV = \frac{N}{3}m < v^2 >$$

However, M/m is simply Avogadro's number i.e. $N_A = M\,m^{-1}$. The gas constant R is related with Boltzmann constant and Avogadro number. $k_B = R/N_A$

Equating these equations, we have

$$nRT = \frac{N}{3}m\langle v^2\rangle = \frac{N}{3}mv_{rms}^2 \quad \text{..........(13.15)}$$

$$mv_{rms}^2 = \frac{3nRT}{N}$$

where $\frac{N}{n} = N_A$ is the number of molecules in one mole and it is Avogadro number.

Boltzmann factor $$mv_{rms}^2 = \frac{3RT}{N_A}$$

or $$\frac{1}{2}mv_{rms}^2 = \frac{3RT}{2N_A}$$

or $$v_{rms}^2 = \frac{3RT}{mN_A}$$

$$v_{rms} = \sqrt{\frac{3RT}{mN_A}} \quad \text{............(13.16)}$$

We rewrite the pressure equation as

$$\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}k_BT \quad \text{............(13.17)}$$

In this equation, $1/2mv_{mrs}^2$ is the average translational kinetic energy and it is denoted by <K.E.>. Therefore, the above equation is written as:

$$P = \frac{2}{3}\langle K.E._T\rangle$$

$$\langle K.E._T\rangle = \frac{1}{2}mv_{rms}^2$$

For your information

Kinetic theory helps to understand the interaction between pollutants and the atmosphere. This knowledge is used to develop the systems to monitor the quality of air.

Substituting v_{rms}^2 value in Eq. (13.17)

$$\frac{2}{3}\left(\frac{1}{2}mv_{rms}^2\right) = k_BT \quad \text{............(13.18)}$$

or $$\langle K.E._T\rangle = \frac{3}{2}k_BT$$

The average translational kinetic energy of the molecules in a gas is given by a simple constant times the temperature. So, if this model is accurate, the temperature of a gas is a direct measure of the average translational kinetic energy of its molecules.

For an air molecule at room temperature (300 K), the quantity k_BT is (1.38×10^{-23} J $K^{-1} \times$ 300 K) = 4.14×10^{-21} J. For such a small quantity, we use electron volt instead of joule (J). The electron volt is the kinetic energy of an electron that has been accelerated through a voltage difference of one volt: 1 eV = 1.6×10^{-19} J.

Boltzmann constant is 8.62×10^{-5} e V K^{-1}, so at room temperature, value of $k_B T = (8.62 \times 10^{-5}$ e V $K^{-1})(300$ K$) = 0.026$ e V.

Daily Life Examples of Kinetic Theory of Gases

Examples from daily life are the popcorn and pressure in tyre of a car. When popcorn kernel is heated, the moisture trapped in popcorn kernel becomes steam. The pressure in the kernel is increased and it ruptures the corn cover releasing the gelatinous starch. It becomes solid after cooling.

When air is pumped into the tyres of a car, the number of air molecules increases which raises the air pressure inside the tyres. We know that more molecules result in frequent molecular collisions with tyre's walls due to limited volume. The pressure gives the tyre its hardness and bears the car's weight without deflating.

Fig. 13.3

13.5 KINETIC THEORY AND STATISTICAL PHYSICS

So far we know that kinetic theory of gases is used to determine the pressure of the ideal gas using the gas laws assuming the thermal or random motion of molecules. For large number of molecule, we use statistical physics. It is the branch of physics that uses probability theory. Accordingly, atoms and molecules of a system may exist in different energy states (levels) E_1, E_2, E_3, ……,etc due to different speeds.

For your information

In 1860 James Clark Maxwell (1831–1879) derived an expression that describes the distribution of molecular speeds within the system at thermal equilibrium. About 60 years later, experiments were performed to confirm Maxwell's predictions.

Boltzmann derived the Boltzmann kinetic equation. This equation describes the dynamic processes in gases having large number of molecules. Boltzmann constant is also related to the population of atoms in two levels.

$$\frac{N_2}{N_1} = e^{-\Delta E / k_B T} \quad \ldots\ldots\ldots\ldots(13.19)$$

where $(e^{-\Delta E / k_B T} =$ Boltzmann factor$)$ N_1 and N_2 are the population of lower and higher energy states, $\Delta E = (E_2 - E_1)$ is the energy difference between these states. The number density is directly proportional to the pressure. This equation is known as the **Boltzmann distribution law** and is important in describing the statistical mechanics of a large number of molecules. It states that the probability of finding the molecules in a particular energy state varies exponentially as the negative of the energy divided by $k_B T$; therefore, more particles reside in lower energy states than in higher ones.

In a gas container, a molecule undergoes billions of collisions every second. Each collision changes the speed of molecule thereby changing the kinetic energy, but kinetic theory concludes that average kinetic energy at temperature T is:

$$E_{av} = \frac{3}{2} k_B T \quad \ldots\ldots\ldots\ldots(13.20)$$

where k_B is effectively the gas constant per molecule.

13.6 STELLAR EVOLUTION

The kinetic theory of gases, when extended to astrophysical systems like stars and galaxies, helps us understand the evolution of stars in a galaxy or gas atoms in a stellar atmosphere.

Stellar evolution deals with the star changes over the time. Star can have range of lifetime from a few million years to trillions of years. The life of a star depends on the mass of the star. The most massive have longer life which is even longer than the current age of the universe. All stars are formed from the clouds of gas and dust which are often called nebulae or molecular clouds. This is also called a proto-star. As the cloud contracts, its density and temperature increase due to the rise in K.E. of particles (K.E. $\propto T$). After a millions of years, these proto-stars can become a star having achieved a state of equilibrium. The rms velocity of gases in the star increases with rise of density. This in turn increases the average kinetic energy and finally temperature increases with time. Figure 13.4 represents the competition between gravity and pressure.

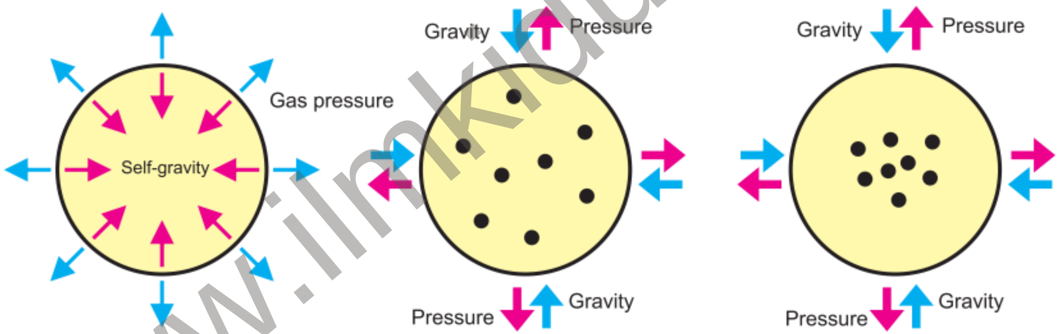

Figure 13.4: Gravity compression is balanced by pressure outward. Greater gravity compresses the gas, making it denser and hotter

As the temperature increases, the internal pressure also rises. A stable star is formed when the inward gravitational force is exactly balanced by the outward pressure. This condition is described by the hydrostatic equation; $\frac{\Delta P}{\Delta r} = -\frac{GM_r \rho_r}{r^2}$.

This equation ensures that the star neither collapses under gravity nor expands indefinitely.

P is the pressure inside the star (force per unit area), $\Delta P/\Delta r$ is the rate of change of pressure with radius (how pressure changes from the centre outward), r is distance from the centre of the star, G is gravitational constant (6.67×10^{-11} N m^2 kg^{-2}), M_r is mass enclosed within radius r, ρ_r is density at radius r, and negative sign indicates that pressure decreases outward, while gravity pulls inward. There are two possibilities if the balance between pressure force and gravitational force is not maintained.

(i) Gravitational force > Internal pressure (ii) Gravitational force < Internal pressure

As the temperature within a star's core rises, nuclear fusion is initiated, allowing hydrogen nuclei to combine and form helium. This process releases an immense amount of energy, which sustains the star and powers it for the majority of its lifetime. The continuous energy production not only maintains equilibrium against gravitational collapse but also leads to a gradual increase in internal temperature. As a consequence, the star begins to expand.

Do you know?

A parsec is a unit of distance that is used by astronomers as an alternative to the light-year, just as kilometres are being used as an alternative to miles. In fact, one parsec is approximately 3.26 light years, or almost 19 trillion miles (31 trillion km)

With further increases in temperature and changes in core composition, the star may expand significantly to become a red giant. At this stage, stars possessing at least about half the mass of the Sun are capable of initiating helium fusion in their cores, producing heavier elements such as carbon and oxygen. In more massive stars, the process continues with the fusion of even heavier elements in successive stages, forming increasingly complex nuclei up to iron.

The final fate of a star depends primarily on its mass. Once a star exhausts its nuclear fuel, it can no longer support itself against gravitational collapse. In low and intermediate mass stars, the core contracts into a dense white dwarf, while the outer layers are expelled into space, forming a planetary nebula. In contrast, stars with masses roughly ten times greater than that of the Sun undergo a dramatic supernova explosion. During this event, the inert iron core collapses under gravity, leading to the formation of dense remnants such as neutron stars or black holes.

Red dwarf stars, due to their low mass and highly efficient fuel consumption, follow a much slower evolutionary pure The universe is not yet old enough for any red dwarf to have completed its life cycle. However, theoretical models indicate that these stars will gradually become hotter and more luminous over time before eventually exhausting their hydrogen fuel and evolving into low mass white dwarfs.

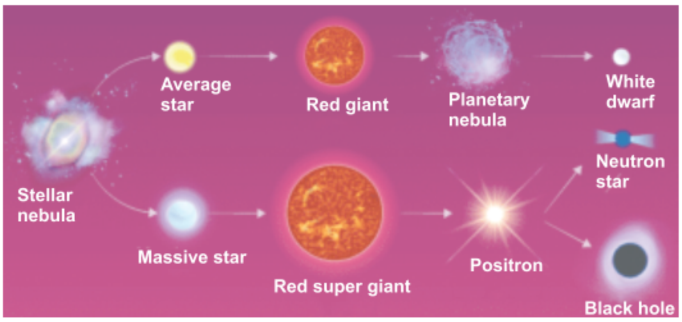

Fig 13.5: Life cycle of a star

Neutron Stars

Neutron stars are among the densest known objects in the universe, second only to black holes. The nearest are many parsecs away, making direct study difficult. Due to their extremely high density, matter inside neutron stars behaves like a degenerate gas and their strong gravitational fields make them challenging to model. According to estimates by NASA, there may be up to a billion neutron stars in the Milky Way galaxy. Many of the neutron stars observed so far are relatively young and rotate rapidly, emitting beams of radiation. These are known as pulsars.

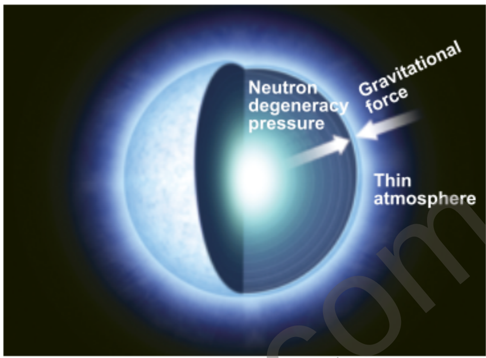

Fig.13.6: Section of neutron star

Scientists believe that pulsar radiation is produced when strong magnetic fields channel matter toward the magnetic poles of neutron stars. When a star collapses to form a neutron star, not only is its mass compressed, but its magnetic field is also greatly intensified. Magnetic fields represented by field lines become stronger as these lines are squeezed closer together during the collapse of the stellar core.

Our Star (SUN)

The Sun is the nearest star to Earth and the main source of heat and light for our planet. It was formed about 4.6 billion years ago from a huge cloud of gas and dust called a solar nebula. Due to gravitational contraction, the temperature at the centre increased and a protostar was formed. When the core became extremely hot, nuclear fusion started in which hydrogen changed into helium and released a large amount of energy. The life stages of the Sun are: nebula, protostar, main sequence, red giant, planetary nebula, and white dwarf. At present, the Sun is in the main sequence stage, where it is producing heat and light by converting hydrogen into helium. In thermal physics, the Sun is important because it is a natural source of enormous heat energy and transfers this energy mainly by radiation.

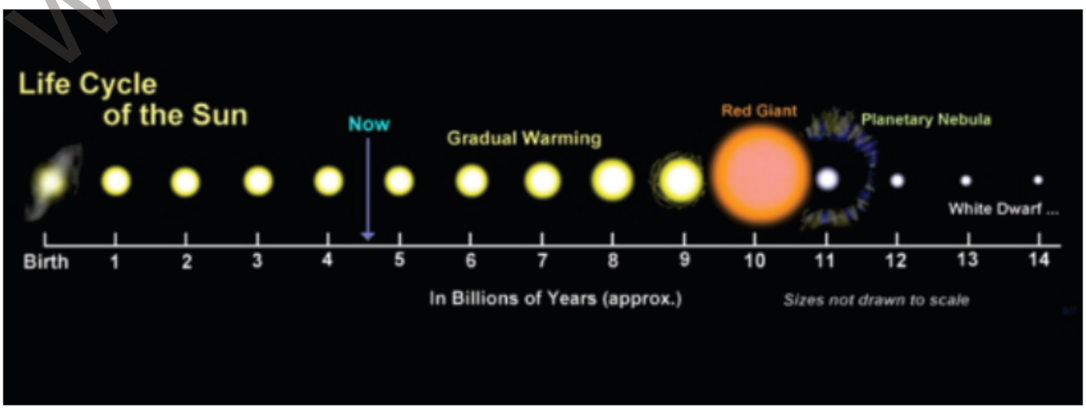

Fig 13.5: Life cycle of the Sun

Example 13.3 A star has mass 2×10^{30} kg and radius 7×10^{8} m. Calculate gravitational pressure and compare it with internal pressure 1.0×10^{14} Pa.

Solution As $P_g \approx \dfrac{GM^2}{R^4}$, therefore,

$$P_g = \frac{(6.67\times 10^{-11}\,\text{Nm}^2\,\text{kg}^{-2})(2\times 10^{30}\,\text{kg})^2}{(7\times 10^{8}\,\text{m})^4}$$

$$= \frac{6.67\times 4\times 10^{49}}{2.401\times 10^{35}} \approx 1.3\times 10^{14}\ \text{Pa}$$

i.e., $P_g = P_{\text{internal}}$

For your information

A neutron star is so dense that one teaspoon of its material would have a mass over 5.5×10^{12} kg, about 900 times the mass of the Great Pyramid of Giza. The entire mass of the Earth at neutron star density would fit into a sphere 305 m in diameter.

As both pressures are nearly equal, therefore, the star is in stable equilibrium.

Example 13.4 A massive star has mass 4×10^{30} kg, radius 5×10^{8} m and internal pressure 1×10^{14} Pa. Check whether the star is stable.

Solution As $P_g \approx \dfrac{GM^2}{R^4}$, therefore,

$$P_g = \frac{(6.67\times 10^{-11}\,\text{Nm}^2\,\text{kg}^{-2})(4\times 10^{30}\,\text{kg})^2}{(5\times 10^{8}\,\text{m})^4} = \frac{6.67\times 16\times 10^{49}}{6.25\times 10^{34}} \approx 1.7\times 10^{15}\ \text{Pa}$$

Gravity is stronger, therefore, the star will contract and may eventually collapse (possible neutron star or black hole formation).

Example 13.5 A star has mass 1×10^{30} kg, radius 1×10^{9} m and internal pressure 5×10^{14} Pa. Determine the state of the star.

Solution As $P_g \approx \dfrac{GM^2}{R^4}$, therefore,

$$P_g = \frac{(6.67\times 10^{-11}\,\text{Nm}^2\,\text{kg}^{-2})(1\times 10^{30}\,\text{kg})^2}{(1\times 10^{9}\,\text{m})^4} = \frac{6.67\times 10^{49}}{10^{36}} \approx 6.67\times 10^{13}\ \text{Pa}$$

Internal pressure dominates, therefore, the star will expand and may evolve into a red giant.

QUESTIONS

Multiple Choice Questions

Choose the correct answer:

13.1 Which condition is necessary for most gases to behave nearly ideally?

(a) Low temperature and low pressure
(b) High temperature and low pressure
(c) Constant temperature and low pressure
(d) High temperature and constant pressure

13.2 v_{rms} of the gas molecule is 300 m s^{-1}. If its absolute temperature is reduced to half and molecular weight is doubled, then v_{rms} will become:
(a) 75 m s^{-1} (b) 150 m s^{-1} (c) 300 m s^{-1} (d) 600 m s^{-1}

13.3 The rms speed of the gas at 27 °C is v. If temperature of the gas is raised to 327 °C, the rms speed of the gas is:
(a) v (b) $v/\sqrt{2}$ (b) $v\sqrt{2}$ (c) $3v$

13.4 Ratio of the rms velocities of O_2 and H_2 at equal temperature will be:
(a) 1:1 (b) 1:4 (c) 2:1 (d) 4:1

13.5 In a closed room, there is a gas with pressure of 3.2×10^5 N m^{-2}. If the gas density is 6 kg m^{-3}, what is the effective speed of each gas particle?
(a) 400 m s^{-1} (b) 300 m s^{-1} (c) 250 m s^{-1} (d) 330 m s^{-1}

13.6 Which statement is not the statement of kinetic theory of gases?
(a) Molecules interact during collisions
(b) Molecules are in continuous random motion
(c) Collisions are of short duration
(d) Molecules are tiny hard sphere undergoing inelastic collisions

13.7 Which equation ensures that a star remains stable against gravitational collapse?
(a) Mass continuity equation (b) Energy gravitational equation
(c) Hydrostatic equilibrium equation (d) Energy transport equation

13.8 What happens to a star when its gravitational force becomes greater than its internal pressure?
(a) The star expands and becomes a red giant
(b) The star contracts and its core temperature increases
(c) The star remains stable with no changes
(d) Nuclear fusion stops immediately

Short Answer Questions

13.1 Do all gases have the same kinetic energy at the same temperature? If yes, give example.

13.2 A vessel is fitted with a mixture of two different gases. Will the mean kinetic energy per molecule of gases be equal? Explain briefly.

13.3 If density of a gas is doubled, keeping all other factors unchanged, what will be the effect on the pressure of the gas?

13.4 A gas enclosed in a container is heated up. What is the effect of pressure on gas molecules?

13.5 The number of molecules of a gas in a container is doubled. What will be the effect on the rms speed?

13.6 Explain briefly, why it is not possible to increase the temperature of the gas while keeping its volume and pressure constant.

13.7 On reducing the volume of a gas at constant temperature, the pressure of the gas increases. Explain briefly on the basis of kinetic theory of gases.

13.8 How does the balance between gravitational force and internal pressure determine whether a start expands or contracts during its evolution?

Constructed Response Questions

13.1 What will be the kinetic energy near absolute zero? Explain.

13.2 What is the average kinetic energy of the oxygen and nitrogen molecules in a room at room temperature?

13.3 Consider a gas enclosed in a sealed container. By what factor does the gas temperature change if: (a) the volume and pressure are both doubled? (b) the volume is halved and the pressure is tripled?

13.4 Is it possible to boil water at room temperature without heating? Explain.

13.5 Explain how the drop of ink in a beaker containing water will behave.

Comprehensive Questions

13.1 Derive the expression of pressure exerted by the gas on the walls of the container.

13.2 Write the expression for rms speed, and most probable speed of a gas molecule.

13.3 Explain the Boltzmann distributions law on the basis of statistical physics.

13.4 Describe the pressure role on the neutron star.

13.5 Show that temperature of a gas is a direct measure of the average translational K.E. of its molecules.

13.6 Explain stellar evolution in terms of the balance between gravitational pressure and internal pressure, and how stellar mass determines whether a star ends as white dwarf, neutron star or black hole.

Numerical Problems

13.1 Calculate the rms velocity of hydrogen molecules at standard temperature and pressure (STP). Density of hydrogen at STP; $\rho = 8.957 \times 10^{-2}$ kg m^{-3}. Density of mercury = 13600 kg m^{-3}, g = 9.8 m s^{-2}. **Ans:** 1.84×10^{3} m s^{-1}

13.2 Determine the pressure of oxygen gas at 0 °C if the density of oxygen is 1.44 kg m^{-3} at STP and rms speed at STP is 456.4 m s^{-1}. **Ans:** 1.0×10^{5} N m^{-2}

13.3 At what temperature will the rms speed of the molecules of gas be three times its value at STP? **Ans:** 2184 °C

13.4 If a gas is at temperature 80 °C and pressure 5×10^{-10} N m^{-2}, having 1 m^{3} volume, what is the number of molecules per cubic metre, where Boltzmann's constant is 1.38×10^{-23} J K^{-1}. **Ans:** 1.02×10^{11} molecules

13.5 The temperature and pressure in the Sun's atmosphere are 2.00×10^{6} K and 0.0300 Pa respectively. Calculate the rms speed of free electrons (mass of the electron = 9.11×10^{-31} kg). **Ans:** 9.5×10^{6} m s^{-1}

13.6 A star has a mass enclosed within radius *r* equal to 2.0×10^{30} kg, density 1500 kg m^{-3} and radius 7.0×10^{8} m. Calculate the gravitational force inside the star.

Ans: 2.94×10^{2} N on mass *m*