Unit 07 Kinematics of Motion in a Straight Line

INTRODUCTION

Kinematics is the branch of mechanics that describes the motion of objects without considering the causes of motion. In this unit, we will explore how objects move along a straight line by studying key physical quantities such as displacement, velocity, and acceleration. Through sketching and interpreting displacement–time and velocity–time graphs, we will develop a visual understanding of motion. The unit also introduces the use of rates of change to analyze how displacement, velocity, and acceleration vary over time. Using differentiation and integration with respect to time, we will solve simple problems related to motion. For situations involving constant acceleration, we will apply appropriate kinematic formulae to determine unknown quantities. Finally, the concepts learned will be applied to real-life situations, such as the motion of vehicles, free fall, vertical projectile motion, relative motion, and the creation of realistic movement in animation.

7.1 Rectilinear Motion

The motion of a particle along a straight line is called rectilinear motion.

7.1.1 Scalars

Scalar quantities are those that are completely described by magnitude only (size or numerical value). Scalars do not have direction.

Examples:

- **Distance:** Total length of the path travelled, regardless of direction.
- **Speed:** Rate at which distance is covered per unit time, irrespective of direction.

7.1.2 Vectors

Vector quantities are described by both magnitude and direction.

Examples:

- **Displacement:** Displacement is change in position of a particle represented by a vector from initial to final position.
- **Velocity:** Rate of change of displacement with respect to time in a specified direction (i.e., speed in a given direction). It is vector quantity and its magnitude gives speed.
- **Acceleration:** Rate of change of velocity with respect to time, including direction.

7.1.3 Displacement, Average Velocity and Acceleration

When a particle undergoes rectilinear motion, its displacement, velocity, and acceleration can have only one of two possible directions, along the line of motion.

In such cases, we do not use arrows or other vector symbols to represent direction; instead, we use positive and negative signs to indicate it:

- One direction along the line is taken as positive.
- The opposite direction is taken as negative.

This simplifies the representation of displacement, velocity, and acceleration in rectilinear motion.

Average Velocity: The average velocity (v) of an object is defined as the total displacement divided by the total time taken:

$$v_{avg} = \text{Average velocity} = \frac{\text{Total displacement}}{\text{Total Time}} = \frac{\Delta x}{\Delta t}$$

Average Speed: The average speed of an object is defined as the total distance travelled divided by the total time taken:

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total Time}}$$

Recall!

a body goes and come back starting position

- Displacement = 0
- Distance 0

Example 2 A man walks 100 meters east in 2 minutes, then turns around and walks 60 meters west in 1 minute. Find his speed and velocity.

Solution: Total distance = 100 + 60 = 160 meters

Total time = 2 + 1 = 3 minutes

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total Time}} = \frac{160}{3} = 53.3 \text{ meters/min}$$

Taking east as positive and west as negative:

Displacement eastward x_1 = 100 meters

Displacement westward x_2 = – 60 meters

Total displacement = $x_1 + x_2$ = 100 – 60 = 40 meters

$$v_{avg} = \text{Average velocity} = \frac{\text{Total displacement}}{\text{Total Time}} = \frac{40}{3} = 13.33 \text{ meters/min (eastward)}$$

Instantaneous Velocity: The instantaneous velocity is the rate of change of displacement with respect to time at a given instant. If $x(t)$ is the displacement of a particle at any time t, then instantaneous velocity $v(t)$ at any time t is given by:

$$v(t) = \frac{dx}{dt}$$

Instantaneous Speed: Instantaneous speed is the magnitude of instantaneous velocity:

$$\text{Instantaneous speed} = |v(t)| = \left|\frac{dx}{dt}\right|$$

Example 2 Let $x(t) = t^3 - 6t^2$ be the position of a particle moving along a straight line at any time t, where x is in meters and t is in seconds. Find the velocity and the speed of the particle at any time t and at $t = 1$ second.

Solution: Given: $x(t) = t^3 - 6t^2$

Differentiate with respect to t:

$$\frac{dx}{dt} = 3t^2 - 12t$$

Therefore, the velocity at any time t is:

$$v(t) = 3t^2 - 12t \quad \ldots (1)$$

Speed at any time t is:

$$\text{Speed} = |v(t)| = |3t^2 - 12t| \quad \ldots (2)$$

Substitute $t = 1$ into equations (1) and (2):

$$v(1) = 3 - 12 = -9 \text{ meter/second and Speed} = |v(1)| = |-9| = 9 \text{ m/s}$$

Acceleration: Rate of change of velocity with respect to time is called acceleration. If $\boldsymbol{v(t)}$ is velocity of a particle at any time $\boldsymbol{t}$, then its acceleration $\boldsymbol{a(t)}$ is given by:

$$a(t) = \frac{dv}{dt}$$

Since $v = \frac{dx}{dt}$, the above expression can also be written as:

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

If velocity $\boldsymbol{v}$ of the particle is given as a function of displacement $\boldsymbol{x}$:

Using the Chain Rule:

$$a = \frac{dv(x)}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$$

But since $v = \frac{dx}{dt}$, we get:

$$a = v\frac{dv}{dx}$$

Units of Acceleration: If displacement is measured in meters and time is in seconds, then unit of acceleration is meters per second square *i.e* m/s^2.

7.1.4 Displacement-Time Graph

Displacement-time graphs are important tools in kinematics, because they provide a visual representation of an object's motion. Here's why we discuss them:

- They give a clear picture of how an object's position changes over time.
- They help us easily determine whether an object is moving, stationary or accelerating.

In a displacement-time graph, the displacement (x) is plotted on the y-axis, and time (t) is plotted on the x-axis.

Key Features of a Displacement-Time Graph

1. Slope of the graph = $\left(\frac{dx}{dt}\right)$, represents velocity of the particle.

- If velocity is uniform i.e.,

  $v = \frac{dx}{dt}$ = Slope of the graph = constant,

  the displacement-time graph is a straight line.

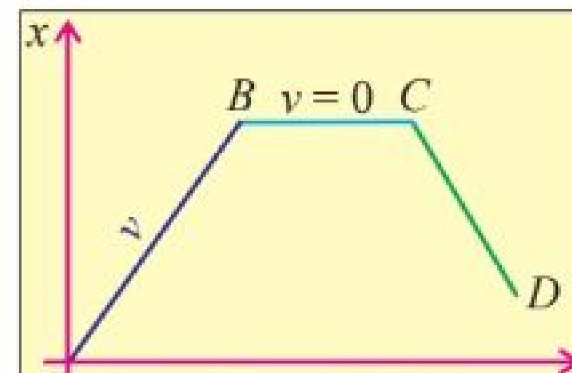

Figure 7.1

  - If $v = \frac{dx}{dt} = 0$, the particle is at rest and the displacement-time graph is a horizontal line, which reflects that displacement remains constant with time. In Figure 7.1, the particle is at rest from point B to point C.

Recall!
- Positive slope forward motion,
- Negative slope backward motion.

  - If $v = \frac{dx}{dt}$ = constant > 0, the particle moves forward, and the displacement-time graph is a rising straight line; that is, displacement increases as time increases. In Figure 7.1, the particle is moving with positive uniform velocity from point A to point B.
  - If $v = \frac{dx}{dt}$ = constant < 0, the particle moves in the opposite direction, and the displacement time graph is a falling straight line; that is, displacement decreases as time increases. In Figure 7.1, the particle is moving with uniform negative velocity from point C to point D.

Example 3 A cyclist rides in a straight line for 20 minutes. He then stops and waits for 30 minutes, after which he returns in a straight line to the starting point in 10 minutes. Figure 7.2 shows the displacement-time graph for his journey.

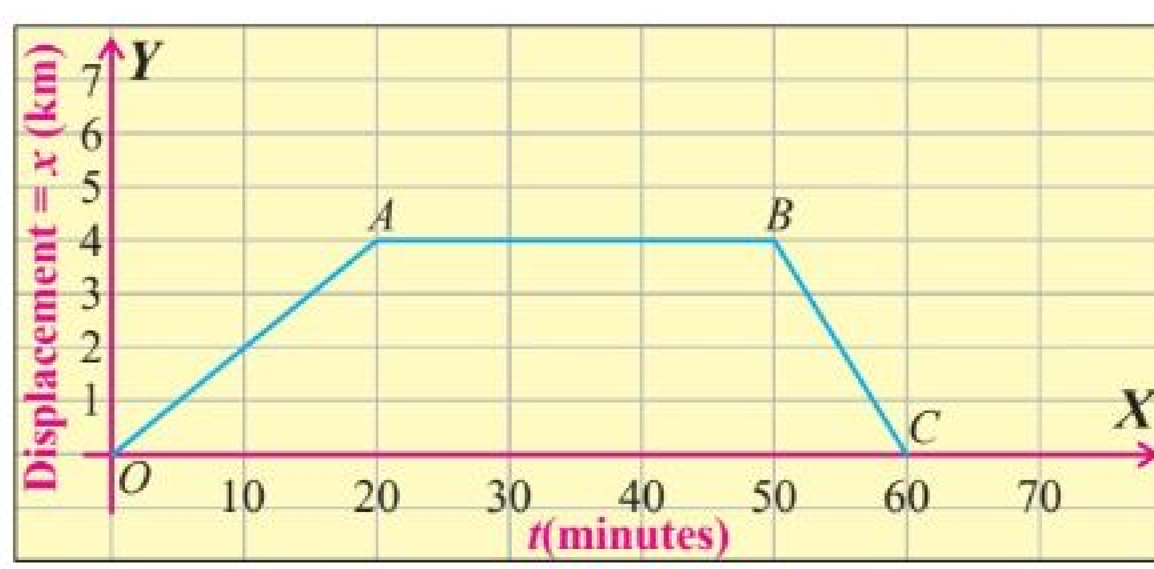

Figure 7.2

(a) Calculate the average velocity for each stage of the journey in km/h.
(b) Calculate the average velocity for the whole journey.
(c) Find the average speed for the whole journey.

Solution: (a) Journey from O to A:

Time = 20 mins, displacement = 4 km

$$\text{Average velocity} = \frac{\text{Displacment}}{\text{Time}} = \frac{4}{20} = \frac{1}{5} \text{ km/min} = 12 \text{ km/h}$$

Journey from A to B:

Time = 30 mins, displacement = 0 km

$$\text{Average velocity} = \frac{\text{Displacment}}{\text{Time}} = \frac{0}{30} = 0 \text{ km/min}$$

$$\Rightarrow 0 \times 60 = 0 \text{ km/h} \quad \Rightarrow \quad \text{object is stationary.}$$

Journey from B to C:

Time = 10 mins, displacement = – 4 km

$$\text{Average velocity} = \frac{\text{Displacment}}{\text{Time}} = \frac{-4}{10} = -\frac{2}{5} \text{ km/min} = -24 \text{ km/h}$$

(b) The displacement for the whole journey is **0**, so average velocity is also **0**.

(c) Total time = 60 mins = 1 h ; Total distance = 4 + 4 = 8 km

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{8}{1} \text{ km/h} = 8 \text{ km/h}$$

Example 4 Javed drives from his home to Murree. He drives for 2 hours at an average velocity of 50 km/h. He then stops for lunch before continuing to Murree. Figure 7.3 shows a displacement-time graph for his journey.

(a) Work out the displacement of Murree from Javed's home.

(b) Work out his average velocity for his whole journey.

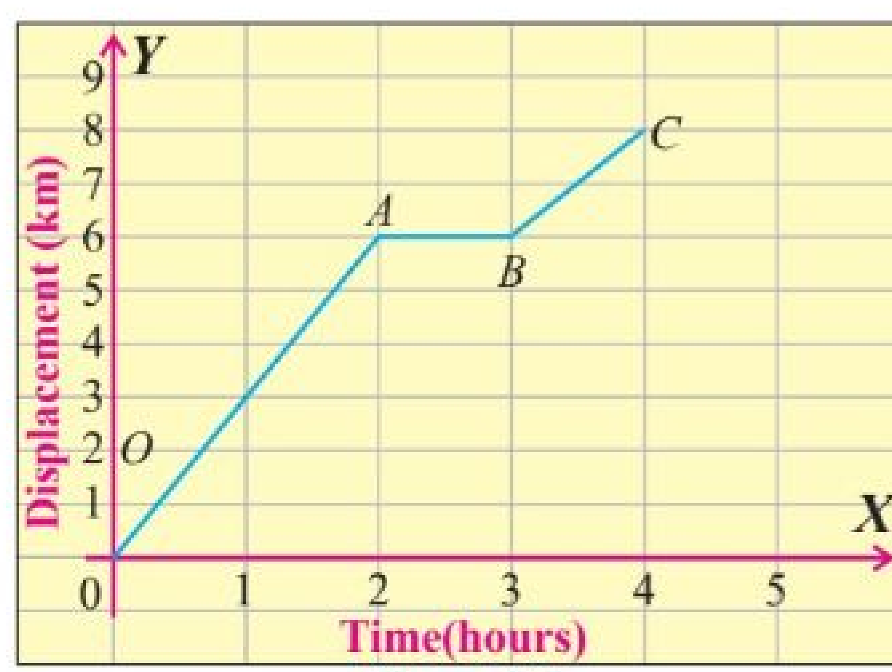

Figure 7.3

Solution: (a) Journey from O to A:

Time = 2 h ; average velocity = 50 km/h

$$\text{Since,} \quad \text{average velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$50 = \frac{\text{Displacement}}{2}$$

Displacement = 50 × 2 = 100 km

Scale on vertical axis from graph:

$$6 \text{ units} = 100 \text{ km}$$

So, $$1 \text{ unit} = \frac{100}{6} = \frac{50}{3} \text{ km}$$

Journey from B to C:

$$\text{vertical rise} = 2 \text{ units}$$

$$\text{Displacement} = 2 \times \frac{50}{3} = \frac{100}{3} \text{ km}$$

$$\text{Total displacement of Murree from Javed's home} = 100 + \frac{100}{3}$$

$$= \frac{400}{3} \simeq 133.3 \text{ km}$$

(b) Average velocity:

Time = 2 h (first drive) + 1 h(lunch stop) + 1 h(second drive) = 4 h

Total displacement = 133.3 km

$$\text{Average velocity} = \frac{133.3}{4}$$

$$\approx 33.33 \text{ km/h}$$

7.1.5 Velocity-Time Graph

Velocity-time-graph helps us visually understand how the velocity of an object changes with time. It is used to:

- Analyze uniform or non-uniform motion
- Determine acceleration
- Calculate displacement
- Study motion in different time intervals clearly and effectively

In a velocity-time graph, the velocity (v) is plotted on the y-axis, and time (t) is plotted on the x-axis.

Key Features of Velocity-Time Graph

1. Slope of the graph $\left(\frac{dv}{dt}\right)$, represents acceleration of the particle.

- If a particle is moving with constant acceleration, its velocity-time graph is a straight line.
  - If $a = \frac{dv}{dt} = 0$, the velocity-time graph is a horizontal straight line. In Figure 7.4, the particle is moving with uniform velocity from point B to point C.

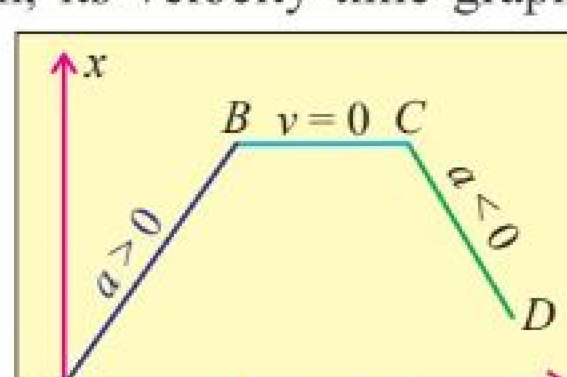

Figure 7.4

- If $a = \frac{dv}{dt}$ = constant > 0, the velocity of the particle is increasing with increase in time, and the velocity-time graph is a rising straight line. In Figure 7.4, the particle is moving with positive uniform acceleration from point A to point B.

Recall!
- Displacement = Area under velocity- time graph
- Acceleration = Slope of velocity time graph

- If $a = \frac{dv}{dt}$ = constant < 0, the velocity of the particle decreases with increase in time, and the velocity-time graph is a falling straight line. In Figure 7.4, the particle is moving with uniform negative acceleration (retardation) from point C to point D.

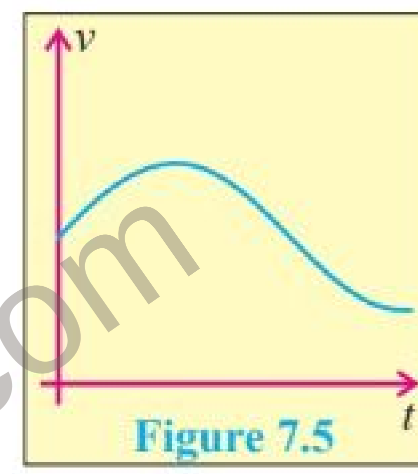

Figure 7.5

- If a particle is moving with variable acceleration, then its velocity-time graph is not a straight line. (See Figure 7.5)

2. If A is the area under the velocity-time graph and above the time-axis from $t = t_1$ to $t = t_2$, then from integral calculus:

$$A = \int_{t_1}^{t_2} v(t)dt$$

Recall!
If $y = f(x) \geq 0$ for $a \leq x \leq b$, then the area under the curve $y = f(x)$, above the x-axis, and between the vertical lines $x = a, x = b$ is equal to $\int_a^b y\,dx$.

Since $v = \frac{dx}{dt}$,

we have:

$$A = \int_{t_1}^{t_2} \frac{dx}{dt} dt = \int_{t_1}^{t_2} dx = x(t_2) - x(t_1)$$

Thus, A = displacement of the particle during the time interval from t_1 to t_2.

Note: If velocity-time graph lies both above and below the time axis, and A_1 is area above time axis and graph, while A_2 is the area below time axis and graph, then:

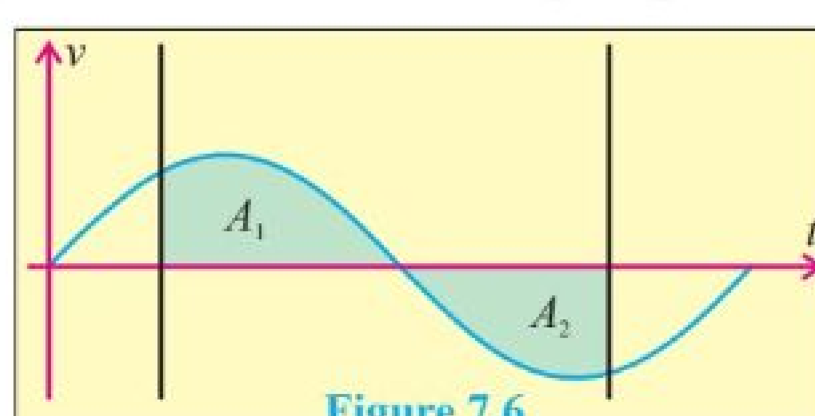

Figure 7.6

Displacement $= A_1 - A_2$

Distance $= A_1 + A_2$

Example 5 Figure 7.7 is the velocity-time graph for a cyclist traveling in a straight line. (The time is in minutes and velocity is in meters per minute).

(a) Calculate the total displacement of the cyclist from his starting position at the end of the 12 minutes.

(b) Determine the uniform deceleration during the final 4 minutes of motion.

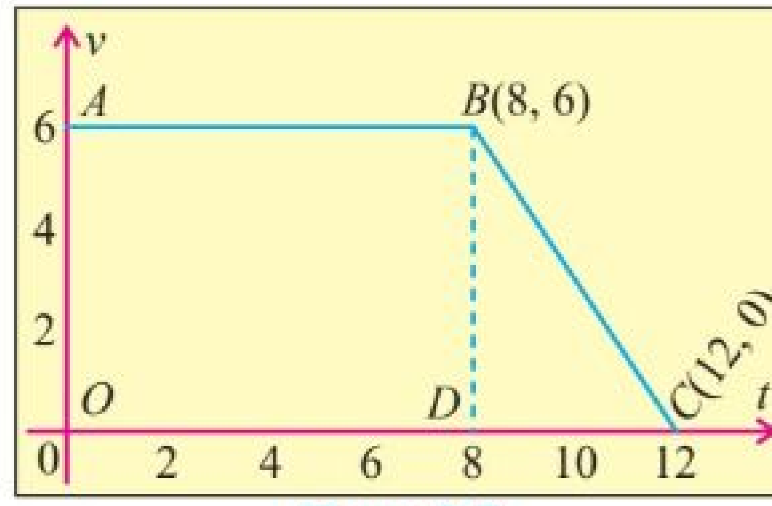

Figure 7.7

Solution: Let x be the total displacement.

The displacement is equal to the area under the velocity–time graph, which is the area of trapezium $OABC$:

$$x = \text{Area of trapezium } OABC = \frac{1}{2}(a+b)h \qquad \ldots(1)$$

Where:

a = length of lower side $\overline{OC} = 12$,

b = length of upper side $\overline{AB} = 8$, and

h = vertical distance between $\overline{OC}$ and $\overline{AB} = 6$

Substitute the values of a, b and h into equation (1):

$$x = \frac{1}{2}(12+8)\times 6 = 60 \text{ meters}$$

(b) Let a be the acceleration during the final 4 minutes of motion, then:

$$a = \text{slope of } \overline{BC} = \frac{0-6}{12-8}$$

$$= -\frac{6}{4} = -\frac{3}{2}\ \text{m/min}^2$$

Area of Trapezium

> **Area of Trapezium**
>
> A trapezium (trapezoid) is a four-sided plane figure that has exactly one pair of parallel sides. If a and b are the lengths of parallel sides and h is the perpendicular distance between them, then the area is given by:
>
> $$\text{Area} = \frac{1}{2}(a+b)\times h$$

Example 6 Ali drives a car 400 km in a straight line. Figure 7.8 is a sketch of a velocity– time graph representing his motion. Ali starts with a velocity of u km/h and accelerates to a velocity of 70 km/h in 3 hours. He then maintains his velocity for 3 hours, after which he slows down uniformly to rest over the next $\frac{1}{2}$ hour. Find:

(a) The value of u

(b) The acceleration of Ali during the first 3 hours of his drive.

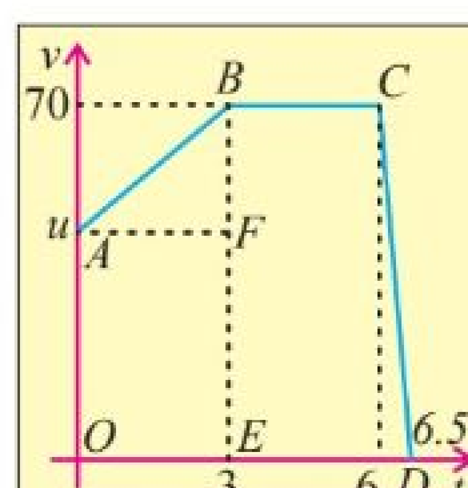

Figure 7.8

Solution: (a) Given that the total displacement is 400 km.

Since the displacement is the area under the velocity-time graph:

$$x = \text{area of trapezium OABE} + \text{area of trapezium BCDE}$$

$$x = \frac{1}{2}(a+b)\times h_1 + \frac{1}{2}(c+d)\times h_2 \qquad \dots (1)$$

Where: $a = |OA| = u$ $\quad b = |BE| = 70$ $\quad h_1 = |OE| = 3$

$c = |BC| = 6 - 3 = 3$ $\quad d = |ED| = 6.5 - 3 = 3.5$ $\quad h_2 = b = 70$

Substitute these values into equation (1):

$$x = \frac{1}{2}(u+70)\times 3 + \frac{1}{2}(3+3.5)\times 70$$

$$= \frac{1}{2}(3u + 210 + 455) \quad = \frac{1}{2}(3u+665)$$

From equation (1) substitute $x = 400$:

$$400 = \frac{1}{2}(3u+665)$$

$$3u = 135 \qquad u = \frac{135}{3} \qquad = 45\,\text{km/h}$$

(b) Let a be the acceleration during the first 3 hours of his drive:

$$a = \text{slope of } \overline{AB} = \frac{|BF|}{|AF|} \qquad \dots (1)$$

Where $|BF| = |BE| - |EF| = 70 - u$, $|BF| = 70 - 45 = 25$, $|AF| = |OE| = 3$

Substitute $|BF| = 25$ and $|AF| = 3$ into equation (1):

$$a = \frac{25}{3} \approx 8.33\ \text{km/h}^2$$

7.1.6 Motion with Constant Acceleration

To study the motion of a particle moving with constant acceleration, we consider four quantities: displacement, time, velocity, and acceleration. Usually, some of these quantities are known and the remaining are to be determined. To relate these quantities, we use the equations of motion. Suppose a particle moves along a straight line with constant acceleration a. At time $t = 0$, the particle is at point O (i.e, $x = 0$) and has initial velocity u. Since acceleration is the instantaneous rate of change in velocity with respect to time, we have:

$$\frac{dv}{dt} = a \quad \Rightarrow \quad dv = a\,dt$$

Integrating both sides:

$$\int dv = \int a\,dt$$

$$v = at + c \text{ (where } c \text{ is the constant of integration)} \qquad \dots(1)$$

Using the initial condition $v = u$ at $t = 0$, substitute into equation (1):

$$u = 0 + c \quad \Rightarrow \quad c = u$$

Substitute back into equation (1):

$$v = u + at \qquad \ldots(2)$$

Since velocity is the instantaneous rate of change of displacement with respect to time:

$$v = \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = u + at \Rightarrow dx = (u + at)dt$$

Integrating both sides:

$$\int dx = \int (u + at)dt$$

$$x = ut + a\frac{t^2}{2} + c_1 \qquad \ldots(3)$$

Using the initial condition $x = 0$ at $t = 0$, substitute into equation (3):

$$0 = 0 + 0 + c_1 \quad \Rightarrow \quad c_1 = 0$$

Substitute back into equation (3):

$$x = ut + \frac{1}{2}at^2 \qquad \ldots(4)$$

Now from equation (2):

$$t = \frac{v - u}{a}$$

Substitute this into equation (4) to eliminate t:

$$x = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

By simplification, we get:

$$2ax = v^2 - u^2 \qquad \ldots(5)$$

The three equations of motion for a particle moving with constant acceleration are:

- $v = u + at$
- $x = ut + \frac{1}{2}at^2$
- $2ax = v^2 - u^2$

These equations are frequently used to describe and analyze the motion of a particle under constant acceleration.

Example 7 A car starts from rest and accelerates uniformly at 3 m/s^2. Find the velocity of the car after 5 seconds.

Solution: Given that $u = 0$ (car stats from vert) $a = 3m/s^2$ $t = 5$, $v = ?$

To find v, we use $v = u + at$

By substituting the values of u, a and t, we get:

$$v = 0 + 3 \times 5 = 15 \text{ m/s.}$$

Example 8 A car accelerates from 10 m/s to 30 m/s in 4 seconds. Find the distance covered during this time.

Solution: Given that $v = 30$ m/s, $u = 10$ m/s, $t = 4$ s.

First, we find its acceleration, using:

$$v = u + at$$

Substitute the known values: $30 = 10 + a \times 4$

To find distance x, we use

$$2ax = v^2 - u^2 \qquad \ldots (1)$$

$$\Rightarrow 4a = 20 \quad \Rightarrow \quad a = \frac{20}{4} = 5 \text{ m/s}^2$$

By substituting the values of u, a and v in (1), we get:

$$2 \times 5x = 30^2 - 10^2 = 800 \quad \Rightarrow \quad x = \frac{800}{10} = 80 \text{meter}$$

Example 9 Find the position and the velocity of a particle moving along a straight line at any time t seconds, given that starts from rest at $t = 0$ and is subject to an acceleration $a(t) = t^2 + \cos t$.

Solution: Given: $a(t) = t^2 + \cos t$

$$\frac{dv}{dt} = t^2 + \cos t \Rightarrow dv = (t^2 + \cos t)dt \Rightarrow \int dv = \int (t^2 + \cos t)dt \quad \boxed{\because a(t) = \frac{dv}{dt}}$$

$$v(t) = \frac{1}{3}t^3 + \sin t + c$$

Using the initial condition $v = 0$ at $t = 0$:

$$0 = 0 + 0 + c \quad \Rightarrow c = 0$$

So, $$v(t) = \frac{1}{3}t^3 + \sin t$$

$$\frac{dx}{dt} = \frac{1}{3}t^3 + \sin t \Rightarrow \int dx = \int \left(\frac{1}{3}t^3 + \sin t\right)dt \quad \boxed{\because a(t) = \frac{dv}{dt}}$$

$$x(t) = \frac{1}{12}t^4 - \cos t + c_1$$

Example 9 $= 0$ at $t = 0$:

$0 = 0 - 1 + c_1 \Rightarrow c_1 = 1$ So, $x(t) = \frac{1}{12}t^4 - \cos t + 1$

Example 10 The acceleration (in m/s^2) of a particle is given by $a(t) = \cos t + \sin t$.

Given that the velocity is $2\,\text{m/s}$ at $t = 0$, find the expression for velocity $v(t)$.

Solution: Given: $a(t) = \cos t + \sin t$

$$\frac{dv}{dt} = \cos t + \sin t \Rightarrow \int dv = \int(\cos t + \sin t)\,dt \qquad \boxed{\because a(t) = \frac{dv}{dt}}$$

$$v(t) = \sin t - \cos t + c$$

Using initial condition $v = 2$ at $t = 0$: $2 = 0 - 1 + c \Rightarrow c = 3$

So, $\quad v(t) = \sin t - \cos t + 3$

7.1.7 Motion of a Free Particle Moving Along a Vertical Line

When a particle moves under the influence of gravity alone and air resistance is neglected, the motion is called free fall or free vertical motion. This motion takes place along a vertical straight line, either upward or downward. The acceleration due to gravity is denoted by ***g*** and is approximately:

$$g = 9.8 \text{ m/s}^2$$

If a particle is projected upward, its velocity decreases uniformly due to gravity and becomes zero at the highest point. If the particle moves downward, it accelerates toward the ground at the rate g. The equations of motion for constant acceleration can also be used in vertical motion by replacing ***a*** with ***g*** for downward motion and with **–*g*** for upward motion.

Example 11 (Free Fall from Rest)

A stone is dropped from the top of a tower 45 m high. Find the time taken to reach the ground and the final velocity on impact.

Solution: Given: $u = 0$, $x = 45$ m, $a = g = 9.8$ m/s^2

Use the equation of motion:

$$x = ut + \frac{1}{2}gt^2$$

Substitute values: $45 = 0 + \frac{1}{2}(9.8)t^2$

Solve for t:

$$t = \sqrt{\frac{45 \times 2}{9.8}} \approx 3.03\ s$$

To find the final velocity on impact, use the equation:

$$v = u + gt$$

Substitute the known values:

$$v = 0 + 9.8 \times 3.03 \approx 29.7 \text{ m/s}$$

Example 12 (Upward Throw)

A ball is thrown vertically upward with a speed of 20 m/s. Find, the maximum height attained and the time to reach the maximum height.

Solution: Given: $u = 20$ m/s, $v = 0$ m/s (at top), $a = g = -9.8$ m/s^2

Use the equation of motion:

$$v = u + gt$$

By substituting the values of u, v and g, we get:

$$0 = 20 + (-9.8)t$$

$$t = \frac{20}{9.8} \approx 2.04 \text{ s}$$

To find maximum height, use the equation of motion:

$$x = ut + \frac{1}{2}gt^2$$

By substituting the values of u, t and g, we get:

$$x = 20 \times 2.04 + \frac{1}{2}(-9.8)(2.04)^2$$

$$\approx 20.4 \text{ m}$$

EXERCISE 7.1

Figure 7.9

1. Junaid covers 100 km in the first 2 hours, stops for 1 hour, and then drives 60 km in the next 2 hours. The displacement-time graph is shown in Figure 7.9.
   (a) What is Junaid's average velocity?
   (b) What is his velocity during the last phase of the journey?
2. The displacement-time graph of a moving car is shown in Fig. 7.10. The car changes its direction during the journey, and its motion occurs in three distinct phases.
   (a) What is the car's velocity in each phase?
   (b) When did the car pass its starting point again?
   (c) What is the total distance travelled?
   (d) What is the average velocity?

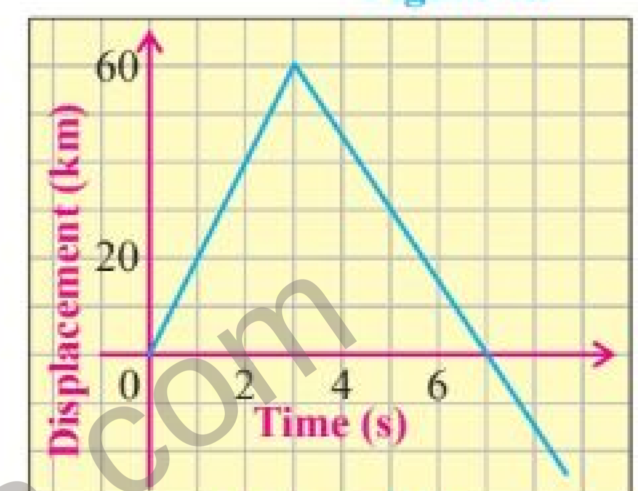

Figure 7.10

3. Figure 7.11 is the velocity-time graph of Aslam's motion as he runs along a straight track.
   For the first 5 seconds, he accelerates uniformly from rest to a velocity of 8 m/s. He then maintains this velocity for a further 8 seconds. Find:
   (a) The rate at which Aslam accelerates.
   (b) The displacement from the starting point of Aslam after 13 seconds.

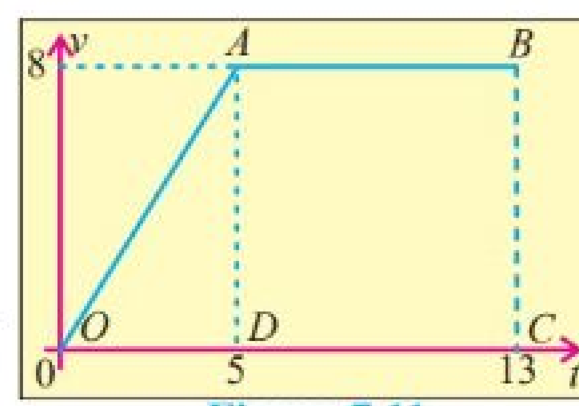

Figure 7.11

4. A car is travelling along a straight road. At $t = 0$ seconds, the car passes a point A with a velocity of 12 m/s, and this velocity is maintained up to $t = 35$ seconds. The driver then applies the brakes, and the car decelerates uniformly, coming to rest at the point B at $t = 40$ seconds.
   (a) Sketch a velocity–time graph to illustrate the motion of the car.
   (b) Find the displacement from A to B.
5. Figure 7.12 shows the velocity–time graph of an object moving in a straight line over a 12-second time interval.
   (a) At what time did the object change its direction of motion?
   (b) Calculate the total distance travelled and the displacement of the object during the 12 seconds.

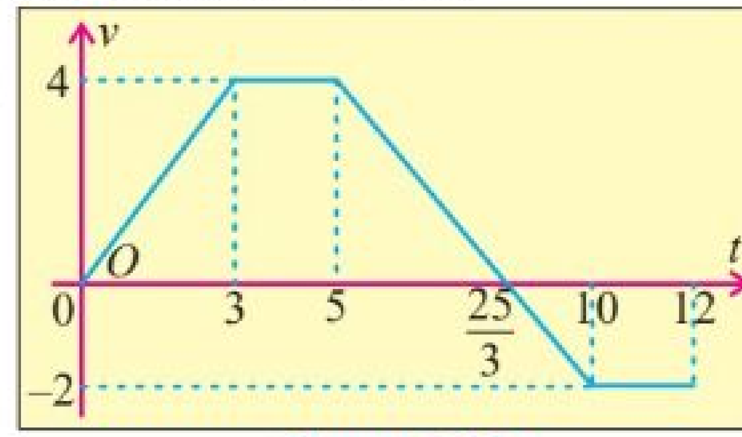

Figure 7.12

6. A vehicle moving with a speed of 36 km/h is stopped by applying brakes and comes to rest in 10 seconds. Find the retardation and distance covered during this time.
7. A train moving at 54 km/h is accelerated uniformly to 90 km/h in 20 s. Find the acceleration and the distance covered in this time.
8. A particle moving in a straight line starts from rest and is accelerated uniformly to attain a velocity of 30 km/h in 5 seconds. Find the distance travelled by the particle in the last three seconds.
9. A particle is projected vertically upwards with a velocity 100 m/s, and another is let fall from a height of 200 m at the same time. Find the height of the point where they meet each other.
10. A ball is thrown vertically upward with a velocity of 20 m/s. Find, the maximum height reached, and the time taken to return to the thrower's hand.
11. A rock is dropped from a cliff. One second later, another rock is dropped from the same height.
    (a) What is the distance between the two rocks 2 seconds after the first rock is dropped?
    (b) At what time will the distance between them be 14.7 m?
12. The acceleration of a particle is $a(t)=\dfrac{1}{1+t}$ for $t \geq 0$. If the velocity is 0 m/s when $t=0$, find the velocity at $t=5$ seconds.
13. The motion of a robotic arm moving along a straight track is described by the velocity function
$$v(t) = 6 + t - t^2$$
where $v(t)$ is in meters per second and t is the time in seconds, valid for $0 \leq t \leq 5$. Find the distance travelled by the arm.