UNIT 10

Practical Geometry of Circles

Students' Learning Outcomes

After completing this unit, the students will be able to:

- Locate the centre of a given circle.
- Draw a circle passing through three given non-collinear points.
- Complete the circle:
  - by finding the centre,
  - without finding the centre,

  when a part of its circumference is given.
- Draw a tangent to a given arc, without using the centre, through a given point *P* when *P* is
  - the middle point of the arc.
  - at the end of the arc.
  - outside the arc.
- Draw a tangent to a given circle from a point *P* when *P* lies
  - on the circumference.
  - outside the circle.
- Draw two tangents to a circle meeting each other at a given angle.
- Apply concepts of practical geometry of a circle to real-life world problems (such as athletic tracks, recreational parks, Ferris wheels and mechanical machines).

INTRODUCTION

This unit focuses on the key geometric properties and constructions related to circles and their practical applications. Students will learn how to accurately locate the centre of a circle and draw a circle passing through three given non-collinear points. The unit further guides students in completing a circle using various methods, whether or not the centre is known and even when only a part of the circumference is given. A major focus is placed on constructing tangents to arcs and circles from different positions of a given point, both with and without using the centre. Additionally, students will gain hands-on experience in drawing two tangents that meet at a specific angle. To bridge theory with real-world relevance, this unit highlights how these geometric concepts are applied in everyday structures such as athletic tracks, Ferris wheels and engineering designs. Through these activities, students will enhance their precision and spatial reasoning of geometry in the physical world.

10.1 Construction of a Circle

The construction of a circle is a fundamental concept in practical geometry. A circle can be drawn using a compass when its centre and radius are known. However, in more advanced constructions, a circle can also be constructed through three given non-collinear points, as these points uniquely determine a circle. In some cases, a circle may need to be completed when only a part of its circumference is given, either by finding the centre or using geometric techniques without it.

10.1.1 Locate the Centre of a Given Circle

Given: *A* circle.

Required: To locate the centre of the circle.

Steps of Construction:

1. Take any two chords $\overline{AB}$ and $\overline{CD}$.
2. Draw $\overleftrightarrow{PQ}$ as right bisector of $\overline{AB}$.
3. Draw $\overleftrightarrow{RS}$ as right bisector of $\overline{CD}$.
4. $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ intersect each other at point O.
5. Point O is the centre of the circle.

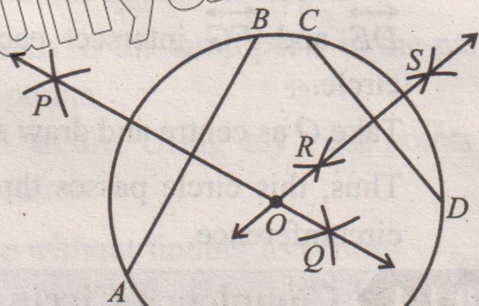


Fig. 10.1.1

Note

- Points that lie on the same line are collinear.
- Points that do not lie on the same line are non-collinear.

10.1.2 Draw a Circle Passing through Three Given Non-collinear Points

Given: Points A, B and C are non-collinear.

Required: Draw a circle passing through points A, B and C.

Steps of Construction:

1. Join B to A and C.
2. Draw $\overleftrightarrow{DE}$ as right bisector of $\overline{AB}$.
3. Draw $\overleftrightarrow{FG}$ as right bisector of $\overline{BC}$.

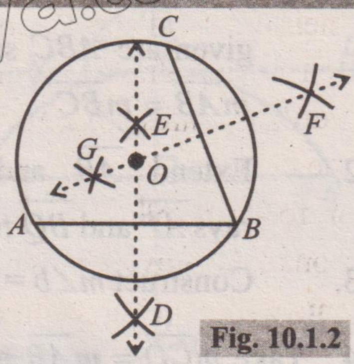


Fig. 10.1.2

4. $\overleftrightarrow{DE}$ and $\overleftrightarrow{FG}$ intersect each other at point O. Now O, is centre of the required circle.
5. Take O as centre and draw a circle of radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$, which is the required circle.

10.1.3 Complete a Circle by Finding its Centre, When a Part of Its Circumference is Given

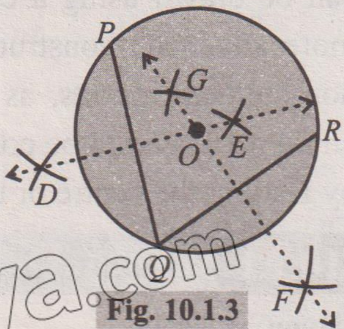


Fig. 10.1.3

Given: PQR is an arc of a circle.

Required: To complete the circle by finding its centre.

Steps of Construction:

1. Join Q to P and R.
2. Draw $\overleftrightarrow{DE}$ as right bisector of $\overline{PQ}$.
3. Draw $\overleftrightarrow{FG}$ as right bisector of $\overline{QR}$.
4. $\overleftrightarrow{DE}$ and $\overleftrightarrow{FG}$ intersect each other at point O. So, O is the centre of the circle.
5. Take O as centre and draw a circle of radius $m\overline{OP} = m\overline{OQ} = m\overline{OR}$.

   Thus, this circle passes through points P, Q and R on the given part of the circumference.

10.1.4 Complete a Circle without Finding its Centre, When a Part of its Circumference (Arc) is Given

Given: $\overparen{ABC}$ is a part of circumference of a circle.

Required: Complete a circle of which $\overparen{ABC}$ is a part without finding the centre of the circle.

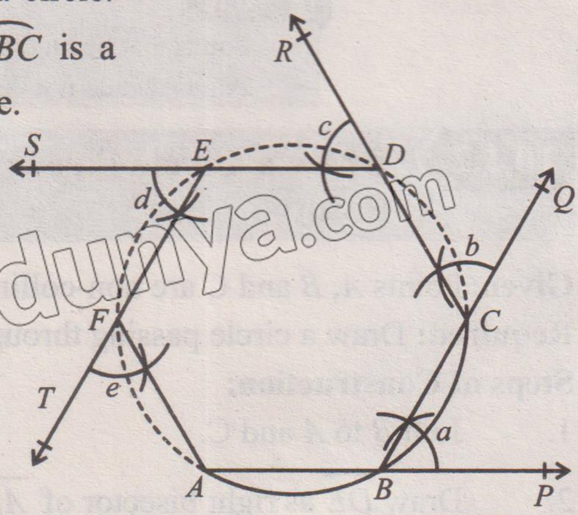


Fig. 10.1.4

Steps of Construction:

1. Take $\overline{AB}$, $\overline{BC}$ two chords of the given arc ABC such that $m\overline{AB} = m\overline{BC}$.
2. Extend $\overline{AB}$ and $\overline{BC}$ to form rays $\overrightarrow{AP}$ and $\overrightarrow{BQ}$ respectively.
3. Construct $m\angle b = m\angle a$.
4. Take $m\overline{CD} = m\overline{AB} = m\overline{BC}$ and construct $m\angle c = m\angle a = m\angle b$.

5. Similarly, construct $\angle d$ and $\angle e$.
6. Now, draw arcs on $\overline{CD}$, $\overline{DE}$, $\overline{EF}$ and $\overline{FA}$ with smooth hand.
   Thus, the required circle is completed.

EXERCISE 10.1

1. Construct a circle with the help of given radius and verify its centre by construction:
   (i) $r = 1.5$ cm (ii) $r = 1.7$ cm (iii) $r = 2$ cm
2. Take any three non-collinear points P, Q, R and construct a circle passing through these points.
3. Draw an arc ABC and complete a circle by finding its centre.
4. Draw an arc PQR and complete a circle without finding its centre.
5. Take any three non-collinear points (locations of the lamp posts in the park), construct a circle passing through all these points.
6. A part of the Ferris wheel rim is visible as an arc. Using any three points on the arc, construct the circle by finding its centre.
7. ABC an arc of a fountain, complete a circle without finding its centre.

10.2 Tangent to a Circle

10.2.1 To Draw a Tangent to a Given Arc without using the Centre Through a Given Point P

Case I When point P is the middle point of the arc

Given: $\widehat{APB}$ is an arc where P is midpoint of the arc.

Required: Draw tangent to the arc at point P.

Steps of Construction:

1. Join end points A and B of the arc.
2. Draw right bisector $\overleftrightarrow{LM}$ of $\overline{AB}$, it passes through point P.

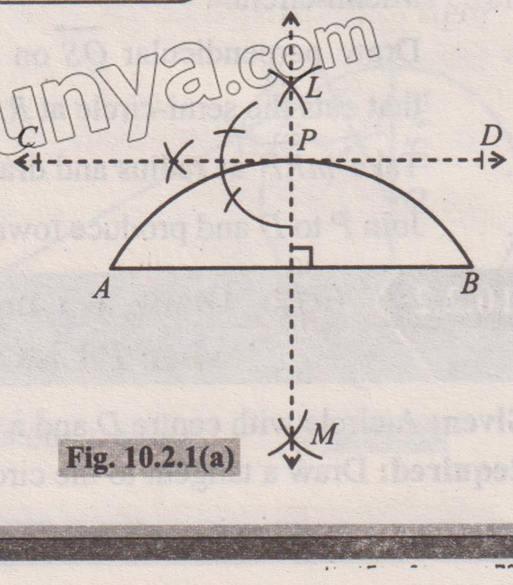


Fig. 10.2.1(a)

3. Construct right angle MPC and produce $\overline{CP}$ in the direction of P.

So, $\overleftrightarrow{CPD}$ is the required tangent.

Case II **When point P is an endpoint of the arc.**

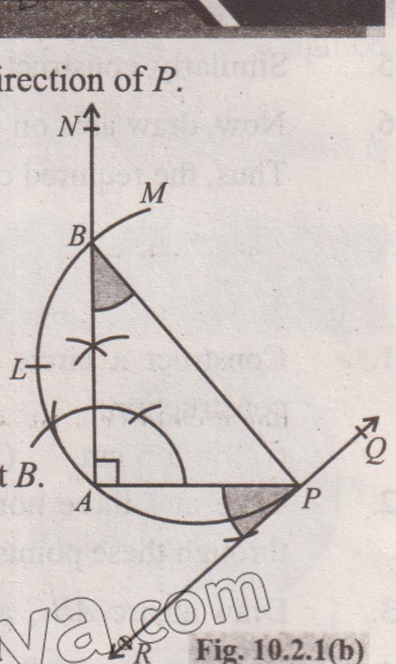


Fig. 10.2.1(b)

Given: $\overset{\frown}{PLM}$ is an arc, where P is its endpoint.

Required: Draw tangent to the arc PLM at point P.

Steps of Construction:

1. Take any point A on the arc and join P to A.
2. Draw right angle PAN so that $\overrightarrow{AN}$ cuts the arc at point B.
3. Join point B to P.
4. Construct $m\angle APR = m\angle ABP$.
5. Produce $\overline{RP}$ towards P, so $\overleftrightarrow{RPQ}$ is the required tangent.

Case III **When Point P is outside the arc.**

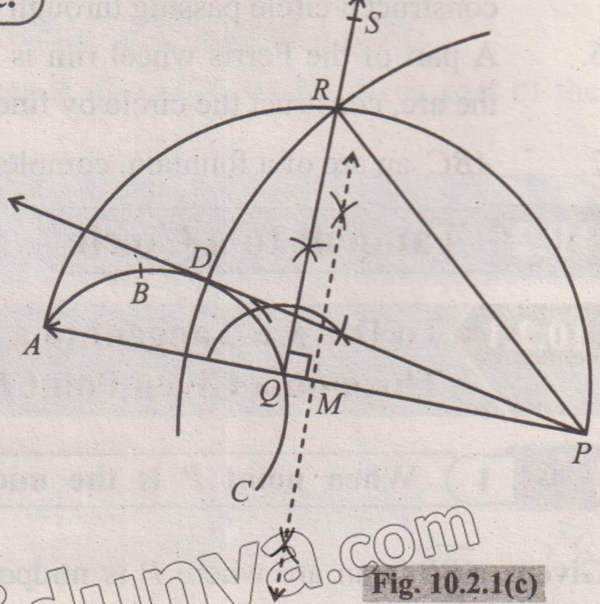


Fig. 10.2.1(c)

Given: $\overset{\frown}{ABC}$ is an arc and a point P is outside it.

Required: To draw a tangent to the arc from point P.

Steps of Construction:

1. Join A to P, $\overline{AP}$ cuts the arc at Q.
2. Find the midpoint M of $\overline{AP}$.
3. Take $\overline{AMP}$ as diameter and draw a semi-circle.
4. Draw perpendicular $\overrightarrow{QS}$ on $\overline{AP}$ that cuts the semi-circle at R.
5. Take $m\overline{RP}$ as radius and draw an arc that cuts $\overset{\frown}{ABC}$ at D.
6. Join P to D and produce towards D, so $\overrightarrow{PD}$ is the required tangent at D.

10.2.2 (a) Draw a Tangent to a given Circle from a Point P when P Lies on the Circumference.

Given: A circle with centre D and a point P on its circumference.

Required: Draw a tangent to the circle at point P.

Steps of Construction:

1. Join centre D with the point P. Now, $\overline{DP}$ is the radial segment.
2. Draw a perpendicular $\overleftrightarrow{AB}$ at point P to the radial segment $\overline{DP}$.

So, $\overleftrightarrow{APB}$ is the required tangent.

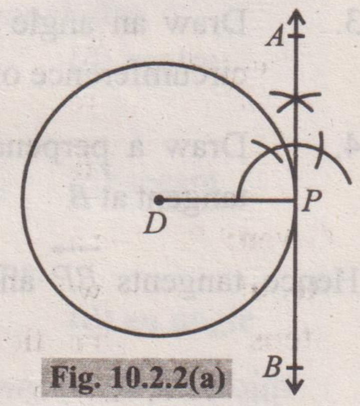


Fig. 10.2.2(a)

b Draw a tangent to the circle from a point P when P lies outside the circle.

Given: A circle with centre O and a point P that lies outside the circle.

Required: To draw a tangent to the circle from point P.

Steps of Construction:

1. Join centre O to point P.
2. Find M, the mid-point of $\overline{OP}$.
3. Take M as centre and draw a semi-circle with radius as $m\overline{OM} = m\overline{MP}$.
   This semi-circle cuts the circumference at point T.
4. Join P to T and produce towards T.

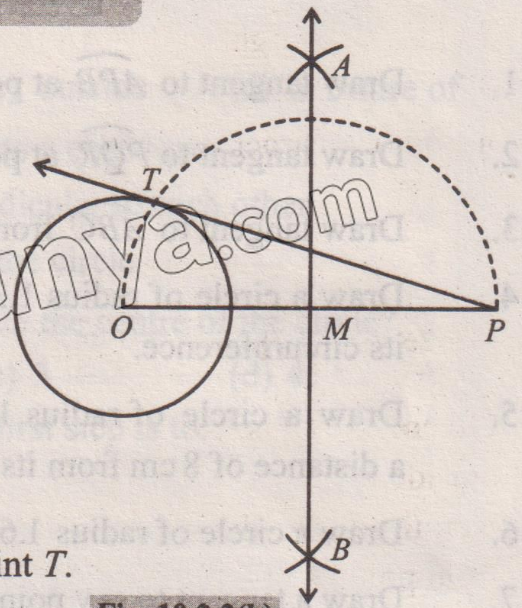


Fig. 10.2.2(b)

Thus, $\overrightarrow{PT}$ is the required tangent and length of tangent is measured as $m\overline{PT}$.

10.2.3 Draw Two Tangents to a Given Circle Meeting each other at a Given Angle

Example 1 Draw a circle with radius 1.5 cm. Draw two tangents to this circle that meet an angle of 30°.

Solution

Given: A circle with centre O, of radius 1.5 cm.

Required: Draw two tangent to the given circle that should meet at 30°.

Steps of Construction:

1. Draw a diameter $\overline{AOB}$.
2. Draw a perpendicular at B on $\overline{AOB}$, this is one tangent.

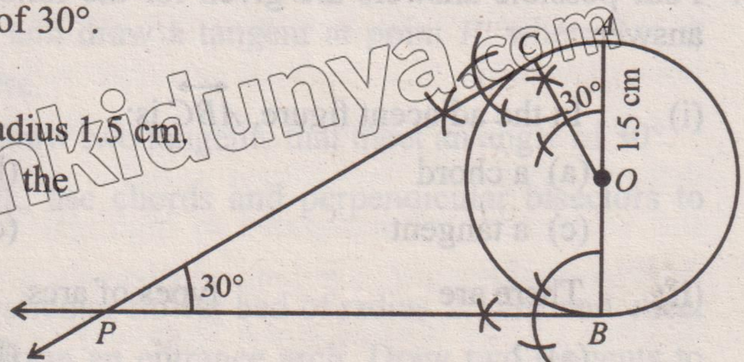


Fig. 10.2.3

3. Draw an angle of 30° at O, as $m\angle AOC = 30°$, where C is a point on the circumference of the given circle.

4. Draw a perpendicular at C on $\overline{OC}$ as $\overrightarrow{CP}$ that meets the already drawn tangent at B.

Hence, tangents $\overrightarrow{BP}$ and $\overrightarrow{CP}$ are the required tangents that meet at $m\angle CPB = 30°$.

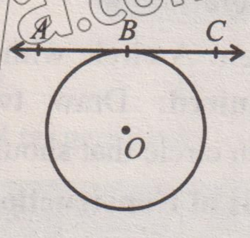
EXERCISE 10.2

1. Draw tangent to $\overparen{APB}$ at point P, when P is midpoint of the arc.
2. Draw tangent to $\overparen{PQR}$ at point P, when P is endpoint of the arc.
3. Draw tangent to $\overparen{ABC}$ from a point P, when P is outside the arc.
4. Draw a circle of radius 1.3 cm and draw a tangent at point P, when P lies on its circumference.
5. Draw a circle of radius 1.5 cm and draw a tangent at point P, when P is at a distance of 8 cm from its centre.
6. Draw a circle of radius 1.6 cm. Draw two tangents that meet an angle of 30°.
7. Draw a tangent to any point on a circular part of the track having radius = 2 cm.
8. From a pulley point, construct two tangents to a machine wheel having $r = 2.1$cm, such that the angle between them is 30°.

REVIEW EXERCISE 10

1. Four possible answers are given for the following questions. Choose the correct answer:

(i) In the adjacent figure, $\overleftrightarrow{ABC}$ is:



(a) a chord (b) an arc

(c) a tangent (d) a secant

(ii) There are ____________ types of arcs.

(a) 2 (b) 3

(c) 4 (d) 5

(iii) Right bisector of the chord of a circle always passes through the:
(a) diameter (b) non-collinear points (c) radius (d) centre

(iv) A circle has only one:
(a) chord (b) centre (c) diameter (d) secant

(v) The point where two tangents meet outside a circle forms:
(a) a semicircle (b) a diameter (c) a radius (d) an angle

(vi) Two equal tangents from a point to a circle can be drawn when the point is _________ the circle.
(a) on (b) inside (c) outside (d) at centre of

(vii) Tangents drawn from a single external point to a circle are:
(a) unequal in length (b) perpendicular to each other
(c) equal in length (d) inside the circle

(viii) At least how many chords are needed to locate the centre of the circle?
(a) 1 (b) 2 (c) 3 (d) 4

(ix) To draw a tangent at middle point of an arc, first step is to:
(a) join endpoints of an arc
(b) draw radius
(c) draw a line perpendicular to chord
(d) draw an angle

(x) The angle between the radius and a tangent at the point of contact is:
(a) 30° (b) 60° (c) 90° (d) 120°

2. Draw a circle of radius 1.4 cm and draw a tangent at point P, when P lies on its circumference.
3. Draw a circle of radius 1.2 cm and draw a tangent at point P, when P is at a distance of 5 cm from the centre.
4. Draw a circle of radius 1.7 cm. Draw two tangents that meet an angle of 30°.
5. Take a part of the circular track, use chords and perpendicular bisectors to complete a circular track.
6. Two decorative fences touch the circular flower bed of radius 2.1 cm and meet outside it at an angle of 30° to form an entrance arch. Draw two tangents to the flower bed from the point where the fences meet at an angle of 30°.